Discreete Assignment No 2

Qno: 1

Ans: R is Reflexive as all reflexive pairs are present
i.e: (a,a), (b,b), (c,c), (d,d)

Qno: 2

a) $a=b$

Pairs: $\{(0,0), (1,1), (2,2), (3,3)\}$.

b) $a+b=4$

Pairs: $\{(1,3), (2,2), (3,1), (4,0)\}$

c) $a>b$

Pairs: $\{(1,0), (2,0), (2,1), (3,0), (3,1), (3,2), (4,0), (4,1), (4,2), (4,3)\}$.

d) $a|b$

$R = \{(1,0), (1,1), (1,2), (1,3), (2,0), (2,2), (3,0), (3,3), (4,0)$

e) GCD $(a,b) = 1$

$R = \{(0,1), (0,3), (1,0), (1,1), (1,2), (1,3), (2,1), (2,3), (3,1), (3,2),$
$(4,1), (4,3)\}$

f) LCM $(a,b) = 2$

$R = \{(1,2), (2,1), (2,2)\}$

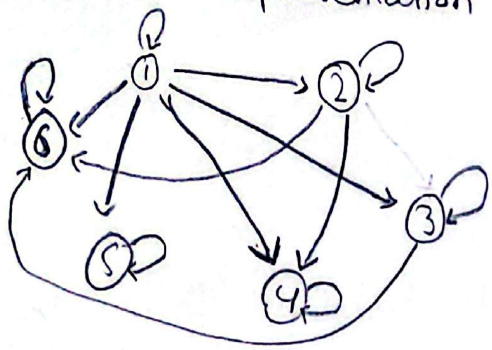
Qno:3

$R = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,2)\ (2,4),(2,6),(3,3),(3,6)$
$(4,4),(5,5),(6,6)\}$

Matrix Representation

$$M_R = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Graphical Representation



__Determinig:__

Reflexive : Yes. Contains (1,1), (2,2)...etc.
Symmentric: No. (1,2) exists but (2,1) does not
Antisymmentric: Yes. If a|b, b|a, then a=b exists
Transitive : Yes. Ex# (1,2) and (2,4) then there exist (1,4).
It is a Partial order Relation.

Q no: 4

a)

Reflexive: No

Symmentric: No

AntiSymmentric: No

Transitive: Yes, all paths leads to an edge. Ex: (2,3) and (3,2) then (2,2) exists.

b)

Reflexive: Yes, All four exists.

Symmentric: Yes, (1,2) and (2,1) exists.

AntiSymmentric: No.

Transitive: Yes, (1,2) and (2,1) exists and also (1,1) exists.

c)

Reflexive:- No.

Symmentric:- Yes.

AntiSymmentric:- No.

Transitive No.

d)

Reflexive: No.

Symmentric: No.

AntiSymmentric: Yes. No pair of (a,b) has its reverse (b,a) when $a \neq b$.

Transitive: No

e)

R:- Yes

S:- Yes

A.S: Yes. (Vacously)

T: Yes. (Vacously)

~~Ans:~~
f)
R : No.
S : No.
A.S; No.
T : No.

Q no. 5

a)
R: No.
S: No.
Anti.S: Yes. If a is taller than b and b is taller than A is impossible.
A.S: Yes.
Irref: Yes.
T: Yes, if a is taller than b and b is taller than c then a is taller than c.

b)
R : Yes
S : Yes
Anti.S: No
A.S: No
Irref: No
I: Yes

c)
R : Yes.
S : Yes.
Anti.S: No.
As : No.
Irref: No.
I: Yes.

d)

R : Yes
S : Yes
Anti.S: No
A.S: No
Irref: No
T: No

Qno:6

a) $R_a = \{(a,b) \mid a=b\}$

b) $R_b = \{(a,b) \mid a \le b \text{ and } a+b \text{ is even}\}$

c) $R_c = \{(0,0),(1,1),(2,2)\}$

d) $R_d = \{(1,2),(2,3)\}$

Qno:7

$R1 = \{(2,1),(3,1),(3,2)\}$
$R2 = \{(1,1),(2,1),(2,2),(3,1),(3,2),(3,3)\}$
$R3 = \{(1,2),(1,3),(2,3)\}$
$R4 = \{(1,1),(1,2),(1,3),(2,2),(2,3),(3,3)\}$
$R5 = \{(1,1),(2,2),(3,3)\}$
$R6 = \{(1,2),(1,3),(2,1),(2,3),(3,1),(3,2)\}$

a) $R2 \cup R4 = \{(1,1),(1,2),(1,3),(2,1),(2,2),(3,1),(3,2),(3,3)\}$

b) $R3 \cup R6 = R6$

c) $R_3 \cap R_6 = R_3$

d) $R_4 \cap R_6 = R_3$

e) $R_3 - R_6 = \phi$

f) $R_6 - R_6 = R_1$

g) $R_2 \oplus R_6$

Sol.

$(R_2 - R_6) \cup (R_6 - R_2)$

$R_2 - R_6 = R_5$

$R_6 - R_2 = R_3$

$R_5 \cup R_3 = R_4$ Ans

f) $R_3 \oplus R_5$

$R_3 - R_5 = R_3$

$R_5 - R_3 = R_5$

$R_3 \cup R_5 = R_4$ Ans

h) $R_2 \circ R_1 = R_1$

i) $R_6 \circ R_6 = R_6 \rightarrow$ Universal Relation on $\mathbb{R}$.

Q no 3 (a)

i) $\{(1,1), (1,2), (1,3)\}$

Ans: $M_R = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

iii) $M_R = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

ii) $M_R = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

iv) $M_R = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

b)

(i) $R = \{(1,1), (1,3), (2,2), (3,1), (3,3)\}$

(ii) $R = \{(1,1), (2,2), (3,2)\}$

(iii) $R = \{(1,1), (1,2), (1,3), (2,1), (2,3), (3,1), (3,2), (3,3)\}$

Q no: 9

a) Reflexivity:- (Check if $L(a) = L(a)$. Since the length of any string is always equal to itself, $l(a) = L(a)$ is always true. The Relation R is Reflexive.

Symmentry:-

b)? By fundamental property of equality (if $x = y$ then $y = x$), then it follow $l(a) = L(b)$. The relation R is symmentric.

