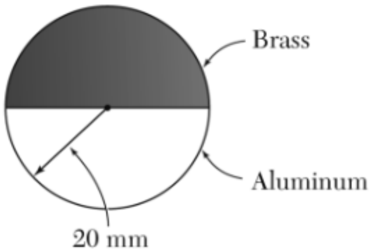
11. The composite beam shown is formed by bonding togethr brass rod and an aluminium rod of semicircular cross sections. The modulus of elasticity is 100 GPa for the brass and 70 GPa for the aluminium. Knowing that the composite beam is bent about a horizontal axis by a moment of 900 N·m, determine the maximum stress in the brass and in the aluminium. [−159.4 MPa, 129.7 MPa]



In [1]:
```
EB = 100e9
EA = 70e9

n = EB/EA
display(n)
```

```
1.4285714285714286
```

Let the area of the aluminium-section be A_A and that of the brass-section be A_B.

The transformed brass-section will be formed by "multiplying" the area of the brass semi-circle by n. Let the area of transformed brass-section be A_{Bt}.

It is extremely important to note that the transformed brass-section is such that the original semi-circle just gets stretched horizontally with its height remaining intact.

In [2]:
```
from math import pi
```

In [3]:
```
r = 20  #in millimetre

A_B = 0.5*pi*r**2  #Area of brass-section before transformation
A_A = A_B  #Area of aluminium-section

A_Bt = A_B*n  #Area of the brass-section after transformation
```

Note that the centroid of each of the semi-circular sections of radius, r is at a height of $h = \dfrac{4r}{3\pi}$ from the interface between brass and aluminium above the centre.

Before transformation, the centroid of the whole cross-section is at the centre of the circle. However, after transformation, the centroid will no longer be located at the centre of the circle because the area of the brass-section is now n-times the semi-circle area.

It is important to note that the centroid of the transformed brass-section will still be at a height of $h = \dfrac{4r}{3\pi}$ from the brass-aluminium interface above the centre because the original semi-circle is only stretched horizontally as stated earlier.

Using the centroids of the transformed brass-section and the aluminium section individually, we will find out the centroid of this overall transformed area, $\bar{y}_t$.

In [4]:

```
h = 4*r/(3*pi)
ybar_t = ( A_Bt*h + A_A*(-h) )/(A_Bt + A_A)
display(ybar_t)
```

```
1.4979288761590153
```

The above result means that the centroid of the overall transformed area lies at a height of approximately 1.5 mm above the brass-aluminium interface. The neutral axis of the overall transformed section will pass through this centroid.

Next, we need to determine the area moments of inertia of the brass-section and the aluminium-section.

For this, we will determine the area moments of inertia about their own centroids, and then determine the area moments of inertia about an axis passing through the centroid of the overall transformed section (this axis is nothing but the neutral axis of the transformed section).

First, for the aluminium-section, let $I_{A,\text{base}} = \dfrac{1}{2}\left(\dfrac{\pi r^4}{4}\right)$ represent the area moment of inertia about an axis passing through the base of the semi-circle, i.e. through the brass-aluminium interface.

Let $\bar{I}_A$ represent the area moment of inertia through its own centroid.
$\bar{I}_A = I_{A,\text{base}} - A_A h^2$

In [5]:

```
I_Abase = 0.5*(pi*r**4/4)
Ibar_A = I_Abase - A_A*h**2
display(Ibar_A)
```

```
17561.11370343453
```

Next, for the *untransformed* brass-section, let $I_{B,\text{base}} = \frac{1}{2}\left(\frac{\pi r^4}{4}\right)$ be the area moment of inertia about an axis pass through the base of the semi-circle, i.e. through the brass-aluminium interface.

Let $\bar{I}_B$ be the area moment of inertia through its own centroid. $\bar{I}_B = I_{B,\text{base}} - A_B h^2$

In [6]:
```
I_Bbase = 0.5*(pi*r**4/4)
Ibar_B = I_Bbase - A_B*h**2
display(Ibar_B)
```

```
17561.11370343453
```

Note that the above result for the brass-section is *before* the transformation.

We recall that after transformation, the area gets multiplied by the factor n, but this transformation occurs in such a way that the semi-circle is only stretched horizontally. That means the area moment of inertia of the transformed section is obtained in a straightforward manner by multiplying the area moment of inertia of the untransformed section by the factor n.

Thus, the area moment of inertia of the *transformed* brass-section about its own centroid will be $\bar{I}_{Bt} = n\bar{I}_B$.

In [7]:
```
Ibar_Bt = n*Ibar_B
display(Ibar_Bt)
```

```
25087.30529062076
```

Next, we have to determine the area moments of inertia of the aluminium-section and the *transformed* brass-section about the neutral axis (that passes through the overall centroid).

For the aluminium-section, the distance between the overall centroid and its own centroid is: $d_A = \bar{y}_t - (-h)$.

For the transformed brass-section, the distance between the overall centroid and its own centroid is: $d_{Bt} = h - \bar{y}_t$.

Then, the area moment of inertia of the aluminium-section about the neutral axis will be: $I_A = \bar{I}_A + A_A d_A^2$.

And, the area moment of inertia of the transformed brass-section about the neutral axis will be: $I_{Bt} = \bar{I}_{Bt} + A_{Bt} d_{Bt}^2$.

In [8]:
```
d_A = ybar_t - (-h)
d_Bt = h - ybar_t

I_A = Ibar_A + A_A*d_A**2
I_B = Ibar_Bt + A_Bt*d_Bt**2

display(I_A, I_B)
```

```
80219.57649701425
68948.22924612655
```

The area moment of inertia of the overall transformed section will be: $I = I_A + I_B$.

In [9]:
```
I = I_A + I_B
display(I)
```

```
149167.8057431408
```

Referred to the neutral axis, the top of transformed brass-section will be at $y_B = r - \bar{y}_t$.

And, again referred to the neutral axis, the bottom of the aluminium-section will be at $y_A = -(r + \bar{y}_t)$.

The normal stress at the top of the transformed brass-section will be: $\sigma_{Bt} = -\dfrac{My_B}{I}$

And, the magnitude of the normal stress at the bottom of the aluminium-section will be: $\sigma_A = -\dfrac{My_A}{I}$

In [10]:
```
y_B = r - ybar_t
y_A = -(r + ybar_t)

M = 900e3 # unit of N-mm

sigma_Bt = -M*y_B/I
sigma_A = -M*y_A/I

display(sigma_Bt, sigma_A)
```

```
-111.6317554481597
129.70718374620057
```

The above result is in units of N/mm^2 or, equivalently MPa.

However, one last important thing to do is to determine the normal stress for the brass-section in the untransformed form. That is done by multiplying by the factor n: $\sigma_B = n\sigma_{Bt}$

In [11]:
```
sigma_B = n*sigma_Bt
display(sigma_B)
```

```
-159.47393635451385
```