MTH – 522

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1) The PM2.5 is an important index for air pollution. A PM2.5 index of over 100 is harmful. To decide if the air over an industry zone is harmful, one took PM2.5 readings at 35 randomly picked spots in the zone. The readings are 119, 120, 102, 94, 79, 119, 92, 85, 96, 120, 101, 84, 97, 115, 107, 125, 117, 98, 91, 80, 122, 108, 122, 111, 109, 90, 83, 101, 118, 107, 99, 80, 102, 97, 96.

   Is the air in the industry zone harmful? Assume $\alpha = 0.02$.

Ans:

Based on the provided PM2.5 readings and assuming a significance level of $\alpha = 0.02$, here's the statistical analysis to determine if the air in the industry zone is harmful: we can perform a one-sample t-test.

Data:

- PM2.5 readings: 119, 120, 102, 94, 79, 119, 92, 85, 96, 120, 101, 84, 97, 115, 107, 125, 117, 98, 91, 80, 122, 108, 122, 111, 109, 90, 83, 101, 118, 107, 99, 80, 102, 97, 96
- Sample size (n) = 35

Hypothesis Testing:

- Null hypothesis (H0): The average PM2.5 level in the industry zone is less than or equal to 100.
- Alternative hypothesis (H1): The average PM2.5 level in the industry zone is greater than 100.

First, we calculate the sample mean ($\bar{x}$): 102.45. {3586/35}

Calculate sample standard deviation (s): 13.59

Conduct one-tailed t-test:

- Use the formula $t = (\bar{x} - \mu 0) / (s / \sqrt{n})$, where $\mu 0 = 100$ (null hypothesis value).
- Calculate $t = 1.28$ with 34 degrees of freedom ($df = n - 1$).

Compare the calculated t-value to the critical t-value:

→ If the calculated t-value is greater than the critical t-value, reject H0.

→ If the calculated t-value is less than or equal to the critical t-value, do not reject H0.

Find p-value:

- Use a t-distribution table with df = 34 and look up the probability associated with t = 1.28.
- The p-value is approximately 0.104.

Conclusion:

- Since the p-value (0.104) is greater than the significance level ($\alpha = 0.02$), we fail to reject the null hypothesis.
- We cannot conclude that the air in the industry zone is harmful at the 0.02 significance level.

2) You are given a coin. You do not know if it is a fair coin or not. You want to figure out a way so that you can make a sound statement about the fairness of the coin.

Ans:

Regretfully, using a single coin and no prior knowledge, there is no foolproof "solution" to ascertain the fairness of a coin. To boost your trust in your assessments of its fairness, you might, nevertheless, employ statistical techniques. Here are two methods:

1. Testing Hypotheses:

Describe your theories:

Hypothesis null (H0): The coin is fair because there is a 0.5 chance of heads.

Hypothesis alternative (H1): The coin is unjust (heads probability $\neq 0.5$).

Select a significance threshold (α): This is the likelihood of rejecting the null hypothesis (often $\alpha = 0.05$) if it is true.

N times, flip the coin: Your results are more dependable the more flips you get.

Determine the percentage of heads ($\hat{p}$): Divide the number of heads by n.

Select a test for statistics: You can apply several tests, such as the Chi-square test, Normal approximation test, or Binomial test, depending on n and your confidence in the fairness assumption. Everyone has their own presumptions and constraints.

Determine the p-value by: This is the likelihood that, under the null hypothesis, findings at least as dramatic as yours will be observed.

Analyze the outcomes:

You are unable to reject the null hypothesis if the p-value is higher than α. With the selected confidence level, it is impossible to infer that the coin is unfair.

Rejecting the null hypothesis is necessary if the p-value is less than α. You have evidence that the coin is unfair, but more research may be required.

Note: Took the reference from the internet and few of them from AI's