

# The Report on Summer Project 2018

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on

*Measurement of the surface tension and viscosity of a liquid  
using the capillary waves as diffraction grating*

Presented by

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# *Measurement of the surface tension and viscosity of a liquid using the capillary waves as diffraction grating*

Amlan Datta\*

## **Abstract**

Apart from the conventional method of measuring surface tension and viscosity of a liquid, we are taking the help of optics to measure those quantities. Capillary waves are generated on the liquid surface and the crests formed are used for diffraction grating. The dispersion relation of capillary waves has been used to determine the surface tension of water. For viscosity we have used the fact of dependence of amplitude of the capillary waves on the root mean square voltage of the signal driving the capillary waves. The surface tension and viscosity of distilled water is verified and how surface tension varies with the addition of any kind of salt, surfactant and alcohol to water is also observed. There is an interesting trend in the variation of surface tension of water with the variation of concentration of salt solution and alcohol-water mixture.

## **1 Introduction**

In this project our main objective is to measure the surface tension and viscosity, the two major physical properties of a liquid, in a non-conventional way. First we should have neat idea of what is surface tension and viscosity.

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## 1.1 Surface tension

Surface tension is the elastic tendency of a fluid surface which makes it acquire the least surface area possible. Let a liquid film is formed between the bent wire BAC and the straight wire XY as shown in Figure 1. Let  $F$  be the force acting normally to maintain the equilibrium.

Suppose  $\gamma$  is the force exerted by the straight wire per unit length then,

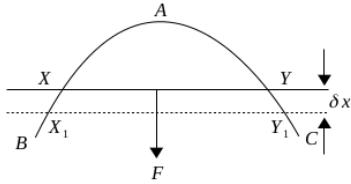


Figure 1.1: Surface tension

$$F = 2\gamma l$$

The surface tension of a liquid is represented by  $\gamma$  (or alternatively by  $\sigma$  or  $\alpha$ ). (the factor 2 comes into action due to the presence of two surfaces)

Also  $\gamma$  is known as the free surface energy per unit area of a liquid[1].

## 1.2 Viscosity

The viscosity of a fluid is the measure of its resistance to gradual deformation by shear stress or tensile stress. It is the property of a fluid which opposes the relative motion between two surfaces of the fluid that are moving at different velocities. In simple terms, viscosity means friction between the molecules of fluid. A fluid that has no resistance to shear stress is known as an ideal or inviscid fluid. Zero viscosity is observed only at very low temperatures in super-fluids. Otherwise, all fluids have positive viscosity and are technically said to be viscous or viscid. And we are interested only in viscous fluid as almost all the liquids what we get easily from mother nature or we generally prepare or use are viscid.

Now consider the Figure 1.2. The velocity of each layer of the volume element of that liquid is different as already mentioned there exists friction in between two layers of the liquid so, there is relative motion between two layers. The top most layer will

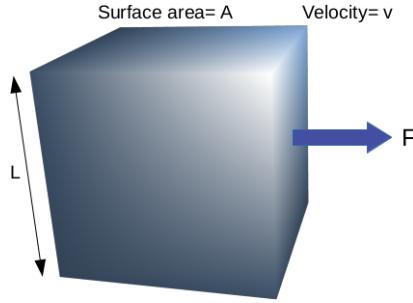


Figure 1.2: A cubical volume element of a liquid under the horizontal force  $F$ . The uppermost layer has the maximum velocity  $v$  and the bottom most layer is considered to be static. The liquid opposes the motion due to the given force by its viscous property.

move with the velocity of  $v$  but the velocity of bottom most layer is zero. Now a force  $F$  is applied to move the liquid to move with velocity  $v$ . The amount of force required depends on several factors:

- $F$  is proportional to the area  $A$  of the layer. This relationship holds good, since  $A$  is directly proportional to the amount of liquid being moved.
- For a given area, greater speed requires larger force. So the force  $F$  is proportional to the speed ( $v$ )
- The force is also inversely proportional to the perpendicular distance  $L$  between the top and bottom layers ( $F$  varies with  $1/L$ ). The larger the distance  $L$ , the smaller is the force required to achieve a given speed with a given contact area.  $L$  is like a lever arm, and the greater the lever arm, the less force that is needed.
- And  $F$  is directly proportional to the viscous nature of the fluid or coefficient of viscosity,  $\eta$ . The greater the viscosity, the greater the force required to move the fluid

These four dependencies are combined into the equation giving the relation,

$$F = \eta v \frac{A}{L}$$

$$\Rightarrow \boxed{\eta = \frac{FL}{vA}}$$

And this  $\eta$  is known as the coefficient of viscosity.

## 2 Capillary waves as reflection diffraction grating

In general we use grating plate or an optical drive like CD or DVD to perform diffraction grating but here we are using the capillary waves generated on the water surface as our diffraction grating. Unlike the transmission diffraction grating in which the light passes through an enormous number of slits located inside the glass plate, in reflection gratings the light is reflected from a series of parallel, very close and smooth surfaces. The crests formed on the capillary waves are acting as the grating plate. The space we used in conventional grating experiment, is determined here by the wavelength of the capillary waves.

## 3 Experimental setup

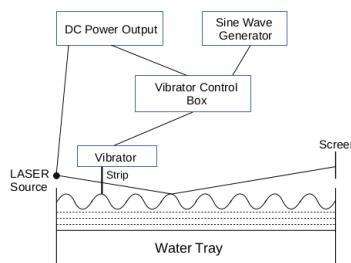


Figure 3.1: Schematic diagram of the experimental setup

A schematic diagram of the experiment is shown in the Figure 3.1. The arrangement of the experiment is not difficult. What are the key parts of the entire apparatus?

- A laser source,
- A non-stick coated metallic tray with drainage pipe,
- A volumetric flask of 500 ml,
- A screen,
- An opaque glass used as attenuating the reflected laser beam,
- An optical bench,
- A vibrator control box,

- A tablet PC to run an application called “Signal Generator”,
- A light sensor assembly attached to a digital vernier calliper,
- A multimeter with banana cables,
- A DC regulated power supply,
- Plastic cover.

The water tray is placed on top of the wooden board whose horizontal level can be adjusted using the levelling screws on which the wooden board is supported. On the same board a laser source is fitted from which the light beam will graze the surface of the liquid contained in the tray. There is an optical bench on which there are two fixed metal rails along which the screen or the light sensor assembly can move back and forth. A meter scale is attached along the track of the capillary wave exciter. This capillary wave exciter is nothing but a loudspeaker with a plastic strip attached to it. This loud speaker is driven by an application called ”Signal Generator” run on Android platform through the tablet PC. The vertical position of the vibrator is adjusted in such a way that only the lower part of the plastic strip is hardly dipped in the liquid surface. The laser beam is of 635 nm wavelength.

First of all it is made sure that the water tray is perfectly horizontal such that liquid depth should be same everywhere. The tray is then filled with the liquid upto the level that there is no meniscus is formed at the edges. Then the laser is adjusted using the screw attached to the apparatus such that the grazing angle of the light beam is less than or equal to  $4^\circ$ . For surface tension measurement the vibrator is fixed at the position of 8 cm in the meter scale. The sinusoidal signal is generated by the application ”Signal generator”. The amplitude of the capillary waves are controlled by the vibrator control box. there in a multimeter connected with the vibrator control box to measure the  $V_{rms}$  of the signal driving the capillary wave exciter. Then the diffraction pattern is formed on the screen which is between the rails in the optical bench. A lens is used to get a bigger pattern on the screen and the photographs are taken to analyse the diffraction pattern. To measure the distance between maxima of the diffraction pattern, a software called *ImageJ*, Java-based image processing program, is used.

While measuring the viscosity we do not need the screen or the camera or *ImageJ*. In

this case capillary waves are generated so that the diffraction pattern is visible on the small white screen. The light sensing hole vertically adjusted to have the first order maxima on the hole. Now the  $V_{rms}$  is noted down. Now the vibrator is moved gradually with the fixed frequency. But the amplitude of the wave is adjusted with the vibrator control box so that the intensity of the first order maxima remains constant.  $V_{rms}$  are noted for different position of the vibrator.

For the calculating the magnification factor of the lens, for any particular frequency photograph is taken one with lens and another without lens.

The distances between the maxima measured using *ImageJ* are in pixels. So we need to convert it into the standard distance measuring unit. For that calibration, one photograph is taken of a ruled sheet(containing parallel lines of known separation) placing on the screen. Then the pixels are converted into standard unit of distance.

The apparatus should be always covered with the plastic cover as the experiment is very vibration sensitive. For example, if there is a mild airflow inside the room by air condition then the capillary wave formation will be disturbed and there will be flickering in the diffraction pattern.

## 4 Theory for measuring Surface tension of liquid

For measurement of surface tension using capillary waves, the very first thing we need is the understanding of dispersion relation of capillary waves. The dispersion relation of capillary wave looks very simple but its derivation is bit complicated. So let us begin from the scratch and will prove the dispersion relation in the following sections.

