AIFA Resolution

29/01/2024

Clause: A special form

- Literal – A single proposition or its negation
  - P, ~P

- A clause is a disjunction of literals
  - $P \vee Q \vee \sim R$

- Can we convert any proposition to a clausal form?

Converting compound proposition to clausal form

- Consider the sentence (wff)
  - $\sim(A \rightarrow B) \vee (C \rightarrow A)$
- Eliminate the implication sign
  - $\sim(\sim A \vee B) \vee (\sim C \vee A)$
- Eliminate double negation and reduce scope of "not" signs (De-Morgan Law)
  - $(A \wedge \sim B) \vee (\sim C \vee A)$
- Convert to conjunctive normal form by using distributive and associative laws
  - $(A \vee \sim C \vee A) \wedge (\sim B \vee \sim C \vee A)$
  - $(A \vee \sim C) \wedge (\sim B \vee \sim C \vee A)$
- Two clauses
  - $(A \vee \sim C)$
  - $(\sim B \vee \sim C \vee A)$

Why are we so interested in clausal form?

↓

Helps us in applying interesting inference mechanism: **Resolution**

Resolution: Inference Mechanism

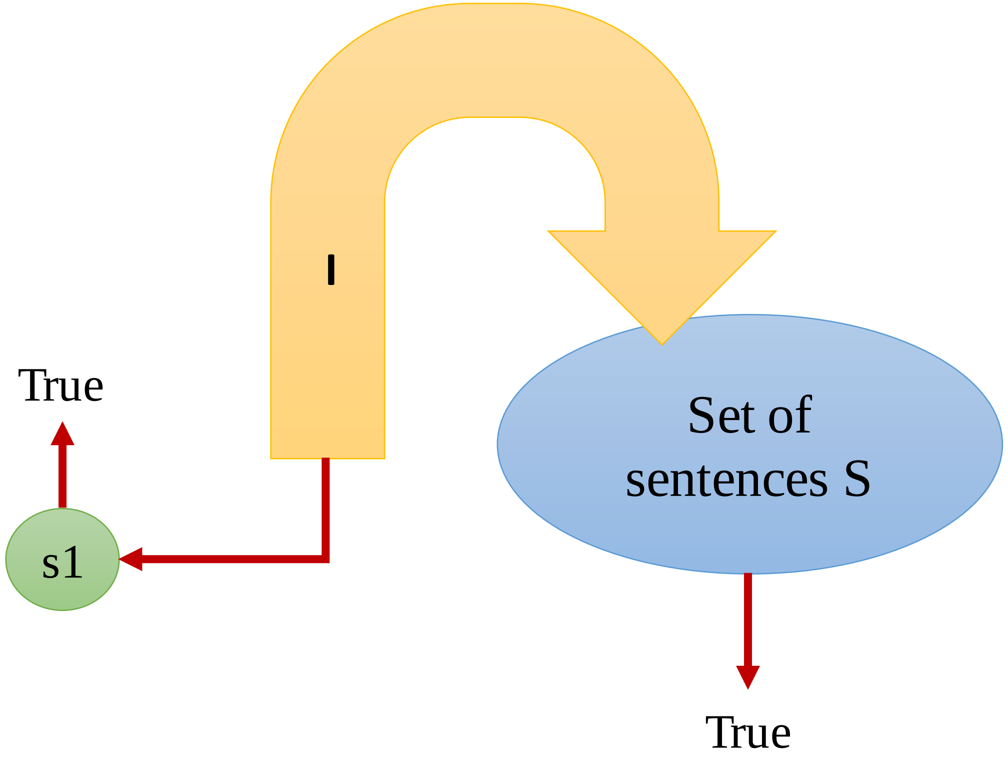
- Objective:
  - Learn to prove new facts given a set of facts
  - Given a set of facts proving a fact means proving the **logical entailment**
- A sound inference mechanism

