Q1

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\sigma_y\right]$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\sigma_x\right]$$

$$\varepsilon_{xy} = \frac{\tau_{xy}}{2G}$$

$$\varepsilon_{avg} = \frac{1}{2}(\varepsilon_x + \varepsilon_y)$$

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \varepsilon_{xy}^2}$$

$$\varepsilon_{1,2} = \varepsilon_{avg} \pm R = -732\times10^{-6},\ -1644\times10^{-6}$$

$$\tan 2\theta_p = \frac{2\varepsilon_{xy}}{\varepsilon_x - \varepsilon_y}$$

$$2\theta_p = -28.072^\circ,\ 151.9275^\circ$$

$$\theta_p = -14.036^\circ,\ 75.964^\circ$$

OR

$$\theta_p = 75.964^\circ,\ 165.964^\circ$$

Q2 $\sigma_1 = \frac{pr}{t} = 93.33$ MPa

$\sigma_2 = \frac{pr}{2t} = 46.67$ MPa

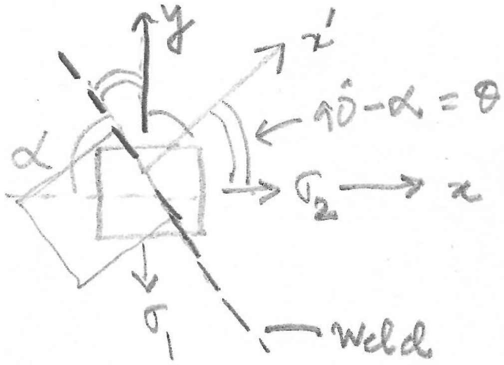
(a) $\tau_{\text{in-plane}} = \frac{1}{2}(\sigma_1 - \sigma_2) = 23.33$ MPa

$\tau_{\text{out-plane}} = \frac{1}{2}(\sigma_1 - 0) = 46.67$ MPa

(b) $\sigma_x = \sigma_2$

$\sigma_y = \sigma_1$

$\theta = 90^\circ - \alpha$ (Parallel to the weld)



$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \underbrace{\cancel{\tau_{xy}}}_{0}\sin 2\theta$$

$= 62.02$ MPa (normal stress on a plane ∥ to weld)

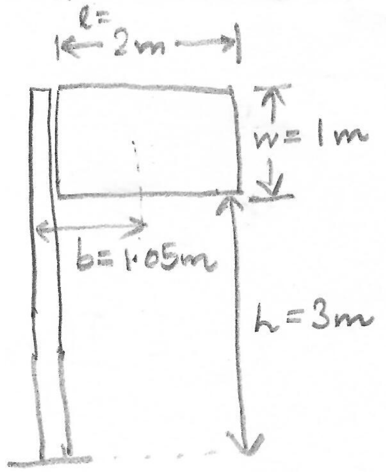
$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos\{2(\theta + 90^\circ)\} + \underbrace{\cancel{\tau_{xy}}}_{0}\sin\{2(\theta+90^\circ)\}$$

$= 77.98$ MPa (normal stress on a plane ⊥ to weld)

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \underbrace{\cancel{\tau_{xy}}}_{0}\cos 2\theta$$

$= 21.93$ MPa (shear stress on a plane ∥ and ⊥ to weld)

Q3



$p = 1\ \text{kPa}$

$P = p\,l\,w = \times 1 \times 2 \times 1\ \text{kN} = 2\ \text{kN}$

$d_2 = 110\ \text{mm}$

$d_1 = 90\ \text{mm}$

$$I = \frac{1}{4}\pi\left(r_2^4 - r_1^4\right) = \frac{1}{64}\pi\left(d_2^4 - d_1^4\right)$$

$$= 3.966 \times 10^6\ \text{mm}^4$$

$$= 3.966 \times 10^{-6}\ \text{m}^4$$

Moment due to wind pressure at the base : $M = P\left(h + w/2\right)$

$$= 2\,\text{kN} \times (3 + 0.5)\ \text{m}$$

$$= 7\ \text{kN}\cdot\text{m}$$

Torque on the sign post : $T = Pb$

$$= 2\,\text{kN} \times 1.05\ \text{m}$$

$$= 2.10\ \text{kN}\cdot\text{m}$$

Shear force on the sign post c/s : $V = P = 2\ \text{kN}$

For A:

$$\sigma^A = \frac{My}{I}$$

$$= \frac{M d_2/2}{I} = 97.069\ \text{MPa}$$

$$\tau^A = \frac{T d_2/2}{J} = 14.56\ \text{MPa}$$ (only due to torsion)

$$\tau^A_{max} = \sqrt{\left(\frac{\sigma^A_{xx} - \sigma^A_{yy}}{2}\right)^2 + \tau^A_{yx}}$$ ← Max in-plane shear stress

$$\boxed{\tau^A_{max} = 50.67\ \text{MPa}}$$

For B:

$$\sigma^B = 0 \quad (\because y = 0)$$

$$\tau^B_{yx} = \frac{T(d_2/2)}{J} - \frac{VQ}{It}$$

$$Q = \frac{4r_2}{3\pi} \times \frac{1}{2}\pi r_2^2 - \frac{4r_1}{3\pi} \times \frac{1}{2}\pi r_1^2$$

$$= \frac{2}{3}(r_2^3 - r_1^3)$$

$$= \frac{2}{3}\frac{(d_2^3 - d_1^3)}{8}$$

$$= \frac{1}{12}(d_2^3 - d_1^3)$$

$$= 5.017 \times 10^4 \text{ mm}^3 = 50167 \text{ mm}^3$$

$$= 5.017 \times 10^{-5} \text{ m}^3 \longrightarrow \frac{VQ}{It} = 1.265 \text{ MPa}$$

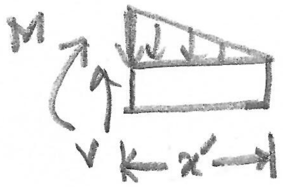
$t \rightarrow d_2 - d_1$

$$\tau^B_{yx} = \cancel{19} \; 13.295 \text{ MPa}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma^B_{xx} - \sigma^B_{yy}}{2}\right)^2 + {\tau^B_{yx}}^2}$$

$$= 13.295 \text{ MPa}$$

Q4



$$q = q_0\left(\frac{L-x}{L}\right)$$

$$M + \left(\frac{1}{2} q x'\right)\frac{x'}{3} = 0$$

$x' = L - x$

$$\Rightarrow M = -\frac{q x'^2}{6} = -\frac{q_0}{6L}(L-x)^3$$

$$EI\frac{d^4y}{dx^4} = -q = -\frac{q_0}{L}(L-x)$$

$\big\downarrow$ Integrate 4 times

$$y = -\frac{w_0}{EIL}\left(L\frac{x^4}{24} - \frac{x^5}{120} + C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3 x + C_4\right)$$

@ $x = 0$, $y = 0 \Rightarrow C_4 = 0$

@ $x = 0$, $\frac{dy}{dx} = 0 \Rightarrow C_3 = 0$

@ $x = L$, $\frac{d^2y}{dx^2} = 0 \Rightarrow 0 = -\frac{w_0}{EIL}\left(L\frac{L^2}{2} - \frac{L^3}{6} + C_1 L + C_2\right)$

$$\Rightarrow \frac{L^3}{3} + C_1 L + C_2 = 0$$

@ $x = L$, $\frac{d^3y}{dx^3} = 0 \Rightarrow 0 = -\frac{w_0}{EIL}\left(L^2 - \frac{L^2}{2} + C_1\right)$

$$\Rightarrow C_1 = -L^2/2$$

$$\therefore C_2 = L^3/6$$

$$y = -\frac{w_0 x^2}{120EIL}\left(10L^3 - 10L^2x + 5Lx^2 - x^3\right)$$

Q5

~~[illegible]~~

$$\frac{d\sigma_{rr}}{dr} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0$$

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Students need to show the intermediate steps as discussed in class.

