INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

End-Autumn Semester Examination 2022-23

Date of examination: Nov. 16, 2022 **Session:** FN **Duration:** 3 hrs **Full Marks:** 100
Subject No.: ME21203/ME31013 **Subject:** Mechanics of Solids
Department: Mechanical Engineering
Specific charts, graph paper, log book, etc. required: NO
Special Instructions (if any): Answer all the parts of a question together.

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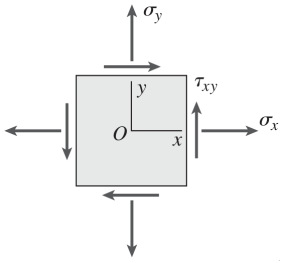
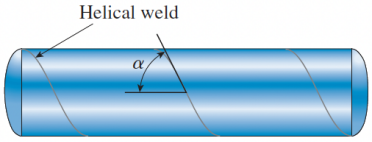
1. An element in *plane stress* is subjected to stresses $\sigma_x = -150$ MPa, $\sigma_y = -210$ MPa, $\tau_{xy} = -16$ MPa. The material is brass with $E = 100$ GPa and $\nu = 0.34$. Determine the principal strains and the maximum in-plane shear strains within the xy plane, together with their corresponding directions. **[10 marks]**

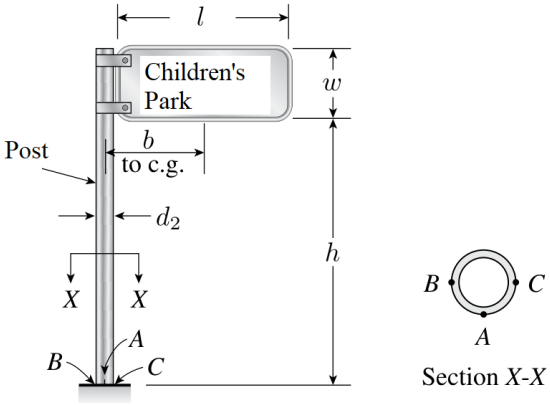


2. A pressurized steel tank is constructed with a helical weld that makes an angle $\alpha = 55°$ with the longitudinal axis. The tank has inner radius $r = 0.6$ m, wall thickness $t = 18$ mm, and internal pressure $p = 2.8$ MPa. For the steel, $E = 200$ GPa and $\nu = 0.30$. For the cylindrical part of the tank (neglecting the end caps), determine the following quantities:
   (a) The maximum in-plane and out-of-plane shear stresses.
   (b) The normal and shear stresses acting on planes parallel and perpendicular to the weld.

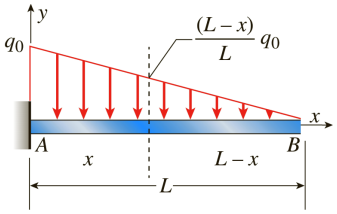
   [2 + 8 = 10 marks]



3. Consider the sign post shown in the figure. The post has an inner diameter of $d_1 = 90$ mm and outer diameter of $d_2 = 110$ mm. The sign has a length of $l = 2$ m and a width of $w = 1$ m. The lower edge of the sign is at a height of $h = 3$ m above the base. The centre of gravity of the sign is at a distance of $b = 1.05$ m from the axis of the post. The wind pressure acting perpendicularly to the face of the sign is $p = 1$ kPa. Determine the maximum in-plane shear stresses (in MPa) at the points A and B located on the outer surface at the base of the post. **[14 marks]**

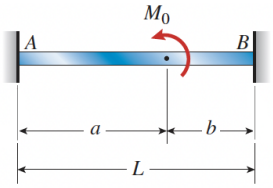


4. A cantilever beam of constant flexural rigidity (EI) is subjected to a linearly varying distributed load, $q(x)$ as shown in the figure. The highest intensity of this distributed load is $q(0) = q_0$. Determine the expression of the beam deflection, $y(x)$, by solving the fourth-order differential equation for deflection of beams together with the use of appropriate boundary conditions. Your answer MUST be expressed in the form: $y = -\dfrac{q_0 x^2}{KEIL}\left(aL^3 + bL^2x + cLx^2 + dx^3\right)$, where the values of K, a, b, c, and d are to be determined. **[12 marks]**

