Week 1 Practice Exercises

<u>Remark</u>: For the sake of easy reading and minimizing errors in writing bits, it is recommended that we write bits in group of four and separate them with spaces. Example: Instead of writing 10011101, it is better to write it is 1001 1101.

1. Convert the decimal numbers 125, 87 and 177 to binary as unsigned binary numbers.

Solution

Operation	Quotient	Remainder
125/2	62	1
62/2	31	0
31/2	15	1
15/2	7	1
7/2	3	1
3/2	1	1
1/2	0	1

Therefore 125 **= 111 1101**

Operation	Quotient	Remainder
87/2	43	1
43/2	21	1
21/2	10	1
10/2	5	0
5/2	2	1
2/2	1	0
1/2	0	1

Therefore 87 **= 101 0111**

Operation	Quotient	Remainder
177/2	88	1
88/2	44	0
44/2	22	0
22/2	11	0
11/2	5	1
5/2	2	1
2/2	2	0
1/2	0	1

Therefore 177 = **1011 0001**

2. Express the unsigned binary representation of the integers 125, 87 and 177 as bytes.

Solution

125 **= 0111 1101**, 87 = **0101 0111**, and 177 = **1011 0001**

3. What are the decimal numbers represented by the unsigned binary numbers **10 0101**, **0000**, and **1 0101 0111**

Solution

10 0101 = $1*2^5 + 0*2^4 + 0*2^3 + 1*2^2 + 0*2^1 + 1 = 32 + 4 + 1 = 37$

0000 = $0*2^3 + 0*2^2 + 0*2^1 + 0 = 0 + 0 + 0 + 0 = 0$

1 0101 0111 = $2^8 + 2^6 + 2^4 + 2^2 + 2^1 + 1 = 256 + 64 + 16 + 4 + 2 + 1 = 343$

4. Find the sign and magnitude representations of the integers -125, 343 and 87 as Bytes.

Solution

For -125,

- We first convert 125 to unsigned binary to get **111 1101**.
- Then add extra left most bit for sign. Since -125 is negative decimal, we set the sign bit to **1**.
- Therefore the answer in Byte is **1111 1101.** The sign bit is shown in blue.

For 343,

- We first convert 343 to unsigned binary to get **1 0101 0111**
- This has 9 bits. But we **ONLY** have a **Byte**. Therefore we will take **ONLY the 7 bits starting from the right most bit** (shown in red) and then add a sign bit. Since 343 is positive decimal, we set the sign bit to **0**.

- Therefore the answer in Byte is **0101 0111. But wait a minute; is this byte in sign and magnitude representation really the decimal number 343? The answer is NO. Actually as a byte in sign and magnitude representation, it represents 87.**
- **Conclusion: Because 343 cannot fit in a byte in sign and magnitude representation, attempting to put 343 in a byte as a sign and magnitude representation actually changes the value to 87.**

For 87,

- We first convert 87 to unsigned binary to get **101 0111**
- Then add extra left most bit for sign. Since 87 is positive decimal, we set the sign bit to **0**.
- Therefore the answer in Byte is **0101 0111.** The sign bit is shown in blue.

5. Find the two's complement representations of the integers -125, 343, 128 and -128, 87, -87 as Bytes.

Solution

For -125,

- We first convert 125 to unsigned binary in Byte pattern to get **0111 1101**
- Then flip all the bits to get **1000 0010**
- Finally add 1 to the result to get **1000 0011. This is the required answer.**

For 343,

- We convert 343 to unsigned binary in Byte Pattern to get **1 0101 0111**. And that is our answer. **But wait a minute!** This has nine bits and the left most bit will be lost because of an overflow. Hence it will give rise to the binary **0101 0111** in a Byte pattern. This surely is not 343. In fact it is the decimal 87. Hence we conclude that we cannot store 343 in a Byte as two's complement; and if we attempt to do so what we get is 87.

For 128,

- We first convert 128 to unsigned binary to get **1000 0000**. And this is our answer. **But wait a minute!** This two's complement binary representation starts with a **1** and must be a representation of a negative decimal. In fact it is the representation of the decimal -128. Once again we conclude that we cannot store 128 in a Byte as two's complement; and if we attempt to do so what we get is -128.

