Name: _____

Answer the questions in the spaces provided on the following pages. If you run out of room for an answer, continue on the back of the page. Show **all** your work!

1. Prove the following using proof by induction

$$\text{For all positive integers, } n,\ 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

2. Prove the following using proof by induction

$$\text{For all integers } n \geq 1,\ 1 + 3 + 5 + \cdots + (2n - 1) = n^2$$

3. Prove the following using proof by induction

$$\text{For all integers } n \geq 1,\ 2 + 4 + 6 + \cdots + 2n = n^2 + n$$

4. *(Challenge)* Prove the following using proof by induction

$$\text{For all integers } n \geq 1,\ \sum_{i=1}^{n} i\,(i!) = (n+1)! - 1$$

5. *(**Mega** Challenge)* Prove the following using proof by induction

For all integers $n \geq 1$, 3 is a factor of $4^n - 1$