# Fuzzy Arithmetic Operation

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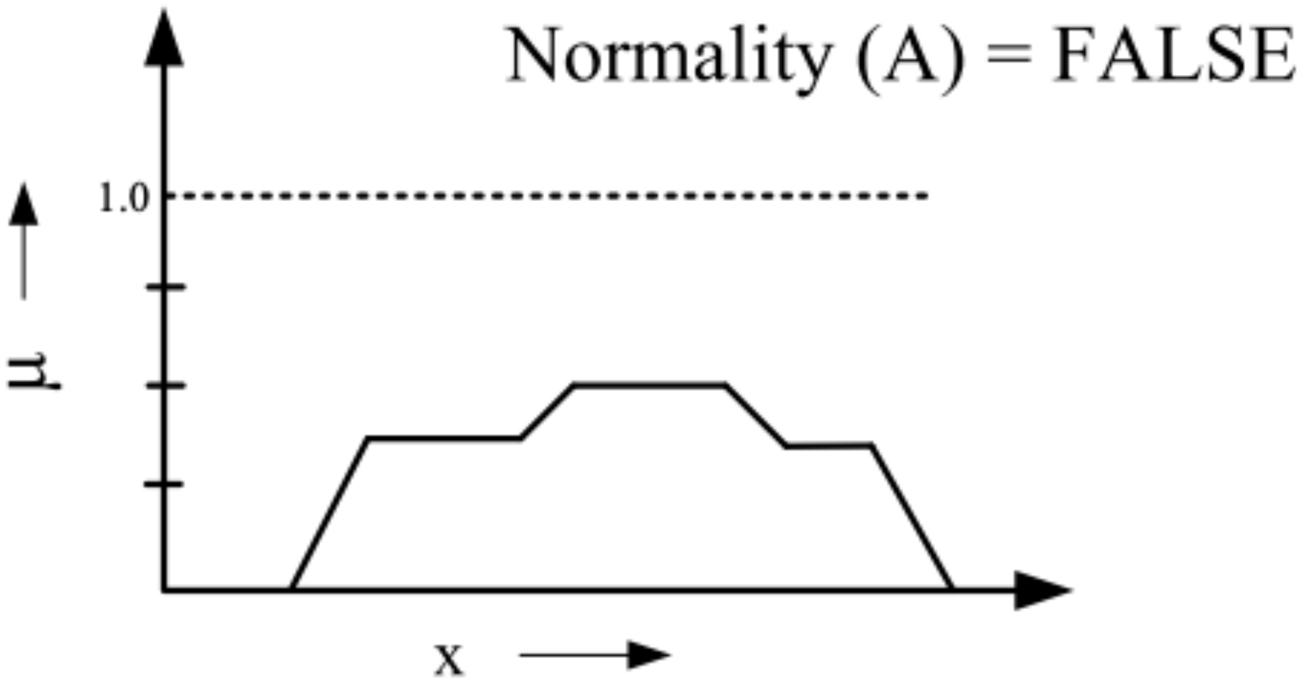
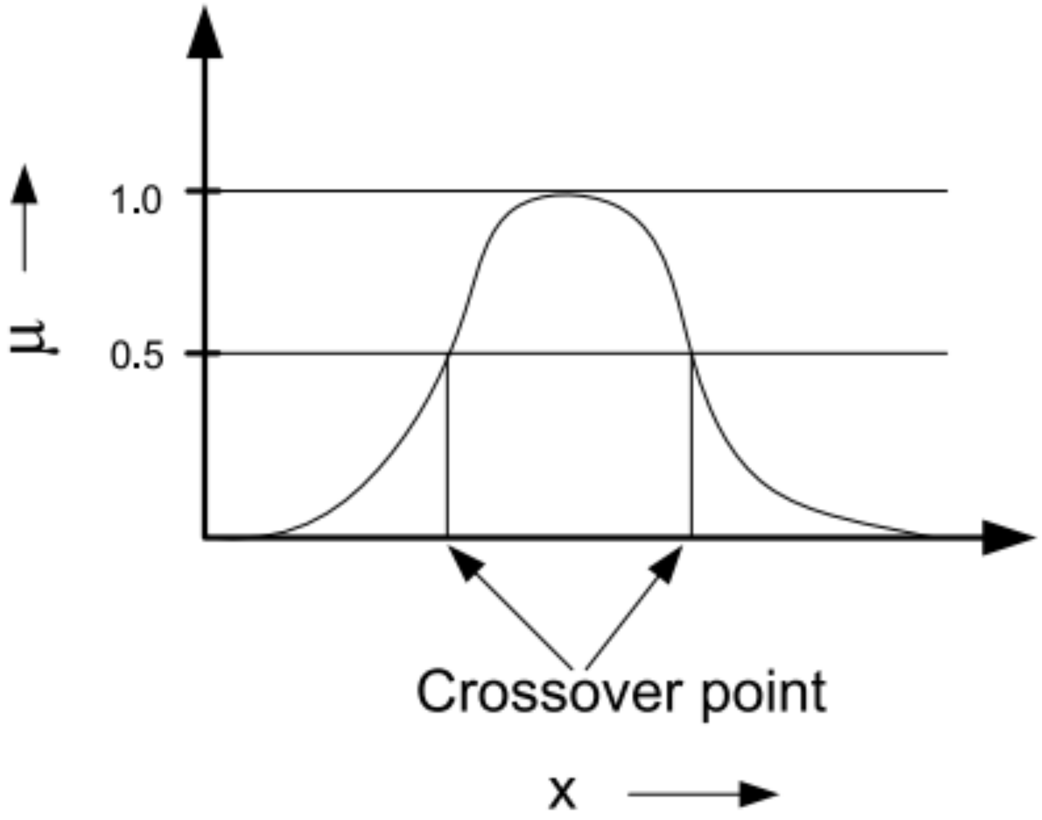
# Normality

- A fuzzy set A is a normal if its core is non-empty. In other words, we can always find a point $x \in X$ such that $\mu A(x) = 1$.



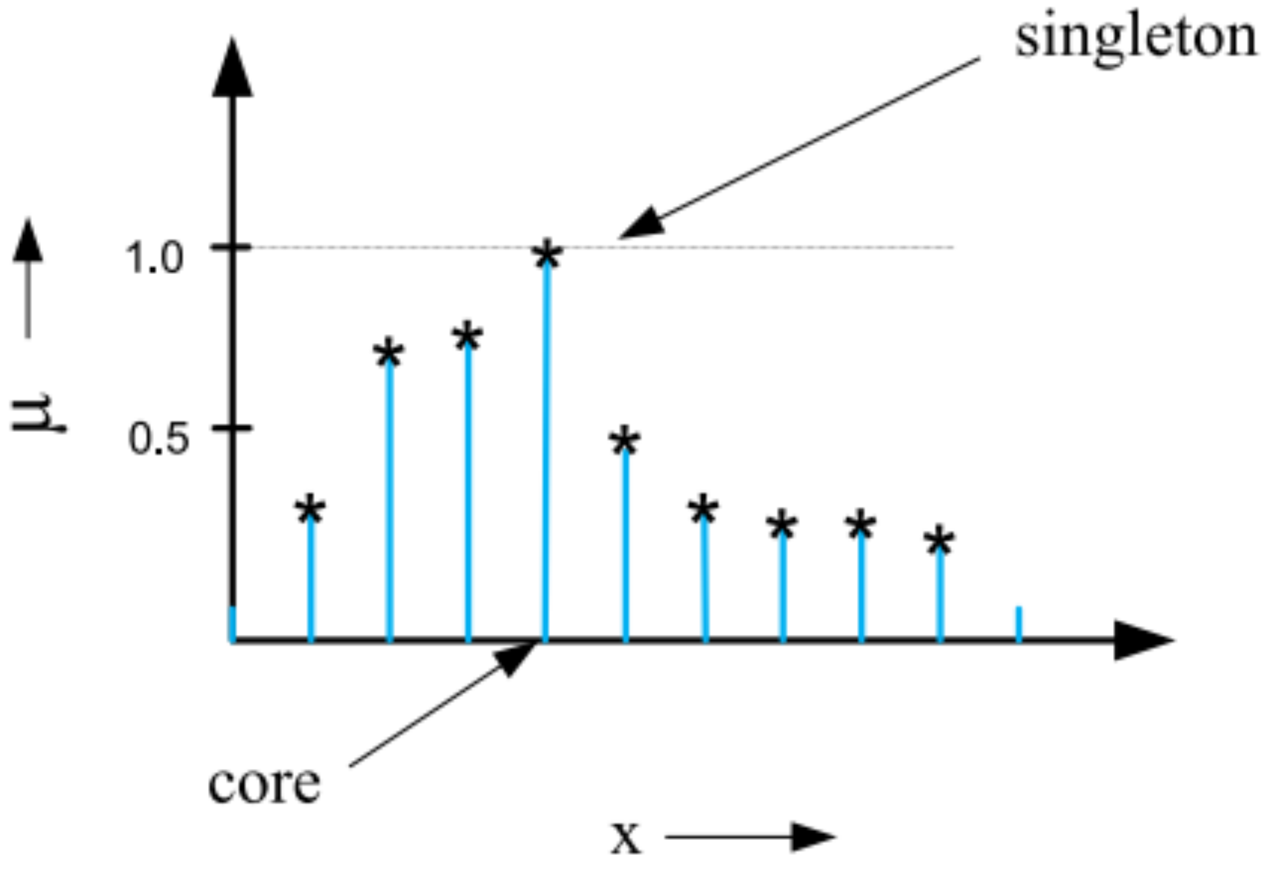
# Crossover points

- A crossover point of a fuzzy set A is a point $x \in X$ at which $\mu A(x) = 0.5$. That is Crossover (A) = $\{x|\mu A(x) = 0.5\}$