### 4.1 The pressure difference across any curved surface free from discontinuities: General approach

We consider an arbitrary surface of volume element of a liquid as in Figure 4.1. Let  $ABCD$  be the the surface of that arbitrary volume element. The angle between adjacent sides are  $90^\circ$ . Normals to the surface at  $A$  and  $B$  meet at  $O_1$  and at  $B$  and  $C$  meet at  $O_2$ .

$$AO_1 = BO_1 = r_1 \text{(say)}$$

$$CO_2 = BO_2 = r_2 \text{(say)} \text{ and } AB = dx \ BC = dy$$

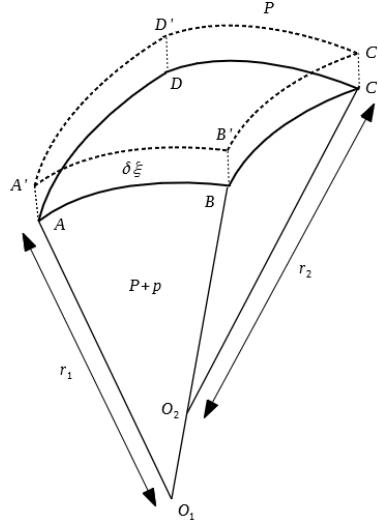


Figure 4.1: Any arbitrary curved volume element

Let the surface be convex and  $P$  be the pressure in the convex side and  $(P + p)$  be the pressure in the concave side. Let  $\gamma$  be the surface tension i.e. free surface energy per unit area(can be shown in a very simple way [1]). Suppose the liquid element is expanded from  $ABCD$  to the new surface  $A'B'C'D'$  temperature remaining constant. Let  $\delta\xi$  be the displacement of the each point on the surface.

So the force on two sides of the surface is  $P.\text{area}(ABCD)$  and  $(P + p).\text{area}(ABCD)$ .

Hence the resultant force acting on the surface is  $(P+p).\text{area}(ABCD) - P.\text{area}(ABCD) = p.(ABCD)$  and work-done is  $p.(ABCD)\delta\xi$ .

The increase in surface energy is,

$$\begin{aligned} &= \gamma\text{area}(A'B'C'D') - \text{area}(ABCD) \\ &= \gamma(A'B'.B'C' - AB.BC) \end{aligned}$$

Also,

$$\begin{aligned} \frac{A'B'}{r_1 + \delta\xi} &= \frac{AB}{r_1} \\ \Rightarrow A'B' &= AB\left(1 + \frac{\delta\xi}{r_1}\right) \end{aligned} \tag{1}$$

and,

$$\begin{aligned} \frac{B'C'}{r_2 + \delta\xi} &= \frac{BC}{r_2} \\ \Rightarrow B'C' &= BC\left(1 + \frac{\delta\xi}{r_2}\right) \end{aligned} \tag{2}$$

$\therefore$  Increase in surface energy is

$$\begin{aligned}
 &= \gamma \left( AB \cdot BC \left( 1 + \frac{\delta\xi}{r_1} \right) \left( 1 + \frac{\delta\xi}{r_2} \right) - AB \cdot BC \right) \\
 &\approx \gamma \left( \frac{1}{r_1} + \frac{1}{r_2} \right) \cdot \delta\xi AB \cdot BC \text{ (neglecting the higher order term)}
 \end{aligned}$$

Now,

$$\text{Work-done} = \text{Change in Surface energy}$$

so,

$$\begin{aligned}
 p \cdot \text{area}(ABCD) \delta\xi &= \gamma \delta\xi \left( \frac{1}{r_1} + \frac{1}{r_2} \right) AB \cdot BC \\
 \Rightarrow p &= \gamma \left( \frac{1}{r_1} + \frac{1}{r_2} \right)
 \end{aligned}$$

So, the excess pressure is, 
$$p = \gamma \left( \frac{1}{r_1} + \frac{1}{r_2} \right)$$

Let us apply the above equation to investigate the mechanical equilibrium of two adjoining media. We assume that no external forces act either on the surface (of separation) or on the media themselves. The pressure is constant on each body. Hence we can write,

$$\frac{1}{r_1} + \frac{1}{r_2} = \text{constant}$$

when this condition is applied to the liquid film supported between rigid frame, the constant on the right hand side must be zero. The  $\frac{1}{r_1} + \frac{1}{r_2}$  must be constant on the both side of the film. But at a given point  $r_1$  and  $r_2$  will have opposite signs, so  $\frac{1}{r_1} + \frac{1}{r_2} = 0$ .

If we consider the infinitesimally small element  $dx$  and  $dy$  for the given surface, the total change in the surface area will be

$$\delta f = \int \delta\xi \left( \frac{1}{r_1} + \frac{1}{r_2} \right) dx dy \quad (3)$$

## 4.2 Approach to excess pressure using surface integral

We have that total change in the surface area is

$$\delta f = \int \delta\xi \left( \frac{1}{r_1} + \frac{1}{r_2} \right) dx dy \quad (3)$$

$$= \int \delta\xi \left( \frac{1}{r_1} + \frac{1}{r_2} \right) df \quad (4)$$

$\therefore A'B' = dx \left(1 + \frac{\delta\xi}{r_1}\right)$  and  $B'C' = dy \left(1 + \frac{\delta\xi}{r_2}\right)$  (from Figure 4.1)

$$\begin{aligned}\therefore \text{change in area} &= \left( dxdy \left(1 + \frac{\delta\xi}{r_1}\right) \left(1 + \frac{\delta\xi}{r_2}\right) - dxdy \right) \\ &\approx dxdy \left( \frac{\delta\xi}{r_1} + \frac{\delta\xi}{r_2} \right)\end{aligned}$$

Now consider the Figure 4.1. Let  $z = \xi(x, y)$  be nearly flat surface[2] i.e.  $|\frac{\delta\xi}{\delta x}|, |\frac{\delta\xi}{\delta y}| \ll 1$ .

The area of the surface is given by

$$f = \int_S \sqrt{1 + \left(\frac{\partial\xi}{\partial x}\right)^2 + \left(\frac{\partial\xi}{\partial y}\right)^2} dxdy \quad (5)$$

for small  $\xi$  we approximately get eq. 5 as,

$$f = \int_S \left( 1 + \frac{1}{2} \left( \frac{\partial\xi}{\partial x} \right)^2 + \frac{1}{2} \left( \frac{\partial\xi}{\partial y} \right)^2 \right) dxdy \quad (6)$$

The variation in  $f$  is given by,

$$\delta f = \int_S \left( \frac{\partial\xi}{\partial x} \cdot \frac{\partial\delta\xi}{\partial x} + \frac{\partial\xi}{\partial y} \cdot \frac{\partial\delta\xi}{\partial y} \right) dxdy \quad (7)$$

By integration by parts of eq. 7,

$$\delta f = - \int_S \left( \frac{\partial^2\xi}{\partial x^2} + \frac{\partial^2\xi}{\partial y^2} \right) \delta\xi dxdy \quad (8)$$

But we already have that the total change change in area of surface of separation is

$$\delta f = \int_S \left( \frac{1}{r_2} + \frac{1}{r_1} \right) \delta\xi dxdy \quad (3)$$

. So by comparing eq. 3 and eq. 8 we get,

$$\begin{aligned}\left( \frac{1}{r_1} + \frac{1}{r_2} \right) &= - \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \xi \\ \boxed{\therefore p = -\gamma (\nabla_x^2 + \nabla_y^2) \xi} \quad (9)\end{aligned}$$

If the surface curves downwards in all direction at a given point then  $(\nabla_x^2 + \nabla_y^2) \xi < 0$ , and the excess pressure will be positive below the surface.

### 4.3 Capillary waves and velocity potential

Let us consider some volume of fluid. The total force acting on this volume is equal to  $-\oint p d\vec{f}$ . Transforming into volume integral we get,

$$\begin{aligned}
-\oint p \mathrm{d}f &= - \int \vec{\nabla} p \mathrm{d}V \\
\Rightarrow \vec{F} &= - \int \vec{\nabla} p \mathrm{d}V \\
\Rightarrow \mathrm{d}\vec{F} &= - \int \vec{\nabla} p \mathrm{d}V \\
\Rightarrow \mathrm{d}m \frac{\mathrm{d}\vec{V}}{\mathrm{d}t} &= - \int \vec{\nabla} p \mathrm{d}V \\
\Rightarrow \rho \mathrm{d}V \frac{\mathrm{d}\vec{V}}{\mathrm{d}t} &= - \int \vec{\nabla} p \mathrm{d}V \\
\Rightarrow \rho \frac{\mathrm{d}\vec{V}}{\mathrm{d}t} &= - \vec{\nabla} p
\end{aligned} \tag{10}$$

Now,

$$\mathrm{d}x \left( \frac{\partial V}{\partial x} \right) + \mathrm{d}y \left( \frac{\partial V}{\partial y} \right) + \mathrm{d}z \left( \frac{\partial V}{\partial z} \right) = \mathrm{d}\vec{r} \cdot \vec{\nabla} V$$

and,

$$\begin{aligned}
\mathrm{d}V &= \mathrm{d}t \left( \frac{\partial V}{\partial t} \right) + \mathrm{d}x \left( \frac{\partial V}{\partial x} \right) + \mathrm{d}y \left( \frac{\partial V}{\partial y} \right) + \mathrm{d}z \left( \frac{\partial V}{\partial z} \right) \\
\therefore \frac{\mathrm{d}V}{\mathrm{d}t} &= \left( \frac{\partial V}{\partial t} \right) + \frac{\mathrm{d}x}{\mathrm{d}t} \left( \frac{\partial V}{\partial x} \right) + \frac{\mathrm{d}y}{\mathrm{d}t} \left( \frac{\partial V}{\partial y} \right) + \frac{\mathrm{d}z}{\mathrm{d}t} \left( \frac{\partial V}{\partial z} \right) \\
\Rightarrow \frac{\mathrm{d}V}{\mathrm{d}t} &= \frac{\partial V}{\partial t} + (\vec{V} \cdot \vec{\nabla} V)
\end{aligned} \tag{11}$$

From eq. 10 and eq. 11,

$$\frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \vec{\nabla} \vec{V} = -\frac{1}{\rho} \vec{\nabla} p \tag{12}$$

If the fluid is in gravitational field then an additional force  $\rho g$  will act. So the eq. 12 becomes,

$$\frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \vec{\nabla} \vec{V} = -\frac{1}{\rho} \vec{\nabla} p + \vec{g} \tag{13}$$

For irrotational fluid(streamline steady flow),  $\vec{\omega} = \vec{\nabla} \times \vec{V} = 0$ . For such functions where curl is zero,  $\vec{V} = \vec{\nabla} \Psi$  where  $\Psi$  is a scalar function which is considered as the velocity potential of the wave.

