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ECE - 6255

ASSIGNMENT - 5

DIGITAL SPEECH PROCESSING

1) Recursive formula for minimum phase signal (1)

Step 1
change to z-Transform

$x(n) \rightarrow X(z)$

Complex cepstrum is given by-

$\hat{x}(n) \Leftrightarrow \hat{X}(z) = \log[X(z)]$

On differentiating

$$\frac{d\hat{X}(z)}{dz} = \frac{d}{dz}\left[\log[X(z)]\right] = \frac{X'(z)}{X(z)}$$

$$\Rightarrow \frac{d\hat{X}(z)}{dz}(X(z)) = X'(z)$$

multiplying both sides by $-z$,

$$-z\frac{d\hat{X}(z)}{dz}(X(z)) = -z X'(z)$$

Taking inverse z-Transform

$$n\hat{x}(n) * x(n) = n x(n)$$

$$n x(n) = \sum_{k=-\infty}^{\infty} \hat{x}(k)\, x(n-k)\, k \quad \text{(opening convolution)}$$

$$x(n) = \sum_{k=-\infty}^{\infty} \hat{x}(k)\, x(n-k)\left(\frac{k}{n}\right)$$

Since this is minimum phase system $\hat{x}(n) = 0$ for all $n < 0$

$$\Rightarrow x(n) = \sum_{k=0}^{n} \hat{x}[k]\, x(n-k) \left[\frac{k}{n}\right]$$

Opening / Splitting when $k=n$

$$\rightarrow x(n) = \sum_{k=0}^{n-1} \hat{x}(k)\, x(n-k) \left(\frac{k}{n}\right) + \hat{x}(n)\, x[0]\,(1)$$

Reordering

$$\hat{x}(n)\, x(0) = x(n) - \sum_{k=0}^{n-1} \hat{x}(k)\, x(n-k) \left(\frac{k}{n}\right)$$

$$\boxed{\hat{x}[n] = \frac{x(n)}{x[0]} - \sum_{k=0}^{n-1} \hat{x}(k) \frac{x(n-k)}{x(0)} \left(\frac{k}{n}\right)} \quad n>0$$

Now to prove $\hat{x}(0) = \log(x(0))$, $\hat{x}(n) = 0$, $n<0$

Lets assume $x(n)$, $n=0, \ldots N-1$

$$x(n) = x(0)\,\delta[n] + x(1)\,\delta(n-1) + \ldots + x(N-1)\,\delta(N-1)$$

$$= x(0)\left[\delta(n) + \frac{x(1)}{x(0)}\delta(n-1) + \ldots + \frac{x(N-1)}{x(0)}\delta(N-1)\right]$$

Taking z-Transform

$$X(z) = \sum_{n=0}^{N-1} x(n) z^{-n} = G \prod_{k=\phi}^{N_{21}} (1-a_k z^{-1}) \prod_{k=1}^{N_{22}} (1-b_k z)$$

Gain $= x(0)$ — prod 0s inside and outside unit circle.

For min. phase all zeros inside unit circle so, second term goes

$$\Rightarrow \boxed{\hat{x}(n) = \log(G) = \log(x(0))}$$

3] $$H(e^{j\omega}) = \frac{G \prod_{q=1}^{Q} (1 - s_q e^{j\phi_q} e^{-j\omega})(1 - s_q e^{-j\phi_q} e^{-j\omega})}{\prod_{p=1}^{P} (1 - r_p e^{j\theta_p} e^{-j\omega})(1 - r_p e^{-j\theta_p} e^{-j\omega})}$$ (2)

$G > 0, |s_q| > 1, |r_p| < 1$

$$\hat{H}(e^{j\omega}) = \log [H(e^{j\omega})]$$

$$= \log \left[ \frac{G \prod_{q=1}^{Q} (1 - s_q e^{j\phi_q} e^{-j\omega})(1 - s_q e^{-j\phi_q} e^{-j\omega})}{\prod_{p=1}^{P} (1 - r_p e^{j\theta_p} e^{-j\omega})(1 - r_p e^{-j\theta_p} e^{-j\omega})} \right]$$

$$= \log G + \sum_{q=1}^{Q} \left[ \log (1 - s_q e^{j\phi_q} e^{-j\omega}) + \log (1 - s_q e^{-j\phi_q} e^{-j\omega}) \right]$$
$$- \sum_{p=1}^{P} \left( \log (1 - r_p e^{j\theta_p} e^{-j\omega}) + \log (1 - r_p e^{-j\theta_p} e^{-j\omega}) \right)$$

$$\log(1 - u) = \sum_{n=1} \frac{(-1)^{n+1}}{n} u^n \qquad |u| < 1$$

$$\hat{h}(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \hat{H}(e^{j\omega}) e^{j\omega n} d\omega$$

Log power series

$$\sum_{q=1}^{Q} \log(1 - s_q e^{j\phi_q} e^{-j\omega}) = \sum_{n=1}^{Q} \frac{(-1)^{n+1}}{n} (-s_q e^{j\phi_q})^n$$

$$\sum_{q=1}^{Q} \log (1 - s_q e^{-j\phi_q} e^{-j\omega}) = \sum_{n=1} \frac{(-1)^{n+1}}{n} (-s_q e^{-j\phi_q})^n$$

$$\sum_{p=1}^{P} \log(1 - r_p e^{+j\theta_p} e^{-j\omega}) = \sum_{n=1}^{P} \frac{(-1)^{n+1}}{n} (-r_p e^{j\theta_p})^n$$

$$\sum^{P} \log(1 - r_p e^{-j\theta_p} e^{-j\omega}) = \sum_{n=1}^{P} \frac{(-1)^{n+1}}{n} (-r_p e^{-j\theta_p})^n$$

$$\Rightarrow \boxed{\hat{h}(n) = \log(G) \text{ when } n = 0}$$

$$\hat{h}(n) = \sum_{q=1}^{Q} \frac{(-1)^{n+1}}{n} [-s_q]^n \left(e^{j\phi_q n} + e^{-j\phi_q n}\right)^n$$

$$+ \sum_{p=1}^{P} \frac{(-1)^n}{n} (-r_p)^n \left(e^{j\theta_p n} + e^{-j\theta_p n}\right)$$

$$= \sum_{q=1}^{Q} \frac{(-1)^{2n}(-1)}{n} (s_q)^n [\cos \phi_q n] 2$$

$$+ \sum_{p=1}^{P} \frac{(-1)^{2n}}{n} (r_p)^n (\cos \theta_p n] 2$$

$$\Rightarrow \hat{h}(n) = -\frac{2}{n} \sum_{q=1}^{Q} (s_q)^n \cos(\phi_q n) \quad n < 0 \quad \text{as } (s_q| > 1)$$

$$\hat{h}(n) = \frac{2}{n} \sum_{p=1}^{P} (r_p)^n \cos(\theta_p n) \quad n > 0 \quad (r_p| < 1)$$

4] $n = 0, 1 \ldots N-1$

$y(n) = \alpha^n x(n)$

complex cepstra $\hat{x}(n)$ $\hat{y}(n)$

a) $0 < \alpha < 1$

$$Y(z) = \sum_{n=0}^{N-1} \alpha^n x(n) z^{-n} \quad [z \text{ transform}]$$

$$= \sum_{n=0}^{N-1} (\alpha z^{-1})^n x(n)$$

$$= X(z/\alpha)$$

This means that $\hat{y}(n) = \alpha \hat{x}(n)$ then zeros of $\hat{x}(n)$ if they occur at $z_0, \ldots z_{N-1}$ then zeros of $Y(z)$ occur at $\alpha z_0, \alpha z_1 \ldots \alpha z_{N-1}$

b) $y(n)$ is not minimum phase when 0 is either on unit circle or outside unit circle.

Let max $|z|$ be max. 0 magnitude. Then $|\alpha|$ is

$$|\alpha| \max|z| \geq 1 \Rightarrow |\alpha| \geq \frac{1}{\max|z|}$$

c) $y(n)$ is not max phase when all zeros of $Y(z)$ lie outside unit circle.

