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FUNDAMENTALS OF COMPUTER GRAPHICS (CSIT304)

PLANE CURVES - INTRODUCTION

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INTRODUCTION

- The simplest curves (lines) and surfaces (planes) discussed earlier.
- There are conics and quadric surfaces.
  - They have been around for about 2000 years.
- We shall discuss these in this part of the course.
- Background knowledge
  - Consulting Calculus and/or geometry books should be very helpful.
  - Linear algebra book should also helpful.

PLANE CURVE

- A multitude of techniques are available for drawing and designing curves manually.
- Tools are used to aid the designer.
  - No single tool is sufficient for all tasks.
- Software tools for curve design and generation in computer graphics.
- Two-dimensional curve generation techniques are discussed here.

REPRESENTING CURVES

- Point based representation

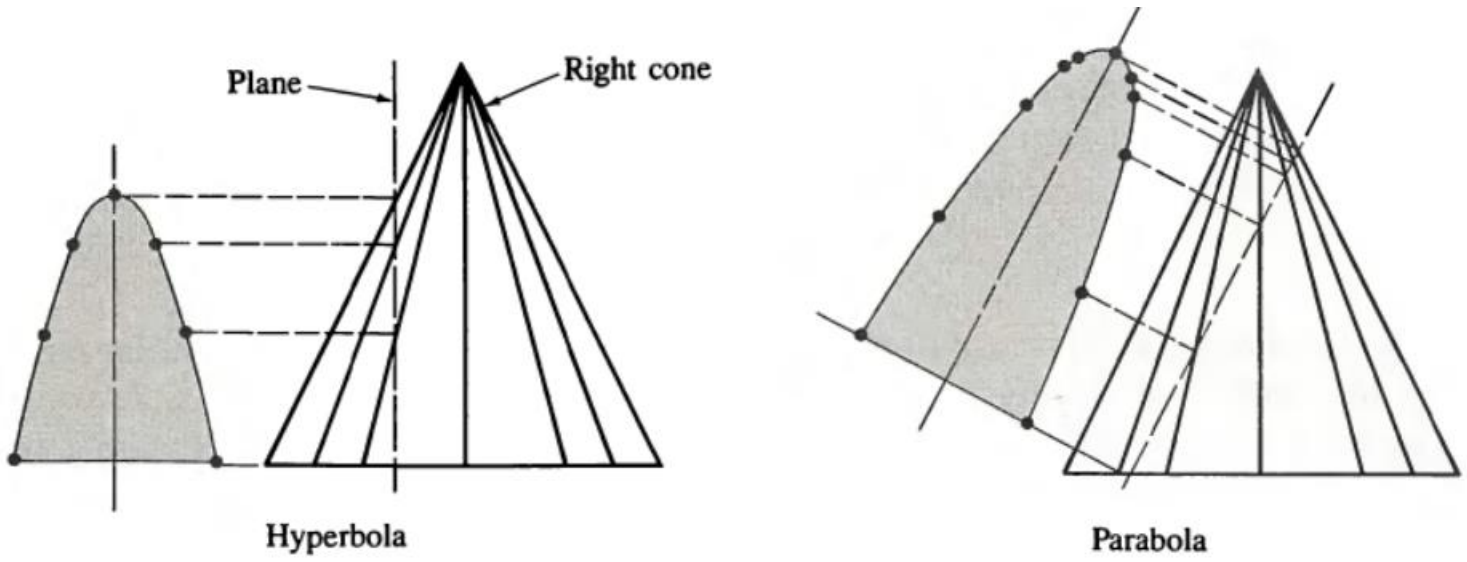
Observe the spacing of the points along the curve.

