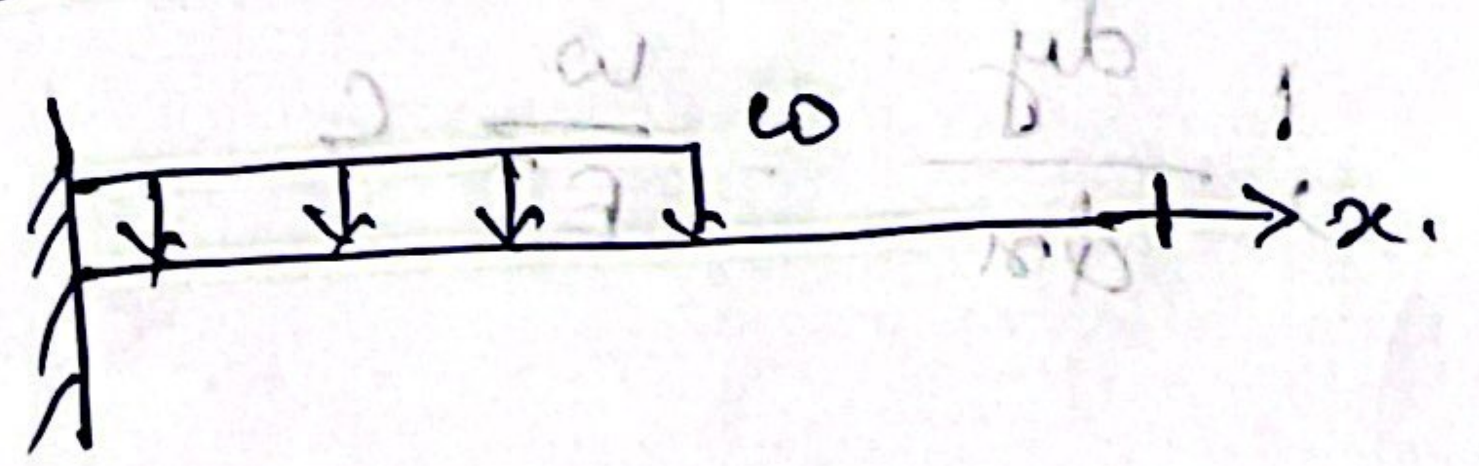
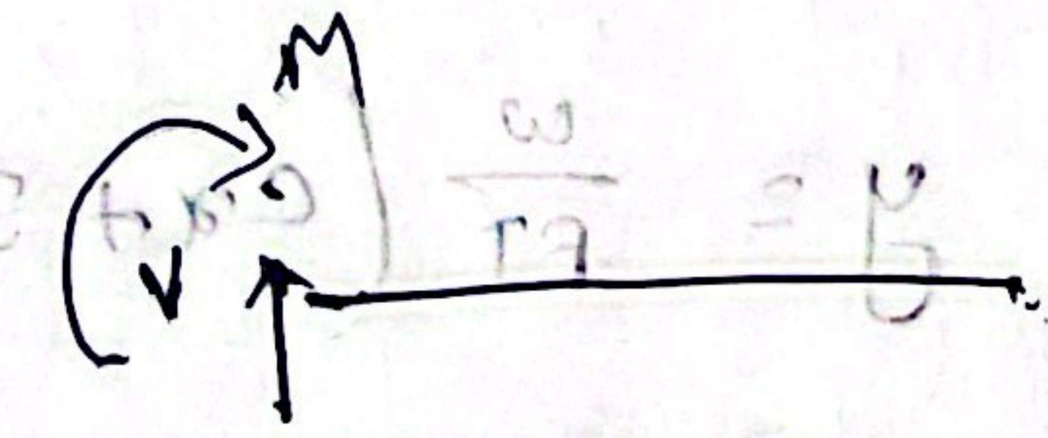
moment method.





