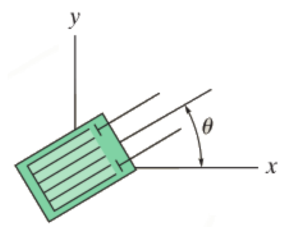
INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

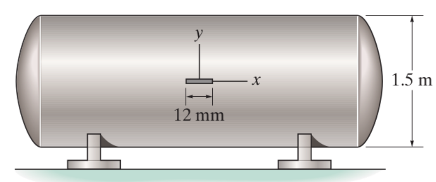
End-Autumn Semester Examination 2022-23

Date of examination: Nov. 20, 2023 **Session:** AN **Duration:** 3 hrs **Full Marks:** 100
Subject No.: ME21203 **Subject:** Mechanics of Solids
Department: Mechanical Engineering
Specific charts, graph paper, log book, etc. required: NO
Special Instructions (if any): Answer all the parts of a question together.

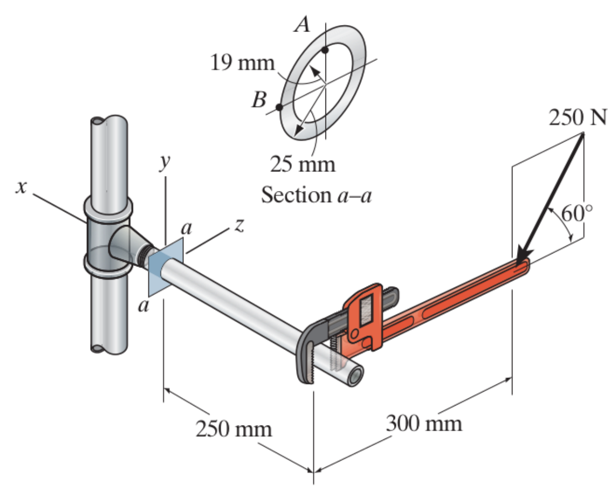
1. A point in a body under plane stress conditions has a state of stress given by non-zero σ_{xx} and σ_{yy} but with $\tau_{xy} = 0$. The material constants are E and ν. A strain gauge placed at the point is orientated in such a way that its reading of the normal strain (along its own axis) is dependent only on σ_{yy} but not on σ_{xx}. Show that the orientation θ is given by $\tan\theta = \dfrac{1}{\sqrt{\nu}}$. **[10 marks]**



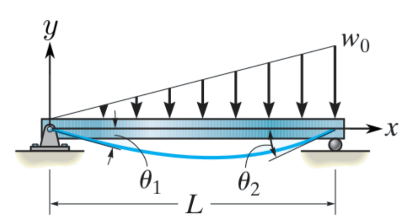
2. The strain gauge is placed on the surface of a thin-walled steel boiler (a kind of pressure vessel) as shown. The boiler wall has an inner diameter of 1.5 m and a wall thickness of 12 mm. For the steel, $E = 200$ GPa and $\nu = 0.3$.
   (a) If the strain gauge is 12 mm long, determine the pressure in the boiler when the gauge undergoes an elongation of 5×10^{-3} mm.
   (b) Determine the maximum *in-plane* shear strain in the wall.
   [10 + 2 = 12 marks]



3. Determine the normal stress and the shear stress at the point A on the cross-section of the pipe at section $a - a$. **[16 marks]**

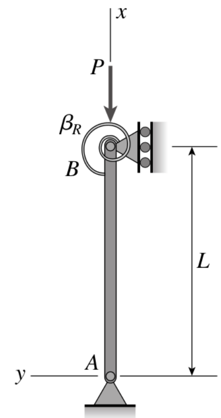


4. A simply-supported beam of constant flexural rigidity (EI) is subjected to a linearly varying distributed load, $w(x) = \dfrac{w_0 x}{L}$. Determine the expression of the beam deflection, $y(x)$, by solving the 2nd-order differential equation for deflection of beams together with the use of appropriate boundary conditions. Your answer MUST be expressed in the form: $y = -\dfrac{w_0 x}{KEIL}\left(ax^4 + bx^3L + cx^2L^2 + dL^3x + eL^4\right)$, where the values of K, a, b, c, d, and e are to be determined. K must be positive. **[14 marks]**