- Analytical representation **[ADVANTAGES?]**
  - Non-parametric (Explicit, Implicit)
  - Parametric

IMPLICIT REPRESENTATION (NON-PARAMETRIC)

- A general second-degree implicit equation written as:

$$ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0$$



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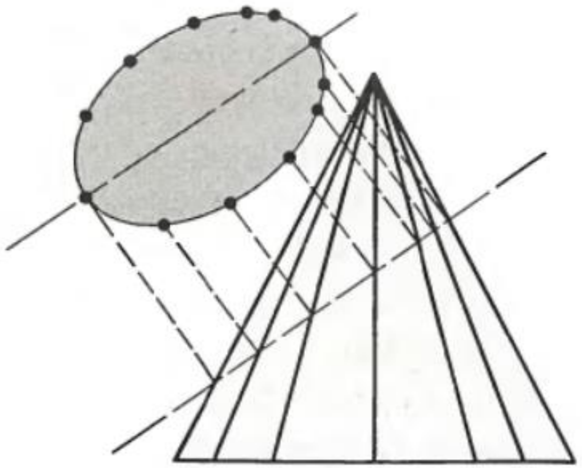
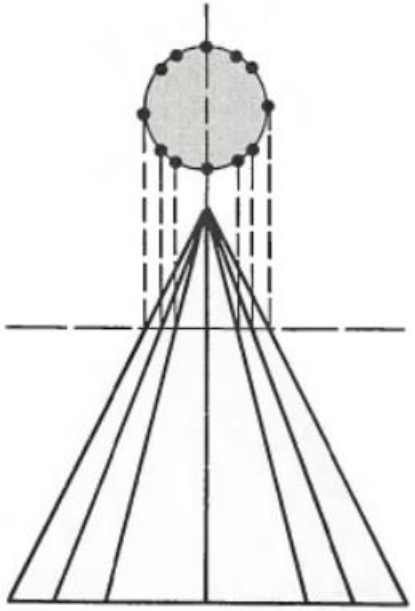


Ellipse



Circle

PARAMETRIC CURVE

- Each coordinate of a point on a curve is represented as a function of a single parameter.
- The position vector of a point on the curve is fixed by the value of the parameter.
- For a two-dimensional curve with t as the parameter, a point on the curve

Cartesian Coordinate

$$x = x(t)$$
$$y = y(t)$$

The position vector

$$P(t) = [x(t) \quad y(t)]$$

EXAMPLE OF PARAMETRIC CURVE

- The simplest parametric 'curve' representation is for a straight line.

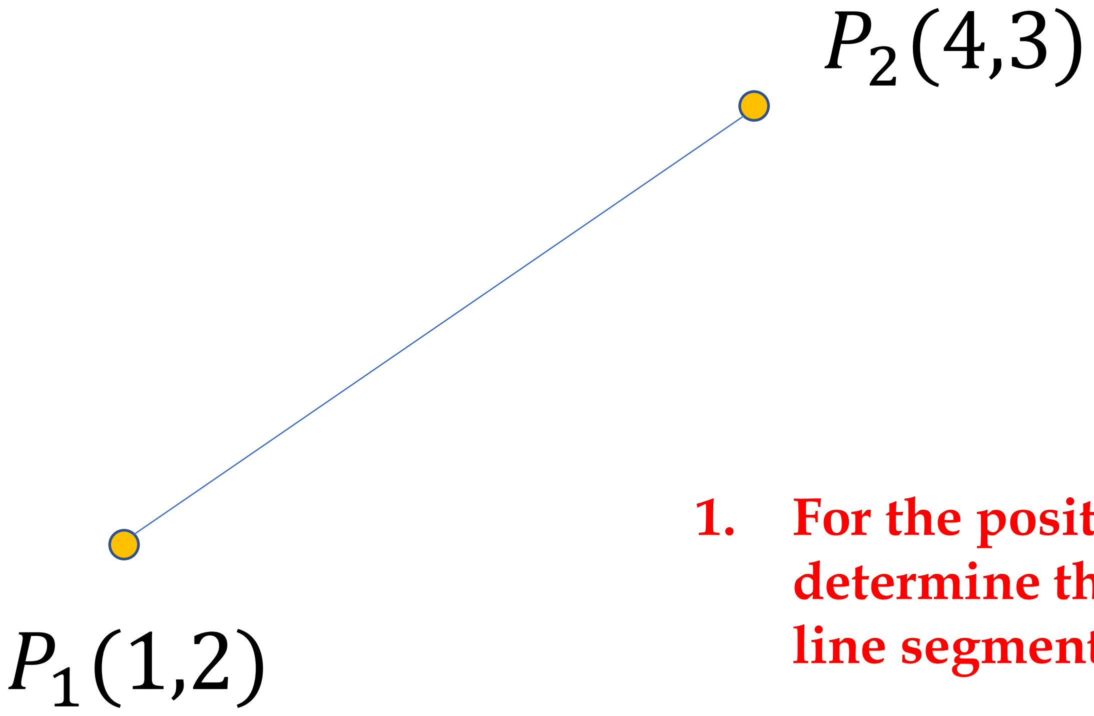
$$P(t) = P_1 + (P_2 - P_1)t \qquad 0 \le t \le 1$$