Now consider

amplitude,  $a \ll \lambda$ , wavelength

For gravity-capillary waves, we can neglect  $\vec{V} \cdot \vec{\nabla} \vec{V}$  w.r.t  $\frac{\partial \vec{V}}{\partial t}$  in eq. 13. This can be shown by comparing the order of  $\vec{V} \cdot \vec{\nabla} \vec{V}$  and  $\frac{\partial \vec{V}}{\partial t}$ . We have  $\vec{V} \cdot \vec{\nabla} \vec{V} = \frac{a}{T} \cdot \frac{a}{T} \cdot \frac{1}{\lambda} = \frac{1}{\lambda} \left(\frac{a}{T}\right)^2$  and  $\frac{\partial \vec{V}}{\partial t} = \frac{a}{T} \frac{1}{T} = \frac{1}{a} \left(\frac{a}{T}\right)^2$ . Now since  $a \ll \lambda$  so  $\vec{V} \cdot \vec{\nabla} \vec{V} \ll \frac{\partial \vec{V}}{\partial t}$ . Therefore, we now have

$$\frac{\partial \vec{V}}{\partial t} = -\frac{1}{\rho} \vec{\nabla} p + \vec{g}$$

Putting  $\vec{V} = \vec{\nabla} \Psi$  we get the following,

$$\begin{aligned} \vec{\nabla} \frac{\partial \Psi}{\partial t} &= -\frac{1}{\rho} \vec{\nabla} p - \nabla g z \\ \Rightarrow \vec{\nabla} \left( \frac{\partial \Psi}{\partial t} \frac{p}{\rho} + g z \right) &= 0 \end{aligned} \quad (14)$$

Let  $f(t) = \frac{\partial \Psi}{\partial t} \frac{p}{\rho} + g z$ . Putting  $f(t) = 0$  in the eq. 14 we get

$$\begin{aligned} \frac{\partial \Psi}{\partial t} \frac{p}{\rho} + g z &= 0 \\ \therefore p &= -\rho \left( g z + \frac{\partial \Psi}{\partial t} \right) \end{aligned}$$

Now the pressure just below the surface is  $P + P_0$  instead of  $P_0$ , so the dynamic boundary condition is replaced by

$$P = P_0 + p \text{ for } z = \xi(x, y)$$

Since  $p$  is positive at crest and negative at trough, surface tension collaborates with the gravity in attempting to flatten the surface.

So, the waves completely dominated by surface tension are termed as *Capillary waves*.

And we now have the equation of excess pressure as,

$$P - P_0 = -\rho_0 \left( g_0 z + \frac{\partial \psi}{\partial t} \right) \quad (15)$$

Now let us talk about the velocity potential  $\psi$  and the possible solutions of that wave function. We know  $V_z = \frac{\partial \xi}{\partial t}$ . But also  $V_z = \frac{\partial \psi}{\partial z}$  due to surface boundary condition,

$$\therefore \left( \frac{\partial \xi}{\partial t} \right) = \left( \frac{\partial \psi}{\partial z} \right)_{z=\xi}, \quad \xi \text{ is very small.} \quad (16)$$

At the surface  $P = P_0$  and for  $z = \xi(x, y)$ ,

$$\begin{aligned} -\rho_0 g_0 \xi - \rho_0 \frac{\partial \psi}{\partial t} &= 0 \\ \therefore \xi &= -\frac{1}{g_0} \frac{\partial \psi}{\partial t} \end{aligned} \quad (17)$$

So, from eq. 16 and eq. 17 we obtain,

$$\left( \frac{\partial \psi}{\partial z} + \frac{1}{g_0} \frac{\partial^2 \psi}{\partial t^2} \right)_{z=\xi} = 0 \quad (18)$$

For  $\xi \rightarrow 0$ ,

$$\psi = f(z) \cos(kx - \omega t)$$

We know that[2]  $\nabla^2 \psi = 0$  so

$$\frac{d^2 f}{dz^2} - k^2 f = 0 \quad (19)$$

The above differential eq. 19 has the solutions  $e^{kz}$  and  $e^{-kz}$ . We take the positive one as the latter gives an unlimited increase of  $\psi$  as we go into the interior of the fluid as fluid occupies the region  $z < 0$ . So the desired solution of the  $\psi$  is written as,

$$\boxed{\psi = A e^{kz} \cos(kx - \omega t)} \quad (20)$$

#### 4.4 Deep water capillary gravity waves and Dispersion relation

Surface tension generally becomes important for the waves having very small wavelengths(already discussed in previous sections) which except for some special case, may be assumed to be deep water waves[2].

We already have the eq. 15 for excess pressure and also from the eq. 20 we have the velocity potential of the capillary wave as

$$\psi = A e^{kz} \cos(kx - \omega t) \quad (21)$$

From 9 and 15 we get,

$$\rho_0 \left( g_0 z + \frac{\partial \psi}{\partial t} \right) - \gamma \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \xi = 0 \quad (22)$$

Putting  $z = \xi(x, y, t)$  the eq. 22 becomes

$$\rho_0 \left( g_0 \xi + \frac{\partial \psi}{\partial t} \right) - \gamma \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \xi = 0 \quad (23)$$

Also we have the boundary condition from eq. 16. So differentiating the eq. 23 w.r.t  $t$  and plugging the boundary condition by eq. 16 we get,

$$\rho_0 \left( g_0 \frac{\partial \psi}{\partial z} + \frac{\partial^2 \psi}{\partial t^2} \right) - \gamma \frac{\partial}{\partial z} \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right) = 0 \quad (24)$$

For our ease let us calculate few things first.

$$\begin{aligned}\frac{\partial^2 \psi}{\partial x^2} &= -k^2 \psi \\ \frac{\partial^2 \psi}{\partial y^2} &= 0 \\ \frac{\partial^2 \psi}{\partial t^2} &= -\omega^2 \psi \\ \frac{\partial \psi}{\partial z} &= k \psi\end{aligned}$$

So plugging above relations into eq.24 after doing some tedious algebra we finally get somewhat desired relation,

$$\boxed{\omega^2 = g_0 k + \frac{\gamma k^3}{\rho_0}} \quad (25)$$

Now, we know that  $R_c = \sqrt{\frac{\gamma}{\rho_0 g_0}}$  = capillary constant. This shows that the only effect of the surface tension is to increase the gravitational acceleration from  $g_0$  to  $g = g_0 (1 + k^2 R_c^2)$ .

Surface tension collaborates with gravity and becomes more important than gravity when  $kR_c \gg 1$  or,  $\lambda \lesssim 2\pi R_c$ .

So for shorter wavelengths the gravity can be neglected and after implementing the approximation the eq. 25 becomes[3, 4]

$$\begin{aligned}\boxed{\omega^2 = \frac{\gamma k^3}{\rho_0}} \quad (26) \\ \Rightarrow \gamma = \frac{\rho_0 \omega^2}{k^3}\end{aligned}$$

<sup>1</sup>

## 4.5 Application of dispersion relation of capillary waves in the experiment

The surface particle motion is bit complicated, but for the capillary waves propagation, we can simply consider a sinusoidal disturbance and it can be considered as,

$$y = A \cos(\omega t - kx) \quad (27)$$

---

<sup>1</sup>(For more detailed derivation of dispersion relation of capillary waves check[1])

where  $y$  is the displacement of the surface in the vertical plane,  $x$  is the along the path the wave is propagating and  $k$  is the wave number of the capillary wave, and  $k = \frac{2\pi}{\lambda_s}$  where  $\lambda_s$  is the wavelength of the capillary wave.

The laser beam is incident on the liquid surface consisting the capillary waves and this surface serves as the reflection diffraction grating for our experiment and Fraunhofer diffraction spots are visible on the screen. The wavelength of the propagating capillary waves  $\lambda_s$  serves as the grating space.

The grating equation [3] for both reflection and transmission is,

$$\lambda_s[\cos \theta - \cos(\theta + \theta_m)] = m\lambda \quad (28)$$

where  $\theta$  is the grazing angle of incidence of the laser beam,  $\theta_m$  is the diffraction angle of the  $m^{th}$  order spot observed on the screen placed at a distance  $l$  from the point of reflection. The grating eq. 28, for the first order maxima i.e. for  $m = \pm 1$  gives

$$\lambda_s[\cos \theta - \cos(\theta + \theta_1)] = \lambda \quad (29)$$

and,

$$\lambda_s[\cos \theta - \cos(\theta - \theta_1)] = -\lambda \quad (30)$$

Using eq. 29 and eq. 30 we get,

$$\lambda_s \sin \theta \sin \theta_1 = \lambda \quad (31)$$

where  $\theta_1$  is the diffraction angle of the first order spot.