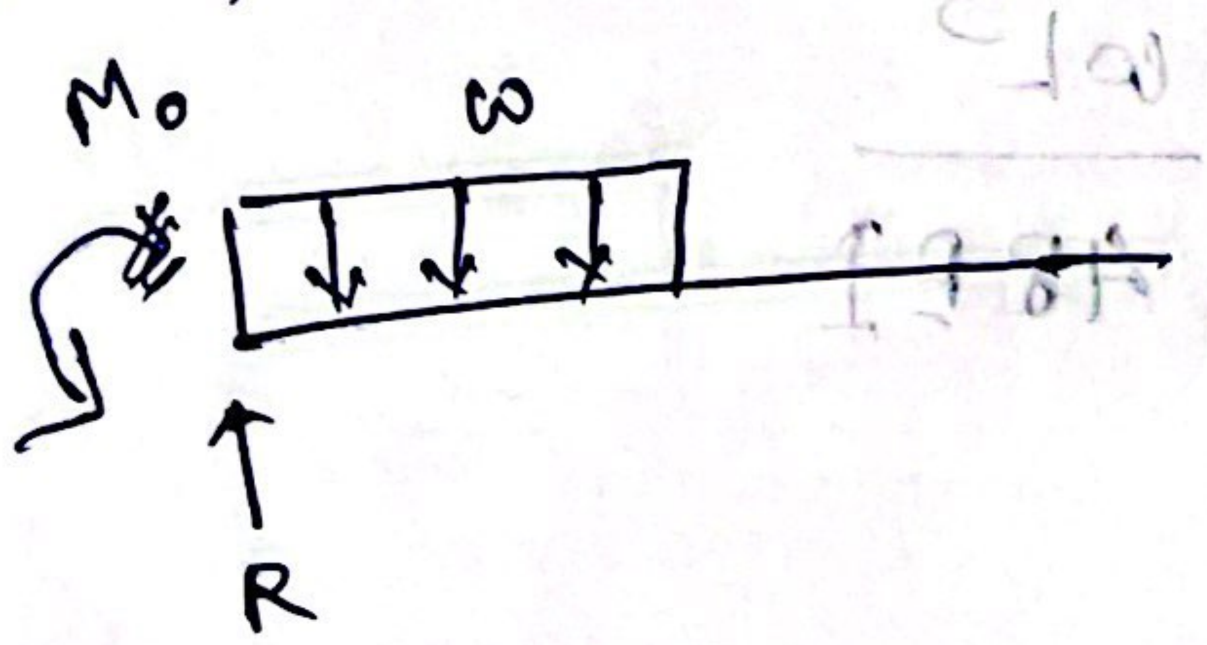
$M = 0$

$$\frac{d^2y}{dx^2} = \frac{1}{EI}(M)$$

$$\frac{dy}{dx} = \frac{1}{EI}\cancel{x}(C)$$

$$y = \frac{1}{EI}(Cx + D) \quad (2)$$

FBD

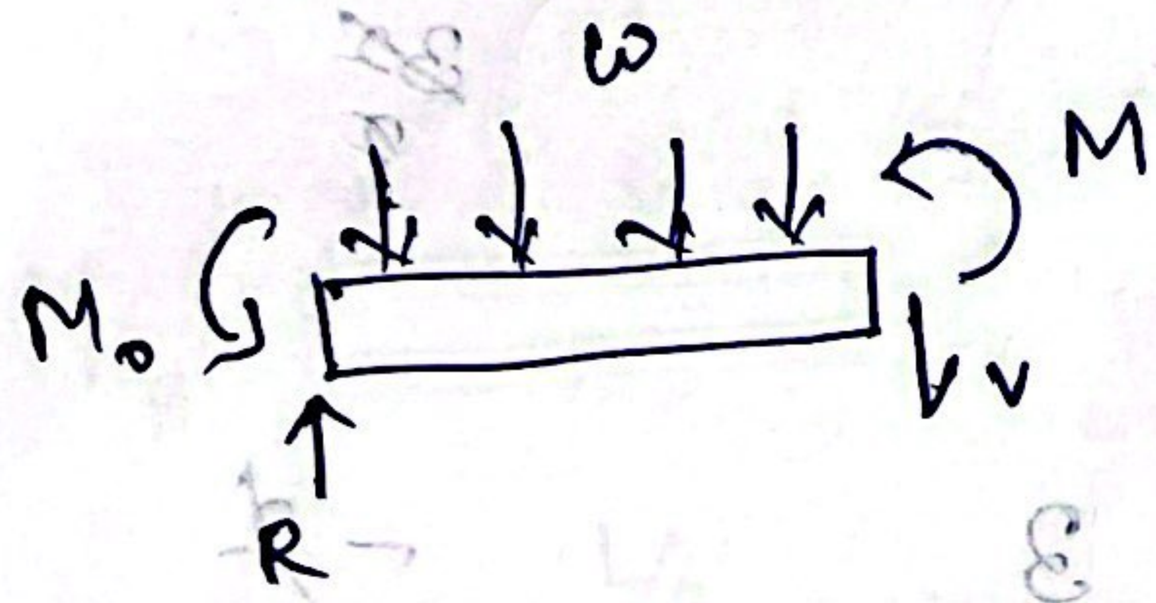


$$R = \frac{\omega L}{2}$$

$$M_o = \frac{\omega L}{2} \cdot \frac{L}{4} = \frac{\omega L^2}{8}$$

Bending Moment

for $x < L/2$



$$M = \frac{\omega L}{2}x - \frac{\omega L^2}{8} - \frac{\omega x^2}{2} \quad (3)$$

$$\frac{d^2y}{dx^2} = \frac{\omega}{EI}\left(\frac{Lx}{2} - \frac{L^2}{8} - \frac{x^2}{2}\right)$$

method & B.C.

$$(2) \quad \frac{dy}{dx} = \frac{\omega}{EI}\left(\frac{Lx^2}{4} - \frac{L^2}{8}x - \frac{x^3}{6} + \cancel{C}^{\,0}\right)$$

final

$$(2) \quad y = \frac{\omega}{EI}\left(\frac{Lx^3}{12} - \frac{L^2x^2}{16} - \frac{x^4}{24} + \cancel{D}^{\,0}\right)$$

$C = 0$ as $\left.\frac{dy}{dx}\right|_{x=0} = 0$

$D = 0$ as $y|_{x=0} = 0$

for $x < L/2$

$$y = \frac{\omega x^2}{EI}\left(-\frac{L^2}{16} + \frac{Lx}{12} - \frac{x^2}{24}\right)$$

at $x = L/2$

$$y = \frac{\omega L^4}{4EI}\left(-\frac{1}{16} + \frac{1}{2\times 12} - \frac{1}{24\times 4}\right)$$

$$= \frac{\omega L^4}{4EI}\left(\cancel{\frac{-3+4-1}{48}} - \frac{1}{32}\right)$$

$$\frac{dy}{dx} = \frac{\omega L^3}{EI}\left(\frac{1}{4\times 4} - \frac{1}{16} - \frac{1}{6\times 8}\right)$$

$$= \frac{\omega L^3}{EI}\left(\frac{-1}{48}\right)$$

$$y = \frac{w}{EI}\left(Cx + D\right) \quad ; \quad \frac{dy}{dx} = \frac{w}{EI}C$$

at $x = L/2$

$$y = \frac{-wL^4}{4 \times 32\, EI} \quad , \quad \frac{dy}{dx} = -\frac{wL^3}{48 EI} \qquad (2)$$

$$\frac{-\cancel{w}L^4}{32\cancel{EI}} = \frac{\cancel{w}}{\cancel{EI}}\left(C\frac{L}{2} + D\right) \quad ; \quad \frac{-wL^3}{48EI} = \frac{w}{EI}C$$

$$\Rightarrow \frac{-L^4}{48 \times 2} + D = \frac{-L^4}{4 \times 32} \quad ; \quad \Rightarrow C = \frac{-L^3}{48}$$

$$\frac{3\,\cancel{48}\times\cancel{2}}{\cancel{32}}$$

$$\Rightarrow D = L^4\left(\frac{1}{48 \times 2} - \frac{1}{4 \times 32}\right)$$

$$= +\frac{L^4}{\cancel{48}\,384} \qquad \frac{1-3}{48 \times 2} = \frac{-\cancel{2}}{48 \times \cancel{2}}$$

