Decision Trees

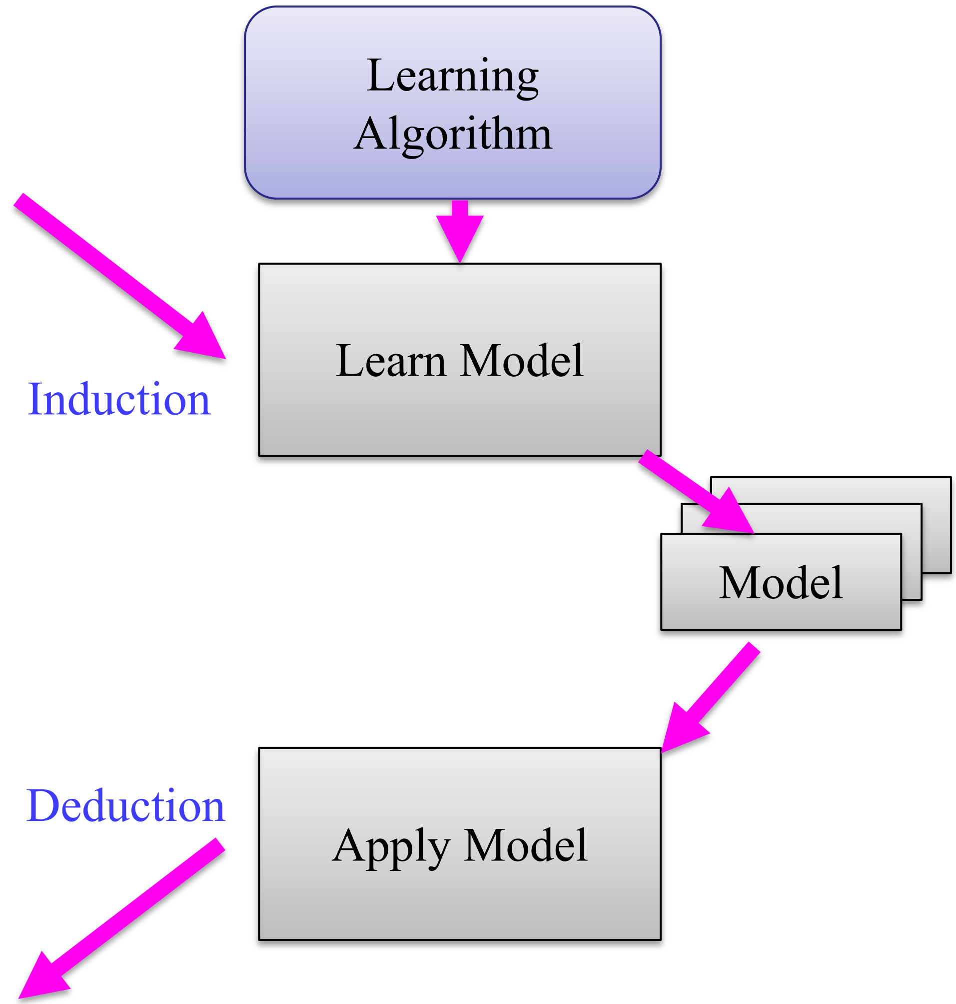
15/04/2024

Koustav Rudra

Illustrating Classification Task

Tid	Attrib1	Attrib2	Attrib3	Class
1	Yes	Large	125k	No
2	No	Medium	100k	No
3	No	Small	70k	No
4	Yes	Medium	120k	No
5	No	Large	95k	Yes
6	No	Medium	60k	No
7	Yes	Large	220k	No
8	No	Small	85k	Yes
9	No	Medium	75k	No
10	No	Small	90k	Yes

Training Set



Tid	Attrib1	Attrib2	Attrib3	Class
1	No	Small	55k	?
2	Yes	Medium	80k	?
3	Yes	Large	110k	?
4	No	Small	95k	?
5	No	Large	67k	?

Test Set

Intuition behind a decision tree

- Ask a series of questions about a given record
  - Each question is about one of the attributes
  - Answer to one question decides what question to ask next (or if a next question is needed)
  - Continue asking questions until we can infer the class of the given record

Example of a Decision Tree

categorical categorical continuous class

Tid	Refund	Loan Status	Taxable Income	Cheat
1	Yes	Medium	125k	No
2	No	Small	100k	No
3	No	Medium	70k	No
4	Yes	Small	120k	No
5	No	Large	95k	Yes
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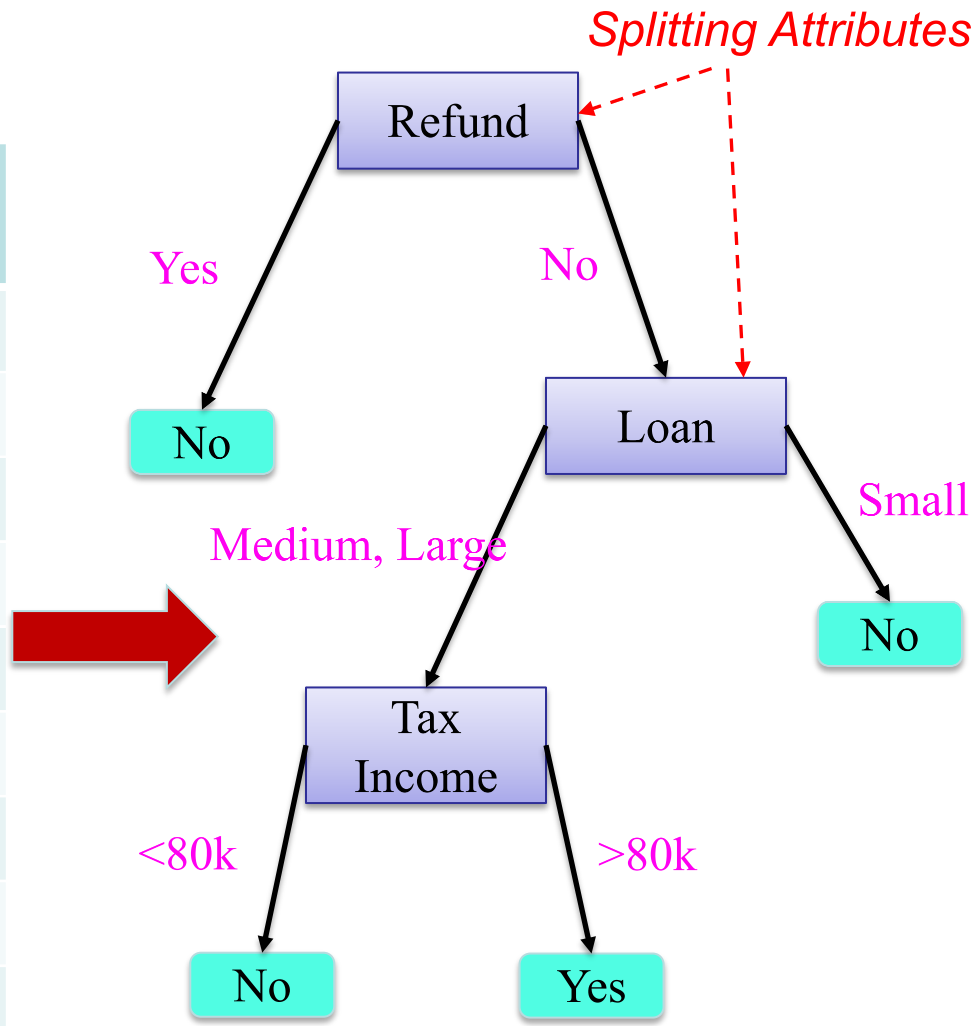