The angles  $\theta$  and  $\theta_1$  are calculated by measuring the distances  $l$ ,  $h$ , and  $d$  as shown in the Figure 4.2. Here  $\sin \theta = h/\sqrt{h^2 + l^2}$  and  $\sin \theta_1 = d/2\sqrt{h^2 + l^2}$ , where  $h$  is the perpendicular height of the zeroth order spot or the commonly known as central maxima from the liquid surface level,  $l$  is the distance between the point of reflection of laser beam on the liquid surface and the screen, and  $d$  is the distance between the positive and negative first order diffraction spot on the screen. From eq. 31 we obtain the expression for the wave number  $k$  as [3],

$$k = \frac{2\pi}{\lambda_s} = \frac{\pi h d}{\lambda(h^2 + l^2)} = \frac{\pi d}{h \lambda} \sin^2 \theta \quad (32)$$

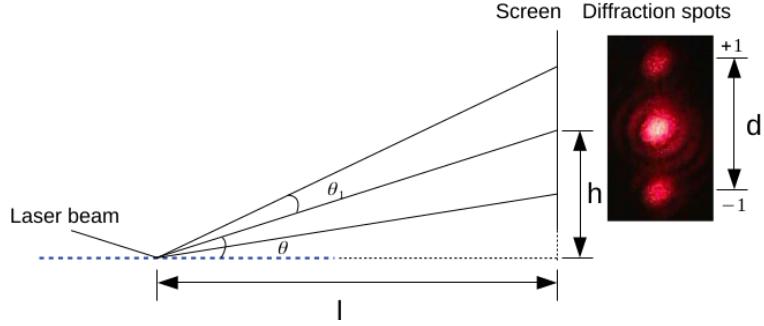


Figure 4.2: Schematic diagram of diffraction grating using the capillary waves on generated on the liquid surface. Diffraction spots are visible on the screen placed at a distance  $l$  from the point of reflection and one photograph of the diffraction pattern is shown in the figure.

## 5 Theory for measuring Viscosity of liquid

The surface tension waves are damped due to the viscosity of water. The wave amplitude,  $h$ , decreases exponentially with the distance,  $s$ , measured from the vibrator,

$$h = h_0 e^{-\delta s} \quad (33)$$

where,  $h_0$  is the amplitude at the vibrator position and  $\delta$  is the attenuation constant. Experimentally, amplitude  $h_0$  can be related to the voltage ( $V_{rms}$ ) applied to the vibrator assembly as,

$$h_0 \propto (V_{rms})^{0.4} \quad (34)$$

Now plugging the eq. 34 into eq. 33 with the proportionality constant,  $D$ ,

$$h = D(V_{rms})^{0.4} e^{-\delta s} \quad (35)$$

Taking log on both sides,

$$\begin{aligned} \ln h &= \ln D + 0.4 \log V_{rms} - \delta s \\ \Rightarrow \ln V_{rms} &= \frac{\delta}{0.4} s + \frac{1}{0.4} \ln \left( \frac{h}{D} \right) \\ \Rightarrow \boxed{\ln V_{rms} = \frac{\delta}{0.4} s + K}, \text{ where } K &= \frac{1}{0.4} \ln \left( \frac{h}{D} \right) \end{aligned} \quad (36)$$

Suppose we have  $V_{rms}^1$  and  $V_{rms}^2$  for corresponding  $s_1$  and  $s_2$  respectively. Then by eq.36,

$$\begin{aligned}\ln V_{rms}^1 &= \frac{\delta}{0.4}s_1 + K \text{ and } \ln V_{rms}^2 = \frac{\delta}{0.4}s_2 + K \\ \therefore \ln V_{rms}^1 - \ln V_{rms}^2 &= \frac{\delta}{0.4}(s_1 - s_2) \\ \Rightarrow \boxed{\ln \left( \frac{V_{rms}^2}{V_{rms}^1} \right)} &= \frac{\delta}{0.4}\Delta s\end{aligned}\tag{37}$$

The attenuation constant is related to the viscosity of the liquid as,

$$\begin{aligned}\delta &= \frac{8\pi\eta f}{3\gamma} \\ \Rightarrow \boxed{\eta} &= \frac{3\gamma\delta}{8\pi f}\end{aligned}\tag{38}$$

## 6 Experimental Data and analysis

We have performed the experiment to measure the viscosity of distilled water and the surface tension of

- Distilled water,
- NaCl solution of different concentration,
- Methanol-water mixture for different ratios,
- SDS solution.

By using the dispersion relation of capillary waves eq. 26 and calculating wave number by eq. 32 , we have plotted  $\omega^2\rho$  vs.  $k^3$ . The experimental data gives a straight line after plotting  $\omega^2\rho$  vs.  $k^3$ . From the slope of the best fitted straight line we have obtained the surface tensions of mentioned compounds.

If we take log on both side of the eq.26 we get,

$$\boxed{\ln \omega = \frac{3}{2} \ln k + \frac{1}{2} \ln \frac{\gamma}{\rho}}$$

If we plot  $\ln \omega$  vs.  $\ln k$ , the value of slope  $\frac{3}{2}$  i.e. the dispersion relation  $\omega^2 \propto k^3$  can be verified [3].

## 6.1 Surface tension of Distilled Water

The air-liquid interface dispersion relation for capillary waves on a deep liquid is already discussed and is given by the eq. 26

$$\omega^2 = \frac{\gamma}{\rho} k^3 \quad (39)$$

Rearranging the equation a little bit gives,

$$\boxed{\omega^2 \rho = \gamma k^3} \quad (40)$$

If we can plot  $\omega^2 \rho$  and  $\gamma k^3$  as y-axis and x-axis respectively then from the slope of the fitted line with those experimental data points we can directly get the value of surface tension of distilled water.

$\omega^2 \rho (\times 10^9)$	$k^3 (\times 10^8)$
3.188	2.52
3.833	3.19
5.185	3.94
5.616	4.76
6.935	5.67
8.382	6.65
10.091	7.71
11.854	8.86
13.545	10.08
15.198	11.38

Table 1: This table contains the data for measurement of surface tension of distilled water at room temperature. The frequency is varied from 80 Hz to 170 Hz with an increment of 10 Hz and using eq. 32 corresponding wave numbers are calculated.

The slope of the fitted line in the Figure 6.1 is  $0.07189 \text{ N m}^{-1}$ . Also by eq. 40 the slope will be the surface tension of distilled water. Hence the experimentally obtained surface tension is  $71.89 \text{ mN m}^{-1}$ . Our temperature during experiment was kept at  $25^\circ\text{C}$ . So the widely accepted value[5] of surface tension for that temperature is

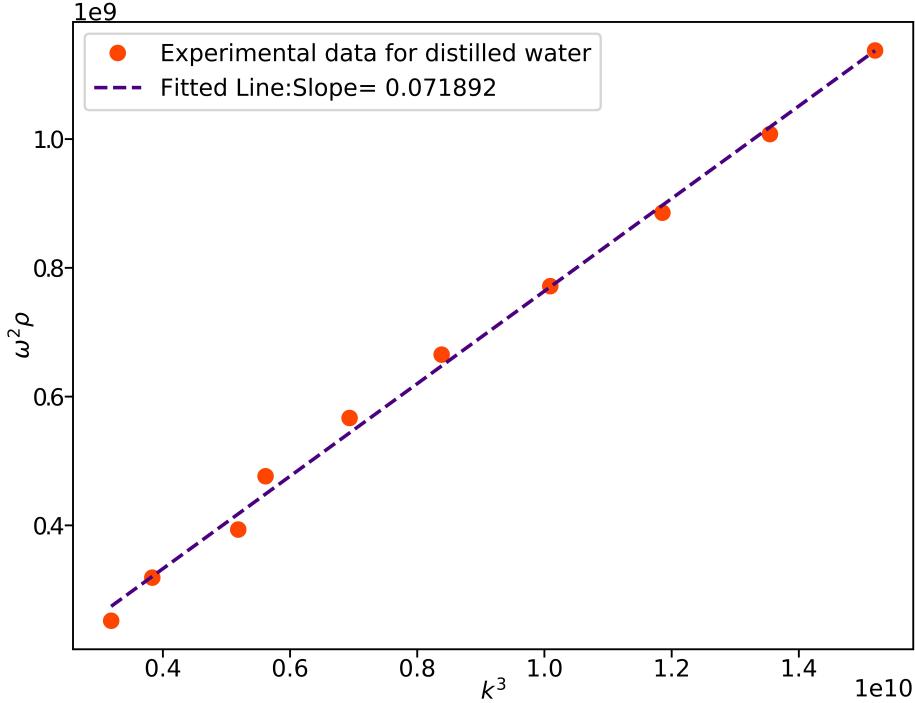


Figure 6.1: The figure shows a linear behaviour between  $\omega^2 \rho$  and  $k^3$ . Experimental data are plotted and fitted with a straight line and the slope of the best fitted straight line is coming out to be  $0.07189 \pm 0.00020 \text{ N m}^{-1}$

$77.99 \pm 0.05 \text{ mN m}^{-1}$ [4].

In our experiment we found out the value of surface tension as  $71.89 \pm 0.20 \text{ mN m}^{-1}$ . The uncertainty is calculated from the fitting of the experimental data points for distilled water.

## 6.2 Viscosity of distilled water

Already mentioned in the Experimental setup section that there is a meter scale attached to the vibrator assembly along which we can move the vibrator back and forth. So we first observed the  $V_{rms}$  for various positions in the scale. But the condition is that the intensity of the first order maxima falling on the light meter sensor must be constant. The amplitude of the capillary waves are that's why adjusted with the help of vibrator control box to keep the intensity constant for different  $V_{rms}$  and it is noted from the multimeter connected to vibrator control box that for which  $V_{rms}$  we are getting same intensity for different distances of the vibrator from the point of reflection

of laser beam on the liquid surface.