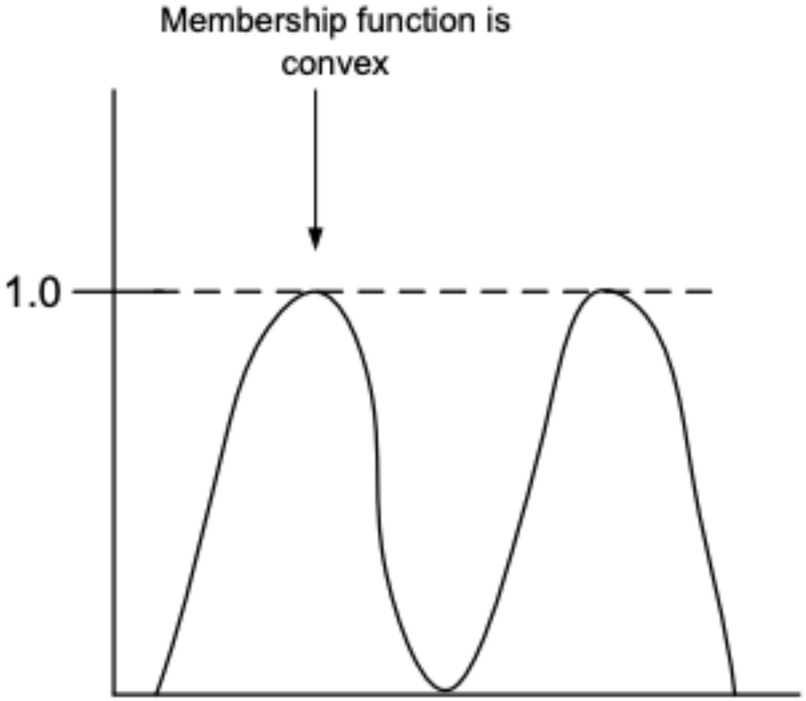
# Fuzzy Singleton

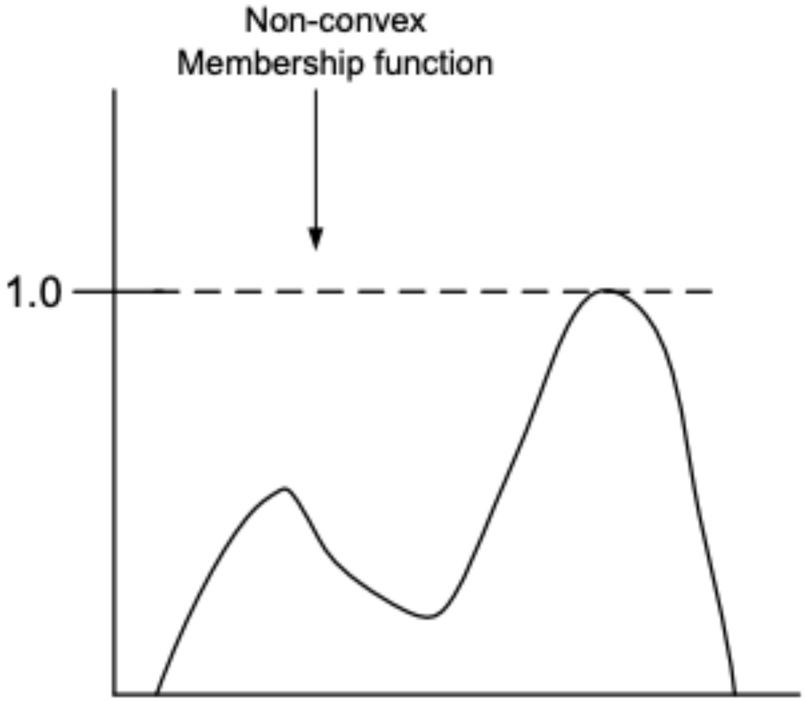
- A fuzzy set whose support is a single point in X with μA(x) = 1 is called a fuzzy singleton. That is |A| = { x | μA(x) = 1}.



# Convexity

- Convexity : A fuzzy set A is convex if and only if for any x1 and x2 ∈ X and any λ ∈ [0, 1] μA (λx1 + (1 -λ)x2) ≥ min(μA(x1), μA(x2))

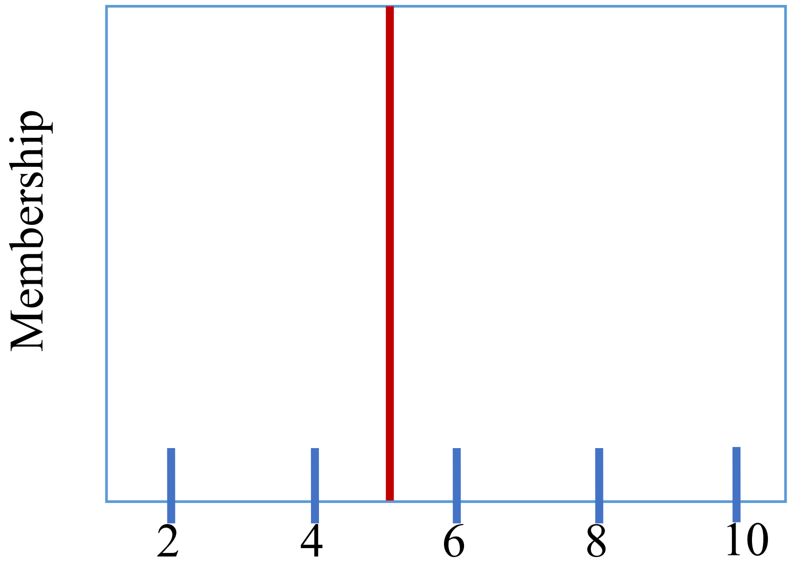


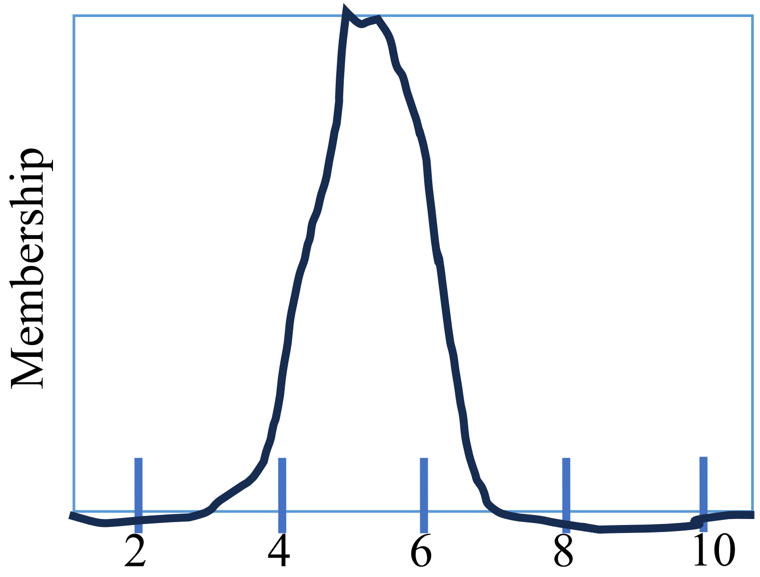
# Fuzzy Number

- A Fuzzy number is a fuzzy set that holds the condition of normality and convexity
- Fuzzy numbers are the most basic types

A crisp number 5 or Fuzzy singleton 5




A Fuzzy number 5



# Arithmetic Operations on Fuzzy Numbers

- There are four types of arithmetic operations that can be performed on fuzzy sets
  - Provided fuzzy sets are qualified for fuzzy numbers

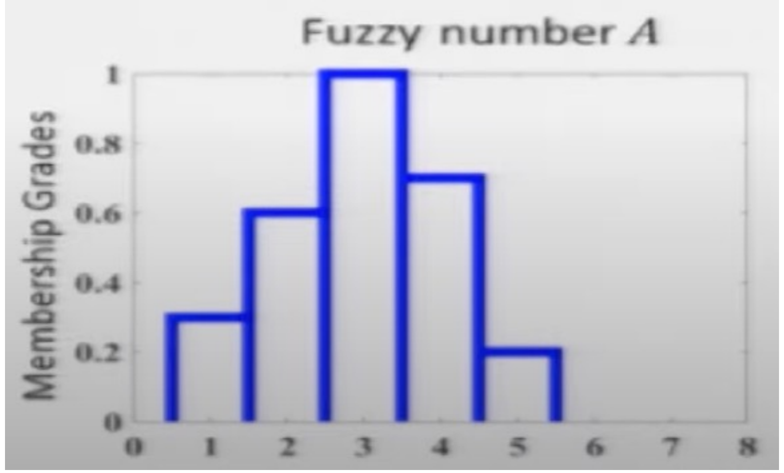
- These operations are
  - Addition on Fuzzy Numbers
  - Subtraction on Fuzzy Numbers
  - Multiplication on Fuzzy Numbers
  - Division on Fuzzy Numbers