Model: Decision Tree

Structure of a decision tree

- Decision tree: hierarchical structure
  - One root node: no incoming edge, zero or more outgoing edges
  - Internal nodes: exactly one incoming edge, two or more outgoing edges
  - Leaf or terminal nodes: exactly one incoming edge, no outgoing edge

- Each leaf node assigned a class label

- Each non-leaf node contains a test condition on one of the attributes

Applying a Decision-Tree Classifier

Tid	Attrib1	Attrib2	Attrib3	Class
1	Yes	Large	125k	No
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3	No	Small	70k	No
4	Yes	Medium	120k	No
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Training Set

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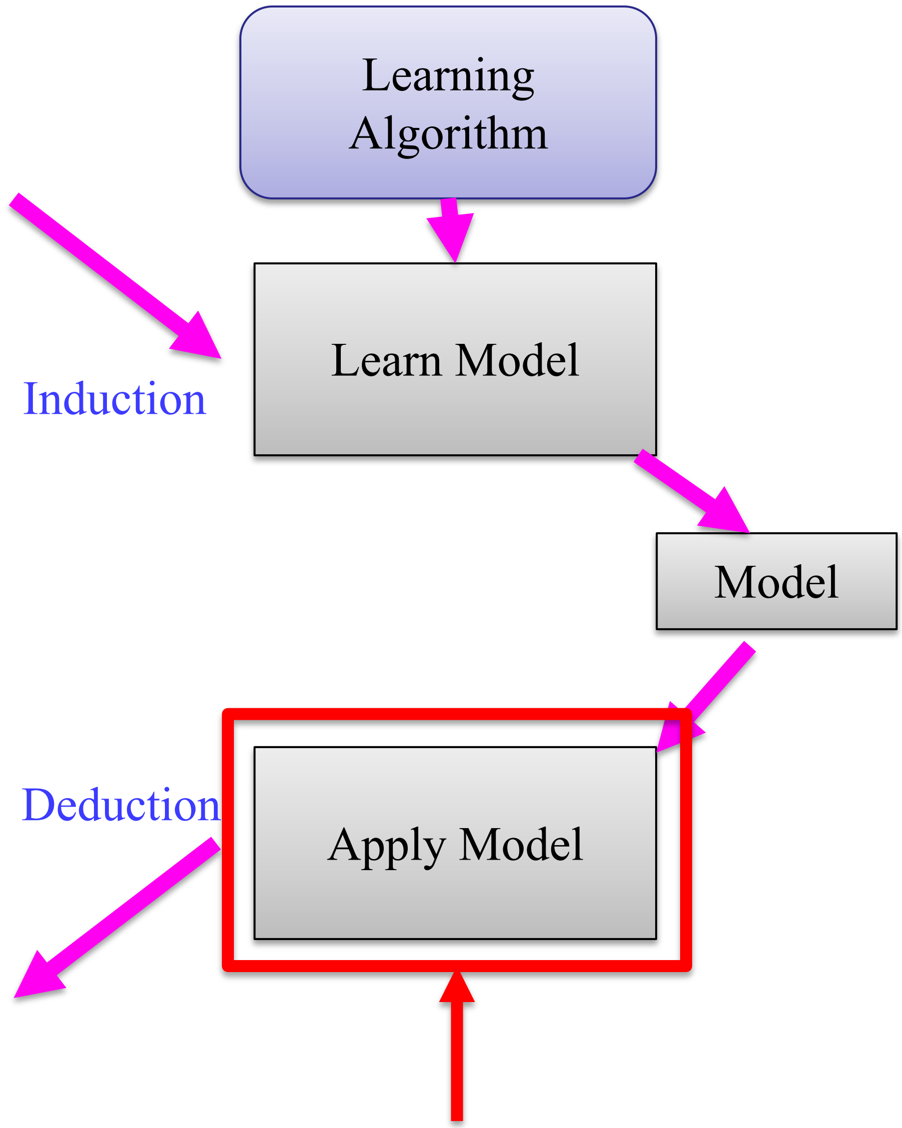
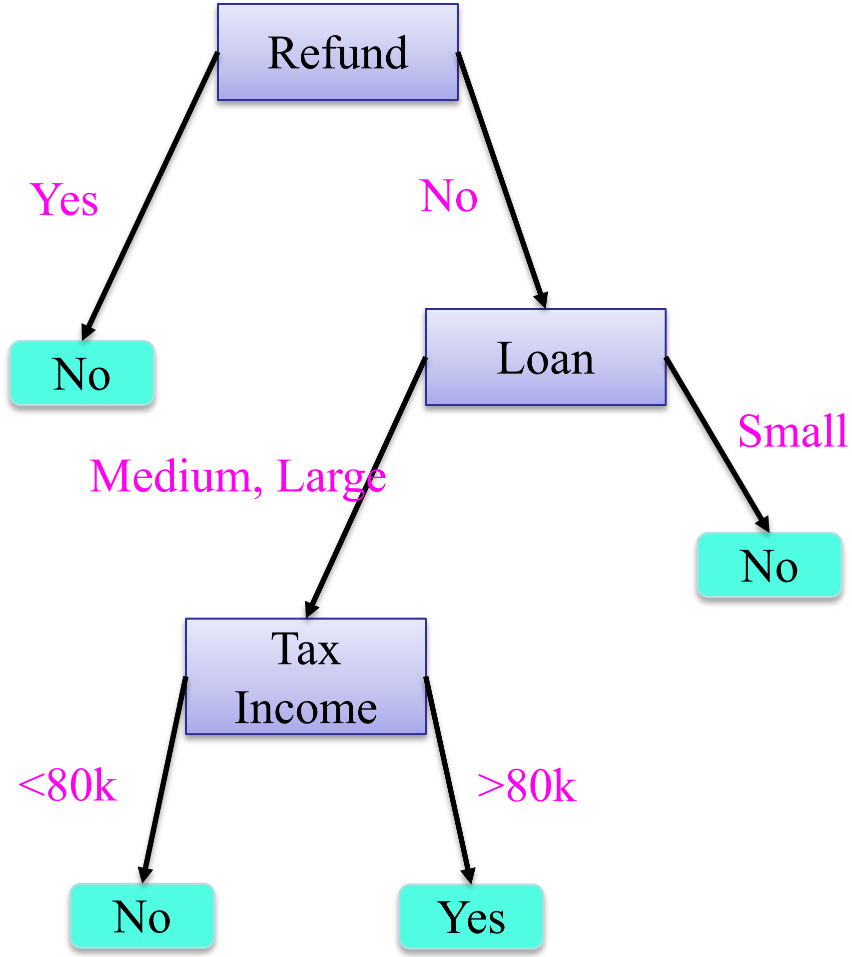
Test Set



