AIFA
A* Analysis

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Algorithm A*: Benefit

- Reduces number of expanded nodes

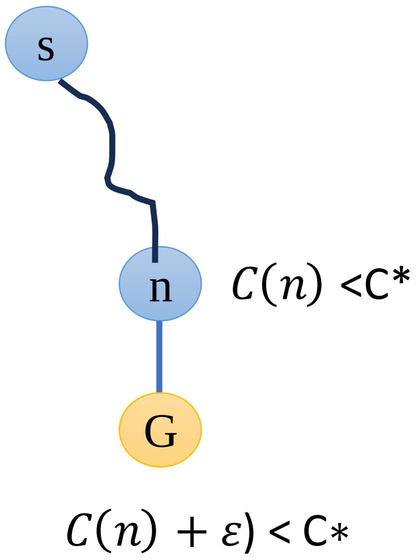
- Performs the lookahead and tells us promising paths

- What about optimality?

Uniform Cost Search

- Claim: If $C(n) < C^*$ (optimal cost) then n must be expanded
- Let algorithm A does not expand n

- For the class of algorithms without any heuristics
  - All states that have cost $< C^*$ will have to be expanded



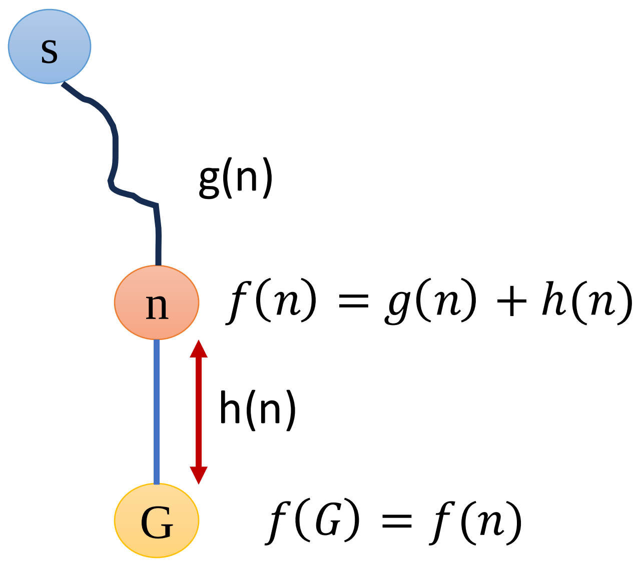
Algorithm A*: Benefit

- Claim: $f(n) < C^*$ then n must be expanded

- The heuristic function underestimates
  - $h(n) \leq h^*(n)$
    - Cost of reaching goal from n
  - All costs are +ve

- If we do not expand n, we can't find the goal

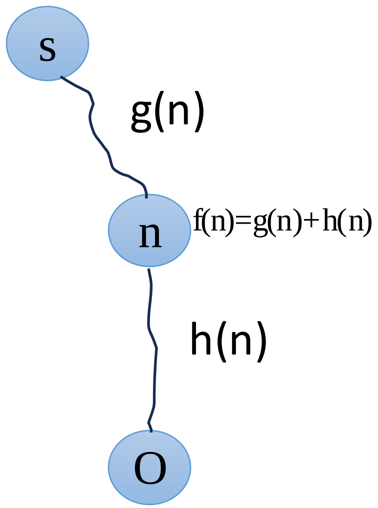
- If we have a state whose cost is less than C*
  - Then every algorithm which guarantees finding optimal solution have to expand it



