ALTERNATING CURRENT

Alternating Quantities (i or V)

(1) An alternating quantity (current i or voltage V) is one whose magnitude changes continuously with time between zero and a maximum value and whose direction reverses periodically.

(2) Some graphical representation for alternating quantities

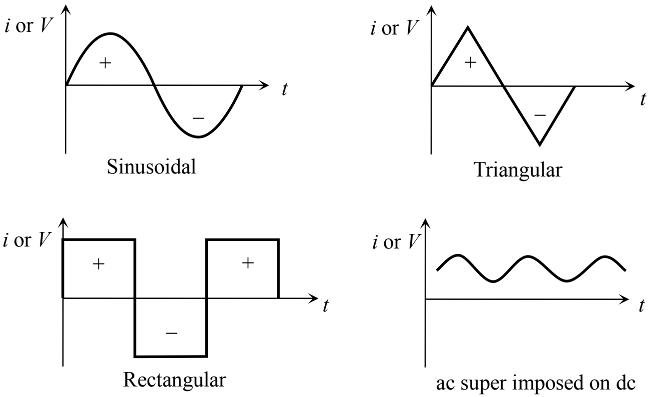


Fig. 24.1

(3) **Equation for i and V** : Alternating current or voltage varying as sine function can be written as

$$i = i_0 \sin\omega t = i_0 \sin 2\pi\nu t = i_0 \sin \frac{2\pi}{T} t$$

and $$V = V_0 \sin\omega t = V_0 \sin 2\pi\nu t = V_0 \sin\frac{2\pi}{T} t$$

where i and V are Instantaneous values of current and voltage, i_0 and V_0 are peak values of current and voltage

ω = Angular frequency in rad/sec,

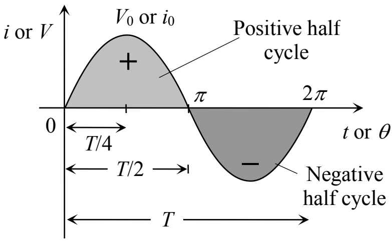


Fig. 24.2

ν = Frequency in Hz and T = time period

(i) The time taken to complete one cycle of variations is called the periodic time or time period.

(ii) Alternating quantity is positive for half the cycle and negative for the rest half. Hence average value of alternating quantity (i or V) over a complete cycle is zero.

(iii) The value of alternating quantity is zero or maximum 2ν times every second. The direction also changes 2(times every second.

(iv) Generally sinusoidal waveform is used as alternating current/voltage.

(v) At $t = \frac{T}{4}$ from the beginning, i or V reaches to their maximum value.

Important Values of Alternating Quantities

(1) **Peak value (i_0 or V_0)** : The maximum value of alternating quantity (i or V) is defined as peak value or amplitude.

(2) **Mean square value** $(\overline{V^2}$ or $\overline{i^2})$: The average of square of instantaneous values in one cycle is called mean square value. It is always positive for one complete cycle. e.g. $\overline{V^2} = \frac{1}{T}\int_0^T V^2 dt = \frac{V_0^2}{2}$ or

$$\overline{i^2} = \frac{i_0^2}{2}$$

(3) **Root mean square (r.m.s.) value** : Root of mean of square of voltage or current in an ac circuit for one complete cycle is called r.m.s. value. It is denoted by V_{rms} or i_{rms}

$$i_{rms} = \sqrt{\frac{i_1^2 + i_2^2 + \ldots}{n}} = \sqrt{\overline{i^2}} = \sqrt{\frac{\int_0^T i^2 dt}{\int_0^T dt}} = \frac{i_0}{\sqrt{2}}$$ = 0.707 i0 = 70.7% i_0

Similarly $V_{rms} = \frac{V_0}{\sqrt{2}} = 0.707\, V_0 = 70.7\%$ of V0

$$\left[\langle \sin^2(\omega t)\rangle = \langle \cos^2(\omega t)\rangle = \frac{1}{2}\right]$$

