INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Mid-Autumn Semester Examination 2023-24

Date of examination: Sep. 20, 2023 **Session:** AN **Duration:** 2 hrs **Full Marks:** 60
Subject No.: ME21203 **Subject:** Mechanics of Solids
Department: Mechanical Engineering
Specific charts, graph paper, log book, etc. required: NO
Special Instructions (if any): Answer all the parts of a question together.

1. Consider a situation where the coordinate axes are oriented along the principal directions. Then the state of stress is given by (you don't have to explain or prove this):

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{p1} & 0 & 0 \\ 0 & \sigma_{p2} & 0 \\ 0 & 0 & \sigma_{p3} \end{bmatrix}$$

A plane which is equally inclined to these coordinate axes is referred to as an octahedral plane, and the unit outward normal to this plane $\hat{\mathbf{n}} = [n_1 \; n_2 \; n_3]^\mathsf{T}$ is such that $|n_1| = |n_2| = |n_3| = 1/\sqrt{3}$ (again, you don't have to prove this).

(a) Show that the expression for the normal stress component on this plane is $\sigma_{nn} = \dfrac{1}{3} I_1$.

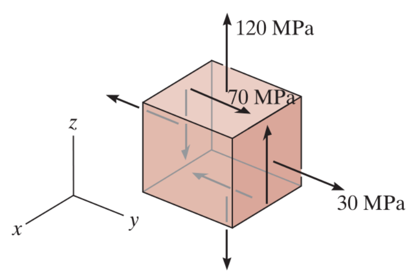
(b) Show that the expression for the shear stress component (i.e. the traction component lying in the same plane that contains the traction vector and the unit outward normal) is $\sigma_{ns} = \dfrac{\sqrt{2}}{3}\left(I_1^2 - 3I_2\right)^{1/2}$.

Here, $I_1 = \text{trace}(\boldsymbol{\sigma})$ and $I_2 = \sigma_{p1}\sigma_{p2} + \sigma_{p2}\sigma_{p3} + \sigma_{p3}\sigma_{p1}$.

[3 + 4 = 7 marks]

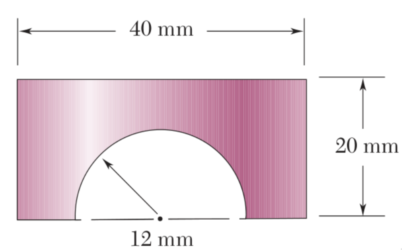
2. The state of stress at a point is depicted on a stress element as shown in the figure. Determine the principal stresses and the absolute maximum shear stress. **[5 marks]**

NOTE: The direction of the arrows represent the *actual physical* direction of the stress components. It is *your* responsibility to take the correct signs following the positivity convention in your calculations.



3. A beam has the cross-section as shown in the figure. The beam material has an allowable stress of 80 MPa in tension and 100 MPa in compression.
   (a) What is the location of the neutral axis (in mm) with respect to the bottom of the cross-section?
   (b) What is the area moment of inertia (or, second moment of area) (in mm^4) of the cross-section about the neutral axis?
   (c) Determine the largest bending moment (in N·m) that can be sustained by the beam.

[2 + 3 + 5 = 10 marks]



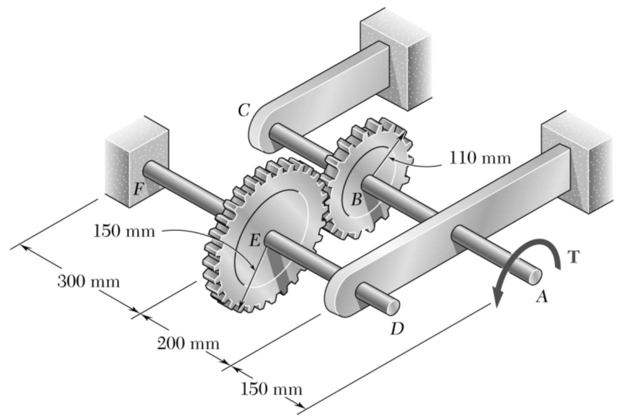
4. Consider a case of plane stress (in the xy-plane) for a material with Young's modulus, E, and Poisson's ratio, ν, and which follows the generalised Hooke's law.

   (a) Show that: $\sigma_{xx} = \dfrac{E}{1-\nu^2}(\varepsilon_{xx} + \nu\varepsilon_{yy})$ and $\sigma_{yy} = \dfrac{E}{1-\nu^2}(\varepsilon_{yy} + \nu\varepsilon_{xx})$.

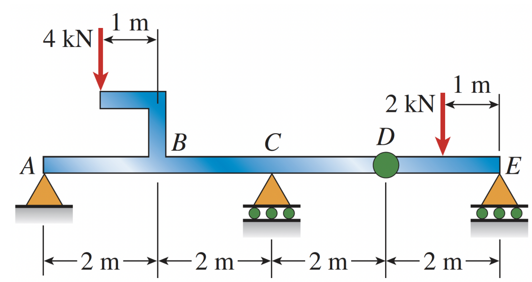
   (b) Derive the strain transformation equations from the stress transformation equations.

[3 + 5 = 8 marks]

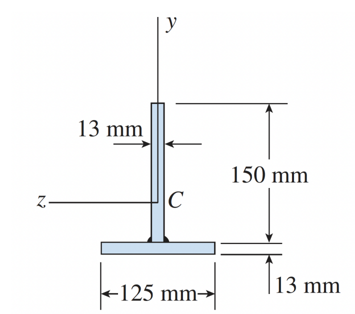
5. Two shafts, each of 22 mm diameter and made of the same material are connected by gears as shown in the figure. Given that $G = 77$ GPa and the shaft at F is fixed, determine the angle through which the end A rotates when a torque T = 130 N·m is applied at A. The slender beam-like supports through which the shafts pass do not provide any resistance to their rotation. Hint: When two gears mesh, the contact forces are equal (not the torques!); additionally the arcs of contact are equal. **[10 marks]**



6. The compound beam $ABCDE$ shown in the figure consists of two beams (AD and DE) joined by a hinged connection at D. The hinge can transmit a shear force but not a bending moment. The loads on the beam consist of a 4 kN force at the end of a bracket attached at point B, and a 2 kN force at the midpoint of beam DE. Draw the shear-force and bending-moment diagrams for this compound beam. **[10 marks]**



7. The T-beam shown in the figure is fabricated by welding together two steel plates. If the allowable load for each weld is 400 kN/m in the longitudinal direction (out of the plane of the paper), what is the maximum allowable shear force V? **[10 marks]**



End of Question Paper

List of useful formulae

- Torsion: $\tau = \dfrac{Tr}{J}$; $\quad \phi = \dfrac{TL}{GJ}$
- Bending: $\dfrac{\mathrm{d}V}{\mathrm{d}x} = -w$ (for w pointed downward);
  $\dfrac{\mathrm{d}M}{\mathrm{d}x} = V$
- Flexure formula: $\sigma = -\dfrac{My}{I}$
- Shear formula: $\tau = -\dfrac{VQ}{It}$
- Stress transformation:
  $\sigma_{x'x'} = \dfrac{\sigma_{xx}+\sigma_{yy}}{2} + \dfrac{\sigma_{xx}-\sigma_{yy}}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$
  $\tau_{x'y'} = -\dfrac{\sigma_{xx}-\sigma_{yy}}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$
- Centroid of a semi-cricle of radius R is located $\dfrac{4R}{3\pi}$ from horizontal base