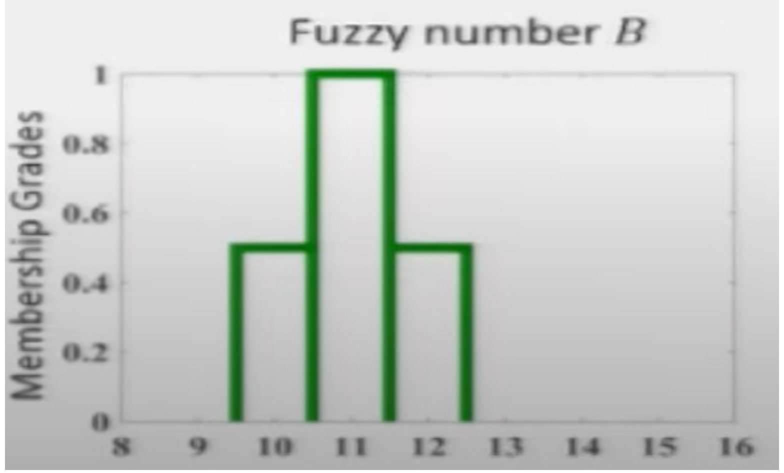
# Addition of Fuzzy Numbers

- Let A and B are two Fuzzy numbers with the universe of discourse X
- If we perform the addition, it results in a new fuzzy number C as,
  - $C = A + B$
- The new fuzzy number C is defined as,
- For discrete: $C = \sum_x \mu_C(x^C)/x^C$
- For continuous: $C = \int \mu_C(x^C)/x^C$
- The membership function values of fuzzy number C are
  - $\mu_C(x^C) = \mu_{A+B}(x^C) = \max\limits_{x^A, x^B}[\mu_A(x^A) \wedge \mu_B(x^B)]$
  - Where $x^C = x^A + x^B; \forall\, x^A, x^B, x^C \in X$
- $C = \sum_x \mu_C(x^C)/x^C = \sum_x \mu_{A+B}(x^C)/x^C = \sum_x \max\limits_{x^A, x^B}[\mu_A(x^A) \wedge \mu_B(x^B)]\,/x^C$

# Addition of Fuzzy Numbers

- Example: Let us consider two fuzzy sets A and B with the universe of discourse $X \in [-20,20]$ as given below. Find the addition of fuzzy numbers A and B
- $A = \frac{0.3}{1} + \frac{0.6}{2} + \frac{1.0}{3} + \frac{0.7}{4} + \frac{0.2}{5}$
- $B = \frac{0.5}{10} + \frac{1.0}{11} + \frac{0.5}{12}$



# Addition of Fuzzy Numbers

- $A = \frac{0.3}{1} + \frac{0.6}{2} + \frac{1.0}{3} + \frac{0.7}{4} + \frac{0.2}{5}$
- $B = \frac{0.5}{10} + \frac{1.0}{11} + \frac{0.5}{12}$
- $C = \sum_x \mu_C(x^C)/x^C = \sum_x \mu_{A+B}(x^C)/x^C = \sum_x \max_{x^A, x^B} [\mu_A(x^A) \wedge \mu_B(x^B)] / (x^A + x^B)$

- For elements $\frac{0.3}{1}$ and $\frac{0.5}{10}$ from Fuzzy number A and B respectively, we have,
  - $(\mu_A(x^A) \wedge \mu_B(x^B))/(x^A + x^B) = (0.3 \wedge 0.5)/(1 + 10) = 0.3/11$
  - Similarly, we will do for all the combinations of elements of Fuzzy number A and B

# Addition of Fuzzy Numbers

- $A = \frac{0.3}{1} + \frac{0.6}{2} + \frac{1.0}{3} + \frac{0.7}{4} + \frac{0.2}{5}$
- $B = \frac{0.5}{10} + \frac{1.0}{11} + \frac{0.5}{12}$
- $C = \sum_x \max_{x^A, x^B} [\mu_A(x^A) \wedge \mu_B(x^B)] \,/ (x^A + x^B)$

$(\frac{0.3}{1}, \frac{0.5}{10}) = \frac{0.3}{11}$
$(\frac{0.3}{1}, \frac{1.0}{11}) = \frac{0.3}{12}$
$(\frac{0.3}{1}, \frac{0.5}{12}) = \frac{0.3}{13}$

$(\frac{0.6}{2}, \frac{0.5}{10}) = \frac{0.5}{12}$
$(\frac{0.6}{2}, \frac{1.0}{11}) = \frac{0.6}{13}$
$(\frac{0.6}{2}, \frac{0.5}{12}) = \frac{0.5}{14}$

$(\frac{1.0}{3}, \frac{0.5}{10}) = \frac{0.5}{13}$
$(\frac{1.0}{3}, \frac{1.0}{11}) = \frac{1.0}{14}$
$(\frac{1.0}{3}, \frac{0.5}{12}) = \frac{0.5}{15}$

$(\frac{0.7}{4}, \frac{0.5}{10}) = \frac{0.5}{14}$
$(\frac{0.7}{4}, \frac{1.0}{11}) = \frac{0.7}{15}$
$(\frac{0.7}{4}, \frac{0.5}{12}) = \frac{0.5}{16}$

$(\frac{0.2}{5}, \frac{0.5}{10}) = \frac{0.2}{15}$
$(\frac{0.2}{5}, \frac{1.0}{11}) = \frac{0.2}{16}$
$(\frac{0.2}{5}, \frac{0.5}{12}) = \frac{0.2}{17}$

# Addition of Fuzzy Numbers

$$A = \frac{0.3}{1} + \frac{0.6}{2} + \frac{1.0}{3} + \frac{0.7}{4} + \frac{0.2}{5}$$
$$B = \frac{0.5}{10} + \frac{1.0}{11} + \frac{0.5}{12}$$
$$C = \sum_{x} \max_{x^A,x^B} [\mu_A(x^A) \wedge \mu_B(x^B)] \,/ (x^A + x^B)$$

- $C = \frac{0.3}{11} + \frac{0.3}{12} + \frac{0.3}{13} + \frac{0.5}{12} + \frac{0.6}{13} + \frac{0.5}{14} + \frac{0.5}{13} + \frac{1.0}{14} + \frac{0.5}{15} + \frac{0.5}{14} + \frac{0.7}{15} + \frac{0.5}{16} + \frac{0.2}{15} + \frac{0.2}{16} + \frac{0.2}{17}$

Rearrange numbers

- $C = \frac{0.3}{11} + \left(\frac{0.3}{12} + \frac{0.5}{12}\right) + \left(\frac{0.3}{13} + \frac{0.6}{13} + \frac{0.5}{13}\right) + \left(\frac{0.5}{14} + \frac{1.0}{14} + \frac{0.5}{14}\right) + \left(\frac{0.5}{15} + \frac{0.7}{15} + \frac{0.2}{15}\right) + \left(\frac{0.5}{16} + \frac{0.2}{16}\right) + \frac{0.2}{17}$

- $C = \frac{0.3}{11} + \frac{0.5}{12} + \frac{0.6}{13} + \frac{1.0}{14} + \frac{0.7}{15} + \frac{0.5}{16} + \frac{0.2}{17}$

# Addition of Fuzzy Numbers

- $A = \frac{0.3}{1} + \frac{0.6}{2} + \frac{1.0}{3} + \frac{0.7}{4} + \frac{0.2}{5}$
- $B = \frac{0.5}{10} + \frac{1.0}{11} + \frac{0.5}{12}$
- $C = \frac{0.3}{11} + \frac{0.5}{12} + \frac{0.6}{13} + \frac{1.0}{14} + \frac{0.7}{15} + \frac{0.5}{16} + \frac{0.2}{17}$

Is Addition symmetric?