(i) The r.m.s. value of alternating current is also called virtual value or effective value.
(ii) In general when values of voltage or current for alternating circuits are given, these are r.m.s. value.
(iii) ac ammeter and voltmeter are always measure r.m.s. value. Values printed on ac circuits are r.m.s. values.
(iv) In our houses ac is supplied at 220 V, which is the r.m.s. value of voltage. It's peak value is $\sqrt{2} \times 200 = 311\,V.$
(v) r.m.s. value of ac is equal to that value of dc, which when passed through a resistance for a given time will produce the same amount of heat as produced by the alternating current when passed through the same resistance for same time.
(4) **Mean or Average value (i_{av} or V_{av})** : The average value of alternating quantity for one

complete cycle is zero.
The average value of ac over half cycle (t = 0 to T/2)

$$i_{av} = \frac{\int_0^{T/2} i\,dt}{\int_0^{T/2} dt} = \frac{2i_0}{\pi} = 0.637 i_0 = 63.7\%$$ of i_0,

Similarly $V_{av} = \frac{2V_0}{\pi} = 0.637 V_0 = 63.7\%$ of V_0.

(5) **Peak to peak value** : It is equal to the sum of the magnitudes of positive and negative peak values
(Peak to peak value = $V_0 + V_0 = 2V_0$

$$= 2\sqrt{2}\,V_{rms} = 2.828\,V_{rms}$$

Phase

Physical quantity which represents both the instantaneous value and direction of alternating quantity at any instant is called it's phase. It's a dimensionless quantity and it's unit is radian.
If an alternating quantity is expressed as $X = X_0 \sin(\omega t \pm \phi_0)$ then the argument of $\sin(\omega t + \phi)$ is called it's phase. Where ωt = instantaneous phase (changes with time) and ϕ_0 = initial phase (constant w.r.t. time)

(1) **Phase difference (Phase constant)** : The difference between the phases of currents and voltage is called phase difference. If alternating voltage and current are given by $V = V_0 \sin(\omega t + \phi_1)$ and $i = i_0 \sin(\omega t + \phi_2)$ then phase difference (= (1 – (2 (relative to current) or $\phi = \phi_2 - \phi_1$ (relative to voltage)

(2) **Time difference** : If phase difference between

alternating current and voltage is (then time difference between them is given as $T.D. = \frac{T}{2\pi} \times \phi$

(3) **Phasor diagram** : A diagram representing alternating current and alternating voltage (of same frequency) as vectors (phasors) with the phase angle between them is called a phasor diagram. While drawing phasor diagram for a pure element (e.g. R, L or C) either of the current or voltage can be plotted along X-axis.

But when phasor diagram for a combination of elements is drawn then quantity which remains constant for the combination must be plotted along X-axis so we observe that

(i) In series circuits current has to be plotted along X-axis.

(ii) In parallel circuits voltage has to be plotted along X-axis.

Measurement of Alternating Quantities

Alternating current shows heating effect only, hence meters used for measuring ac are based on heating effect and are called hot wire meters (Hot wire ammeter and hot wire voltmeter)

Impedance, Reactance, Admittance and Susceptance

(1) **Impedance (Z) :** The opposition offered by ac circuits to the flow of ac through it is defined it's impedance. It’s unit is ohm(Ω).

(2) **Reactance (X)** : The opposition offered by inductor or capacitor or both to the flow of ac through it is defined as reactance. It is of following

two type

(i) **Inductive reactance (X_L)** : Offered by inductive circuit $X_L = \omega L = 2\pi\nu L$ $\nu_{dc} = 0$ so for dc, $X_L = 0$.

Capacitive reactance (X_C) : Offered by capacitive circuit $X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$ for dc XC = (.

