FLAME UNIVERSITY

EVERLASTING learning

FUNDAMENTALS OF COMPUTER GRAPHICS (CSIT304)

THREE-DIMENSIONAL VIEWING

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AFFINE AND PERSPECTIVE GEOMETRY

INTRODUCTION

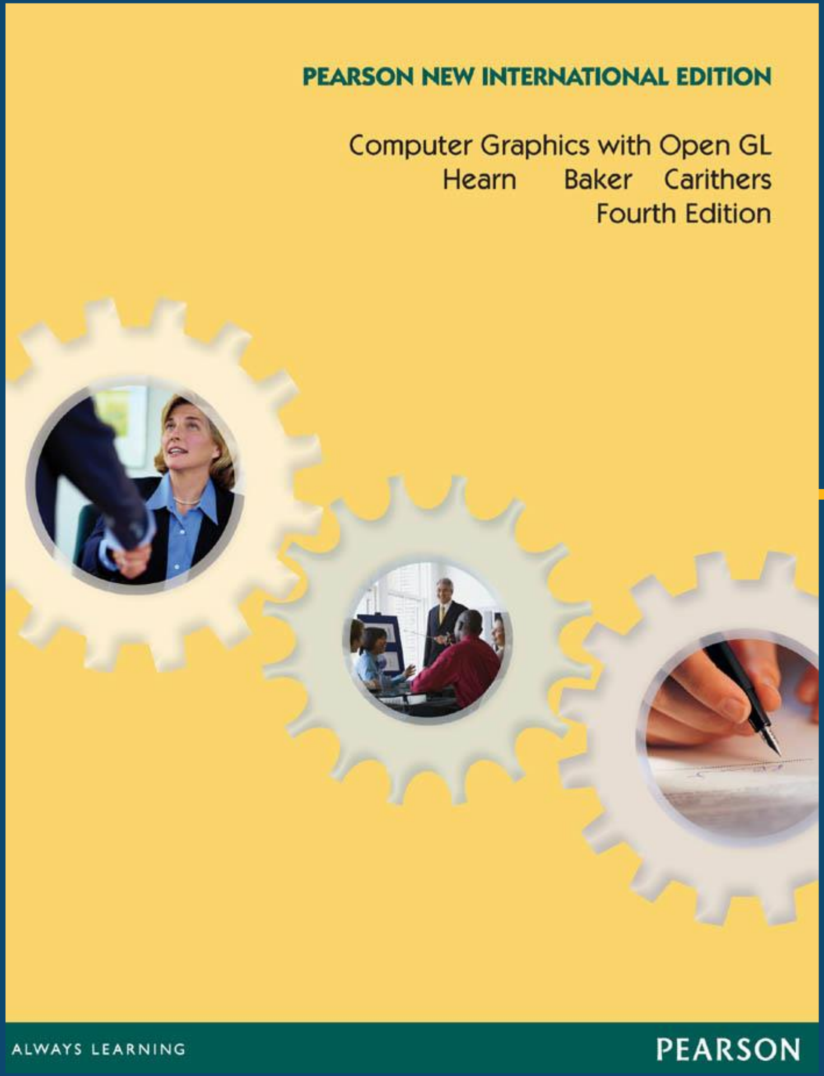
- Geometric theorems have been developed for both perspective and affine geometry.
- The theorems of affine geometry are identical to those for Euclidean geometry.
- In both parallelism is an important concept.
- In perspective geometry, lines are generally nonparallel.

AFFINE TRANSFORMATION

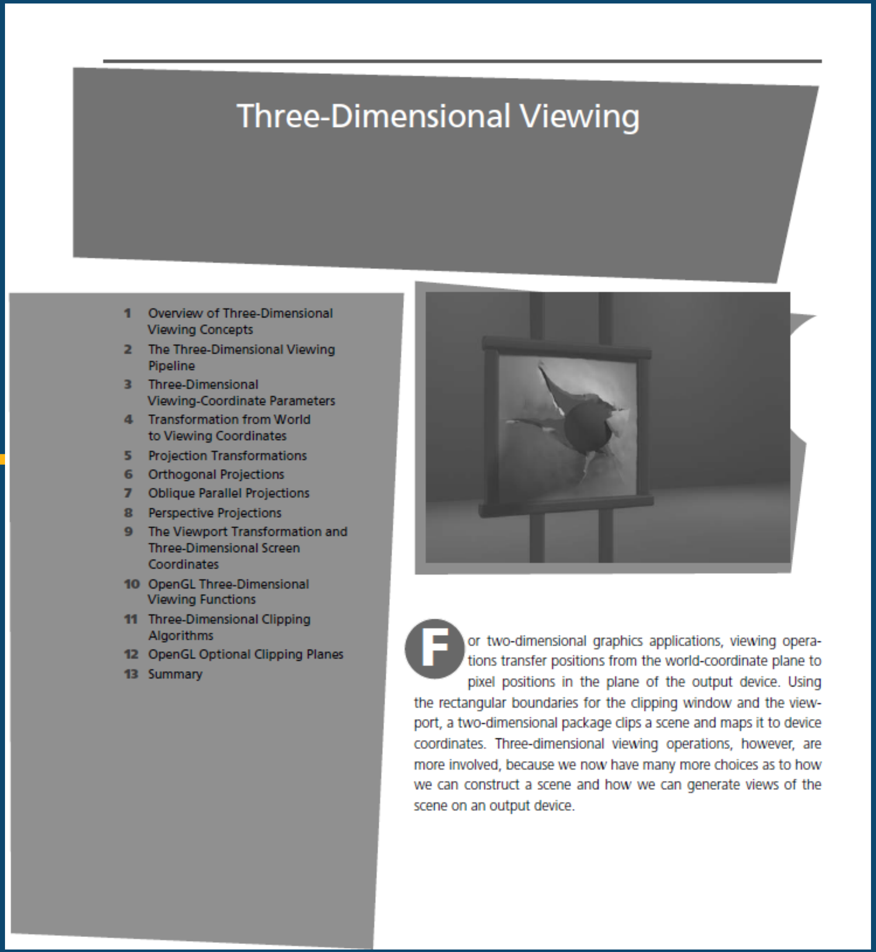
- It is a combination of linear transformations
  - E.g., rotation followed by translation.
- For an affine transformation, the last column in the general 4x4 transformation matrix is [0 0 0 1]
- Otherwise
  - The transformed homogeneous coordinate h is not unity
  - There is not a one-to-one correspondence between the affine transformation and the 4x4 matrix operator.
- It form a useful subset of bilinear transformations
  - Product of two affine transformations is also affine.
- This allows
  - The general transformation of a set of points relative to an arbitrary coordinate system
  - Maintains a value of unity for the homogeneous coordinate ft.

AFFINE AND PERSPECTIVE TRANSFORMATION

- Both affine and perspective transformations are three-dimensional
- They are transformations from one three space to another three space.
- Viewing the results on a two-dimensional surface requires a projection from three space to two space.
- The result is called a plane geometric projection.
- The projection matrix from three space to two space always contains a column of zeros.
- Consequently the determinant of a projective transformation is always zero.



3D VIEWING



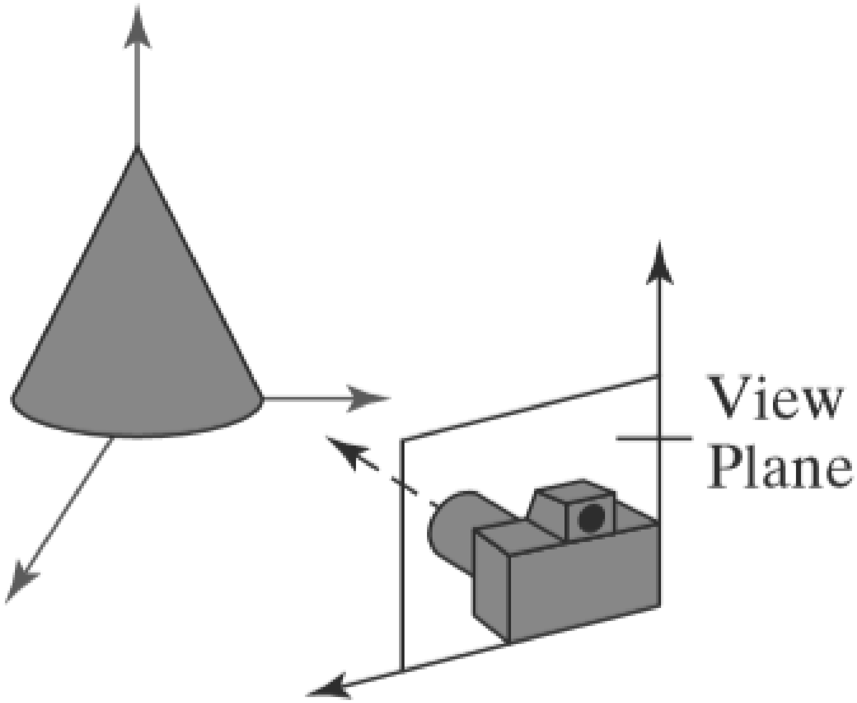
Three-Dimensional Viewing

For two-dimensional graphics applications, viewing operations transfer positions from the world-coordinate plane to pixel positions in the plane of the output device. Using the rectangular boundaries for the clipping window and the viewport, a two-dimensional package clips a scene and maps it to device coordinates. Three-dimensional viewing operations, however, are more involved, because we now have many more choices as to how we can construct a scene and how we can generate views of the scene on an output device.

INTRODUCTION

- Each object in the scene is typically defined with a set of surfaces
- They form a closed boundary around the object interior
- We may also need to specify information about the interior structure of an object
- Ultimately project a specified view of the objects onto the surface of a display device
- Additional routines are there to visualize a 3D scene

VIEWING A 3D SCENE



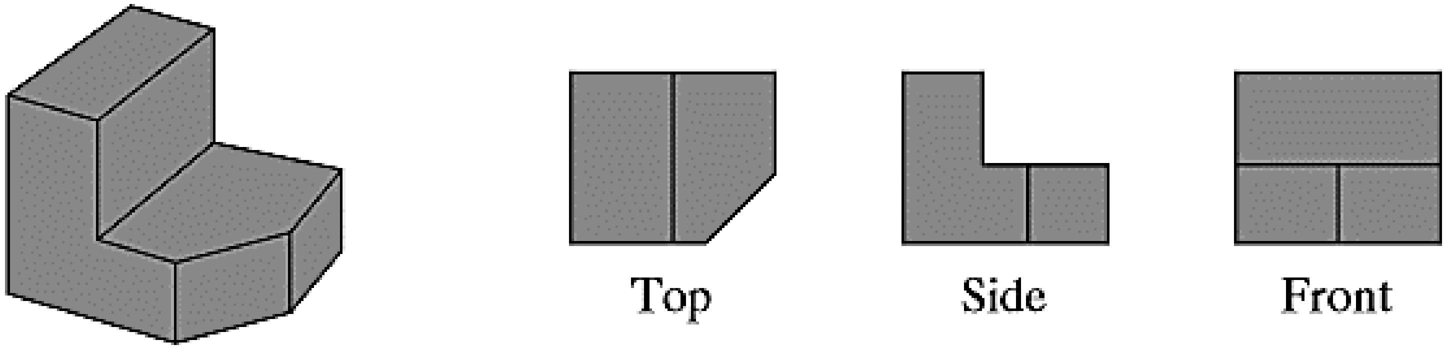
1. Set up a **coordinate reference** for the viewing, or "camera," parameters.
   - Defines the **position** and **orientation** for a *view plane* (or *projection plane*)
   - Corresponds to a camera film plane.

2. Object descriptions are transferred to the viewing reference coordinates and **projected** onto the view plane.

3. Generate a **view of an object** on the output device
   - In wireframe (outline) form, or
   - Apply lighting and surface-rendering techniques (visual realism).

PROJECTION

- We can choose different methods for projecting a scene onto the view plane.
- Parallel Projection
  - Project points on the object surface along parallel lines
  - Used in engineering and architectural drawing store present an object
  - A set of views that show accurate dimensions of the object



PROJECTION

- Plane geometric projections of objects are formed by the intersection of lines called **projectors** with a plane called the **projection plane.**
- Projectors are lines from an arbitrary point called the **center of projection**, through each point in an object.
- If the center of projection is located at
  - A Finite point in three space,
    - the result is a perspective projection.
  - Infinity,
    - All the projectors are parallel and the result is a parallel projection.
- Plane geometric projections provide the basis for descriptive geometry.

PROJECTION

- Perspective projection
  - Project points to the view plane along converging paths
  - Objects farther from the viewing position : appear small
  - Objects of same size nearer to the viewing position: appear large
  - More realistic scene

PROJECTION

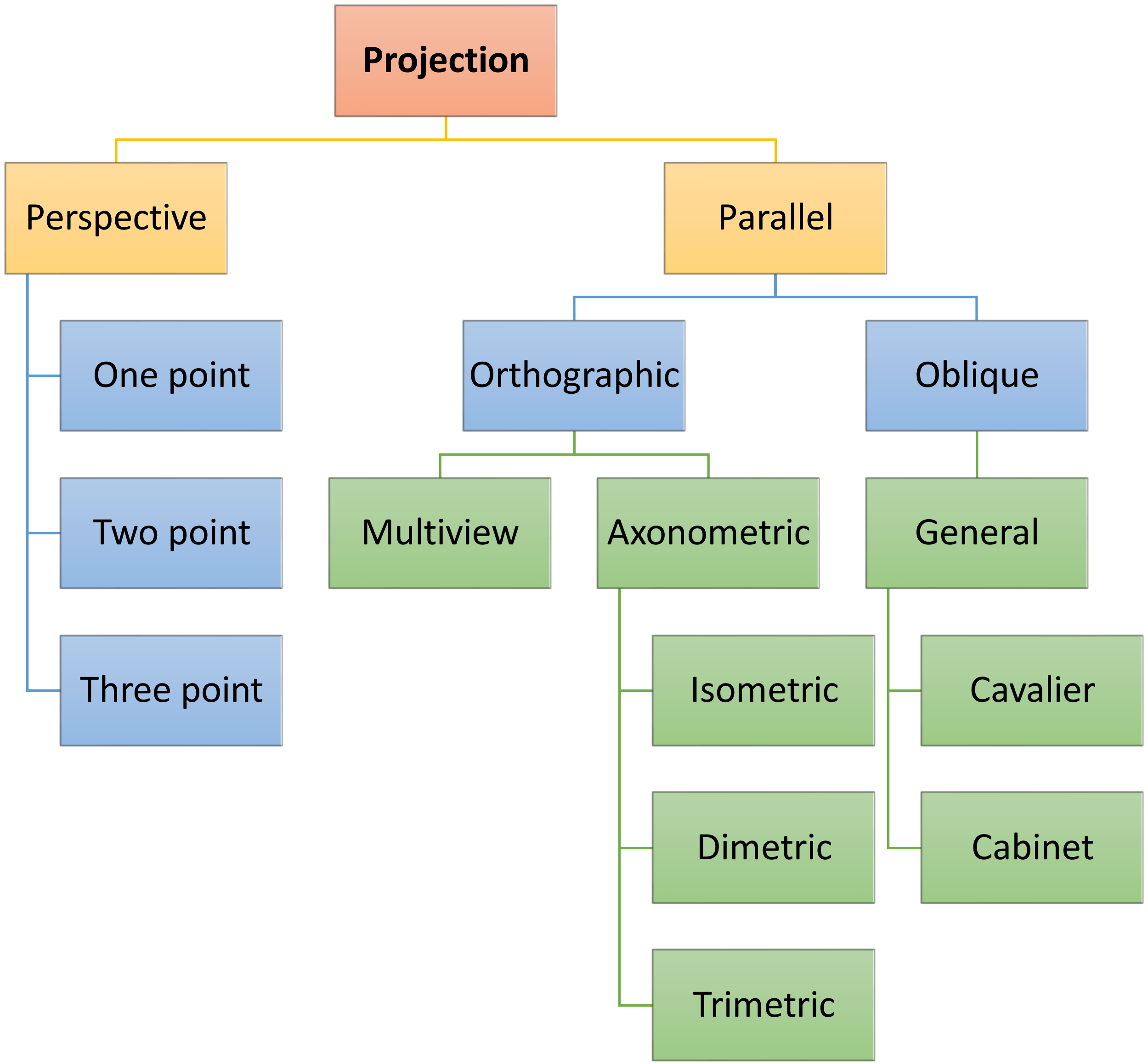
Orthographic

Perspective

COMPARISON OF ORTHOGRAPHIC AND PERSPECTIVE PROJECTIONS

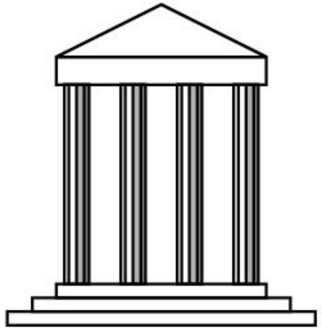
Feature	Orthographic Projection	Perspective Projection
Parallel vs. Converging Lines	Parallel lines remain parallel.	Parallel lines converge towards a vanishing point.
Size Consistency	Objects maintain relative sizes regardless of depth.	Objects appear smaller with distance.
Depth Perception	Limited depth perception.	Strong depth perception due to size changes and converging lines.
Mathematical Representation	Represented using an orthographic projection matrix.	Represented using a perspective projection matrix.
Common Use Cases	Technical drawings, engineering, architectural illustrations.	Computer graphics, video games, movies, virtual reality.
Camera Setup	Simulates an orthographic camera with parallel rays.	Simulates a perspective camera with converging rays.

TYPES OF PROJECTION

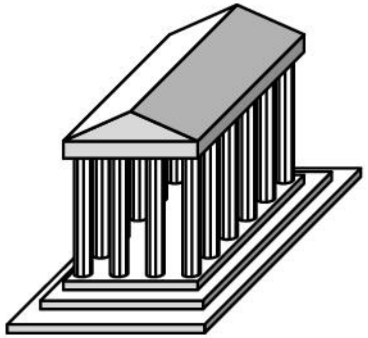


CLASSICAL VIEWINGS

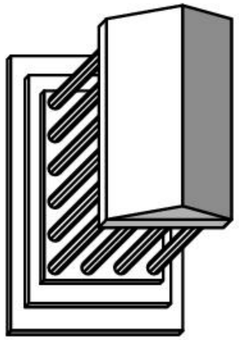
- Hand drawings : Determined by a specific relationship between the object and the viewer.