Let $\min|z|$ be max. 0 magnitude

$$|\alpha| \min|z| > 1 \Rightarrow |\alpha| > \frac{1}{\min|z|}$$

note

$\boxed{5}$ $q(n) = \delta(n) + \alpha\,\delta(n - N_p)$ , $|\alpha| \leq 1$

a) $\alpha = 0.8$

$$Q(z) = 1 + \alpha z^{-N_p}$$

$$\hat{Q}(z) = \log(Q(z)) = \log(1 + \alpha z^{-N_p})$$

log power series

$$\log(1+u) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} u^n \quad (|u| < 1)$$

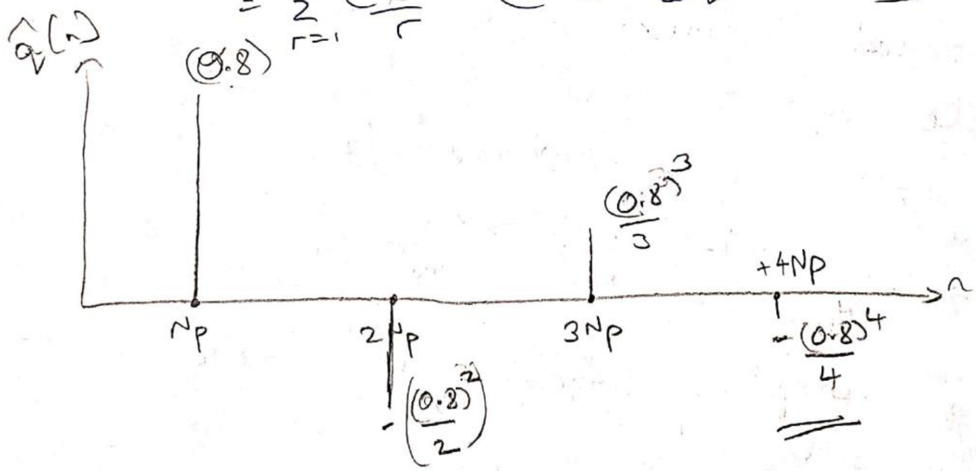
$$\Rightarrow \hat{Q}(z) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} (\alpha z^{-N_p})^n \quad \Rightarrow |\alpha z^{-N_p}| \leq 1$$

$$= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\alpha^n}{n} z^{-N_p n}$$

$$\hat{q}(n) = \sum_{r=1}^{\infty} \frac{(-1)^{r+1}}{r} \alpha^r \delta[n - r N_p]$$

$$= \sum_{r=1}^{\infty} \frac{(-1)^{r+1}}{r} (0.8)^r \delta[n - r N_p]$$

reverting back to time domain



$r=0 \quad k=-\infty$

(4)

5] b $\hat{q}_N(n) = \sum_{r=-\infty}^{\infty} \hat{q}(n+rN)$

From part (a) we have

$$\hat{q}(n) = \sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{k} \alpha^k \delta(n-kN_p)$$

$$\Rightarrow \hat{q}_N(n) = \sum_{r=-\infty}^{\infty} \sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{k} \alpha^k \delta(n+rN-kN_p)$$

$\delta(n+rN-kN_p) = 1$ only when $n+(rN-kN_p) = 0$

$$n = +kN_p - rN$$

Given $N = 6N_p$

$$n = kN_p - 6rN_p$$

$$k - 6r = \frac{n}{N_p} = \text{integer}$$

$\tilde{q}(n)$ is complex cepstrum periodic with period of $N$ samples.

The same will be the case for $N = 6N_p$.
Due to aliasing the amplitudes of the samples will increase.
$\Rightarrow \hat{q}(0)$ will be a non zero term.

$N = 6N_p$

$$\hat{q}(n) = -\left[\frac{(0.8)^6}{6} + \frac{(0.8)^{12}}{12} + \dots\right] = -0.04941 \quad n=0,\ k=6r$$

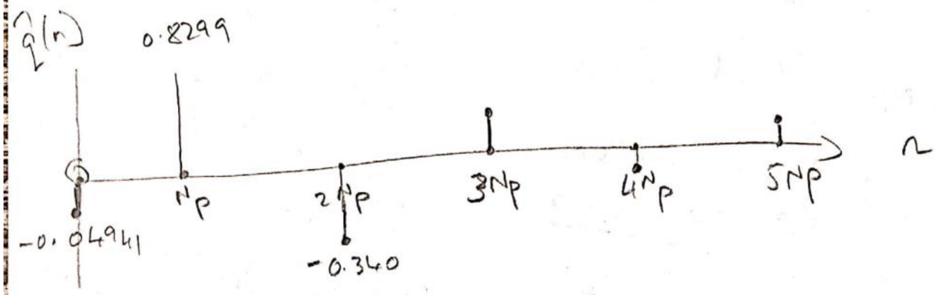
$$= 0.8 + \frac{(0.8)^7}{7} + \dots = 0.8299 \quad n = N_p,\ k = 6r+1$$

$$= -\left[\frac{(0.8)^2}{2} + \frac{(0.8)^8}{8} + \dots\right] = -0.340 \quad n = 2N_p,\ k=6r+2$$

$$= \left[\frac{(0.8)^3}{3} + \frac{(0.8)^9}{9} + \dots\right] \quad n = 3N_p,\ k=6r+3$$

$$= -\left[\frac{(0.8)^4}{4} + \frac{(0.8)^{10}}{10} + \dots\right] \quad n = 4N_p,\ k=6r+4$$

$$= \left[\frac{(0.8)^5}{5} + \frac{(0.8)^{11}}{11} + \dots\right] \quad n = 5N_p,\ k=6r+5$$



5 c) Now, when $N$ is not divisible by $N_p$ (5)

say, $N = \frac{7}{2} N_p$, then due to aliasing, additional samples will appear with seperation less than $N_p$.

$$n = kN_p - rN$$

$$N = \frac{7}{2} N_p$$

$$n = kN_p - \frac{7}{2} N_p r$$

$$k - \frac{7}{2} r = \frac{n}{N_p} = \text{~~integer~~ and non integers as well.}$$

$$\hat{q}_N(n) = \sum_{r=-\infty}^{\infty} \sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{k} \alpha^k \, \delta[n + rN - kN_p]$$

Now $n = 0, \frac{N_p}{2}, N_p, \frac{3N_p}{2}, 2N_p, \frac{5N_p}{2} \dots$

$$\hat{q}_N[n] = \frac{(0.8)^7}{7} - \frac{(0.8)^{14}}{14} \approx 0.268$$

$n = 0$

$k = \frac{7r}{2}$ $\quad r = 2, 4$

$k = 7, 14$

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$$\hat{q}_N[n] = -\frac{(0.8)^4}{4} + \frac{(0.8)^{11}}{11} + \dots = -0.094$$

$n = \frac{N_p}{2}$

$k = \frac{7}{2} r + \frac{1}{2}$

$= \frac{7r+1}{2}$ $\quad r = 1, 3 \dots$

$k = 4, 11$

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$$\hat{q}_N[n] = 0.8 - \frac{(0.8)^8}{8} + \dots$$

$$= 0.78$$

$n = N_p$

$k = \frac{7r}{2} + 1 = \frac{7r+2}{2}$

$k = 1, 8 \dots$ $\quad r = 0, 2, 4.$

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$$\hat{q}_N(n) = \frac{-(0.8)^5}{5} + \frac{(0.8)^{12}}{12} + \dots = 0.0598$$

$n = \frac{3N_p}{2}$

$k = \frac{7r}{2} + \frac{3}{2} = \frac{7r+3}{2}$

$k = 5, 12 \dots$ $\quad r = 1, 3, 5 \dots$

$n = 2N_p$

$k = \frac{7r}{2} + 2 = \frac{7r+4}{2} \qquad r = 0, 2, 4 \dots$

$\Rightarrow k = 2, 9, \dots$

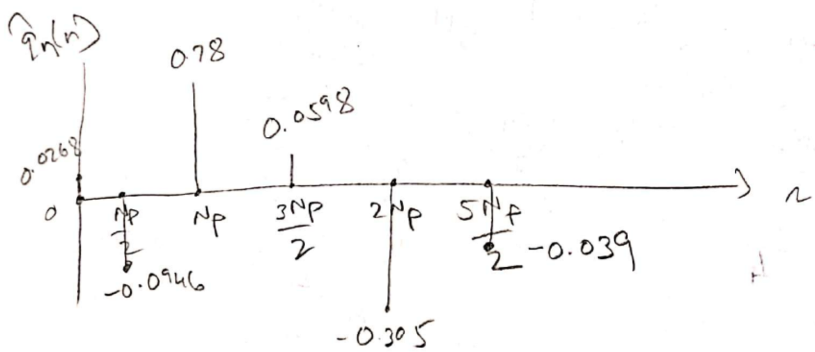
$\Rightarrow \hat{q}_N(n) = -\frac{(0.8)^2}{2} + \frac{(0.8)^9}{9} + \dots \approx -0.305$

$n = \frac{5N_p}{2}$

$k = \frac{7r}{2} + \frac{5}{2} = \frac{7r+5}{2} \qquad r = 1, 3, 5 \dots$

$\Rightarrow k = 6, 13 \dots$

$\Rightarrow \hat{q}_N(n) = -\frac{(0.8)^6}{6} + \frac{(0.8)^{13}}{13} \dots \approx -0.039$



s

(a) $y_1[n] = \delta[n] - 0.85\delta[n-99], \quad 0 \le n \le 400$;