The position vector is represented as: $P(t) = [x(t) \quad y(t)]$

$$x(t) = x_1 + (x_2 - x_1)t$$

$$y(t) = y_1 + (y_2 - y_1)t$$

EXAMPLE (1)



1. For the position vectors P_1 [1 2] and P_2 [4 3] determine the parametric representation of the line segment between them.

2. Also determine the slope and tangent vector of the line segment.

PARAMETRIC REPRESENTATION OF A CIRCLE

- An origin-centered circle of radius r is parametrically represented by

$$x = r\cos\theta \qquad\qquad 0 \le \theta \le 2\pi$$
$$y = r\sin\theta$$

Here θ is the parameter

- The Cartesian coordinates of any point on an origin-centered circle are then

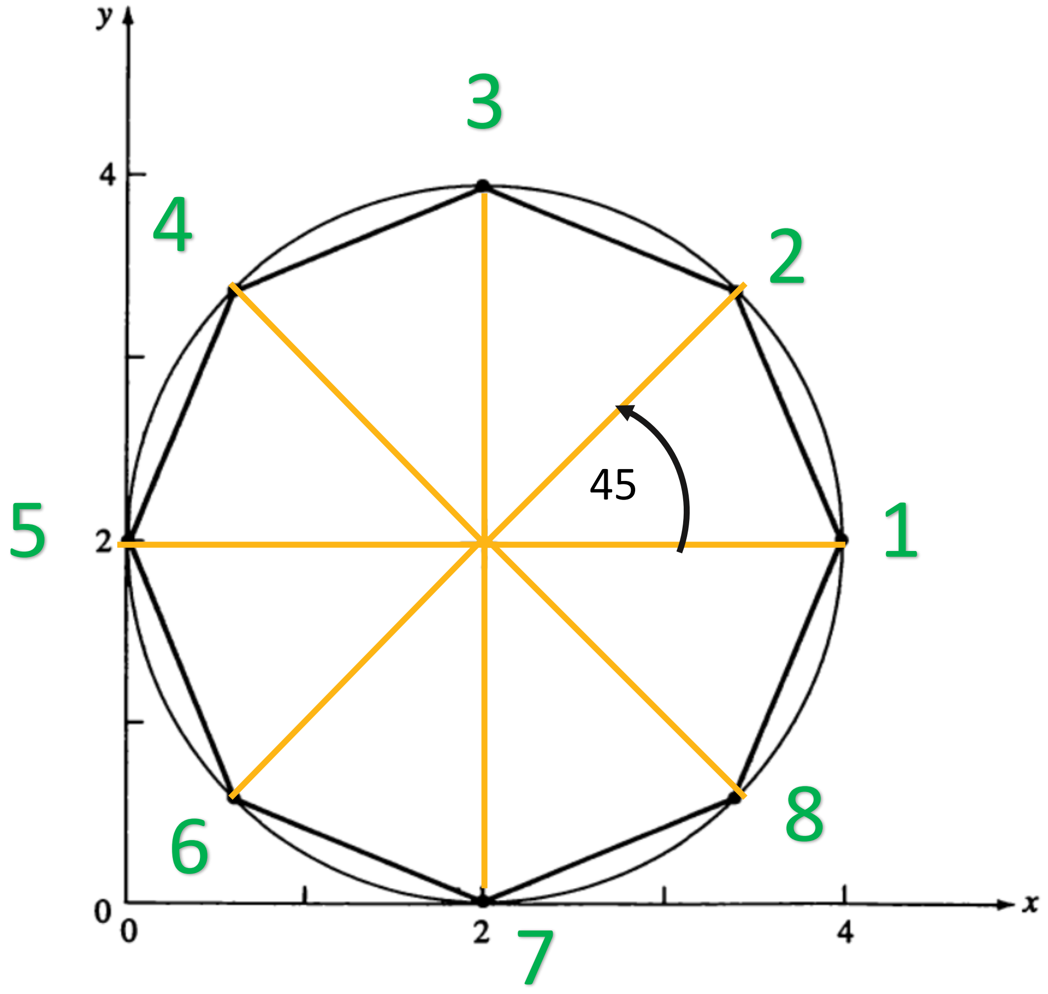
$$x_{i+1} = r\cos(\theta_i + \delta\theta)$$
$$y_{i+1} = r\sin(\theta_i + \delta\theta)$$

