AIFA
Logical Deduction
Propositional Logic

25/01/2024

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Example wffs

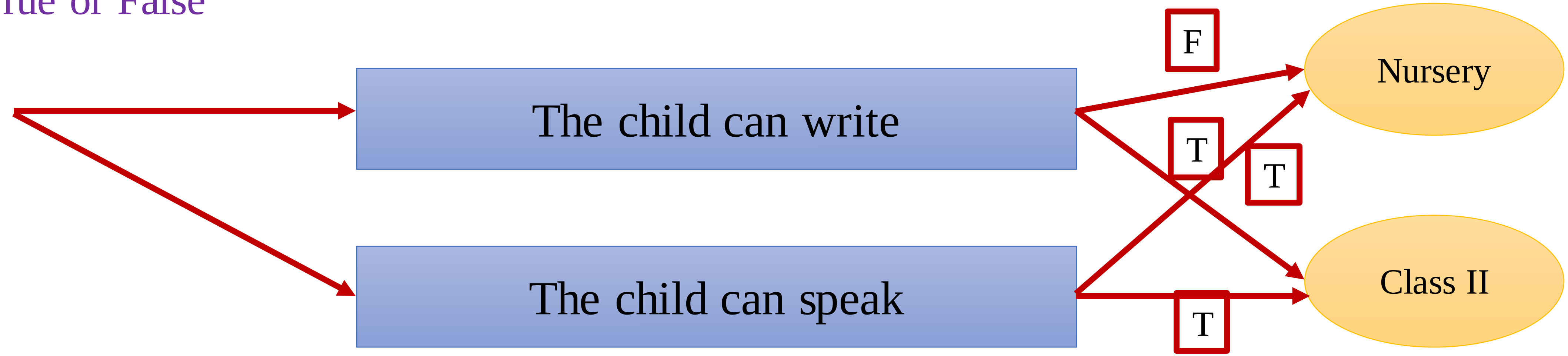
- P
- True
- $P \wedge Q$
- $(P \wedge Q) \rightarrow R$
- $(P \wedge Q) \vee R \rightarrow S$
- $\sim(P \vee Q)$
- $\sim(P \vee Q) \rightarrow R \wedge S$

What does a wff mean --- Semantics?

- Interpretation in a world

- When we interpret a sentence in a world we assign meaning to it and it evaluates to either True or False

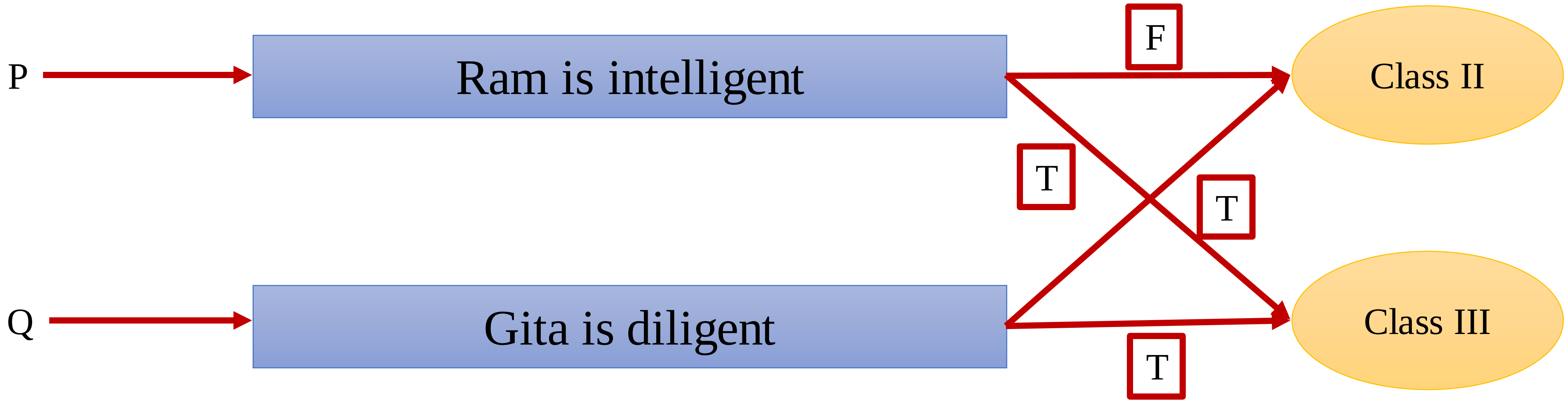
- P



- Same proposition could be interpreted in two different worlds in two different ways

- Interpretation attributes meaning or semantics to propositions

Semantics



- We deal with two symbols P and Q
- Truth values of P and Q depend on the way we interpret it in a particular world

How do we get a meaning?