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \frac{u}{r^2} = 0$$

$$u = r^m$$

$$u = A_1 r + A_2 \frac{1}{r}$$

$$\varepsilon_{rr} = \frac{du}{dr} = A_1 - \frac{A_2}{r^2}$$

$$\varepsilon_{\theta\theta} = \frac{u}{r} = A_1 + \frac{A_2}{r^2}$$

$$\sigma_{rr} = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_{rr} + \nu\varepsilon_{\theta\theta}\right]$$

$$\sigma_{\theta\theta} = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_{\theta\theta} + \nu\varepsilon_{rr}\right]$$

$$\sigma_{rr} = \bar{E}\left[(1-\nu)\left(A_1 - \frac{A_2}{r^2}\right) + \nu\left(A_1 + \frac{A_2}{r^2}\right)\right]$$

$$= \bar{E}\left[A_1 - \frac{A_2}{r^2} - \nu A_1 + \nu\frac{A_2}{r^2} + \nu A_1 + \nu\frac{A_2}{r^2}\right]$$

$$= \bar{E}\left[A_1 - \frac{A_2}{r^2} + 2\nu\frac{A_2}{r^2}\right]$$

$$\sigma_{\theta\theta} = \bar{E}\left[(1-\nu)\left(A_1 + \frac{A_2}{r^2}\right) + \nu\left(A_1 - \frac{A_2}{r^2}\right)\right]$$

$$= \bar{E}\left[A_1 + \frac{A_2}{r^2} - \nu A_1 - \nu\frac{A_2}{r^2} + \nu A_1 - \nu\frac{A_2}{r^2}\right]$$

$$= \bar{E}\left[A_1 + A_2/r^2 - 2\nu A_2/r^2\right]$$

@ $r = r_o$, $u = 0$

@ $r = r_i$, $\sigma_{rr} = -p$

$$A_1 r_o + A_2 \frac{1}{r_o} = 0 \Rightarrow A_2 = -A_1 r_o^2$$

$$\bar{E}\left[A_1 - \frac{A_2}{r_i^2} + 2\nu \frac{A_2}{r_i^2}\right] = -p$$

$$\Rightarrow \bar{E}\left[A_1 + \frac{A_1 r_o^2}{r_i^2} - 2\nu \frac{A_1 r_o^2}{r_i^2}\right] = -p$$

$$\Rightarrow \bar{E} A_1 \left[1 + \frac{r_o^2}{r_i^2}(1-2\nu)\right] = -p$$

$$\Rightarrow \quad A_1 = \frac{-p}{\bar{E}\left[1 + \frac{r_o^2}{r_i^2}(1-2\nu)\right]}$$

$$A_2 = -A_1 r_o^2 = \frac{p r_o^2}{\bar{E}\left[1 + \frac{r_o^2}{r_i^2}(1-2\nu)\right]}$$

$$\sigma_{rr}\Big|_{r=r_o} = \bar{E}\left[A_1 - \frac{A_2}{r_o^2} + 2\nu \frac{A_2}{r_o^2}\right]$$

$$= \bar{E}\left[A_1 + \frac{A_1 \cancel{r_o^2}}{\cancel{r_o^2}} - 2\nu \frac{A_1 \cancel{r_o^2}}{\cancel{r_o^2}}\right]$$

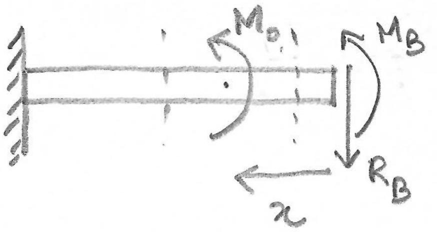
$$= \bar{E} A_1 \left[1 + 1 - 2\nu \right]$$

$$= 2 \bar{E} A_1 (1 - \nu)$$

$$= 2 \bar{E} \frac{(-p)(1-\nu)}{\bar{E}\left[1 + \frac{r_0^2}{r_i^2}(1-2\nu)\right]}$$

$$= \frac{-2p(1-\nu)}{1 + \frac{r_0^2}{r_i^2}(1-2\nu)}$$

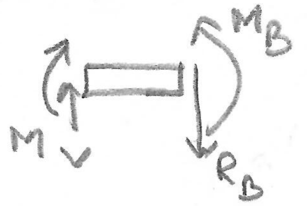
Q6



"Unclamp" the end at B and consider the moment M_B and the force R_B.

