

# AIFA

# Searching With Costs

09/01/2024

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# Search and Optimization

- **Given:** [S, s, O, G]
- **To find:**
  - A minimum cost sequence of transitions to a goal state
  - A sequence of transitions to the minimum cost goal
  - A minimum cost sequence of transitions to a minimum cost goal

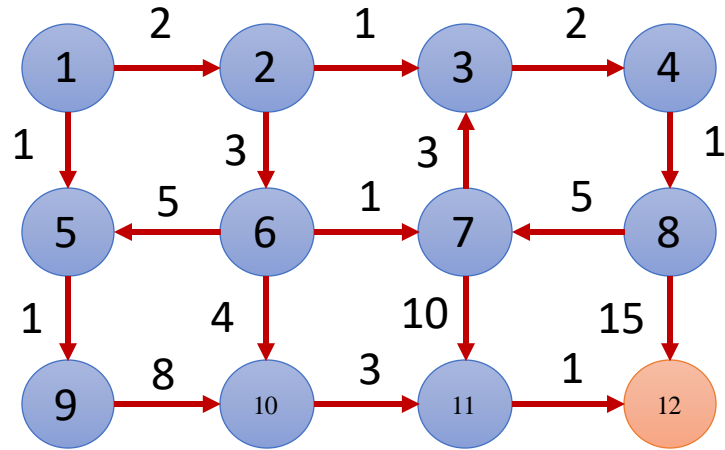
# Search with Cost

- **Initialize:** Set  $OPEN = \{s\}$ ,  $CLOSED = \{\}$ , Set  $C(s) = 0$
- **Fail:**
  - If  $OPEN = \{\}$ , Terminate with failure
- **Select:** Select the minimum cost state,  $n$ , from  $OPEN$  and
  - Save  $n$  is  $CLOSED$
- **Terminate:**
  - If  $n \in G$ , terminate with success

# Search with Cost

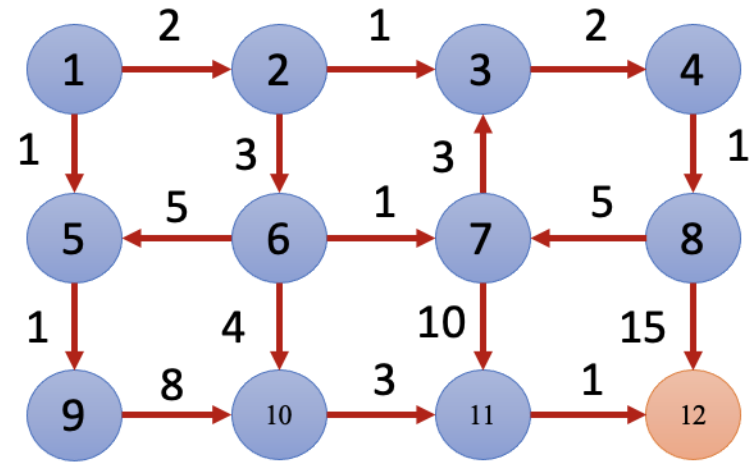
- **Expand:**
  - Generate successors of  $n$  using  $O$
  - For each successor,  $m$ :
    - If  $m \notin [OPEN \cup CLOSED]$ 
      - Set  $C(m) = C(n) + C(n, m)$
      - Insert  $m$  in OPEN
    - If  $m \in [OPEN \cup CLOSED]$ 
      - Set  $C(m) = \min \begin{cases} C(m) \\ C(n) + C(n, m) \end{cases}$
      - If  $C(m)$  has decreased and  $m \in CLOSED$ 
        - Move  $m$  to OPEN
- **Loop:**
  - Go to step 2