- Sentences can be compound propositions

- **Steps:**

  - Interpret each atomic proposition in the **same world**

  - Assign Truth values to each interpretation

  - Compute the Truth value of compound proposition

Example

- P: likes(Akash, Aritra)
- Q: knows(Amit, Adway)

- **World:** Akash and Aritra are friends. Amit and Adway are known to each other.

- $P = T, Q = T$

- $P \land Q = T$
- $P \land {\sim}Q = F$

Validity of a sentence

- If a propositional sentence is true under all possible interpretation, it is <u>VALID</u>

- A sentence is <u>VALID</u> means it is True irrespective of the world in which we interpret it

- $P \vee \sim P$ is always True
  - <u>Tautology</u>

Satisfiability

- An interpretation is a mapping to a world

- A sentence is satisfiable by an interpretation **if**
  - Under that interpretation the sentence evaluates to **True**

- If NO interpretation makes a sentence True then
  - That sentence is called UNSATISFIABLE or INCONSISTENT
  - $P \land \sim P$

- If NO interpretation makes all the sentences in the set to be True then
  - The set of sentences is UNSATISFIABLE or INCONSISTENT

Inference in Propositional Logic

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Objective

- Infer the truth value of a proposition
- Reason towards new facts given a set of propositions
- Prove a proposition given a set of propositional facts

Truth Value Assignment

P	Q	P∧Q	P∨Q	~P	~Q	P→Q
T	T	T	T	F	F	T
T	F	F	T	F	T	F
F	T	F	T	T	F	T
F	F	F	T	T	T	T

De Morgan's Theorem

- $\sim(P \wedge Q) = \sim P \vee \sim Q$
- $\sim(P \vee Q) = \sim P \wedge \sim Q$

P	Q	P∨Q
T	T	T
T	F	T
F	T	T
F	F	F

~(P∨Q)
F
F
F
T

~P	~Q
F	F
F	T
T	F
T	T

~P∧~Q
F
F
F
T

Problem 2

- If P and Q are True, then what is the truth value of following statements?
  - S: $(\sim P \vee Q) \rightarrow P$

P	Q	$\sim P \vee Q$	S
T	T	T	T

Deduction using Propositional Logic: Steps

- Choice of Boolean variables a, b, c, d … which can take values True or False

- **Boolean Formulae developed** using well defined connectors ~, Λ, V, →, etc, whose meaning (semantics) is given by their truth tables

- **Codification of Sentences** of the argument into Boolean Formulae

- Developing the **Deduction Process** as obtaining truth of a Combined Formula expressing the complete argument

- Determining the Truth or Validity of the formula and thereby proving or disproving the argument and Analyzing its truth under various Interpretations.

Problem 1

- If I am the Director then I am well-known. I am the Director. So I am well-known.

Choice of Boolean variables a, b, c, d … which can take values True or False

- **Coding: Variables**
- a: I am the Director
- b: I am well-known

Boolean Formulae developed using well defined connectors ~, Λ, V, →, etc, whose meaning (semantics) is given by their truth tables

Codification of Sentences of the argument into Boolean Formulae

- **Coding the sentences**

1. $a \rightarrow b$
2. a
3. b

Developing the **Deduction Process** as obtaining truth of a Combined Formula expressing the complete argument

- **The final formula for deduction**
- $((a \rightarrow b) \land a) \rightarrow b$

Determining the Truth or Validity of the formula and thereby proving or disproving the argument and Analyzing its truth under various Interpretations.

Proof or Otherwise

a	b	a→b	((a→b)∧a)	((a→b)∧a)→b
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Problem 2

- If I am the Director then I am well-known. I am not the Director. So I am not well-known.

Choice of Boolean variables a, b, c, d … which can take values True or False

- **Coding: Variables**
- a: I am the Director
- b: I am well-known

Boolean Formulae developed using well defined connectors ~, Λ, V, →, etc, whose meaning (semantics) is given by their truth tables

- **Coding the sentences**

1. a→b
2. ~a
3. ~b

Codification of Sentences of the argument into Boolean Formulae

Developing the **Deduction Process** as obtaining truth of a Combined Formula expressing the complete argument

- **The final formula for deduction**
- ((a→b)Λ~a)→~b

Determining the Truth or Validity of the formula and thereby proving or disproving the argument and Analyzing its truth under various Interpretations.