Transitivity:- Since $L(a) = l(b)$ and $L(b) = l(c)$, by transitivity of equality, it follows that $l(a) = l(c)$. Result: The relation R is Transitive.

b)

i) Reflexivity:- $(a-a) = 0$ and m divides 0 $(0 = 0 \cdot m)$ the condition is satisfied. The Relation is Reflexive.

2) Symmentric:-
$$-(a-b) = -(k \cdot m)$$
$$b-a = (-k) \cdot m$$

Since k is an integer, -k is also an integer. Therefore b-a is a multiple of m, which proves $b \equiv a \pmod{m}$. Relation is symmentric

3) Transitivity let $a \equiv b$ and $b \equiv c$

Prove
$$(a-b) + (b-c) = (k \cdot m) + (l \cdot m)$$
$$a - c \equiv (k+l) \cdot m$$

Q no 10

a) $(2^n - 1)$

$n=1 \Rightarrow 2^{(1)} - 1 = 1$
$n=2 \Rightarrow 2^2 - 1 = 3$
$n=3 \Rightarrow 2^3 - 1 = 7$
$n=4 \Rightarrow 2^4 - 1 = 15$
$n=5 \Rightarrow 2^5 - 1 = 31$

b) $10 - \frac{3}{2}n$

Sol:

$n=1 \Rightarrow 10 - \frac{3}{2}1 \Rightarrow 10 - \frac{3}{2} = \frac{17}{2}$

$n=2 \Rightarrow 10 - \frac{3}{\cancel{2}} \times \cancel{2} \Rightarrow 7$

$n=3 \Rightarrow 10 - \frac{3}{2} \times 3 \Rightarrow 10 - \frac{9}{2} = \frac{11}{2}$

$n=4 \Rightarrow 10 - \frac{3}{\cancel{2}} \times \cancel{4}2 \Rightarrow 10 - 6 = 4$

$n=5 \Rightarrow 10 - \frac{3}{2} \times 5 = 10 - \frac{15}{2} = \frac{5}{2}$

c) $\frac{(-1)^n}{2^n}$

Sol:

$n=1 \Rightarrow (-1)/2 \Rightarrow -1/2$
$n=2 \Rightarrow (-1)^2/2^2 = 1/4$
$n=3 \Rightarrow (-1)^3/2^3 = -1/8$
$n=4 \Rightarrow (-1)^4/2^4 = 1/16$
$n=5 \Rightarrow (-1)^5/2^5 = -1/32$

d) $\frac{3n+4}{2n+1}$

Sol.

$n=1 \Rightarrow (3(1)+4)/2(1)+1 = 7/3$

$n=2 \Rightarrow (3(2)+4)/(2(2)+1) = 10/5 = 2$

$n=3 \Rightarrow (3(3)+4)/(2(3)+1) = 13/7$

$n=4 \Rightarrow (3(4)+4)/(2(4)+1) = 16/9$

$n=5 \Rightarrow (3(5)+4)/(2(5)+1) = 19/11$

Q no: 10 (b)

i)

Sol.

$d = a_2 - a_1 = -22-(-15) = -22+15 = -7$

$a = -15$, $n=11$

$\therefore a_n = a+(n-1)(-7)$

$a_{11} = -15+(11-1)(-7)$

$a_{11} = -15+(-70)$

$= -85$ Ans

ii)

Sol. → Arithmetic Sequence

$a = a-42b$

$d = a-39b - a+42b \Rightarrow d = 3b$

$n = 15$

$a_{15} = (a-42b)+(15-1)\,3b$

$= a-42b+42b$

$= a$

iii) → Arithmetic Sequence

$a = 4$

$r = \frac{a_2}{a_1} = \frac{3}{4}$

$n = 17$

$a_{17} = 4\left(\frac{3}{4}\right)^{16} = \frac{3^{16}}{4^{15}}$ Ans

Sol:-

$a = 32$

$r = \frac{16}{32} = 1/2$

$n = 9$

$a_9 = 32(1/2)^{9-1}$

$= 2^5 \frac{1}{2^{8}3} \Rightarrow \frac{1}{8}$ Ans.

Q no. 11 (a)

i) $T_3 = 10$, $T_5 = 2\frac{1}{2}$

Sol:-

$T_3 = 10 \rightarrow$ (i)

$T_5 = 2\frac{1}{2} \rightarrow$ (ii)

Divide 2 by 1

$\frac{T_5}{T_3} = \frac{\cancel{a}r^4}{\cancel{a}r^2} = r^2 = \frac{10}{5/4} \Rightarrow r^2 = \frac{40}{5} 8 \Rightarrow r^2 = 8 \Rightarrow \sqrt{r^2} = \sqrt{8} \Rightarrow r = 2\sqrt{2}$

from (i)

$ar^2 = 10$

$a(2\sqrt{2})^2 = 10$

$a(4\times 2) = 10 \Rightarrow a = \frac{5}{4}$

G.P is $= \frac{5}{4}, \frac{5\sqrt{2}}{2}, 10, \ldots$

(ii) $T_5 = 8$, $T_8 = -\frac{64}{27}$

$\frac{T_8}{T_5} = \frac{\cancel{a}r^7}{\cancel{a}r^4} = r^3 = -\frac{64}{27} \div 8 = -\frac{8}{27} \Rightarrow r = -\frac{2}{3}$

$ar^4 = 8$

$a = \left(\frac{3}{-2}\right)^4 8 \Rightarrow a = \frac{243}{\cancel{16}_2} \times \cancel{8}^1 \Rightarrow a = \frac{243}{2}$

G.P $= \frac{243}{2}, -\frac{243}{3}, \ldots$

Q no. 11 (b)

i) Sol:-

$a+3d=7 \rightarrow eq(i)$

$a+11d=31 \rightarrow eq(ii)$

Subtract eq(i) from (ii)

$$\begin{array}{r} a+11d=31 \\ -a\pm3d=\mp7 \\ \hline 12d=24 \end{array}$$

$d=2$

Put in (i)

$a+3(2)=7$

$a=1$

A.P is $= 1, 3, 5 \ldots$

iii) Sol:-

$a+4d=8$

$$\begin{array}{r} -a\pm7d=-\frac{64}{27} \\ \hline -3d=\frac{216+64}{27} \end{array}$$

$-3d=\frac{280}{27}$

$d=-\frac{280}{81}$

Put in (i)

$a+4\left(-\frac{280}{81}\right)=8$

$a=8+\frac{1040}{81}$

$a=\frac{648+1040}{81}=\frac{1688}{81}$

Q no. 12 a)

First multiple $= 259 \geq 256$

2nd multiple $= 784 \leq 789$

$n=\frac{784-259}{7}+1=76$

$\therefore S=\frac{n}{2}(\text{last}+\text{first})=\frac{76}{2}(259+784)=38\times1043=\boxed{39634}$ Ans

b) $S_n=\frac{n}{2}(\text{first}+\text{last})$

$=\frac{n}{2}\left(\frac{1}{n}+\frac{n^2-n+1}{n}\right)$

$=\frac{n}{2}\left(\frac{n^2-n+2}{n}\right)$

$=\frac{n^2-n+2}{2}$ Ans.