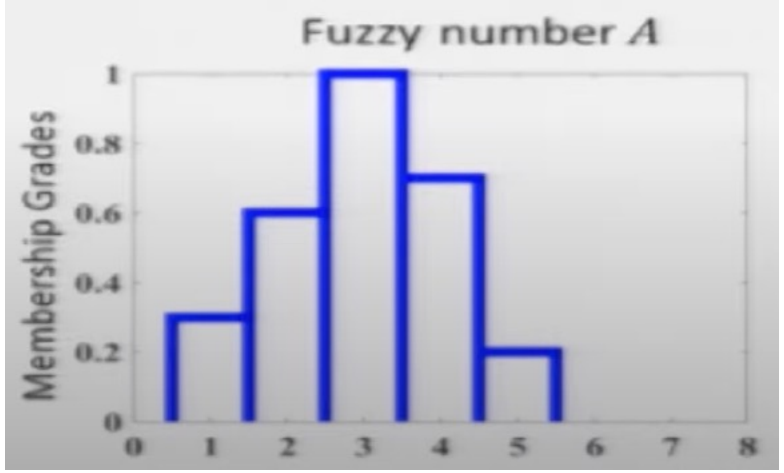

- The spread of resultant has increased
- The uncertainty level in Fuzzy number C is more than Fuzzy number A and B

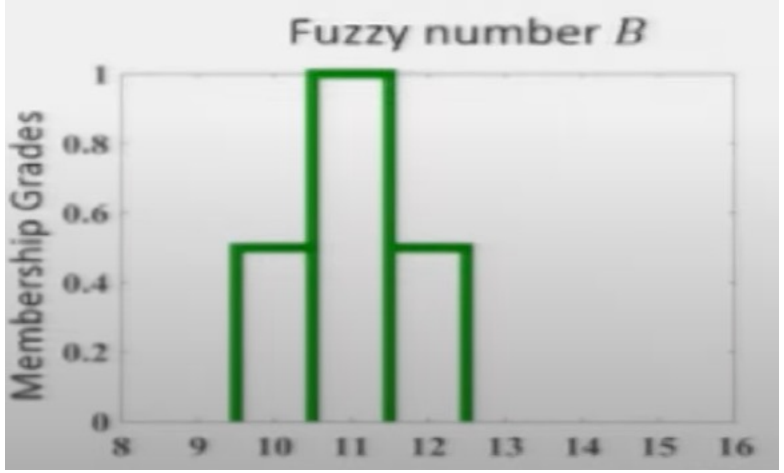
# Subtraction of Fuzzy Numbers

- Let A and B are two Fuzzy numbers with the universe of discourse X
- If we perform the subtraction, it results in a new fuzzy number C as,
  - $C = A - B$
- The new fuzzy number C is defined as,
- For discrete: $C = \sum_x \mu_C(x^C)/x^C$
- For continuous: $C = \int \mu_C(x^C)/x^C$
- The membership function values of fuzzy number C are
  - $\mu_C(x^C) = \mu_{A-B}(x^C) = \max\limits_{x^A, x^B}[\mu_A(x^A) \wedge \mu_B(x^B)]$
  - Where $x^C = x^A - x^B; \forall\, x^A, x^B, x^C \in X$
- $C = \sum_x \mu_C(x^C)/x^C = \sum_x \mu_{A-B}(x^C)/x^C = \sum_x \max\limits_{x^A, x^B}[\mu_A(x^A) \wedge \mu_B(x^B)]\,/x^C$

# Subtraction of Fuzzy Numbers

- Example: Let us consider two fuzzy sets A and B with the universe of discourse $X \in [-20,20]$ as given below. Find the addition of fuzzy numbers A and B
- $A = \frac{0.3}{1} + \frac{0.6}{2} + \frac{1.0}{3} + \frac{0.7}{4} + \frac{0.2}{5}$
- $B = \frac{0.5}{10} + \frac{1.0}{11} + \frac{0.5}{12}$



# Subtraction of Fuzzy Numbers

- $A = \frac{0.3}{1} + \frac{0.6}{2} + \frac{1.0}{3} + \frac{0.7}{4} + \frac{0.2}{5}$
- $B = \frac{0.5}{10} + \frac{1.0}{11} + \frac{0.5}{12}$
- $C = \sum_x \mu_C(x^C)/x^C = \sum_x \mu_{A+B}(x^C)/x^C = \sum_x \max_{x^A, x^B} [\mu_A(x^A) \wedge \mu_B(x^B)] / (x^A - x^B)$

- For elements $\frac{0.3}{1}$ and $\frac{0.5}{10}$ from Fuzzy number A and B respectively, we have,
  - $(\mu_A(x^A) \wedge \mu_B(x^B))/(x^A - x^B) = (0.3 \wedge 0.5)/(1 - 10) = 0.3/(-9)$
  - Similarly, we will do for all the combinations of elements of Fuzzy number A and B

# Subtraction of Fuzzy Numbers

- $A = \frac{0.3}{1} + \frac{0.6}{2} + \frac{1.0}{3} + \frac{0.7}{4} + \frac{0.2}{5}$
- $B = \frac{0.5}{10} + \frac{1.0}{11} + \frac{0.5}{12}$
- $C = \sum_x \max_{x^A, x^B} [\mu_A(x^A) \wedge \mu_B(x^B)] / (x^A - x^B)$

$(\frac{0.3}{1}, \frac{0.5}{10}) = \frac{0.3}{(-9)}$
$(\frac{0.3}{1}, \frac{1.0}{11}) = \frac{0.3}{(-10)}$
$(\frac{0.3}{1}, \frac{0.5}{12}) = \frac{0.3}{(-11)}$

$(\frac{0.6}{2}, \frac{0.5}{10}) = \frac{0.5}{(-8)}$
$(\frac{0.6}{2}, \frac{1.0}{11}) = \frac{0.6}{(-9)}$
$(\frac{0.6}{2}, \frac{0.5}{12}) = \frac{0.5}{(-10)}$

$(\frac{1.0}{3}, \frac{0.5}{10}) = \frac{0.5}{(-7)}$
$(\frac{1.0}{3}, \frac{1.0}{11}) = \frac{1.0}{(-8)}$
$(\frac{1.0}{3}, \frac{0.5}{12}) = \frac{0.5}{(-9)}$

$(\frac{0.7}{4}, \frac{0.5}{10}) = \frac{0.5}{(-6)}$
$(\frac{0.7}{4}, \frac{1.0}{11}) = \frac{0.7}{(-7)}$
$(\frac{0.7}{4}, \frac{0.5}{12}) = \frac{0.5}{(-8)}$

$(\frac{0.2}{5}, \frac{0.5}{10}) = \frac{0.2}{(-5)}$
$(\frac{0.2}{5}, \frac{1.0}{11}) = \frac{0.2}{(-6)}$
$(\frac{0.2}{5}, \frac{0.5}{12}) = \frac{0.2}{(-7)}$

# Subtraction of Fuzzy Numbers

$$A = \frac{0.3}{1} + \frac{0.6}{2} + \frac{1.0}{3} + \frac{0.7}{4} + \frac{0.2}{5}$$

$$B = \frac{0.5}{10} + \frac{1.0}{11} + \frac{0.5}{12}$$

$$C = \sum_{x} \max_{x^A, x^B} [\mu_A(x^A) \wedge \mu_B(x^B)] \,/ (x^A - x^B)$$

- $C = \frac{0.3}{(-9)} + \frac{0.3}{(-10)} + \frac{0.3}{(-11)} + \frac{0.5}{(-8)} + \frac{0.6}{(-9)} + \frac{0.5}{(-10)} + \frac{0.5}{(-7)} + \frac{1.0}{(-8)} + \frac{0.5}{(-9)} + \frac{0.5}{(-6)} + \frac{0.7}{(-7)} + \frac{0.5}{(-8)} + \frac{0.2}{(-5)} + \frac{0.2}{(-6)} + \frac{0.2}{(-7)}$

Rearrange numbers

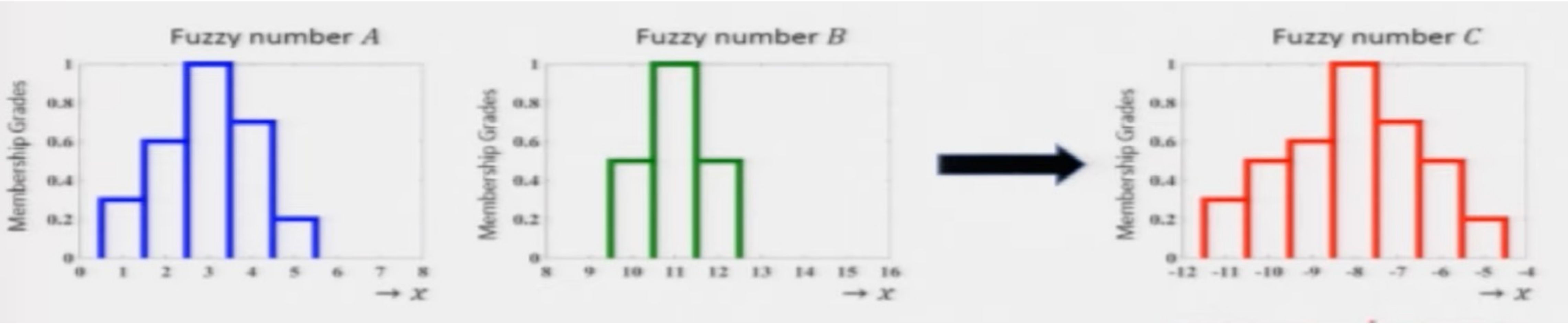
- $C = \frac{0.3}{(-11)} + \left(\frac{0.3}{(-10)} + \frac{0.5}{(-10)}\right) + \left(\frac{0.3}{(-9)} + \frac{0.6}{(-9)} + \frac{0.5}{(-9)}\right) + \left(\frac{0.5}{(-8)} + \frac{1.0}{(-8)} + \frac{0.5}{(-8)}\right) + \left(\frac{0.5}{(-7)} + \frac{0.7}{(-7)} + \frac{0.2}{(-7)}\right) + \left(\frac{0.5}{(-6)} + \frac{0.2}{(-6)}\right) + \frac{0.2}{(-5)}$