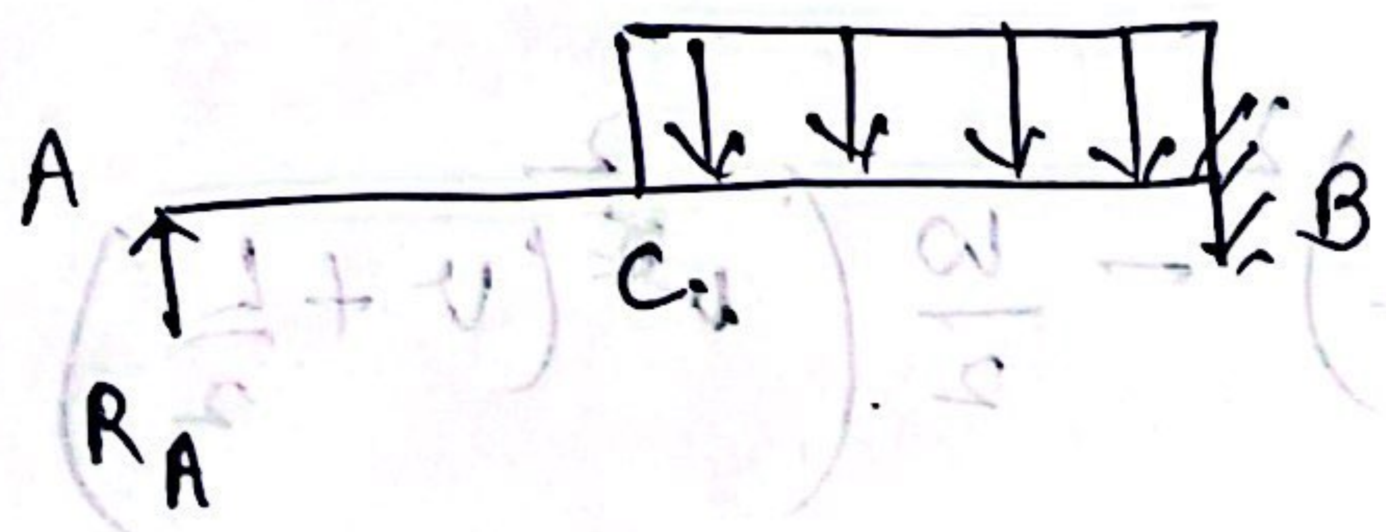
$$y = \frac{w}{48EI}\left(-L^3 x - \frac{L^4}{2}\right)$$

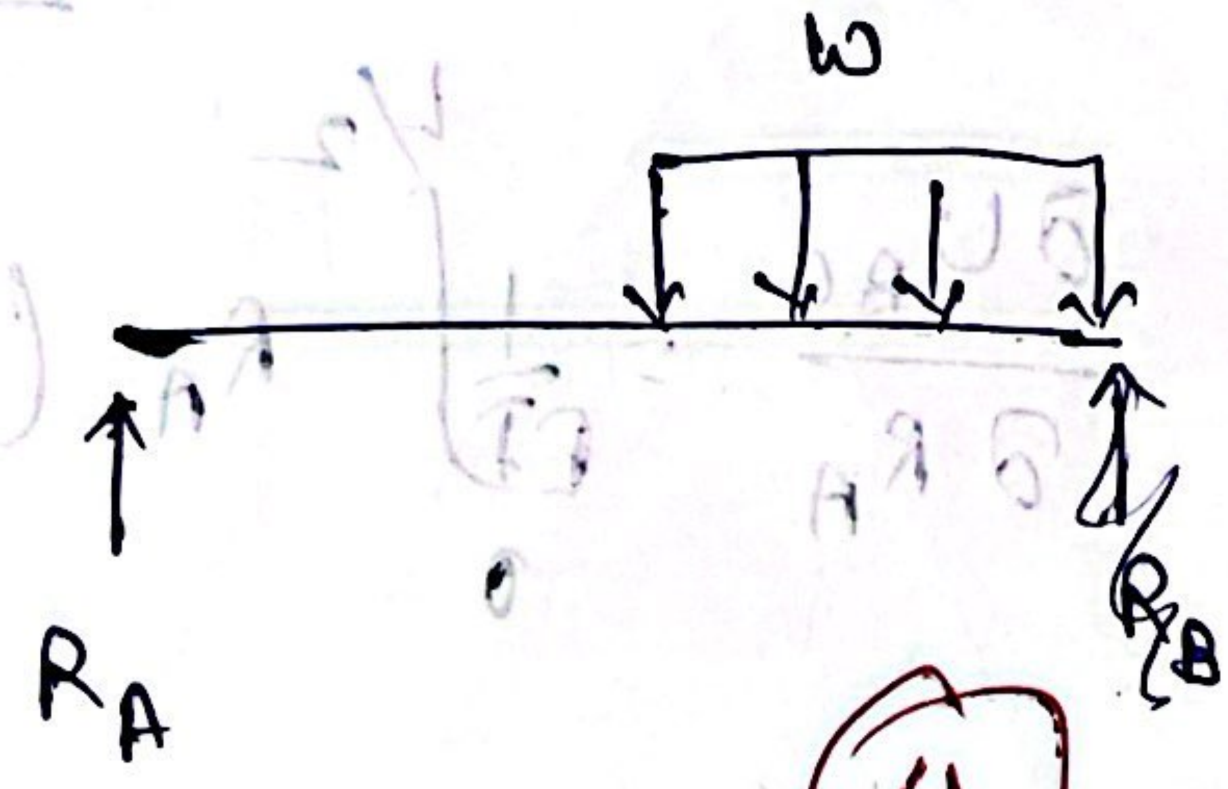
$$= \frac{-wL^3}{48EI}\left(x - L\right)$$

$$y = \frac{w}{EI}\left(\frac{-L^3 x}{48} + \frac{L^4}{384}\right)$$

$$= \frac{-wL^3}{384EI}\left(-L + 8x\right) \qquad (3)$$

(6)