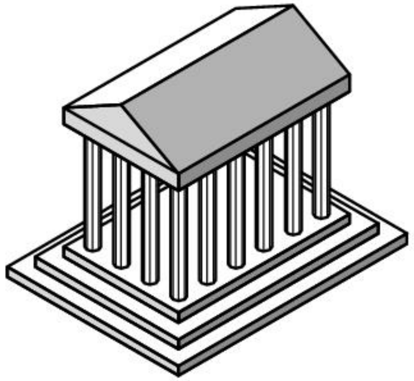
Front elevation



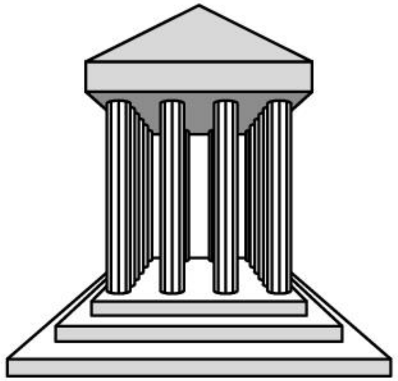
Elevation oblique



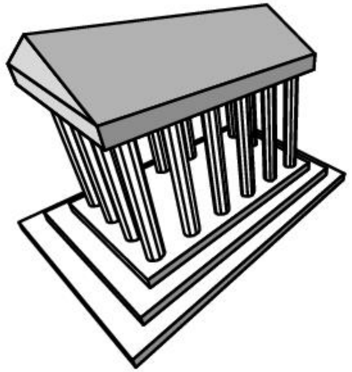
Plan oblique



Isometric

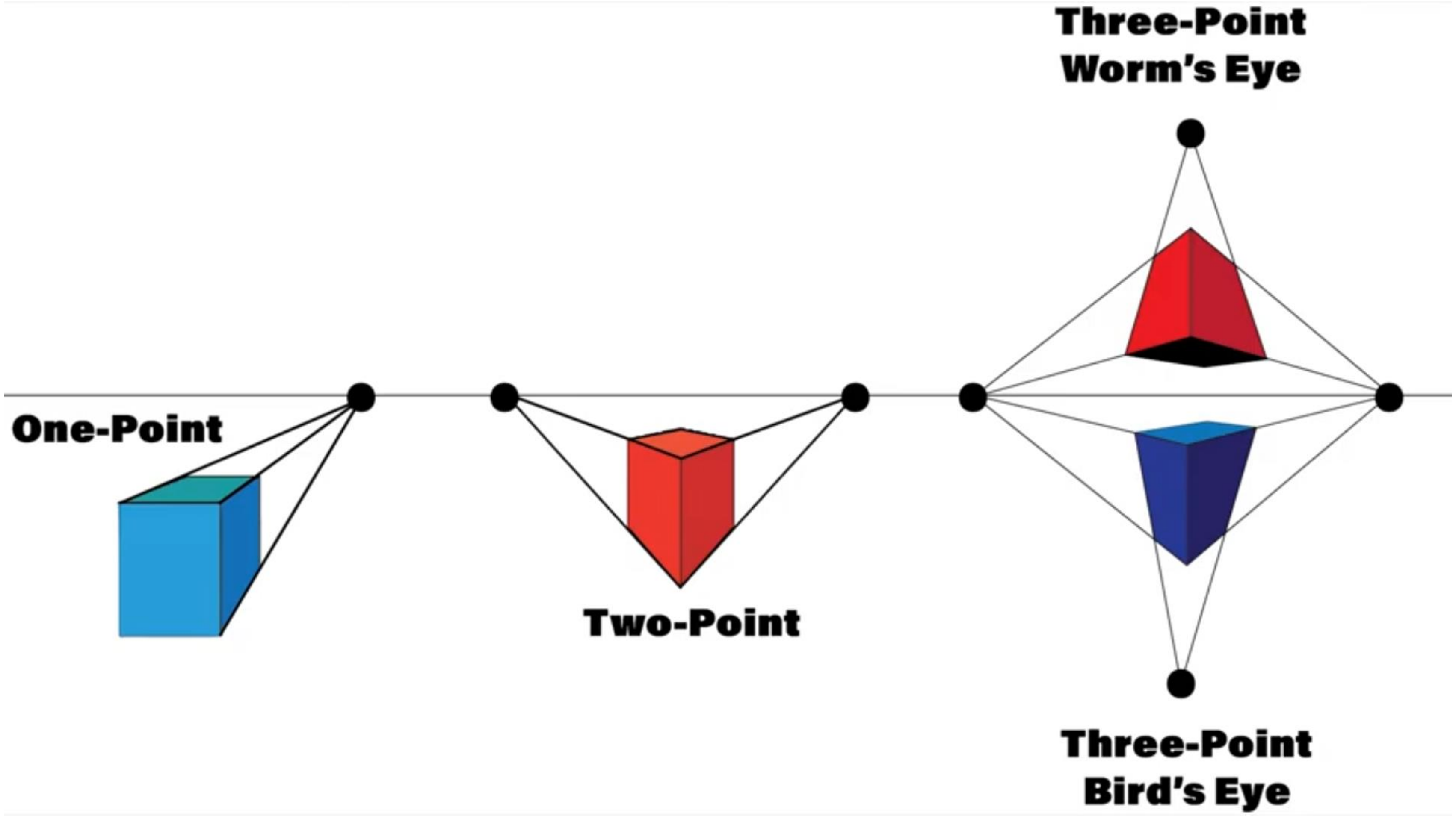


One-point perspective



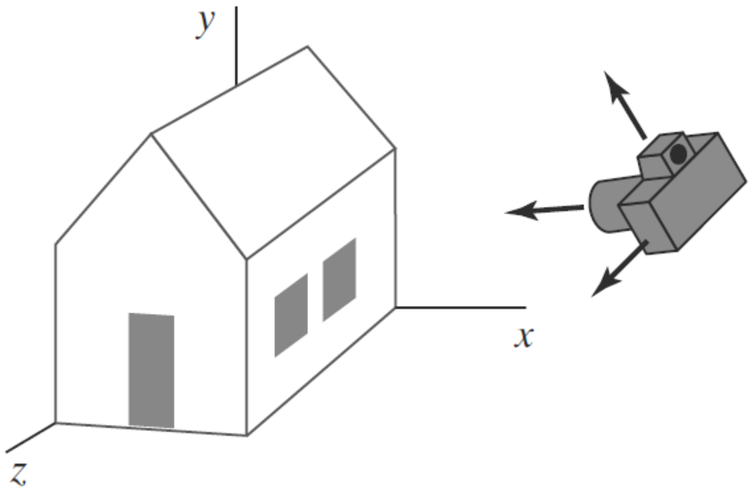
Three-point perspective

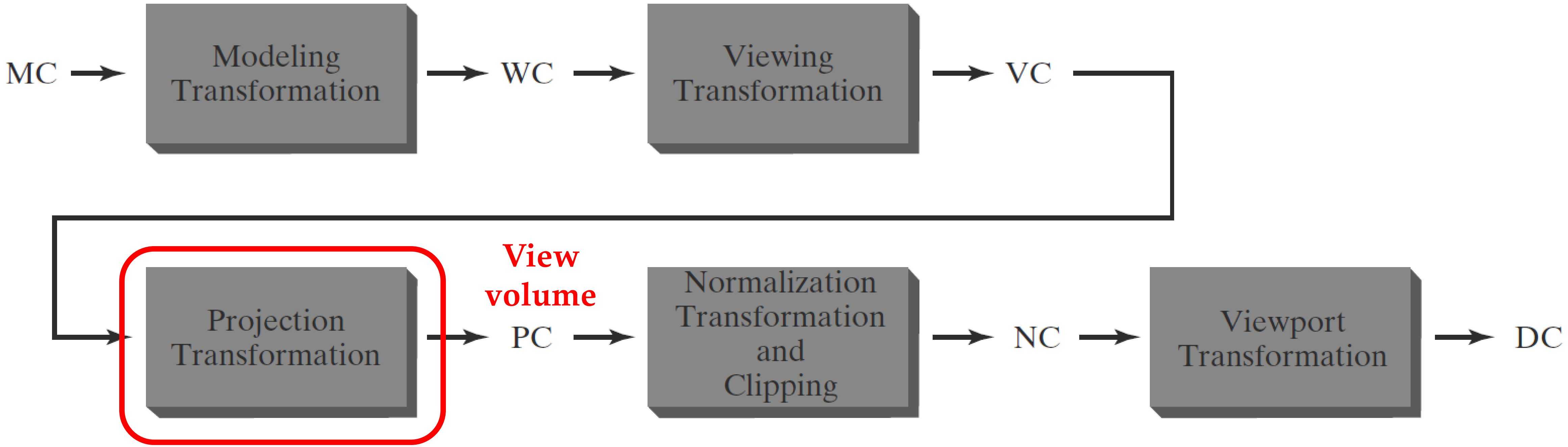
TYPES OF PERSPECTIVE



3D VIEWING PIPELINE

- Procedures for generating a computer-graphics view of a three-dimensional scene
- Analogous to the processes involved in taking a photograph



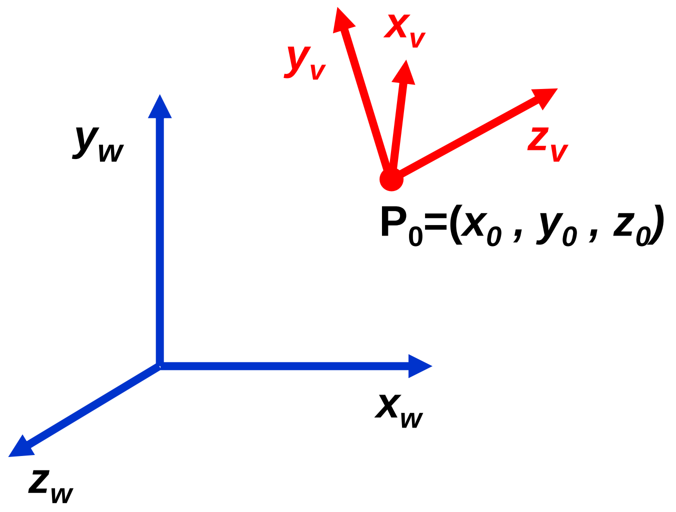


VIEWING COORDINATES

- Generating a view of an object in 3D is similar to photographing the object.
- Whatever appears in the viewfinder is projected onto the flat film surface.
- Depending on the position, orientation and aperture size of the camera corresponding views of the scene is obtained.

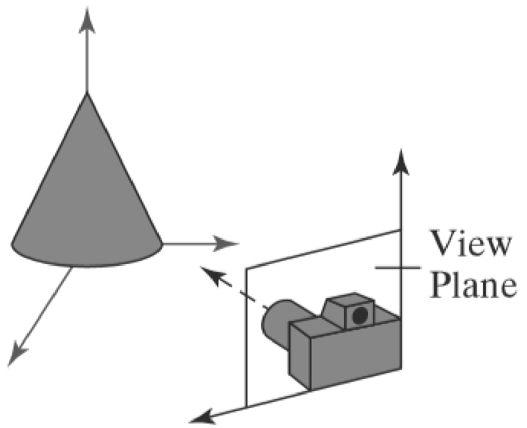
SPECIFYING THE VIEW COORDINATES

- For a particular view of a scene
    - First we establish viewing-coordinate system.



- A view-plane (or projection plane) is set up **perpendicular** to the viewing z-axis.

- World coordinates are transformed to viewing coordinates



- Then viewing coordinates are projected onto the view plane.

SPECIFYING THE VIEW COORDINATES

- To establish the viewing reference frame,
    - We first pick a world coordinate position : **view reference point.**

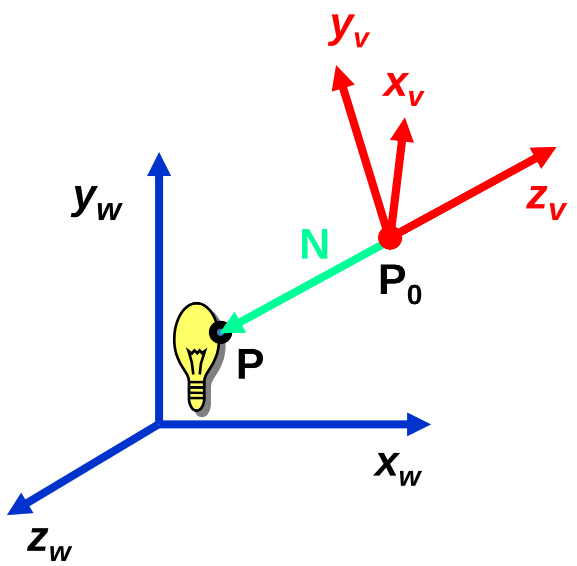
- This point is the origin of the viewing coordinate system.

- E.g., consider a point on an object
    - This point as the position where we aim a camera to take a picture of the object.

SPECIFYING THE VIEW COORDINATES

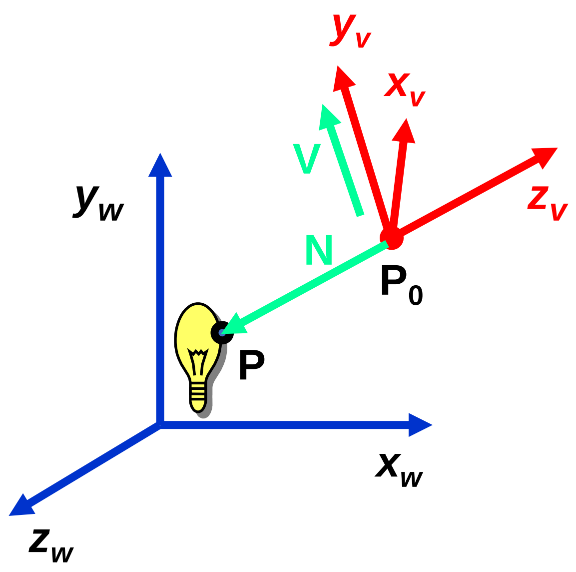
- Select the
  - Positive direction for the viewing z-axis,
  - The orientation of the view plane,
  - By specifying the view-plane normal vector, N.

- Choose a world coordinate position P
  - This point establishes the direction for N.



SPECIFYING THE VIEW COORDINATES

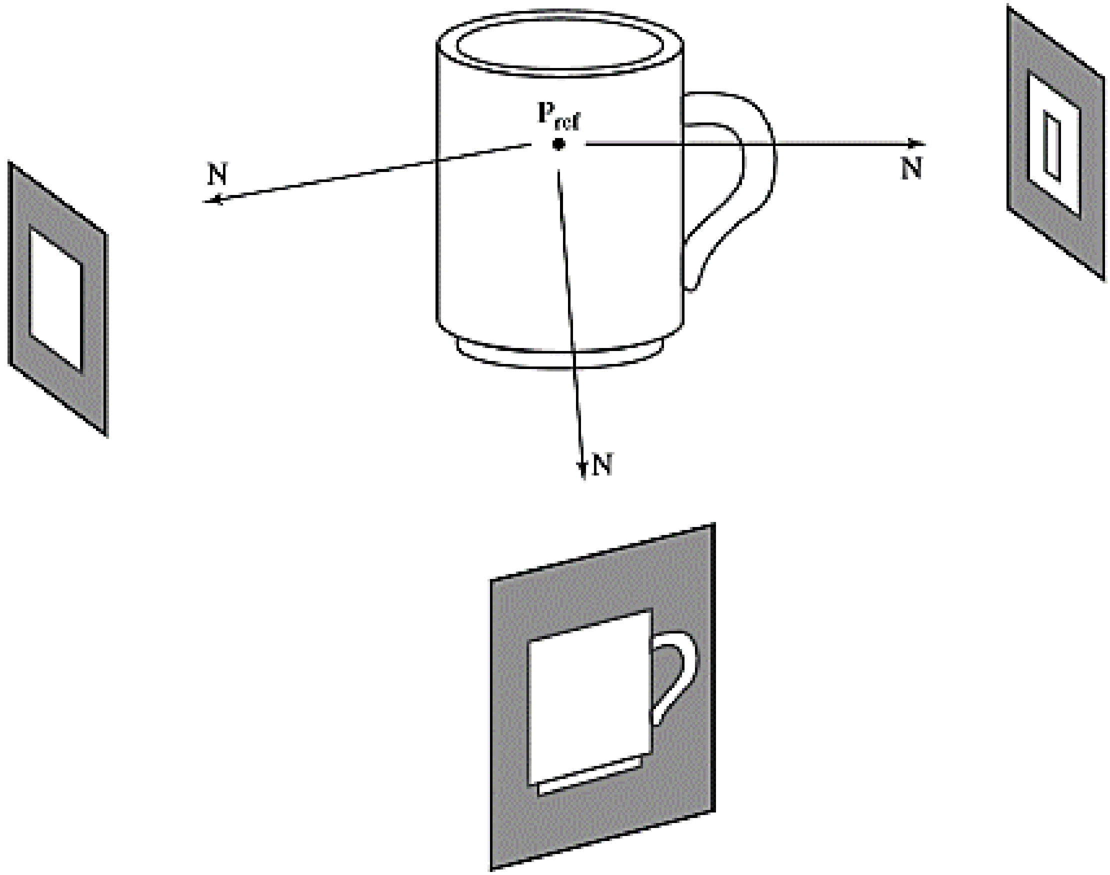
- Finally, choose the **up direction for the view** by specifying view-up vector V.
- This vector is used to establish the positive direction for the yv axis.
- The vector V is perpendicular to N.
- Using N and V, we can compute a third vector U, perpendicular to both N and V, to define the direction for the xv axis.



SPECIFYING THE VIEW COORDINATES

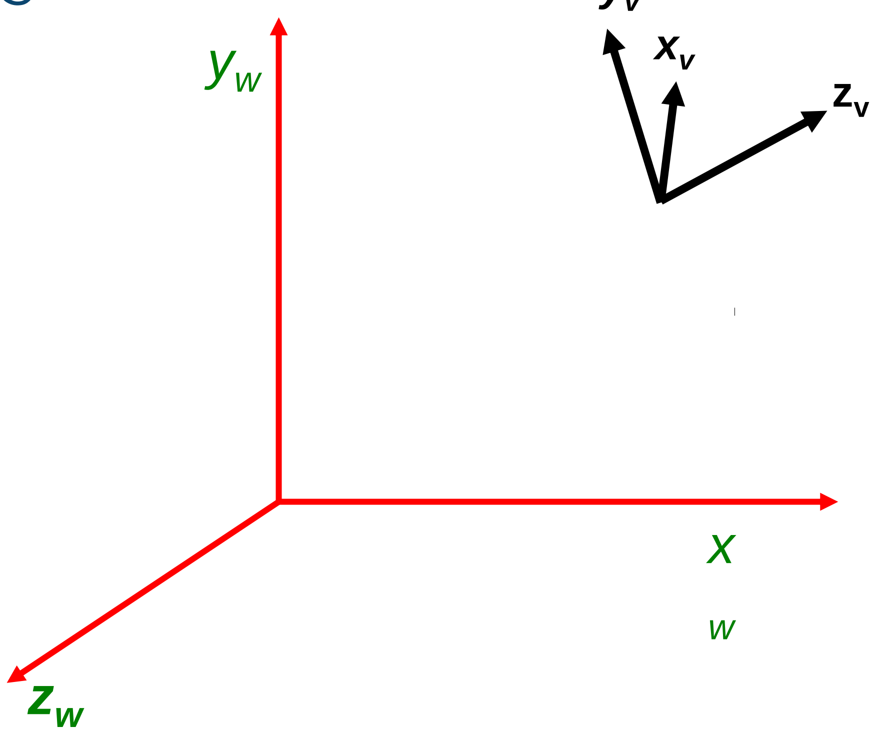
To obtain a series of views of a scene ,

- Keep the the view reference point fixed
- Change the direcion of N.
- Generate views as we move around the viewing coordinate origin.