Position(m)	$V_{rms}$ (volts)
0.130	0.115
0.125	0.105
0.120	0.090
0.115	0.080
0.110	0.072
0.105	0.064

Table 2: The value of  $V_{rms}$  for different positions in the meter scale attached to the vibrator assembly keeping the the intensity of first order maxima constant.

But it will be not accurate to measure the distance between the vibrator strip and the

$\Delta s(m)$	$\ln \left( \frac{V_{rms}}{0.115} \right)$
0.005	0.090971778
0.01	0.245122458
0.015	0.362905494
0.02	0.468266009
0.025	0.586049045

Table 3: Taking  $V_{rms}^1 = 0.115$  volts,  $\ln \left( \frac{V_{rms}}{0.115} \right)$  is calculated for corresponding  $\Delta s$ .

point of reflection of laser beam using a tailor's tape or using a scale and also there is a chance of contamination of the liquid if it somehow comes in contact with the scale or the tailor's tape or the our hands. So it is preferred to use the eq. 37 where we need only the increment of the positions i.e. the intervals we are taking for changing the position of the vibrator. We have taken  $V_{rms}^1 = 0.115$  volts = constant, and calculated  $\ln \left( \frac{V_{rms}^x}{V_{rms}^1} \right)$ , where  $x = 2, 3, \dots$  for corresponding  $\Delta s$ .

By eq. 38 the value of coefficient of viscosity is,

$$\eta = \frac{3 \times 71.89 \times 10^{-3} \times 9.706377}{8\pi \times 100}$$

$$\Rightarrow \boxed{\eta = 8.33 \times 10^{-4} \text{ Pas}} \quad (41)$$

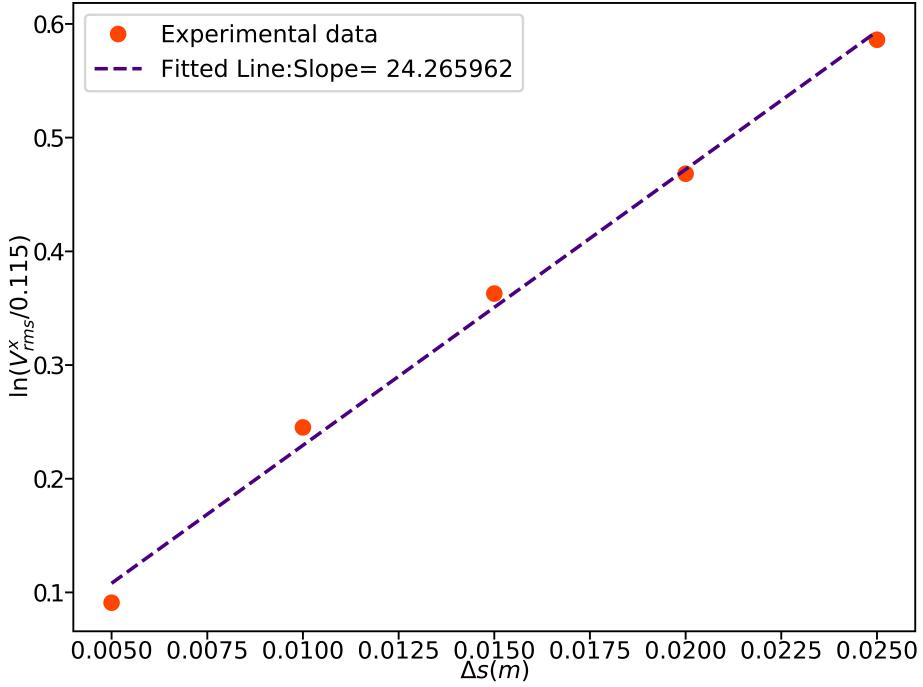


Figure 6.2: The figure shows the linearity of  $\ln\left(\frac{V_{rms}}{0.115}\right)$  with the increment of  $\Delta s$ (m). The blue crosses are the experimental data points corresponding to  $\Delta s$  and the lime coloured straight line is the fitted line for the data. The slope of the fitted line is coming out to be  $24.265\ 96\text{ m}^{-1}$ . Hence the value of attenuation constant,  $\delta$  is  $9.706\ 377\text{ m}^{-1}$

By our experiment the value of viscosity is  $8.33 \pm 0.57 \times 10^{-4}\text{ Pa.s}$ . The known viscosity of distilled water is  $8.90 \times 10^{-4}\text{ Pa.s}$  at  $25^\circ\text{C}$ [6].

## 6.3 NaCl Solution

NaCl is an ionic compound and soluble in water. We have used the following concentrations of NaCl solution:

- 0.30M NaCl solution
- 0.52M NaCl solution
- 1M NaCl solution

### 6.3.1 0.3M NaCl Solution

The 0.3M NaCl solution was prepared in the 500 ml volumetric cylinder. The density of the solution was calculated as  $1035 \text{ kg m}^{-3}$ . The grazing angle of incidence of laser beam was set at  $3.99^\circ$ .

$\omega^2 \rho (\times 10^9)$	$k^3 (\times 10^8)$
5.67464	3.62706
6.2689	4.0189
6.62071	4.43084
7.06828	4.86287
7.75462	5.315
8.48401	5.78722
9.35766	6.27954
10.0772	6.79195

Table 4: This table contains the data for measurement of surface tension of 0.3M NaCl solution of water prepared in 500mL of volumetric cylinder at room temperature. The frequency is varied from 95 Hz to 125 Hz with an increment of 5 Hz and using eq. 32 corresponding wave numbers are calculated.

For 0.3M NaCl solution, from the slope of the fitted line in the Figure 6.3, we see that the surface tension of the solution is coming out to be  $73.32 \pm 0.30 \text{ N m}^{-1}$ .

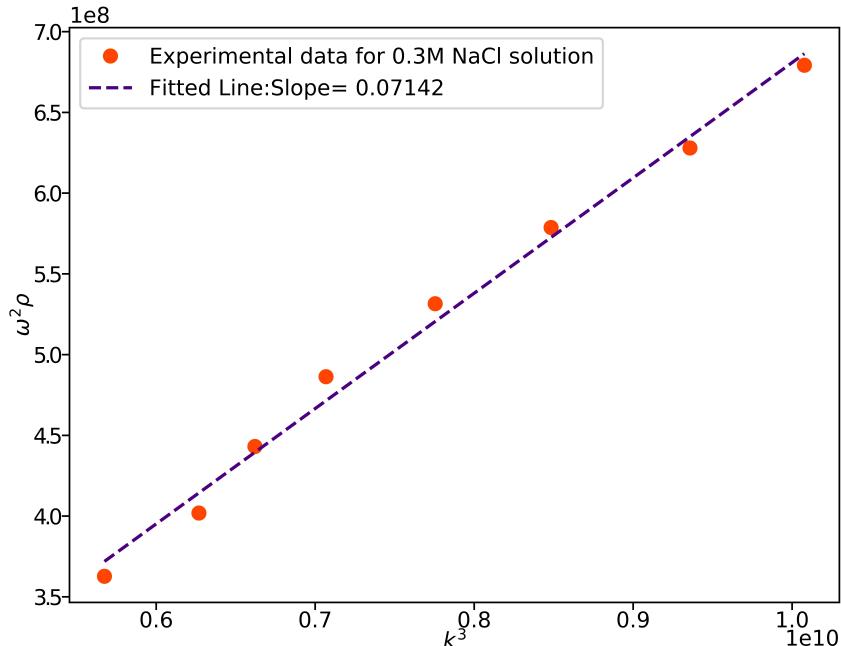


Figure 6.3: The figure shows a linear behaviour between  $\omega^2\rho$  and  $k^3$  for 0.3M NaCl solution of water prepared in 500 ml of volumetric cylinder at room temperature. Experimental data are plotted and fitted with a straight line and the slope of the best fitted straight line is coming out to be  $0.073\ 32 \pm 0.000\ 30\ N\ m^{-1}$ .

### 6.3.2 0.52M NaCl Solution

The 0.52M NaCl solution was prepared in the 500 ml volumetric cylinder. The density of the solution was calculated as  $1061\ kg\ m^{-3}$ . The grazing angle of incidence of laser beam was set at  $3.99^\circ$ .

For 0.52M NaCl solution, from the slope of the fitted line in the Figure 6.4, we see that the surface tension of the solution is coming out to be  $74.74 \pm 0.10\ N\ m^{-1}$

$\omega^2 \rho (\times 10^9)$	$k^3 (\times 10^8)$
1.43392	1.46443
1.7349	1.71867
2.07531	1.99325
2.45756	2.28817
2.88408	2.60343
3.3573	2.93903
3.87964	3.29496

Table 5: This table contains the data for measurement of surface tension of 0.52M NaCl solution of water prepared in 500 ml of volumetric cylinder at room temperature. The frequency is varied from 60 Hz to 90 Hz with an increment of 5 Hz and using eq. 32 corresponding wave numbers are calculated.