Entailment

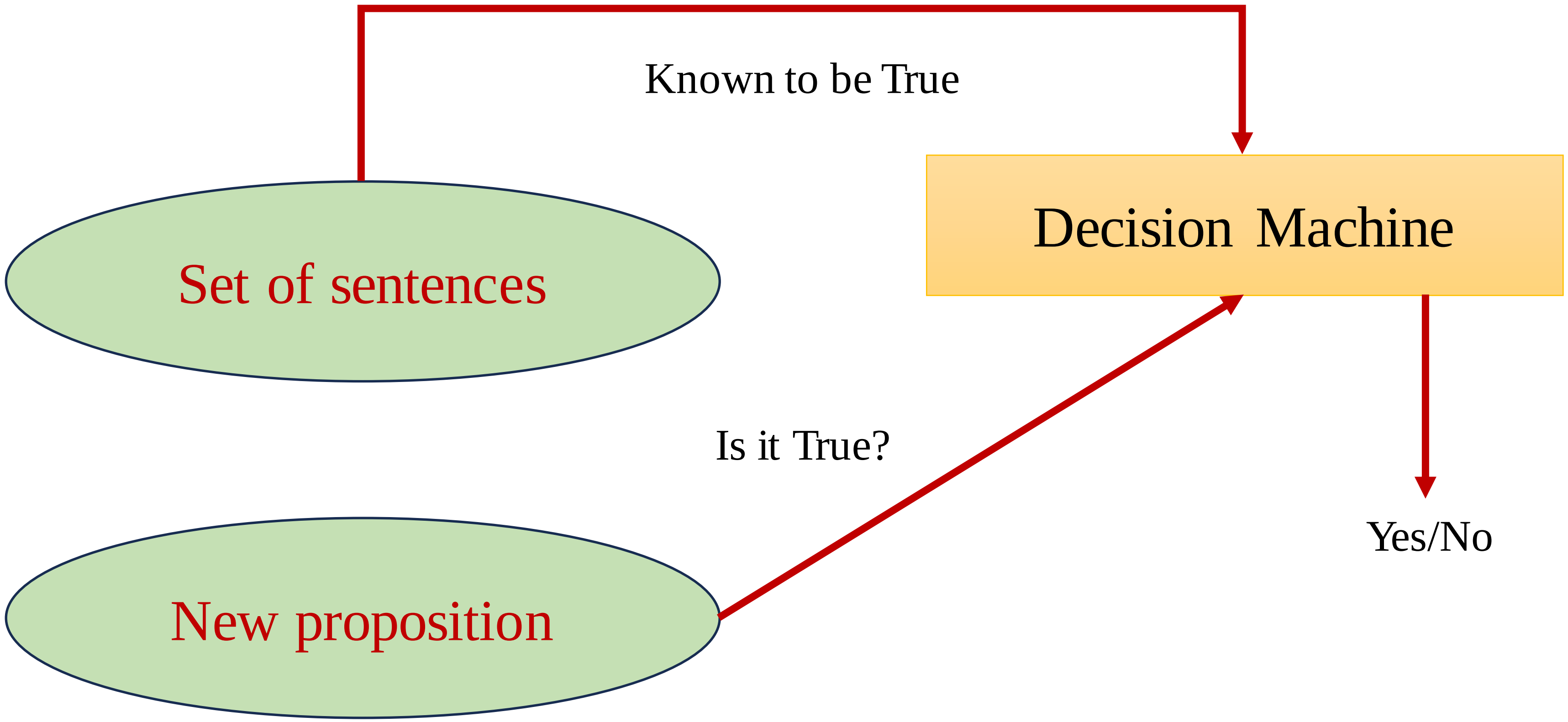
If a sentence s1 has a value True for all interpretations

that make all sentences in a set S True then

- S |- s1
- s1 logically follows from S
- s1 is a logical consequence of S
- S logically entails s1



Inference Mechanism



Resolution

- Suppose x is a literal
- S1 and S2 are two sets of propositional sentences represented in clausal form

- If we have $(x \vee S1) \wedge (\sim x \vee S2)$
  - Then we get $S1 \vee S2$
  - Here $S1 \vee S2$ is the resolvent
  - x is resolved upon

Problem 3

- If a triangle is equilateral then it is isosceles
- If a triangle is isosceles then two sides AB and AC are equal
- If AB and AC are equal then angle B and C are equal
- ABC is an equilateral triangle

- Prove angle B is equal to angle C

Problem 3: Proposition Form

- If a triangle is equilateral then it is isosceles
  - Equilateral(ABC)→Isosceles(ABC)

- If a triangle is isosceles then two sides AB and AC are equal
  - Isosceles(ABC)→Equal(AB,AC)

- If AB and AC are equal then angle B and C are equal
  - Equal(AB,AC)→Equal(B,C)