Q no. 13

a) $a_j = \frac{1}{j}$, $j = 1, 2, 3 \ldots$

$$\sum_{j=1}^{100} \frac{1}{j}$$

b) (i)

$1 - 1 + 1 - 1 + 1 = 1$

(ii)

$(1)^2 + (2)^2 + (3)^2 + (4)^2 + (5)^2 = 1 + 4 + 9 + 16 + 25 = 55$ A

Q no: 14

a)

A.P. = 40, 33, 26 ... a_n

$a = 40$, $d = -7$

$a_5 = 40 + 4(-7)$

$a_5 = 40 - 28$

$a_5 = 12$ feet

$S_{n} = \frac{5}{2}(40 + 12) = (\cancel{52})^{26} \frac{5}{\cancel{2}} = 130$ Am.

b)

P. population = 70,000

Rate = 7%

T = 6 years

$P = P_0 (1 + r)^n$

$P = 70000 (1 + 0.07)^6 = 70000 (1.07)^6 = 70000 \times 1.5 = 105051.$

Q no: 16

i)

- Graph has undirected edges
- Graph has undirected multiple edges
- Graph has no loops
- Hence it is an undirected graph

ii)

- Undirected edges
- No multiple edges
- No loops
- Undirected Simple Graph

iii)

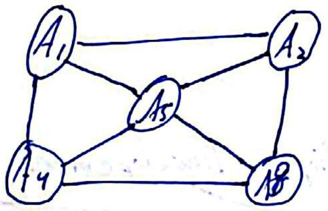
- undirected edges
- multiple edges
- 3 loops
- undirected psedocodes

iv)

- directed graph
- multiple edges
- 2 loops
- directed multigraph

Question 17 (a)

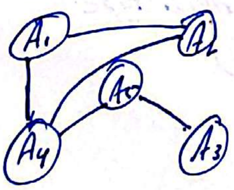
Sol:



Ans

(b)

Sol:



Question 18

(a)(i) = (i) vertices (v) = 5 , edges (e) = 18

(ii) Degrees of Vertices:- $d(a)=5$, $d(b)=6$, $d(c)=5$, $d(d)=5$, $d(e)=3$.

(iii) Neighborhood vertices: $N(a)=\{a,b,c\}$, $N(b)=\{a,c,d,e\}$, $N(c)=\{b,c,d\}$

$N(d)=\{b,c,e\}$, $N(e)=\{a,b,d\}$ Ans

(ii) $V=9$, $E=12$

$D = d(a)=3$, $d(b)=2$, $d(c)=4$, $d(d)=0$, $d(e)=6$, $d(f)=0$,

$d(g)=4$, $d(h)=2$, $d(i)=3$

$N = N(a)=\{c,e,i\}$, $N(b)=(e,h)$, $N(c)=\{a,e,g,i\}$, $N(d)=\phi$,

$N(e)=\{a,b,c,g\}$, $N(f)=\phi$, $N(g)=\{c,e\}$, $N(h)=\{b,i\}$,

$N(i)=\{a,c,h\}$ Ans.

(b)

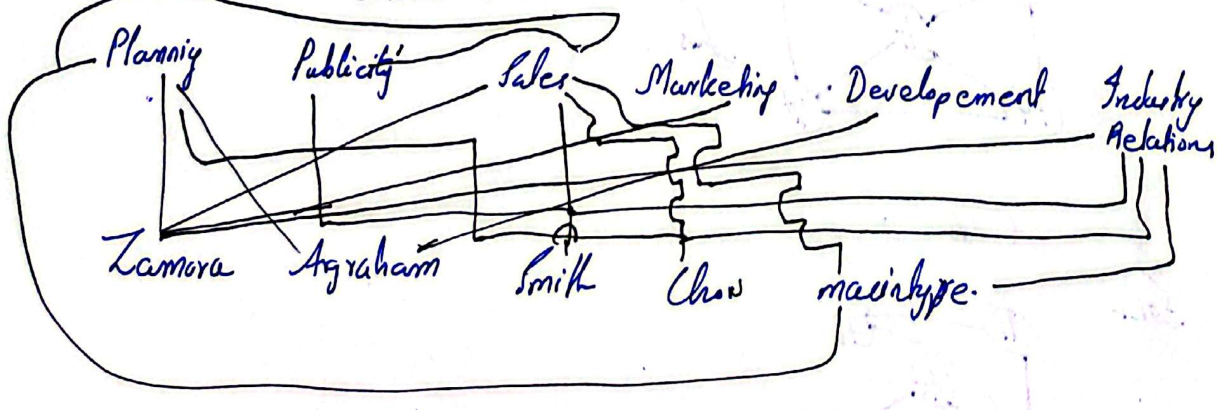
i) Indegree :- $d^-(a)=6$, $d^-(b)=1$, $d^-(c)=2$, $d^-(d)=0$

Outdegree: $d^+(a)=1$, $d^+(b)=5$, $d^+(c)=5$, $d^+(d)=2$, $d^+(e)=0$ Ans.

