MODULE 1
VECTOR CALCULUS

GRADIENT, DIRECTIONAL DERIVATIVES.

1. If $\varphi = xyz$ then find $\nabla\phi$

2. If $\phi = x^3y^2z^4$ then find $\nabla\phi$ and $|\nabla\phi|$ at $(1,-2,1)$

3. If $\phi = x^2 y^2 + y^2 z^2 + z^2x^2$ find $\nabla\phi$ at (1, 1, 1)

4. Find the unit normal vector to the surface $x^3 y^3 z^2 = 4$ at the point (-1, -1, 2).

5. Find the unit normal vector to the surface $x^2 yz + xy^2z + xyz^2 = 3$ at the point (1, 1, 1).

6. Find the angle between the normal's to the surface $xy = z^2$ at the point (4, 1, 2) and (3, 3, -3).

7. Find the angle between the normal's to the surface $x^2 + y^2 - z^2 = 4$ and $z = x^2 + y^2 - 13$ at $(2,1,2)$

8. Find the angle between the normal's to the surface $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - z = 3$ at (2, -1, 2).

9. Find the directional derivative for the surface $\phi = x^2yz^3$ in the direction of $i + j + 2k$ at (1, 1, 1).

10. Find the values of a and b such that the surfaces $ax^2 - byz = (a+2)x$ and $4x^2 y + z^3 = 4$ are orthogonal at the point (1, -1, 2).

11. Find the directional derivative for the surface $\phi = x^2yz + 4xz^2$ along $2i - j - 2k$ at (1, -2, -1)

12. Find the directional derivative for the surface $\varphi = 4xz^3 - 3x^2 y^2 z$ at $(2,-1,2)$ along $2i - 3j + 6k$.

13. Find the directional derivative for the surface $\phi = x\ y^2 + yz^3$ at (2, -1, 1) along the normal to the surface $xy + yz + zx = 3$ at (1, 1, 1)

14. Find the directional derivative for the surface $\phi = xy^2 + yz^3$ at (1, -2, -1) along the normal to the surface $x\log z - y^2 = -4$ at (-1, 2, 1).

PROBLEMS ON DIVERGENCE AND CURL.

15. If $\vec{F} = 2xy^3 z^4 i + 3x^2y^2z^4 j + 4x^2y^3z^3k,$ Find i) $(\nabla \cdot \vec{F})$ ii) $(\nabla \times \vec{F})$.

16. If $\phi = x^2 + y^2 + z^2$ and $\vec{F} = x^2 i + y^2 j + z^2 k$ then find $\nabla\varphi$, $div\,\vec{F}$, $curl\vec{F}$

17. If $\vec{F} = xy^2i + 2x^2 yz\,j - 3y^2 z\,k$ then find $div\,\vec{F}$ and $curl\,\vec{F}$ at (2, 1, 1).

18. Find $div\,\vec{F}$ and $curl\,\vec{F}$ where $\vec{F} = grad\,(x^3 + y^3 + z^3 - 3xyz)$

19. If $\vec{A} = 2x^2 i - 3yz\,j + xz^2 k$ and $\phi = 2z - x^3 y$ find $\vec{A}\cdot\nabla\varphi$ and $(\vec{A}\times\nabla\varphi)$ at (1, -1, 1).

20. If $\vec{F} = grad\,(x^3y + y^3 z + z^3 x - x^2 y^2 z^2)$ find $div\,\vec{F}$ and $curl\,\vec{F}$ at (1, 2, 3)

21. If $\vec{F} = \nabla\left(x y^3 z^2\right)$ find $div\,\vec{F}$ and $curl\,\vec{F}$ at (1, -1, 1)

22. If $\vec{F} = \left(3x^2 y - z\right)i + \left(x z^3 + y^4\right)j - \left(2x^3 z^2\right)k$ find $grad\left(div\,\vec{F}\right)$.

23. If $\vec{F} = (x + y + 1)i + j - (x + y)k$ show that $\vec{F} \cdot curl\,\vec{F} = 0$

24. If $\vec{A} = xz^3\, i - 2x^2 yz\, j + 2yz^4 k$ find $\nabla \cdot \vec{A}$, $\nabla \times \vec{A}$, $\nabla \cdot \left(\nabla \times \vec{A}\right)$

25. If $\vec{F} = x^2 y\, i - 2xz\, j + 2yz\, k$ find $curl\left(curl\,\vec{F}\right)$

26. If $\vec{F} = x^2 y\, i + y^2 z\, j + z^2 y\, k$ find $curl\left(curl\,\vec{F}\right)$

27. Show that $\vec{F} = \dfrac{x\,i + y\,j}{x^2 + y^2}$ is both solenoidal and irrotational.

28. If $\vec{V} = 3xy^2 z^2\, i + y^3 z^2\, j - 2y^2 z^3\, k$ prove that $\vec{V}$ is solenoidal.

29. Show that $\vec{F} = \left(2xy^2 + yz\right)i + \left(2x^2 y + xz + 2yz^2\right)j + \left(2y^2 z + xy\right)k$ is a conservative force field and find its scalar potential ϕ such that $\vec{F} = \nabla\phi$.

30. Show that $\vec{F} = 2xyz^3\, i + x^2 z^3\, j + 3x^2 yz^2\, k$ is an irrotational vector field. Hence find the scalar function ϕ such that $\vec{F} = \nabla\phi$.

31. Show that $\vec{F} = (y + z)i + (z + x)j + (x + y)k$ is an irrotational vector field. Hence find the scalar function ϕ such that $\vec{F} = \nabla\phi$.

32. Show that $\vec{F} = \left(x^2 - yz\right) i + \left(y^2 - zx\right)j + \left(z^2 - xy\right)k$ is an irrotational vector field. Hence find the scalar potential ϕ such that $\vec{F} = \nabla\phi$.

33. Find the constants a, b, c so that the vector function is irrotational

$\vec{F} = (x + 2y + az)i + (bx - 3y + z)j + (4x + cy + 2z)k$

34. Find the constants a, b, c so that the vector function is irrotational

$\vec{F} = (x + y + az)i + (bx + 2y - z)j + (x + cy + 2z)k$

35. Find the constants a, b,c for $\vec{F} = \left(axy + bz^3\right)i + \left(3x^2 - cz\right)j + \left(3xz^2 - y\right)k$ so that the vector function is irrotational hence find the scalar function ϕ such that $\vec{F} = \nabla\,\phi$

ORTHOGONAL CURVILINEAR COORDINATES

36. Prove that Cylindrical coordinate system is orthogonal

37. Express the vector $\vec{A} = z\hat{\imath} - 2x\hat{\jmath} + y\hat{k}$ in cylindrical coordinates

38. Express the vector $\vec{A} = 2x\hat{\imath} - 3y^2\hat{\jmath} + xz\hat{k}$ in cylindrical polar coordinates

39. Prove that Spherical co ordinate system is orthogonal

40. Express the vector $\vec{A} = y\hat{\imath} - z\hat{\jmath} + x\hat{k}$ in spherical coordinate system and hence find $F_r,\ F_\theta,\ F_\varphi$.