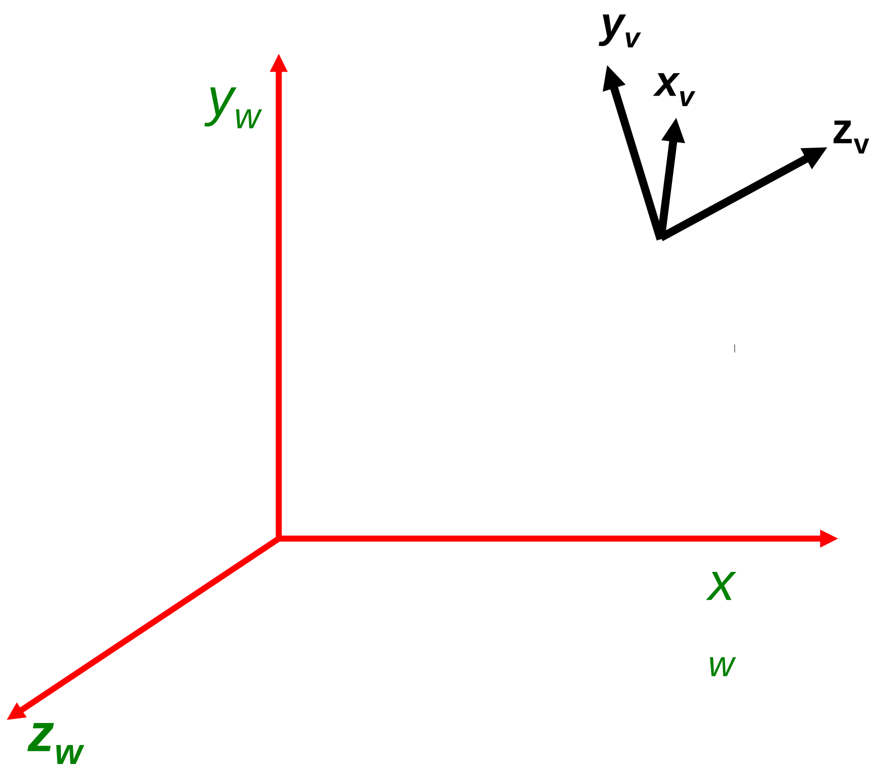
TRANSFORMATION FROM WORLD TO VIEWING COORDINATES

- It is equivalent to transformation that superimpoes the viewing reference frame onto the world frame using the translation and rotation.



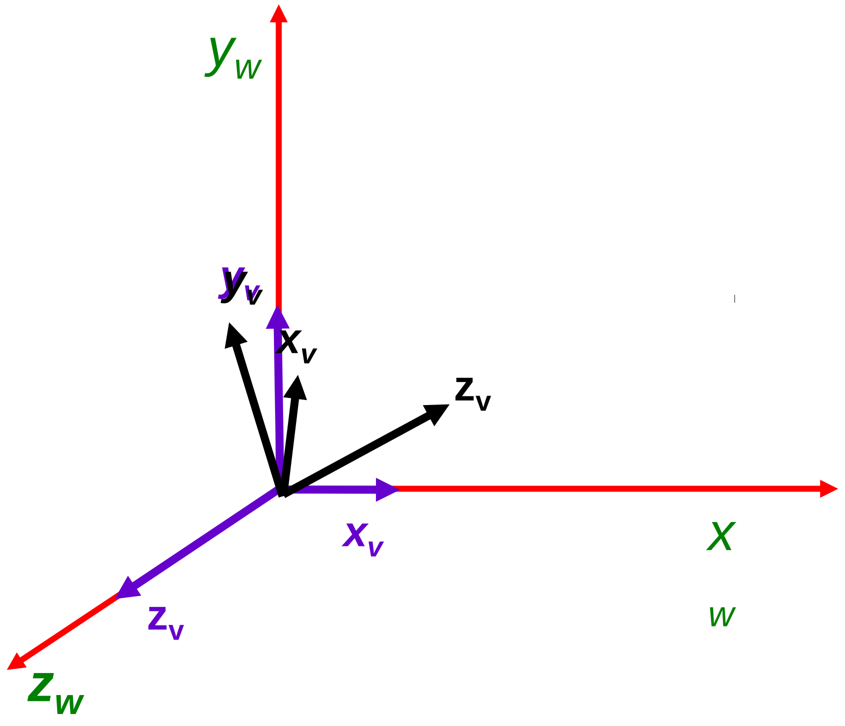
TRANSFORMATION FROM WORLD TO VIEWING COORDINATES

- First, we translate the view reference point to the origin of the world coordinate system



TRANSFORMATION FROM WORLD TO VIEWING COORDINATES

- Second, apply rotations to align the xv, yv and zv axes with the world xw, yw and zw axes, respectively.



TRANSFORMATION FROM WORLD TO VIEWING COORDINATES

- If the view reference point is specified at word position $(\boldsymbol{x_0}, \boldsymbol{y_0}, \boldsymbol{z_0})$, this point is translated to the world origin with the translation matrix $\boldsymbol{T}$.

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & -x_0 \\ 0 & 1 & 0 & -y_0 \\ 0 & 0 & 1 & -z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

TRANSFORMATION FROM WORLD TO VIEWING COORDINATES

- The rotation sequence requires 3 coordinate-axis transformation depending on the direction of N.

- First we rotate around xw-axis to bring zv into the xw -zw plane.

$$\mathbf{R}_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & Cos\theta & -Sin\theta & 0 \\ 0 & Sin\theta & Cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

TRANSFORMATION FROM WORLD TO VIEWING COORDINATES

- Then, we rotate around the world yw axis to align the zw and zv axes.

$$\mathbf{R}_y = \begin{bmatrix} Cos\alpha & 0 & Sin\alpha & 0 \\ 0 & 1 & 0 & 0 \\ -Sin\alpha & 0 & Cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

TRANSFORMATION FROM WORLD TO VIEWING COORDINATES

- The final rotation is about the world zw axis to align the yw and yv axes.

$$\mathbf{R}_z = \begin{bmatrix} Cos\beta & -Sin\beta & 0 & 0 \\ Sin\beta & Cos\beta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

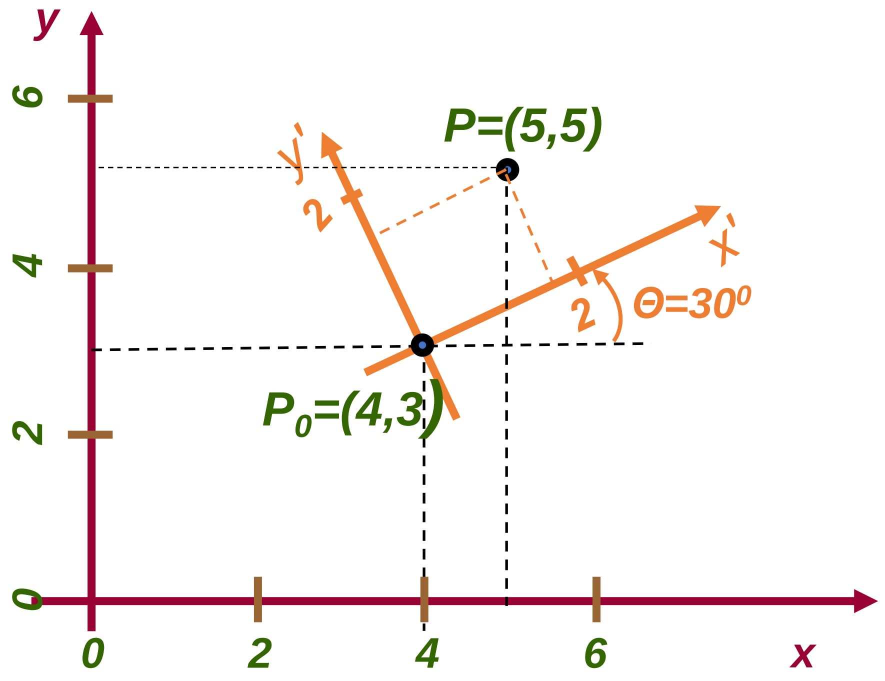
TRANSFORMATION FROM WORLD TO VIEWING COORDINATES

- The complete transformation from world to viewing coordinate transformation matrix is obtaine as the matrix product

$$\mathbf{M}_{wc,vc} = \mathbf{R}_z \cdot \mathbf{R}_y \cdot \mathbf{R}_x \cdot \mathbf{T}$$

What if the rotation angles are not given ?

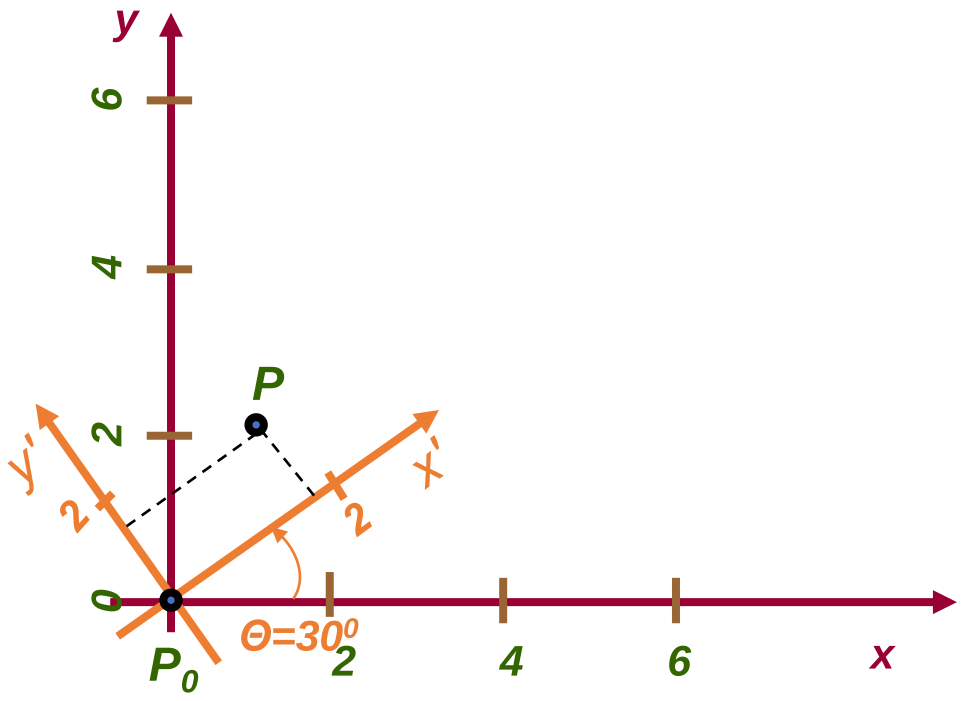
TRANSFORMATION FROM WORLD TO VIEWING COORDINATES: AN EXAMPLE FOR 2D SYSTEM



TRANSFORMATION FROM WORLD TO VIEWING COORDINATES: AN EXAMPLE FOR 2D SYSTEM

Translation:

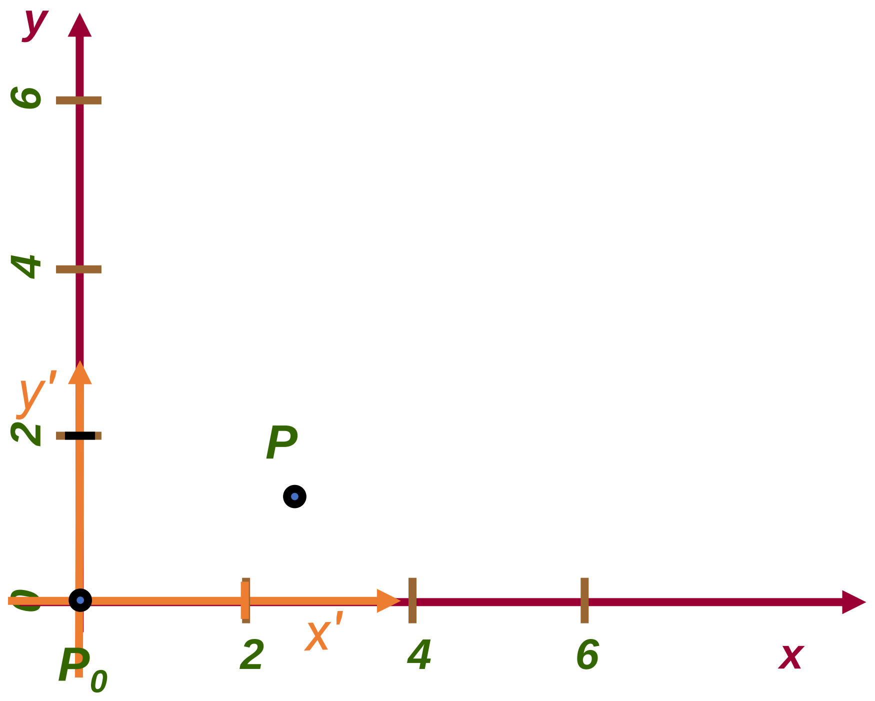
$$\mathbf{T} = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$



TRANSFORMATION FROM WORLD TO VIEWING COORDINATES: AN EXAMPLE FOR 2D SYSTEM

Rotation

$$\mathbf{R} = \begin{bmatrix} 0.866 & 0.5 & 0 \\ -0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



TRANSFORMATION FROM WORLD TO VIEWING COORDINATES: AN EXAMPLE FOR 2D SYSTEM

New coordinates

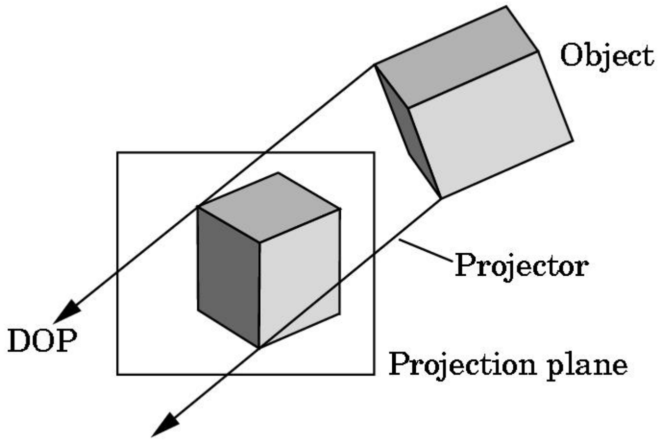
$$\mathbf{M}_{wc.vc} = \begin{bmatrix} 0.866 & 0.5 & 0 \\ -0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

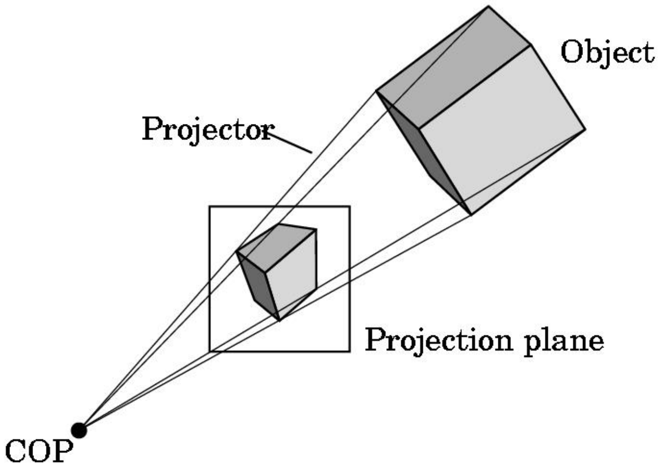
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 0.866 & 0.500 & -4.964 \\ -0.500 & 0.866 & -0.598 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.866 \\ 1.232 \\ 1 \end{bmatrix}$$

PROJECTIONS

- Once WC description of the objects in a scene are converted to VC we can project the 3D objects onto 2D view-plane.

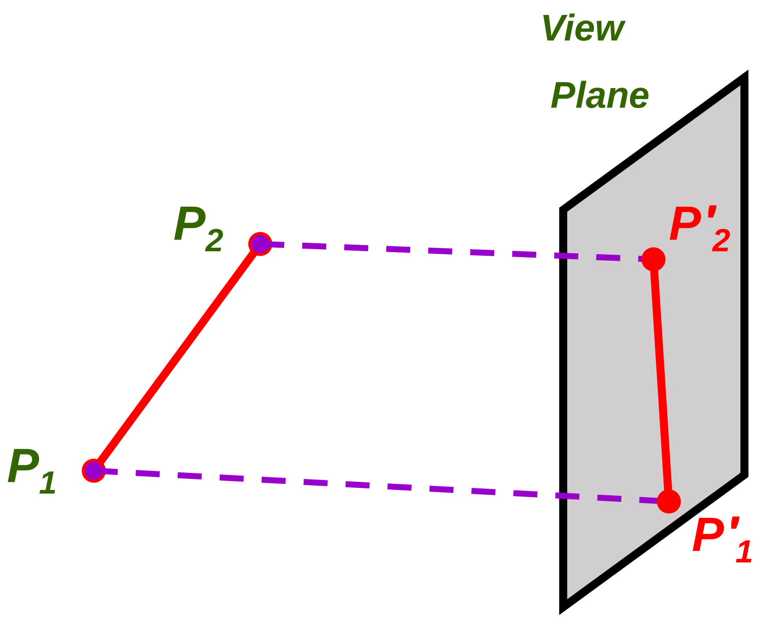
- Two types of projections:
  - Parallel Projection
  - Perspective Projection





PARALLEL PROJECTIONS

- Coordinate Positions are transformed to the view plane along parallel lines.