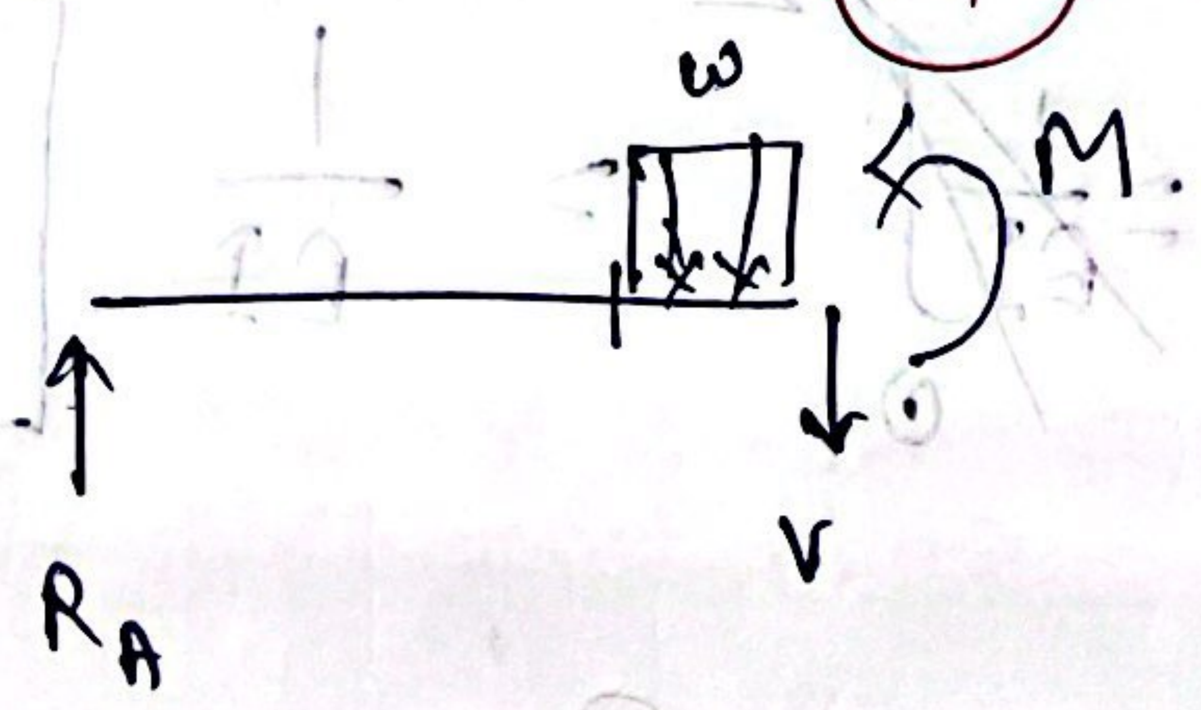
4

4

$U = U_{AC} + U_{BC}$

$\delta_A = \dfrac{\partial U}{\partial R_A} = 0$

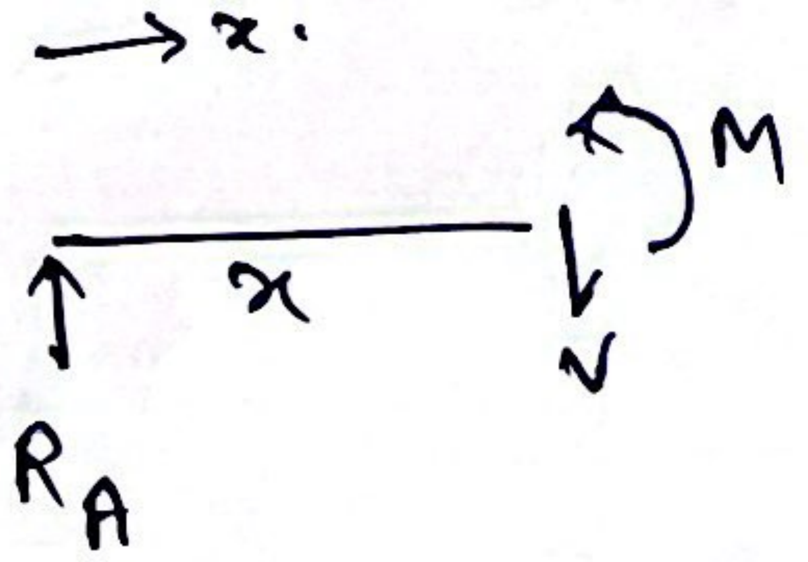
$\Rightarrow \dfrac{\partial U_{AC}}{\partial R_A} + \dfrac{\partial U_{BC}}{\partial R_A} = 0.$



$M = R_A x - \frac{w}{2}(x - L/2)^2$

$\dfrac{\partial M}{\partial R_A} = x$

for part AC,



$\Rightarrow V = R_A$

$M = R_A x \Rightarrow \dfrac{\partial M}{\partial R_A} = x$

$U_{AC} = \displaystyle\int_0^{L/2} \frac{M^2}{2EI}\, dx.$ 2

4

$\dfrac{\partial U_{AC}}{\partial R_A} = \displaystyle\int_0^{L/2} \frac{M}{EI} \frac{\partial M}{\partial R_A}\, dx = \frac{1}{EI}\int_0^{L/2} R_A x^2\, dx = \frac{R_A}{EI}\frac{1}{3}\frac{L^3}{8}$

$= \dfrac{R_A L^3}{24EI}$ 3

$\dfrac{\partial U_{BC}}{\partial R_A} = \dfrac{1}{EI}\displaystyle\int_{L/2}^{L} \left(R_A x - \frac{w}{2}(x - L/2)^2\right) x\, dx$

$= \cancel{\left[\frac{R_A x^3}{2} - \frac{w}{2}\right.}$

Let $v = x - L/2$

$$\frac{\partial U_{BC}}{\partial R_A} = \frac{1}{EI}\int_0^{L/2} R_A\left(v + L/2\right)^2 - \frac{w}{2}\left(v^2\left(v + \frac{L}{2}\right)\right) dx$$

$$= \frac{1}{EI}\left[\frac{R_A}{3}\left(v + \frac{L}{2}\right)^3 - \frac{w}{2}\left(\frac{v^4}{4} + \frac{L}{2}\frac{v^3}{3}\right)\right]$$

$$= \frac{1}{EI}\left[\frac{R_A}{3}L^3 - \frac{w}{2}\left(\frac{L^4}{64} + \frac{L^4}{48}\right) - \frac{R_A L^3}{24}\right]$$

$$= \frac{1}{EI}\left[R_A\frac{L^3}{3} - \frac{7wL^4}{384} - \frac{R_A L^3}{24}\right]$$

