# INTRODUCTION TO CLASSICAL CHAOS

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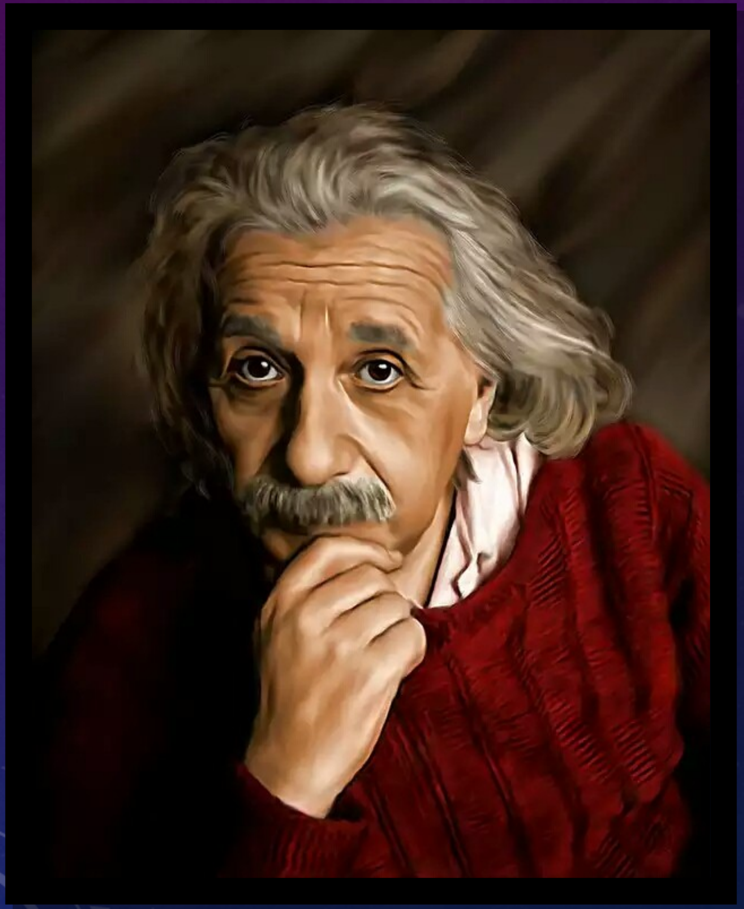
# A PERSPECTIVE ON RANDOMNESS



*" GOD DOESN'T PLAY DICE WITH THE UNIVERSE "*

-Albert Einstein

# DETERMINISM AND RANDOMNESS

DETERMINISTIC SYSTEM

STOCHASTIC SYSTEM

# WHAT IS A DETERMINISTIC SYSTEM?

**A deterministic system is a system in which no randomness is involved in the development of future states of the system. A deterministic model will thus always produce the same output from a given starting condition or initial state.**

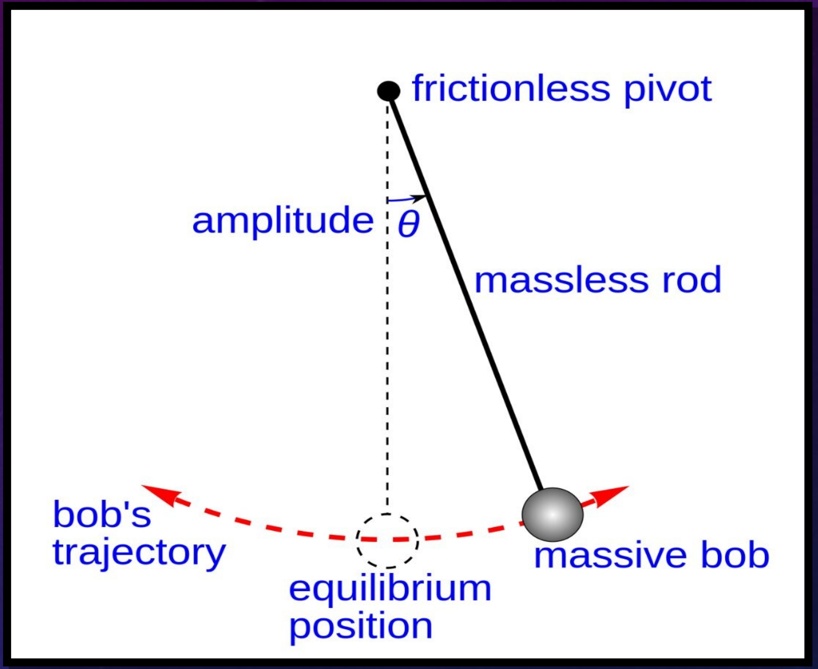
# WHAT IS A STOCHASTIC SYSTEM?