# Search with Cost



OPEN SET	SELECT	GOAL	EXPANDED	CLOSED
[1(0)]	1(0)	N	[2(2),5(1)]	[1(0)]
[2(2),5(1)]	5(1)	N	[2(2),9(2)]	[1(0),5(1)]
[2(2),9(2)]	2(2)	N	[9(2),3(3),6(5)]	[1(0),5(1),2(2)]
[9(2),3(3),6(5)]	9(2)	N	[3(3),6(5),10(10)]	[1(0),5(1),2(2),9(2)]
[3(3),6(5),10(10)]	3(3)	N	[6(5),10(10),4(5)]	[1(0),5(1),2(2),9(2),3(3)]

# Search with Cost

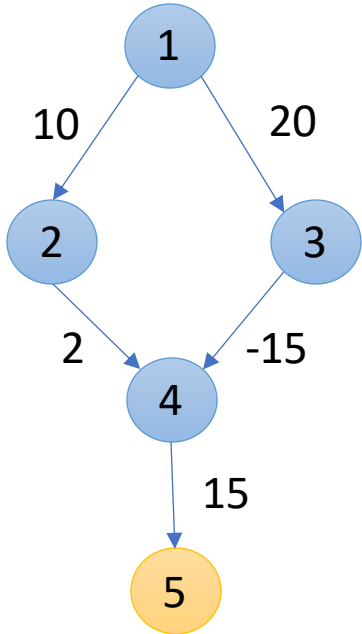


OPEN SET	SELECT	GOAL	EXPANDED	CLOSED
[3(3),6(5),10(10)]	3(3)	N	[6(5),10(10),4(5)]	[1(0),5(1),2(2),9(2),3(3)]
[6(5),10(10),4(5)]	6(5)	N	[10(9),4(5),7(6)]	[1(0),5(1),2(2),9(2),3(3),6(5)]
			•	
			•	
			•	
[12(13)]	12(13)	Y		

# Searching with Cost

- What are advantages of having positive cost?
- What will happen if operators have unit cost?
- What will happen if we have negative edge cost?

# Searching with Cost



OPEN SET	SELECT	GOAL	EXPANDED	CLOSED
[1(0)]	1(0)	N	[2(10),3(20)]	[1(0)]
[2(10),3(20)]	2(10)	N	[3(20),4(12)]	[1(0),2(10)]
[3(20),4(12)]	4(12)	N	[3(20),5(27)]	[1(0),2(10),4(12)]
[3(20),5(27)]	3(20)	N	[4(5),5(27)]	[1(0),2(10), <del>4(12)</del> ,3(20)]
[4(5),5(27)]	4(5)	N	[5(20)]	[1(0),2(10), <del>4(12)</del> ,3(20),4(5)]
[5(20)]	5(20)	Y		



# AIFA

# Branch and Bound

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# Branch and Bound

- We know an **upper bound** on solution cost
- We can do BFS/DFS to find out a goal and its associated cost
  - But it is not guaranteed that it is least cost goal
  - However, it works as an upper bound

# Branch and Bound

- **Initialize:** Set  $\text{OPEN}=\{s\}$ ,  $\text{CLOSED} = \{\}$ , Set  $C(s)=0$ ,  $C^* = \infty$
- **Fail:**
  - If  $\text{OPEN}=\{\}$ , then return  $C^*$
- **Select:** Select a state,  $n$ , from OPEN and save in CLOSED
- **Terminate:**
  - If  $n \in G$ , and  $C(n) < C^*$ , then
    - $C^*=C(n)$  and Go To Step 2

# Branch and Bound

- **Expand:**
  - If  $C(n) < C^*$ , Generate successors of  $n$  using  $O$
  - For each successor,  $m$ :
    - If  $m \notin [OPEN \cup CLOSED]$ 
      - Set  $C(m) = C(n) + C(n, m)$
      - Insert  $m$  in OPEN
    - If  $m \in [OPEN \cup CLOSED]$ 
      - Set  $C(m) = \min \begin{cases} C(m) \\ C(n) + C(n, m) \end{cases}$
      - If  $C(m)$  has decreased and  $m \in CLOSED$ 
        - Move  $m$  to OPEN
- **Loop:**
  - Go to step 2