Algorithm A*

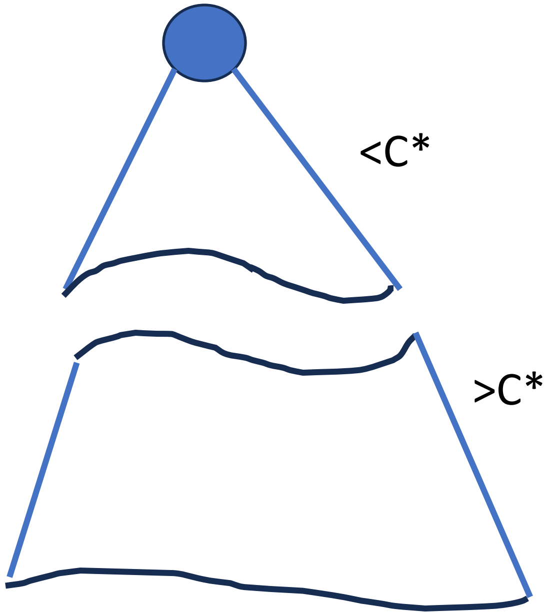
- Initialize: Set OPEN={s}, CLOSED = {}, g(s)=0, f(s) = h(s)
- Fail:
  - If OPEN={}, Terminate with failure
- Select: Select the minimum cost state, n, from OPEN and save in CLOSED
- Terminate:
  - If n∈G, terminate with success

How to break the tie?



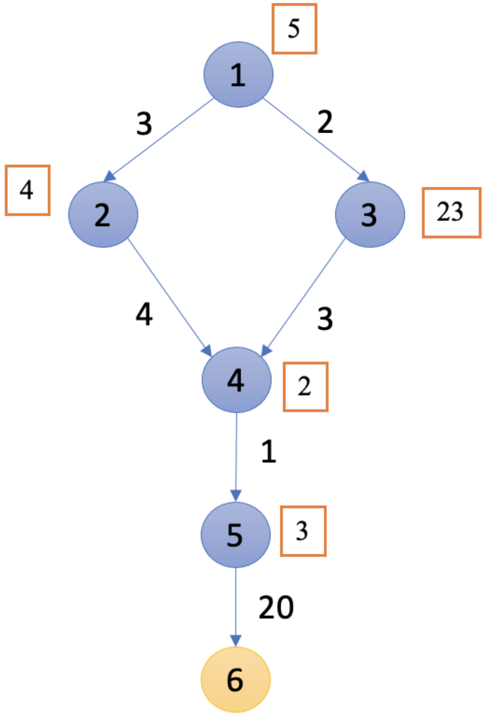
A*: Result

- What is an admissible heuristic?
  - If it always underestimates
  - We always have $h(n) \leq h^*(n)$, where $h^*(n)$ denotes minimum distance to a goal state from n



A*: Result

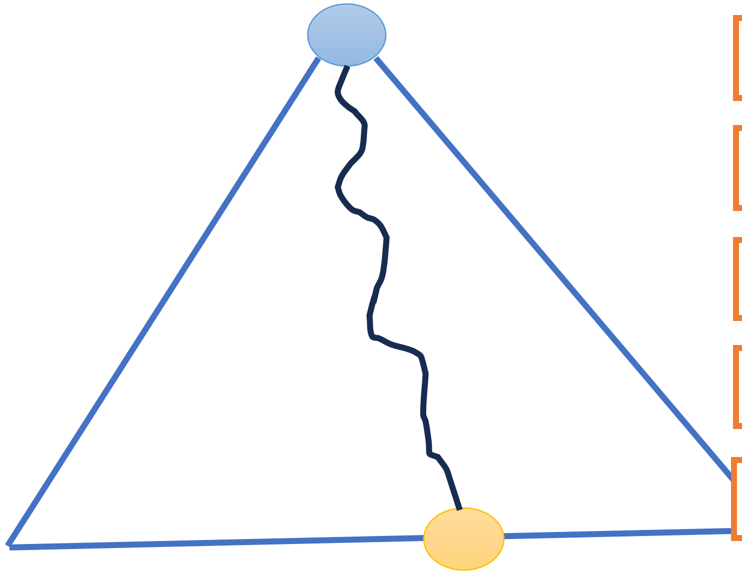
- At any time before A* terminates, there exists in OPEN a state **n**
  - That is on an optimal path from s to a goal state, with
  - $f(n) \leq f^*(s)$