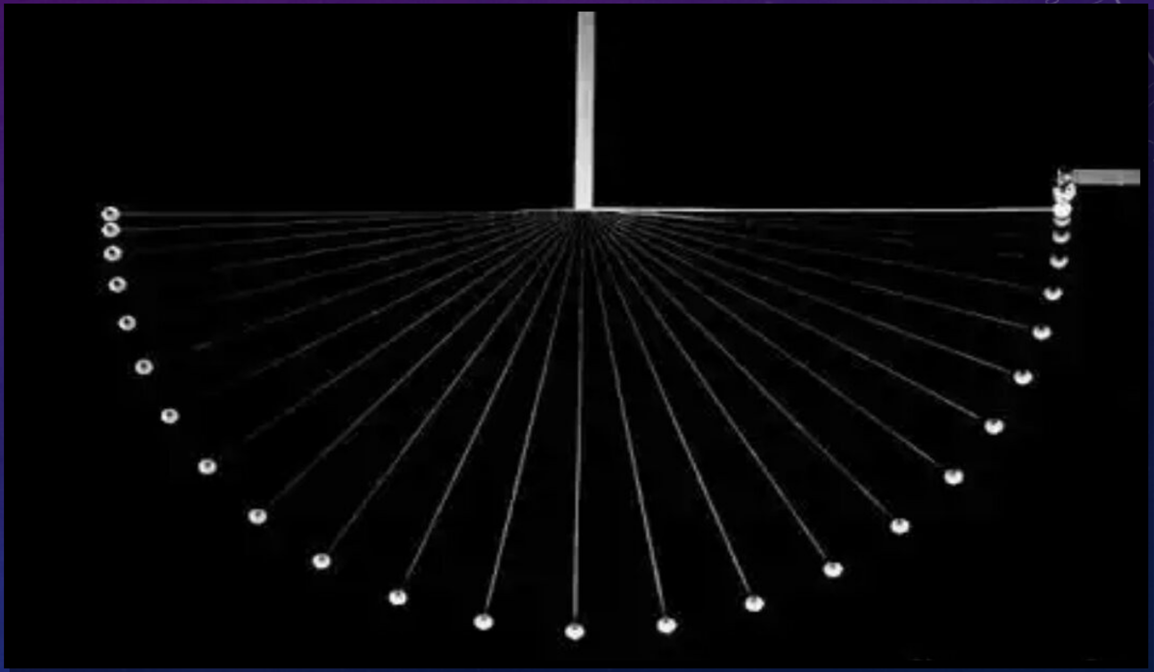
**A Purely stochastic system is a system whose state is randomly determined and cannot be precisely predicted.  Only the probability that the system might end up in a specific state can be predicted.**

# A CHAOTIC SYSTEM

A Chaotic system is essentially a deterministic system whose evolution is predictable in theory if we know all the variables associated with the system at a given instant, but is actually impossible to predict. This is usually due to the large number of variables associated with the system and the complex relations between them. Due to our inability to predict the evolution of a chaotic system,