(3) **Admittance (Y)** : $Z = \frac{V_0}{i_0} = \frac{V_{rms}}{i_{rms}}$ Reciprocal of impedance is known as admittance $\left(Y = \frac{1}{Z}\right)$. It's unit is mho

(4) **Susceptance (S)** : the reciprocal of reactance is defined as susceptance $\left(S = \frac{1}{X}\right)$. It is of two type

(i) inductive susceptance $S_L = \frac{1}{X_L} = \frac{1}{2\pi\nu L}$ and

(ii) Capacitive susceptance, $S_C = \frac{1}{X_C} = \omega C = 2\pi\nu C$.

Power in ac Circuits

In dc circuits power is given by P = Vi. But in ac circuits, since there is some phase angle between voltage and current, therefore power is defined as the product of voltage and that component of the current which is in phase with the voltage.

Thus $\boldsymbol{P = V\, i \cos\phi}$; where V and i are r.m.s. value of voltage and current.

(1) **Instantaneous power** : Suppose in a circuit $V = V_0 \sin\omega t$ and $i = i_0 \sin(\omega t + \phi)$ then

$P_{instantaneous} = Vi = V_0 i_0 \sin\omega t \sin(\omega t + \phi)$

(2) **Average power (True power)** : The average of instantaneous power in an ac circuit over a full

cycle is called average power. It's unit is watt i.e.

$$P_{av} = V_{rms} i_{rms} \cos\phi = \frac{V_0}{\sqrt{2}} \cdot \frac{i_0}{\sqrt{2}} \cos\phi = \frac{1}{2} V_0 i_0 \cos\phi$$

$$= i_{rms}^2 R = \frac{V_{rms}^2 R}{Z^2}$$

(3) **Apparent or virtual power** : The product of apparent voltage and apparent current in an electric circuit is called apparent power. This is always positive $P_{app} = V_{rms} i_{rms} = \frac{V_0 i_0}{2}$

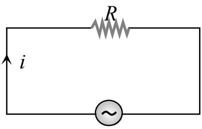
Power Factor

(1) It may be defined as cosine of the angle of lag or lead (i.e. $\cos\phi$)

(2) It is also defined as the ratio of resistance and impedance (i.e. $\frac{R}{Z}$)

(3) The ratio $\frac{\text{True power}}{\text{Apparent power}} = \frac{W}{VA} = \frac{kW}{kVA} = \cos\phi$

Resistive Circuit (R-Circuit)



$V = V_0 \sin\alpha$

Fig. 24.3

(1) Current : $i = i_0 \sin\omega t$

(2) Peak current : $i_0 = \frac{V_0}{R}$

(3) Phase difference between voltage and current : $\phi = 0^o$

(4) Power factor : $\cos\phi = 1$

(5) Power : $P = V_{rms} i_{rms} = \frac{V_0 i_0}{2}$

(6) Time difference : T.D. = 0

(7) Phasor diagram : Both are in same phase

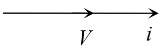


Fig. 24.4

Inductive Circuit (L-Circuit)

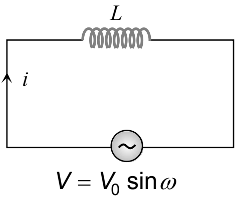


Fig. 24.5

(1) Current : $i = i_0 \sin\left(\omega t - \frac{\pi}{2}\right)$

(2) Peak current :

$$i_0 = \frac{V_0}{X_L} = \frac{V_0}{\omega_L} = \frac{V_0}{2\pi\nu L}$$

(3) Phase difference between

voltage and current $\phi = 90^\circ$ (or $+\frac{\pi}{2}$)

(4) Power factor : $\cos\phi = 0$

(5) Power : P = 0

(6) Time difference : $\text{T.D.} = \frac{T}{4}$

(7) Phasor diagram : Voltage leads the current by $\frac{\pi}{2}$

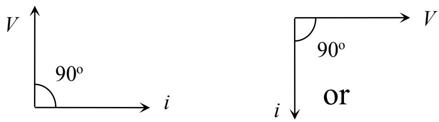


Fig. 24.6

Capacitive Circuit (C-Circuit)