$\cancel{3}$ 4

Now

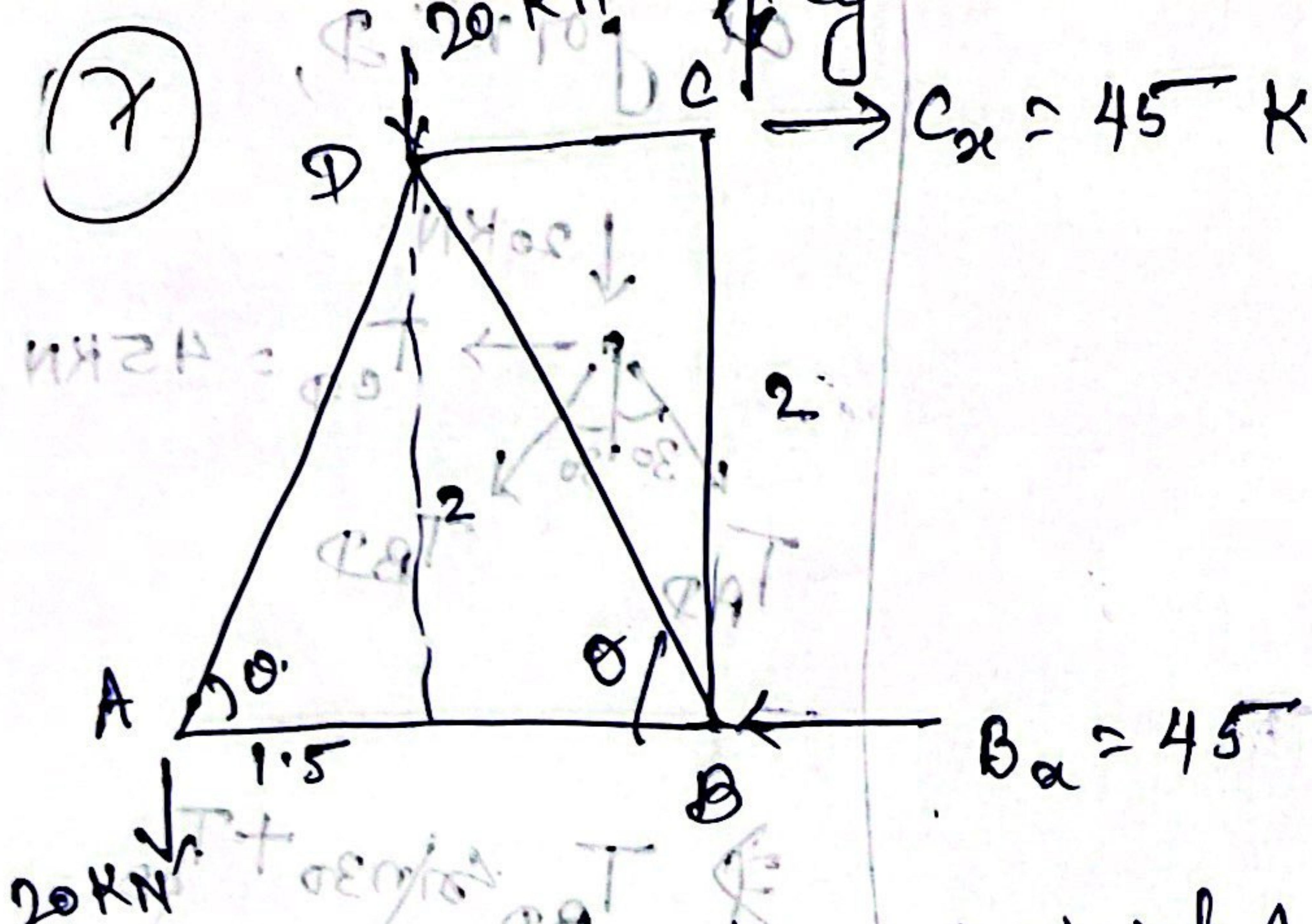
$$\delta_A = \frac{\partial U_{AC}}{\partial R_A} + \frac{\partial U_{BC}}{\partial R_A} = 0$$

$$\Rightarrow \frac{1}{EI}\left(\frac{R_A L^3}{3} - \frac{7wL^4}{384}\right) = 0$$

4

$$\Rightarrow R_A = \frac{7wL^4 \times 3}{\cancel{384}_{128}\, L^3} = \frac{7}{128}wL$$ Ans.

(7)