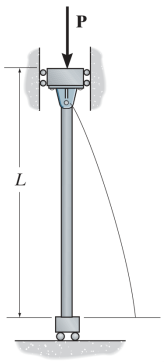


5. Consider a long thick-walled cylinder of inner radius r_i and outer radius r_o. The cylinder is subjected to an internal pressure p. The outer wall of the cylinder is constrained against any movement. The Young's modulus of the wall material is E and the Poisson's ratio is ν. Determine the radial stress at the outer radius. **Note:** All the steps including the solution of the differential equationand the use of the boundary conditions must be clearly shown. **[14 marks]**

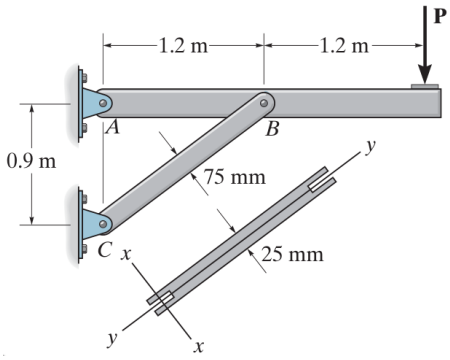
6. A beam AB clamped to walls at both ends is subjected to a moment M_0 as shown in the figure. The Young's modulus is E and the second moment of area is I. Determine the reaction forces and moments at the two ends A and B. **[10 marks]**



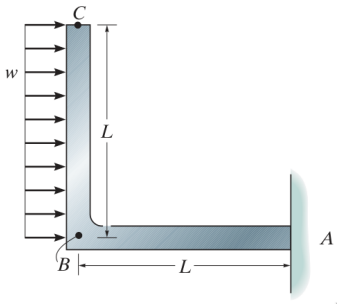
7. A column with constant flexural rigidity (EI) is pinned at its top end ($x = 0$) and has a guided support at the bottom end ($x = L$). Showing all the steps involving the solution of the appropriate differential equation together with the use of the proper boundary conditions, determine the critical buckling load. **[10 marks]**



8. Determine the greatest load, P the frame will support without causing the steel member BC to buckle. Consider the supports at B and C to act as pins for $x - x$ buckling (i.e. buckling in the plane of the paper) and to act as fixed or clamped supports for $y - y$ buckling (i.e. buckling in a plane perpendicular to the paper). For steel, $E = 200$ GPa. **[10 marks]**



9. The L-shaped frame is made from two segments, each of length L and flexural rigidity EI. If it is subjected to the uniformly distributed load, w as shown in the figure, determine the horizontal deflection of the end C, using Castigliano's theorem on deflection. **[10 marks]**



List of useful formulae

- Torsion: $\tau = \dfrac{Tr}{J}$, $\phi = \dfrac{TL}{GJ}$; Bending: $\dfrac{\mathrm{d}V}{\mathrm{d}x} = -w$, $\dfrac{\mathrm{d}M}{\mathrm{d}x} = V$; Flexure: $\sigma = -\dfrac{My}{I}$; Shear: $\tau = -\dfrac{VQ}{It}$
- Stress transformation:

$$\sigma_{x'x'} = \frac{\sigma_{xx} + \sigma_{yy}}{2} + \frac{\sigma_{xx} - \sigma_{yy}}{2}\cos 2\theta + \tau_{xy}\sin 2\theta; \qquad \tau_{x'y'} = -\frac{\sigma_{xx} - \sigma_{yy}}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

- Buckling: Pinned-Pinned Ends: $L_{\text{eff}} = L$; Fixed-Fixed Ends: $L_{\text{eff}} = 0.5L$