(b) $y_2[n] = \sin(0.02\pi n), \quad 0 \le n \le 100$;

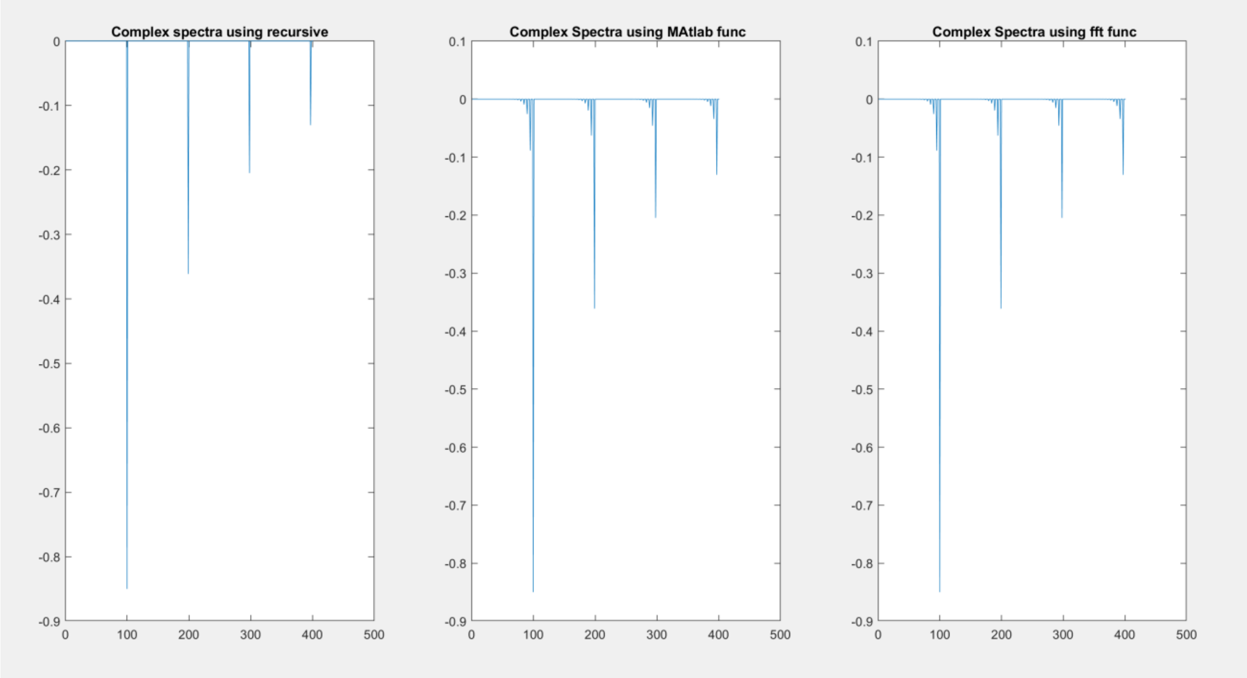
(c) $H(z) = 9(1-3z^{-1}), \;\; h[n] = ???, \; \hat{h}[n] = ???, \quad 0 \le n \le 400$;

2)

a) 1st plot is plotted using the recursive function in question 1.

2nd plot is using the inbuilt matlab cceps function.

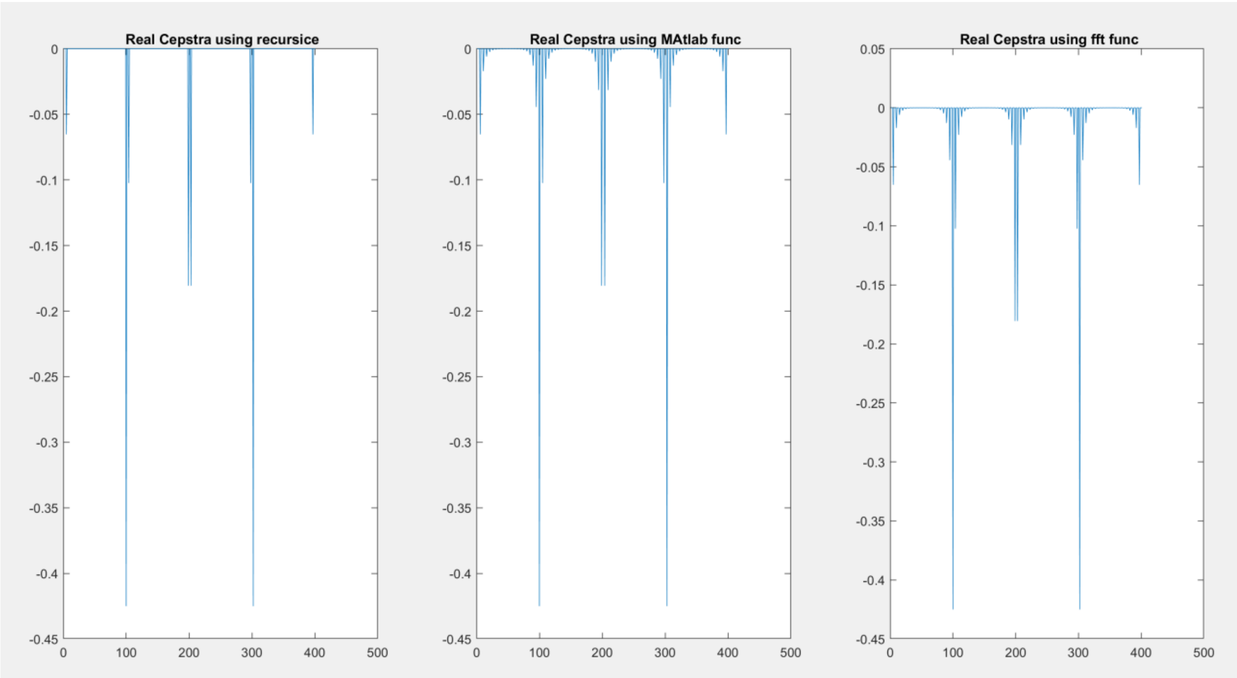
3rd plot is using fft first. log(abs(h)) + sqrt(-1)*unwrap(angle(h));




1st plot is plotted using the formula to plot real Cepstrum from complex

2nd plot is using the inbuilt matlab rceps function.

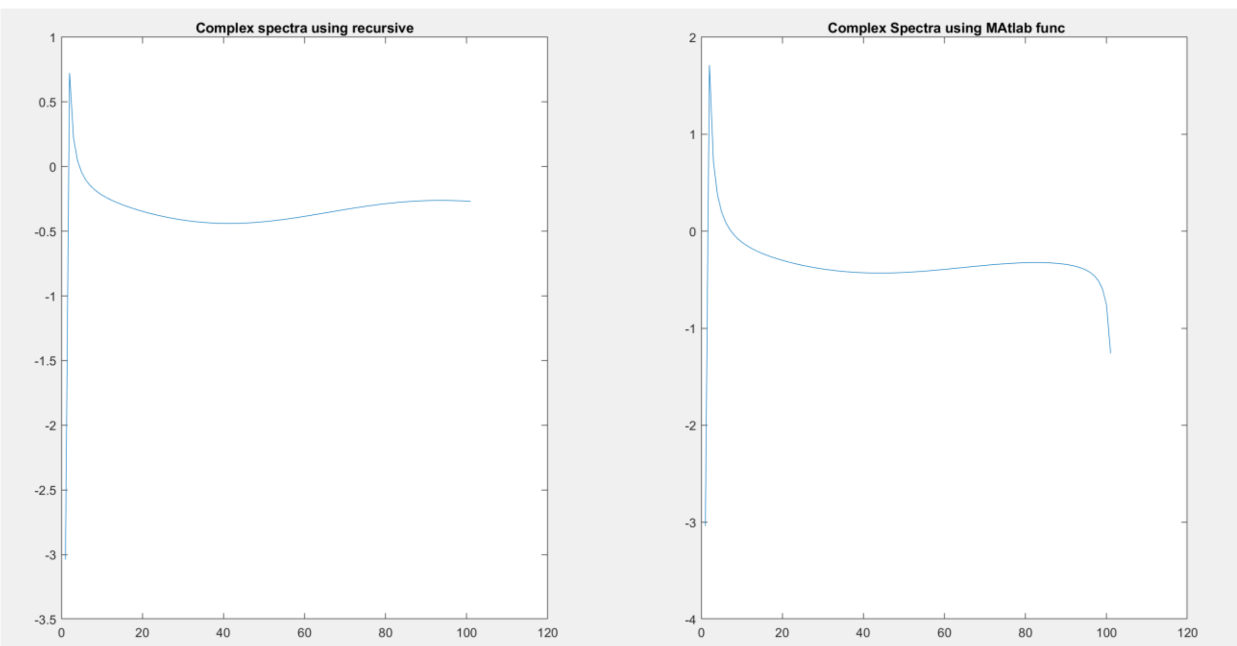
3rd plot is plotted using the formula to plot real Cepstrum from complex




b)