- $C = \frac{0.3}{(-11)} + \frac{0.5}{(-10)} + \frac{0.6}{(-9)} + \frac{1.0}{(-8)} + \frac{0.7}{(-7)} + \frac{0.5}{(-6)} + \frac{0.2}{(-5)}$

# Subtraction of Fuzzy Numbers

- $A = \frac{0.3}{1} + \frac{0.6}{2} + \frac{1.0}{3} + \frac{0.7}{4} + \frac{0.2}{5}$
- $B = \frac{0.5}{10} + \frac{1.0}{11} + \frac{0.5}{12}$
- $C = \frac{0.3}{(-11)} + \frac{0.5}{(-10)} + \frac{0.6}{(-9)} + \frac{1.0}{(-8)} + \frac{0.7}{(-7)} + \frac{0.5}{(-6)} + \frac{0.2}{(-5)}$

Is Subtraction symmetric?

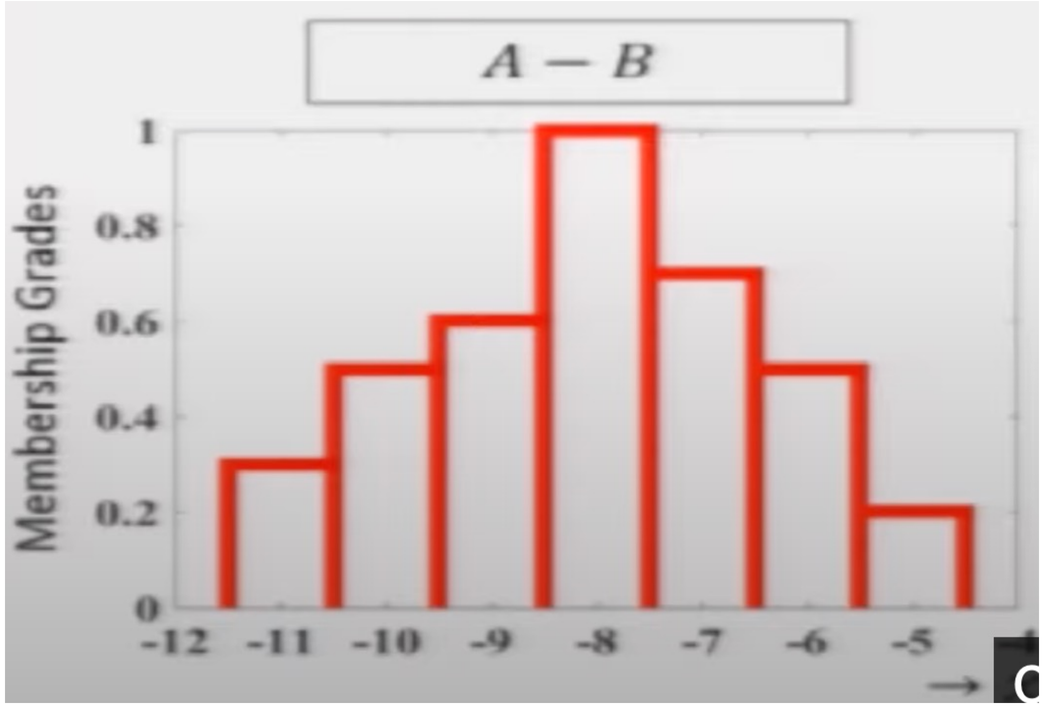



- The spread of resultant has increased
- The uncertainty level in Fuzzy number C is more than Fuzzy number A and B

# Subtraction of Fuzzy Numbers

- $A = \frac{0.3}{1} + \frac{0.6}{2} + \frac{1.0}{3} + \frac{0.7}{4} + \frac{0.2}{5}$
- $B = \frac{0.5}{10} + \frac{1.0}{11} + \frac{0.5}{12}$
- $C = (A - B) = \frac{0.3}{(-11)} + \frac{0.5}{(-10)} + \frac{0.6}{(-9)} + \frac{1.0}{(-8)} + \frac{0.7}{(-7)} + \frac{0.5}{(-6)} + \frac{0.2}{(-5)}$
- $C = (B - A) = \frac{0.2}{5} + \frac{0.5}{6} + \frac{0.7}{7} + \frac{1.0}{8} + \frac{0.6}{9} + \frac{0.5}{10} + \frac{0.3}{11}$

# Subtraction of Fuzzy Numbers

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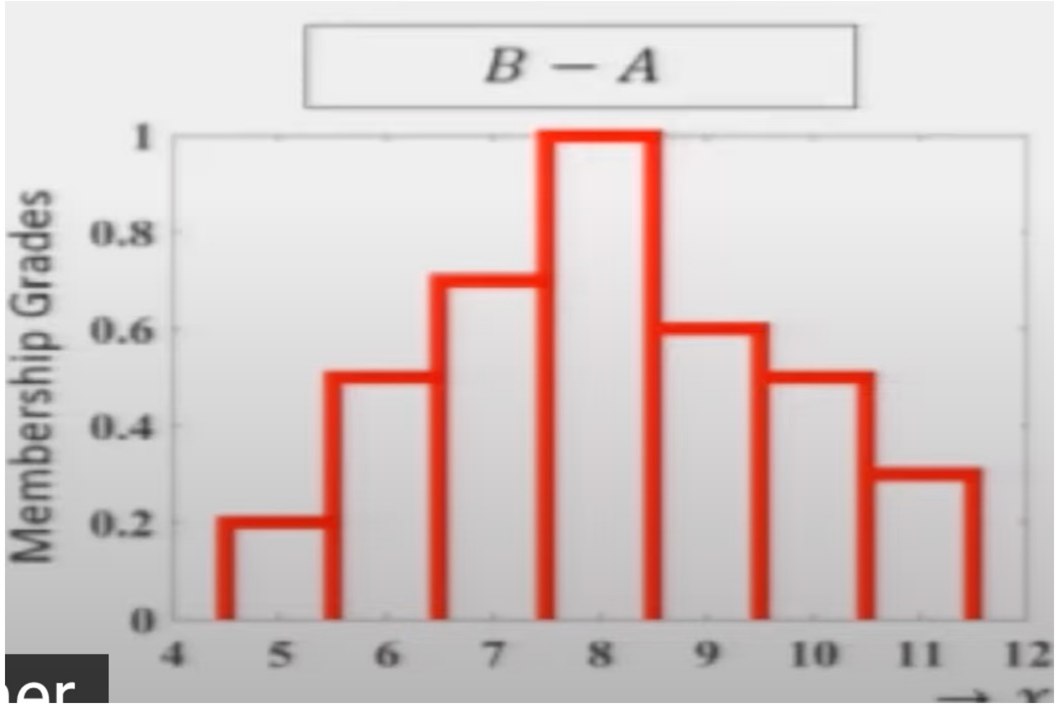



Is Subtraction symmetric?

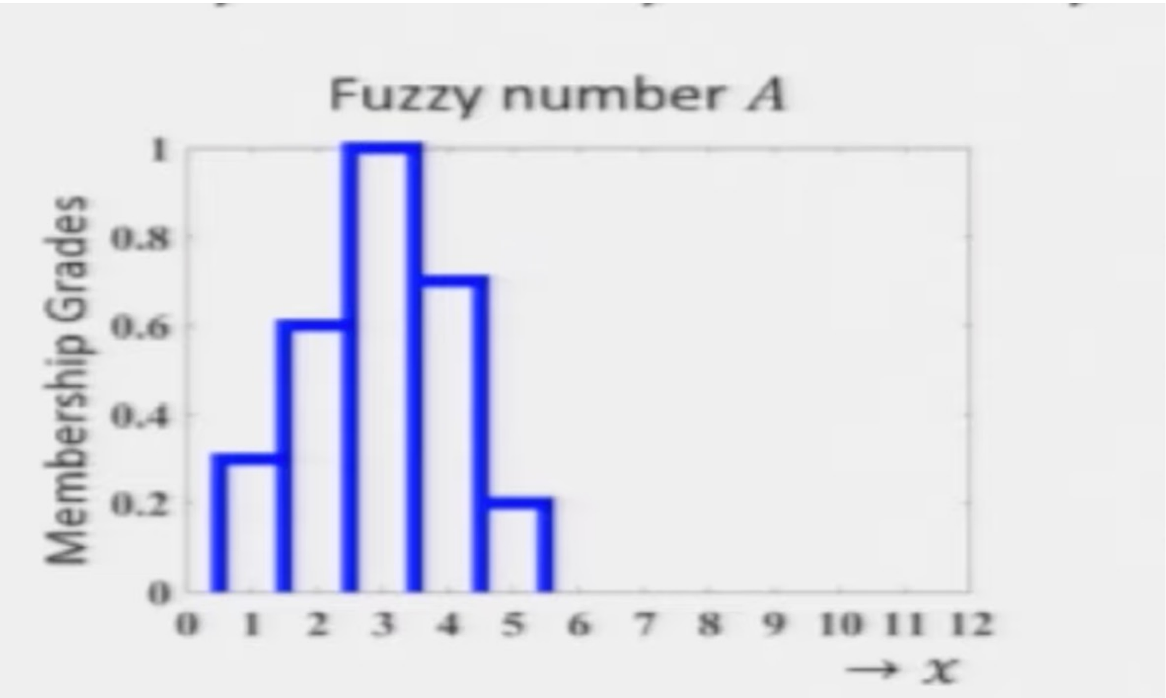
Is Subtraction commutative?

