INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Mid-Autumn Semester Examination 2024-25

Date of examination: Sep. 18, 2024 **Session:** AN **Duration:** 2 hrs **Full Marks:** 60
Subject No.: ME21203 **Subject:** Mechanics of Solids
Department: Mechanical Engineering
Specific charts, graph paper, log book, etc. required: NO
Special Instructions (if any): Answer all the parts of a question together.

1. Consider a situation where the coordinate axes are oriented along the principal directions. Then the state of stress is given by (you don't have to explain or prove this):

$$[\boldsymbol{\sigma}] = \begin{bmatrix} \sigma_{p1} & 0 & 0 \\ 0 & \sigma_{p2} & 0 \\ 0 & 0 & \sigma_{p3} \end{bmatrix}$$

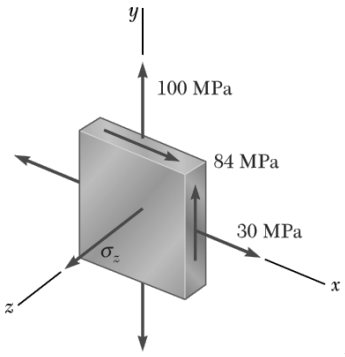
The deviatoric part of the above stress matrix is given by: $[\boldsymbol{\sigma}^D] = [\boldsymbol{\sigma}] - \sigma^m[\mathbf{I}]$, where $[\mathbf{I}]$ is the identity matrix and $\sigma^m = \dfrac{1}{3}(\sigma_{p1} + \sigma_{p2} + \sigma_{p3})$.

A widely used method to represent the stress field in a body is in terms of something called the von Mises stress which is defined as $\sigma_{\text{vonMises}} = \sqrt{\dfrac{3}{2}\left(\sum_{j=1}^{3}\sum_{i=1}^{3}\left(\sigma_{ij}^D\right)^2\right)}$, where σ_{ij}^D is an element of $[\boldsymbol{\sigma}^D]$ in the i-th row and j-th column.

Show that $\sigma_{\text{vonMises}} = \sqrt{I_1^2 - 3I_2}$, where $I_1 = \text{trace}(\boldsymbol{\sigma})$ and $I_2 = \sigma_{p1}\sigma_{p2} + \sigma_{p2}\sigma_{p3} + \sigma_{p3}\sigma_{p1}$.

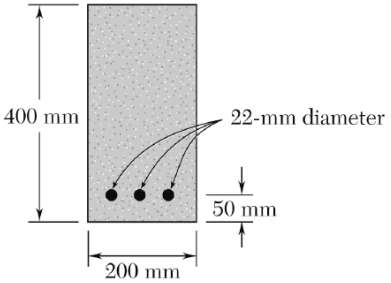
[6 marks]

2. The state of stress at a point is depicted on a stress element as shown in the figure: $\sigma_{xx} = 30$ MPa, $\tau_{xy} = 84$ MPa, and $\sigma_{yy} = 100$ MPa. If $\sigma_{zz} = -60$ MPa, determine *all* the principal stresses and the *absolute* maximum shear stress. **[6 marks]**



3. A concrete beam is reinforced by three steel rods placed as shown in the figure. The Young's modulus for concrete is 20 GPa and that for steel is 200 GPa. The allowable stress for concrete is 9 MPa and that for steel is 140 MPa. The beam is subjected to a bending moment such that the top part of the cross-section is in compression while the bottom part is in tension.
   (a) What is the location of the neutral axis (in mm) with respect to the top of the cross-section?
   (b) What is the area moment of inertia (or, second moment of area) (in mm^4) of the cross-section about the neutral axis?
   (c) Determine the largest bending moment (in N·m) that can be sustained by the beam.

[2 + 3 + 5 = 10 marks]



4. Consider the displacement field given by: $u_x = kxy$, $u_y = kxy$, $u_z = 2k(x+y)z$, where k is a constant.

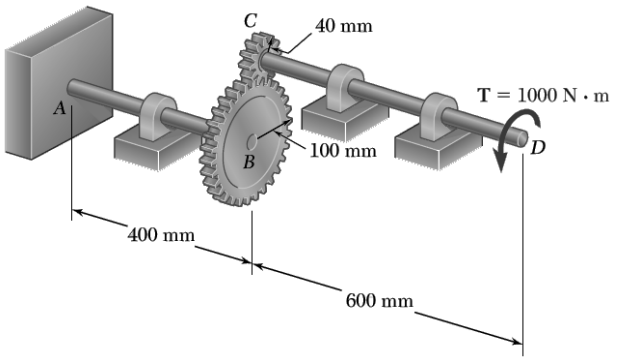
   (a) Determine the strain matrix, using linear strain-displacement relations.