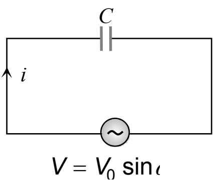


Fig. 24.7

(1) Current : $i = i_0 \sin\left(\omega t + \frac{\pi}{2}\right)$

(2) Peak current :

$$i_0 = \frac{V_0}{X_C} = V_0\omega C = V_0(2\pi\nu C)$$

(3) Phase difference between

voltage and current : $\phi = 90^\circ$ (or $-\frac{\pi}{2}$)

(4) Power factor : $\cos\phi = 0$

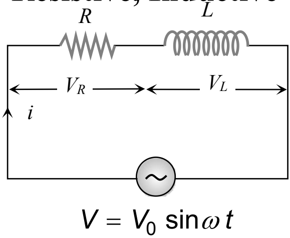
(5) Power : P = 0

(6) Time difference : $TD = \frac{T}{4}$

(7) Phasor diagram : Current leads the voltage by $\pi/2$

Fig. 24.8

Resistive, Inductive Circuit (RL-Circuit)



$V_R = iR$, $V_L = iX_L$

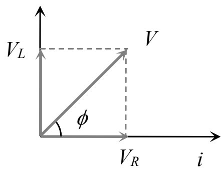


Fig. 24.9

(1) Applied voltage : $V = \sqrt{V_R^2 + V_L^2}$

(2) Impedance : $Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \omega^2 L^2} = \sqrt{R^2 + 4\pi^2 \nu^2 L^2}$

(3) Current : $i = i_0 \sin(\omega t - \phi)$

(4) Peak current $i_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{R^2 + X_L^2}} = \frac{V_0}{\sqrt{R^2 + 4\pi^2 \nu^2 L^2}}$

(5) Phase difference : $\phi = \tan^{-1} \frac{X_L}{R} = \tan^{-1} \frac{\omega L}{R}$

(6) Power factor : $\cos\phi = \frac{R}{\sqrt{R^2 + X_L^2}}$

(7) Leading quantity : Voltage

Resistive, Capacitive Circuit (RC-Circuit)

VR = iR,
VC = iX_C

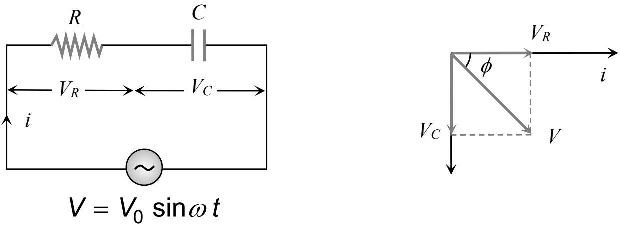


Fig. 24.10

(1) Applied voltage : $V = \sqrt{V_R^2 + V_C^2}$

(2) Impedance : $Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

(3) Current : $i = i_0 \sin(\omega t + \phi)$

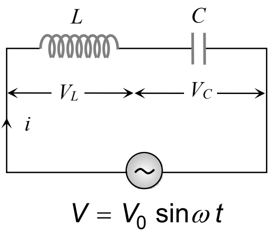
(4) Peak current : $i_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{R^2 + X_C^2}} = \frac{V_0}{\sqrt{R^2 + \frac{1}{4\pi^2\nu^2 C^2}}}$

(5) Phase difference : $\phi = \tan^{-1}\frac{X_C}{R} = \tan^{-1}\frac{1}{\omega CR}$

(6) Power factor : $\cos\phi = \frac{R}{\sqrt{R^2 + X_C^2}}$

(7) Leading quantity : Current

Inductive, Capacitive Circuit (LC-Circuit)