# AIFA

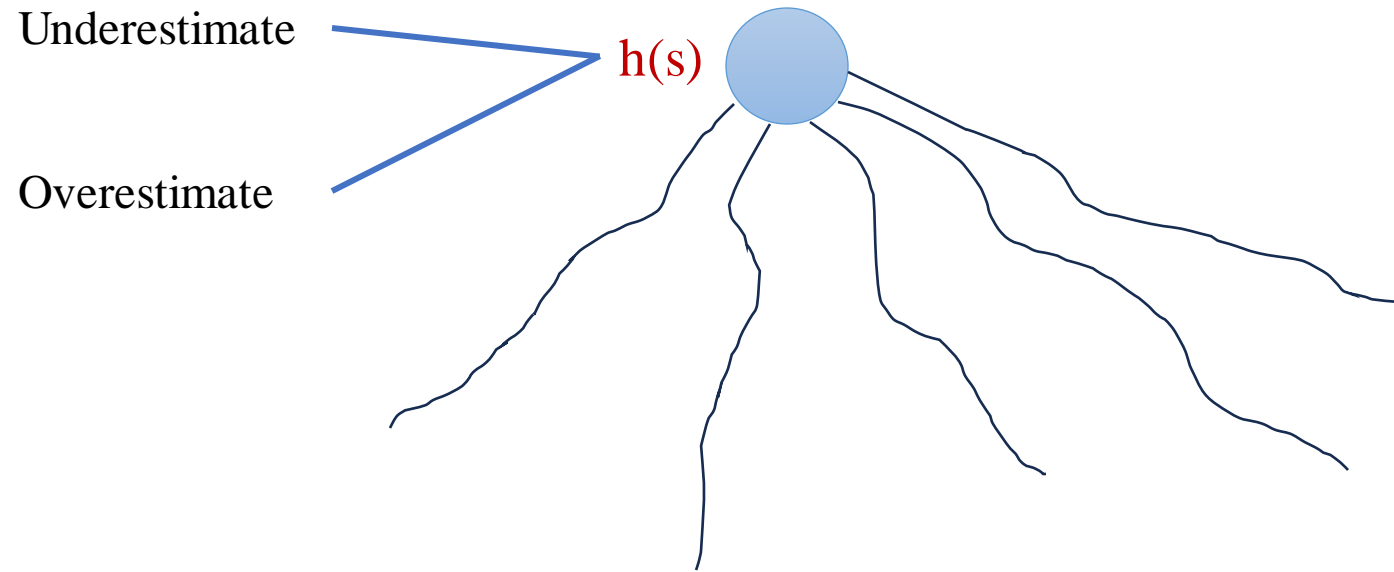
# Informed State Space Search

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# The notion of heuristics

- Heuristics use domain specific knowledge to estimate the quality or potential of partial solutions



# The notion of heuristics

- Examples:
  - Manhattan distance heuristic for 8 puzzle

5	6	7
4	1	8
3	9	

1	2	3
4	5	6
7	8	

$$2 + 0 + 4$$

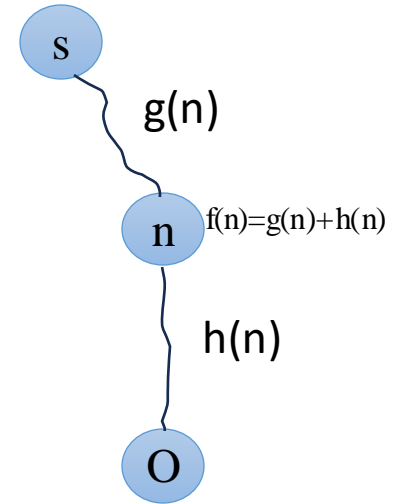
# The informed search problem

- Given:  $[S, s, O, G, h]$  where
  - $S$  is the (implicitly specified) set of states
  - $s$  is the start state
  - $O$  is the set of state transition operators each having some cost
  - $G$  is the set of Goal states
  - $h()$  is a heuristic function estimating the distance to a goal
- To find:
  - A minimum cost sequence of transitions to a goal state



