



INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR
End-Autumn Semester Examination 2024-25

Date of examination: Nov. 19, 2024

Session: AN

Duration: 3 hrs

Full Marks: 100

Subject No.: ME21203

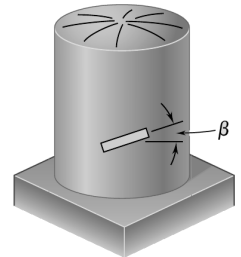
Subject: Mechanics of Solids

Department: Mechanical Engineering

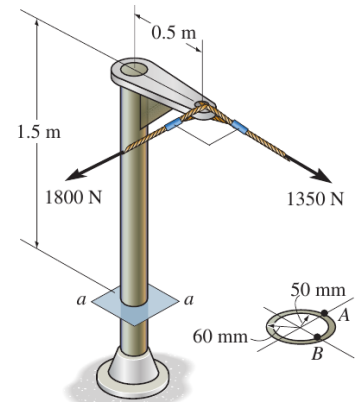
Specific charts, graph paper, log book, etc. required: NO

Special Instructions (if any): Answer all the parts of a question together.

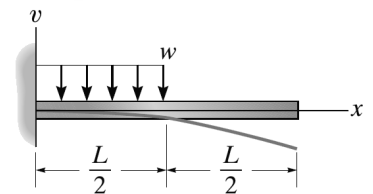
1. A strain gauge forming an angle $\beta = 18^\circ$ with a horizontal plane is used to determine the gauge pressure in the cylindrical steel tank shown (a kind of pressure vessel). The cylindrical wall of the tank is 6 mm thick, has a 600-mm inside diameter, and is made of a steel with $E = 200$ GPa and $\nu = 0.3$. Determine the pressure in the tank indicated by a strain gauge reading of 280×10^{-6} . [12 marks]



2. Determine the normal stress and the shear stress at the point B on the cross-section of the post at section $a - a$. Note that B is a point on the inner boundary of the annulus. [16 marks]



3. A cantilever beam of constant flexural rigidity (EI) is subjected to a load, $w(x)$ as shown in the figure. By solving the 2nd-order differential equations for deflection of beams together with the use of appropriate boundary and other required conditions, determine the deflection $v(x)$.

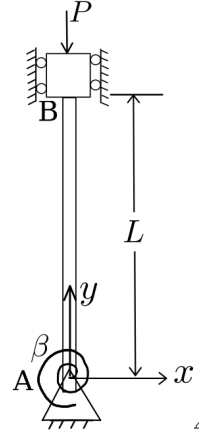


- (a) For the part $0 \leq x \leq L/2$, your answer MUST be of the form: $v = \frac{-wx^2}{K_1 EI} (aL^2 + bLx + cx^2)$, where the values of K_1 , a , b , and c are to be determined. K_1 must be positive.
- (b) For the part $L/2 \leq x \leq L$, your answer MUST be of the form: $v = \frac{-wL^3}{K_2 EI} (pL + qx)$, where the values of K_2 , p , and q are to be determined. K_2 is positive.

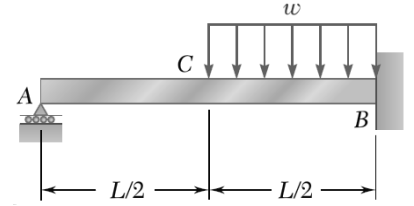
[7 + 7 = 14 marks]

4. A thin annular disk of inner radius r_i and outer radius r_o is fixed at its outer boundary to a rigid support. However axial displacement is not constrained. At the inner boundary, there is a force (per unit area) f acting radially inward. Starting from the governing differential equation for radial displacement in the form: $\frac{d^2 u}{dr^2} + \frac{1}{r} \frac{du}{dr} - \frac{u}{r^2} = 0$, and using the appropriate boundary conditions, determine the radial stress at the outer boundary. [12 marks]

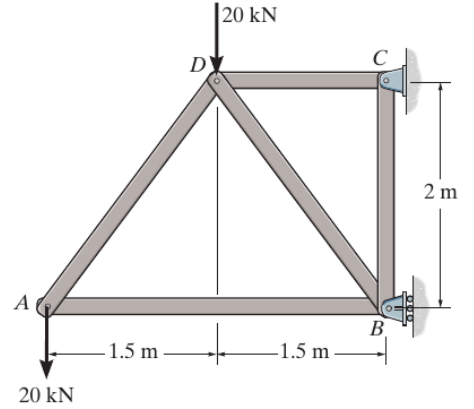
5. Consider a column with constant flexural rigidity (EI) as shown in the figure. The bottom end A ($x = 0$) is pinned and attached with a rotational spring of stiffness β . The top end B ($x = L$) is fixed to the vertical guide. Showing all the steps involving the solution of the 4th-order differential equation together with the use of the proper boundary conditions, determine the critical condition(s) for buckling. **[12 marks]**



6. A beam AB is supported by rollers at end A, is clamped to the wall at B, and is subjected to a distributed load as shown in the figure. The flexural rigidity is EI , a constant. Using Castigliano's theorem, determine the reaction force at the end A. **[12 marks]**



7. The truss is made from steel rods ($E = 200$ GPa). Each rod has a diameter of 30 mm.
- Determine the vertically downward deflection of the joint A, using the principle of virtual work. **[14 marks]**
 - Which of the members are in compression? **[1 mark]**
 - Do the stresses in the compression members exceed the yield stress, $\sigma_y = 345$ MPa? **[2 marks]**
 - Considering pinned-pinned end conditions, can the compression members buckle? **[5 marks]**



— END OF QUESTION PAPER —

List of useful formulae

- Torsion: $\tau = \frac{Tr}{J}$, $\phi = \frac{TL}{GJ}$; Bending: $\frac{dV}{dx} = -w$, $\frac{dM}{dx} = V$; Flexure: $\sigma = -\frac{My}{I}$; Shear: $\tau = -\frac{VQ}{It}$
- Strain transformation:

$$\varepsilon_{x'x'} = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} + \frac{\varepsilon_{xx} - \varepsilon_{yy}}{2} \cos 2\theta + \varepsilon_{xy} \sin 2\theta; \quad \varepsilon_{x'y'} = -\frac{\varepsilon_{xx} - \varepsilon_{yy}}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$
- 2nd-order beam deflection equation: $EI \frac{d^2y}{dx^2} = M$
- 4th-order equation for buckling: $EI \frac{d^4y}{dx^4} + P \frac{d^2y}{dx^2} = 0$
- Buckling: Pinned-Pinned Ends: $L_{\text{eff}} = L$