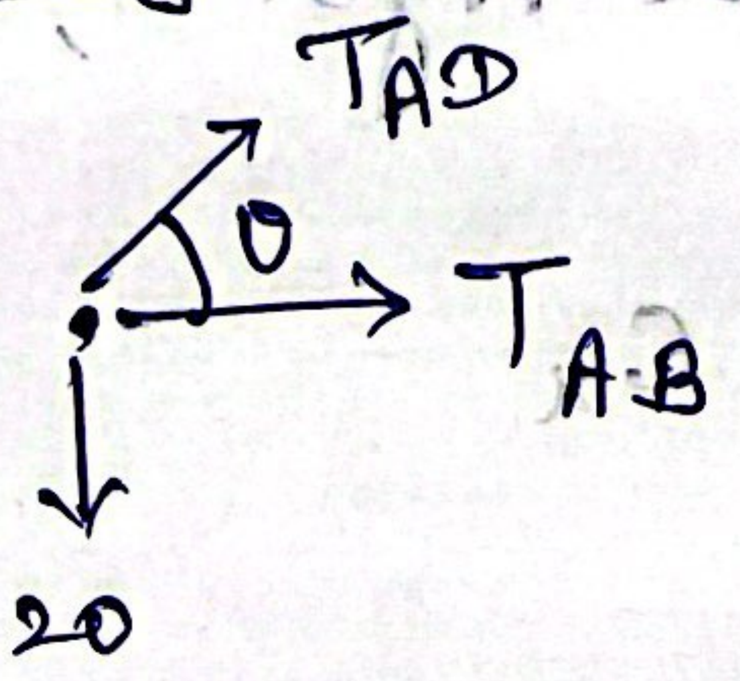


$\tan\theta = \frac{2}{3/2} = \frac{4}{3}$

$\sin\theta = \frac{4}{5}$

$\cos\theta = 3/5$

at joint A,



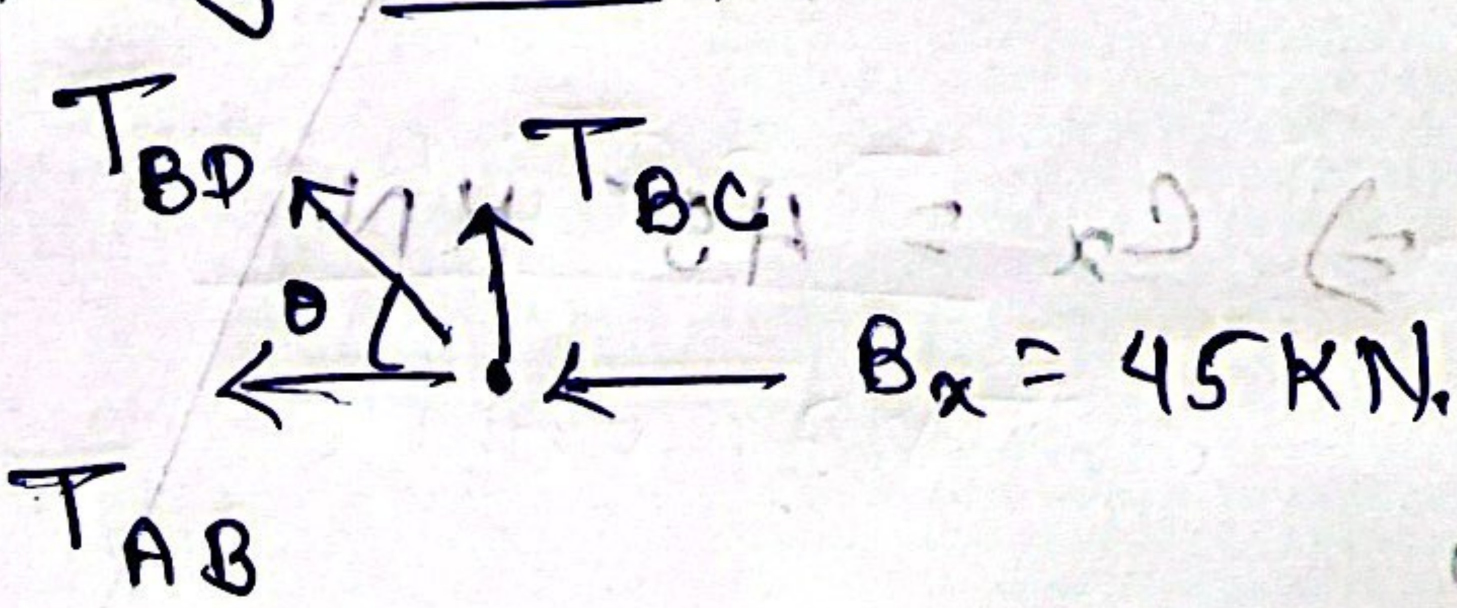
$T_{AD}\sin\theta = 20$

$\Rightarrow T_{AD} = \frac{20}{\sin\theta} = \frac{20}{4} \times 5 = 25$ KN

$T_{AB} + T_{AD}\cos\theta = 0$

$\Rightarrow T_{AB} = -25 \times \frac{3}{5} = -15$ KN

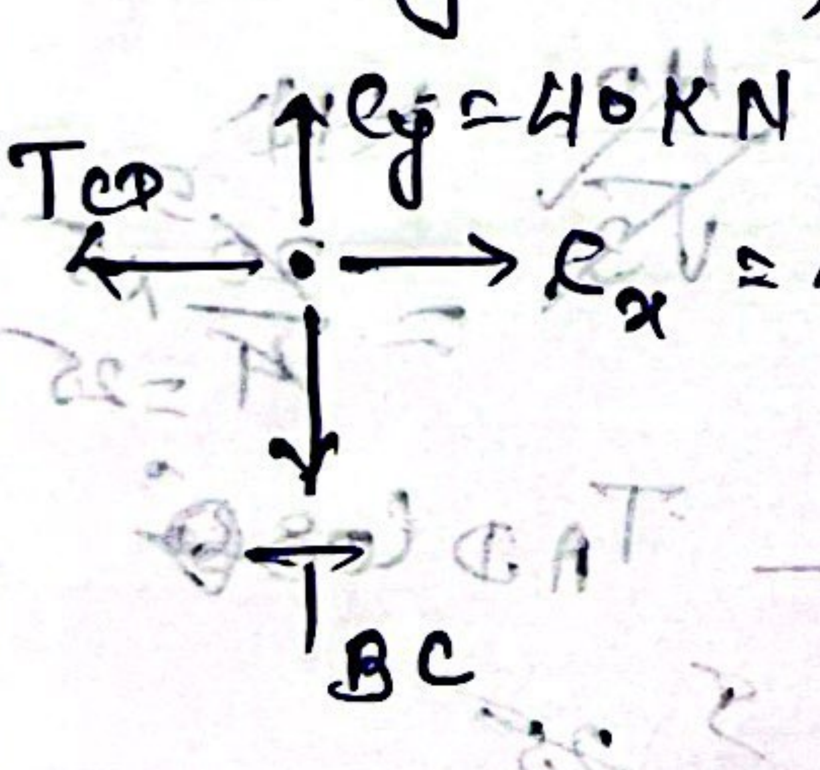
at joint B,



$T_{BC} + T_{BD}\sin\theta = 0$

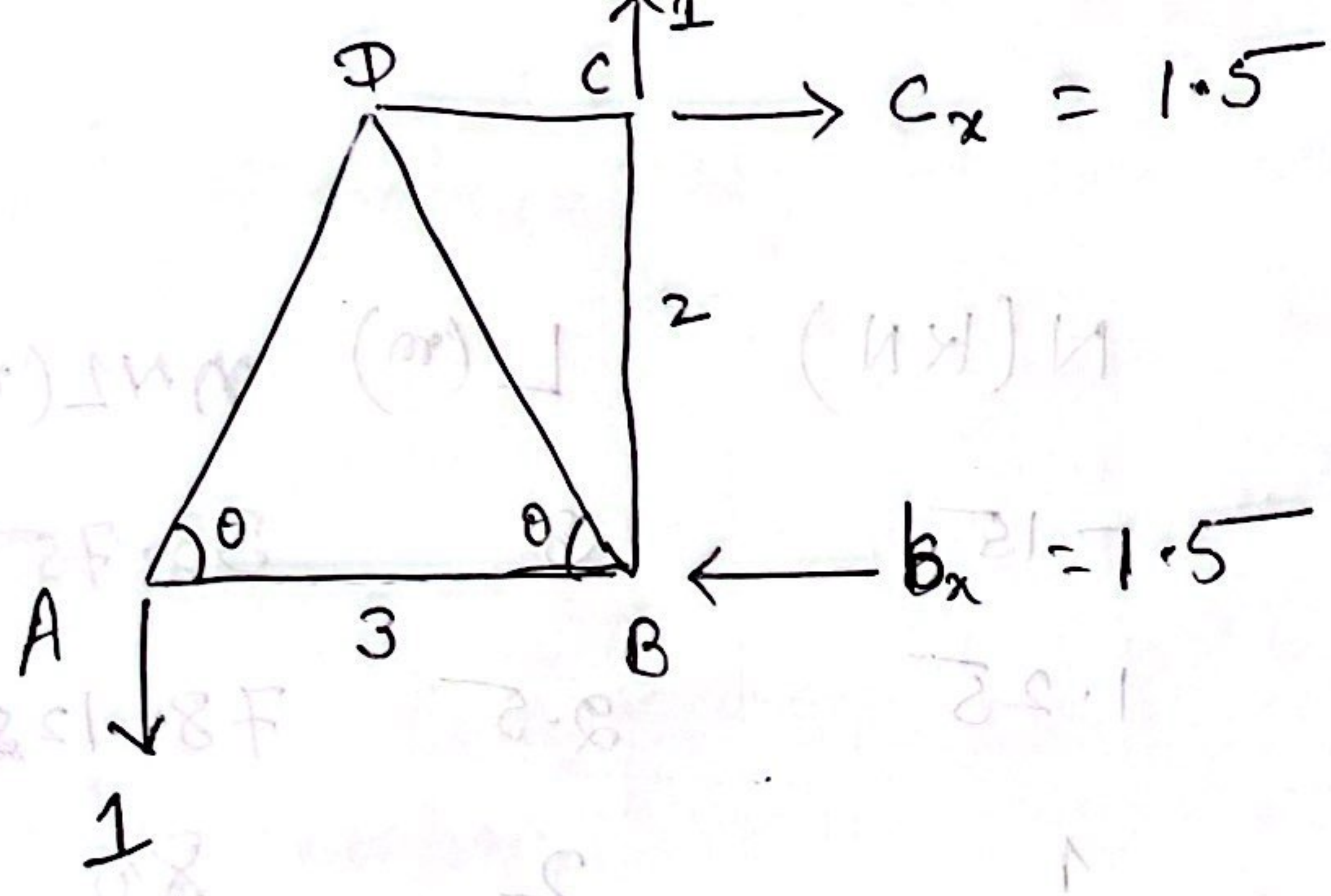
$\Rightarrow T_{BD} = -\frac{T_{BC}}{\sin\theta} = -\frac{40 \times 5}{4} = -50$ KN