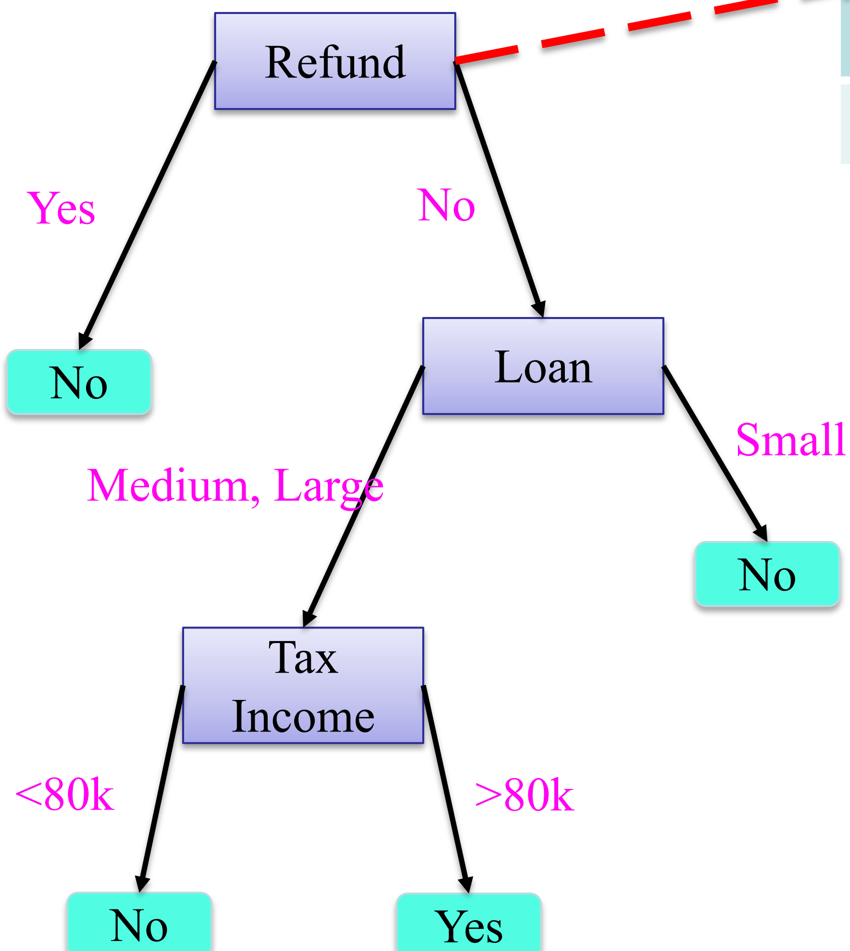
Applying Model to Test Data

Refund	Loan Status	Taxable Income	Cheat
No	Small	80k	?



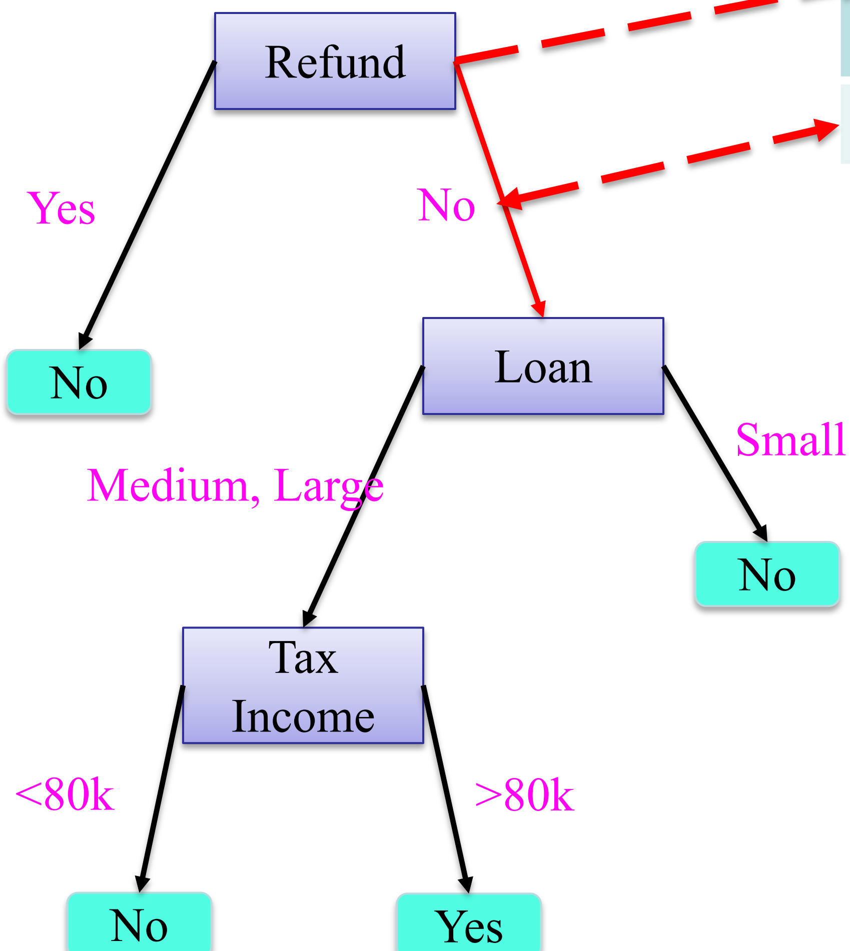
Once a decision tree has been constructed (learned), it is easy to apply it to test data

Applying Model to Test Data



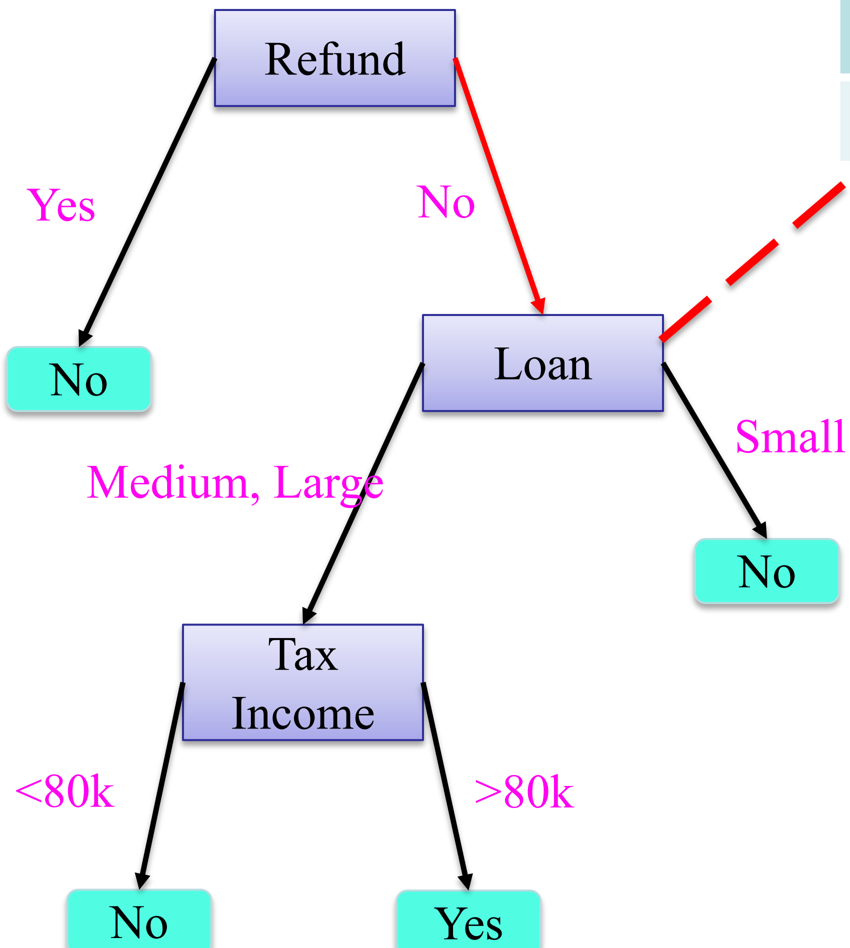
Refund	**Loan Status**	**Taxable Income**	**Cheat**
No	Small	80k	?

Applying Model to Test Data



Refund	Loan Status	Taxable Income	Cheat
No	Small	80k	?