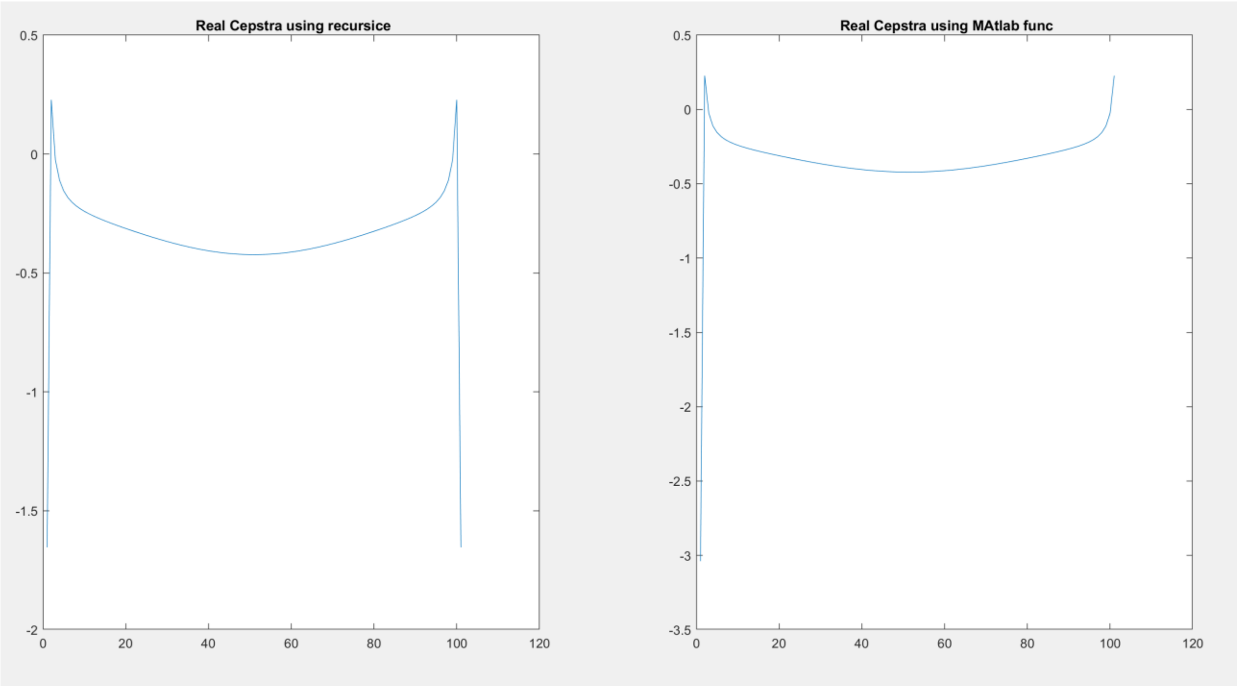
1st plot is using the recursive function in question 1.

2nd plot is using cceps function.



1st plot is using formula to convert from complex to real.

2nd plot is using rceps function.

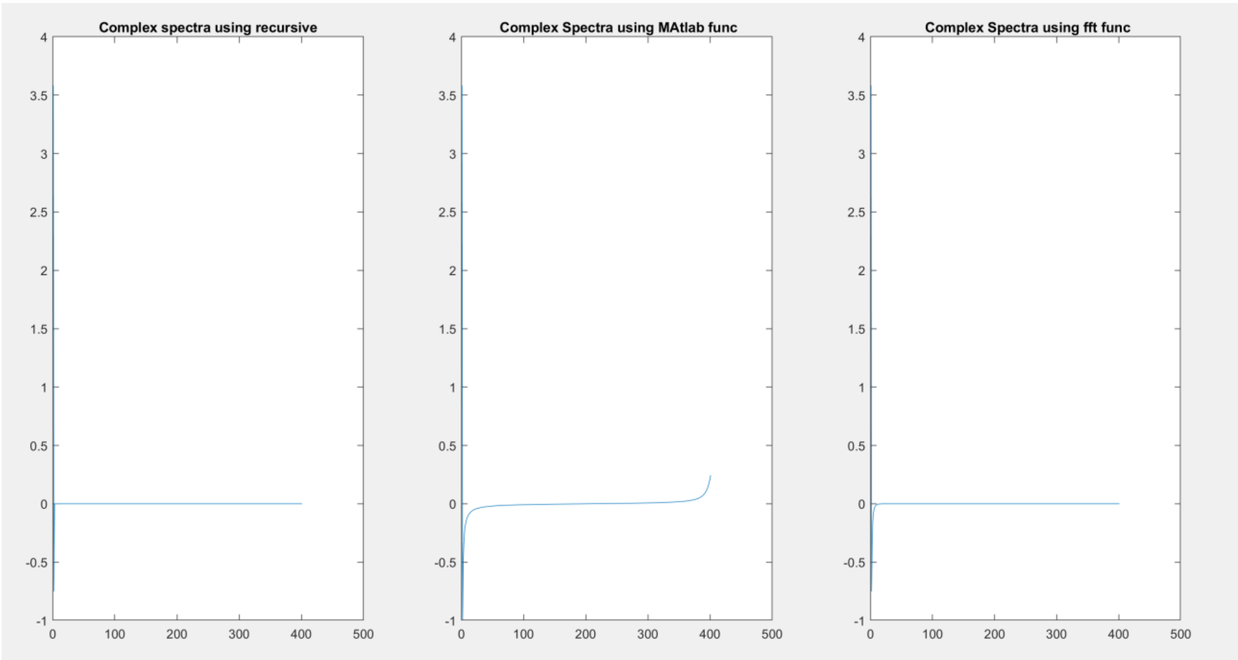



c)

1st plot is plotted using the recursive function in question 1.

2nd plot is using the inbuilt matlab cceps function.

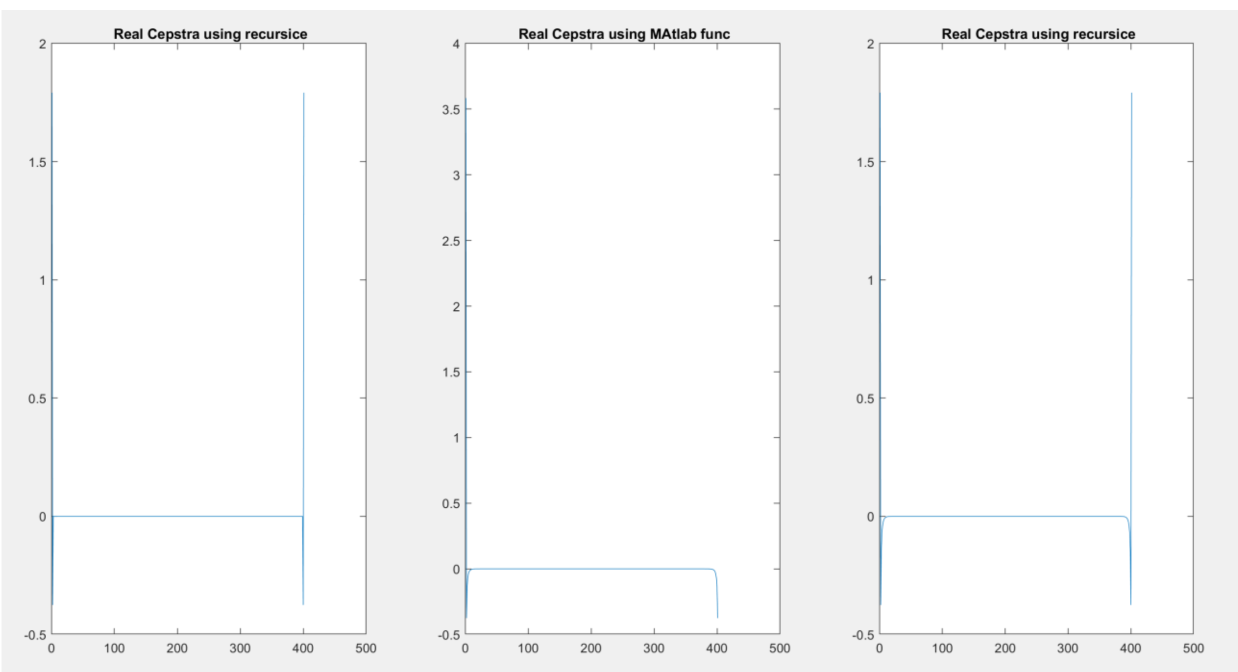
3rd plot is using fft first. log(abs(h)) + sqrt(-1)*unwrap(angle(h));




1st plot is plotted using the formula to plot real Cepstrum from complex

2nd plot is using the inbuilt matlab rceps function.