at joint c,



$T_{CD} = 45$ KN

$T_{BC} = 40$ KN.



$b_x \, 2 = 3 \times 1$

$b_x = \frac{3}{2}$

$t_{AD} \sin\theta \uparrow$

$t_{AD} \cos\theta \rightarrow \cdot \rightarrow t_{AB}$

$\downarrow 1$

$t_{AD} = \frac{1}{\sin\theta} = \frac{5}{4} = 1.25$

$t_{AB} = -t_{AD} \cos\theta = -\frac{\cancel{5}}{4} \times \frac{3}{\cancel{5}} = -0.75$

at C,

$\uparrow 1$

$\leftarrow \cdot \rightarrow 1.5$

$1.5 = t_{CD}$

$t_{BC} = 1.$

at B,

t_{BD} 1

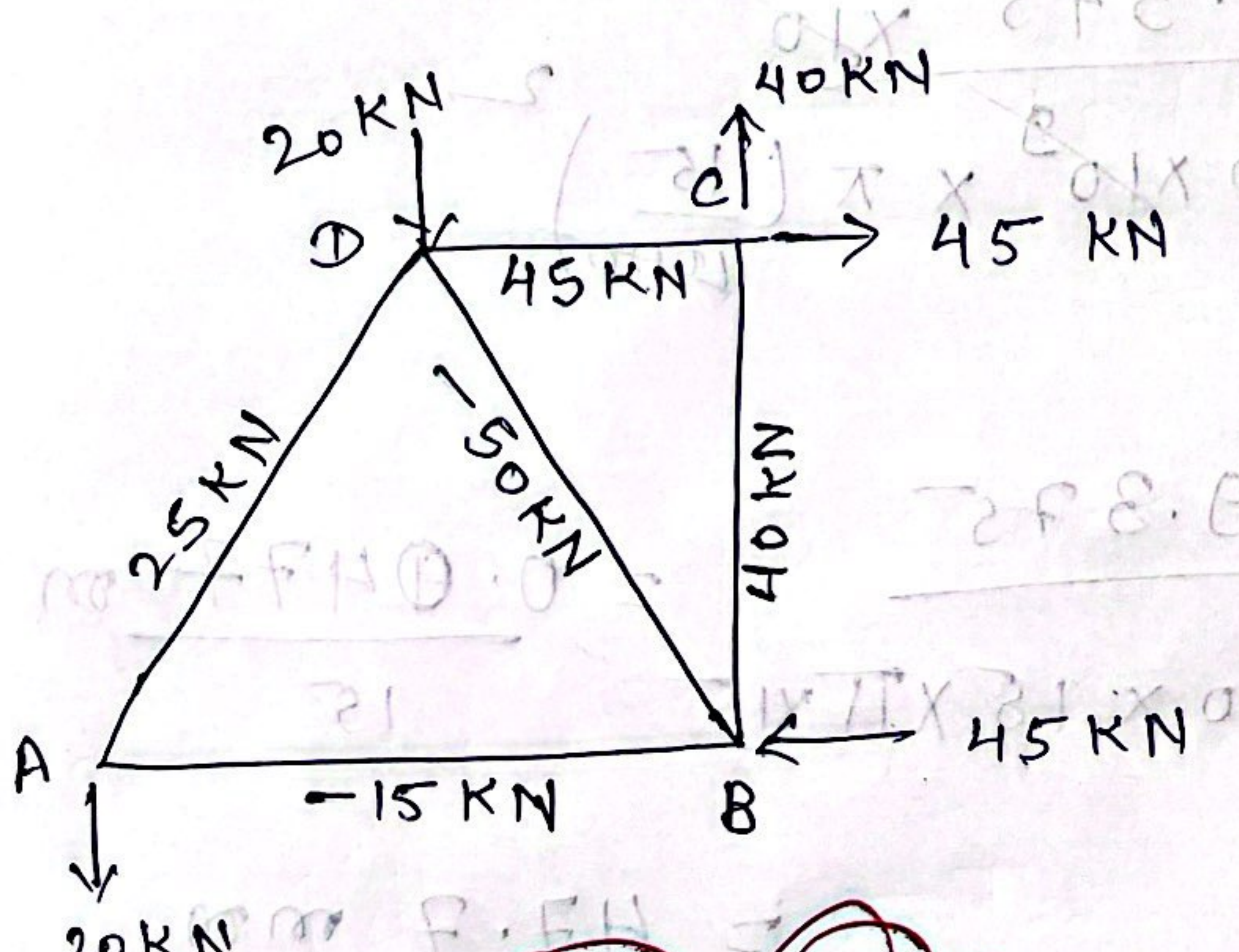
θ

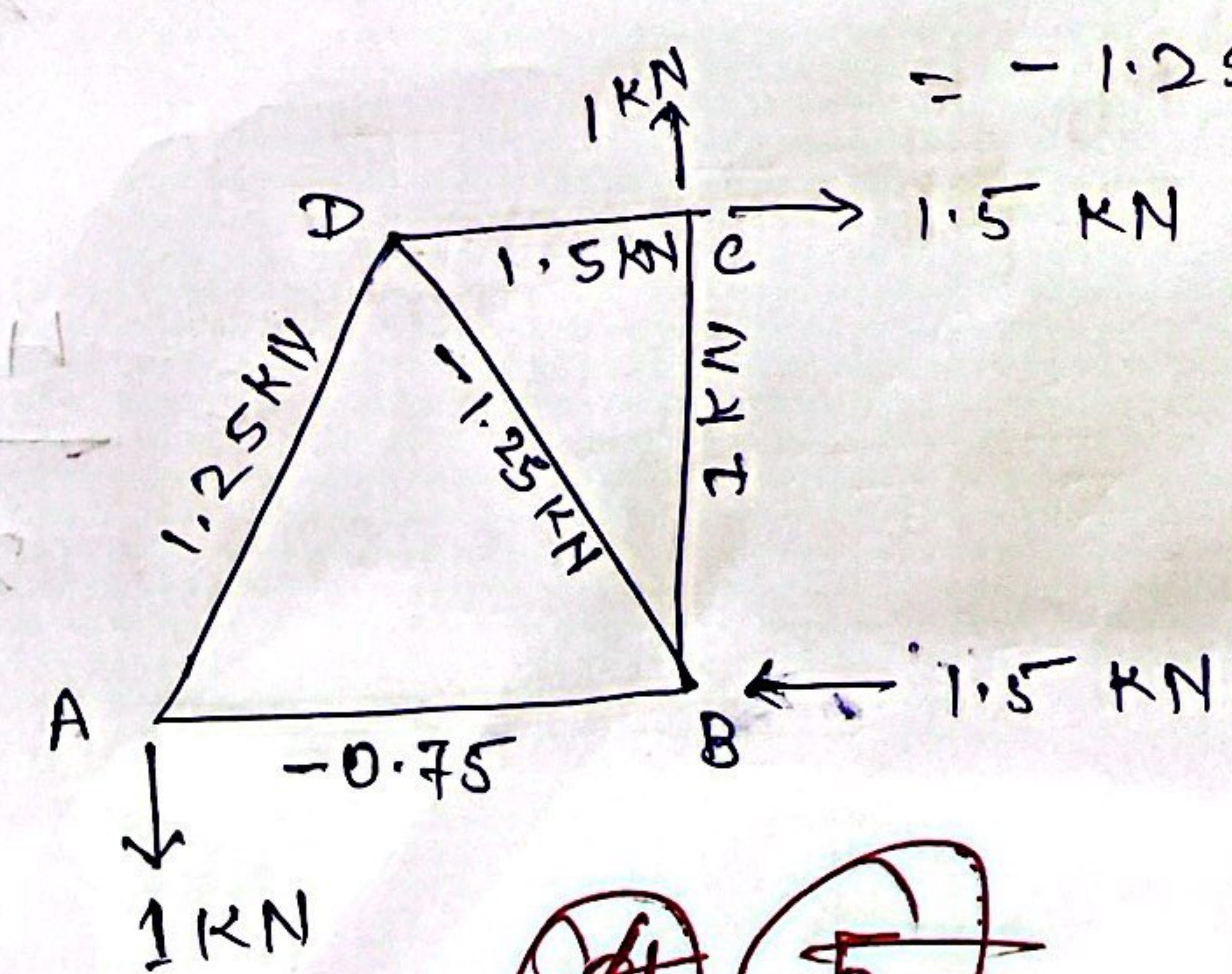
$-0.75 \leftarrow \cdot \leftarrow 1.5$

$t_{BD} \sin\theta = -1$

$\Rightarrow t_{BD} = \frac{-1}{\sin\theta} = -\frac{5}{4}$

$= -1.25$





~~3~~ ~~5~~

5 ~~4~~

~~3~~ ~~5~~

3 ~~4~~

Member	n(KN)	N(KN)	L(m)	nNL(m
AB	−0.75	−15	3	33.75
AD	25	1.25	2.5	78.125
BC	40	1	2	80
CD	45	1.5	1.5	101.25
BD	−50	−1.25	2.5	156.25
				449.375

from the unit dummy load method,

$$1\,\cancel{KN}\cdot\Delta\,\cancel{m} = \frac{449.375\,(KN)^2\,\cancel{m}}{EA}$$

~~7~~ 6

$$\Rightarrow \quad \Delta = \frac{449.375}{EA}\ \underline{KN}$$

$$= \frac{449.375 \times \cancel{10^3}}{200 \times \cancel{10^9} \times \pi \left(\frac{15}{1000}\right)^2}$$

$$= \frac{449.375}{200 \times 15 \times \pi \times 15} = \frac{0.0477}{15}\ m$$

$$= \frac{47.7}{15}\ mm$$

$$= 3.1787\ mm$$