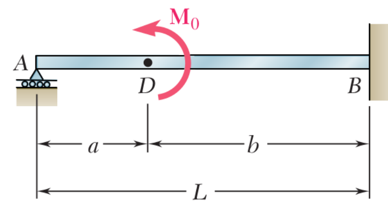


5. A column with constant flexural rigidity (EI) is pinned at both its ends. At the top end ($x = L$), there is a rotational spring of stiffness β_R. Showing all the steps involving the solution of the 4th-order differential equation together with the use of the proper boundary conditions, show that the equation for finding the critical buckling load will be: $\dfrac{\beta_R L}{EI}\left(kL\cot(kL) - 1\right) = k^2L^2$, where $k^2 = \dfrac{P}{EI}$. **[14 marks]**

   NOTE: You don't have to solve this equation for the critical load.



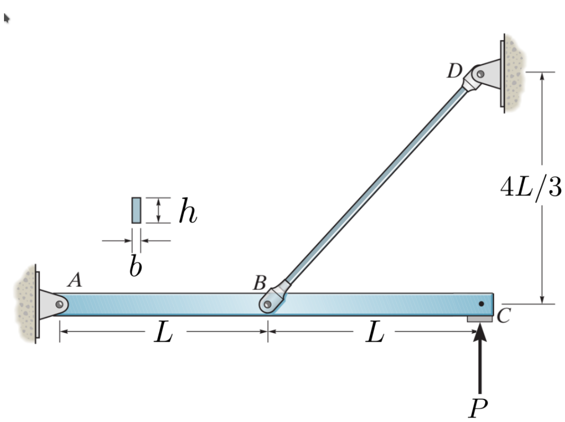
6. A beam AB is supported by rollers at end A, is clamped to the wall at B, and is subjected to a moment M_0 as shown in the figure. The flexural rigidity is EI, a constant. Using Castigliano's theorem, determine the reaction force at the the end A. **[12 marks]**



7. The bar ABC has a rectangular cross-section ($b \times h$) while the attached rod BD has a circular cross-section of diameter d. Both members are made of the same material (Young's modulus, E). A point load P is acting at C in the vertically upward direction.

   (a) Determine the vertically upward deflection of the end C, using the principle of virtual work. Neglect contribution due to shear. **[12 marks]**

   (b) Taking $h = 3b$, determine the relation between b and d such that for a given P, both the part AB and the member BD will have a tendency to buckle together *out of the plane*. **[10 marks]**



End of Question Paper

List of useful formulae

- Torsion: $\tau = \dfrac{Tr}{J}$, $\phi = \dfrac{TL}{GJ}$; Bending: $\dfrac{\mathrm{d}V}{\mathrm{d}x} = -w$, $\dfrac{\mathrm{d}M}{\mathrm{d}x} = V$; Flexure: $\sigma = -\dfrac{My}{I}$; Shear: $\tau = -\dfrac{VQ}{It}$
- Strain transformation:

$$\varepsilon_{x'x'} = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} + \frac{\varepsilon_{xx} - \varepsilon_{yy}}{2}\cos 2\theta + \varepsilon_{xy}\sin 2\theta; \qquad \varepsilon_{x'y'} = -\frac{\varepsilon_{xx} - \varepsilon_{yy}}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

- 2nd-order beam deflection equation: $EI\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = M$
- 4th-order equation for buckling: $EI\dfrac{\mathrm{d}^4y}{\mathrm{d}x^4} + P\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = 0$
- Buckling: Pinned-Pinned Ends: $L_{\text{eff}} = L$; Fixed-Fixed Ends: $L_{\text{eff}} = 0.5L$