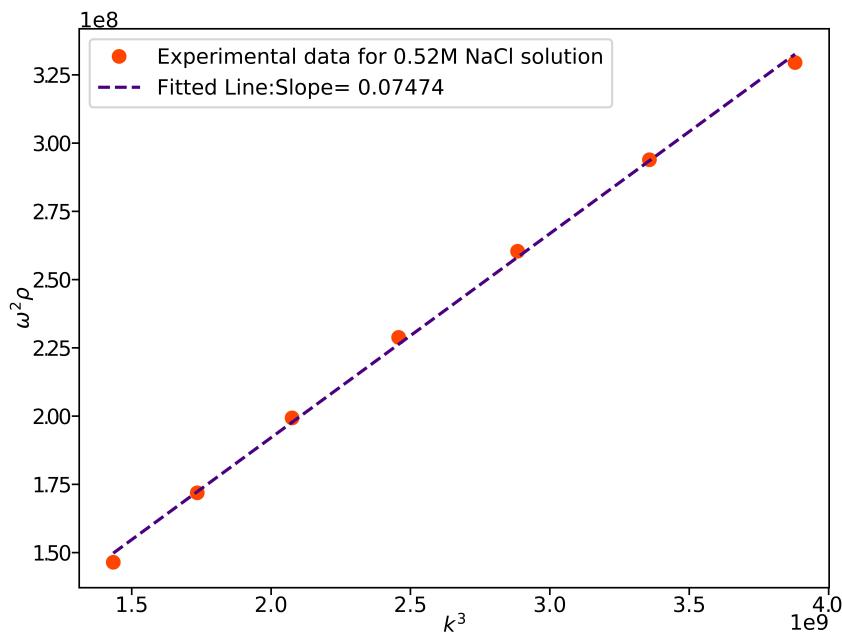


Figure 6.4: The figure shows a linear behaviour between  $\omega^2 \rho$  and  $k^3$  for 0.52M NaCl solution of water prepared in 500mL of volumetric cylinder at room temperature. Experimental data are plotted and fitted with a straight line and the slope of the best fitted straight line is coming out to be  $0.074\,739 \pm 0.000\,010\, N\, m^{-1}$ .

### 6.3.3 1M NaCl Solution

The 1M NaCl solution was prepared in the 500 ml volumetric cylinder. The density of the solution was calculated as  $1117 \text{ kg m}^{-3}$ . The grazing angle of incidence of laser beam was set at  $4.04^\circ$ .

$\omega^2 \rho (\times 10^9)$	$\mathbf{k}^3 (\times 10^8)$
2.84374	3.29496
4.24488	4.06786
5.1737	4.92211
6.41773	5.85771
7.84673	6.87468
8.98778	7.973

Table 6: This table contains the data for measurement of surface tension of 1M NaCl solution of water prepared in 500 ml of volumetric cylinder at room temperature. The frequency is varied from 90 Hz to 140 Hz with an increment of 10 Hz and corresponding wave numbers are calculated using eq. 32.

For 1M NaCl solution, from the slope of the fitted line in the Figure 6.5, we see that the surface tension of the solution is coming out to be  $76.45 \pm 0.30 \text{ N m}^{-1}$

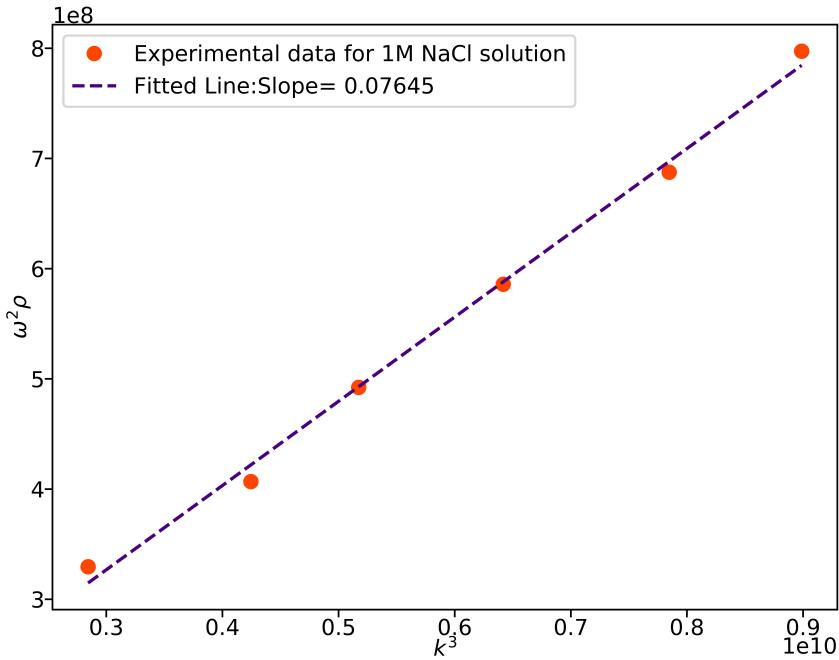


Figure 6.5: The figure shows a linear behaviour between  $\omega^2\rho$  and  $k^3$  for 1M NaCl solution of water prepared in 500mL of volumetric cylinder at room temperature. Experimental data are plotted and fitted with a straight line and the slope of the best fitted straight line is coming out to be  $0.076\ 45 \pm 0.000\ 30\ N\ m^{-1}$ .

#### 6.3.4 Trend in NaCl solution

Adding salt to water increases the surface tension of the water. This property is obvious from our experiment. Here we see a linear increase in the surface tension of the NaCl-water solution with increase in the concentration.

Concentration(M)	Surface tension(mN/m)
0.0	71.89
0.30	73.3
0.52	74.7
1	76.45

Table 7: The table contains the values of surface tension for various concentration of NaCl solution prepared in the 500 mL volumetric cylinder.

The reason behind the increase in surface tension of water with addition of salt

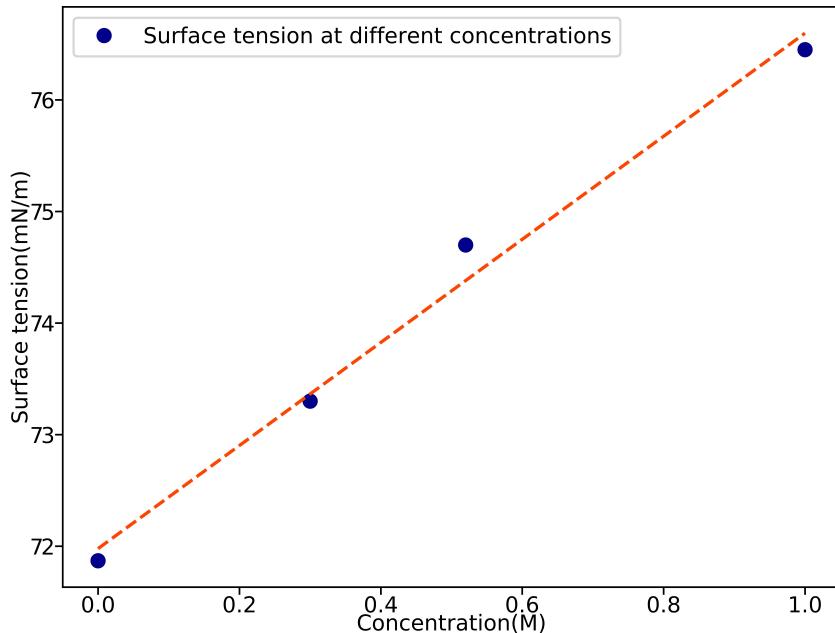


Figure 6.6: This figure shows the linearity of surface tension of salt solution with increase in concentration. The concentration is varied from pure distilled water to 1M NaCl solution.

is simple. We already know surface tension is generated due to the attractive forces present in a liquid. In pure distilled water, the attractive forces are weak hydrogen bonding between oxygen end of one water molecule with hydrogen end of another water molecule. As soon as NaCl is added, it gets dissociated into  $\text{Na}^+$  and  $\text{Cl}^-$  ions which leads to much stronger attraction of water molecules around these ions (hydration) where H<sup>−</sup> ends of water molecules are arranged around negatively charged Cl<sup>−</sup> and oxygen ends will similarly arrange themselves around Na<sup>+</sup>. As the net attractive forces increases between the hydrated charged species as in increasingly add up NaCl, the net surface tension increases.

## 6.4 Methanol-Water mixture

Methanol is miscible in water and forms a homogeneous mixture or solution. Due to small molecular weight is is volatile but when it is in solution in water upto certain level of concentration it is not that much volatile as the pure methanol is. The experiment is done for

- 2% Methanol solution

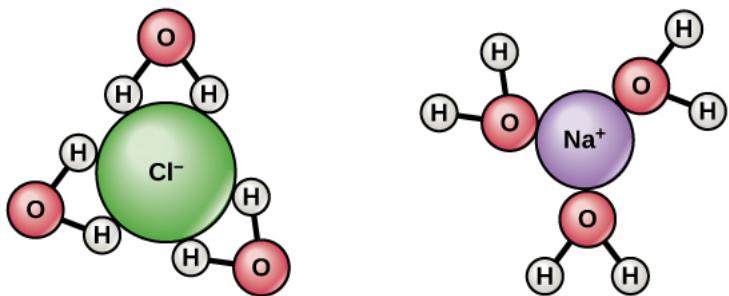


Figure 6.7: Interaction of NaCl molecule with water molecule after dissolution in water. It gets dissociated and each ionic part are covered with the part of opposite polarity of water molecule. It increases the internal interactions and hence increases the surface tension more and more with addition of NaCl in the water.

- 10% Methanol solution
- 25% Methanol solution

Due to volatility above 25% concentration it was little bit difficult to perform the experiment. But may be the experiment can be done by reducing temperature or anything suitable for that concentration.

#### 6.4.1 2% Methanol solution

The methanol- water mixture was prepared in the 500 ml volumetric cylinder. To prepare 2% methanol solution in water we took 10 ml of methanol and the density of the solution was calculated as  $992.9 \text{ kg m}^{-3}$ . The grazing angle of incidence of laser beam was set at  $3.71^\circ$ .