Applying Model to Test Data



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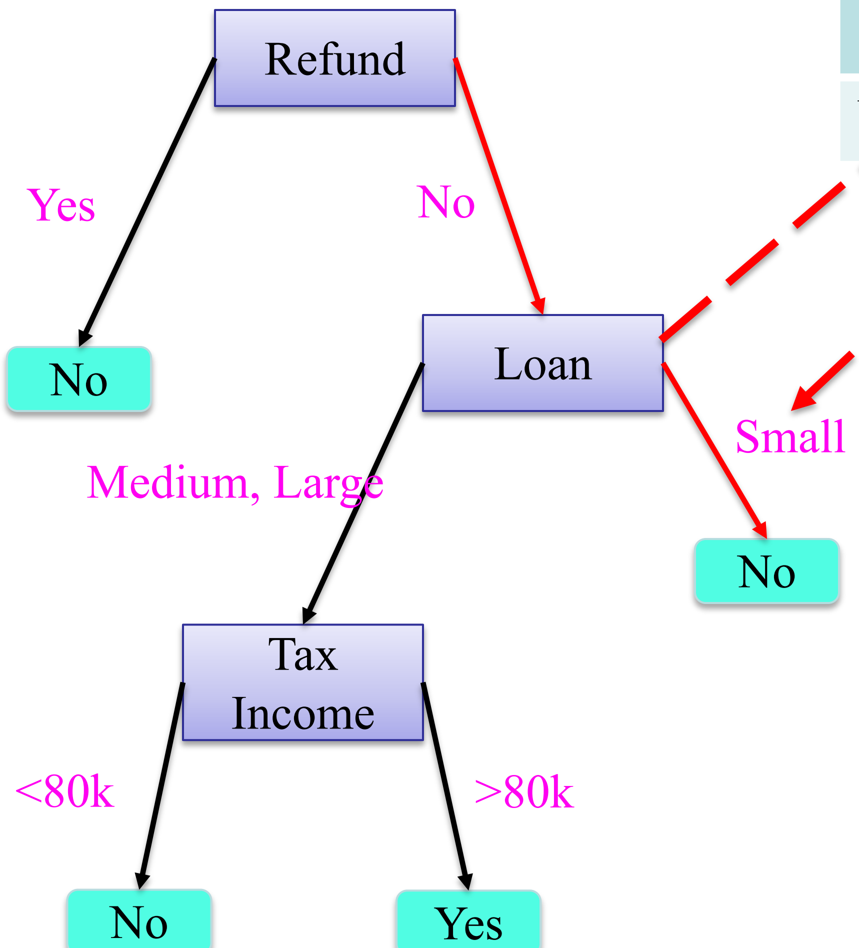
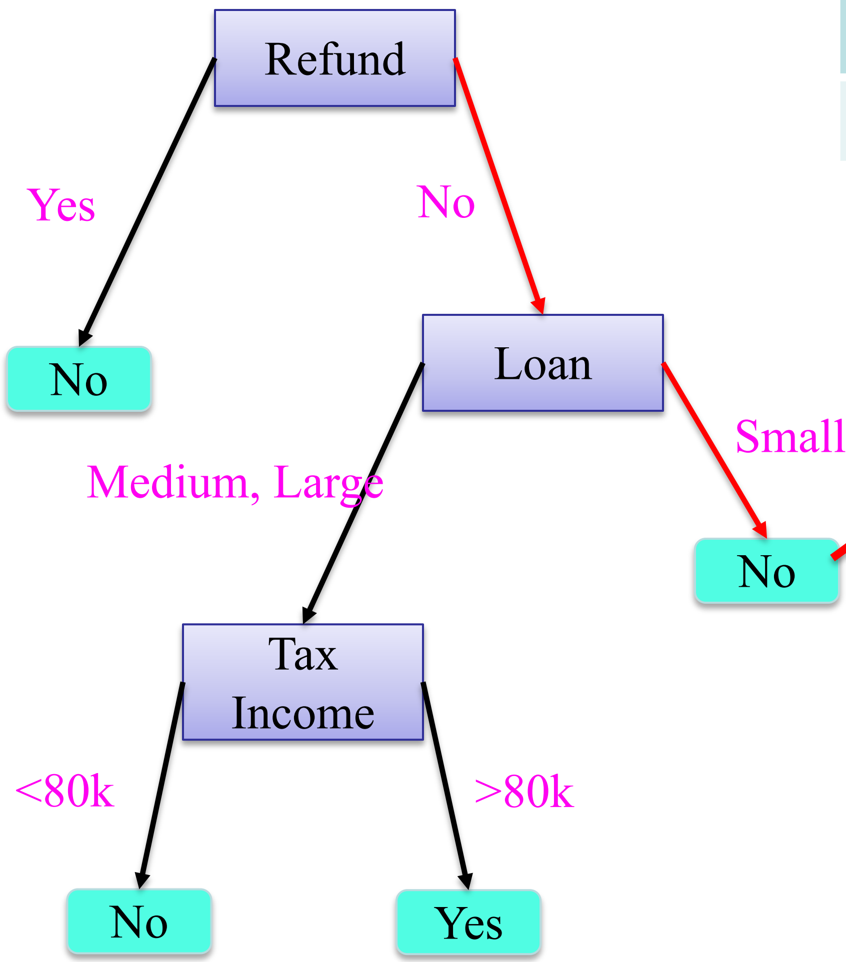
No	Small	80k	?

Applying Model to Test Data



Refund	Loan Status	Taxable Income	Cheat
No	Small	80k	?

Applying Model to Test Data



Refund	Loan Status	Taxable Income	Cheat
No	Small	80k	?

Assign Cheat to “No”

Learning a Decision-Tree Classifier

Tid	Attrib1	Attrib2	Attrib3	Class
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Training Set

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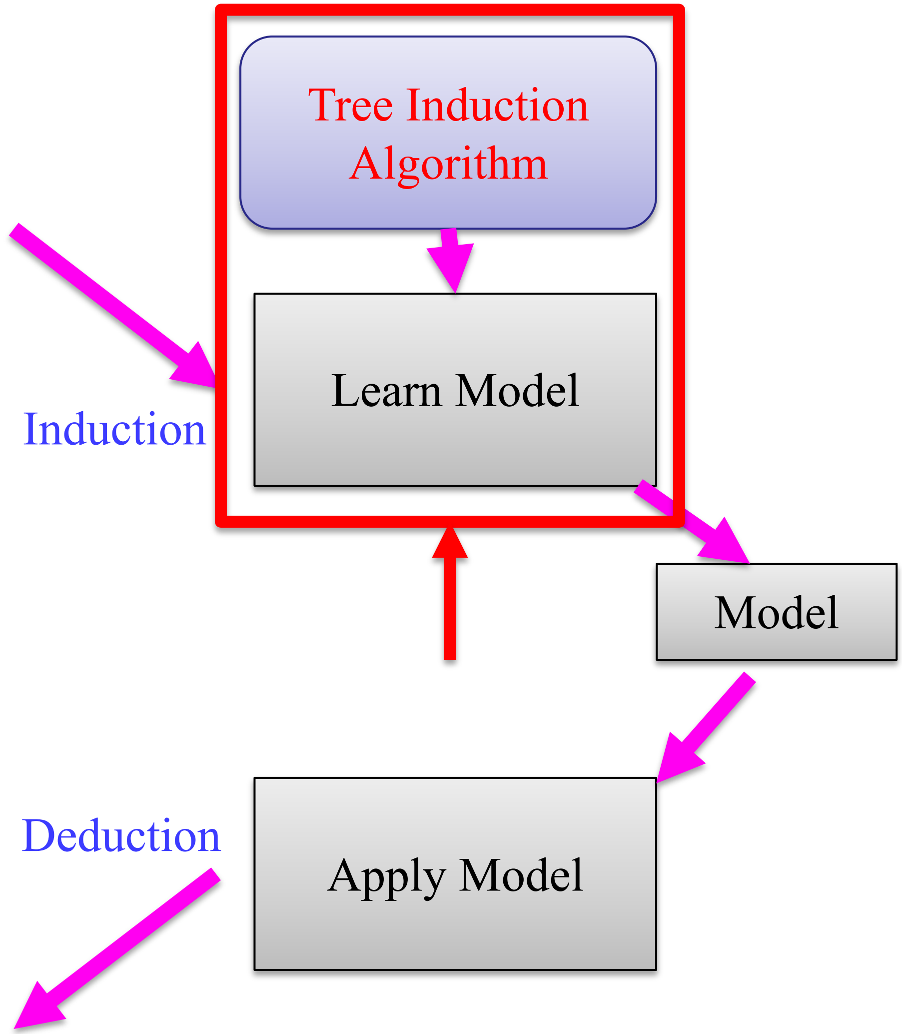
Test Set



How to learn a decision tree?