   (b) What is the normal strain in the direction, $N_x = N_y = N_z = 1/\sqrt{3}$?

   (c) Let the material in which the above strain field exists follow the generalized Hooke's law (E, ν). It is known that ν cannot exceed 1/2. Now, a student claims that on the plane $z = 0$, a state of plane stress exists, i.e. $\sigma_{zz} = 0$, $\sigma_{zx} = 0$, $\sigma_{yz} = 0$. Assumming these 0 values, compare the expression of ε_{zz} from the stress-strain relations with that from the strain matrix, and check if the student's claim leads to a contradiction or not.

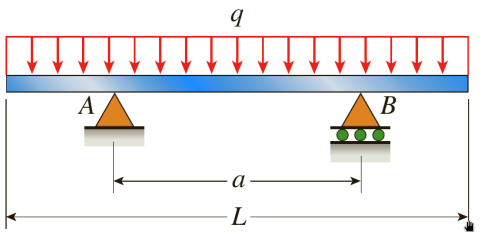
[3 + 2 + 3 = 8 marks]

5. The design of the gear-and-shaft system shown requires that steel shafts of the same diameter be used for both AB and CD. It is further required that $\tau_{\max} \leq 60$ MPa, and that the angle ϕ_D through which end D of shaft CD rotates not exceed 1.5°. Given $G = 77.2$ GPa, determine the required diameter. The bearings through which the shafts pass do not provide any resistance to their rotation. Hint: When two gears mesh, the contact forces are equal (not the torques!); additionally the arcs of contact are equal.

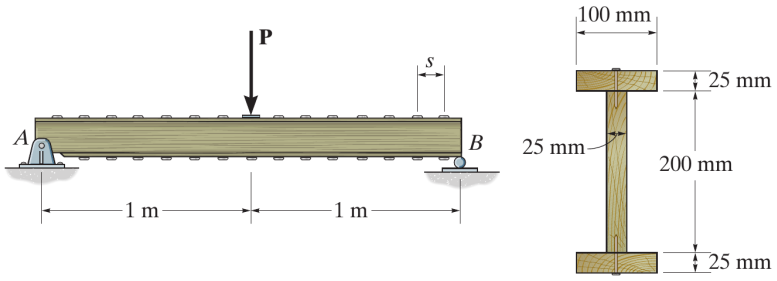


[10 marks]

6. A beam of length L is designed to support a uniform load of intensity q (see figure). If the supports of the beam are placed at the ends, the maximum bending moment in the beam is $qL^2/8$. However, if the supports of tbe beam are moved symmetrically toward the middle of the beam (as shown), the maximum bending moment is reduced. Determine the distance a between the supports so that the maximum bending moment in the beam has the smallest possible numerical value. Draw the shear-force and bending-moment diagrams for this condition. **[12 marks]**



7. The simply-supported beam is built-up from three boards by nailing them together as shown. If $P = 12$ kN, determine the maximum allowable spacing s of the nails to support that load, if each nail can resist a shear force of 1.5 kN. **[8 marks]**



End of Question Paper

List of useful formulae

- Torsion: $\tau = \dfrac{Tr}{J}$; $\quad \phi = \dfrac{TL}{GJ}$
- Bending: $\dfrac{\mathrm{d}V}{\mathrm{d}x} = -w$ (for w pointed downward); $\dfrac{\mathrm{d}M}{\mathrm{d}x} = V$
- Flexure formula: $\sigma = -\dfrac{My}{I}$
- Shear formula: $\tau = -\dfrac{VQ}{It}$
- Stress transformation:
  $\sigma_{x'x'} = \dfrac{\sigma_{xx} + \sigma_{yy}}{2} + \dfrac{\sigma_{xx} - \sigma_{yy}}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$
  $\tau_{x'y'} = -\dfrac{\sigma_{xx} - \sigma_{yy}}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$