(Ans.)

(b) Members which are in compression.

→ AB, BD

1

(c) ~~Compr.~~ Compressive Stress in Compression members,

$$\sigma_{AB} = \frac{\cancel{15} \times 10^{9}}{\pi \times \cancel{15} \times 15} \text{ Pa} = 21.2207 \text{ MPa}$$ 1

$$\sigma_{BD} = \frac{50 \times 10^{9}}{\pi \times 15 \times 15} = 70.7355 \text{ MPa.}$$ 2

yes both Compressive stress are less than.

$\sigma_y = 345$ MPa,

$I = \frac{1}{2}\int r^2 \, dA$

$\frac{1}{2}\int_0^R 2\pi r^3 \, dr$

$\frac{1}{2}\left(2\pi \frac{r^4}{4}\right)$

$= \frac{\pi}{64} d^4$

(d.) for AB, $P_{cr} = \frac{\pi^2 EI}{L^2}$

~~EI =~~ $I = \frac{\pi}{64} d^4 = \frac{\pi}{4} r^4$

$= \frac{\pi}{4} \times 225 \times 225 \text{ mm}^4$

$= \frac{7.9522 \times 10^4}{2} \text{ mm}^4$

$EI = \frac{200 \times 10^9}{2} \times 7.9522 \times 10^4 \times 10^{-12}$

$= \frac{1.5904 \times 10^4}{2}$ ~~mm^4~~ N m^2

2.5

$$P_{cr} = \frac{\pi^2 \times 1.5904 \times 10^4}{2 \times (\cancel{2.375})^2} \text{ N} = \cancel{1.378}\ \cancel{137.805 \text{ N}}\ \frac{17.44}{2} \text{ KN}$$

$= 8.72$ KN.

for member. BD

$$P_{cr} = \frac{\pi^2 \times 1.5904 \times 10^4}{(2.5)^2 \times 2} N$$

2.5

$$= \frac{25.115}{2} \text{ KN. } \underline{\underline{Ans.}}$$

$$= 12.52 \text{ KN}$$

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