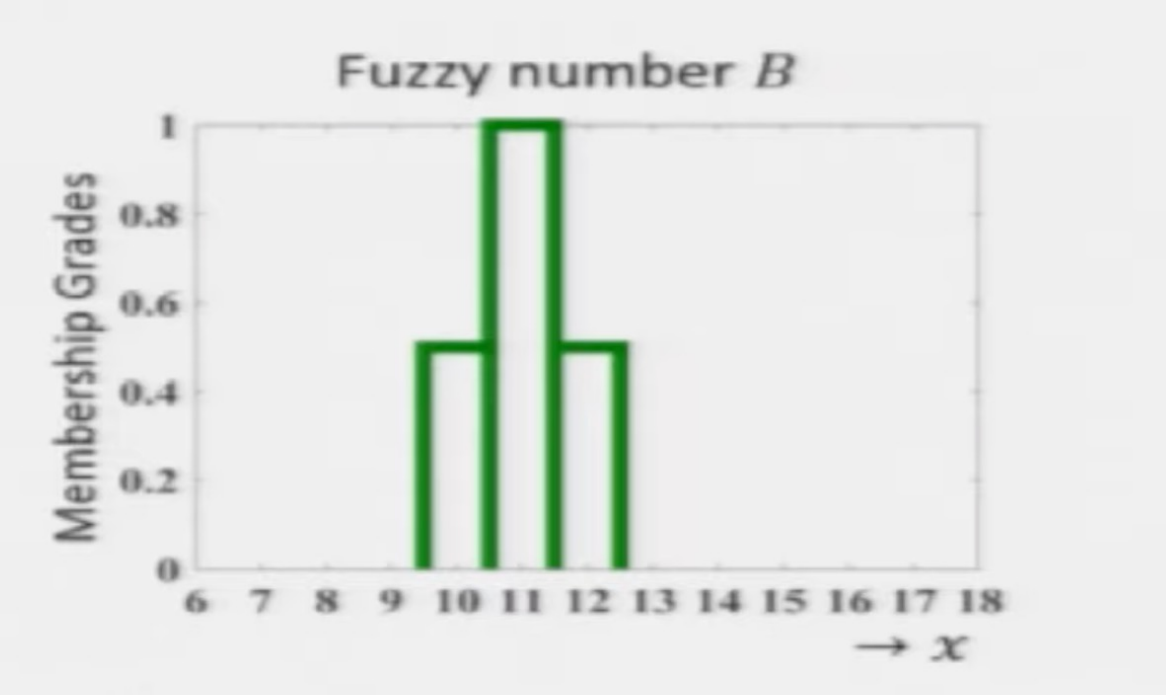
# Multiplication of Fuzzy Numbers

- Let A and B are two Fuzzy numbers with the universe of discourse X
- If we perform the subtraction, it results in a new fuzzy number C as,
  - $C = A * B$
- The new fuzzy number C is defined as,
- For discrete: $C = \sum_x \mu_C(x^C)/x^C$
- For continuous: $C = \int \mu_C(x^C)/x^C$
- The membership function values of fuzzy number C are
  - $\mu_C(x^C) = \mu_{A*B}(x^C) = \max\limits_{x^A, x^B}[\mu_A(x^A) \wedge \mu_B(x^B)]$
  - Where $x^C = x^A * x^B; \forall\, x^A, x^B, x^C \in X$
- $C = \sum_x \mu_C(x^C)/x^C = \sum_x \mu_{A*B}(x^C)/x^C = \sum_x \max\limits_{x^A, x^B}[\mu_A(x^A) \wedge \mu_B(x^B)]\,/x^C$

# Multiplication of Fuzzy Numbers

- Example: Let us consider two fuzzy sets A and B with the universe of discourse $X \in [-15,15]$ as given below. Find the addition of fuzzy numbers A and B
- $A = \frac{0.3}{1} + \frac{0.6}{2} + \frac{1.0}{3} + \frac{0.7}{4} + \frac{0.2}{5}$
- $B = \frac{0.5}{10} + \frac{1.0}{11} + \frac{0.5}{12}$



# Multiplication of Fuzzy Numbers

- $A = \frac{0.3}{1} + \frac{0.6}{2} + \frac{1.0}{3} + \frac{0.7}{4} + \frac{0.2}{5}$
- $B = \frac{0.5}{10} + \frac{1.0}{11} + \frac{0.5}{12}$
- $C = \sum_x \mu_C(x^C)/x^C = \sum_x \mu_{A+B}(x^C)/x^C = \sum_x \max_{x^A, x^B} [\mu_A(x^A) \wedge \mu_B(x^B)] / (x^A - x^B)$

- For elements $\frac{0.3}{1}$ and $\frac{0.5}{10}$ from Fuzzy number A and B respectively, we have,
  - $(\mu_A(x^A) \wedge \mu_B(x^B))/(x^A * x^B) = (0.3 \wedge 0.5)/(1 * 10) = 0.3/10$
  - Similarly, we will do for all the combinations of elements of Fuzzy number A and B

# Multiplication of Fuzzy Numbers

- $A = \frac{0.3}{1} + \frac{0.6}{2} + \frac{1.0}{3} + \frac{0.7}{4} + \frac{0.2}{5}$
- $B = \frac{0.5}{10} + \frac{1.0}{11} + \frac{0.5}{12}$
- $C = \sum_x \max_{x^A, x^B} [\mu_A(x^A) \wedge \mu_B(x^B)] / (x^A * x^B)$

$(\frac{0.3}{1}, \frac{0.5}{10}) = \frac{0.3}{10}$
$(\frac{0.3}{1}, \frac{1.0}{11}) = \frac{0.3}{11}$
$(\frac{0.3}{1}, \frac{0.5}{12}) = \frac{0.3}{12}$

$(\frac{0.6}{2}, \frac{0.5}{10}) = \frac{0.5}{20}$
$(\frac{0.6}{2}, \frac{1.0}{11}) = \frac{0.6}{22}$
$(\frac{0.6}{2}, \frac{0.5}{12}) = \frac{0.5}{24}$

$(\frac{1.0}{3}, \frac{0.5}{10}) = \frac{0.5}{30}$
$(\frac{1.0}{3}, \frac{1.0}{11}) = \frac{1.0}{33}$
$(\frac{1.0}{3}, \frac{0.5}{12}) = \frac{0.5}{36}$

$(\frac{0.7}{4}, \frac{0.5}{10}) = \frac{0.5}{40}$
$(\frac{0.7}{4}, \frac{1.0}{11}) = \frac{0.7}{44}$
$(\frac{0.7}{4}, \frac{0.5}{12}) = \frac{0.5}{48}$

$(\frac{0.2}{5}, \frac{0.5}{10}) = \frac{0.2}{50}$
$(\frac{0.2}{5}, \frac{1.0}{11}) = \frac{0.2}{55}$
$(\frac{0.2}{5}, \frac{0.5}{12}) = \frac{0.2}{60}$

# Multiplication of Fuzzy Numbers

$$A = \frac{0.3}{1} + \frac{0.6}{2} + \frac{1.0}{3} + \frac{0.7}{4} + \frac{0.2}{5}$$

$$B = \frac{0.5}{10} + \frac{1.0}{11} + \frac{0.5}{12}$$

$$C = \sum_{x} \max_{x^A, x^B} [\mu_A(x^A) \wedge \mu_B(x^B)] / (x^A * x^B)$$

- $C = \frac{0.3}{10} + \frac{0.3}{11} + \frac{0.3}{12} + \frac{0.5}{20} + \frac{0.6}{22} + \frac{0.5}{24} + \frac{0.5}{30} + \frac{1.0}{33} + \frac{0.5}{36} + \frac{0.5}{40} + \frac{0.7}{44} + \frac{0.5}{48} + \frac{0.2}{50} + \frac{0.2}{55} + \frac{0.2}{60}$

- Universe of discourse is $X \in [-15,15]$
- Elements out of this range will be discarded

- $C = \frac{0.3}{10} + \frac{0.3}{11} + \frac{0.3}{12}$

# Multiplication of Fuzzy Numbers

- $A = \frac{0.3}{1} + \frac{0.6}{2} + \frac{1.0}{3} + \frac{0.7}{4} + \frac{0.2}{5}$
- $B = \frac{0.5}{10} + \frac{1.0}{11} + \frac{0.5}{12}$
- $C = \frac{0.3}{10} + \frac{0.3}{11} + \frac{0.3}{12}$

Is Subtraction symmetric?