3rd plot is plotted using the formula to plot real Cepstrum from complex



6) The results obtained are very similar to the ones obtained in the class(unit 8).

WE have kept the lifter value as 20 in this case and the magnitudes of complex Cepstrum and low pass liftered values are clearly distinguishable.

The matlab code has been attached with the email.

As seen in the plots attached below. In the first figure I have basically put together all frames and then plotted the outputs.

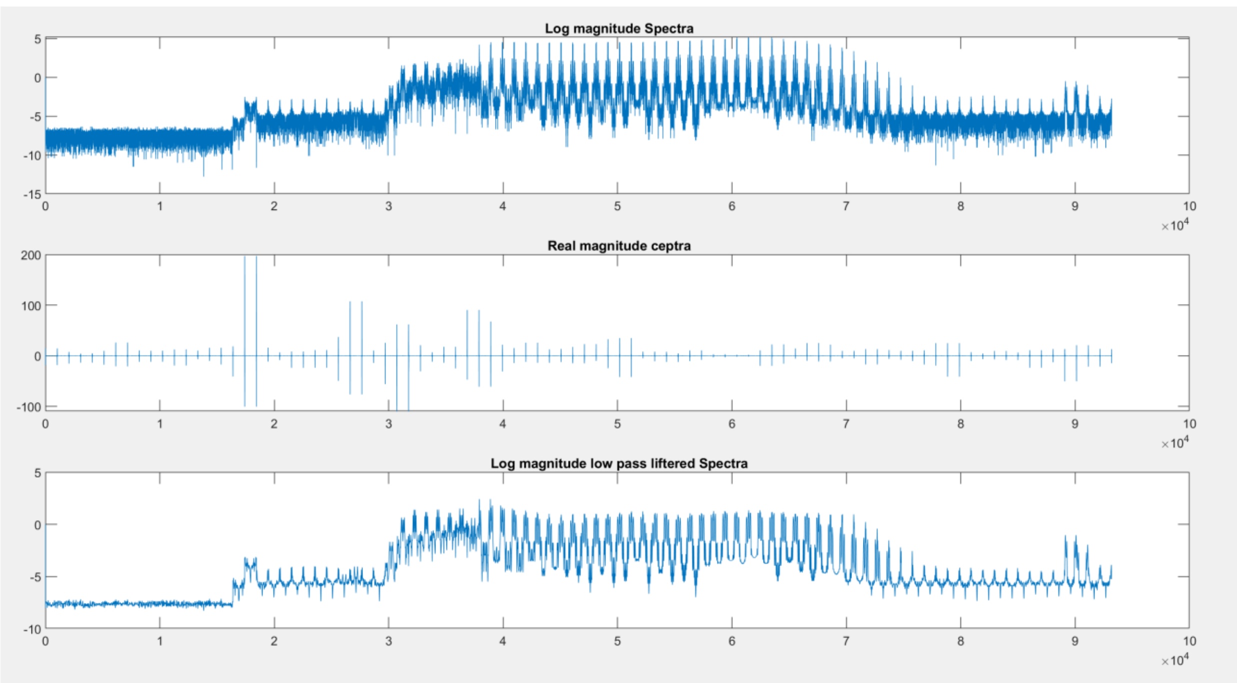
In plot 2, it is a combined plots of all both complex cepstra and low pass liftered cepstra.

In plot 3, it is a plot of first 10 frames of all both complex cepstra and low pass liftered cepstra.

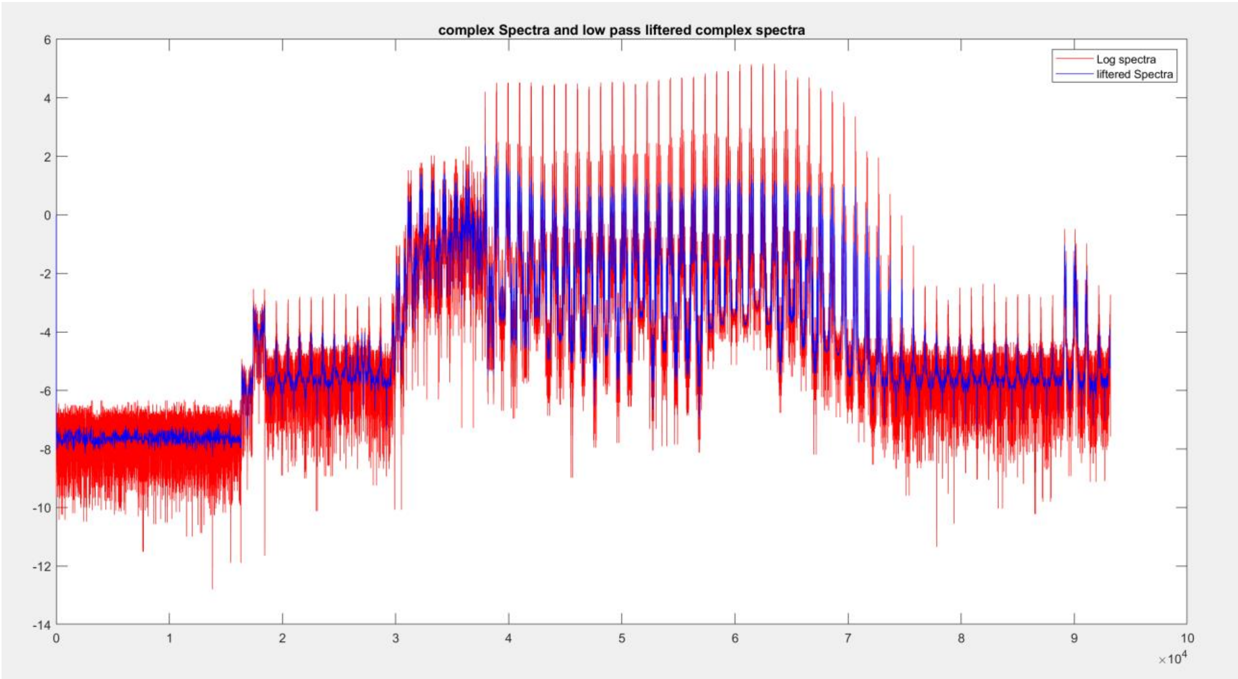
The recording used is also attached along with matlab codes.

It matches with the teaching done in class.

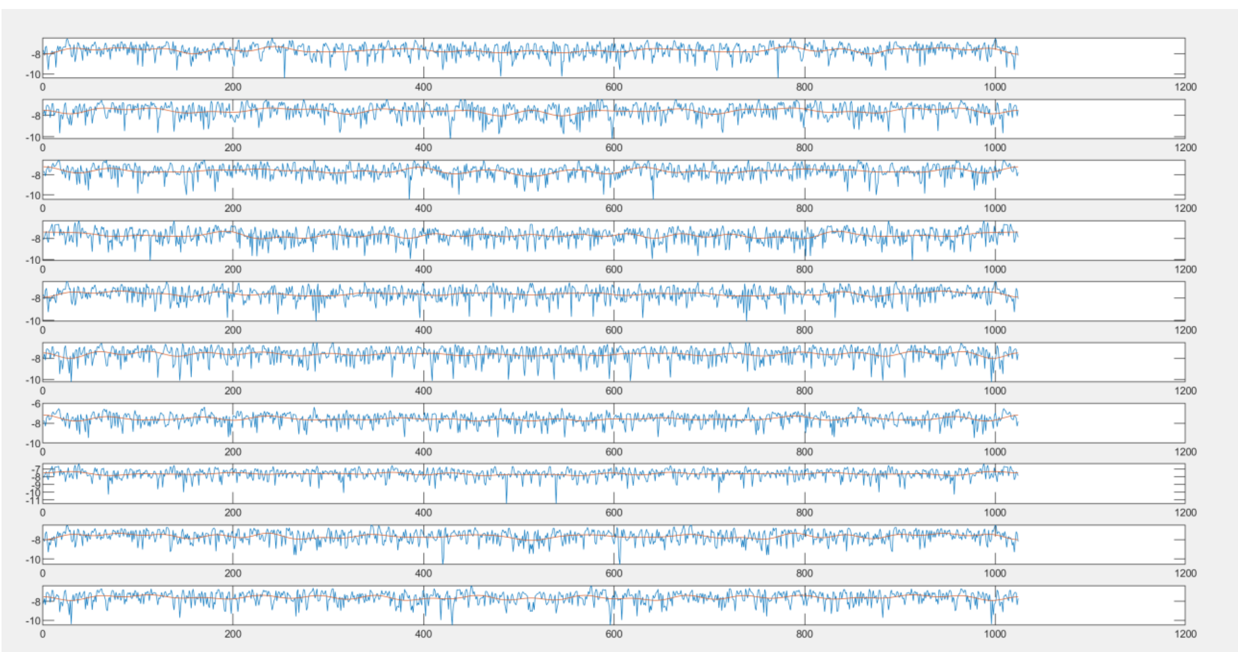
Plot for all frames. 92 in total. FFT size is 1024