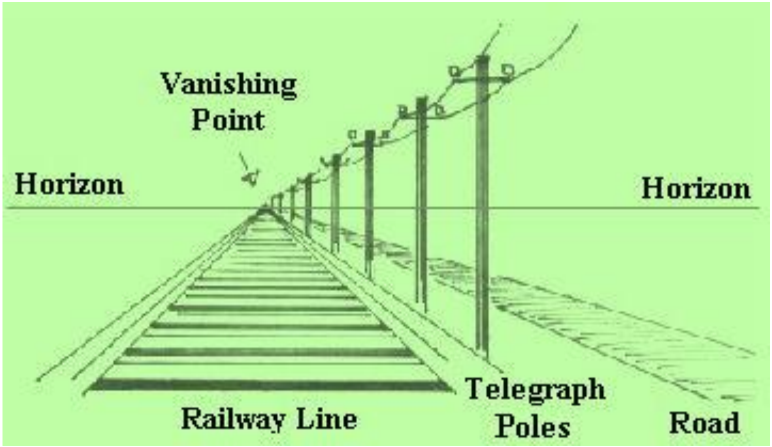
PARALLEL PROJECTIONS

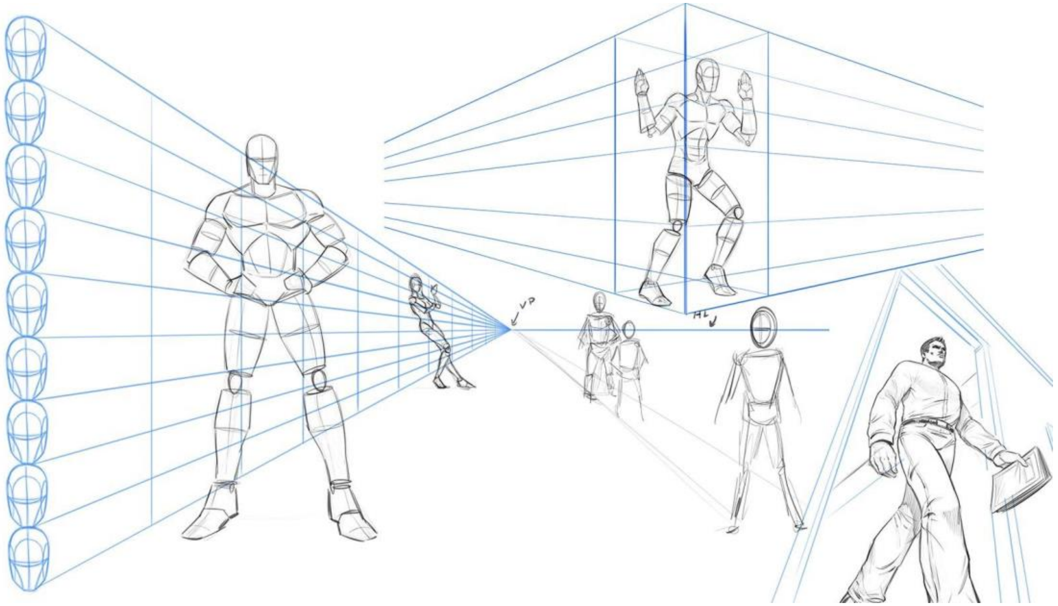
- Orthographic parallel projection
  - The projection is perpendicular to the view plane.
  - This results in a straightforward mapping of 3D coordinates to 2D coordinates, where the depth (distance along the view direction) is not considered.
- Oblique parallel projecion
  - The parallel projection is not perpendicular to the view plane.
  - **Isometric projection** is a specific form where the projection lines are equally spaced and oriented at specific angles (usually 120 degrees).

PERSPECTIVE PROJECTION

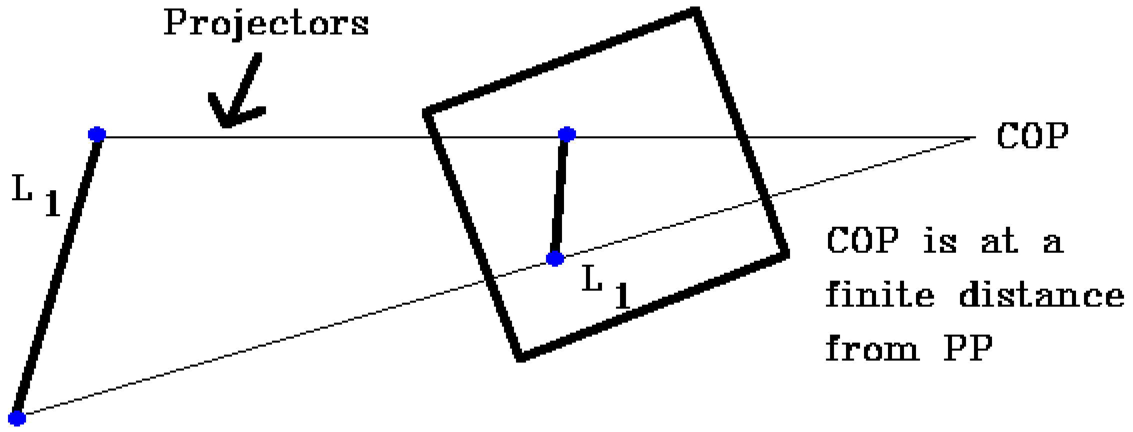
PERSPECTIVE PROJECTION

- Distance from COP to the projection plane is finite
- The projectors are not parallel
- We specify a center of projection (COP)
- Also known as perspective reference point (PRP)
- Perspective foreshortening (illusion)
- Vanishing point

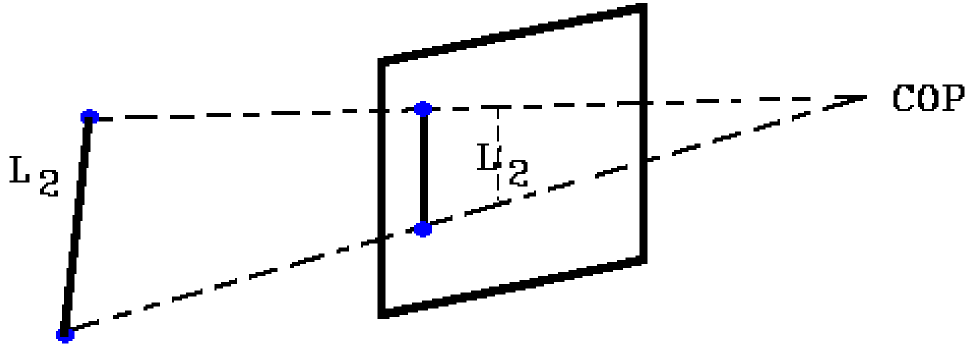




PERSPECTIVE FORESHORTENING

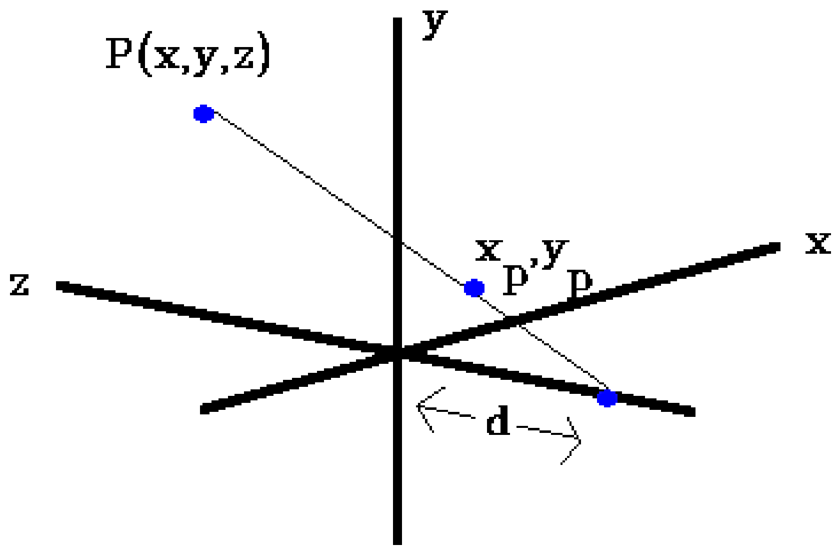


The Perspective viewing projection has a Center of Projection ("eye")



At a finite distance from the projection plane (PP).

COMPUTING THE PERSPECTIVE PROJECTION

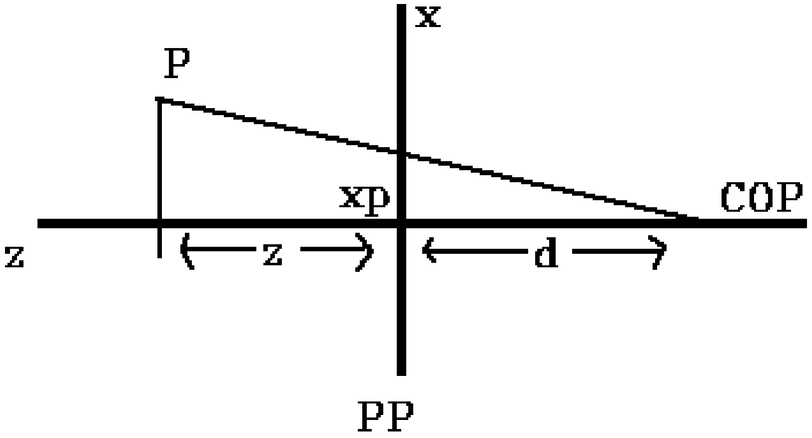


$$x_p = \frac{x}{\frac{z}{d} + 1}$$

$$y_p = \frac{y}{\frac{z}{d} + 1}$$

$$z_p = 0$$

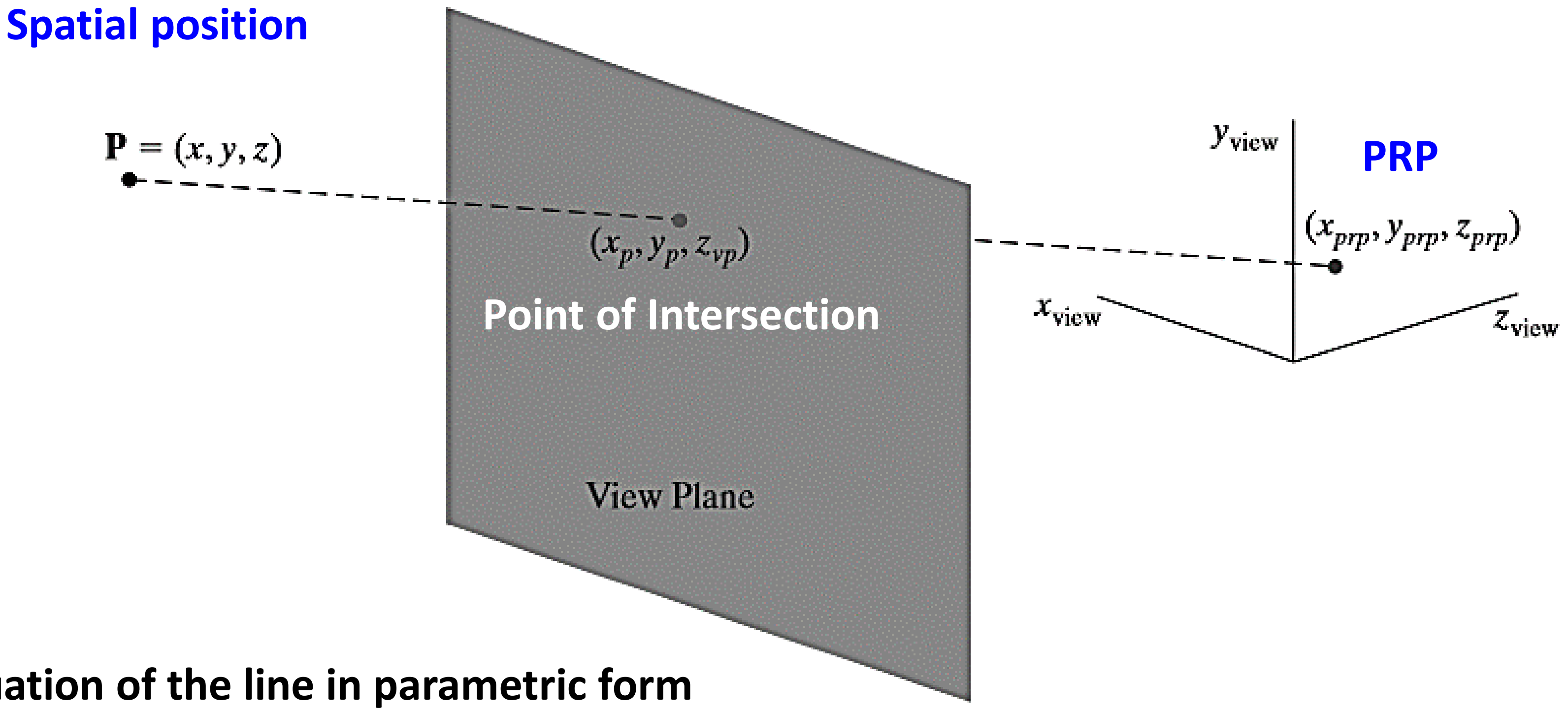
Look at above diagram from y axis



In homogeneous representation

$$\begin{bmatrix} x_h \\ y_h \\ z_h \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{d} & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

GENERALIZED PRP

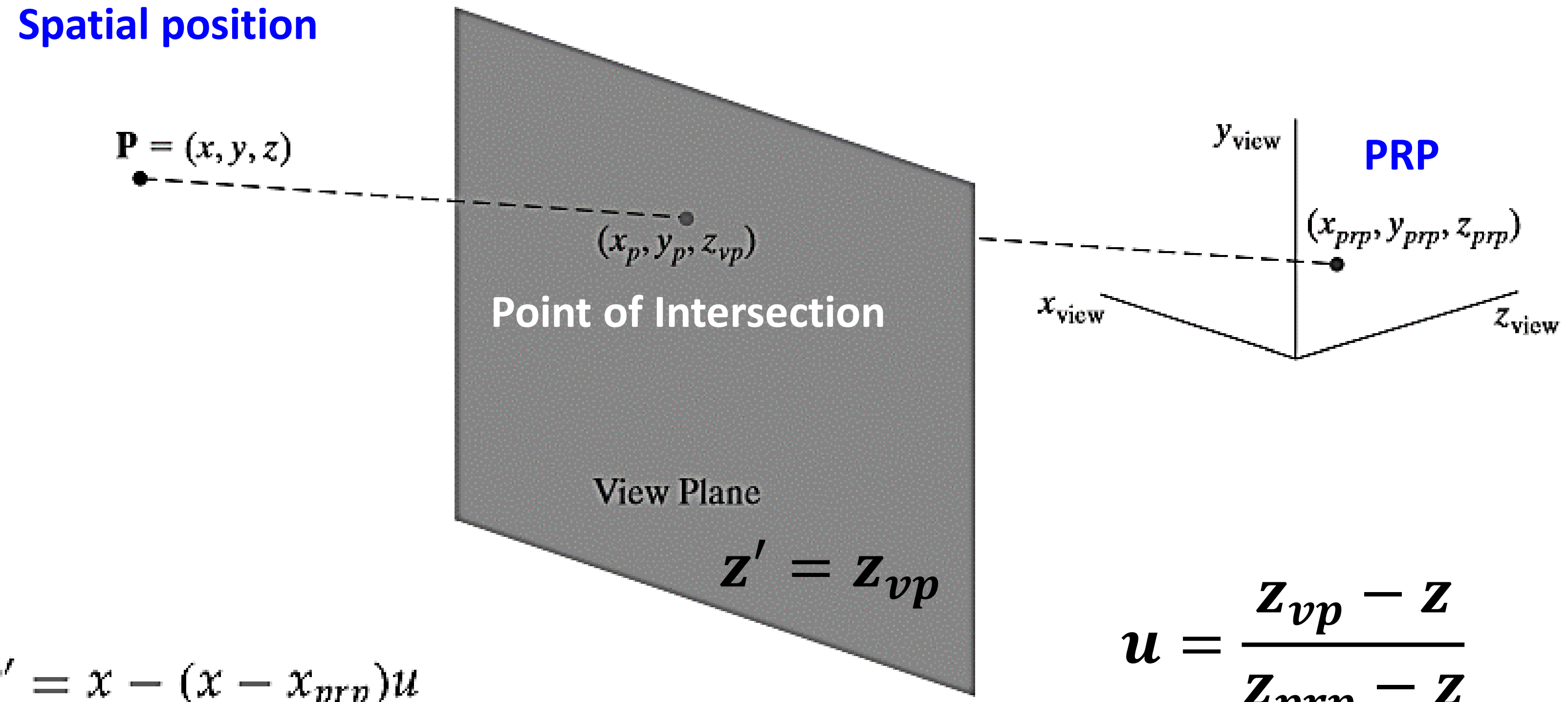


Equation of the line in parametric form

$If\ x', y', z'$ any point on the along the projection line

$$x' = x - (x - x_{prp})u$$
$$y' = y - (y - y_{prp})u \qquad 0 \leq u \leq 1$$
$$z' = z - (z - z_{prp})u$$