From the Figure 6.8 where experimental data are plotted for 2% Methanol solution and fitted with a straight line, the slope of the best fitted graph is coming out to be  $0.06863 \text{ N m}^{-1}$ . So the surface tension of 2% Methanol solution , obtained in this method of experiment, is  $68.63 \pm 0.50 \text{ mN m}^{-1}$ .

#### 6.4.2 10% Methanol solution

The methanol- water mixture was prepared in the 500 ml volumetric cylinder. To prepare 10% methanol solution in water we took 50 ml of methanol and the density of the solution was calculated as  $976.5 \text{ kg m}^{-3}$ . The grazing angle of incidence of laser

$\omega^2 \rho (\times 10^9)$	$k^3 (\times 10^8)$
2.90721	2.20489
3.45671	2.50868
3.96429	2.83206
4.40439	3.17505
5.44528	3.91981
6.05733	4.32159

Table 8: This table contains the data for calculating the surface tension of 2% Methanol solution prepared in a 500 ml volumetric cylinder. The frequency is varied from 75 Hz to 105 Hz with an increment of 5 Hz and corresponding wave numbers are calculated using eq. 32.

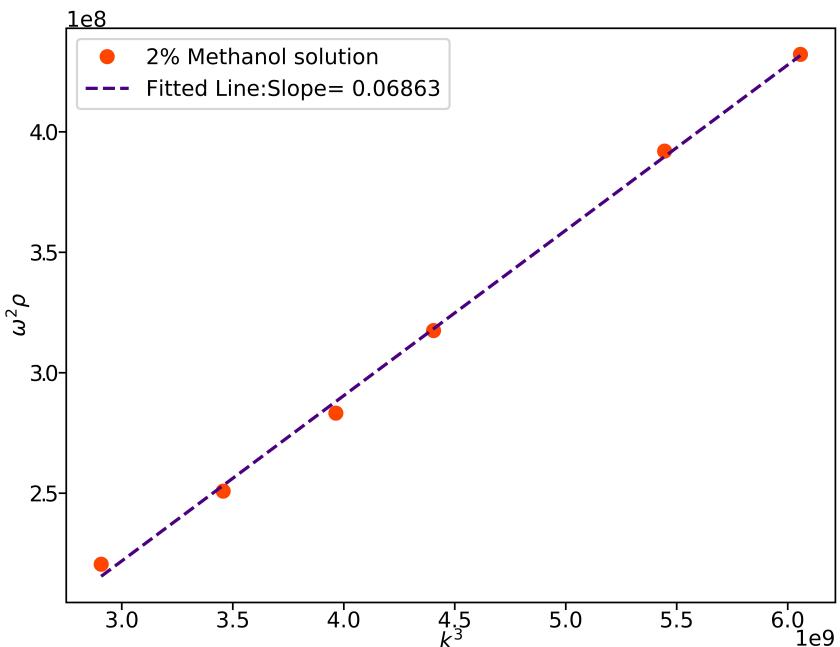


Figure 6.8: The figure shows a linear behaviour between  $\omega^2 \rho$  and  $k^3$  for 2% Methanol solution prepared in 500 ml of volumetric cylinder at room temperature of 25 °C. Experimental data are plotted and fitted with a straight line and the slope of the best fitted straight line is coming out to be  $0.06863 \pm 0.00050 \text{ N m}^{-1}$ .

beam was set at  $3.71^\circ$ .

From the Figure 6.9 where experimental data are plotted for 10% Methanol solution and fitted with a straight line, the slope of the best fitted graph is coming out to be

$\omega^2 \rho (\times 10^9)$	$k^3 (\times 10^8)$
2.87514	1.88898
3.29756	2.16848
4.00574	2.46724
4.43974	2.78529
4.93606	3.1226
5.78344	3.4792

Table 9: This table contains the data for calculating the surface tension of 10% Methanol solution prepared in a 500 ml volumetric cylinder. The frequency is varied from 60 Hz to 95 Hz with an increment of 5 Hz and using eq. 32 corresponding wave numbers are calculated.

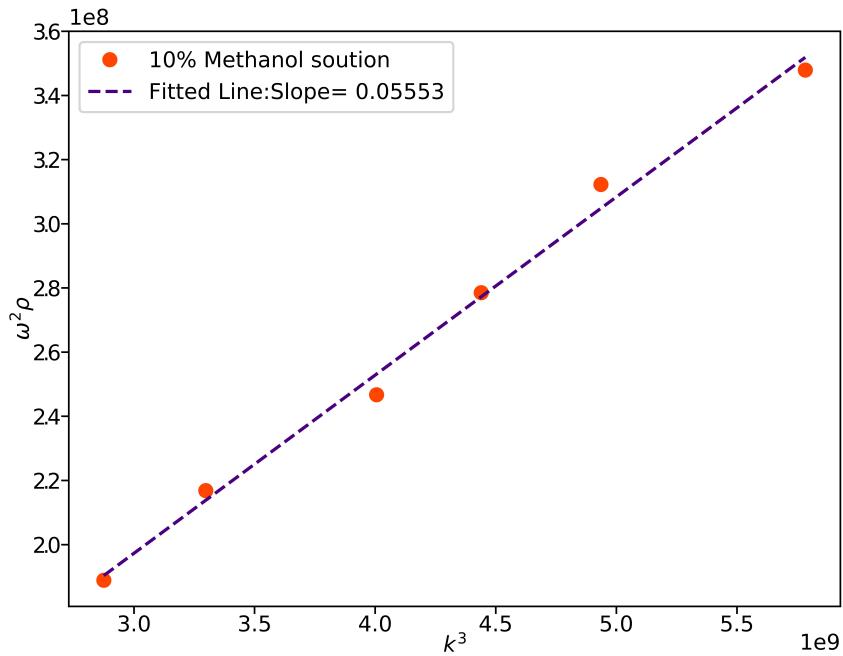


Figure 6.9: The figure shows a linear behaviour between  $\omega^2 \rho$  and  $k^3$  for 10% Methanol solution prepared in 500 ml of volumetric cylinder at room temperature of 25 °C. Experimental data are plotted and fitted with a straight line and the slope of the best fitted straight line is coming out to be  $0.05553 \pm 0.00030 \text{ N m}^{-1}$ .

$0.06863 \text{ N m}^{-1}$ . So the surface tension of 10% Methanol solution , obtained in this method of experiment, is  $55.53 \pm 0.30 \text{ N m}^{-1}$ .

### 6.4.3 25% Methanol solution

The methanol- water mixture was prepared in the 500 ml volumetric cylinder. To prepare 10% methanol solution in water we took 125 ml of methanol and the density of the solution was calculated as  $945.75 \text{ kg m}^{-3}$ . The grazing angle of incidence of laser beam was still set at  $3.71^\circ$ .

From the Figure 6.10 where experimental data are plotted for 25% Methanol solution

$\omega^2 \rho (\times 10^9)$	$\mathbf{k}^3 (\times 10^8)$
3.91427	1.57748
4.86847	1.8295
5.70536	2.10019
6.55217	2.38955
7.47887	2.69758
8.06548	3.02427
9.38023	3.36964
10.6625	3.73367

Table 10: This table contains the data for calculating the surface tension of 25% Methanol solution prepared in a 500 ml volumetric cylinder. The frequency is varied from 60 Hz to 100 Hz with an increment of 5 Hz and corresponding wave numbers are calculated using eq. 32.

and fitted with a straight line, the slope of the best fitted graph is coming out to be  $0.033 \text{ } 10 \text{ N m}^{-1}$ . So the surface tension of 25% Methanol solution , obtained in this method of experiment, is  $33.10 \pm 0.90 \text{ N m}^{-1}$ .

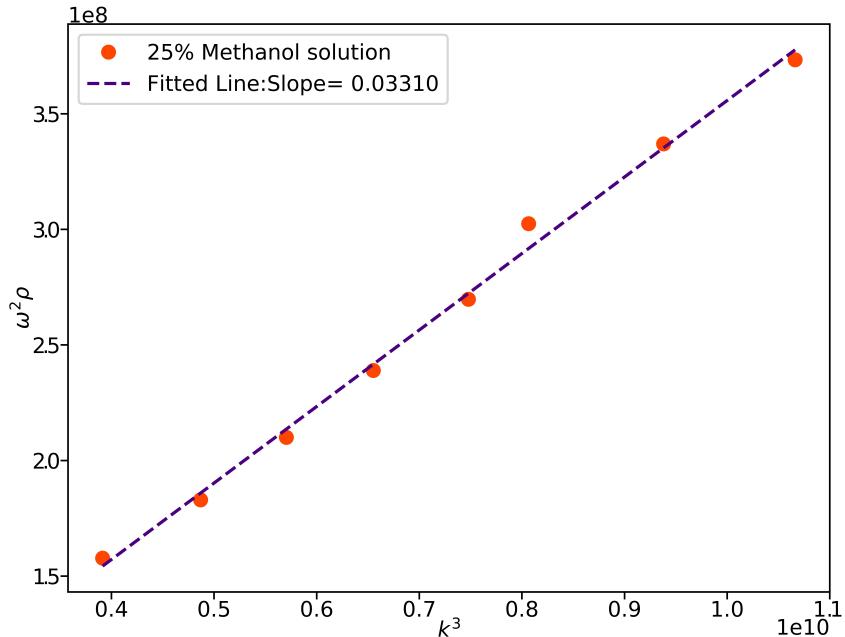


Figure 6.10: The figure shows a linear behaviour between  $\omega^2\rho$  and  $k^3$  for 25% Methanol solution prepared in 500 ml of volumetric cylinder at room temperature of 25 °C. Experimental data are plotted and fitted with a straight line and the slope of the best fitted straight line is coming out to be  $0.033\ 10 \pm 0.000\ 90\ N\ m^{-1}$ .