A Decision Tree

categorical categorical continuous class

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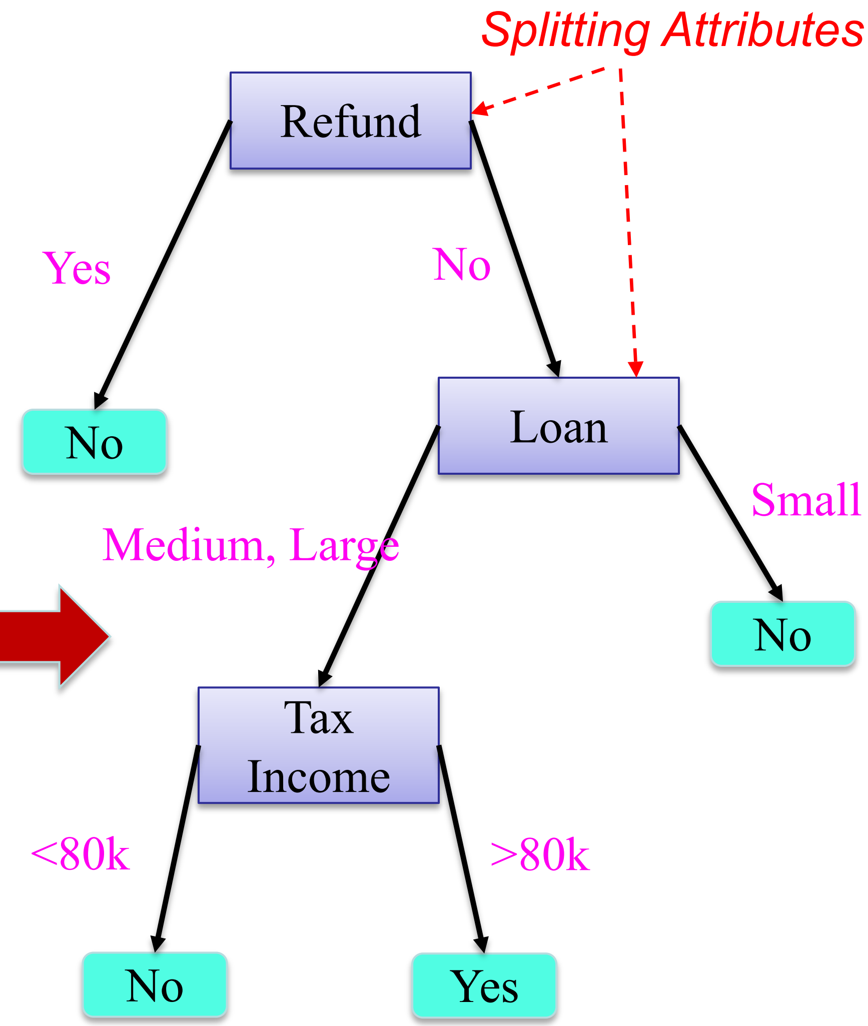
Training data



Model: Decision Tree

Another Decision Tree on same dataset

	categorical	categorical	continuous	class
Tid	**Refund**	**Loan Status**	**Taxable Income**	**Cheat**
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2	No	Small	100k	No
3	No	Medium	70k	No
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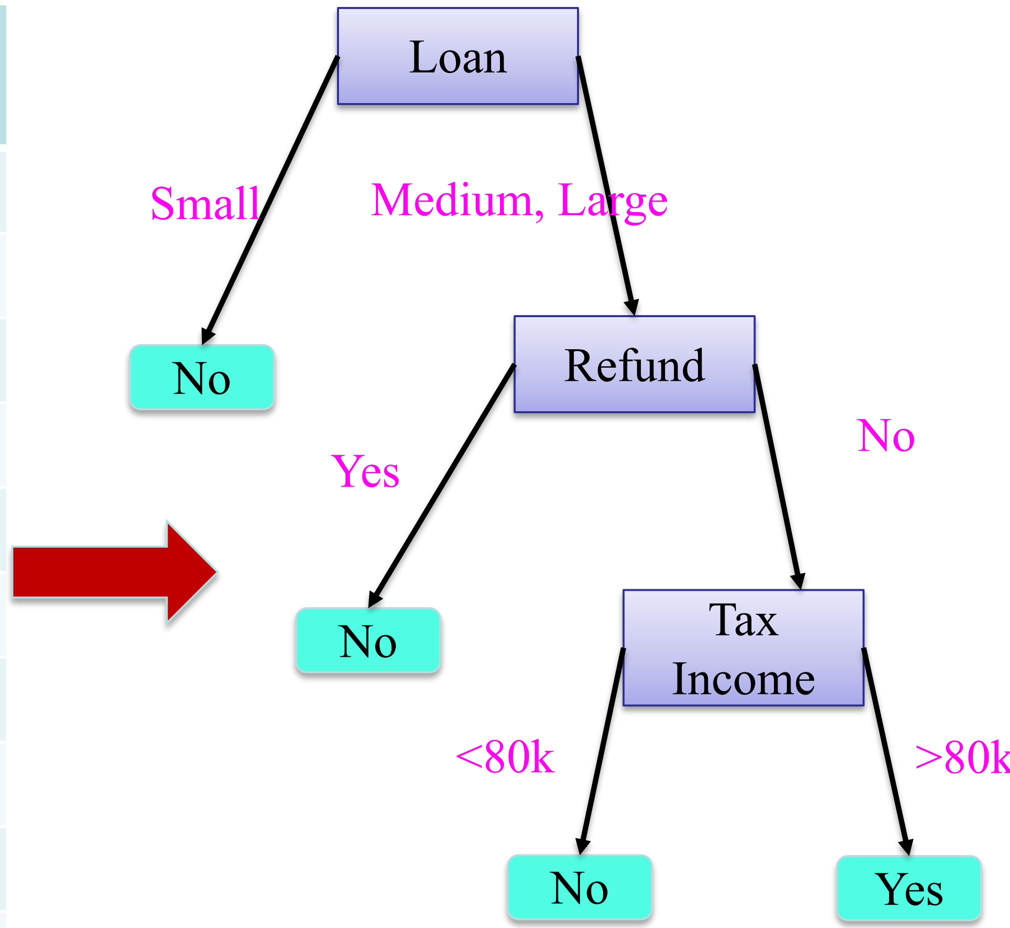
10	No	Medium	90k	Yes



There could be more than one tree that fits the same data!