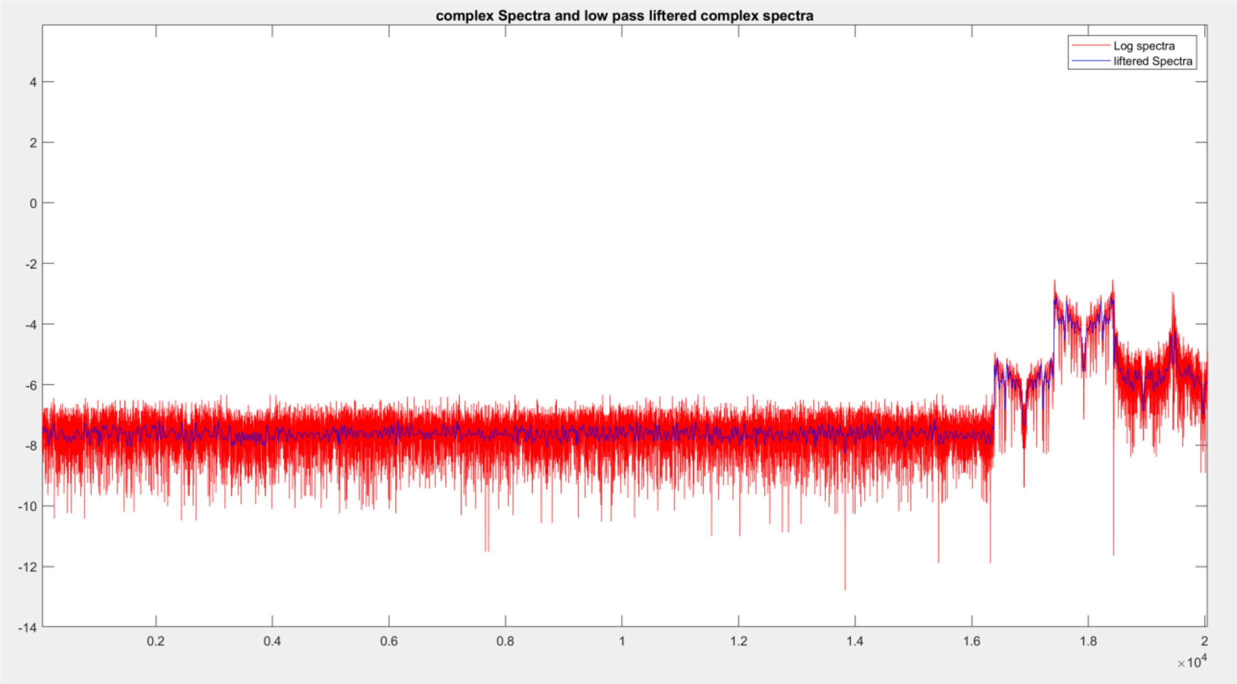

Plot of Complex Ceptra and Low pass liftered ceptra all 92 frames put together.




Plot of Complex spectra and Low pass liftered spectra all 92 frames put together.

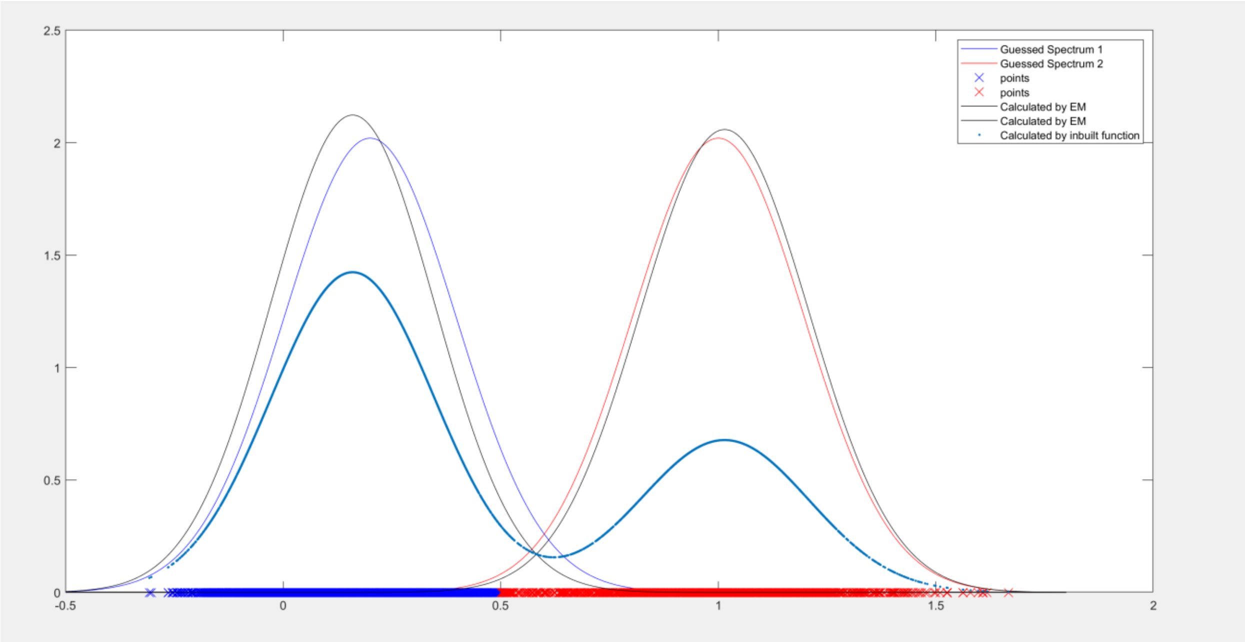


Zoomed in version

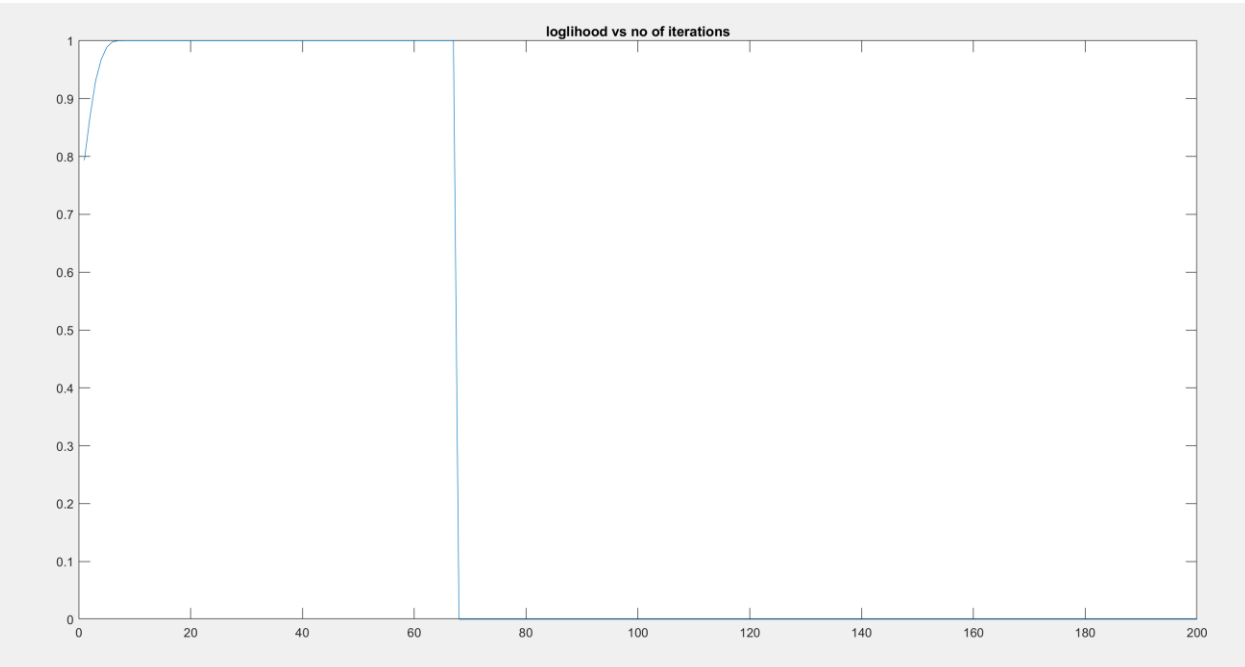

complex Spectra and low pass liftered complex spectra
Log spectra
liftered Spectra
4
2
0
-2
-4
-6
-8
-10
-12
-14
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
×10^4

7) The expectation maximization algo has been coded out. WE have plotted for 2,3,4,5 distributions. We can see that the distribution 2 converges very fast as that's what is clearly seen from histogram. In distributions 3,4,5 convergences occurs late but that is not the ideal distributions.