Does it appear similar to some earlier concepts?

Which approach is more efficient and why?

EXAMPLE

- **Generate a circle of radius 2 with center located at (2,2).**



What are the steps to draw this circle using parametric equation?

$$x_{i+1} = x_i \cos \delta\theta - y_i \sin \delta\theta$$
$$y_{i+1} = x_i \sin \delta\theta + y_i \cos \delta\theta$$

$$\delta\theta = \frac{2\pi}{(n+1-1)} = \frac{2\pi}{n} = \frac{2\pi}{8} = \frac{\pi}{4}$$

CALCULATION OF POINTS ON THE CIRCLE (UNIT CIRCLE AT THE ORIGIN)

- At $\theta = 0$ (First point)

$$x_1 = r\cos\theta_1 = (1)\cos(0) = 1$$
$$y_1 = r\sin\theta_1 = (1)\sin(0) = 0$$

- Compute $\delta\theta$ component

$$\sin\delta\theta = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\cos\delta\theta = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

- The second point

$$x_2 = x_1\cos\delta\theta - y_1\sin\delta\theta$$

$$y_2 = x_1\sin\delta\theta + y_1\cos\delta\theta$$

$$= (1)(\sqrt{2}/2) - 0(\sqrt{2}/2) = (\sqrt{2}/2)$$
$$= (1)(\sqrt{2}/2) + 0(\sqrt{2}/2) = (\sqrt{2}/2)$$

- And so on …

CALCULATION OF POINTS ON THE CIRCLE (UNIT CIRCLE AT THE ORIGIN)

