

Question - 2.

1. entropy $H(Y) = - \sum_{i=1}^n P(Y=Y_i) \log_2 P(Y=Y_i)$

$$P(Y=+) = \frac{3+4+4+1}{21} = \frac{12}{21}$$

$$P(Y=-) = \frac{0+1+3+5}{21} = \frac{9}{21}$$

$$H(Y) \text{ (Entropy)} = - \left(\frac{12}{21} * \log_2(12/21) + \frac{9}{21} * \log_2(9/21) \right)$$

$$= - \left(12/21 * (-0.807) + 9/21 * (-1.224) \right)$$

$$= 0.98$$

2. $P(X_1=T) = \frac{3+4+1+0}{21} = \frac{8}{21}$

$$P(X_2=T) = 1/21$$

$$P(X_1=F) = 13/21$$

$$P(X_2=F) = 1/21$$

$$P(Y=+ / X_1=T) = 7/8 \Rightarrow \left(\frac{7/21}{8/21} \right)$$

$$P(Y=+ / X_2=T) = 7/10$$

$$P(Y=+ / X_1=F) = 5/13$$

$$P(Y=+ / X_2=F) = 5/11$$

$$P(Y=- / X_1=T) = 1/8$$

$$P(Y=- / X_2=T) = 3/10$$

$$P(Y=- / X_1=F) = 8/13$$

$$P(Y=- / X_2=F) = 6/11$$

Let's calculate conditional properties.

$$H(Y|X_1) = - \sum_{i=1}^n P(X_1=x_i) \sum_{j=1}^n P(Y=y_j | X_1=x_i) \log P(Y=y_j | X_1=x_i)$$

$$\approx 0.80$$

$$H(Y|X_2) = -\frac{10}{21} \left(\frac{7}{10} \log(7/10) + \frac{3}{10} \log(3/10) \right) - \frac{11}{21} \left(\frac{5}{11} \log(5/11) + \frac{6}{11} \log(6/11) \right)$$

$$\approx 0.820$$

$$= 0.420 + 0.520$$

$$= 0.940$$

$$\approx 0.94$$

Information
gain

$$\begin{aligned} IG(X_1) &= H(Y) - H(Y|X_1) \\ &= 0.98 - 0.8 \\ &= 0.18 \end{aligned}$$

$$\begin{aligned} IG(X_2) &= H(Y) - H(Y|X_2) \\ &= 0.98 - 0.94 \\ &= 0.04. \end{aligned}$$

3.

