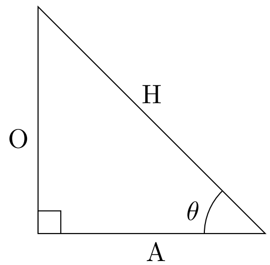
TRIG FUNCTIONS REVIEW

Review

Recall that the trigonometric functions are derived from the ratio of angles in a right triangle.



Given the right triangle to the left, we can define the basic trigonometric functions as follows (fill in the blanks):

$$\sin(\theta) = \frac{O}{H} \qquad \csc(\theta) = \frac{1}{\sin(\theta)} = \frac{H}{O}$$

$$\cos(\theta) = \frac{A}{H} \qquad \sec(\theta) = \frac{1}{\cos(\theta)} = \frac{H}{A}$$

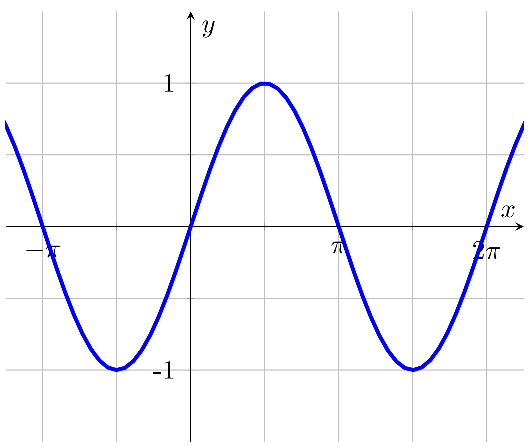
$$\tan(\theta) = \frac{O}{A} \qquad \cot(\theta) = \frac{1}{\tan(\theta)} = \frac{A}{O}$$

Note that O stands for opposite, A for adjacent, and H for hypotenuse

Graphing

Complete the graphs below:

1. $f(x) = \sin(x)$



(a) Period:

Solution: 2π

(b) Amplitude:

Solution: 1

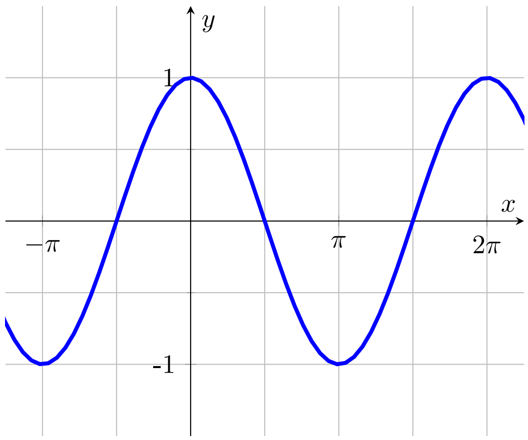
(c) Domain:

Solution: $(-\infty, \infty)$ OR $\mathbb{R}$

(d) Range:

Solution: $[-1, 1]$

2. $f(x) = \cos(x)$



(a) Period:

Solution: 2π

(b) Amplitude:

Solution: 1

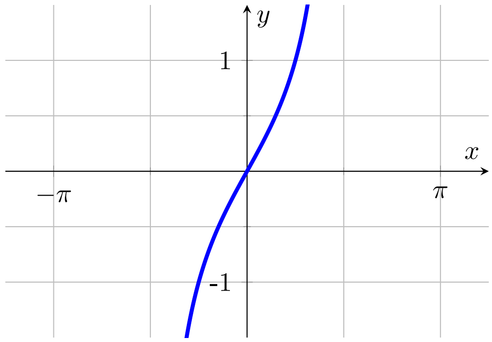
(c) Domain:

Solution: $(-\infty, \infty)$ OR $\mathbb{R}$

(d) Range:

Solution: $[-1, 1]$

3. $f(x) = \tan(x)$ (Graph a single period)



(a) Period:

Solution: π

(b) Domain:

Solution: $(-\frac{n\pi}{2}, \frac{n\pi}{2})\, \forall n \in \mathbb{Z}$

(c) Range:

Solution: $(-\infty, \infty)$ OR $\mathbb{R}$

(d) Vertical Asymptote:

Solution: $x = \frac{n\pi}{2}\, \forall n \in \mathbb{Z}$

**Note that $\mathbb{Z}$ is the set of all integers*

TRIG IDENTITIES

Intro

A large part of what makes trig functions so important is their ability to be transformed from one function to another so easily. Knowing certain trig identities will make your mathematical life/career much easier in the near future. As well as having basic important identities memorized, being able to derive the truthfulness of an identity is also an important skill.