- ABC is an equilateral triangle
  - Equilateral(ABC)

Problem 3: Clausal Form

- Equilateral(ABC)→Isosceles(ABC)
  - ~ Equilateral(ABC)∨Isosceles(ABC)

- Isosceles(ABC)→Equal(AB,AC)
  - ~Isosceles(ABC)∨Equal(AB,AC)

- Equal(AB,AC)→Equal(B,C)
  - ~ Equal(AB,AC)∨Equal(B,C)

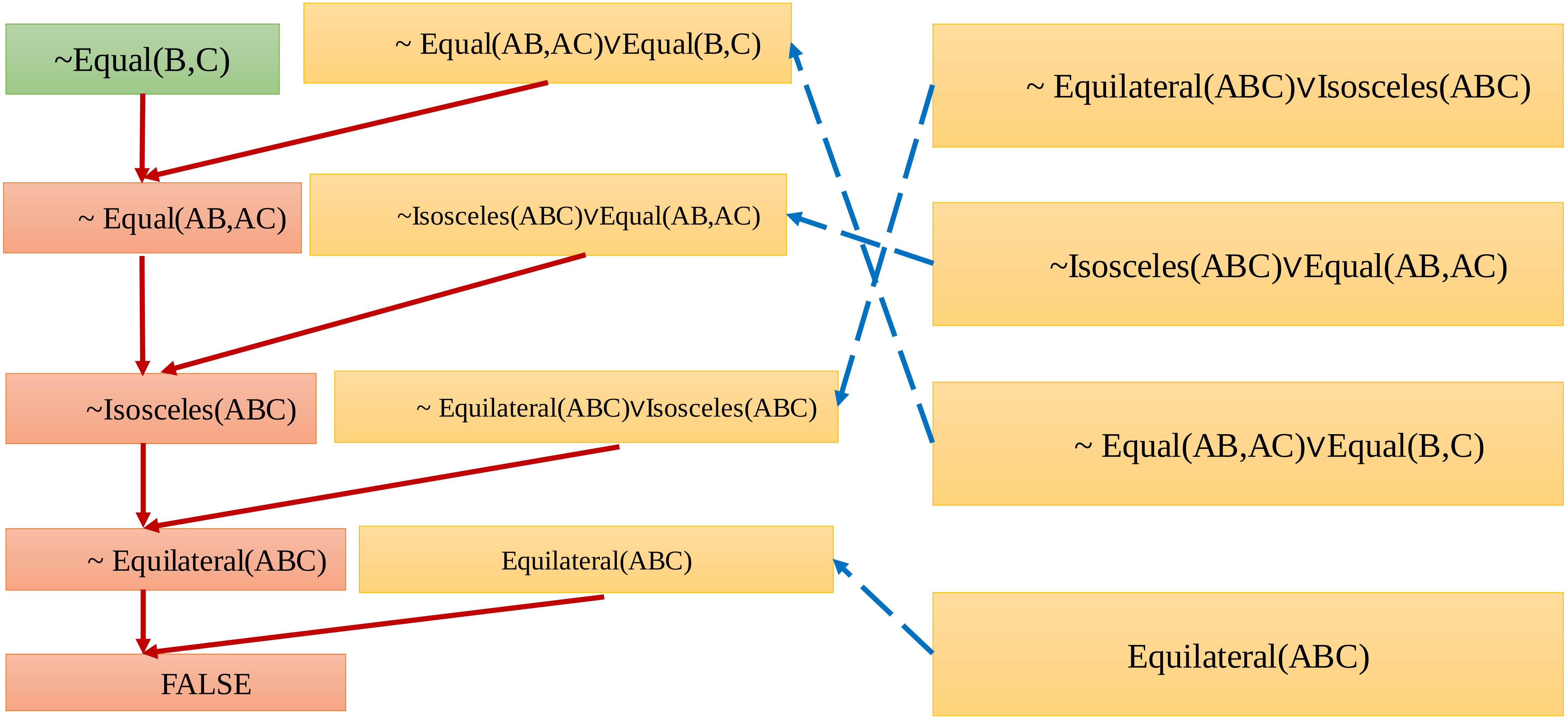
- Equilateral(ABC)

Proof by Refutation

- To Prove: Angle B is equal to Angle C: Equal(B,C)

- Let us disprove: NotEqual(B,C) = ~Equal(B,C)

- φ : $F1 \wedge F2 \wedge \ldots \wedge Fn \rightarrow G$
- φ : $\sim(F1 \wedge F2 \wedge \ldots \wedge Fn) \vee G$
- $\sim\varphi$: $F1 \wedge F2 \wedge \ldots \wedge Fn \wedge \sim G$



We have arrived in contradictory situation that is not supported by given set of facts

AIFA
First Order Logic

29/01/2024

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Objective

- Formulate more types of sentences in logic?