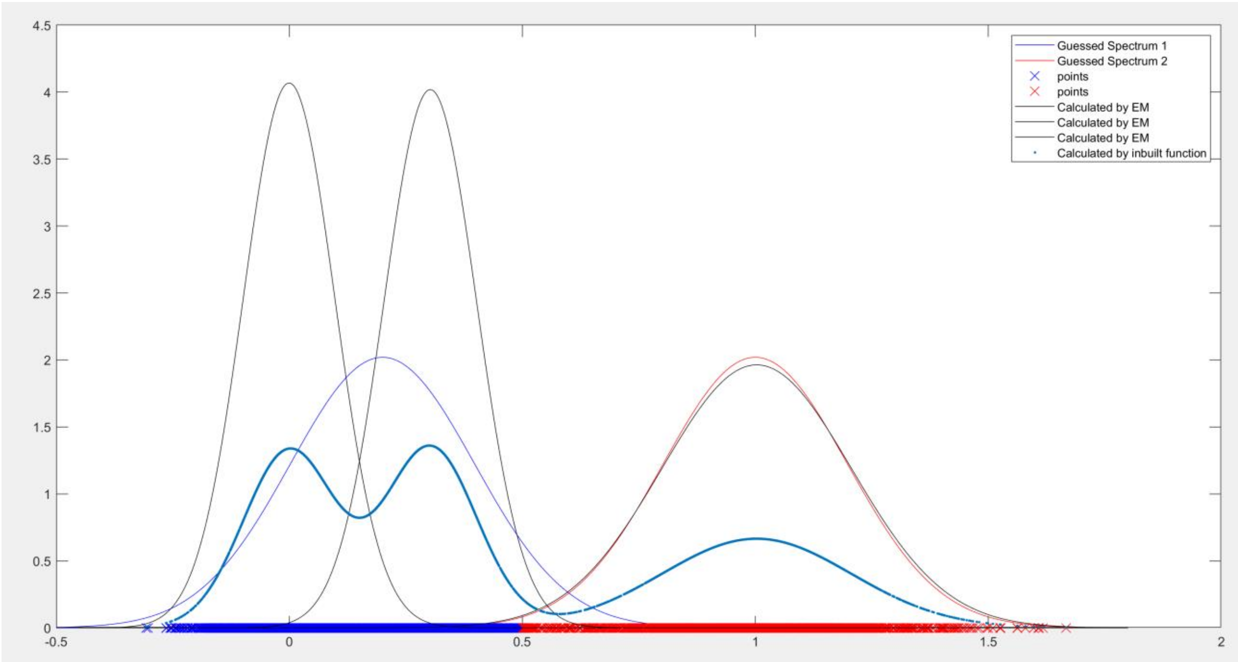
Setting k as 2 distributions to be found

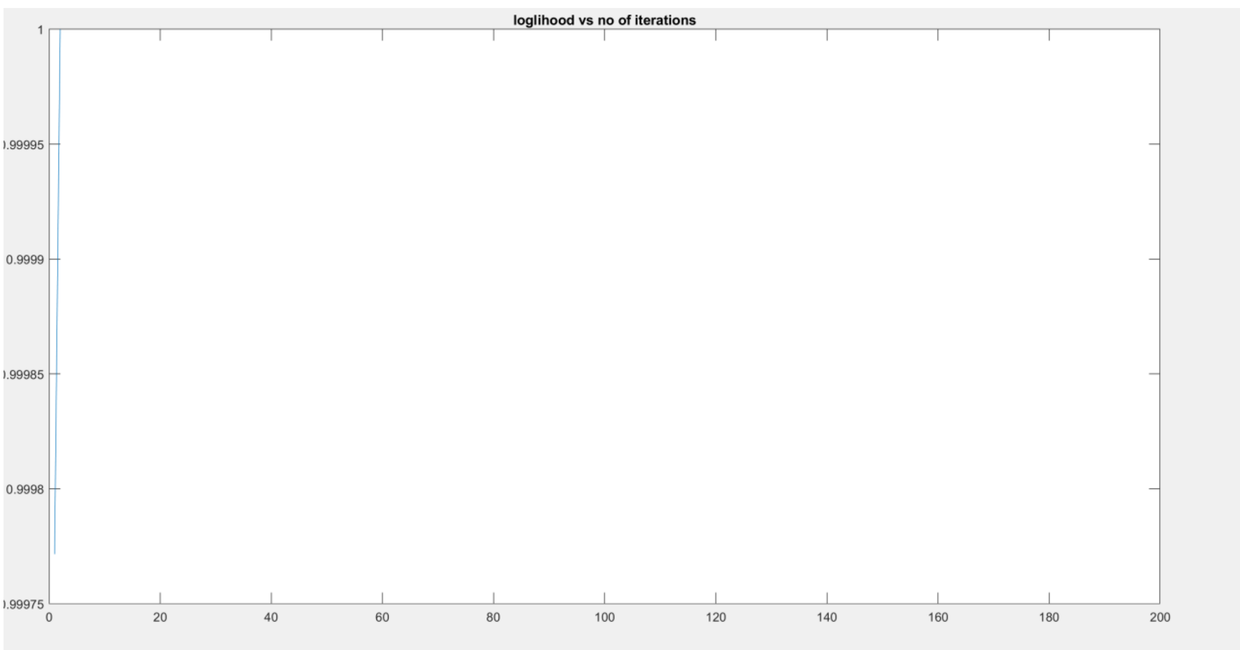



Number of iterations break at around 50-60 so there is a fall to 0.