- All the points on the circle

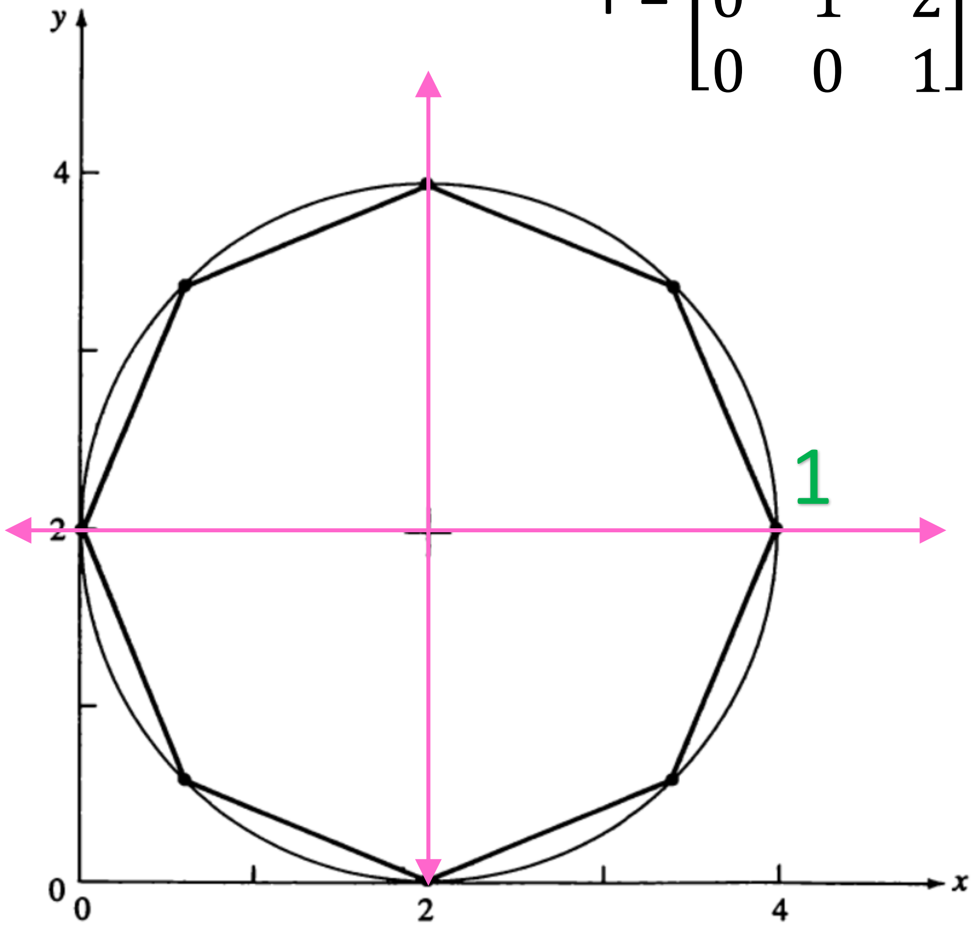
i	x_i	y_i
1	1	0
2	$\sqrt{2}/2$	$\sqrt{2}/2$
3	0	1
4	$-\sqrt{2}/2$	$\sqrt{2}/2$
5	-1	0
6	$-\sqrt{2}/2$	$-\sqrt{2}/2$
7	0	-1
8	$\sqrt{2}/2$	$-\sqrt{2}/2$
9	1	0

RELOCATION TO THE DESIRED POSITION

1. Locally scale the points by a factor of 2

2. Translates the center of the circle to the point (2,2)

$$T = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 2 \\ 0 & 2 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$



$$P_1 = \begin{bmatrix} 2 & 0 & 2 \\ 0 & 2 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}$$

CALCULATION OF POINTS ON THE CIRCLE (RADIUS = 2, CENTER AT (2,2))

- All the points on the circle

i	x_i	y_i
1	1	0
2	$\sqrt{2}/2$	$\sqrt{2}/2$
3	0	1
4	$-\sqrt{2}/2$	$\sqrt{2}/2$
5	-1	0
6	$-\sqrt{2}/2$	$-\sqrt{2}/2$
7	0	-1
8	$\sqrt{2}/2$	$-\sqrt{2}/2$
9	1	0

$\rightarrow$

i	x_i	y_i
1	4	2
2	3.414	3.414
3	2	4
4	0.586	3.414
5	0	2
6	0.586	0.586
7	2	0
8	3.414	0.586
9	4	2

- By restricting the range of the parameter, the algorithm generates circular arcs.

EXERCISE

- Implement the parametric circle generation algorithm discussed in todays class.
- Radius 2 with center located at (2,2).

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THANK YOU