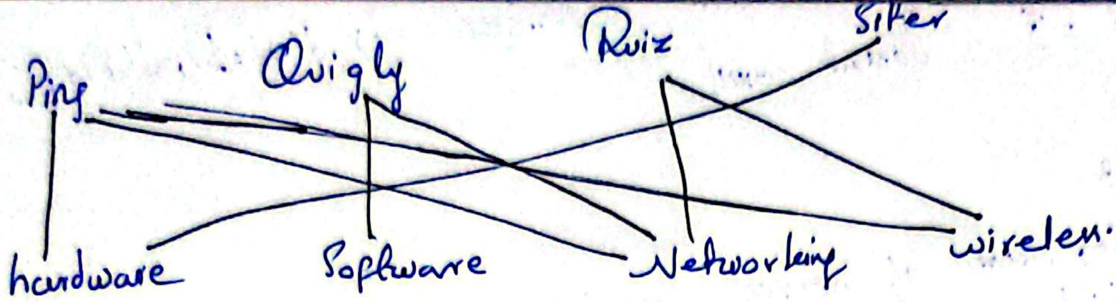
ii) Indegree:- $d^-(a)=2$, $d^-(b)=3$, $d^-(c)=2$, $d^-(b)=4$, ~~$d^-(e)=0$~~

Outdegree:- $d^+(a)=2$, $d^+(b)=4$, $d^+(c)=1$, $d^+(d)=2$, ~~$d^+(e)=0$~~ Ans

Qno:19 (a)



b)



Qno:20

a) Bipartite Graph → Not

b) Bipartite Graph

c) Not Bipartite

d) Not Bipartite.

Qno:21

a) i) Graph G = Bipartite => {u, w}, {v, x, y}

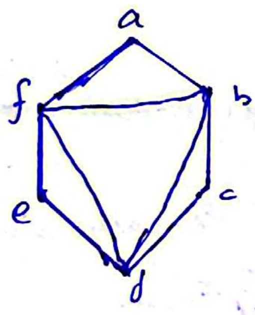
ii) Graph H = Bipartite => {a, d, c}, {b, e, f}

iii) Graph A = Not a Bipartite

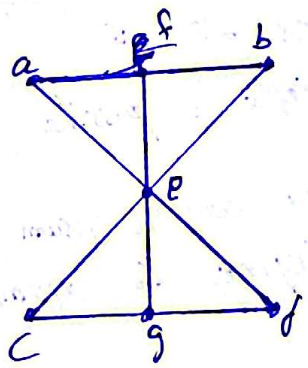
iv) Graph B = Bipartite => {d} {e, f, g, h, i, j, k, l}

Qno: 21 (b)

i)



(ii)



Qno:22 four vertices

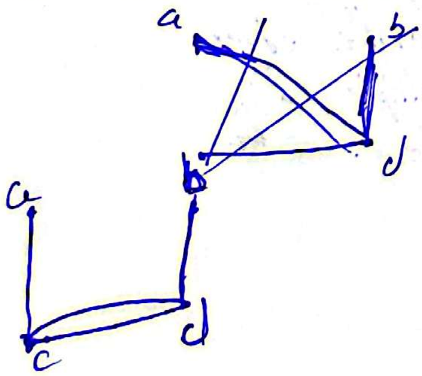
a) d(a) = 1, d(b) = 1, d(c) = 2, d(d) = 3

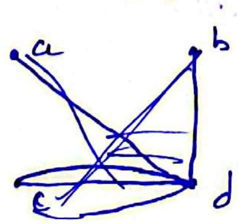
Sol:

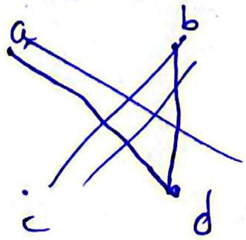
Total degree = 1 + 1 + 2 + 3 = 7

Acc to Handshaking theorem, total degree of a graph should be even. hence it is not a graph.

b) four vertices : d(a) = 2, d(b) = 1, d(c) = 3, d(d) = 3







c) A simple graph with 4 vertices

A simple graph with 4 vertices c) $d(a)=1$, $d(b)=2$, $d(c)=3$ $d(d)=3$ doesn't exist.

Qno:23

a) No, sum of degree must be even and ~~15~~ $15\times3=45$ ~~wid~~ which is odd Ans.

b) Yes, because sum of $4\times3=12$ which is even.

c) People are vertices & handshakes are edges, each person handshakes with 9 people, so total Handshakes would be:

$$T.no = \frac{\text{sum of degree}}{2} = \frac{90}{2} = 45 \text{ handshakes.}$$

d) 5 people with 2 degree = 10 dg sum.
so edges = $\frac{10}{2} = 5$
for 3 people = possible sum 4/2 sum of degree = ~~10 degree~~ ~~Ans~~ $3\times2 = 6$ Ans

e) If edges are 10, sum of degree = $10\times2=20$ & each degree has 4 degree so number of vertices = $\frac{20}{4} = 5$ Ans

Qno:24

a) w → connected
x → Not-connected
y → Connected
z → connected

b) w → cycle free
x → has cycles → circle
y → has triangle so has cycles
z → has cycles

c)
loop free: w, x, y.
has loops: z at c

d)
No loops
No parallel edges
No multiple edges
Undirected.

Question 25:

(i) Vertices = 5 , degree sequence = {4, 3, 3, 2, 2}

Sol:

⇒ Hence isomorphic

$g: V(G) \rightarrow V(G') \Rightarrow g(v_1) = w_1 , \; g(v_2) = w_3 , \; g(v_3) = w_2$

$g(v_4) = w_5 , \; g(v_5) = w_4$

(ii) Vertices = 6 , edges = 7 , degree sequence = 3, 3, 3, 2, 1}

Hence isomorphic

$g: V(G) \rightarrow V(G') = g(v_1) = u_5 , \; g(v_2) = u_4 , \; g(v_3) = u_3$

$g(v_4) = u_1 , \; g(v_5) = u_5$

(iii) vertices: 7 , edges: 9 , degree sequence = 4, 3, 3, 2, 2, 2}

Condition satisfied

Hence Isomorphic.

$g: V(G) \rightarrow V(G') \Rightarrow g(v_1) = u_5 , \; g(v_2) = u_4 , \; g(v_3) = u_3 ,$

$g(v_4) = u_2 , \; g(v_5) = u_7 , \; g(v_6) = u_1$

$g(v_7) = u_6$

(iv) vertices = 5 , edges = 7 , degree sequence different

Hence not isomorphic.