Multiplication of two fuzzy numbers may not be a fuzzy number

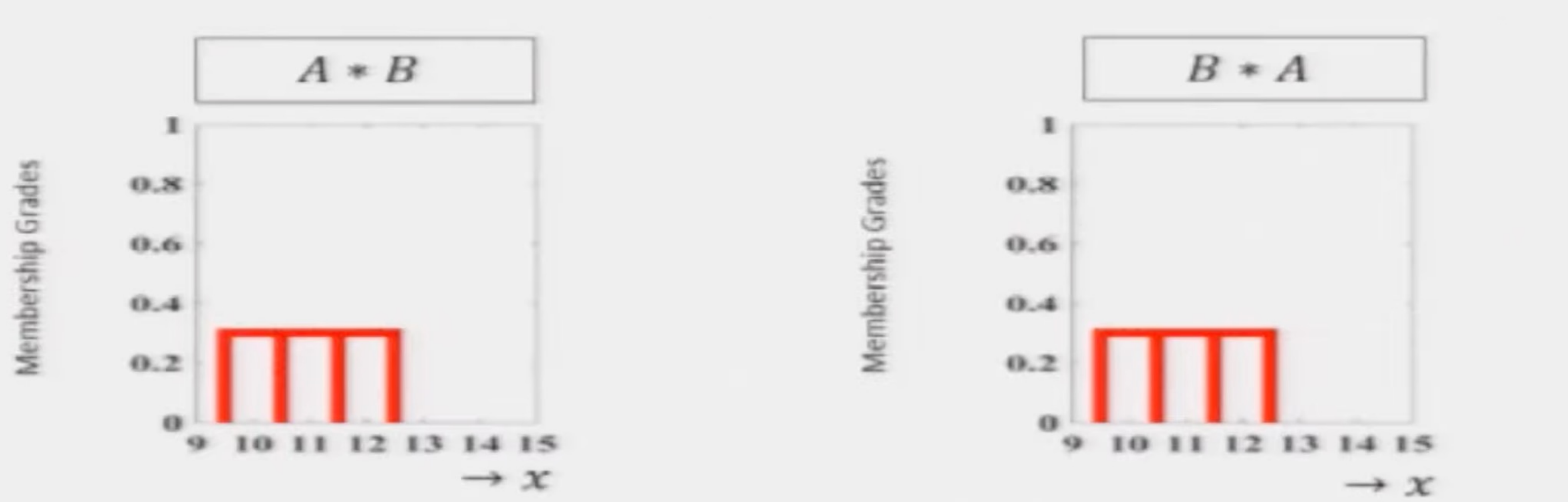
# Multiplication of Fuzzy Numbers

- $A = \frac{0.3}{1} + \frac{0.6}{2} + \frac{1.0}{3} + \frac{0.7}{4} + \frac{0.2}{5}$

- $B = \frac{0.5}{10} + \frac{1.0}{11} + \frac{0.5}{12}$

- $C = A * B = \frac{0.3}{10} + \frac{0.3}{11} + \frac{0.3}{12}$
- $C = B * A = \frac{0.3}{10} + \frac{0.3}{11} + \frac{0.3}{12}$

# Multiplication of Fuzzy Numbers




Is Subtraction symmetric?

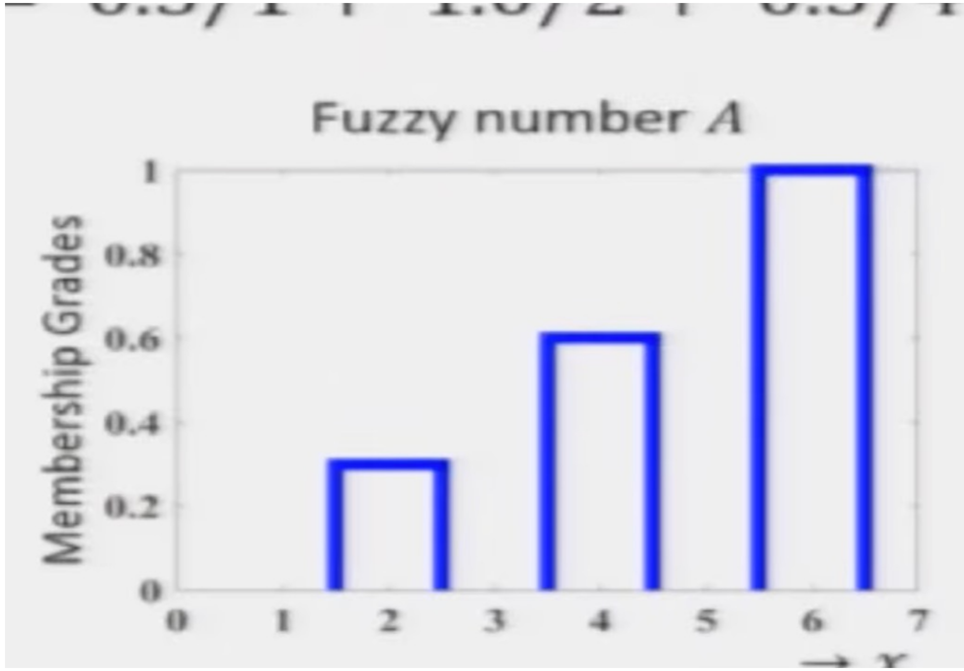
Is Subtraction commutative?

# Division of Fuzzy Numbers

- Let A and B are two Fuzzy numbers with the universe of discourse X
- If we perform the subtraction, it results in a new fuzzy number C as,
  - $C = A \div B$
- The new fuzzy number C is defined as,
- For discrete: $C = \sum_x \mu_C(x^C)/x^C$
- For continuous: $C = \int \mu_C(x^C)/x^C$
- The membership function values of fuzzy number C are
  - $\mu_C(x^C) = \mu_{A\div B}(x^C) = \max_{x^A, x^B}[\mu_A(x^A) \wedge \mu_B(x^B)]$
  - Where $x^C = x^A \div x^B; \forall\, x^A, x^B, x^C \in X$
- $C = \sum_x \mu_C(x^C)/x^C = \sum_x \mu_{A\div B}(x^C)/x^C = \sum_x \max_{x^A, x^B}[\mu_A(x^A) \wedge \mu_B(x^B)]\, /x^C$

# Division of Fuzzy Numbers

- Example: Let us consider two fuzzy numbers A and B with the universe of discourse $X \in \mathbb{N}$ as given below. Find the addition of fuzzy numbers A and B
- $A = \frac{0.3}{2} + \frac{0.6}{4} + \frac{1.0}{6}$
- $B = \frac{0.5}{1} + \frac{1.0}{2} + \frac{0.5}{4}$




# Division of Fuzzy Numbers

- $A = \frac{0.3}{2} + \frac{0.6}{4} + \frac{1.0}{6}$
- $B = \frac{0.5}{1} + \frac{1.0}{2} + \frac{0.5}{4}$

- $C = \sum_x \mu_C(x^C)/x^C = \sum_x \mu_{A\div B}(x^C)/x^C = \sum_x \max_{x^A, x^B}[\mu_A(x^A) \wedge \mu_B(x^B)] / (x^A \div x^B)$

- For elements $\frac{0.3}{2}$ and $\frac{0.5}{1}$ from Fuzzy number A and B respectively, we have,
  - $(\mu_A(x^A) \wedge \mu_B(x^B))/(x^A \div x^B) = (0.3 \wedge 0.5)/(2 \div 1) = 0.3/2$
  - Similarly, we will do for all the combinations of elements of Fuzzy number A and B

# Division of Fuzzy Numbers

- $A = \frac{0.3}{2} + \frac{0.6}{4} + \frac{1.0}{6}$
- $B = \frac{0.5}{1} + \frac{1.0}{2} + \frac{0.5}{4}$
- $C = \sum_x \max_{x^A, x^B} [\mu_A(x^A) \wedge \mu_B(x^B)] / (x^A \div x^B)$

$(\frac{0.3}{2}, \frac{0.5}{1}) = \frac{0.3}{2}$
$(\frac{0.3}{2}, \frac{1.0}{2}) = \frac{0.3}{1}$
$(\frac{0.3}{2}, \frac{0.5}{4}) = \frac{0.3}{0.5}$

$(\frac{1.0}{6}, \frac{0.5}{1}) = \frac{0.5}{6}$
$(\frac{1.0}{6}, \frac{1.0}{2}) = \frac{1.0}{3}$
$(\frac{1.0}{6}, \frac{0.5}{4}) = \frac{0.5}{1.5}$