Proof or Otherwise

a	b	a→b	((a→b)∧~a	((a→b)∧~a)→~b
T	T	T	F	T
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

Reasoning

- Using the given propositions which are assumed to be True
  - Trying to derive new facts which will also be True

- P: It is the month of July
- Q: It rains
- R: P→Q [If it is month of July then it rains]
- Premise: It is the month of July
- Conclude: It rains

Symbolic Deduction

Modus Ponens: One Inference Rule

- $P \rightarrow Q$
- P

- Q

- $P \rightarrow Q = \sim P \vee Q$
- $P \wedge \sim P \vee Q$
- $(P \wedge \sim P) \vee Q$
- $F \vee Q$
- Q

Allows us to deduce the truth of a consequent depending on the truth of the antecedents

Inference Rule: Importance

- We want to develop some mechanical procedures using which we can make the machine infer new facts
- Inference rules can be mechanically applied

- **Rules:**
  - If Not(Not(P)) then P
  - Chain Rule:
    - If P then Q
    - If Q then R
    - If P then R

Rules of Natural Deduction

- Modus Ponens: $(a \rightarrow b)$, a :- therefore b
- **Modus Tollens: $(a \rightarrow b)$, ~b :- therefore ~a**
- **Hypothetical Syllogism: $(a \rightarrow b)$, $(b \rightarrow c)$:- therefore $(a \rightarrow c)$**
- **Disjunctive Syllogism: (a V b), ~a:- therefore b**
- **Constructive Dilemma: $(a \rightarrow b)$ Λ $(c \rightarrow d)$, (a V c) :- therefore (b V d)**
- **Destructive Dilemma: $(a \rightarrow b)$ Λ $(c \rightarrow d)$, (~b V ~d) :- therefore (~a V ~c)**
- **Simplification: a Λ b:- therefore a**
- **Conjunction: a, b:- therefore a Λ b**
- **Addition: a :- therefore a V b**

Inference Mechanisms

- Formal way of inferencing using propositional logic
- Truth Table Method
  - We can find out the truth of any compound proposition when we know the truth values of the individual propositions
- Deductive method
  - Inference rules which are not dependent on any interpretation
  - The propositions will evaluate to True or False based on some interpretation
  - Modus Ponen is one such inference rule
- Resolution
  - Propositions converted into clausal form
  - Negation of the goal, convert to clausal form
  - Iteratively apply propositions and prove NULL

Automated Reasoning

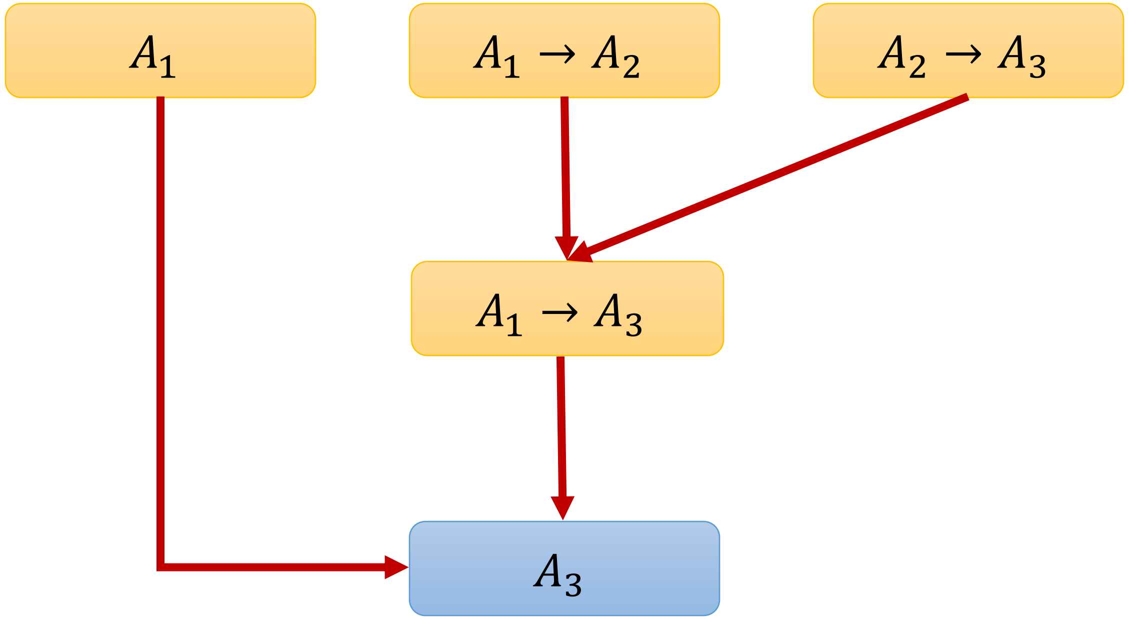
- In general, the inference problem is NP-complete [Cook's Theorem]