Take cut sections to determine bending moments:

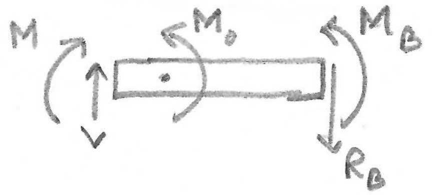
For $0 < x < b$:



$$M - M_B + R_B x = 0$$

$$\Rightarrow M = M_B - R_B x$$

For $b < x < L$:



$$M - M_0 - M_B + R_B x = 0$$

$$\Rightarrow M = M_0 + M_B - R_B x$$

Using Castigliano's theorem:

$$\underset{0}{\Delta_B} = \int_0^b \frac{M}{EI}\frac{\partial M}{\partial R_B}\,dx + \int_b^L \frac{M}{EI}\frac{\partial M}{\partial R_B}\,dx$$

$$\Rightarrow 0 = \int_0^b \frac{(M_B - R_B x)}{EI}(-x)\,dx + \int_b^L \frac{(M_0 + M_B - R_B x)}{EI}(-x)\,dx$$

$$\Rightarrow 0 = -M_B\frac{b^2}{2} + R_B\frac{b^3}{3} - (M_0 + M_B)\frac{L^2 - b^2}{2} + R_B\frac{L^3 - b^3}{3}$$

$$\Rightarrow 0 = -M_0\frac{L^2 - b^2}{2} - M_B\frac{L^2}{2} + R_B\frac{L^3}{3} \quad — \quad (1)$$

$$\cancel{\theta_B} = \int_0^b \frac{M}{EI}\frac{\partial M}{\partial M_B}\,dx + \int_b^L \frac{M}{EI}\frac{\partial M}{\partial M_B}\,dx$$

0

$$\Rightarrow 0 = \int_0^b \frac{(M_B - R_B x)}{EI}(1)\,dx + \int_b^L \frac{(M_0 + M_B - R_B x)}{EI}(1)\,dx$$

$$\Rightarrow 0 = M_B b - R_B \frac{b^2}{2} + (M_0 + M_B)(L-b) - R_B \frac{L^2 - b^2}{2}$$

$$\Rightarrow 0 = M_0(L-b) + M_B L - R_B \frac{L^2}{2} \quad -(2)$$

From (1): $-M_B \frac{L^2}{2} + R_B \frac{L^3}{3} = M_0 \frac{L^2 - b^2}{2} \quad -(3)$

From (2): $-M_B L + R_B \frac{L^2}{2} = M_0(L-b)$

$$\Rightarrow -M_B \frac{L^2}{2} + R_B \frac{L^3}{4} = M_0 \frac{L^2 - bL}{2} \quad -(4)$$

(3) - (4)

$$\Rightarrow R_B \frac{L^3}{12} = -M_0 \frac{b^2}{2} + M_0 \frac{bL}{2}$$

$$\Rightarrow R_B = \frac{6M_0 b}{L^3}(-b + L) = \frac{6M_0 b a}{L^3}$$

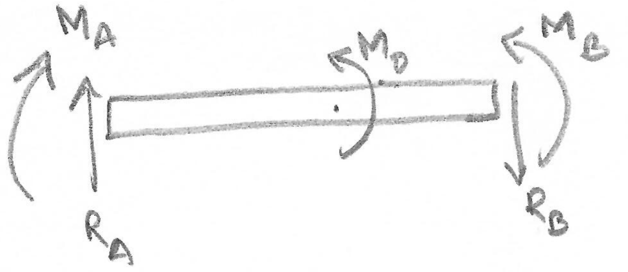
Substituting this R_B in (2)

$$0 = M_0(L-b) + M_B L - \frac{L^2}{2} \times \frac{6M_0 b}{L^3}(L-b)$$

$$\Rightarrow M_B L = -M_0(L-b) + \frac{3M_0 b}{L}(L-b)$$

$$\Rightarrow M_B L = M_0(L-b)\left[-1 + \frac{3b}{L}\right]$$

$$\Rightarrow M_B = M_0 \frac{(L-b)}{L}\left[-1 + \frac{3b}{L}\right] = M_0 \frac{(L-b)(3b-L)}{L^2} = M_0 \frac{a(3b-L)}{L^2}$$



Taking momant about A:

$$M_A - M_0 - M_B + R_B L = 0$$

$$\Rightarrow M_A = M_0 + M_B - R_B L$$

$$= M_0 + \frac{M_0 a(3b-L)}{L^2} - \frac{6M_0 ab}{L^3} L$$

$$= M_0 \left[1 + \frac{a(3b-L)}{L^2} - \frac{6ab}{L^2} \right]$$

$$= \frac{M_0}{L^2} \left[L^2 + 3ab - aL - 6ab \right]$$

$$= \frac{M_0}{L^2} \left[L^2 - aL - 3ab \right]$$

$$= \frac{M_0}{L^2} \left[Lb - 3ab \right]$$

$$= \frac{M_0 b(L-3a)}{L^2}$$

And $\sum F_y = 0$

$$\Rightarrow R_A = R_B$$

$$\Rightarrow R_A = \frac{6M_0 ab}{L^3}$$

Q7

$$EI\frac{d^4y}{dx^4} + P\frac{d^2y}{dx^2} = 0 \xrightarrow[\text{steps}]{\text{show}} y = A\sin kx + B\cos kx + Cx + D$$

$$k^2 = P/EI$$

@ $x=0$, $y=0$

@ $x=0$, $\frac{d^2y}{dx^2}=0$

@ $x=L$ $\frac{dy}{dx}=0$

@ $x=L$, $\frac{d^3y}{dx^3}=0$

Alternatively,

$$EI\frac{d^2y}{dx^2} + Py = 0$$

show steps : $y = A\sin kx + B\cos kx$

@ $x=0$, $y=0$

@ $x=L$, $\frac{dy}{dx}=0$

$$\frac{dy}{dx} = Ak\cos kx - Bk\sin kx + C$$

$$\frac{d^2y}{dx^2} = -Ak^2\sin kx - Bk^2\cos kx$$

$$\frac{d^3y}{dx^3} = -Ak^3\cos kx + Bk^3\sin kx$$

@ $x=0$, $y=0 \Rightarrow \cancel{B}\ B + D = 0 \Rightarrow D=0$

@ $x=0$, $\frac{d^2y}{dx^2}=0 \Rightarrow B=0$

@ $x=L$, $\frac{dy}{dx}=0 \Rightarrow Ak\cos kL + C = 0$

@ $x=L$, $\frac{d^3y}{dx^3}=0 \Rightarrow -Ak^3\cos kL = 0$

$x=0$, $y=0$

$\Rightarrow B=0$

$x=L$, $\frac{dy}{dx}=0$

$\Rightarrow Ak\cos kL = 0$

$A=0 \Rightarrow$ trivial sol^n

$\therefore \cos kL = 0$

$\Rightarrow kL = (2n+1)\frac{\pi}{2}$

~~cos k~~ $\cos kL = 0$

$$kL = (2n+1)\frac{\pi}{2}, \quad n = 0, 1, 2, \ldots$$

$$k = \frac{\pi}{2L}$$

$$k^2 = \frac{\pi^2}{4L^2}$$

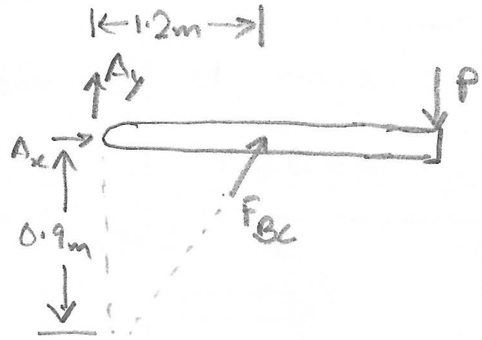
$$\frac{P}{EI} = \frac{\pi^2}{4L^2}$$

Q8 Length of member BC $= \sqrt{0.9^2 + 1.2^2}$ m

$L_{BC} = 1.5$ m

To find force in member BC:

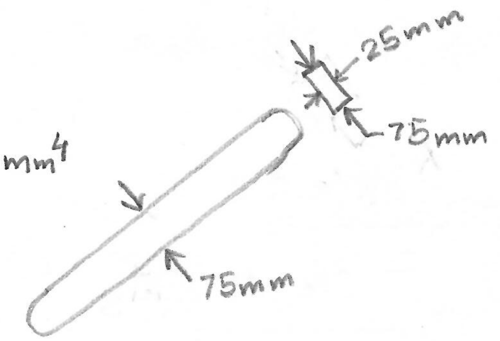
$\sum M_A = 0$



$\Rightarrow F_{BC} \frac{0.9}{1.5} \times 1.2 - P(2.4) = 0$

$\Rightarrow F_{BC} = \frac{\overset{5}{\cancel{1.5}} \times \overset{2}{\cancel{2.4}}}{\underset{3}{\cancel{0.9}} \times \cancel{1.2}} P = \frac{10}{3} P$