$(\frac{0.6}{4}, \frac{0.5}{1}) = \frac{0.5}{4}$
$(\frac{0.6}{4}, \frac{1.0}{2}) = \frac{0.6}{2}$
$(\frac{0.6}{4}, \frac{0.5}{4}) = \frac{0.5}{1}$

# Division of Fuzzy Numbers

$$A = \frac{0.3}{2} + \frac{0.6}{4} + \frac{1.0}{6}$$
$$B = \frac{0.5}{1} + \frac{1.0}{2} + \frac{0.5}{4}$$
$$C = \sum_{x} \max_{x^A, x^B} [\mu_A(x^A) \wedge \mu_B(x^B)] \,/ (x^A \div x^B)$$

- $C = \frac{0.3}{2} + \frac{0.3}{1} + \frac{0.3}{0.5} + \frac{0.5}{4} + \frac{0.6}{2} + \frac{0.5}{1} + \frac{0.5}{6} + \frac{1.0}{3} + \frac{0.5}{1.5}$

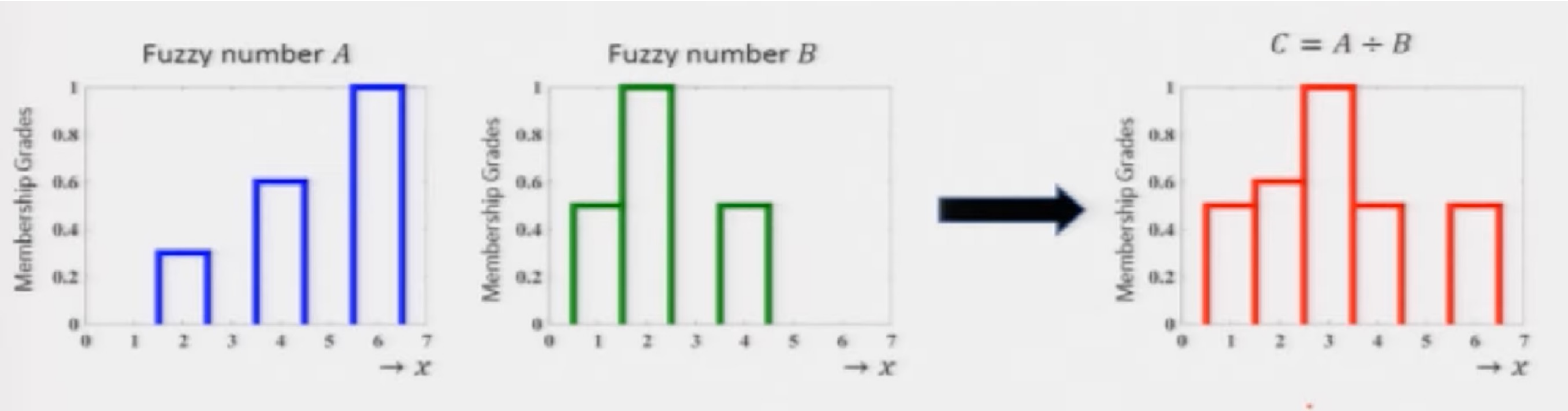
Rearrange numbers

- $C = \frac{0.3}{0.5} + \left(\frac{0.3}{1} + \frac{0.5}{1}\right) + \frac{0.5}{1.5} + \left(\frac{0.3}{2} + \frac{0.6}{2}\right) + \frac{1.0}{3} + \frac{0.5}{4} + \frac{0.5}{6}$
- $C = \frac{0.3}{0.5} + \frac{0.5}{1} + \frac{0.5}{1.5} + \frac{0.6}{2} + \frac{1.0}{3} + \frac{0.5}{4} + \frac{0.5}{6}$
- $C = \frac{0.5}{1} + \frac{0.6}{2} + \frac{1.0}{3} + \frac{0.5}{4} + \frac{0.5}{6}$

# Division of Fuzzy Numbers

- $A = \frac{0.3}{2} + \frac{0.6}{4} + \frac{1.0}{6}$
- $B = \frac{0.5}{1} + \frac{1.0}{2} + \frac{0.5}{4}$
- $C = \frac{0.5}{1} + \frac{0.6}{2} + \frac{1.0}{3} + \frac{0.5}{4} + \frac{0.5}{6}$

Is Division symmetric?

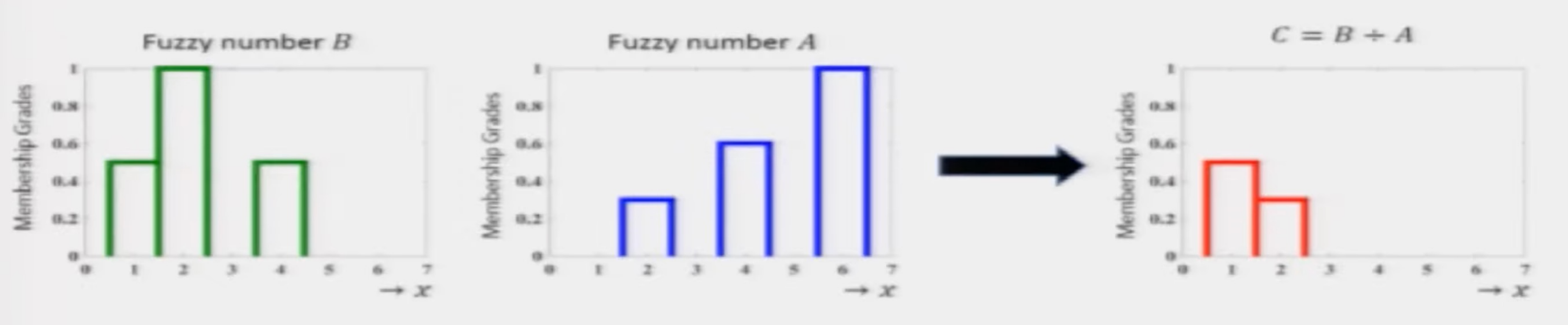



Division of two fuzzy numbers may not be a fuzzy number

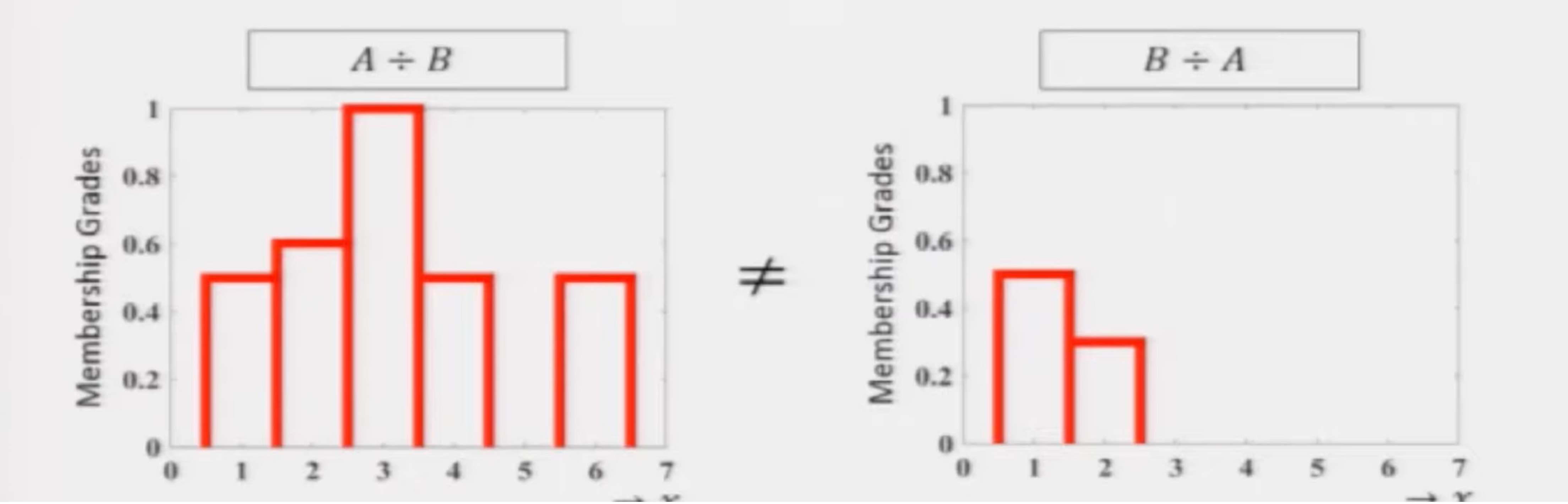
# Division of Fuzzy Numbers

- $A = \frac{0.3}{2} + \frac{0.6}{4} + \frac{1.0}{6}$
- $B = \frac{0.5}{1} + \frac{1.0}{2} + \frac{0.5}{4}$

- $C = A \div B = \frac{0.5}{1} + \frac{0.6}{2} + \frac{1.0}{3} + \frac{0.5}{4} + \frac{0.5}{6}$
- $C = B \div A = \frac{0.5}{1} + \frac{0.3}{2}$



# Division of Fuzzy Numbers




Is Division symmetric?

Is Division commutative?

# Thank You