- If we restrict ourselves to Horn sentences , then repeated use of Modus Ponens gives us a polytime procedure.
  - Horn sentences are of the form:
    - $F1 \wedge F2 \wedge \ldots \wedge Fn \rightarrow G$

  - Forward chaining
  - Backward chaining

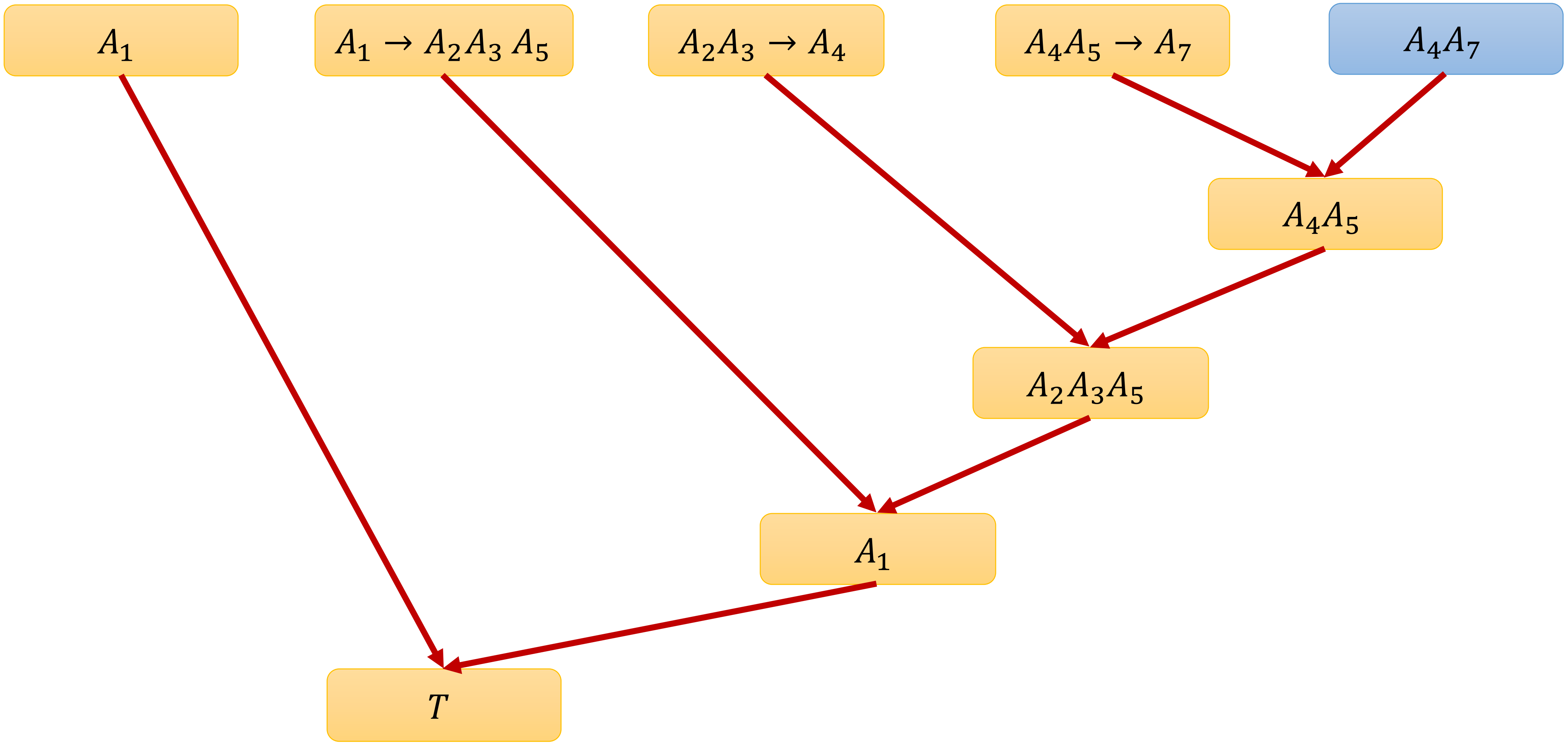
Automated Reasoning

- Forward Chaining



Automated Reasoning

- Backward chaining



Thank You