Decision Tree Induction

- Many Algorithms:
  - Hunt's Algorithm (one of the earliest)
  - CART
  - ID3, C4.5
  - SLIQ,SPRINT

General Structure of Hunt's Algorithm

- Let D_t be the set of training records that reach a node t
- General Procedure:
  - If D_t contains records that all belong the same class y_t, then t is a leaf node labeled as y_t
  - If D_t is an empty set, then t is a leaf node labeled by the default class y_d
  - If D_t contains records that belong to more than one class, *use an attribute test to split the data into smaller subsets*. Recursively apply the procedure to each subset

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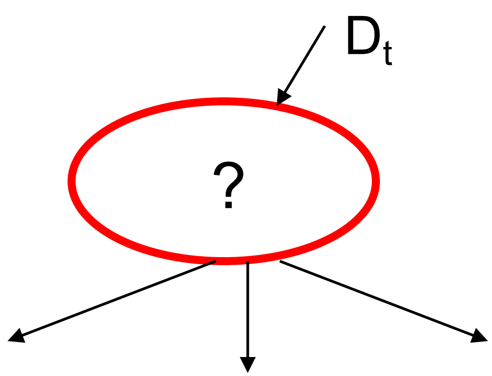

Hunt's Algorithm

Don't
Cheat + Cheat

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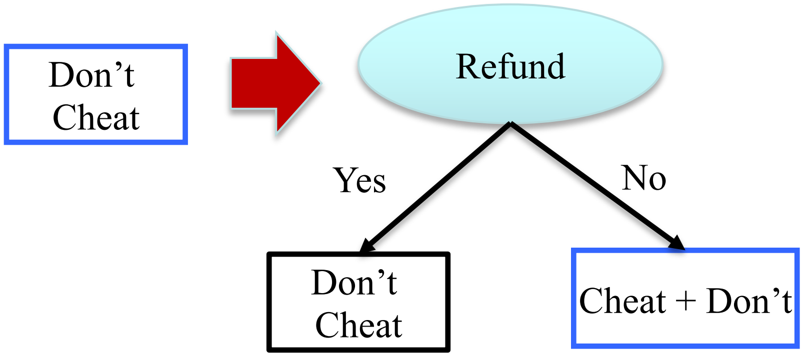
10	No	Medium	90k	Yes

Default class is "Don't cheat" since it is the majority class in the dataset

Hunt's Algorithm



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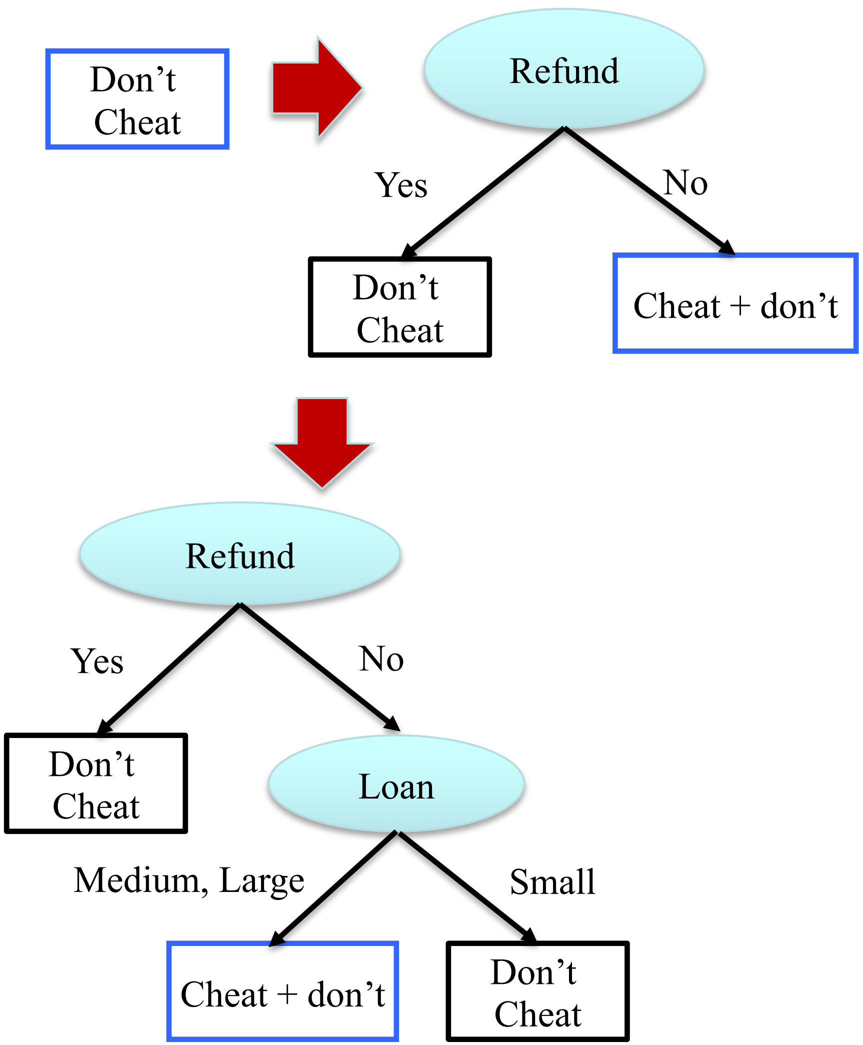
10	No	Medium	90k	Yes

For now, assume that "Refund" has been decided to be the best attribute for splitting in some way (to be discussed soon)

Hunt's Algorithm



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Hunt's Algorithm

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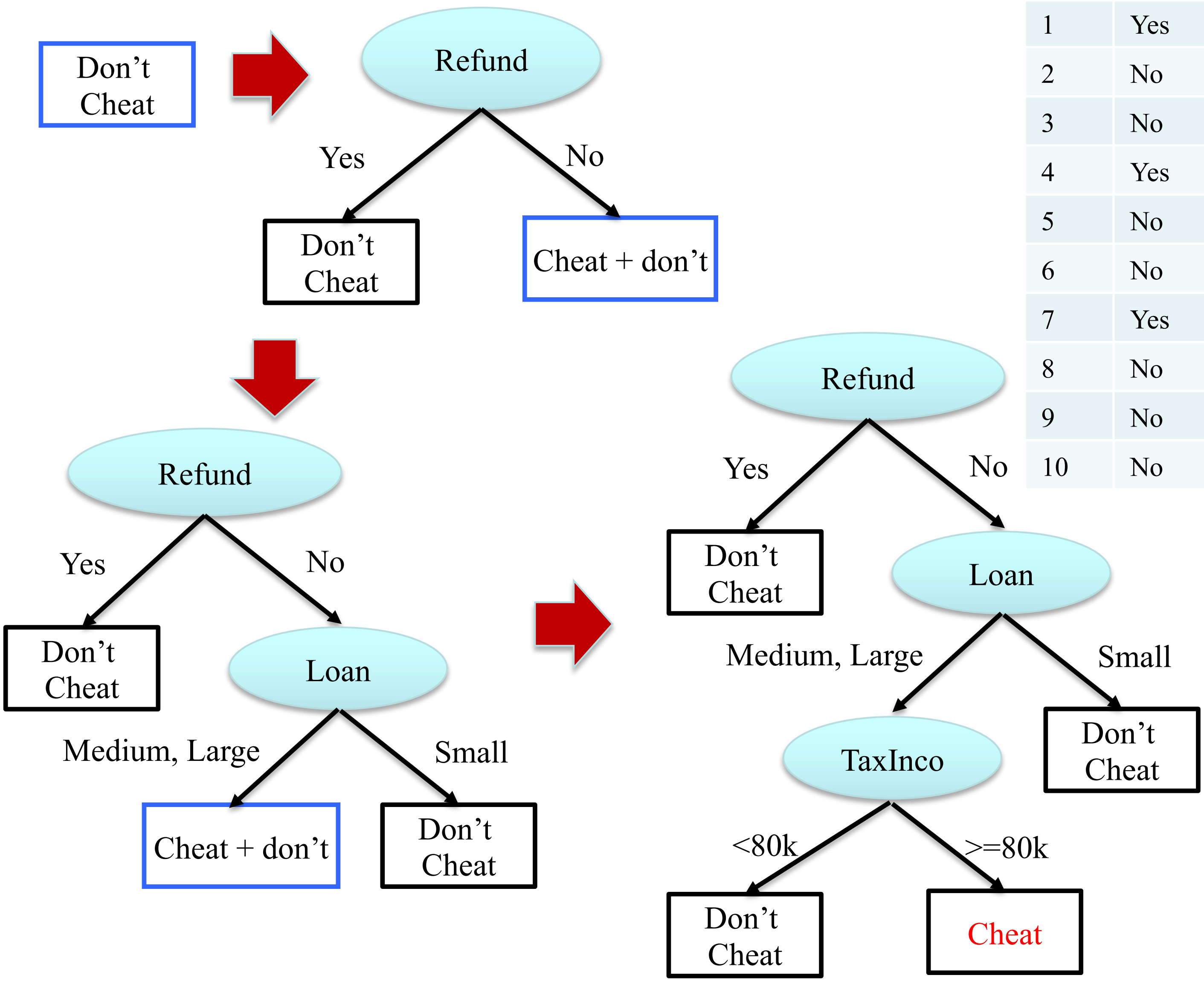