For -128,

- We first convert 128 to unsigned binary in Byte pattern to get **1000 0000**
- Then flip all the bits to get **0111 1111**
- Finally add 1 to the result to get **1000 0000. This is the required answer.**

For 87,

- We convert 87 to unsigned binary in Byte pattern to get **0101 0111. This is the required answer.**

For -87,

- We first convert 87 to unsigned binary in Byte pattern to get **0101 0111**
- Then flip all the bits to get **1010 1000**
- Finally add 1 to the result to get **1010 1001. This is the required answer.**

6. Convert the following two's complement Byte patterns to decimal: **1000 0000, 1010 1001,** and **0101 0111**

Solution

For **1000 0000**,

- It starts with a 1 therefore it is negative decimal. Find its two's complement to get **1000 0000**

- Convert this Byte pattern to decimal to get 128. The required decimal is therefore -128.

For **1010 1001**,

- It starts with a 1 therefore it is negative decimal. Find its two's complement to get **0101 0111**
- Convert this Byte pattern to decimal to get 87. The required decimal is therefore -87.

For **0101 0111**,

- It starts with a **0**. Therefore it is positive decimal. Converting it to decimal by expanding it with powers of two gives 87. Hence we conclude that **0101 0111** in two's complement is 87 in decimal.

7. Perform the operation 125 – 87, -128 + 37 and -87 – 37 in two's complement using a Byte pattern and show that your answer is consistent with what we would expect if the arithmetic is performed in decimal.

Solution

For 125 – 87, we first write it as 125 + -87. Then we have

- 125 in two's complement in Byte pattern is **0111 1101**
- -87 in two's complement in Byte pattern is **1010 1001**
- Adding the binaries gives the result **1 0010 0110**
- The left most extra bit will be lost (overflow). Therefore the answer will be **0010 0110.** This binary corresponds to the decimal 38. In fact, performing the operation in decimals shows that 125 – 87 = 38. Hence we conclude that the arithmetic we performed using two's complement gives a correct answer.

For -128 + 37, we have

- -128 in two's complement in Byte pattern is **1000 0000**
- 37 in two's complement in Byte pattern is **0010 0101**
- Adding the binaries gives the result **1010 0101**
- Therefore the answer will be **1010 0101.** This binary corresponds to the decimal -91. In fact, performing the operation in decimals shows that -128 + 37 = -91. Hence we conclude that the arithmetic we performed using two's complement gives a correct answer.

For -87 – 37, we first write it as -87 + -37. Then we have

- -87 in two's complement in Byte pattern is **1010 1001**
- -37 in two's complement in Byte pattern is **1101 1011**
- Adding the binaries gives the result **1 1000 0100**
- The left most extra bit will be lost (overflow). Therefore the answer will be **1000 0100.** This binary corresponds to the decimal -124. To this end, performing the operation in decimals shows that -87 - 37 = -124. Hence we conclude that the arithmetic we performed using two's complement gives a correct answer.

8. Given the binary 1101 1100 0100 1111 0111 1011, write it in Hexadecimal format.
9. Given the Hexadecimal A68D4F, write it as binary.
10. Given the decimal 173. Write it down as unsigned binary in a Byte. Also write it down as Hexadecimal.
11. Given the Byte binary 1101 0011. What value does it represent if it is
   a. Unsigned binary?
   b. Sign and Magnitude binary?
   c. Two's complement binary?
   d. Ascii code?

12. <u>Challenge</u>

a. Give the two's complement representation of -206 as a BYTE pattern.

b. What signed decimal number does your BYTE in part (a) actually represent?

c. Give the two's complement representation of 322 as a BYTE pattern?

d. What signed decimal number does your BYTE in part (c) actually represent?

e. Perform the binary addition of your answers for parts (a) and (c) and give your binary answer as a BYTE?

f. Does the binary addition operation of part (e) give rise to an overflow?

g. What signed decimal number does your BYTE in part (e) represent?

h. Surprise surprise!!! Observe that your answer for part (g) is actually the correct sum of the signed decimal numbers -206 + 322. How did this happen? That is although your BYTE memories were not able to store the signed decimal numbers -206 and 322; the addition operation of the BYTEs however gave rise to correct answer. How?