<u>For x-x buckling:</u>



$L_{eff} = L_{BC}$; $I = \frac{1}{12}(25)(75)^3$ mm^4

$P_{cr}^{xx} = \frac{\pi^2 E I}{L_{eff}^2}$

$= 771.06$ kN

<u>For y-y buckling:</u>

$L_{eff} = 0.5L$; $I = \frac{1}{12}(75)(25)^3$ mm^4

$P_{cr}^{yy} = \frac{\pi^2 E I}{L_{eff}^2}$

$= 342.69$ kN

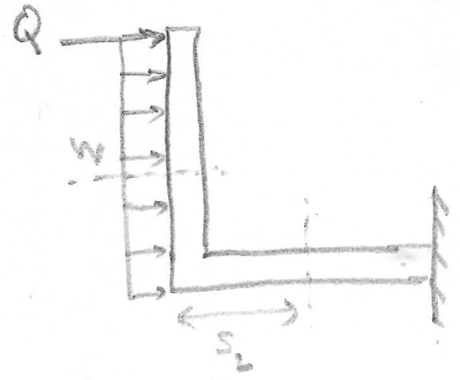
For max. allowable load, we must choose $\min(P_{cr}^{xx}, P_{cr}^{yy})$

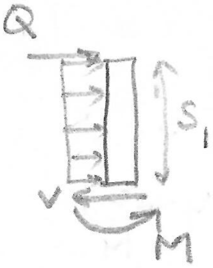
$= P_{cr}^{yy}$

$\therefore F_{BC} = P_{cr}^{yy} = \frac{10}{3} P$

$\Rightarrow P = \frac{3}{10} P_{cr}^{yy} = 102.81$ kN

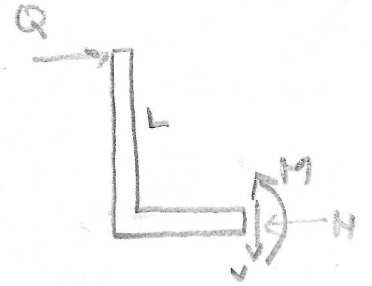
Q9





$$M - \frac{ws_1^2}{2} - Qs_1 = 0$$

$$\Rightarrow M = Qs_1 + \frac{ws_1^2}{2}$$



$$M - QL - \int_0^L w\,ds_1\,(L-s_1) = 0$$

$$N = Q + wL$$

$$\Rightarrow M - QL - \frac{wL^2}{2} = 0$$

$$\Rightarrow M = QL + \frac{wL^2}{2}$$

$$U = \int_0^L \frac{M^2}{2EI}\,ds_1 + \int_0^L \frac{M^2}{2EI}\,ds_2 + \int_0^L \frac{N^2}{2EA}\,ds_2$$

$$\Delta_t = \left.\frac{\partial U}{\partial Q}\right|_{Q=0} = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial Q}\,ds_1 + \int_0^L \frac{M}{EI}\frac{\partial M}{\partial Q}\,ds_2 + \int_0^L \frac{N}{EA}\frac{\partial N}{\partial Q}\,ds_2$$

$$= \int_0^L \frac{\left(ws_1^2/2\right)}{EI}\,s_1\,ds_1 + \int_0^L \frac{\left(wL^2/2\right)}{EI}\,L\,ds_2 + \int_0^L \frac{wL}{EA}(1)\,ds_2$$

$$= \frac{w}{2EI}\int_0^L s_1^3\,ds_1 + \frac{wL^3}{2EI}\int_0^L ds_2 + \frac{wL^2}{EA}$$

$$= \frac{wL^4}{8EI} + \frac{wL^4}{2EI} + \frac{wL^2}{EA}$$

$$= \frac{5wL^4}{8EI} + \frac{wL^2}{EA}$$