# Algorithm A\*

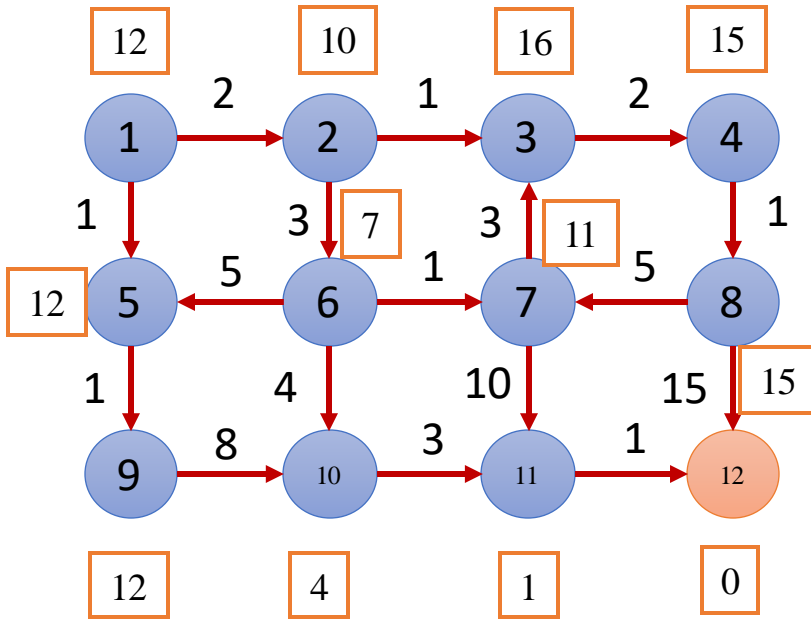
- **Initialize:** Set  $OPEN = \{s\}$ ,  $CLOSED = \{\}$ ,  $g(s)=0$ ,  $f(s) = h(s)$
- **Fail:**
  - If  $OPEN = \{\}$ , Terminate with failure
- **Select:** Select the minimum cost state,  $n$ , from  $OPEN$  and save in  $CLOSED$
- **Terminate:**
  - If  $n \in G$ , terminate with success



# Algorithm A\*

- **Expand:**
  - For each successor,  $m$ , of  $n$ :
    - If  $m \notin [\text{OPEN} \cup \text{CLOSED}]$ 
      - Set  $g(m) = g(n) + C(n, m)$
      - Set  $f(m) = g(m) + h(m)$
      - Insert  $m$  in OPEN
    - If  $m \in [\text{OPEN} \cup \text{CLOSED}]$ 
      - Set  $g(m) = \min \begin{cases} g(m) \\ g(n) + C(n, m) \end{cases}$
      - Set  $f(m) = g(m) + h(m)$
      - If  $f(m)$  has decreased and  $m \in \text{CLOSED}$ 
        - Move  $m$  to OPEN
- **Loop:**
  - Go to step 2

# Algorithm A\*



OPEN SET	SELECT	GOAL	EXPANDED	CLOSED
[1(12)]	1(12)	N	[2(12),5(13)]	[1(12)]
[2(12),5(13)]	2(12)	N	[5(13),3(19),6(12)]	[1(12),2(12)]
[5(13),3(19),6(12)]	6(12)	N	[5(13),3(19),7(17),10(13)]	[1(12),2(12),6(12)]
[5(13),3(19),7(17),10(13)]	5(13)	N	[3(19),7(17),10(13),9(14)]	[1(12),2(12),6(12),5(13)]
[3(19),7(17),10(13),9(14)]	10(13)	N	[3(19),7(17),9(14),11(13)]	[1(12),2(12),6(12),5(13),10(13)]

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•  
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Thank You