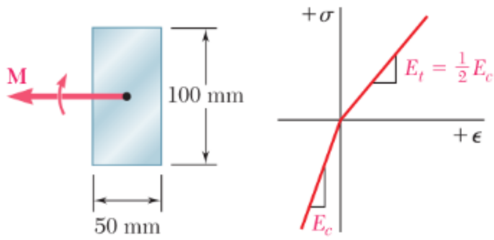
12. The rectangular beam shown is made of a plastic for which the value of the modulus of elasticity in tension is one-half of its value in compression. For a bending moment $M = 600$ N·m, determine the maximum tensile and compressive stresses. [6.15 MPa, −8.69 MPa]



Let the breadth of the cross-section be b and height be h. And, let the neutral axis be located at a height x from the bottom. Then, the height of the material above the neutral axis will be $(h - x)$.

Above the neutral axis, the material will be in compression (Young's modulus: E_c) and below the neutral axis, the material will be in tension (Young's modulus: $E_t = 0.5E_c$). Then the ratio $n = E_c/E_t = 2$.

Let the material above the neutral axis (in compression) be transformed by stretching it horizontally by the factor n.

The area of the transformed part, i.e. the part above the neutral axis is $A_c = nb(h - x)$. And, its centroid lies at a height of $y_c = (h - x)/2$ above the neutral axis.

The area of the part below the neutral axis is $A_t = bx$. And, its centroid lies at $y_t = -x/2$.

Let the origin be placed on the neutral axis. Then, we have:

$$A_c y_c + A_t y_t = (A_c + A_t) \times 0$$

because the neutral axis passes through the origin.

In [1]:
```
import sympy as sym
```

In [2]:
```
B, H, X, N = sym.symbols('b, h, x, n', positive=True)

Ac = N*B*(H-X)
yc = (H-X)/2

At = B*X
yt = -X/2

eq1 = sym.Eq(Ac*yc + At*yt,0)
display(eq1)
```

$$bn\left(\frac{h}{2} - \frac{x}{2}\right)(h - x) - \frac{bx^2}{2} = 0$$

Solving the above equation and using $n = 2$ gives us $x = (2 - \sqrt{2})h$ as the viable root.

In [3]:
```
from math import sqrt
```

In [4]:
```
b = 50
h = 100
n = 2

x = (2-sqrt(2))*h
display(x)
```

```
58.57864376269048
```

In [5]:
```
I1 = 1/12*b*x**3 + b*x*(x/2)**2
I2 = n*1/12*b*(h-x)**3 + n*b*(h-x)*((h-x)/2)**2

I = I1 + I2
display(I)
```

```
5719095.841793664
```

For the material above the neutral axis, the stress for the untransformed part is obtained by multiplying the stress for the transformed part by n.

In [7]:
```
M = 600e3 # units in N-mm

ytop = h-x
ybottom = -x

sigma_top = -n*M*ytop/I  # units in N/mm^2 or equivalently MPa
sigma_bottom = -M*ybottom/I # units in N/mm^2 or equivalently MPa

display(sigma_top, sigma_bottom)
```

```
-8.691168824543144

6.14558441227157
```

In [ ]: