AIFA: Reasoning Under Uncertainty

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Bayesian Network

- A set of random variables makes up the nodes of the network
  - Variables may be discrete or continuous

- A set of directed links or arrows connects pairs of nodes
  - Arrows represent probabilistic dependence among variables

- An arrow from X →Y indicates X is parent of Y

- Each node X_i *has a conditional probability distribution* $P(X_i|Parents(X_i))$
  - Quantifies the effect of the parents on the node

- The graph has no directed cycles (DAG)

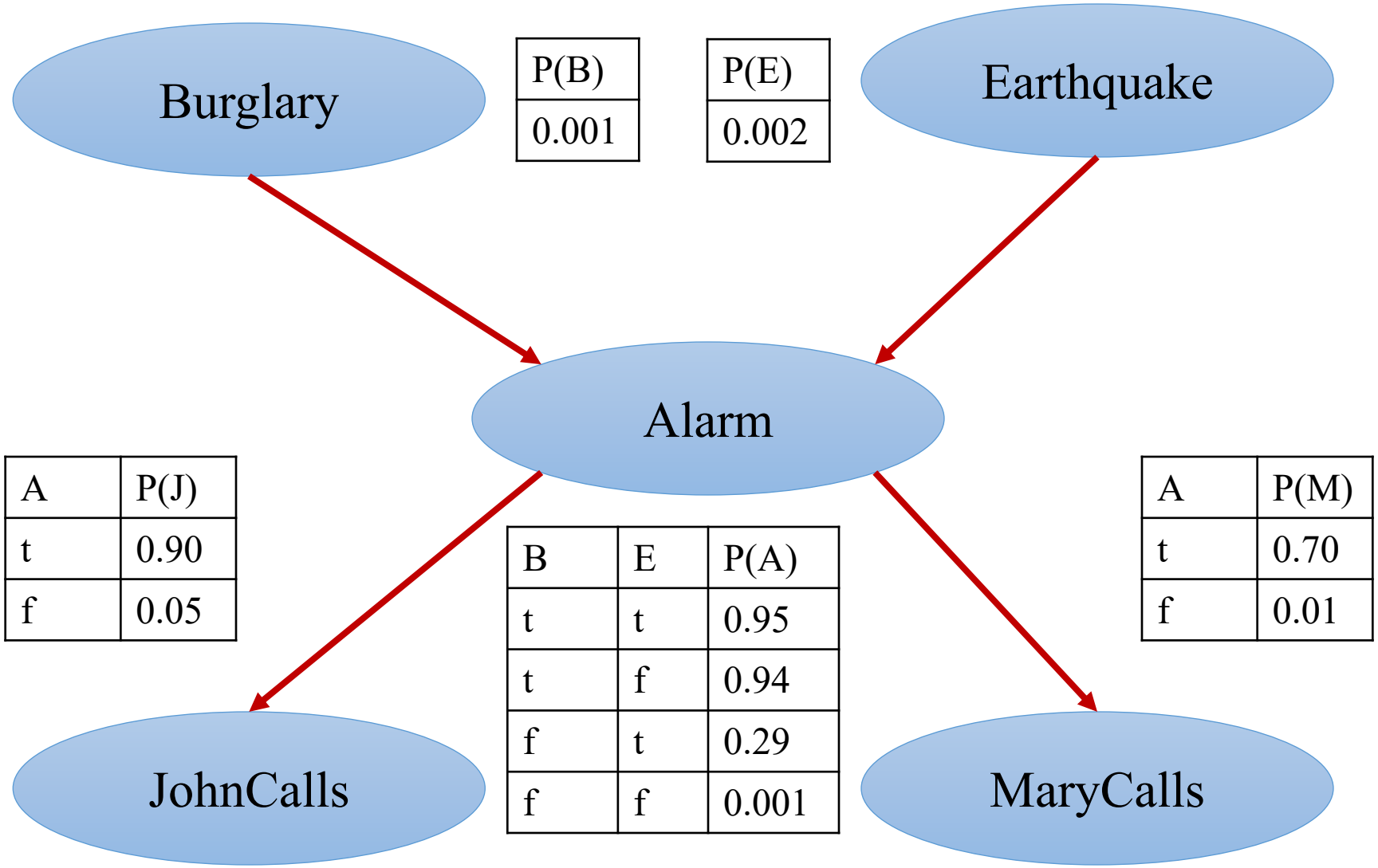
Example

- Burglar alarm at home
  - Fairly reliable at detecting a burglary
  - Responds at times to minor earthquakes

- Two neighbors, on hearing alarm, calls police
  - John always calls when he hears the alarm
  - But sometimes confuses the telephone ringing with the alarm and calls then too

  - Mary likes loud music
  - But sometimes misses the alarm altogether

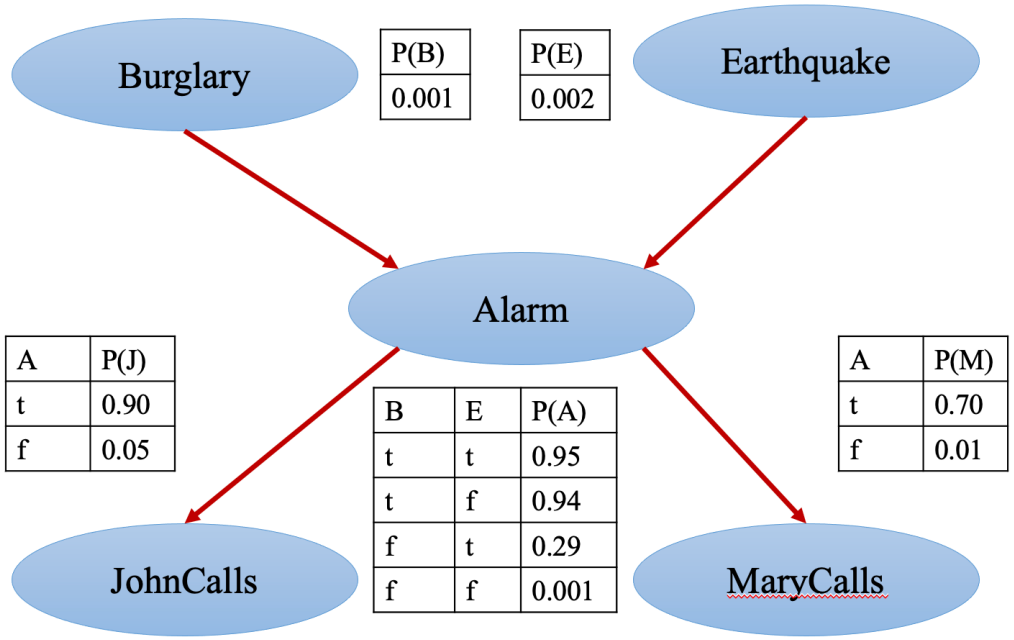
Belief Network



Joint probability distribution

- $P(x_1, \ldots, x_n) = \prod_{i=1}^{n} P(x_i | Parents(x_i))$

- $P(J \wedge M \wedge A \wedge \sim B \wedge \sim E)$
  - $P(J|A)$ *
  - $P(M|A)$ *
  - $P(A|\sim B \wedge \sim E)$ *
  - $P(\sim B)$ *
  - P($\sim E$)



- $P(J \wedge M \wedge A \wedge \sim B \wedge \sim E) = 0.9 \times 0.7 \times 0.001 \times 0.999 \times 0.998 = 0.00062$

- $P(J) = ?$

Conditional Independence

- $P(x_1, \dots, x_n) = P(x_n|x_{n-1}, \dots, x_1)P(x_{n-1}, \dots, x_1)$
- $P(x_1, \dots, x_n) = P(x_n|x_{n-1}, \dots, x_1)P(x_{n-1}|x_{n-2}, \dots, x_1) \dots P(x_2|x_1)P(x_1)$
- $P(x_1, \dots, x_n) = \prod_{i=1}^{n} P(x_i|x_{i-1}, \dots, x_1)$

- The belief network represents conditional independence:
  - $P(x_i|x_i, \dots, x_1) = P(x_i|Parents(x_i))$

How to construct this network?

Conditional Independence

- $P(J, M, A, B, E) = P(J|M, A, B, E)P(M, A, B, E)$
- $P(J, M, A, B, E) = P(J|A)P(M|A, B, E)P(A, B, E)$
- $P(J, M, A, B, E) = P(J|A)P(M|A)P(A|B, E)P(B, E)$
- $P(J, M, A, B, E) = P(J|A)P(M|A)P(A|B, E)P(B)P(E)$

How does ordering matter?

Conditional Independence

Conditional Independence Relation

- If every undirected path from a node in X to a node in Y is d-separated by a given set of evidence nodes E
  - X and Y are conditionally independent given E

- A set of nodes E d-separates two set of nodes X and Y if every undirected path from a node in X to a node in Y is blocked given E

Conditional Independence Relation

- A path is blocked given a set of nodes E if there is a node Z on the path for which one of three conditions hold:
  - Z is in E and Z has one arrow on the path leading in and one arrow out
  - Z is in E and Z has both path arrows leading out
  - Neither Z nor any descendent of Z is in E and both path arrows lead into Z

Conditional Independence Relation: Summary

- A path is blocked given a set of nodes E if there is a node Z on the path for which one of three conditions hold:
  - Z is in E and Z has one arrow on the path leading in and one arrow out
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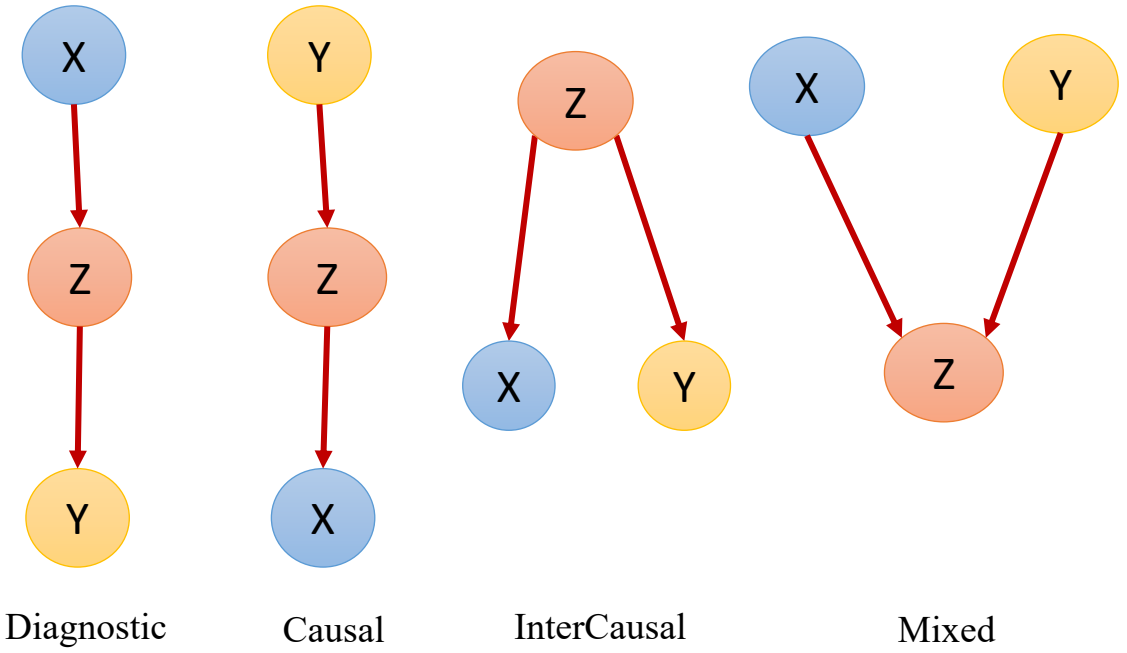
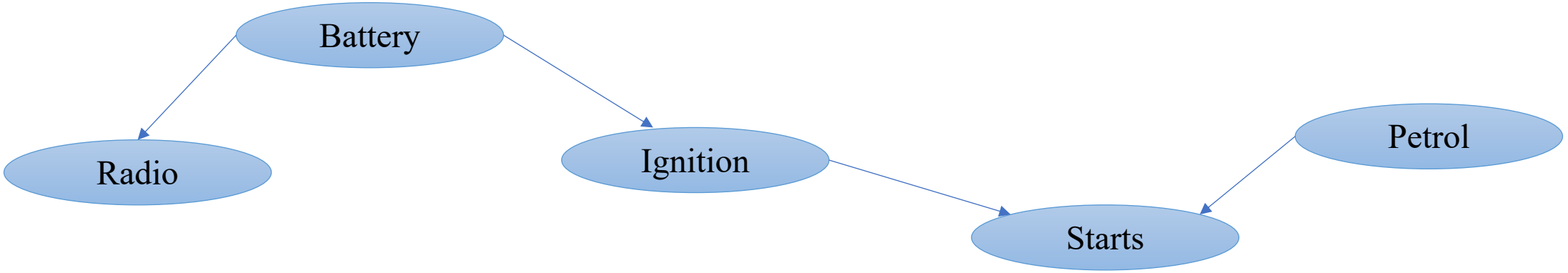
- If every undirected path from a node in X to a node in Y is **d-separated** by a given set of evidence nodes E
  - X and Y are conditionally independent given E

- A set of nodes E **d-separates** two set of nodes X and Y if every undirected path from a node in X to a node in Y is **blocked** given E

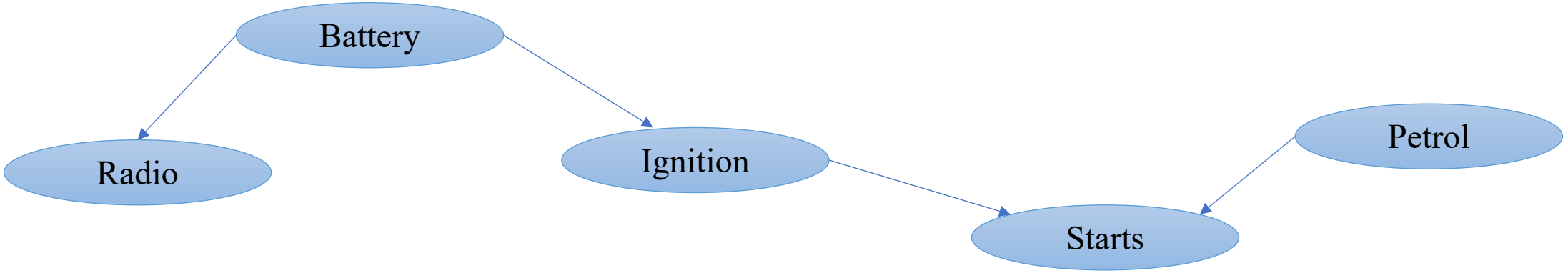


Conditional Independence Relation



- Whether there is petrol and whether the radio plays are independent given evidence about whether the ignition takes place
- Petrol and Radio are independent if it is known whether the battery works

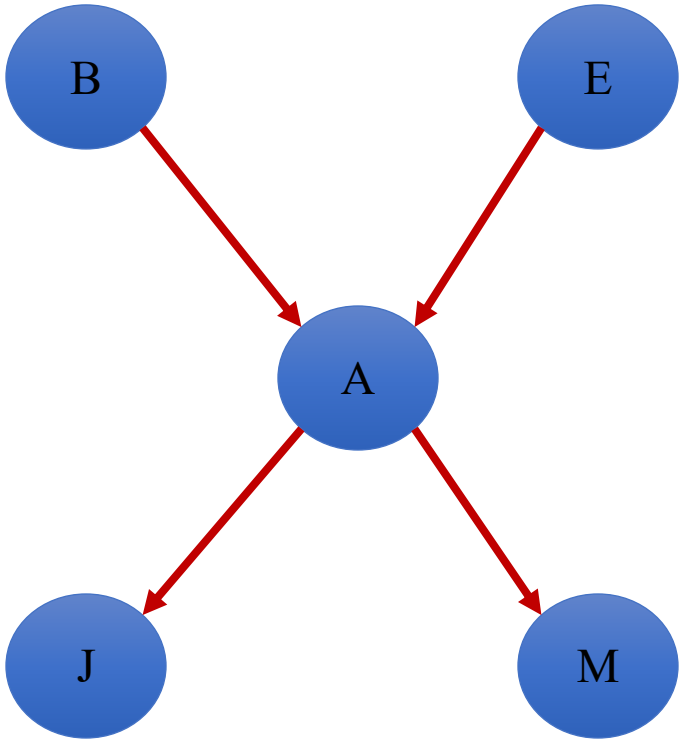
Conditional Independence Relation



- Petrol and Radio are independent given no evidence at all
- But they are dependent given evidence about whether the car starts
- If the car does not start, then radio works is an increase evidence that car is out of petrol

Bayesian Network: Topological Semantics

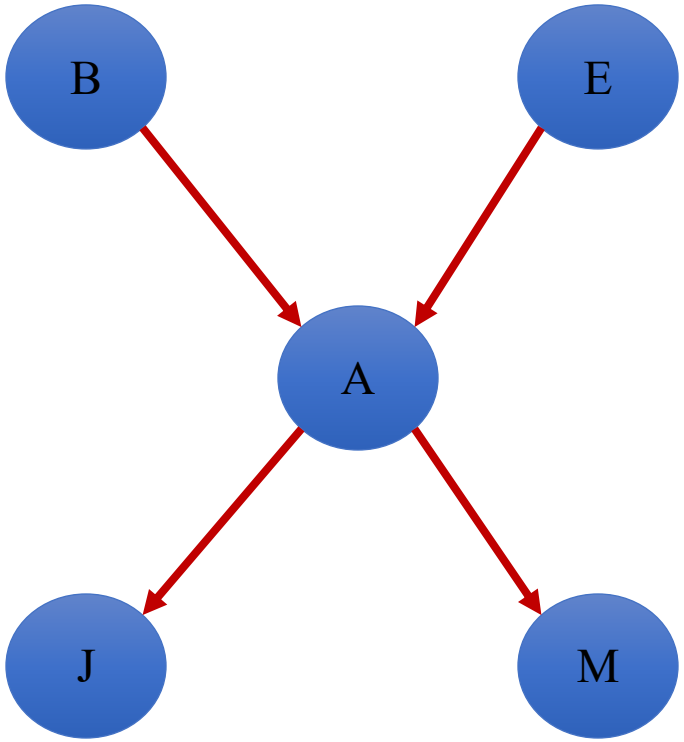
- A node is conditionally independent of its non-descendants, given its parents



JohnCalls is independent of Burglary and Earthquake given the value of Alarm

Bayesian Network: Topological Semantics

- A node is conditionally independent of all other nodes in the network, given its parents, children, and children's parents
  - Markov Blanket



Burglary is independent of JohnCalls and MaryCalls given the value of Alarm and Earthquake

Thank You