INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Mid-Autumn Semester Examination 2022-23

Date of examination: Sep. 20, 2022 **Session:** AN **Duration:** 2 hrs **Full Marks:** 60
Subject No.: ME21203/ME31013 **Subject:** Mechanics of Solids
Department: Mechanical Engineering
Specific charts, graph paper, log book, etc. required: NO
Special Instructions (if any): Answer all the parts of a question together.

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1. Consider the general 3D state of stress. The problem of finding the principal stresses reduces to the eigenvalue problem (in matrix form):

$$\begin{bmatrix} \sigma_{xx} - \sigma_p & \tau_{xy} & \tau_{zx} \\ \tau_{xy} & \sigma_{yy} - \sigma_p & \tau_{yz} \\ \tau_{zx} & \tau_{yz} & \sigma_{zz} - \sigma_p \end{bmatrix} \begin{bmatrix} n_x^p \\ n_y^p \\ n_z^p \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$

where σ_p denotes the principal stress and n_x^p, n_y^p, and n_z^p denote the components of the unit normal to the plane on which the principal stress occurs.

(a) Show that for non-trivial solutions, an equation of the following form is satisfied:

$$\sigma_p^3 - I_1\sigma_p^2 + I_2\sigma_p - I_3 = 0.$$

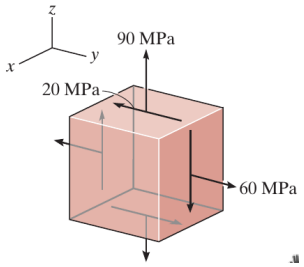
What are the expressions of I_1, I_2, and I_3?

(b) I_1, I_2, and I_3 are referred to as the stress invariants. What do you think is the motivation behind calling them "invariants", i.e. unchanging quantities?

(c) Let $\vec{T}^{(x)}$, $\vec{T}^{(y)}$, and $\vec{T}^{(z)}$ be traction vectors on planes that are perpendicular, respectively, to the x, y, and z axes. Show that the sum of the squares of the magnitudes of these traction vectors, i.e. $|\vec{T}^{(x)}|^2 + |\vec{T}^{(y)}|^2 + |\vec{T}^{(z)}|^2$ is a quantity that is expressible in terms of the stress invariants. (You must derive this expression clearly.)

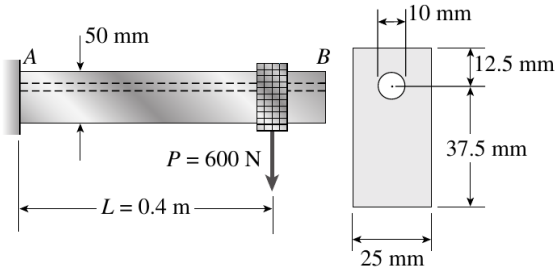
[3 + 1 + 3 = 7 marks]

2. The state of stress at a point is depicted on a stress element as shown in the figure. Determine the principal stresses and the absolute maximum shear stress. **[5 marks]**

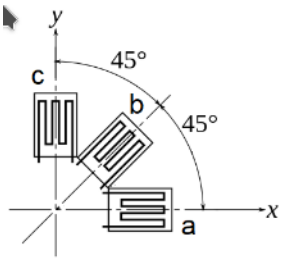


NOTE: The direction of the arrows represent the *actual physical* direction of the stress components. It is *your* responsibility to take the correct signs following the positivity convention in your calculations.

3. A cantilever beam AB has a rectangular cross-section with a circular hole in it. The hole is drilled longitudinally throughout the beam length as shown in the figure. The beam suppports a load of $P = 600$ N. Determine the flexural (or, bending) stress at (a) the top of the beam, and (b) the top of the hole. **[8 marks]**