VL = iXL,
VC = iXC

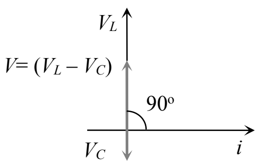


Fig. 24.11

(1) Applied voltage : $V = V_L - V_C$

(2) Impedance : $Z = X_L - X_C = X$

(3) Current : $i = i_0 \sin\left(\omega t \pm \frac{\pi}{2}\right)$

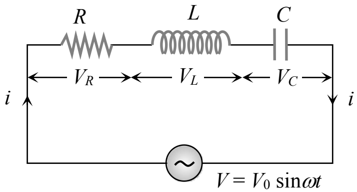
(4) Peak current : $i_0 = \frac{V_0}{Z} = \frac{V_0}{X_L - X_C} = \frac{V_0}{\omega L - \frac{1}{\omega C}}$

(5) Phase difference : $\phi = 90^o$

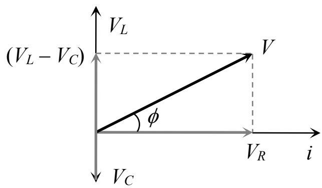
(6) Power factor : $\cos\phi = 0$

(7) Leading quantity : Either voltage or current

Series RLC-Circuit



$V_R = iR,\ V_L = iX_L,\ V_C = iX_C$



Phasor diagram

Fig. 24.12

(1) **Equation of current** : $i = i_0 \sin(\omega t \pm \phi)$; where $i_0 = \frac{V_0}{Z}$

(2) **Equation of voltage** : From phasor diagram

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

(3) **Impedance of the circuit** :

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

(4) **Phase difference** : From phasor diagram

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R} = \frac{\omega L - \frac{1}{\omega C}}{R} = \frac{2\pi\nu L - \frac{1}{2\pi\nu C}}{R}$$

(5) **If net reactance is inductive** : Circuit behaves as LR circuit

(6) **If net reactance is capacitive** : Circuit behave as CR circuit

(7) **If net reactance is zero** : Means $X = X_L - X_C = 0$

$\Rightarrow X_L = X_C$. This is the condition of resonance

(8) **At resonance** (series resonant circuit)

(i) $X_L = X_C \Rightarrow Z_{min} = R$ i.e. circuit behaves as resistive circuit

(ii) $V_L = V_C \Rightarrow V = V_R$ i.e. whole applied voltage appeared across the resistance

(iii) Phase difference : $\phi = 0^o \Rightarrow$ p.f. $= \cos\phi = 1$

(iv) Power consumption $P = V_{rms}\, i_{rms} = \frac{1}{2} V_0 i_0$

(v) Current in the circuit is maximum and it is $i_0 = \frac{V_0}{R}$

(vi) These circuit are used for voltage amplification and as selector circuits in wireless telegraphy.

(9) **Resonant frequency** (Natural frequency)

At resonance $X_L = X_C \Rightarrow \omega_0 L = \frac{1}{\omega_0 C} \Rightarrow$

$$\omega_0 = \frac{1}{\sqrt{LC}} \frac{rad}{sec} \Rightarrow \nu_0 = \frac{1}{2\pi\sqrt{LC}} Hz \text{ (or } cps)$$

(Resonant frequency doesn't depend upon the resistance of the circuit)

(10) **Half power frequencies and band width** :
The frequencies at which the power in the circuit is half of the maximum power (The power at resonance), are called half power frequencies.

(i) The current in the circuit at half power frequencies (HPF) is $\frac{1}{\sqrt{2}}$ or 0.707 or 70.7% of

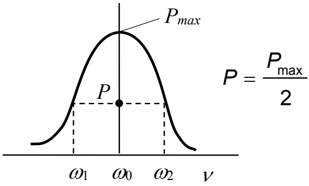


Fig. 24.13

maximum current (current at resonance).

(ii) There are two half power frequencies

(a) $\omega_1 \rightarrow$ called lower half power frequency. At this frequency the circuit is capacitive.

(b) $\omega_2 \rightarrow$ called upper half power frequency. It is greater than ω_0. At this frequency the circuit is inductive.

(iii) Band width ((() : The difference of half power frequencies ω_1 and ω_2 is called band width ((() and $\Delta\omega = \omega_2 - \omega_1$. For series resonant circuit it can be proved $\Delta\omega = \left(\frac{R}{L}\right)$

(11) **Quality factor (Q-factor) of series resonant circuit**

(i) The characteristic of a series resonant circuit is determined by the quality factor (Q - factor) of the circuit.

(ii) It defines sharpness of i - ν curve at resonance when Q - factor is large, the sharpness of resonance curve is more and vice-versa.

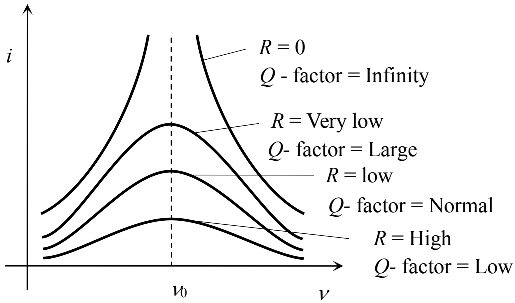
(iii) Q - factor also defined as follows

$$\text{Q - factor} = 2\pi \times \frac{\text{Max. energy stored}}{\text{Energy dissipation}}$$

$$= \frac{2\pi}{T} \times \frac{\text{Max. energy stored}}{\text{Mean power dissipated}} = \frac{\text{Resonant frequency}}{\text{Band width}} = \frac{\omega_0}{\Delta\omega}$$

(iv) Q - factor $= \dfrac{V_L}{V_R}$ or $\dfrac{V_C}{V_R} = \dfrac{\omega_0 L}{R}$ or $\dfrac{1}{\omega_0 CR}$

$$\Rightarrow Q\text{-factor} = \frac{1}{R}\sqrt{\frac{L}{C}}$$



Resonance curve

Fig. 24.14

Parallel RLC Circuits

$$i_R = \frac{V_0}{R} = V_0 G$$

$$i_L = \frac{V_0}{X_L} = V_0 S_L$$

$$i_C = \frac{V_0}{X_C} = V_0 S_C$$

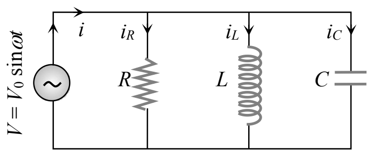


Fig. 24.15

(1) C**urrent and phase difference** : From phasor diagram current $i = \sqrt{i_R^2 + (i_C - i_L)^2}$ and phase difference

$$\phi = \tan^{-1}\frac{(i_C - i_L)}{i_R} = \tan^{-1}\frac{(S_C - S_L)}{G}$$

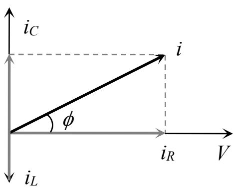


Fig. 24.16

(2) **Admittance (Y) of the circuit** : From equation of current

$$\frac{V_0}{Z} = \sqrt{\left(\frac{V_0}{R}\right)^2 + \left(\frac{V_0}{X_L} - \frac{V_0}{X_C}\right)^2}$$

$$\Rightarrow \quad \frac{1}{Z} = Y = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2} = \sqrt{G^2 + (S_L - S_C)^2}$$

(3) **Resonance** : At resonance (i) $i_C = i_L \Rightarrow$ $i_{\min} = i_R$

(ii) $\frac{V}{X_C} = \frac{V}{X_L}$ ($S_C = S_L \Rightarrow \Sigma S = 0$

(iii) $Z_{\max} = \frac{V}{i_R} = R$

(iv) $\phi = 0 \Rightarrow$ p.f. $= \cos\phi = 1 =$ maximum

(v) Resonant frequency $\Rightarrow \nu = \frac{1}{2\pi\sqrt{LC}}$

(4) **Parallel LC circuits** : If inductor has resistance (R) and it is connected in parallel with capacitor as shown