Important Identities

These are the identities that you will see most often while solving problems, so it's important to recognize them and have them memorized:

Pythagorean Identities

$$\sin^2(x) + \cos^2(x) = 1 \qquad 1 + \cot^2(x) = \csc^2(x) \qquad \tan^2(x) + 1 = \sec^2(x)$$

Double Angle Identities*

$$\cos(2x) = \cos^2(x) - \sin^2(x) \qquad \sin(2x) = 2\sin(x)\cos(x)$$

**Note that there are actually several different double angle identities for cosine, however, I recommend memorizing this form*

Half Angle Identities

$$\sin^2(x) = \frac{1-\cos(2x)}{2} \qquad \cos^2(x) = \frac{1+cos(2x)}{2}$$

Proving Identities

When verifying trig identities its important to remember one key rule: **you cannot treat them as equations!!!** What this means is that you can't move terms from one side to another in order to reach a true statment, instead you must manipulate each side individually and independently until you can reach an equality by showing that both the left hand side and the right hand side of the equal sign are equivalent to the same quantity.

Now, for some practice use known trig identities and definitions to prove the following:

1. $\dfrac{\sec x}{\cos x} - \dfrac{\tan x}{\cot x} = 1$

Solution:

$$\begin{aligned}
\frac{\sec x}{\cos x} - \frac{\frac{\sin x}{\cos x}}{\frac{\cos x}{\sin x}} &= 1 \\
\frac{1}{\cos^2} - \left(\frac{\sin x}{\cos x}\frac{\sin x}{\cos x}\right) &= 1 \\
\frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} &= 1 \\
\frac{1 - \sin^2 x}{\cos^2 x} &= 1 \\
\frac{\cos^2 x}{\cos^2 x} &= 1 \\
1 &= 1
\end{aligned}$$

2. $\sec(x) - \tan(x)\sin(x) = \dfrac{1}{\sec(x)}$

Solution:

$$\begin{aligned}
\sec x - \tan x \sin x &= \frac{1}{\sec x} \\
\frac{1}{\cos x} - \frac{\sin x}{\cos x}\sin x &= \frac{1}{\sec x} \\
\frac{1}{\cos x}\left(1 - \sin^2 x\right) &= \frac{1}{\sec x} \\
\frac{1}{\cos x}\left(\cos^2 x\right) &= \frac{1}{\sec x} \\
\cos x &= \frac{1}{\sec x} \\
\frac{1}{\sec x} &= \frac{1}{\sec x}
\end{aligned}$$

3. $\dfrac{\sec x \sin x}{\tan x + \cot x} = \sin^2 x$

Solution:

$$\begin{aligned}
\frac{\sec x \sin x}{\tan x + \cot x} &= \sin^2 x \\
\frac{1}{\cos x}\frac{\sin x}{\tan x + \cot x} &= \sin^2 x \\
\frac{\sin x}{\cos x\left(\tan x + \cot x\right)} &= \sin^2 x \\
\frac{\sin x}{\cos x\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)} &= \sin^2 x \\
\frac{\sin x}{\sin x + \frac{\cos^2 x}{\sin x}} &= \sin^2 x \\
\frac{\sin x}{\frac{1}{\sin x}\left(\sin^2 x + \cos^2 x\right)} &= \sin^2 x \\
\sin x\left(\frac{\sin x}{\sin^2 x + \cos^2 x}\right) &= \sin^2 x \\
\sin x\left(\frac{\sin x}{1}\right) &= \sin^2 x \\
\sin^2 x &= \sin^2 x
\end{aligned}$$