Q no 26

A ⇒ E = 5   F ⇒ E = 4   K ⇒ E = 4   M ⇒ E = 4   R ⇒ E = 5

V = 5   V = 5   V = 5   V = 5   R = 5

Seq = 2, 3, 3, 1, 1   Seq = 1, 2, 3, 1, 1   Seq = 1, 1, 4, 1, 1   Seq = 2, 2, 2, 1, 1   Seq = 2, 3, 3, 1, 1

S ⇒ E = 4   T ⇒ E = 4   V ⇒ E = 4   X ⇒ E = 4   Z ⇒ E = 4

V = 5   V = 5   V = 5   V = 5   V = 5

Seq = 1, 2, 2, 2, 1   Seq = 3, 1, 1, 2, 1   Seq = 2, 2, 2, 1, 1   Seq 4, 1, 1, 1, 1   Seq = 1, 2, 2, 2, 1

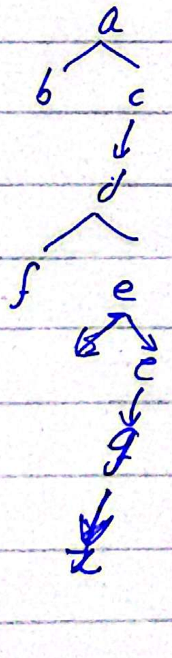
K & X are isomorphic

S, M, V & Z are isomorphic

F E Γ are isomorphic

Qno 27(a) ~~Shortest path = 2+1+3+2+2+2 = 20~~ Ans.

(b)	D(b)	D(c)	D(d)	D(e)	D(f)	D(g)	D(z)
a	4,a	3,a					
ac	4,a						
acb			6,c	9,c			
acbd			6,c	9,d			
acbdc				7,d	11,d		
acbdce					11,d	12,e	
acbdeef							8,f
acbdeefg						12,e	18,f
acbdefgz							

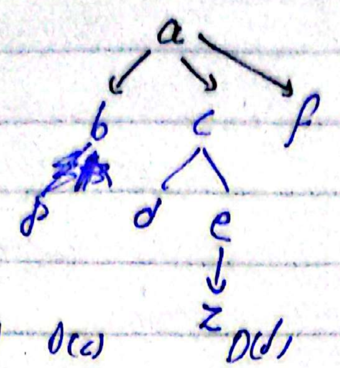


path a to z = a-c-d-e-g-h

length = 3+3+1+5+2+4 = 16

Ans: 27

(c)



	D(b)	D(c)	D(d)	D(e)	D(f)	D(z)
a	12,a	12,a	(12,a)			
ac	12,a		(12,a)	(13,c)	(21,c)	
acb			(12,a)	(13,c)	21,c	
acbf				(13,c)	21,c	21,f
acbfc					21,c	21,e
acbfez					21,C	
acbfezd						

path = a-c-e-z
= 10+3+6 = 19 Ans.

a) ABCDA = 125
ABDCA = 140
ACDBA = 140
ADBCA = 155
So min is ABCDA = 125

b) ABCDA = 108
ADBCA = 141
ADCBA = 119
ACBDA = 141
ABCDA = 97
Min is = 97

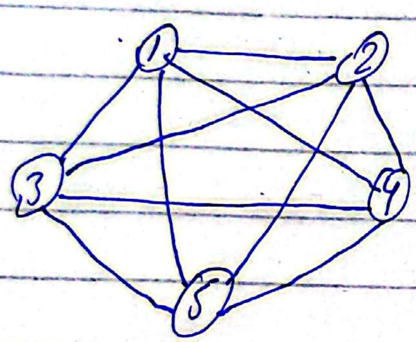
c) ADBCA = 95
ADCBA = 80
ABCDA = 90
ABDCA = 95
Min is = 80.

Qno.29

part (a) This is euler path & A (Source) & (Destination) E have odd degrees & every other have even, so yes it is possible.

Yes, It is possible because A & B have odd edges.

Every degree has even degree so it is Euler-Circuit.



2 → 5 → 1 → 4 → 3 → 2.

Qno.30

a) i) $V_1 - V_2 - V_3 - V_7 - V_4 - V_5 - V_6 - V_2 - V_1$ (H-Circuit)
$V_1 - V_0 - V_6 - V_4 - V_7 - V_6 - V_2 - V_3$ (H-path)

(ii) b - a - c - h - g - h - f - g - n ⇒ (H-circuit) → No path.
~~a - b - c - d - g - f - e~~

(iii) a - b - gc - d - e - f - g - a H-circuit.
a - b - c - d - g - f - c (H path)

b) i) a-h-g-f-e-~~d~~-c-b-a (H-circuit)

~~(ii) No~~ a-b-h-g-f-e-d-c-~~b-c~~

c) (ii) No circuit. d-e-f-g-j-c-b-a (H.path)

d)(iii) H-circuit a-x-b-y-c-z

H-path a-x-by-c-z

Q.no:31

(a)

i) It is eular circuit as every vertex has even degree.

ii) Not a Eular circuit because some vertex has odd degree.

(iii) It is eular circuit.

(b)

i) Path doesn't exist as v and w has odd degree.

(ii) Yes, it is Eular-path because only v & w has odd degree.

Q.no: 32 (a)

i)

	e_1	e_2	e_3	e_4	e_5	e_6	e_7
v_1	1	1	1	0	0	0	0
v_2	0	0	0	0	1	1	1
v_3	0	1	1	1	0	0	0
v_4	0	0	0	$\cancel{0}$	1	0	0
v_5	0	0	0	0	0	1	0
v_6	1	0	0	0	0	0	1

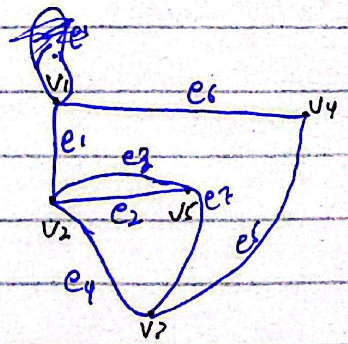
$v.$

ii

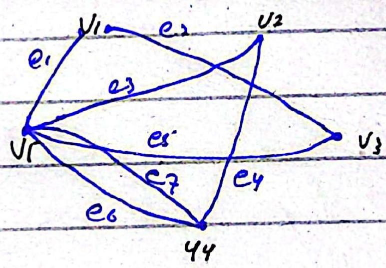
	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
v_1	1	1	1	0	0	0	0	0
v_2	0	1	ϕ	1	0	1	1	0
v_3	0	0	0	1	1	0	0	0
v_4	0	0	0	0	0	0	1	1
v_5	0	0	0	0	1	1	0	0