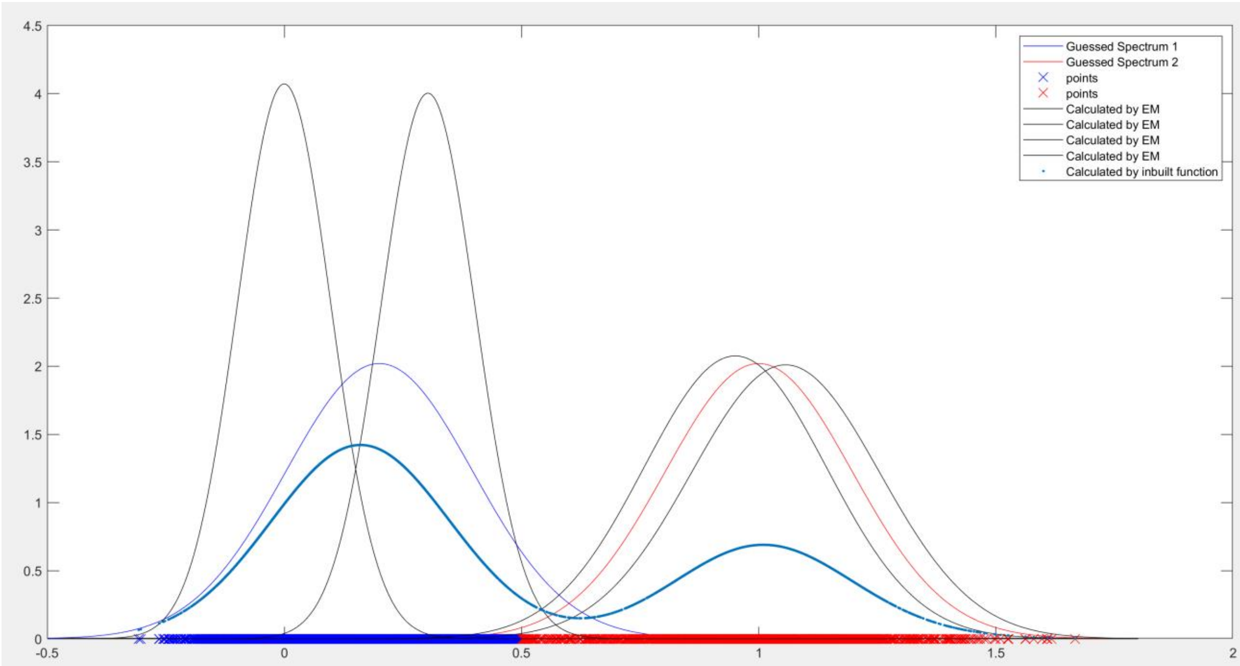


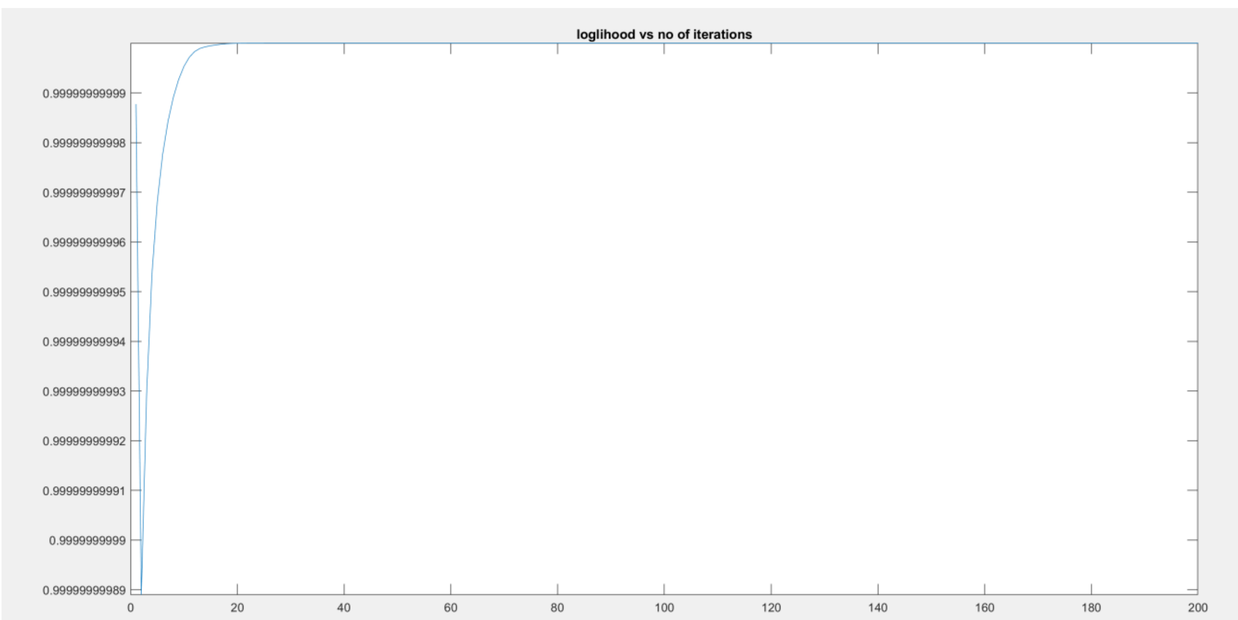

3 distributions to be found



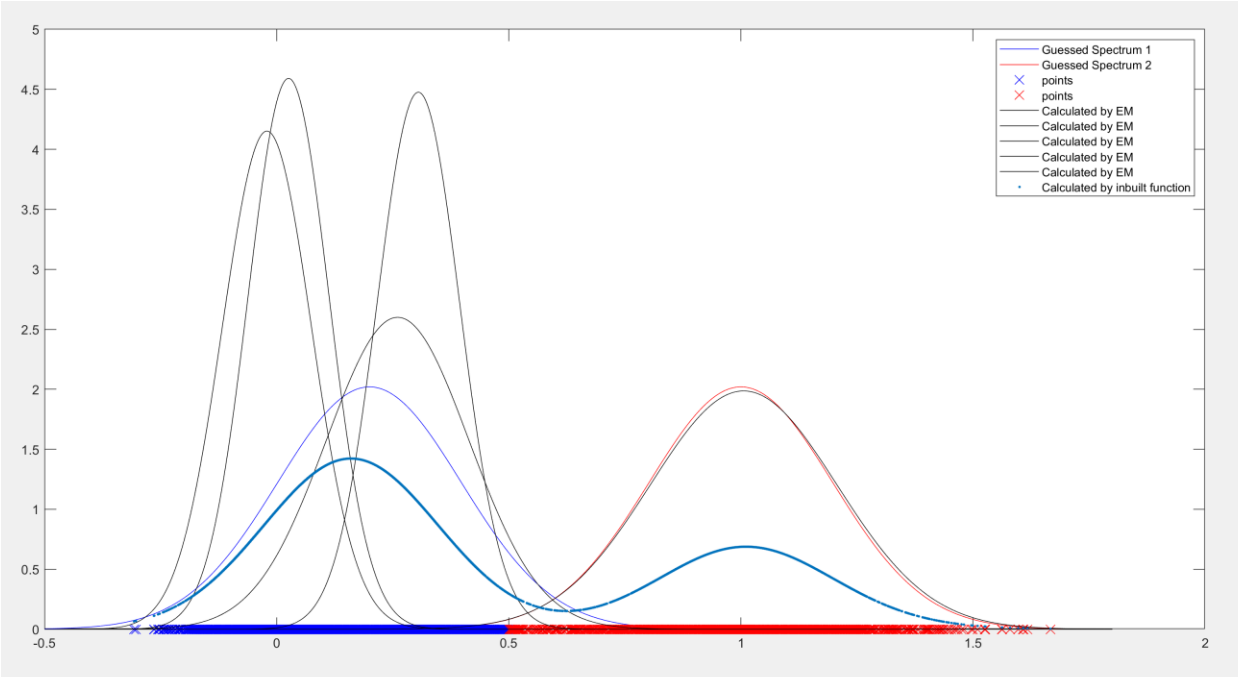



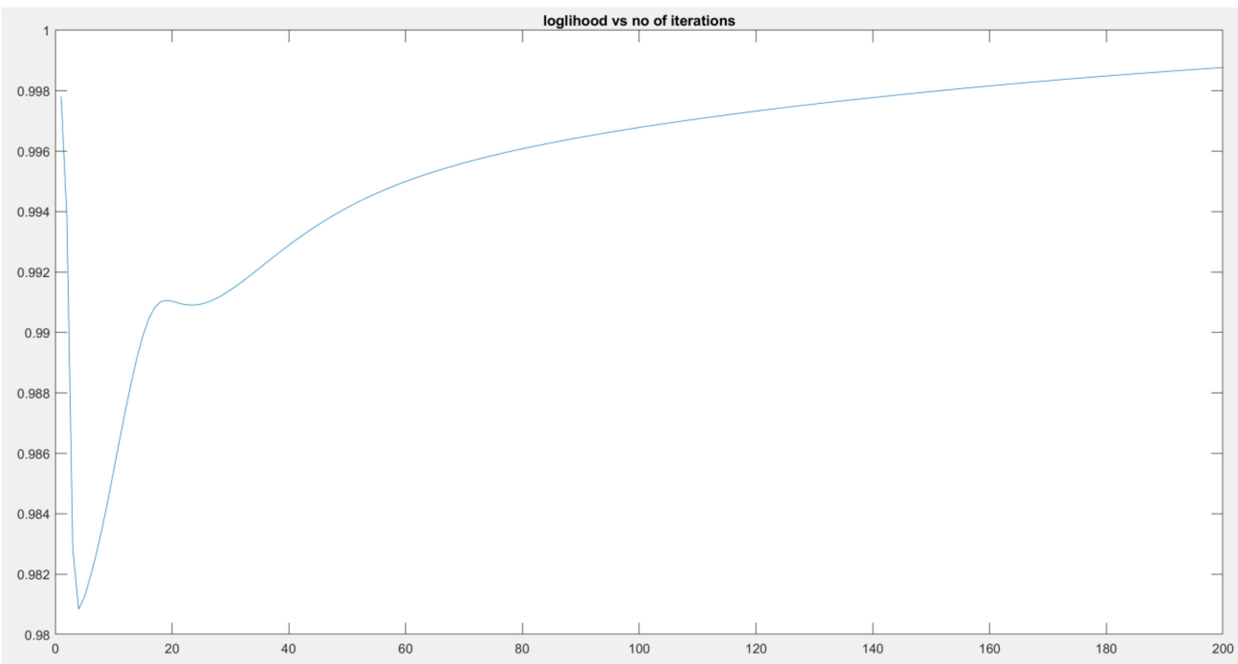

4 distributions to be found




5 distributions to be found


Guessed Spectrum 1
Guessed Spectrum 2
points
points
Calculated by EM
Calculated by EM
Calculated by EM
Calculated by EM
Calculated by EM
Calculated by inbuilt function


loglihood vs no of iterations