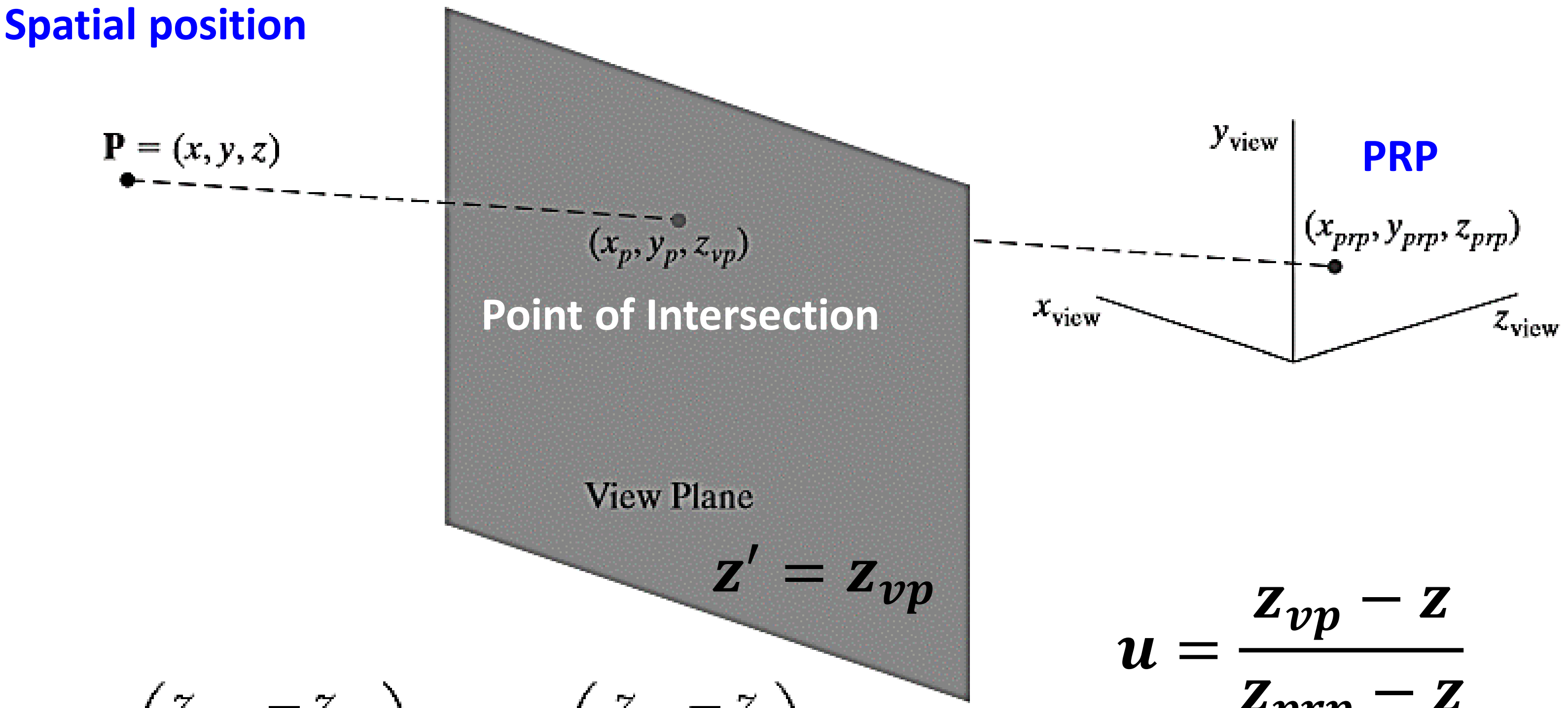
GENERALIZED PRP



$$u = \frac{z_{vp} - z}{z_{prp} - z}$$

$$x' = x - (x - x_{prp})u$$
$$y' = y - (y - y_{prp})u \qquad 0 \le u \le 1$$
$$z' = z - (z - z_{prp})u$$

GENERALIZED PRP



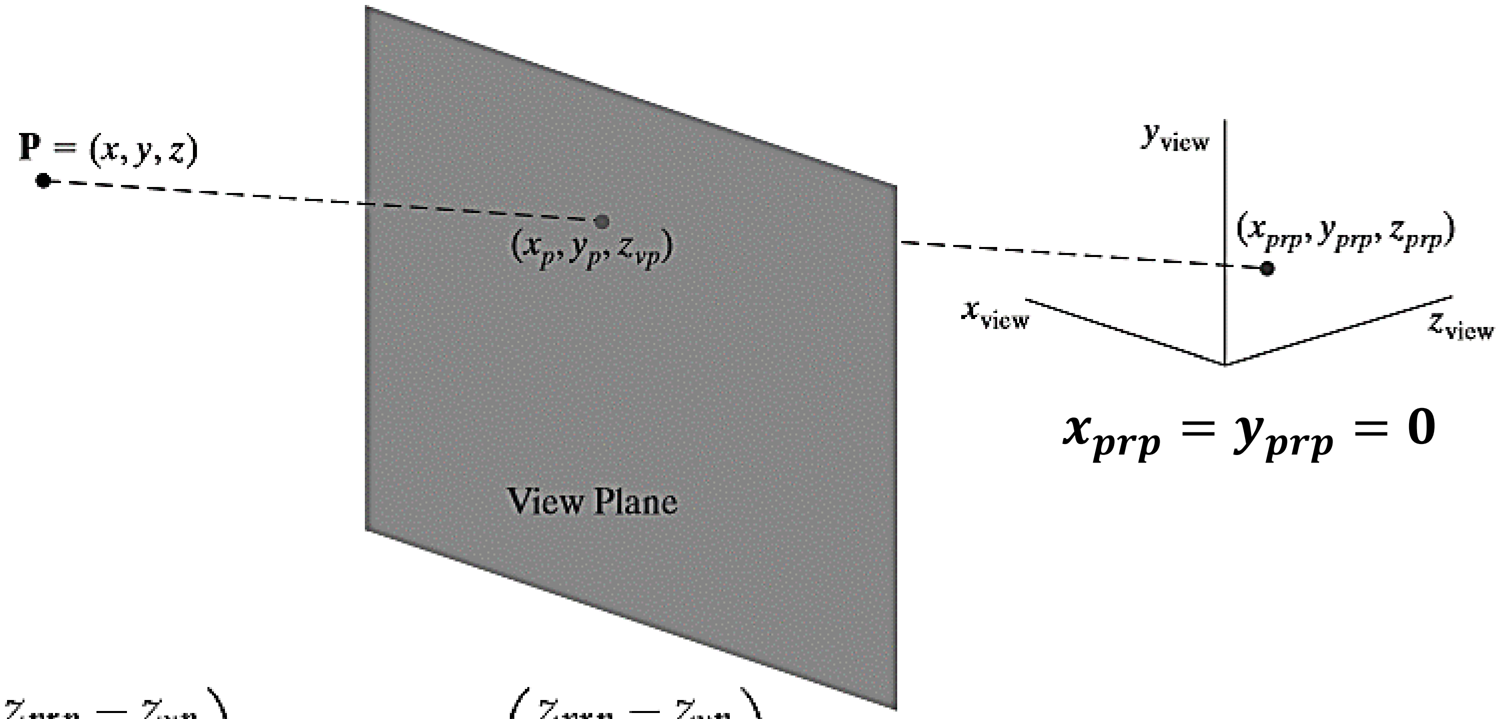
$$u = \frac{z_{vp} - z}{z_{prp} - z}$$

$$x_p = x\left(\frac{z_{prp} - z_{vp}}{z_{prp} - z}\right) + x_{prp}\left(\frac{z_{vp} - z}{z_{prp} - z}\right)$$

$$y_p = y\left(\frac{z_{prp} - z_{vp}}{z_{prp} - z}\right) + y_{prp}\left(\frac{z_{vp} - z}{z_{prp} - z}\right)$$

SPECIAL CASES

- Case 1: Projection reference point along z_{view} axis

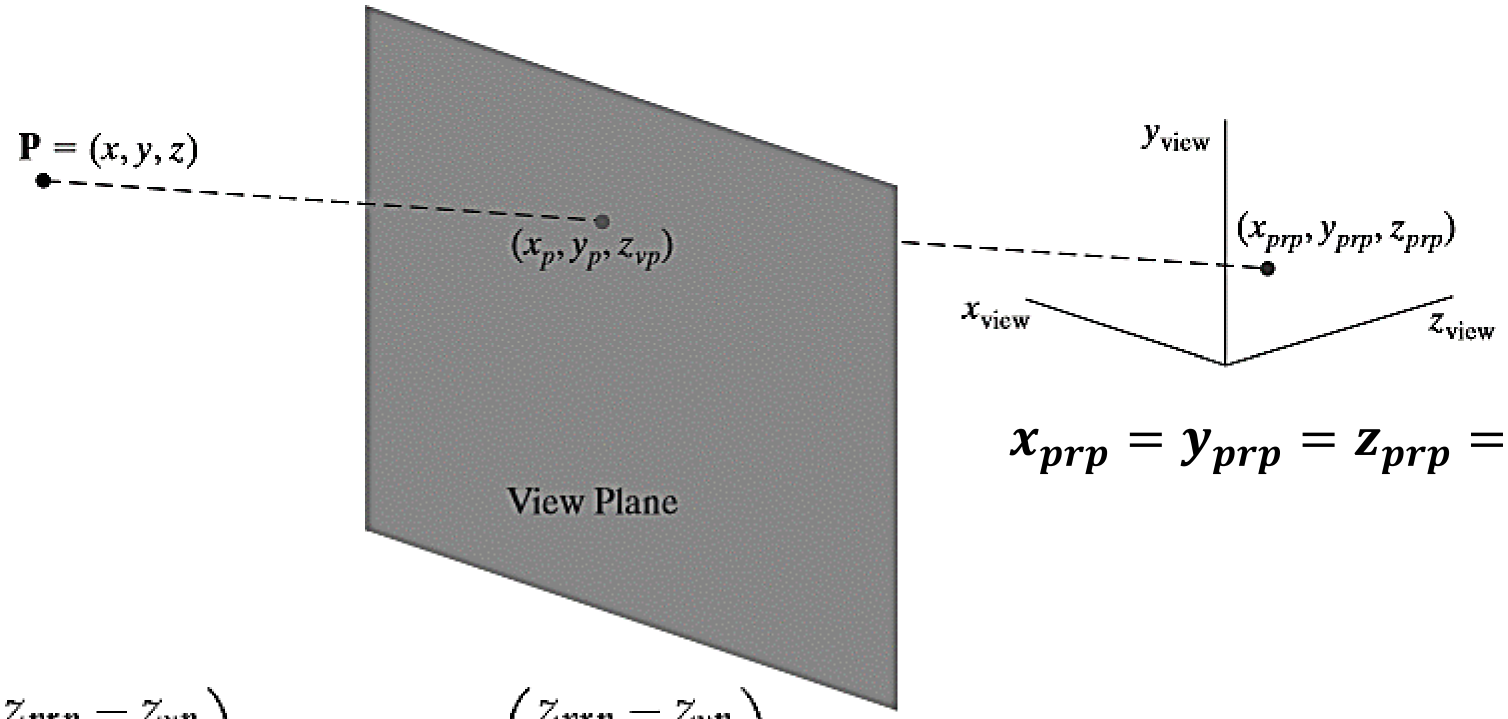


$$x_{prp} = y_{prp} = 0$$

$$x_p = x\left(\frac{z_{prp} - z_{vp}}{z_{prp} - z}\right), \qquad y_p = y\left(\frac{z_{prp} - z_{vp}}{z_{prp} - z}\right)$$

SPECIAL CASES

- Case 2: Projection reference point is fixed at the coordinate origin

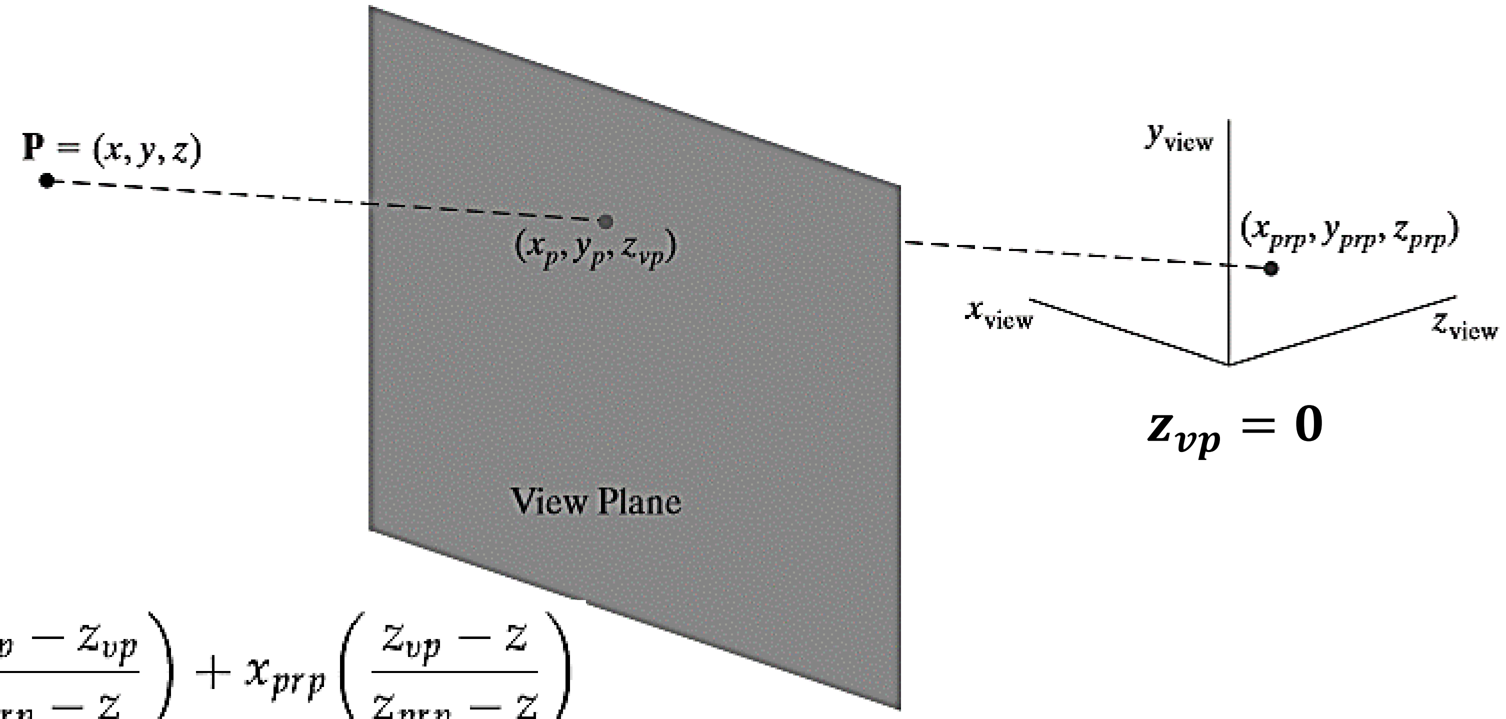


$$\boldsymbol{x_{prp} = y_{prp} = z_{prp} = 0}$$

$$x_p = x\left(\frac{z_{prp} - z_{vp}}{z_{prp} - z}\right), \qquad y_p = y\left(\frac{z_{prp} - z_{vp}}{z_{prp} - z}\right)$$

SPECIAL CASES

- Case 3: If the view plane is the uv plan



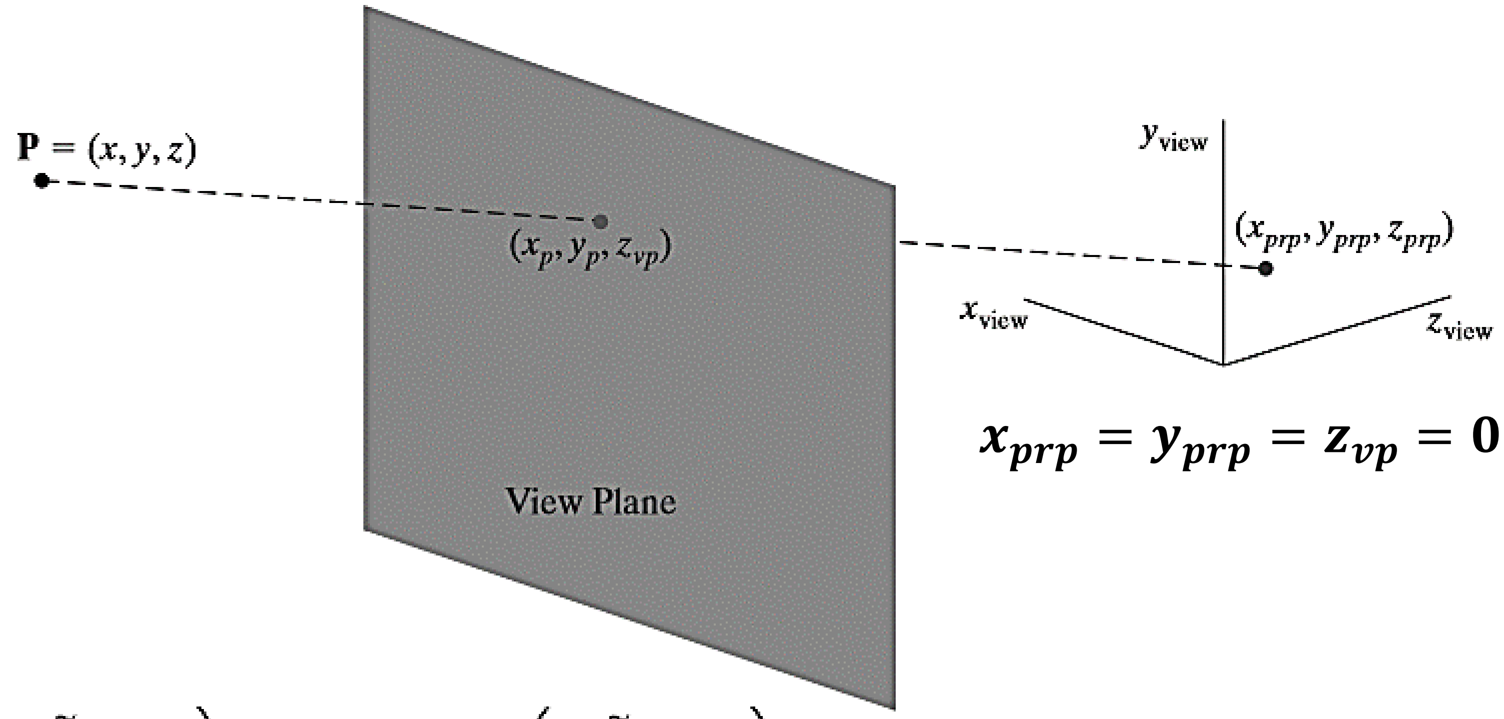
$$z_{vp} = 0$$

$$x_p = x\left(\frac{z_{prp} - z_{vp}}{z_{prp} - z}\right) + x_{prp}\left(\frac{z_{vp} - z}{z_{prp} - z}\right)$$

$$y_p = y\left(\frac{z_{prp} - z_{vp}}{z_{prp} - z}\right) + y_{prp}\left(\frac{z_{vp} - z}{z_{prp} - z}\right)$$