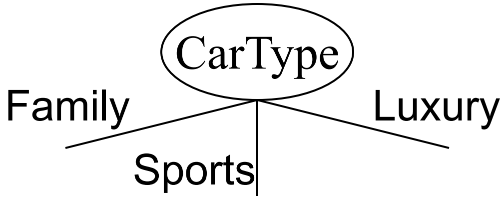
How to Specify Test Condition?

- Depends on attribute types
  - Nominal: two or more distinct values (special case: binary) E.g., Loan status: {small, medium, large}
  - Ordinal: two or more distinct values that have an ordering. E.g. shirt size: {S, M, L, XL}
  - Continuous: continuous range of values

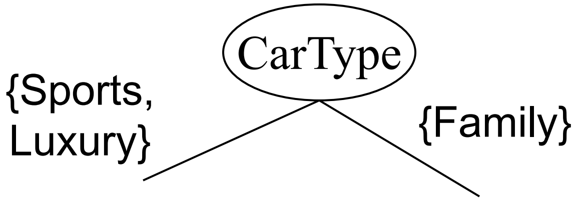
- Depends on number of ways to split
  - 2-way split
  - Multi-way split

Splitting Based on Nominal Attributes

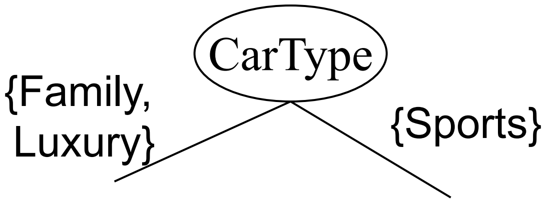
- Multi-way split: Use as many partitions as distinct values.



- Binary split: Divides values into two subsets.
  Need to find optimal partitioning

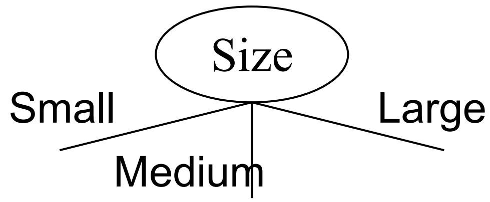


OR

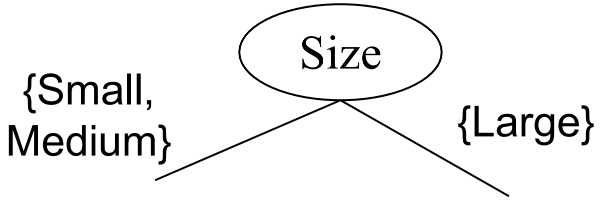


Splitting Based on Ordinal Attributes

- Multi-way split: Use as many partitions as distinct values.



- Binary split: Divides values into two subsets.
  Need to find optimal partitioning.



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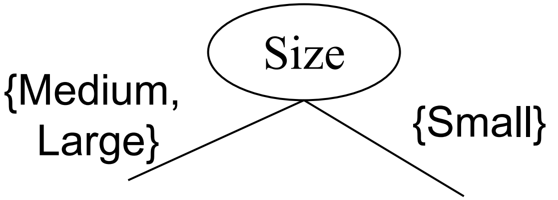
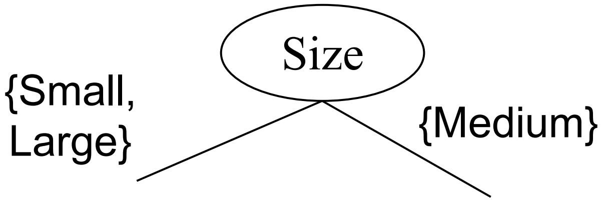


- What about this split?



Tree Induction

- Greedy strategy
  - Split the records based on an attribute test that optimizes certain criterion

- Issues
  - Determine how to split the records
    - How to specify the attribute test condition?
    - How to determine the best split?
  - Determine when to stop splitting

Decision Trees

Finding Best Attribute

15/04/2024

Measures of Node Impurity

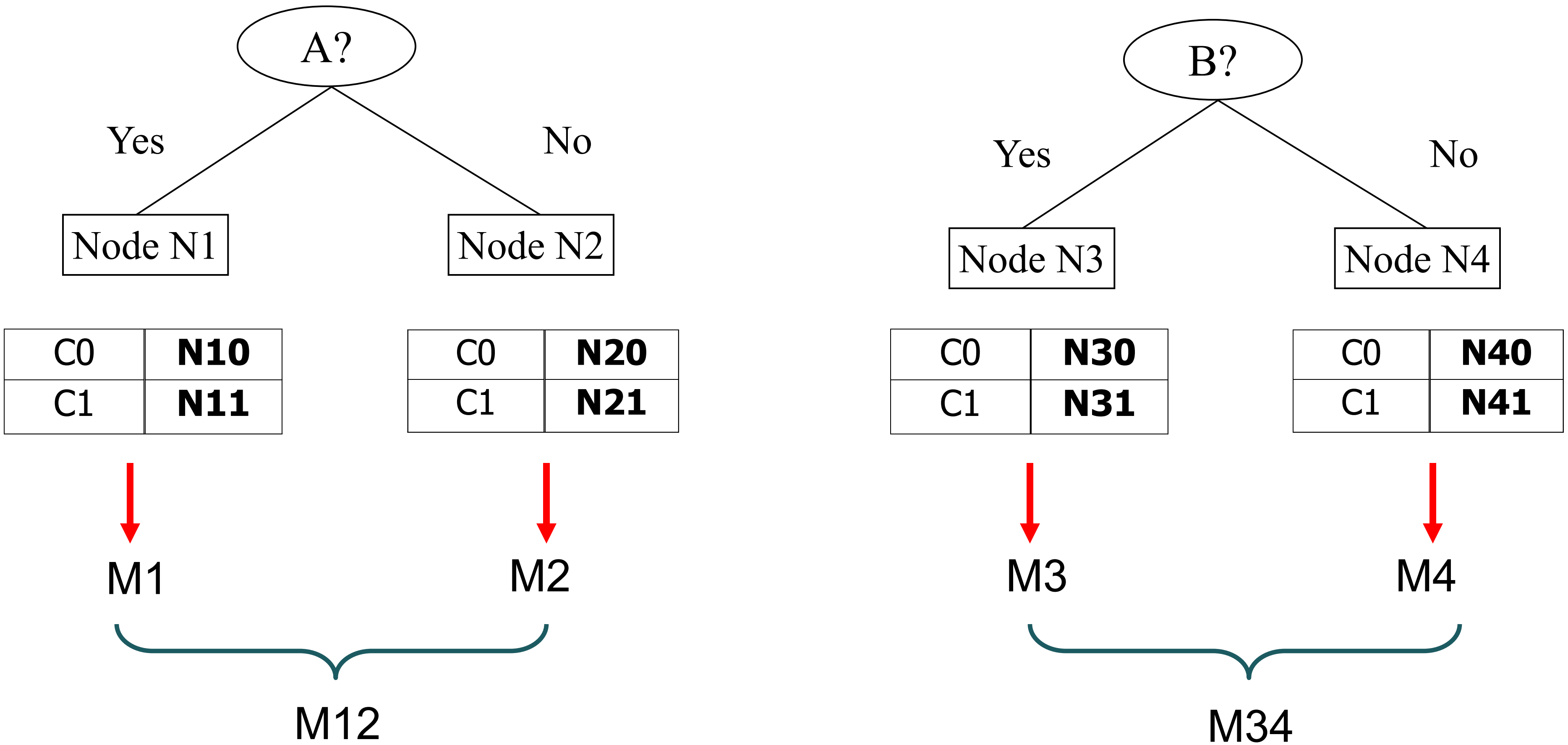
- Gini Index
- Entropy
- Misclassification error