- Write correct predicate logic formulae

Limitations of propositional logic

- All dogs are faithful
- Tommy is a dog
- Therefore, Tommy is faithful

How to represent and infer this in propositional logic?

- P: All dogs are faithful
- Q: Tommy is a dog
- Can we claim? $P \land Q \rightarrow$ Tommy is faithful

- Machine does not know what does "all dogs" mean?

Limitations of propositional logic

- Anil is a hardworking student
  - Hardworking_Anil

- Anil is an intelligent student
  - Intelligent_Anil

- Anil scores high marks
  - Score_High_Mark_Anil

- If Anil is hardworking and Anil is intelligent, then Anil scores high marks
  - Hardworking_Anil $\wedge$ Intelligent_Anil $\rightarrow$ Score_High_Mark_Anil

- What about Akash and Asish?

Limitations of propositional logic

- Anil is a hardworking student
- Anil is an intelligent student

- All students who are hardworking and intelligent scores high marks

- For all x such that x is a student and x is intelligent and x is hardworking then x scores high marks
  - x is a variable
  - Need power to write such sentences

The problem of Infinite Model

- Propositional logic, we have to restrict ourselves to constants
- In general, propositional logic can deal with only a finite number of propositions

- If there are only three students Anil Akash Asish, then
  - P: Anil is intelligent
  - Q: Akash is intelligent
  - R: Asish is intelligent
- All students are intelligent: $P \wedge Q \wedge R$

- If a new student joins the class?
- How long should we go on?
- Limitation: Finiteness of statements

First Order Logic

- Generalization of propositional logic that allows us to express and infer arguments in infinite models like

  - All men are mortal
  - Some birds cannot fly
  - At least one planet has life on it

Syntax of FOL

- The syntax of First-Order Logic can be defined in terms of
  - Terms
  - Predicates
  - Quantifiers

Term

- A term denotes some object other than **true** or **false**

- Tommy is a dog
- All men are mortal

term

- Terms can be constants as well as variables
- In proposition logic, we only have constants

Term: Constants & Variables

- A constant of type W is a name that **denotes a particular object** in a set W
  - Example: 5, Anil

- A variable of type W is a name that can **denote any element** in the set W
  - Examples: $x \in N$ denotes a Natural number
  - s denotes the name of a student

Functions

- A functional term of arity n takes n objects of type W_1 to W_n as inputs and returns an object of type W

  - f(W_1, W_2, … , W_n)
  - plus(3,4) = 7
    - Two objects of type constant from the set of Natural numbers

Functional Term

Constants

Functions: Example

- Let plus be a function that takes two arguments of type Natural number and returns a Natural number

- Valid Functional Terms:
  - plus(2,3)
  - plus(5,plus(7,3))

- Invalid Functional Terms:
  - plus(0,-1)
  - Plus(1.2,3.3)

Predicates

- Predicates are like functions except that their return type is **true** or **false**

- Example:
  - gt(x,y) is true iff x>y
  - Here gt is a predicate symbol that takes two arguments of type natural number
  - gt(3,4) is a valid predicate but gt(3,-4) is not

Types of Predicates

- A predicate with no variable is a proposition
  - Anil is a student

- A predicate with one variable is called a property
  - student(x) is true iff x is student
  - mortal(y) is true iff y is mortal

Formulation of Predicates

- Let P(x,y,…) and Q(x,y,…) are two predicates

- Then so are
  - $P \wedge Q$
  - $P \vee Q$
  - $\sim P$
  - $P \rightarrow Q$

Predicate Examples

- If x is a man then x is mortal
  - $man(x) \rightarrow mortal(x)$
  - $\sim man(x) \vee mortal(x)$

- If n is a natural number then n is either even or odd
  - $natural(n) \rightarrow even(n) \vee odd(n)$

Thank You