(b)

i)



iii



Qno:33 (Part(a))

1)

	a	b	c	d
a	1	1	1	1
b	0	0	0	1
c	1	1	0	0
d	0	1	1	1

list

a → a, b, c, d

b → d

c → a, b

d → b, c, d

	a	b	c	d	e
a	0	1	0	1	0
b	1	0	1	1	1
c	0	1	1	0	0
d	1	0	0	0	1
e	0	0	1	0	1

list

a → b, d

b → a, c, d, e

c → b, c

d → a, e

e → c, e

(iii) (a)

	a	b	c	d
a	0	3	0	1
b	3	0	1	0
c	0	1	0	3
d	1	0	3	0

a → b, b, b, d
b → a, a, a, c
c → b, d, d, d
d → a, c, c, c

(iv)

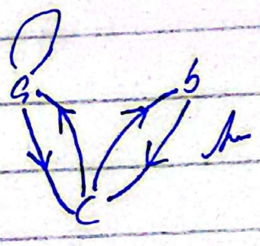
	a	b	c	d
a	1	0	2	1
b	0	1	1	2
c	2	1	1	0
d	1	2	0	1

a → a, c, c
b → b, c, d, d
c → a, a, b, c
d → a, b, b, d

(b)

i)

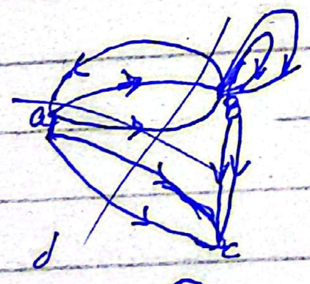
	a	b	c
a	1	0	1
b	0	0	1
c	1	1	1

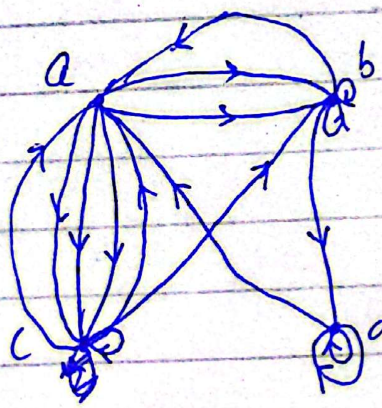


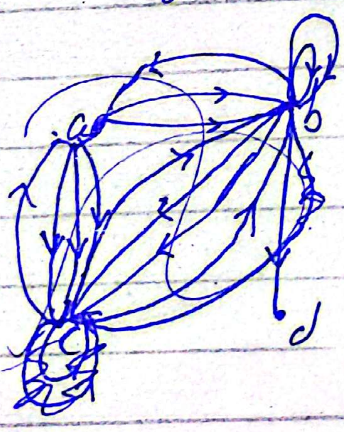
ii)

	a	b	c	d
a	0	2	3	0
b	1	2	2	1
c	2	1	1	0
d	1	0	0	2

Graph







Question 34

a) i) Complete bip~~artite~~artite graph is never planar ≤

$e \le 3v - 6$ $\therefore v = 5$

$6 \le 15 - 6$

$6 \le 9$

iii) It has no circuit of length to 3 so, not a planar $e \le 3v - 4$

$6 \le 18 - 4$

$6 \le 14$

Euler not applicable.

b) $G \Rightarrow v = 4$, edges $= 6$, deg-seq. 3, 3, 3, 3, odd degree.

$R = 4$, ~~A E V~~ $R = E - V + 2 \Rightarrow 6 - 4 + 2 = 4 = 4$

$H \Rightarrow v = 5$, $E = 9$, $R = 5 \Rightarrow R = E - V + 2 \Rightarrow 5 = 5$

$J \Rightarrow v = 5$, $E = 10$, $R = 7$

$R = E - V + 2 \Rightarrow 7 = 7$

Question. 35

a) 3   0   5   u, v   d

k, l   s, m, t, x, y   a, b, c, d, e, f, h, i   every letter except x, y.

j, l, q, r, g, s, x, y, u, w, z, p

Q 36) a) i) $v_0 \to v_5 \to v_6 \to v_7 \to v_2$ total = 6?

$\hookrightarrow v_4 \to v_3 \to v_1$

(ii) $v_0 \to v_1 \to v_3 \to v_2 \to v_7 \to v_6 \to v_5$

(b)

i) d,f = 1

a,c = 2

a,b = 3

b,c = 3 x

c,d = 4

d,f = 1

c,a = 6

c,f = 6 x

$a \to b$

$\hookrightarrow c \to d \to f$

length

2 + 4 + 1 =

(ii) (g, f) = 1

a,b = 2

a,c = 2

a, b = 3

b,c = 3

b, g = 4

b, c = 4

f, g = 4

g, e = 5

h, e = 7

c, e = 7

c, d = 16

d, e = 11 X

d, f = 12 X

$g \to f \to e \to a \to$

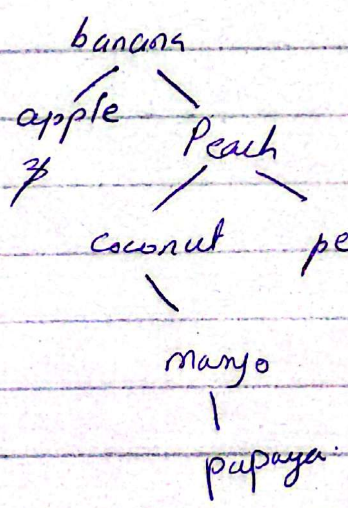
$b \to c \to d$

1 + 3 + 3 + 4 + 7 +

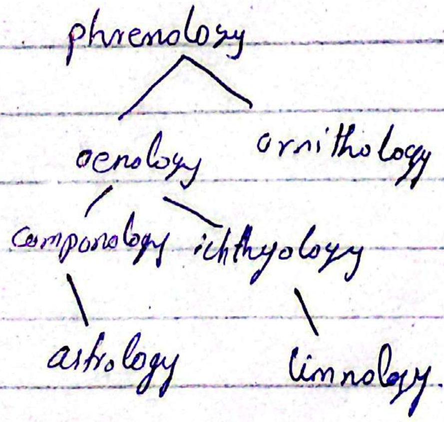
10 = 28 Ans

Ans: 37

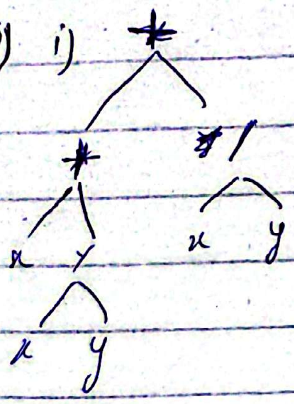
a)