#### 6.4.4 Trend in Methanol-water mixture

In case of alcohols, just as it happens in case of many other biological molecules, the basic solubility rule that like dissolves like is a bit more complexed. Each alcohol consists of a carbon chain (always nonpolar) and a OH group (which is polar). For ethanol for example the chemical formula looks lie this: C<sub>2</sub>H<sub>5</sub>OH. Ethanol has a 2 carbon chain and a OH group. As water is polar it attracts OH group. Carbon chain on the other hand as nonpolar is repelled. Solubility of alcohols is therefore determined by the stronger of the two forces.

Because of the strength of the attraction of the OH group, first three alcohols (methanol, ethanol and propanol) are completely miscible. We have used methanol having the chemical formula CH<sub>3</sub>OH. By its chemical formula it is evident that methanol is completely miscible in water as the nonpolar carbon chain part is very small.

The interesting fact here is that the decrease of surface tension of water with the addition of methanol does not show the same linear trend as previously observed for

% concentration	Surface tension(mN m <sup>-1</sup> )
0	71.87
2	68.63
10	55.53
25	33.10
100	22.50

Table 11: This table contains the values of surface tension of Methanol-water mixture for different concentration of methanol in the water prepared in the 500 ml of a volumetric cylinder varying from pure distilled water to 25% methanol solution in water.

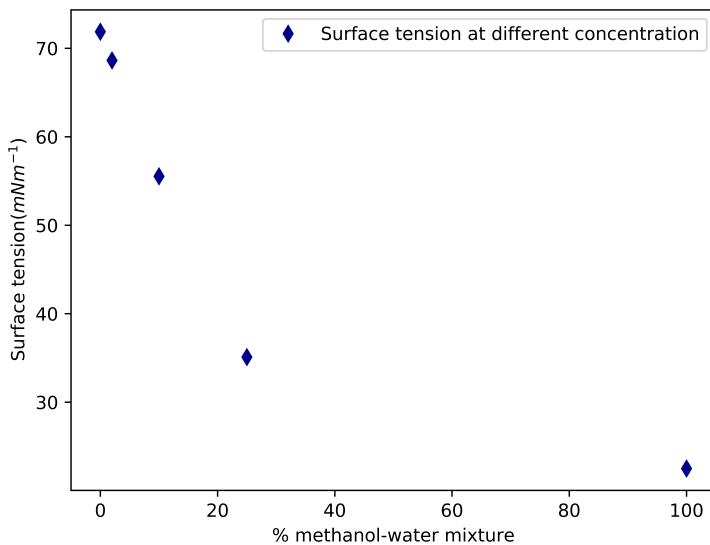


Figure 6.11: This figure shows the reduction of surface tension of distilled water with increase in the concentration of methanol. Linearity is not maintained here just like the case for NaCl solution.

NaCl solution[7] and that is obvious from Figure 6.11. The trend the solutions are following is complex in nature. As already mentioned the solubility of methanol in water is complex also there are several factor including polar interaction of methanol and water and H-bonding [8] because of which the there is an anomalous behaviour in the surface tension of methanol-water mixture of different concentration.

## 6.5 SDS Solution

Sodium dodecyl sulfate or commonly known as SDS has the molecular formula  $\text{NaC}_{12}\text{H}_{25}\text{SO}_4$  and consists of a 12-carbon tail attached to a sulfate group(see Figure 6.12), that is, it is the sodium salt of a 12-carbon alcohol that has been esterified to sulfuric acid. As a result of its hydrocarbon tail, and its anionic "head group", it has amphiphilic properties that allow it to form micelles, and so act as a detergent.

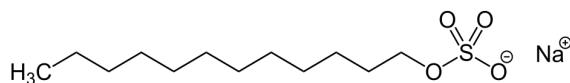


Figure 6.12: Skeletal formula of Sodium dodecyl sulfate or SDS ( $\text{NaC}_{12}\text{H}_{25}\text{SO}_4$ ) having a hydrophobic 12-carbon tail and a hydrophilic anionic head group.

We took 4 grams of SDS to make 0.027M solution in 500 ml of a volumetric cylinder. The density of the solution was {need to calculate} and the grazing angle of incidence of the laser beam was  $3.78^\circ$

$\omega^2 \rho (\times 10^9)$	$k^3 (\times 10^8)$
1.53215	0.994856
2.18152	1.43259
2.86605	1.94992
3.98299	2.54683
4.81037	3.22333
6.36011	3.97942

Table 12: This table contains the data for calculating the surface tension of 0.027M SDS solution prepared in a 500 ml volumetric cylinder. The frequency is varied from 50 Hz to 100 Hz with an increment of 10 Hz and using eq. 32 corresponding wave numbers are calculated.

For 0.027M SDS solution, from the slope of the fitted line in the Figure 6.13, we see that the surface tension of the solution is coming out to be  $62.46 \pm 0.20 \text{ mN m}^{-1}$ . Here we notice that after adding SDS to the distilled water, the surface tension is reduced from  $71.87 \text{ mN m}^{-1}$  straight to  $62.46 \text{ mN m}^{-1}$ . SDS is a surfactant and so, it must lower

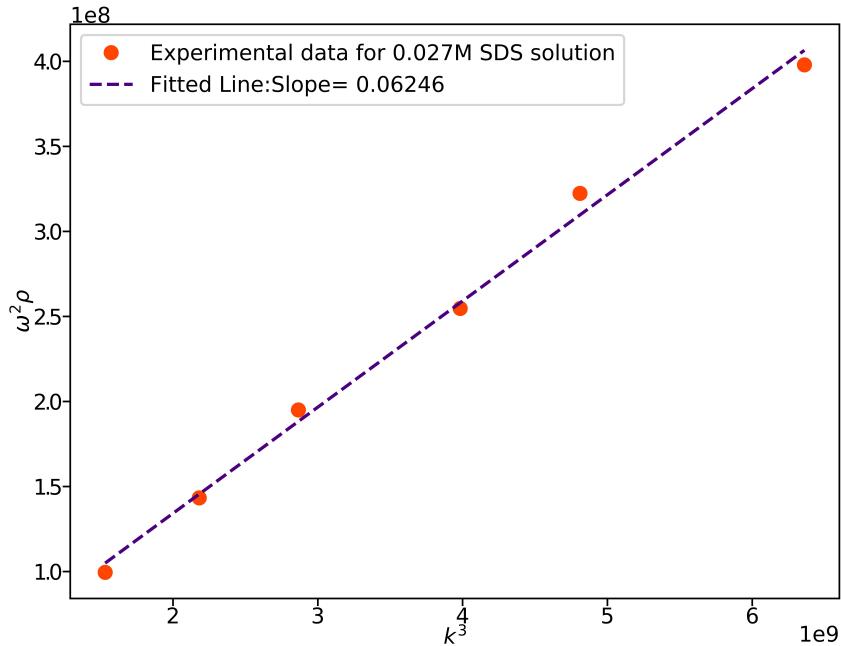


Figure 6.13: The figure shows a linear behaviour between  $\omega^2\rho$  and  $k^3$  for 0.027M SDS solution prepared in 500 ml of volumetric cylinder. Experimental data are plotted and fitted with a straight line and the slope of the best fitted straight line is coming out to be  $0.062\,46 \pm 0.000\,20\text{ N m}^{-1}$ .

down the surface tension of the water and experimentally this property is verified.

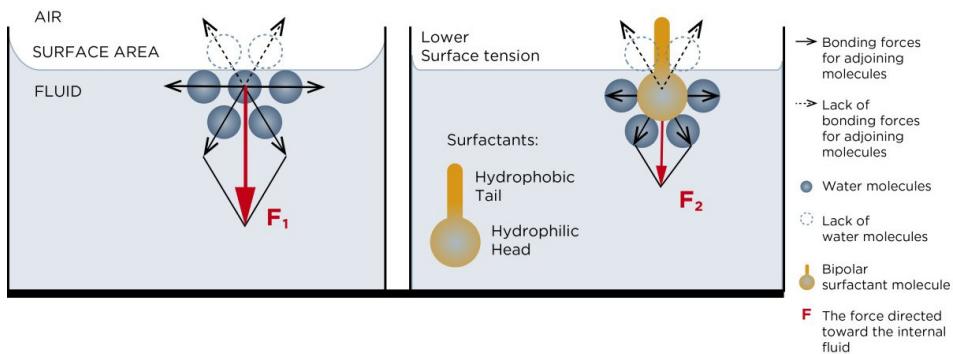


Figure 6.14: This figure shows the interaction of a surfactant molecule with a water molecule and how it reduces the surface tension of water.

## 7 Summary

The surface tension water is successfully verified using the method of using capillary waves as diffraction grating and the value is  $71.89 \text{ mN m}^{-1}$ . Also the coefficient of viscosity of distilled water has been verified. While performing the experiment for NaCl we observe a linear trend in the graph of surface tension vs. concentration. The interaction between the NaCl and H<sub>2</sub>O leads to this linear trend. But surprisingly there is no linear trend for the case of methanol water mixture. We see a decay like curve for increasing concentration of methanol in water. Why this is happening is bit complicated to explain in short. For surfactant molecules, they decreases the surface tension of water and this property is also observed in this project. The value of surface tension has gone straight to  $62.46 \text{ mN m}^{-1}$  from  $71.89 \text{ mN m}^{-1}$  for only 0.027M SDS solution in water.

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