(i) **At resonance**

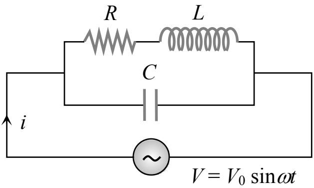


Fig. 24.17

(a) $Z_{\max} = \frac{1}{Y_{\min}} = \frac{L}{CR}$

(b) Current through the circuit is minimum and $i_{\min} = \frac{V_0 CR}{L}$

(c) $S_L = S_C \Rightarrow \frac{1}{X_L} = \frac{1}{X_C} \Rightarrow X = \infty$

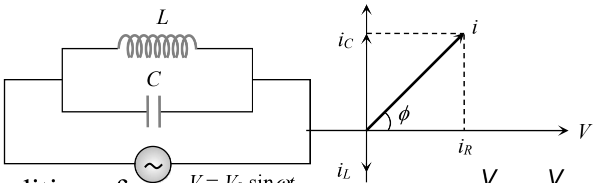
(d) Resonant frequency $\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \frac{rad}{sec}$ or

$$\nu_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}\ Hz$$

(Condition for parallel resonance is $R < \sqrt{\frac{L}{C}}$)

(e) Quality factor of the circuit $= \frac{1}{CR} \cdot \frac{1}{\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$. In the state of resonance the quality factor of the circuit is equivalent to the current amplification of the circuit.

(ii) **If inductance has no resistance** : If R = 0 then circuit becomes parallel LC circuit as shown



Condition of resonance : $i_C = i_L \Rightarrow \frac{V}{X_C} = \frac{V}{X_L}$

Fig. 24.18

$\Rightarrow X_C = X_L$. At resonance current i in the circuit is zero and impedance is infinite. Resonant frequency : $\nu_0 = \frac{1}{2\pi\sqrt{LC}}\ Hz$

Wattless Current

In an ac circuit R = 0 $\Rightarrow \cos\phi = 0$ so $P_{av} = 0$ i.e. in resistance less circuit the power consumed is zero. Such a circuit is called the wattless circuit and the current flowing is called the wattless current.

or

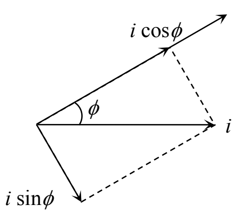
The component of current which does not contribute to the average power dissipation is called wattless current

(i) The average of wattless component over one cycle is zero

(ii) Amplitude of wattless current = $i_0 \sin\phi$

and r.m.s. value of wattless current =

$$i_{rms} \sin\phi = \frac{i_0}{\sqrt{2}} \sin\phi .$$



It is quadrature (90°) with voltage.

Fig. 24.19

Choke Coil

Choke coil (or ballast) is a device having high inductance and negligible resistance. It is used to control current in ac circuits and is used in fluorescent tubes. The power loss in a circuit

containing choke coil is least.

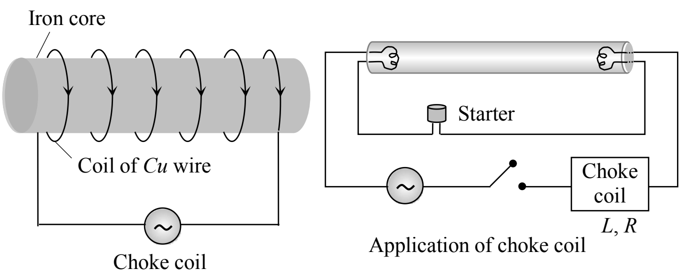


Fig. 24.20

(1) It consist of a Cu coil wound over a soft iron laminated core.

(2) Thick Cu wire is used to reduce the resistance (R) of the circuit.
(3) Soft iron is used to improve inductance (L) of the circuit.
(4) The inductive reactance or effective opposition of the choke coil is given by $X_L = \omega L = 2\pi\nu L$
(5) For an ideal choke coil r = 0, no electric energy is wasted i.e. average power P = 0.
(6) In actual practice choke coil is equivalent to a R– L circuit.
(7) Choke coil for different frequencies are made by using different substances in their core.
For low frequency L should be large thus iron core choke coil is used. For high frequency ac circuit, L should be small, so air cored choke coil is used.