INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

End-Spring Semester Examination 2023-24

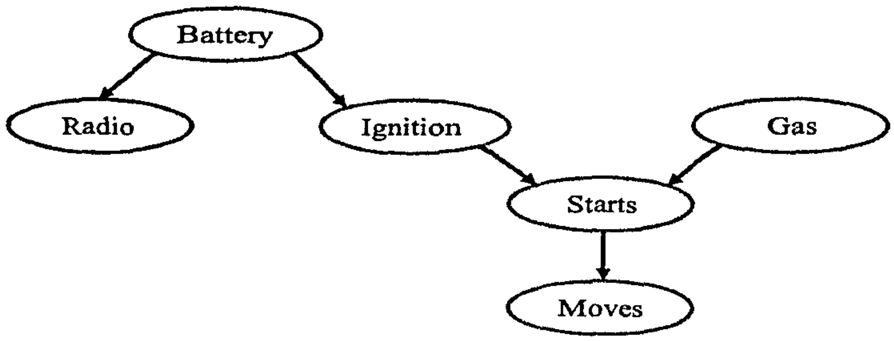
Date of Examination: 23/04/2024 **Session:** AN **Duration:** 3 Hrs **Full Marks: 100**

Subject No. AI61005 **Subject:** Artificial Intelligence: Foundations and Applications

Department/Center/School: Centre of Excellence in Artificial Intelligence

Specific charts, graph paper, log book etc., required NO

Special Instructions (if any) : Answer all the parts of a question in same place. Your answer should be precise and to the point for full credit. Detailed steps must be shown.



1. Here is a graphical model of a car's electrical system and engine that models the starting of a car. Each of the variables are binary in nature. Battery can be in two states – working (B1) and not working (B0). Radio is either working (R1) or not (R0). The ignition is working (I1) or not (I0). The Gas represents whether it is available (G1) or not(G0). The car either starts (S1) or does not start(S0). Finally, it is moving (M1) or not (M0). The conditional probability table is given below:

$P(B0) = 0.20$ $P(B1) = 0.80$

$P(G1) = 0.995$ $P(G0) = 0.005$

$P(R1|B1) = 0.99$ $P(R1|B0) = 0.05$

$P(I1|B1) = 0.998$ $P(I1|B0) = 0.01$

$P(S0|G0,I0) = 0.95$ $P(S0|G0,I1) = 0.35$ $P(S0|G1,I0) = 0.40$ $P(S0|G1,I1) = 0.01$

$P(S1|G0,I0) = 0.05$ $P(S1|G0,I1) = 0.65$ $P(S1|G1,I0) = 0.60$ $P(S1|G1,I1) = 0.99$

$P(M1|S1) = 0.998$ $P(M1|S0) = 0.03$

(a) Based on the above data, compute the values of following three equations based on enumeration. Show the computation tree:

(i) $P(R0,B1,I1,S1,G0,M1)$ [2]

(ii) $P(M1|B0)$ [3]

(iii) $P(B0|M1)$ [3]

(b) How many independent values are contained in the joint probability distribution for six Boolean nodes, assuming that no conditional independence relations are known to hold among them? [2]

(c) How many independent probability values do the above network tables contain? [2]

(d) Is the network a polytree? Give reasons? [2]

(e) If the network is a polytree, how to break this property through insertion of minimum number of additional variables? Otherwise, if the network is not a polytree, what are the minimum changes required to make it a polytree? [2]

(f) Which of the followings are asserted by the network structure? Give reasons. [2+2]

(i) $P(I,G|S) = P(I|S)P(G|S)$

(ii) P(B,S|I) = P(B|I)P(S|I)

2. **(a)** Suppose you have to predict a dependent variable y that depends on attribute x. You are given following set of four instances, with respect to attribute x and outcome value y: (2,5); (3,8); (5,10); (7,12). If you use linear regression to fit the best line/hypothesis with respect to x1 i.e., h(x) = $h_\theta(x) = \theta_0 + \theta_1 x$, using the process of sum-squared error (SSE) minimization, what will be the values of θ_1 and θ_0 after one round? The learning rate 0.001. Initial parameter values 0.5? [3]

Day	Outlook	Temp	Humidity	Wind	Tennis?
1	Sunny	Hot	High	Weak	No
2	Sunny	Hot	High	Strong	No
3	Overcast	Hot	High	Weak	Yes
4	Rain	Mild	High	Weak	Yes
5	Rain	Cool	Normal	Weak	Yes
6	Rain	Cool	Normal	Strong	No
7	Overcast	Cool	Normal	Strong	Yes
8	Sunny	Mild	High	Weak	No
9	Sunny	Cool	Normal	Weak	Yes
10	Rain	Mild	Normal	Weak	Yes
11	Sunny	Mild	Normal	Strong	Yes
12	Overcast	Mild	High	Strong	Yes
13	Overcast	Hot	Normal	Weak	Yes
14	Rain	Mild	High	Strong	No
15	Sunny	Hot	High	Weak	No

Table 1

(b) Consider Table 1, which contains data about weather condition for tennis playing. We wish to build a decision tree classifier to predict whether a day is suitable for tennis playing. We consider the two classes 'Yes' and 'No'.

(i). Calculate the Entropy of the root node of a decision tree with the given training data. [2]

(ii). Build the decision tree using the ID3 algorithm using information gain. Show all steps. Assume log base is 2. [10]

(iii). Use the decision tree to classify the following two unknown points <P16: Sunny, Cool, High, Strong> and <P17: Rain, Mild, Normal, Strong> Show the calculations. [1]

(c) Consider the following prolog codes

• Different(X, X) :- !, fail.	• Different(X, X) :- fail.	• Different(X, X) :- fail.
• Different(X,Y).	• Different(X,Y).	• Different(X,Y) :- X=\=Y.
Code(a)	Code(b)	Code(c)

(i). Write the outputs for Different(3,3) for code a, b, and c. [2]

(ii). Write the outputs for Different(3,5) for code a, b, and c. [2]

3. **(a)** Consider weather forecasting scenario in figure 1. We have three states SUN(S), WIND(W), HELL (H). We begin in state S and MDP is shown in Figure 1. Answer following questions:

(i). Suppose you run a value iteration algorithm with discount factor 0.8. Fill up Table 2 for next time step. [5]

(ii). Assuming above values of states $V^2(S), V^2(W), V^2(H)$ computed in (i), what is the implied policy π at this step. Write the values of $\pi(S), \pi(W), \pi(H)$. [3]

(iii). Suppose the weather adopts the policy a1 from all the states i.e., $\pi_1(S), \pi_1(W), \pi_1(H) = a1$. Evaluate the value of each state given this policy π_1. Calculate $V_{\pi_1}(S), V_{\pi_1}(W), V_{\pi_1}(H)$. [5]

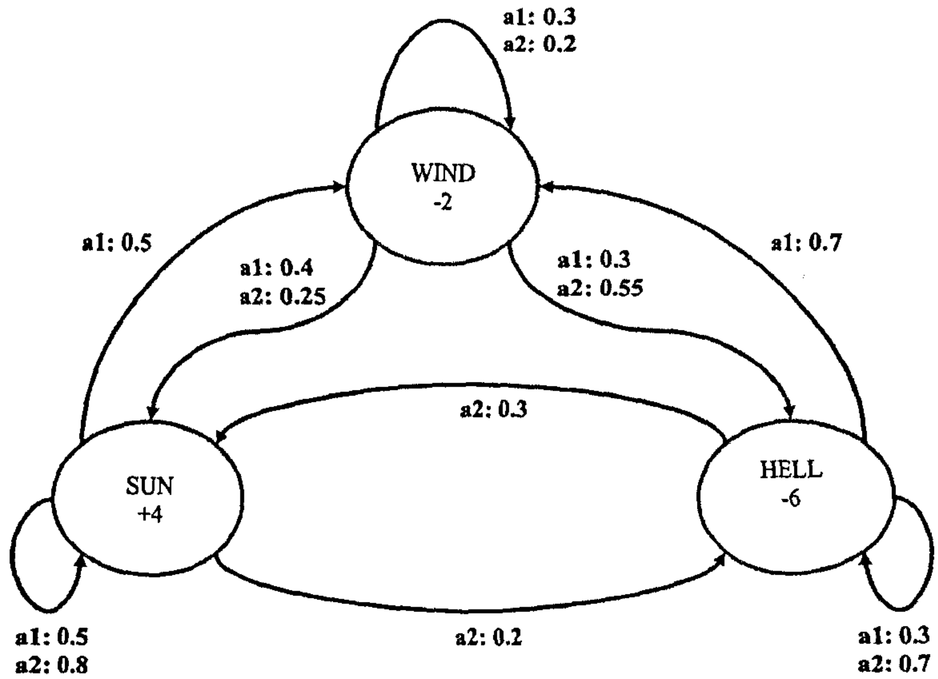
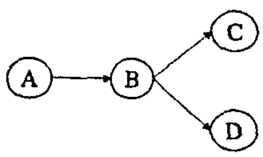


Figure 1

t	$V^t(S)$	$V^t(W)$	$V^t(H)$
1	+4	-2	-6
2			

Table 2

(iv). Starting with π_1 as in (iii), do a policy update to find new policy π_2 (one step of policy iteration). Find the value of $\pi_2(S), \pi_2(W), \pi_2(H)$. [3]



A	P(A)
+a	$^1/_5$

A	B	P(B|A)
+a	+b	$^1/_4$
-a	+b	$^1/_3$

B	C	P(C|B)
+b	+c	$^1/_5$
-b	+c	$^3/_5$

B	D	P(D|B)
+b	+d	$^3/_4$
-b	+b	$^1/_3$

Figure 2

(b) Using following samples (which were generated using likelihood weighting from Figure 2), estimate P(+a|-c,-d) using likelihood weighting or state why it can't be computed. [4]

+a -b -c -d　　　　-a +b -c -d　　　　+a -b -c -d

4. A computing system contains seven (7) clients and four (4) servers. Each client has a specific allocated time slot during which it can run. You are constrained by the fact that one server can run one code at a time.

The clients running times are:

- C1: run from 8-9 AM　　C2: run from 8:45-9:45 AM　　C3: run from 8:30-9:30 AM
- C4: run from 9-9:15 AM　　C5: run from 9:30-9:45 AM
- C6: run from 9-10:30 AM　　C7: run from 10-10:30 AM

The servers are:

- S1 can run C1, C2, C3, C4, C5, C6, C7
- S2, S3, S4 can run C2, C3, C4, C5, C6, C7

(a) Formulate this problem as a CSP problem in which there is one variable per class, stating the domains, and constraints. Constraints should be specified formally and precisely, in an explicit way. [4]
(b) Draw the constraint graph associated with your CSP [2]
(c) Search for a solution using basic backtracking. Only check whether any new assignment violates no constraint with previous assignments. As a tie breaker assign a client to a server based on alphabetical order. **Continue up to the first time you try and fail to assign any value for C7.** [4]
Fill out this worksheet (Table 3) as you draw your search tree. There may be more rows than you need.
(i). Every time you assign a variable or remove a variable from the propagation queue, fill out a new row in the table. (The same variable might appear in more than one row, especially if you have to backtrack.)
(ii). In that row, indicate **which variable you assigned or de-queued;** write its **assigned** value if it has one (e.g. X=x), otherwise just write its name (X). In the second column, list the **values that were just eliminated from neighboring variables** as a result. If no values were just eliminated, write **NONE** instead.
(iii). If your search has to backtrack after assigning or de-queuing a variable: first, **finish listing** all values eliminated from neighboring variables in the current row. Next, check the backtrack box in that row. Then, continue with the next assignment in the following row as usual.
(iv). At some point, you might add several variables to your propagation queue at once. Break ties by adding variables to your propagation queue **in alphabetical order.**

Table 3

	Var assigned or dequeued	**List all values eliminated from neighbouring variables**	**Backtrack?**
Ex	X	Y≠B Z≠C (example)	R
1			..
2			..

(d) Fill the worksheet (Table 3) but now implement backtracking with Forward checking and Minimum Value Ordering. Break ties **in alphabetical order.** [5]

5. **(a)** Consider a temperature system that follows membership functions as follows:
Very Cold: Trapezoidal (a<0, b<=0, c=7, d=10)
Cold: Trapezoidal (a=7, b=10, c=16, d=20)
Normal: Trapezoidal (a=16, b=20, c=26, d=30)
Hot: Trapezoidal (a=26, b=30, c=36, d=40)
Very Hot: Trapezoidal (a=36, b=40, c=46, d>46)
(i). Compute membership value for normal and hot for input x=29? [2.5]
(ii). Compute membership value for hot and very hot for input x=39? [2.5]

(b) Consider a Sugeno-type fuzzy inference system with the following rules:
Rule 1: If x is A1 and y is B1, then z = 2x + 3y + 5
Rule 2: If x is A2 and y is B2, then z = 4x - 2y + 10
Given the following membership functions:
A1: Triangular (1, 3, 5)
A2: Trapezoidal (4, 6, 8, 10)
B1: Gaussian (mean = 4, sigma = 1)
B2: Bell (a = 2, b = 0.5, c = 6) [membership function: $\frac{1}{1+(\frac{x-c}{a})^{2*b}}$]
If x = 4 and y = 6, calculate the output value z using the inference system. [10]

(c) For Fuzzy production rule R: If height is TALL, then speed is HIGH, with known membership function of: $\mu_{TALL}^{(height)} = \{\frac{0.4}{5'}, \frac{0.6}{6'}, \frac{0.9}{7'}\}$ $\mu_{HIGH}^{(speed)} = \{\frac{0.2}{6mps}, \frac{0.4}{7mps}, \frac{0.9}{8mps}\}$

(i) Determine the relational matrix for the rule R. [5]

(ii) When the observed membership function $\mu_{MORE\ or\ LESS\ TALL}^{(height)} = \{\frac{0.3}{5'}, \frac{0.9}{6'}, \frac{0.7}{7'}\}$ is given, find $\mu_{MORE\ or\ LESS\ HIGH}^{(speed)}$ using max-min composition operator. [5]