

Name: _____

Answer the questions in the spaces provided on the following pages. If you run out of room for an answer, continue on the back of the page. Show **all** your work!

1. Prove the following identity: $\frac{\sec^2 x}{\sec^2 x - 1} = \csc^2 x$

Solution:

$$\begin{aligned}\csc^2 x &= \frac{\sec^2 x}{\sec^2 x - 1} \\ &= \frac{\sec^2 x}{\tan^2 x} \\ &= \sec^2 x \cot^2 x \\ &= \frac{1}{\cos^2 x} \frac{\cos^2 x}{\sin^2 x} \\ &= \frac{1}{\sin^2 x} \\ &= \csc^2 x\end{aligned}$$

2. Prove the following identity: $\csc^2 x \tan^2 x - 1 = \tan^2 x$

Solution:

$$\begin{aligned}\tan^2 x &= \csc^2 x \tan^2 x - 1 \\ &= \frac{1}{\sin^2 x} \frac{\sin^2 x}{\cos^2 x} - 1 \\ &= \frac{1}{\cos^2 x} - 1 \\ &= \sec^2 x - 1 \\ &= \tan^2 x\end{aligned}$$

3. Prove the following identity: $\tan^2 x \sin^2 x = \tan^2 x - \sin^2 x$

Solution:

$$\begin{aligned}
 \tan^2 x \sin^2 x &= \tan^2 x - \sin^2 x \\
 &= \tan^2 x - \sin^2 x \left(\frac{\cos^2 x}{\cos^2 x} \right) \\
 &= \tan^2 x - \frac{(\sin^2 x)(\cos^2 x)}{\cos^2 x} \\
 &= \tan^2 x - \frac{\sin^2 x}{\cos^2 x} (\cos^2 x) \\
 &= \tan^2 x - \tan^2 x \cos^2 x \\
 &= \tan^2 x (1 - \cos^2 x) \\
 &= \tan^2 x \sin^2 x
 \end{aligned}$$

4. Prove the following identity: $\frac{\cos x + 1}{\sin^3 x} = \frac{\csc x}{1 - \cos x}$

Solution:

$$\begin{aligned}
 \frac{\cos x + 1}{\sin^3 x} &= \frac{\csc x}{1 - \cos x} \\
 &= \frac{1}{\sin x} \frac{\cos x + 1}{\sin^2 x} \\
 &= (\csc x) \frac{\cos x + 1}{1 - \cos^2 x} \\
 &= (\csc x) \frac{\cos x + 1}{(1 - \cos x)(1 + \cos x)} \\
 &= (\csc x) \frac{1}{1 - \cos x} \\
 &= \frac{\csc x}{1 - \cos x}
 \end{aligned}$$

5. **Bonus extra hard question:** Prove the following identity: $\frac{1 + \tan^2 x}{\sec x} = \sqrt{\frac{1}{\frac{1}{2}(1 - \cos(2x))}}$

Solution:

$$\frac{1 + \tan^2 x}{\sec x} = \sqrt{\frac{1}{\frac{1}{2}(1 - \cos(2x))}}$$

$$\frac{1 + \tan^2 x}{\sec x} = \sqrt{\frac{1}{2 - 1 - \cos(2x)}}$$

$$\frac{1 + \tan^2 x}{\sec x} = \sqrt{\frac{1}{\frac{1}{2}(2 - 1 - \cos(2x))}}$$

$$\frac{1 + \tan^2 x}{\sec x} = \sqrt{\frac{1}{1 - \frac{1 - \cos(2x)}{2}}}$$

$$\frac{1 + \tan^2 x}{\sec x} = \sqrt{\frac{1}{1 - \sin^2 x}}$$

$$\frac{1 + \tan^2 x}{\sec x} = \sqrt{\frac{1}{\cos^2 x}}$$

$$\frac{1 + \tan^2 x}{\sec x} = \sqrt{\sec^2 x}$$

$$\frac{\sec^2 x}{\sec x} = \sec x$$

$$\sec x = \sec x$$