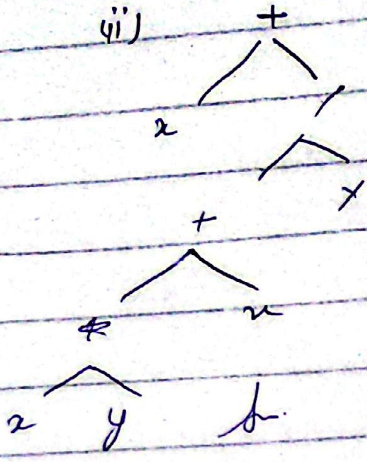
b)



1(b) i)



(ii)



Question # 38

a) i) pre-order: a, b, e, k, l, m, f, g, g, h, r, s, c, c, d

iii) pre-order: a, b, d, e, i, j, m, n, o, f, h, l, p

i) Inorder = k, e, l, m, b, f, r, n, s, g, a, c, o, h, d, i, p, j, q

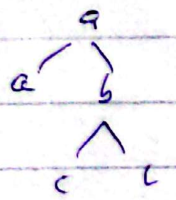
ii) Inorder = d, b, i, e, m, j, n, o, a, f, c, k, h, g, l.

i) ~~pos~~ postorder: k, l, m, e, f, r, s, n, g, b, c, o, l, i, p, q, j, d, a

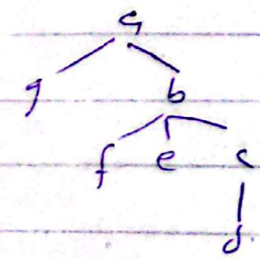
iii) post-order: d, i, m, n, o, j, e, b, f, g, p, c, a.

(b)

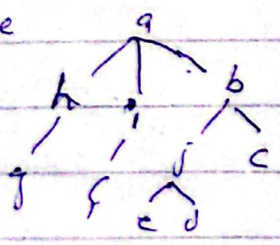
i) Remove (d,b), (c,e)



ii) Remove (g,b), (g,f), (f,e), (c,e), (e,d), (r,d), (g,d)



iii) Remove



Ans Qno.39 (a) 10000-1

a) Edges = vertices - 1 = 9999

b) Edges = 2l = 2(1000) = 2000 edges

c) Total vertices = mi + 1

d) n = mi + 1 = 100 - 33 = 67 Ans

Qno:40

a) $(x+xy)+(x/y)$

prefix: $++x * xy / xy$

postfix: $x x y * + x y / +$

$x+((xy+x)/y)$

prefix: $+x/+*xyxy$

postfix: $xxy*x+y/+$

b) i) prefix: ~~+-98/62~~ ~~+-↑32↑23/62 = 4-2=2~~

$+-98/625+13 = 4$

ii) postfix: $121* 14 *-124/-3$

Qno:41 (a)

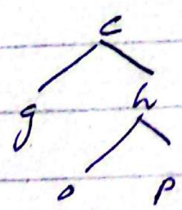
No, it does not follow the condition that the parents should have 0 or m childs.

(b)

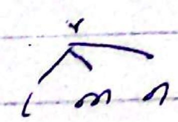
No, because it has leaves on level 1 to level 8 & height = 5. So, acc to BBT we should only have leaves on 5^{th} or 4^{th} level.

(c)

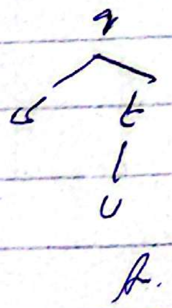
i) Subtree of c



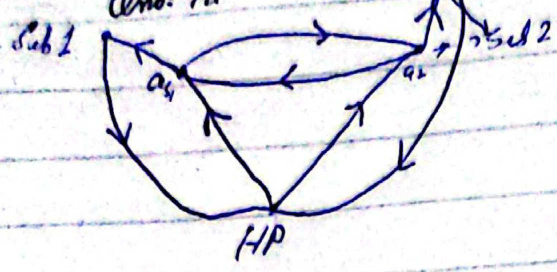
ii) Subtree of f



Subtree of E



Ans: 42 (a)



(b) GS1 ⇒ indegree = 2 outdegree = 2
GS2 ⇒ indegree = 2 ——— = 2
HP ⇒ —— = 2 —— = 2
Sub1 ⇒ —— = 1 —— = 2
Sub2 ⇒ —— = 1 ..... = 2

(c) Neither it is equivalence nor partial order because for both we need reflexive property.

Question 44

(a)

N	~~#~~	A	B	C	D	E	F	S
H		(3,H)	(2,H)	(5,H)	–	–	–	–
HB				(3,B)	(6,B)	–	–	
HBDA				(3,B)	(8,B)	(4,D)		
HBDA					(8,B)	(7,C)		
HBDAC					(7,C)	(7,D)		
HBDACE								(11,E)
HBDACEF								(9,F)

Shortest path is 9 for H.

Q.no: 44 (b)

b) deg(H) = 3 , deg(A) = 2 , deg(B) = 3
deg(C) = 2 , deg(D) = 3 , deg(E) = 4
deg(F) = 3 , deg(G) = 2

→ odd degree vectors (H, B, D, F).

→ As it doesn't have even degree so it is not euler.

(C)

By writing each vertex we cannot reach to the source or to any other vertex like home to school crossing every vertex so its neither hamilton circuit nor path.

Q.no: 45 (a)

a) ⇒ no of vertices = 8 , vertices = 12
Sequence of degree = 3, 3, 3, 3, 3, 3, 3, 3

⇒ Mapping ::

r(A) = 1 r(d) = 4 r(y) = 7
r(B) = 2 r(w) = 5 r(z) = 8
r(C) = 3 r(x) = 6

This proves that G and H are isomorphic.

b)

i) No it is not bipartite graph because each vertex or disjoint set. u = {A, B, C, D} is not connected to all vertex or ? distinct disjoint set.

ii) It is not euler circuit because vertices degree should be even and here it is odd. Yes, hamilton circuit exists: 1 → 2 → 4 → 7 → 6 → 5 → 8 → 3 → 1 A.