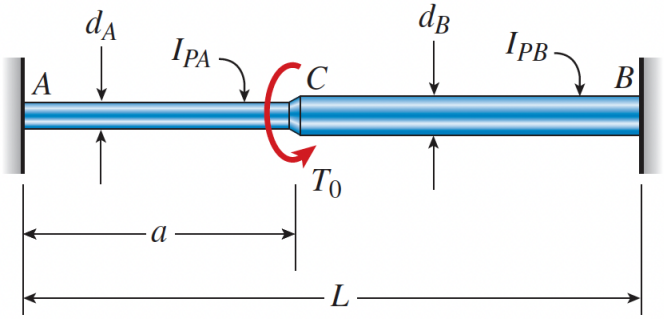
4. Consider the strain rosette, shown in the figure, that is attached to the surface of a panel that is in a *plane stress* condition. The material of the panel is an aluminium alloy with Young's modulus, $E = 72$ GPa and Poisson's ratio, $\nu = 0.33$. The strain readings are $\varepsilon_a = 0.00250$, $\varepsilon_b = 0.00140$, and $\varepsilon_c = -0.00125$.
   (a) Determine the values (with proper signs) of *all* the strain components.
   (b) What is the change in the angle between two short line segments which are initially oriented along the "a" and the "c" strain gauges?



[6 + 1 = 7 marks]

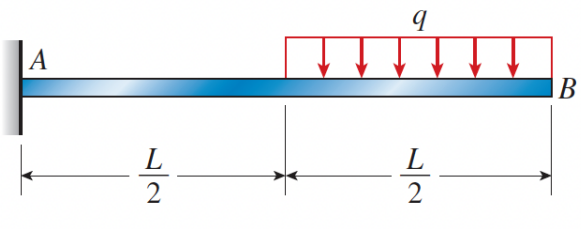
5. For a material that follows the generalized Hooke's law, prove that at a point in the xy plane, the directions of the principal stresses coincide with those of the principal strains. **[3 marks]**

6. A stepped shaft ACB is held against rotation at ends A and B and subjected to a torque T_0 acting at section C (see figure). The two segments of the shaft (AC and CB) have diameters d_A and d_B, respectively, and polar moments of inertia I_{PA} and I_{PB}, respectively. The shaft has length L and segment AC has length a.
   (a) For what ratio a/L will the maximum shear stresses be the same in both segments of the shaft?
   (b) For what ratio a/L will the internal torques be the same in both segments of the shaft?

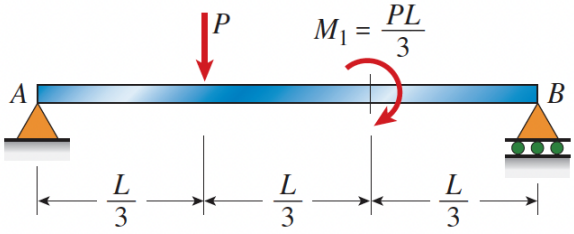


[5 + 5 = 10 marks]

7. Draw the shear force and bending moment diagrams for the two beams shown in the figure. You **MUST** follow the positivity convention for shear force and bending moment as discussed in class.



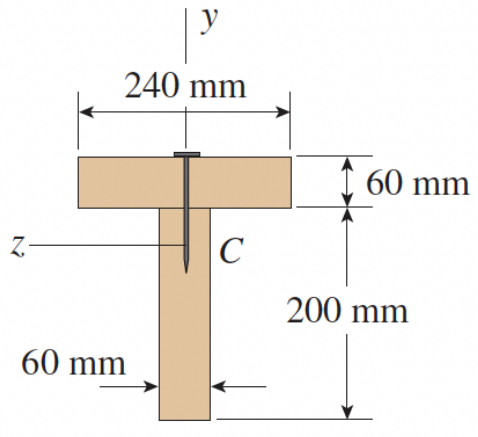
(a)



(b)

[5 + 5 = 10 marks]

8. A beam of a **T** cross section is formed by nailing together two boards having the dimensions shown in the figure. If the total shear force V acting on the cross section is 1500 N and each nail may carry 760 N in shear, what is the maximum allowable nail spacing s (in the x direction)?



[10 marks]

End of Question Paper

List of useful formulae

- Torsion: $\tau = \dfrac{Tr}{J}; \qquad \phi = \dfrac{TL}{GJ}$
- Bending: $\dfrac{\mathrm{d}V}{\mathrm{d}x} = -w; \qquad \dfrac{\mathrm{d}M}{\mathrm{d}x} = V$
- Flexure formula: $\sigma = -\dfrac{My}{I}$
- Shear formula: $\tau = -\dfrac{VQ}{It}$
- Stress transformation:

$$\sigma_{x'x'} = \frac{\sigma_{xx} + \sigma_{yy}}{2} + \frac{\sigma_{xx} - \sigma_{yy}}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$
$$\tau_{x'y'} = -\frac{\sigma_{xx} - \sigma_{yy}}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$