SPECIAL CASES

- Case 4: Case 2 + Case 4

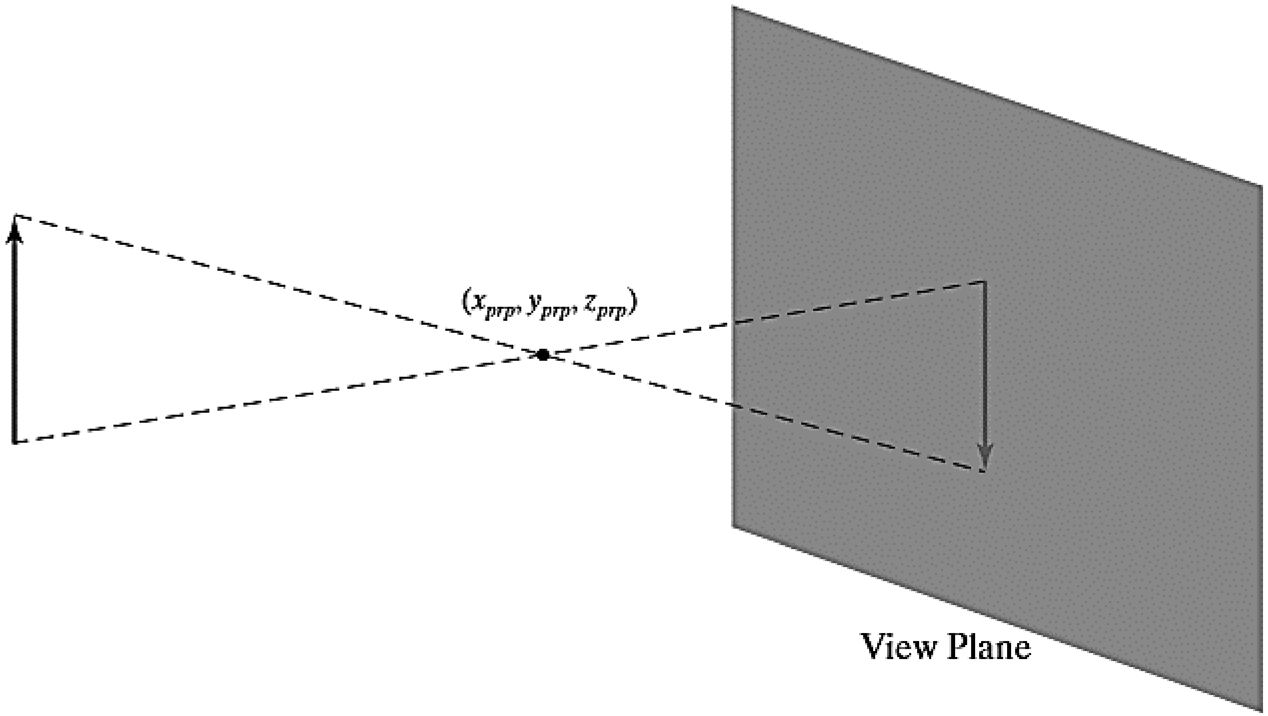


$$x_{prp} = y_{prp} = z_{vp} = 0$$

$$x_p = x\left(\frac{z_{prp}}{z_{prp} - z}\right), \qquad y_p = y\left(\frac{z_{prp}}{z_{prp} - z}\right)$$

POSITIONING THE PRP AND VP

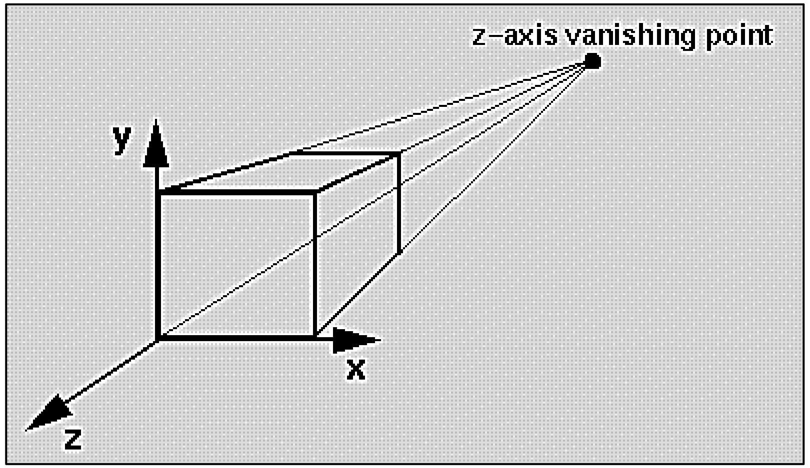
The view plane could be placed anywhere except at the projection point

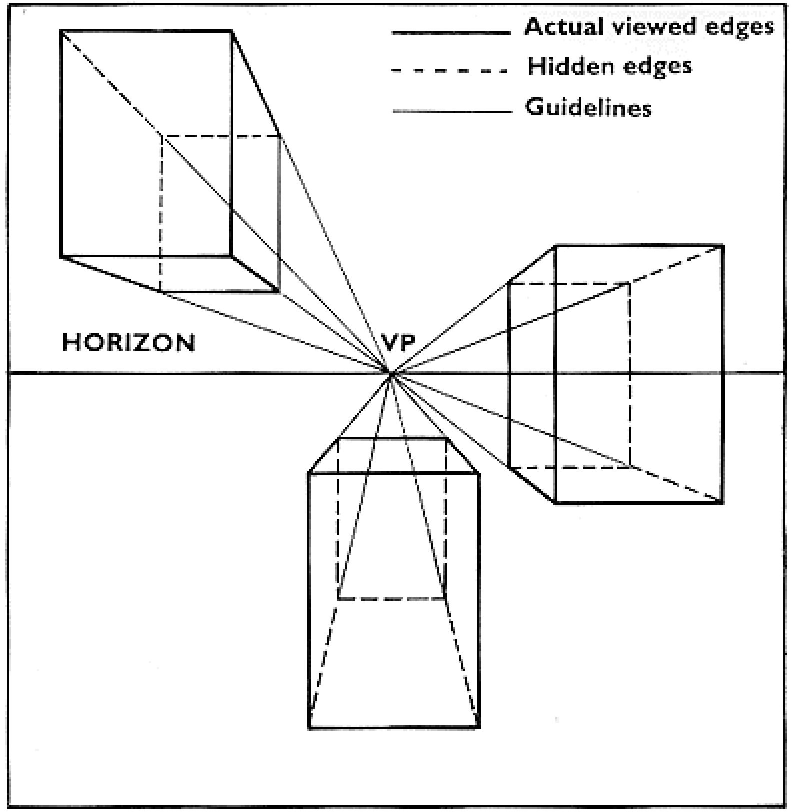


VANISHING POINT

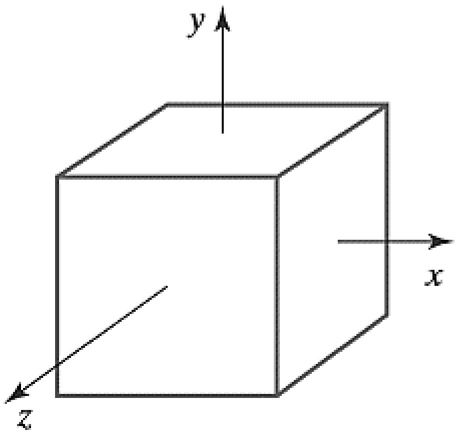
- A scene is projected onto a view plane using a perspective mapping
- Lines parallel to the view plane remains parallel
- Parallel lines in the scene that are not parallel to the view plane are projected into converging lines
- There point of convergence is called the vanishing point

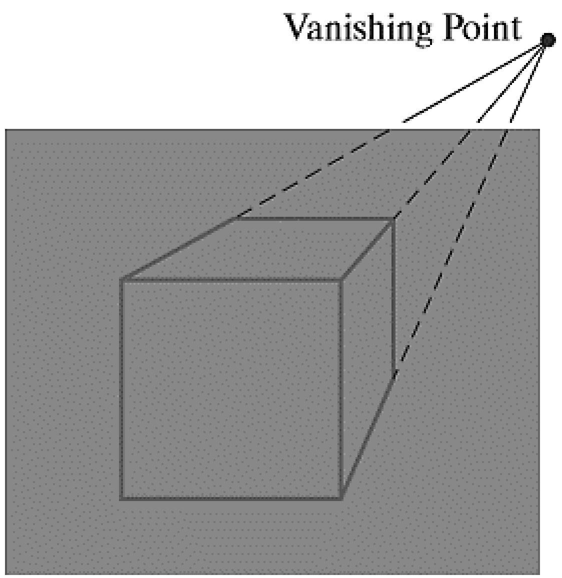
Z-AXIS VANISHING POINT



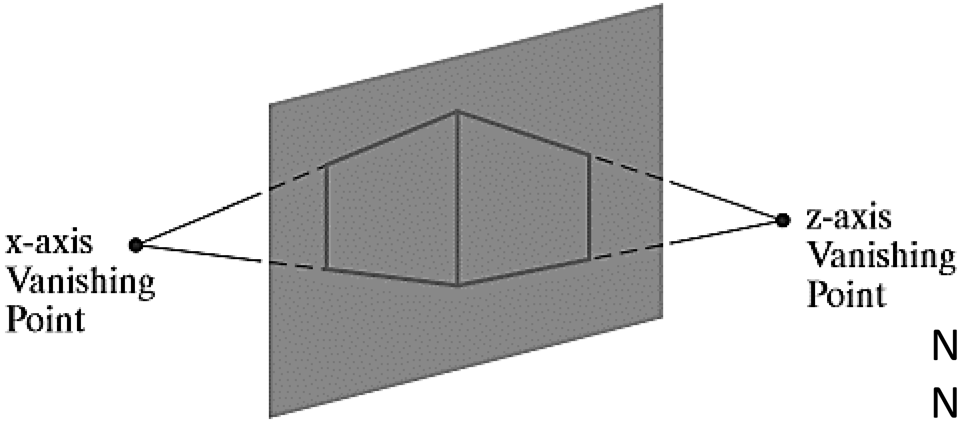


ILLUSTRATIONS



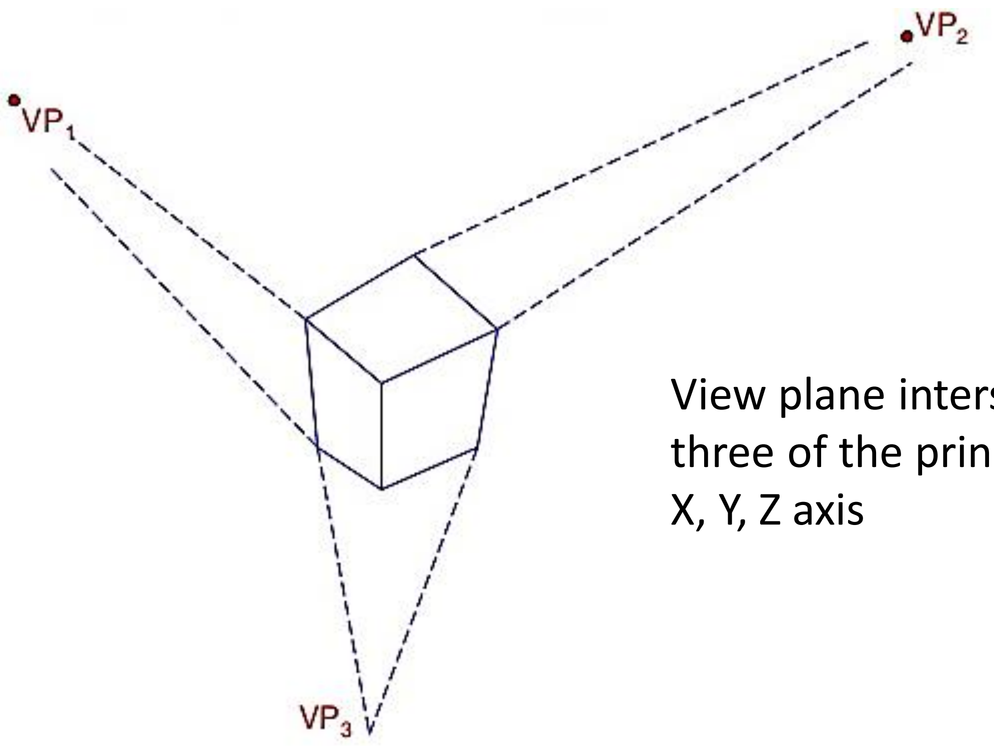


one-point, two-point, three-point projections



Number of principal vanishing points = Number of principal axes that intersect the view plane.

THREE POINT PERSPECTIVE PROJECTION

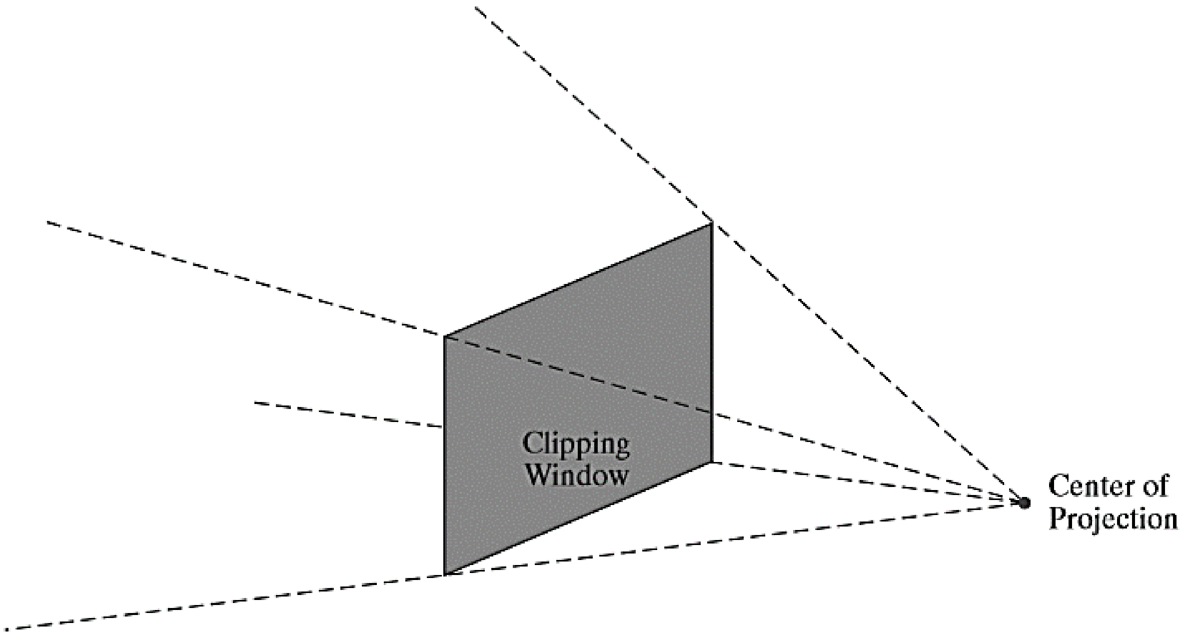


View plane intersects all three of the principal axis X, Y, Z axis

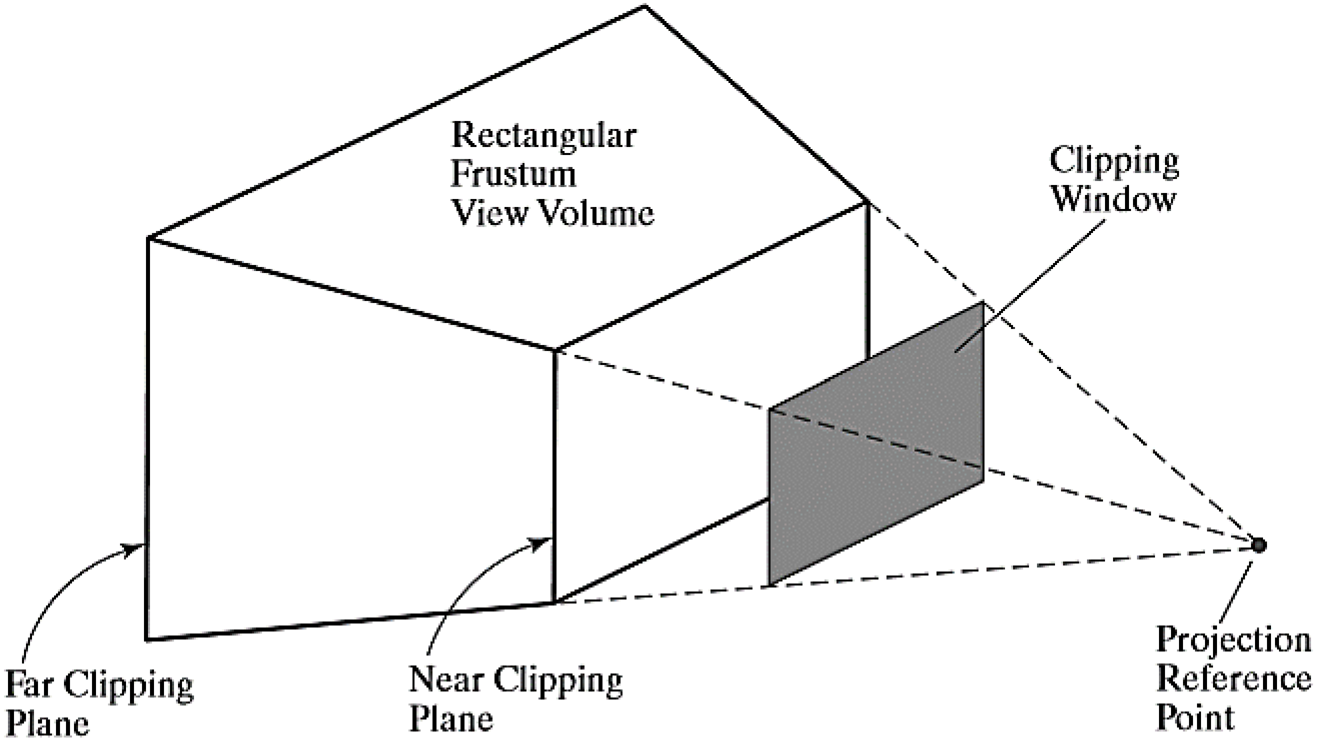
PERSPECTIVE-PROJECTION VIEW VOLUME

- Specifying the position of a rectangular clipping window on the view plane

- Forms a view volume that is an infinite rectangular pyramid with its apex at the center of projection

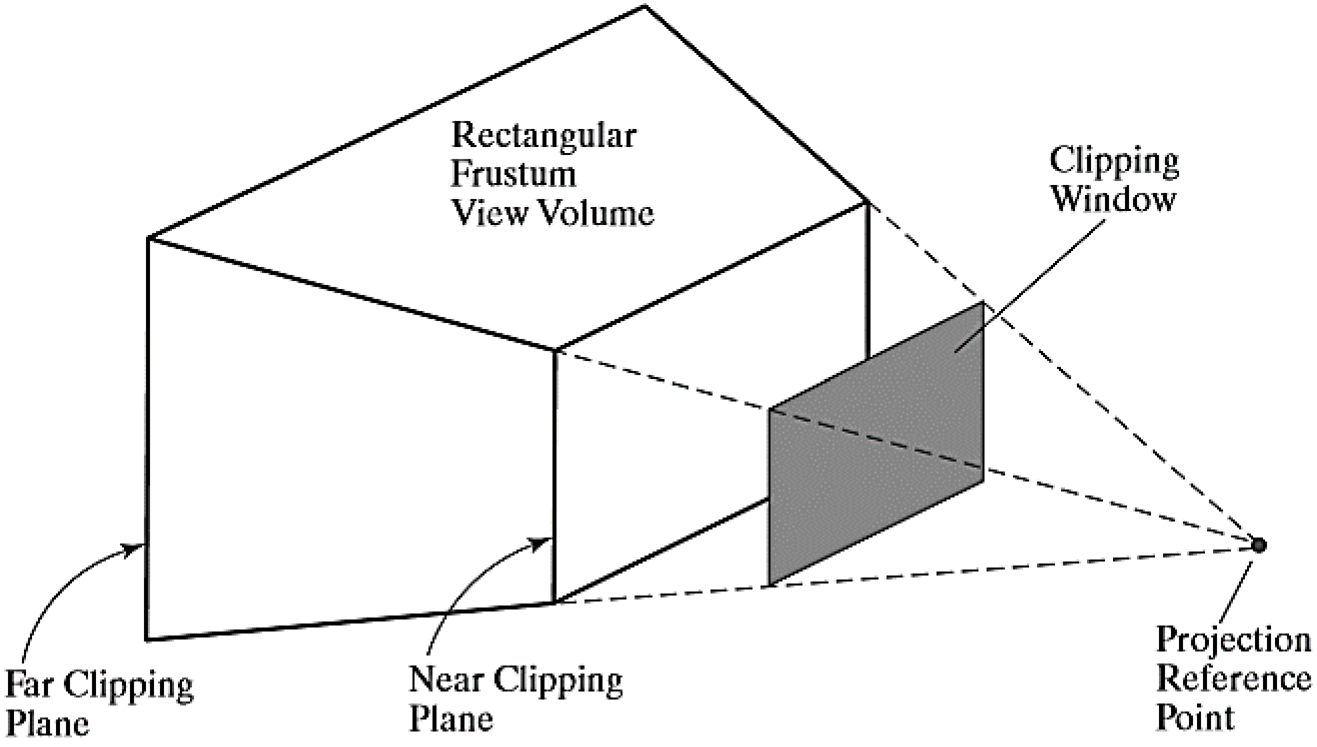


FRUSTUM



Rectangular
Frustum
View Volume
Clipping
Window
Far Clipping
Plane
Near Clipping
Plane
Projection
Reference
Point

FRUSTUM



Rectangular
Frustum
View Volume
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Far Clipping
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Near Clipping
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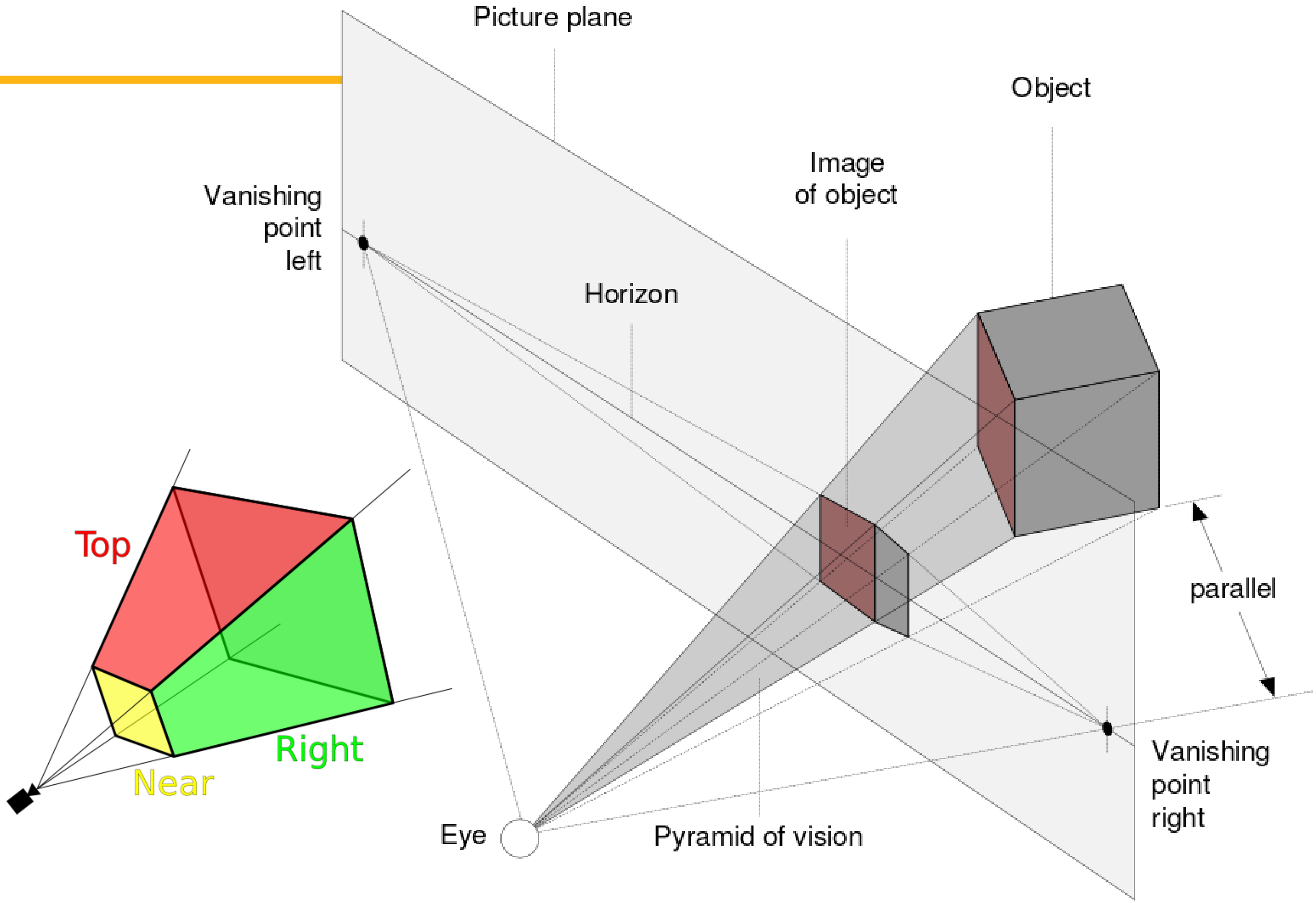
Point

FRUSTUM



Picture plane
Object
Image
of object
Vanishing
point
left
Horizon
Top
parallel
Right
Near
Vanishing
point
right
Eye
Pyramid of vision

PERSPECTIVE-PROJECTION TRANSFORMATION MATRIX

- Homogeneous-coordinate representation to express the perspective-projection equations in the form

$$x_p = \frac{x_h}{h}, \qquad y_p = \frac{y_h}{h} \qquad \text{where} \qquad h = z_{prp} - z$$

We've already derived the expressions for x and y

$$x_p = x\left(\frac{z_{prp} - z_{vp}}{z_{prp} - z}\right) + x_{prp}\left(\frac{z_{vp} - z}{z_{prp} - z}\right)$$

$$y_p = y\left(\frac{z_{prp} - z_{vp}}{z_{prp} - z}\right) + y_{prp}\left(\frac{z_{vp} - z}{z_{prp} - z}\right)$$

Homogeneous coordinate representation is given by

$$x_h = x(z_{prp} - z_{vp}) + x_{prp}(z_{vp} - z)$$

$$y_h = y(z_{prp} - z_{vp}) + y_{prp}(z_{vp} - z)$$

PERSPECTIVE-PROJECTION TRANSFORMATION MATRIX

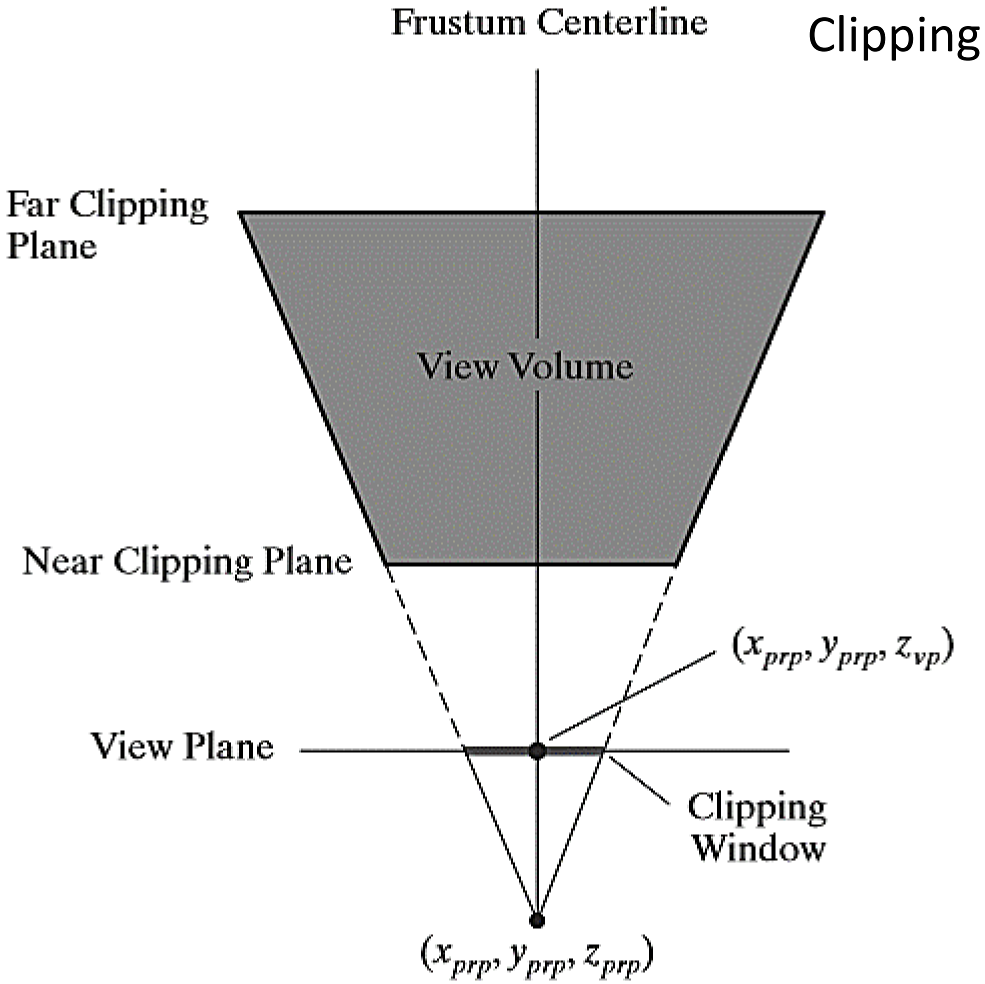
- Homogeneous coordinates using the perspective-transformation matrix can be written as:

$$\mathbf{P}_h = \mathbf{M}_{\text{pers}} \cdot \mathbf{P}$$

$$\mathbf{M}_{\text{pers}} = \begin{bmatrix} z_{prp} - z_{vp} & 0 & -x_{prp} & x_{prp} z_{prp} \\ 0 & z_{prp} - z_{vp} & -y_{prp} & y_{prp} z_{prp} \\ 0 & 0 & s_z & t_z \\ 0 & 0 & -1 & z_{prp} \end{bmatrix}$$

- Frustum view volume can have any orientation
- How to handle that?

SYMMETRIC PERSPECTIVE-PROJECTION FRUSTUM



Clipping window can be represented as:

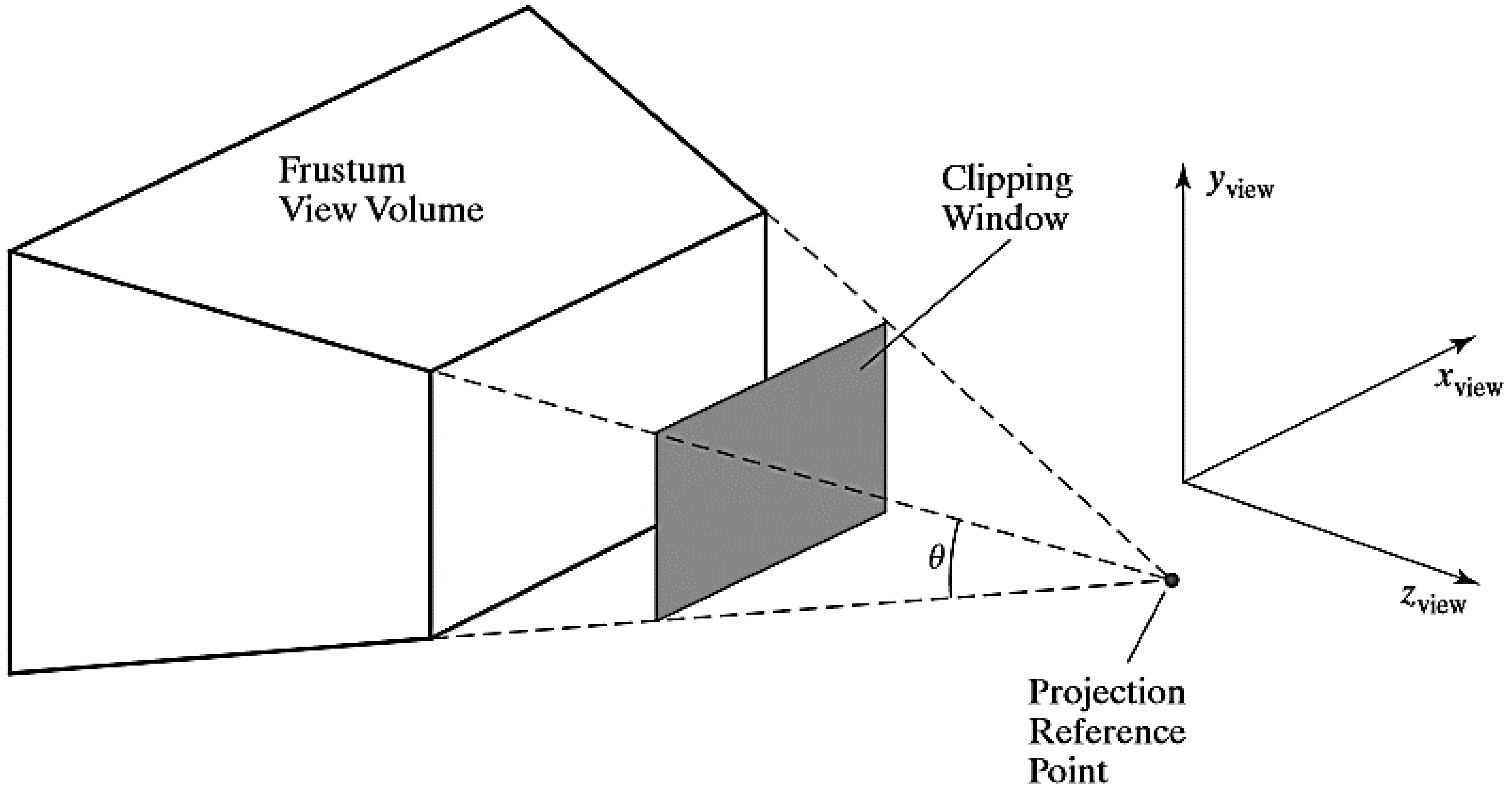
$$xw_{\min} = x_{prp} - \frac{\text{width}}{2},$$

$$yw_{\min} = y_{prp} - \frac{\text{height}}{2},$$

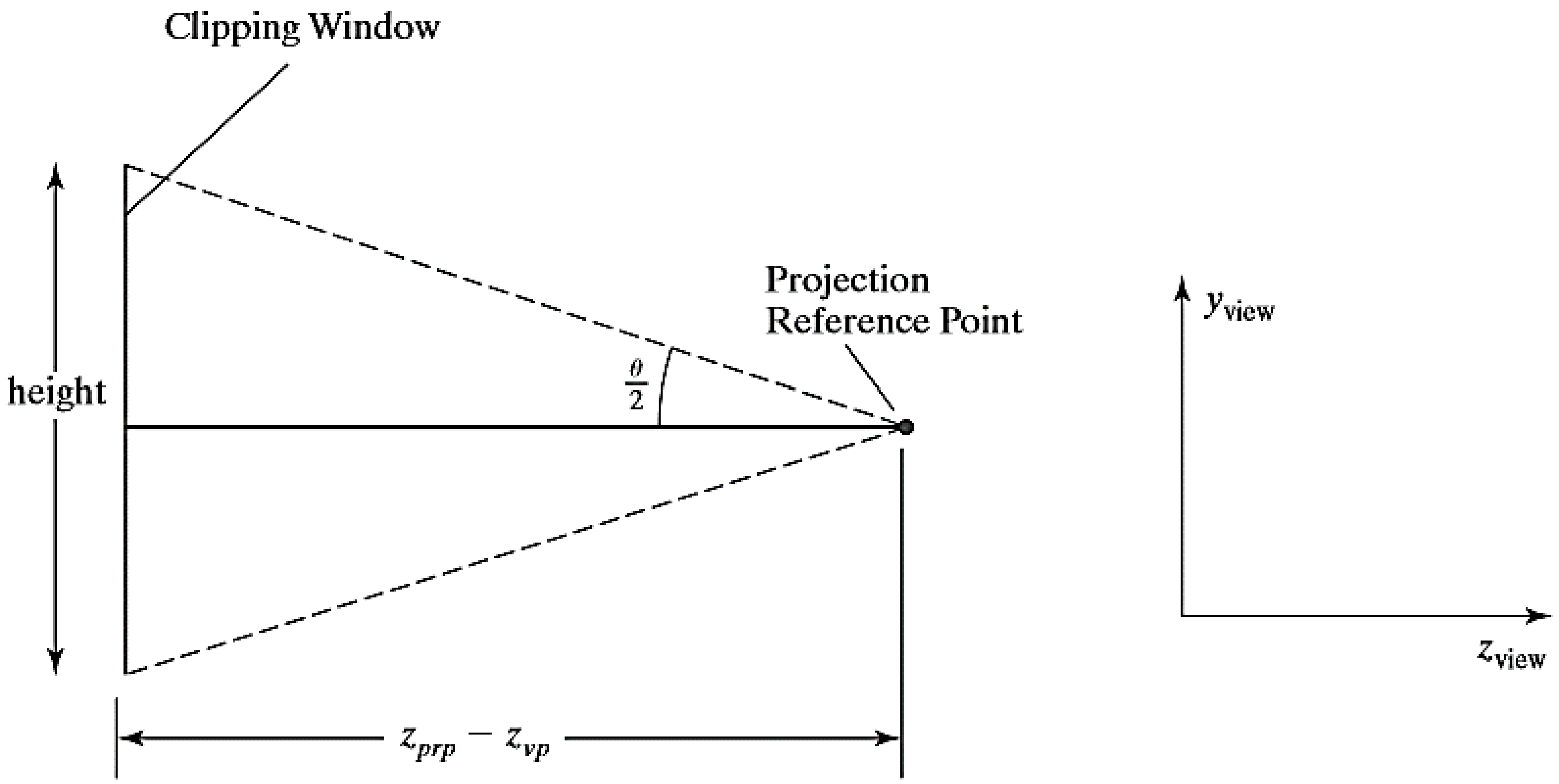
$$xw_{\max} = x_{prp} + \frac{\text{width}}{2}$$