$A \times (B \cap C) = (A \times B) \cap (A \times C)$

Ans: 16

a)

Sol:

Given function can be written as:

$f = \{(a,b),(b,a),(c,c),(d,d)\}$

(ii) Yes, it is injective because every domain extracts different ranges.

Yes, Surjective

Yes Bijective

i) Range $= \{a,b,c,d\}$, Codomain $= \{a,b,c,d\}$, Domain $= \{a,b,c,d\}$

(iii) Yes, inverse exists.

b) $f(a)=b$, $f(b)=b$, $f(c)=d$, $f(d)=c$

(i) Range $=\{b, b, d, c\}$, Domain $=\{a, b, c, d\}$

(ii) No, Both a and b map to b. Surjective? No (element a of codomain is not an image. So not Bijective

(iii) No inverse b/c Not one $\longrightarrow$ one.

c) $f: a \longrightarrow d$, $b \longrightarrow b$, $f(c)=c$, $f(d)=d$

i) Range $=\{d, b, c, d\}$, Domain $=\{a, b, c, d\}$

(ii) ~~No: bo~~ Injective: No, both a and d maps d.

Surjective: No, (element a of codomain is not image)

Bijective: No

(iii) No, inverse

d) $f(a)=c$, $f(b)=a$, $f(c)=b$, $f(d)=d$

i) Range $=\{c, a, b, d\}$, Domain $=\{a, b, c, d\}$.

(ii) Injective; Yes

Surjective; Yes

Bijective: Yes.

(iii) Inverse exists.

(47) Let $f(x) = \left\lceil \frac{x^2}{3} \right\rceil$ find $f(S)$ if:

i) $S=\{-2, -1, 0, 1, 2, 3\}$

$f(-2)=4/3$, $f(-1)=1/3$, $f(0)=0$, $f(1)=1/3$, $f(2)=4/3$, $f(3)=9/3=3$

$f(S)=\{4/3, 1/3, 0, 1/3, 4/3, 3\}$.

(ii) $S=\{0, 1, 2, 3, 4, 5\}$

$f(0)=0$, $f(1)=1/3$, $f(2)=4/3$, $f(3)=3$, $f(4)=16/3$, $f(5)=25/3$

$f(S)=\{0, 1/3, 4/3, 3, 16/3, 25/3\}$.

iv) $S = \{2, 6, 10, 14\}$

Sol.:

$f(2) = 4/3$, $f(6) = 12$, $f(10) = 100/3$, $f(14) = 196/3$

$f(S) = \{4/3, 12, 100/3, 196/3\}$

Q no.

(b)

i) $\lceil \frac{3}{4} \rceil = \lceil 0.75 \rceil = 1$ Ans.

(ii) $\lfloor \frac{7}{8} \rfloor = 0.875 = 0$ Ans

(iii) $\lceil -\frac{3}{4} \rceil = -0.75 = 0$ Ans

iv) $\lfloor -\frac{7}{8} \rfloor = -0.875 = -1$ Ans.

(v) $\lceil 3 \rceil = 3$ Ans.

(vi) $\lfloor -1 \rfloor = -1$ Ans

(vii) $\lfloor \frac{1}{2} + \lceil \frac{3}{2} \rceil \rfloor = \lfloor 0.5 + 2 \rfloor = \lfloor 2.5 \rfloor = 2$ Ans.

(viii) $\lfloor \frac{1}{2} \cdot \lceil \frac{5}{2} \rceil \rfloor = \lfloor 0.5 \cdot \lceil 2.5 \rceil \rfloor = \lfloor (0.5) \cdot 2 \rfloor = 1$ Ans.

c)

i) $\lfloor -x \rfloor = -\lceil x \rceil$

Sol.:

Let $n = \lceil x \rceil$

By definition of $\lceil \ \rceil$

$n - 1 < x \leq n$

Multiply (-1) reverses inequalities

$-n + 1 > -x \geq -n$

we have

$-n \leq -x < -n + 1$

This means $-n$ is the greatest integer less than or equal to $-x$

$\lfloor -x \rfloor = -n = -\lceil x \rceil$

2) $\lceil -x \rceil = -\lfloor x \rfloor$

Similarly as above

Let $m = \lfloor x \rfloor$

$m \le x < m+1$

Multiply -1

$-m \ge -x \ge -m-1$

$-m-1 < -x \le -m$

So, $-m$ is the smallest integer $\ge x$.

$\lceil -x \rceil = -m = -\lfloor x \rfloor$

Q.no 48 $f(a) = 2a+3$, $g(a) = 3a+2$

Function	Composition Res	Injective?	Surjective? (onto $\mathbb{Z}$)	Invertible
$f \circ g$	$6a+7$	Yes	No	No
$g \circ f$	$6a+11$	Yes	No	No
f	$2a+3$	Yes	No	Yes (if codomain $\equiv$ odd ints)
g	$3a+2$	Yes	No (only ints $\equiv 2$ mod 3)	Yes (if codomain $= \{n \equiv 2 \bmod 3\}$).

CamScanner

Qno: 49

a) Functions:-

i) Temperature Conversion:- Functions convert one measurement to another ; eg celcius → Fahrenheit

2. Salary Calculation:- A company uses a function to calculate salary based on hours worked and hourly rate.

b) Relations

i) Social Networks: "Is -friends-with" or "follows" represent a relation b/w users.

(ii) E-commerce : The relation b/w customers and products purchased helps recommend items.

c) Sequence:

i) Daily whether Data:- Recording temperature each day forms a numerical sequence.

ii) Traffic light timming: The time interval for signal.

d) Series:

Loan / Emi calculation : monthly payments are based on the formula for sum of a geometric seq where interest is applied.

Population -Growth models:-

Used to predict total population over time.

e) Graph Theory:-

i) Google Theory:- Roads and intersections are modeled as vertices & edges to find shortest-paths.

iii) Social media Networks:-

Friendships are designed using Graph theory.

f) Trees:

i) File System Organization

Folders and files in computers are structured as a tree.

ii) Decision-Making /AI:-

Decision trees are used in machine learning and game algorithms.