OPEN SET	SELECT	GOAL	EXPANDED	CLOSED
[3(25),5(11)]	5(11)	N	[3(25),6(28)]	[1(5),2(7),4(9),5(11)]
[3(25),6(28)]	3(25)	N	[6(28),4(7)]	[1(5),2(7),~~4(9)~~,5(11),3(25)]
[6(28),4(7)]	4(7)	N	[6(28),5(9)]	[1(5),2(7),~~4(9)~~,~~5(11)~~,3(25),4(7)]
[6(28),5(9)]	5(9)	N	[6(26)]	[1(5),2(7),~~4(9)~~,~~5(11)~~,3(25),4(7),5(9)]
[6(26)]	6(26)	Y		

A*: Result

- At any time before A* terminates, there exists in OPEN a state **n**
  - That is on an optimal path from s to a goal state, with
  - $f(n) \leq f^*(s)$



OPEN SET	SELECT	GOAL	EXPANDED	CLOSED
[3(25),5(11)]	5(11)	N	[3(25),6(28)]	[1(5),2(7),4(9),5(11)]
[3(25),6(28)]	3(25)	N	[6(28),4(7)]	[1(5),2(7),~~4(9)~~,5(11),3(25)]
[6(28),4(7)]	4(7)	N	[6(28),5(9)]	[1(5),2(7),~~4(9)~~,~~5(11)~~,3(25),4(7)]
[6(28),5(9)]	5(9)	N	[6(26)]	[1(5),2(7),~~4(9)~~,~~5(11)~~,3(25),4(7),5(9)]
[6(26)]	6(26)	Y		

A*: Result

- If there is a path from s to a goal state, A* terminates

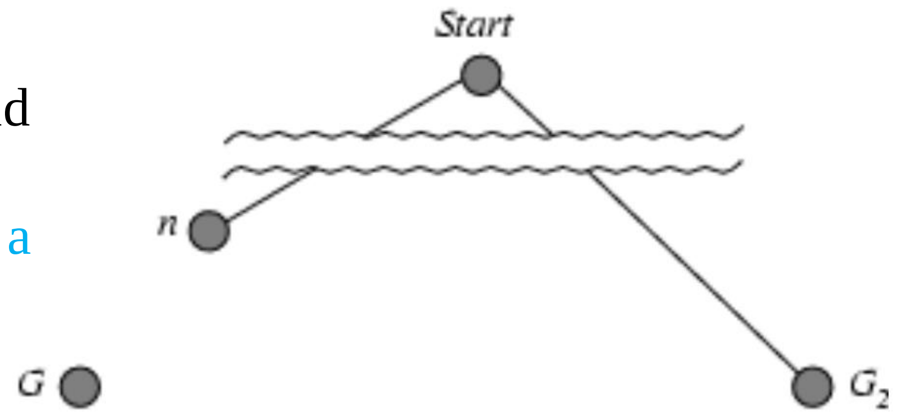
- What is the worst case scenario?

- Approximation algorithm

A*: Result

- Algorithm A* is admissible
  - If there is a path from s to a goal state,
  - A* terminates by finding an optimal path

A* Optimality



- Suppose some suboptimal goal path G2 has been generated and is in OPEN
- Let n be an unexpanded node in OPEN such that n is on a shortest path to an optimal goal G

$h(G_2) = h(G) = 0$

$f(G_2) = g(G_2)$ $\quad$ $\mathrm{f}(G) = g(G)$

$g(G_2) > g(G)$ $\quad$ G_2 is suboptimal

$f(G_2) > f(G)$

$h(n) \leq h^*(n)$ $\quad$ h is admissible, h^* is minimal distance

$g(n) + h(n) \leq g(n) + h^*(n)$

$\mathrm{f}(n) < f^*(G)$ $\quad$ $f(G_2) > \mathrm{f}(n)$ $\quad$ A* will never select G_2 for expansion

A*: Result

- Algorithm A* is admissible
  - If there is a path from s to a goal state,
  - A* terminates by finding an optimal path

- If A1 and A2 are two versions of A* such that A2 is more informed than A1
  - A1 expands at least as many states as does A2

A*: Result

- We have two good heuristic functions h1 and h2 but do not know which one is more informed

- How much effort should we put in computing heuristics?

Thank You