$$yw_{\max} = y_{prp} + \frac{\text{height}}{2}$$

FIELD OF VIEW ANGLE

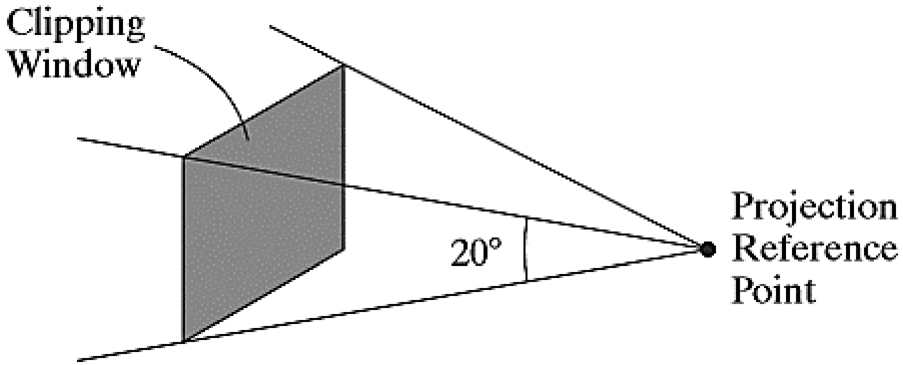


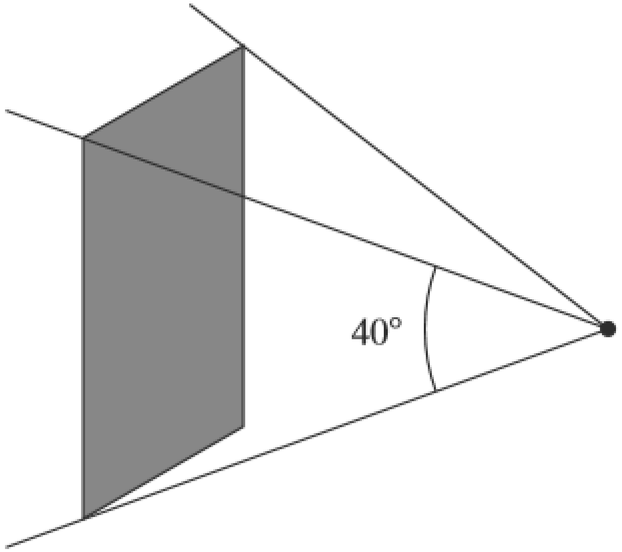
CLIPPING W.R.TO FIELD OF VIEW

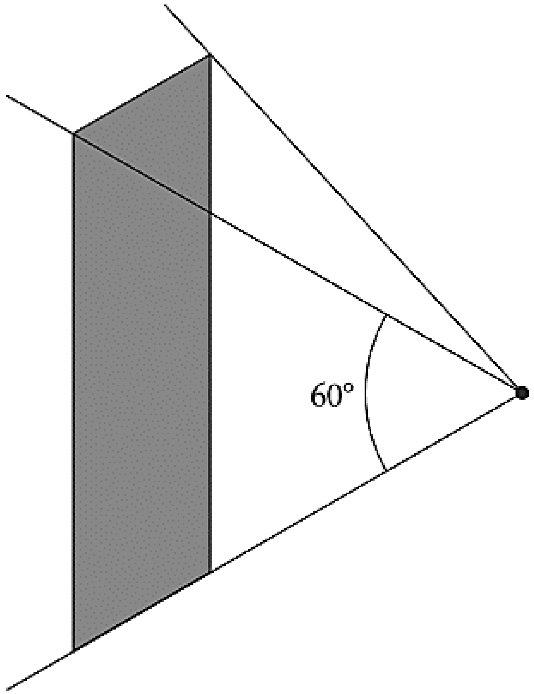


$$height = 2(z_{prp} - z_{vp}) \tan\left(\frac{\theta}{2}\right)$$

EFFECT OF FIELD OF VIEW ANGLE

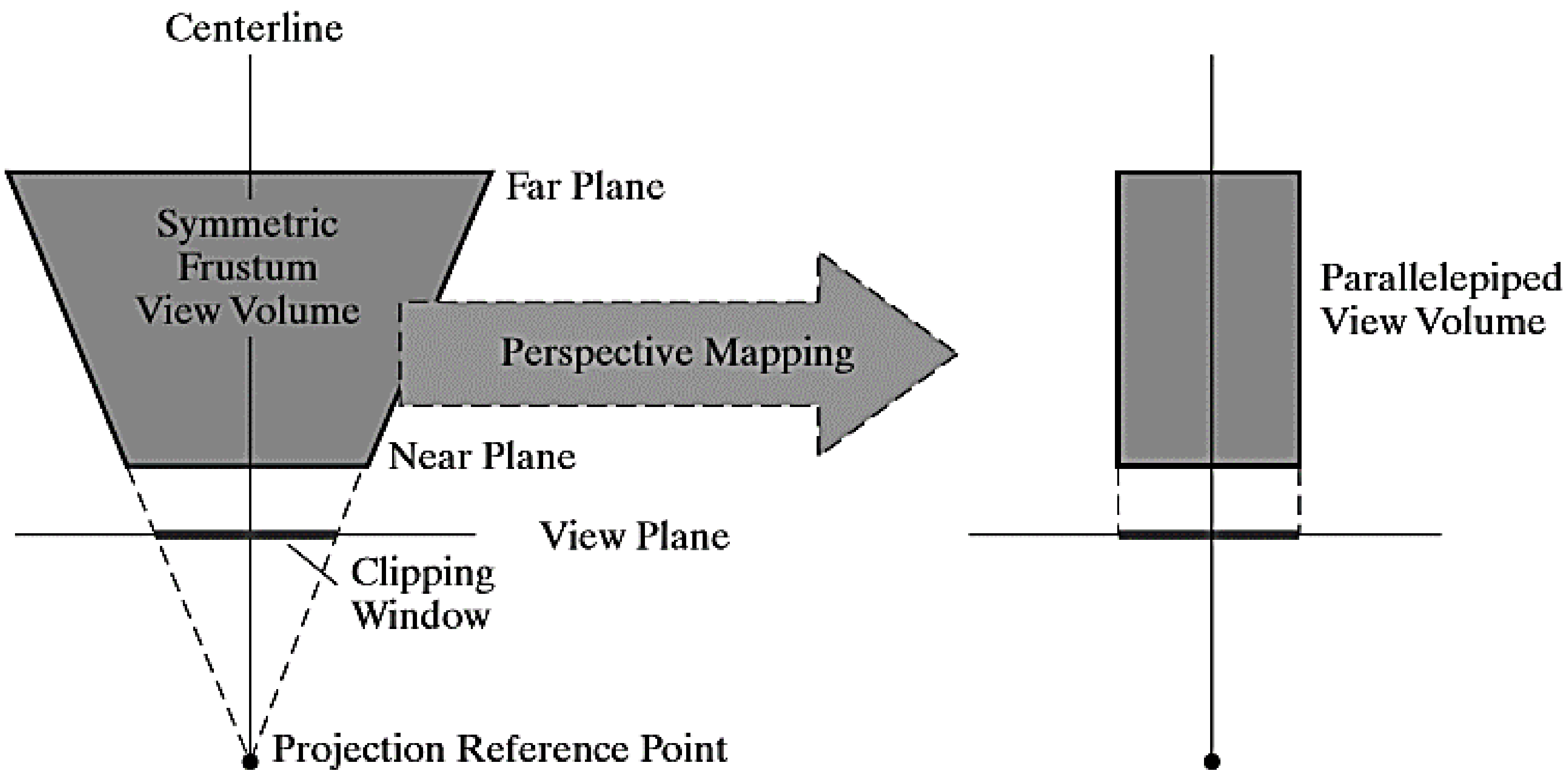






Increasing the size of the field-of-view angle increases the height of the clipping window and ***increases the perspective-projection foreshortening.***

MAPPING OF POINTS WITHIN THE FRUSTUM



OBLIQUE → SYMMETRIC FRUSTUM

- Oblique perspective-projection matrix for converting coordinate positions in a scene to homogeneous orthogonal projection coordinates

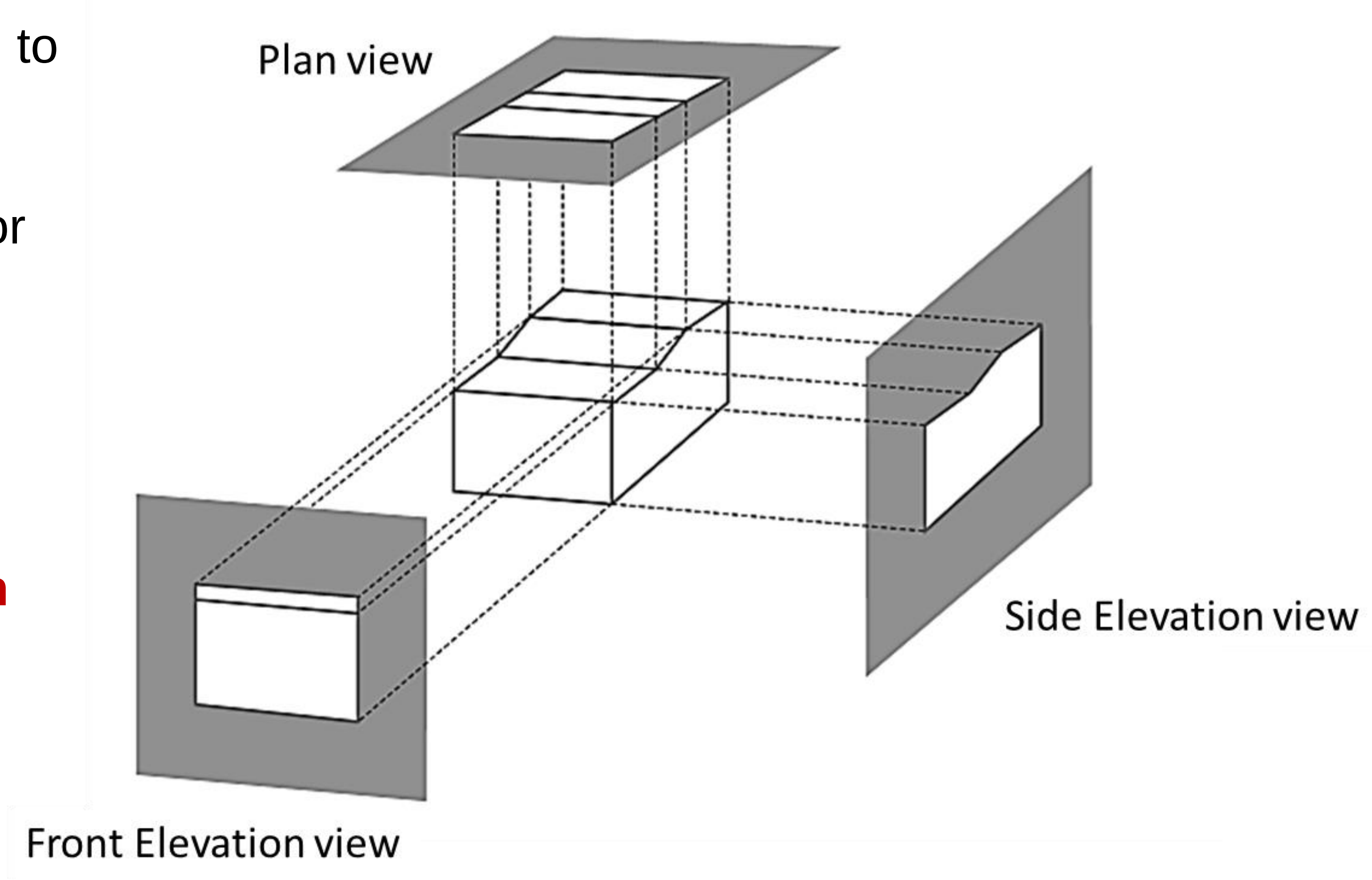
$$\mathbf{M}_{\text{obliquepers}} = \mathbf{M}_{\text{pers}} \cdot \mathbf{M}_{z\,\text{shear}}$$
$$= \begin{bmatrix} -z_{\text{near}} & 0 & \dfrac{xw_{\min} + xw_{\max}}{2} & 0 \\ 0 & -z_{\text{near}} & \dfrac{yw_{\min} + yw_{\max}}{2} & 0 \\ 0 & 0 & s_z & t_z \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

ORTHOGONAL PROJECTIONS

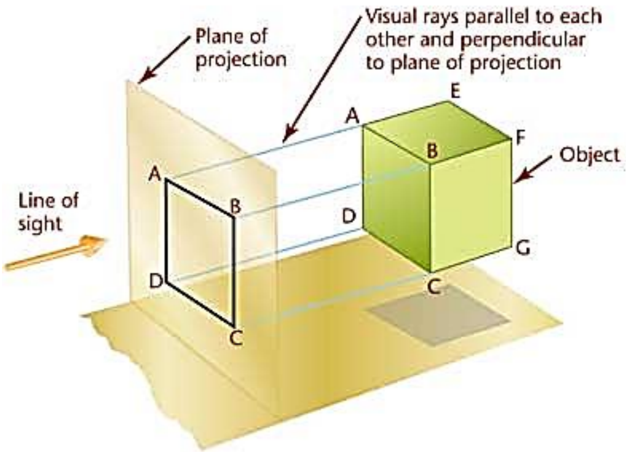
ORTHOGONAL PROJECTION

- A transformation of object descriptions to a view plane along lines that are all parallel to the view-plane normal vector **N** is called an **orthogonal projection**

- This produces a parallel-projection transformation in which the **projection lines are perpendicular to the view plane**.

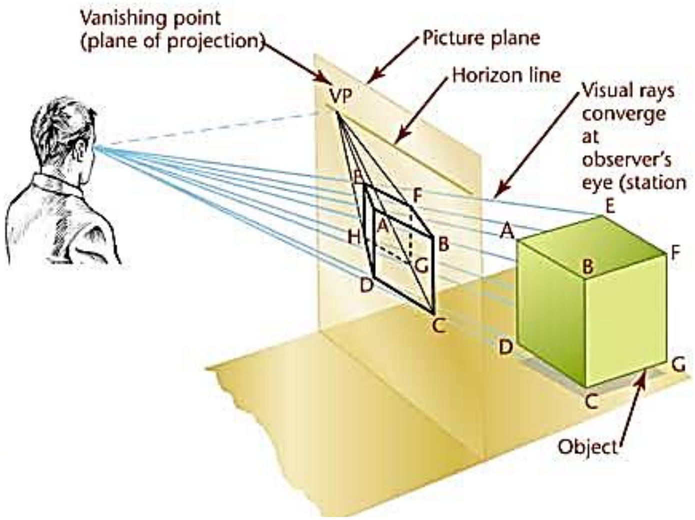


PROJECTION



Multiview projection

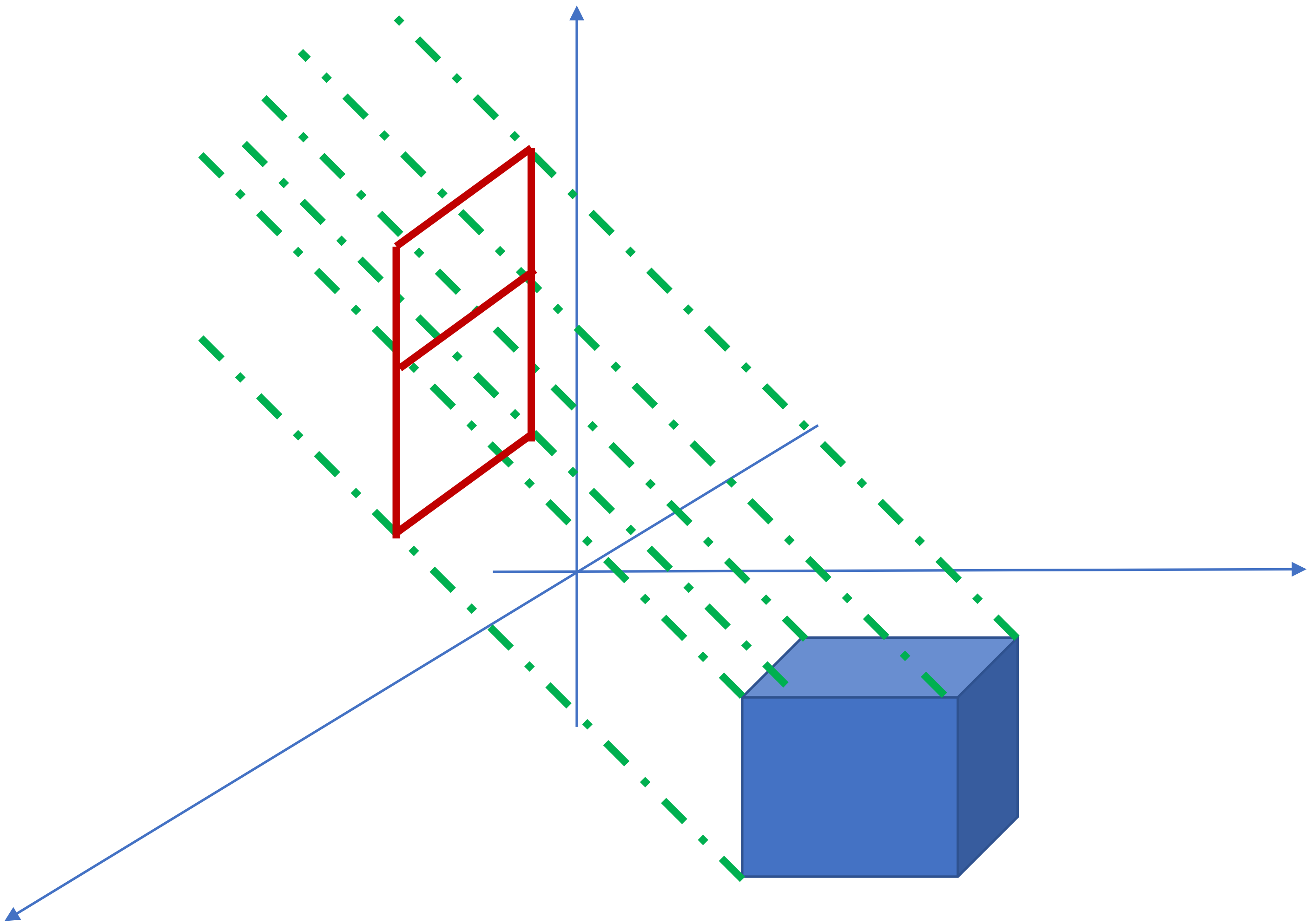
Axonometric



Oblique

Perspective

PARALLEL PROJECTION



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