How to Find the Best Split

Before Splitting:

C0	**N00**
C1	**N01**

→ M0



Gain = M0 – M12 vs M0 – M34

Measures of Node Impurity

- Gini Index
- Entropy
- Misclassification error

Alternative Splitting Criteria based on INFO

- Entropy at a given node t:

$$Entropy(t) = -\sum_j p(j \mid t) \log_2 p(j \mid t)$$

  - $p(j \mid t)$ is the relative frequency of class j at node t

- Measures homogeneity of a node

- Entropy of sample S: Average optimal number of bits to encode information about certainty/uncertainty about S

Examples for computing Entropy

$$Entropy(t) = -\sum_j p(j\,|\,t)\log_2 p(j\,|\,t)$$

C1	**0**
C2	**6**

P(C1) = 0/6 = 0   P(C2) = 6/6 = 1

Entropy = – 0 log 0 – 1 log 1 = – 0 – 0 = 0

C1	**1**
C2	**5**

P(C1) = 1/6   P(C2) = 5/6

Entropy = – (1/6) $\log_2$ (1/6) – (5/6) $\log_2$ (1/6) = 0.65

C1	**2**
C2	**4**

P(C1) = 2/6   P(C2) = 4/6

Entropy = – (2/6) $\log_2$ (2/6) – (4/6) $\log_2$ (4/6) = 0.92

Alternative Splitting Criteria based on INFO

- Entropy at a given node t:

$$\boxed{Entropy(t) = -\sum_{j} p(j\,|\,t)\log_2 p(j\,|\,t)}$$

  - *p(j | t)* is the relative frequency of class j at node t

- Measures homogeneity of a node
  - Maximum ($\log n_c$) when records are equally distributed among all classes
    - implying least information
  - Minimum (0.0) when all records belong to one class,
    - implying most information

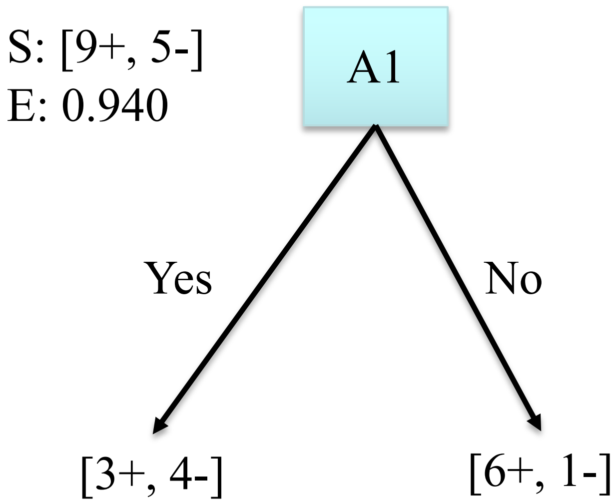
Information Gain

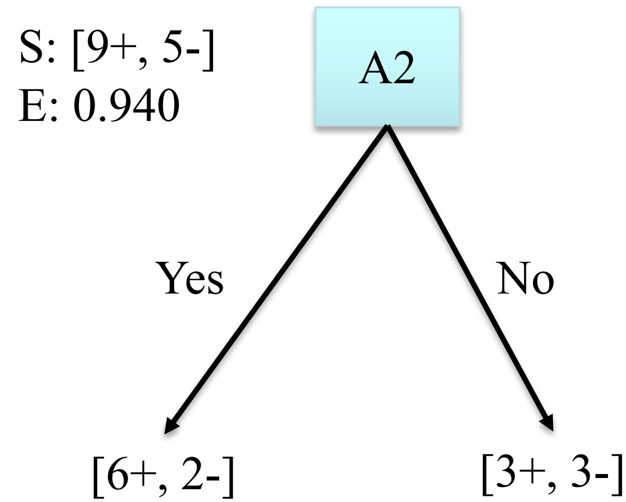
- Measures how well a given attribute separates the training examples according to their target classification

- This measure is used to select among the candidate attributes at each step while growing the tree

- Gain is measure of how much we can reduce uncertainty (Value lies between [0,1])

Information Gain





Entropy(3+,4-) = - (3/7)log(3/7) – (4/7)log(4/7) = 0.985

Entropy(6+,1-) = - (6/7)log(6/7) – (1/7)log(1/7) = 0.592

Gain(S,A1) = 0.940 – (7/14)*0.985 – (7/14)*0.592 = 0.151

Entropy(6+,2-) = - (6/8)log(6/8) – (2/8)log(2/8) = 0.811

Entropy(3+,3-) = - (3/6)log(3/6) – (3/6)log(3/6) = 1.0

Gain(S,A2) = 0.940 – (8/14)*0.811 – (6/14)*1.0 = 0.048

Splitting Based on INFO...

- Information Gain:

$$GAIN_{split} = Entropy(p) - \left(\sum_{i=1}^{k} \frac{n_i}{n} Entropy(i) \right)$$

  - Parent Node p is split into k partitions;
  - n_i is number of records in partition i
- Measures Reduction in Entropy achieved because of the split Choose the split that achieves most reduction (maximizes GAIN)
- Used in ID3 and C4.5

- **Disadvantage:** Tends to prefer splits that result in large number of partitions, each being small but pure

Splitting Based on INFO...

- Gain Ratio:

$$GainRATIO_{split} = \frac{GAIN_{Split}}{SplitINFO}$$

$$SplitINFO = -\sum_{i=1}^{k} \frac{n_i}{n} \log \frac{n_i}{n}$$

  - Parent Node, p is split into k partitions
  - n_i is the number of records in partition i

- Adjusts Information Gain by the entropy of the partitioning (SplitINFO)
  - Higher entropy partitioning (large number of small partitions) is penalized!
- Used in C4.5

- Designed to overcome the disadvantage of Information Gain

Stopping Criteria for Tree Induction

- Stop expanding a node when all the records belong to the same class
- Stop expanding a node when all the records have similar attribute values (if different class values, then usually assign the majority class)
- Early termination, usually to prevent overfitting (to be discussed later)

ID3

ID3(Examples, Target_attribute, Attributes)

- Create a Root node for the tree
- If all examples are positive, Returns single-node tree Root with label +
- If all examples are negative, Returns single-node tree Root with label –
- If Attributes is empty, Returns single-node tree Root, with label = most common value of Target_attribute in Examples

ID3

ID3(Examples, Target_attribute, Attributes)

- Begin
  - A ← Best attribute from Examples
  - The decision attribute for Root ← A
  - For each possible value, v_i, of A,
    - Add a new branch below Root, corresponding to A= v_i
    - Examples_v_i subset of examples with v_i = A
    - If Examples_v_i is empty
      - Add a leaf node with label = most common value of Target_attribute in Examples
    - Else below this new branch add the subtree
      - ID3(Examples_v_i, Target_attribute, Attributes-{A})
- End
- Return Root

Thank You