# A DETERMINISTIC SYSTEM: THE SIMPLE PENDULUM

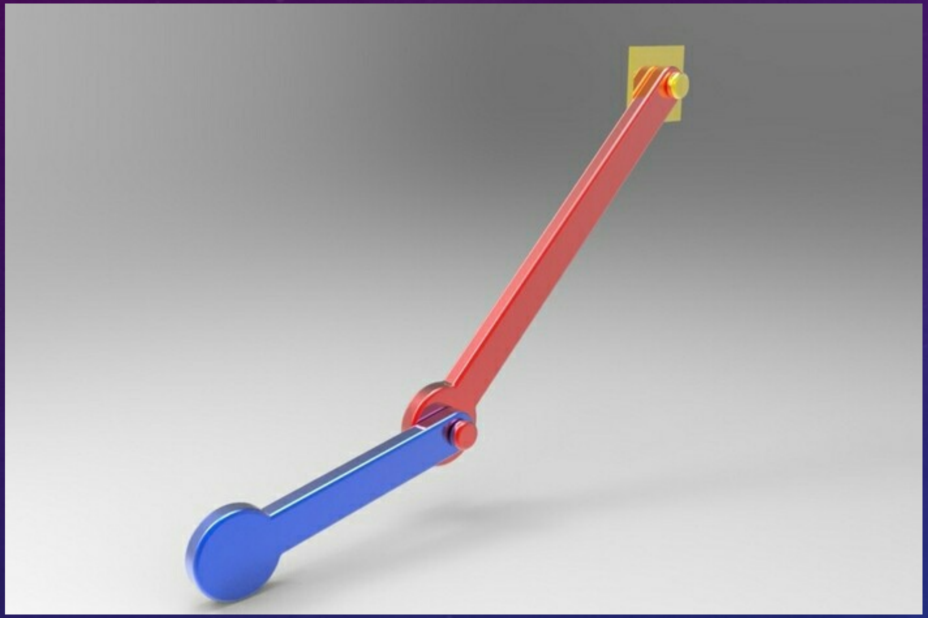


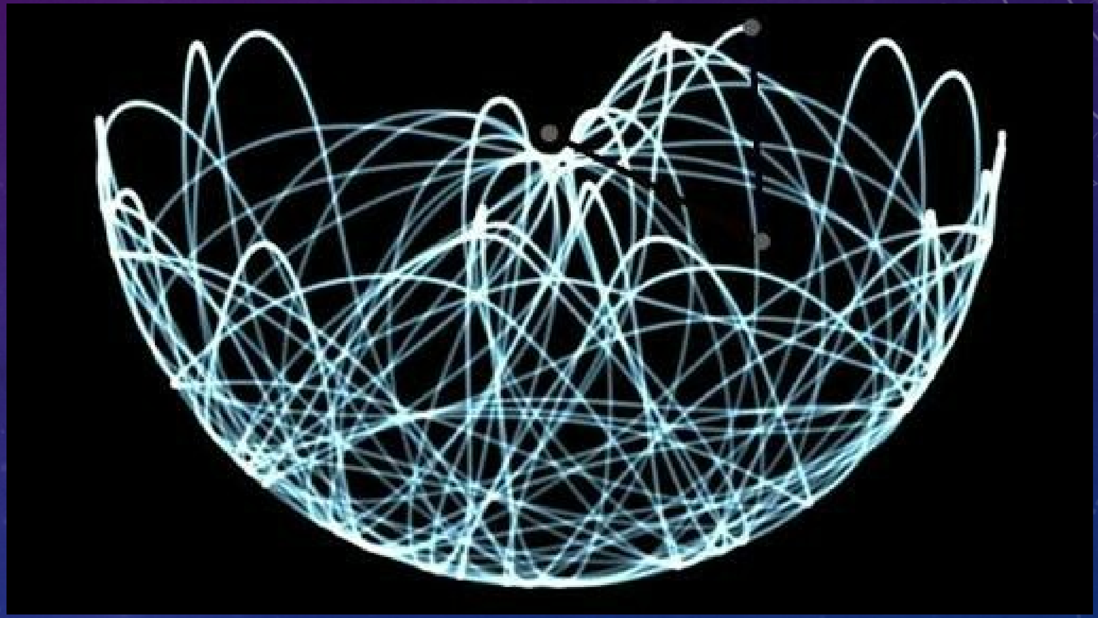



**We can obtain a simple equation to predict the evolution (position) of the simple pendulum. The equation is a function of time and The angular displacement from a fixed position, given by:**

$$y = A\sin\sqrt{\frac{k}{m}}\,t$$

# A SIMPLE CHAOTIC SYSTEM: THE DOUBLE PENDULUM





A double pendulum Is a simple pendulum with another simple pendulum attached to  it’s end. The double pendulum is highly sensitive to initial conditions and shows chaotic behavior. A plot of the double pendulum is shown in the second image.

# SENSITIVITY TO INITIAL CONDITIONS

**Chaotic systems are very sensitive to initial conditions, even a small change in the intial conditions (the state of the system when the prediction about it's future is made) results in a significantly different evolutionary path.**

**Illustration of sensitivity is an abstract mathematical example:**

**Let F(x) be a function of x and x€N (the set of natural numbers)**
**And F(x)= pi^x**
**Let's choose two approximate values for pi:**
**Pi=3.1416 and Pi= 3.1415**
**As an example let is consider F(10) , now the function yields two different values for each value of Pi:**
**I.e.F(10)= 93650 and F(10)= 93620 respectively.**

**One can expect something similar in systems which start with variables which differ slightly.**

# SOME IMPORTANT TERMS

1. Perturbations
2. Phase space
3. Attractors (And Strange Attractor)
4. Limit Cycles

## PERTURBATIONS

Perturbations are interactions of the system which bring about deviations in the evolution of a system from an idealized path which are quantifiable.

## PHASE SPACE

The Phase space of a dynamical system is a space in which all possible states of the system are represented.

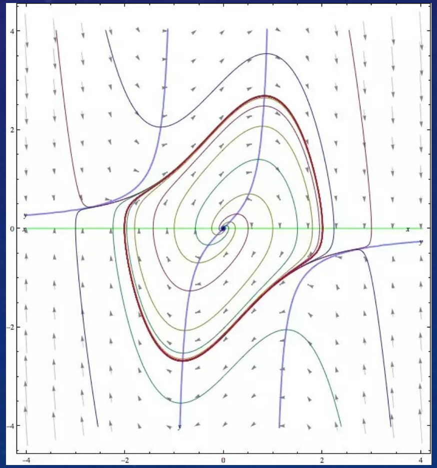
# ATTRACTORS

If a mechanical system evolves towards a particular point / a collection of points / a region in n- dimensional space with the passage of time, then this point/ set of points /…. Constitutes an Attractor.

Visual representation of a strange attractor.

# LIMIT CYCLE

A limit cycle is a  closed trajectory for the time evolution of a system. Atleast one other trajectory of the system spirals into a limit cycle as time tends to limits (+/- ve infinity)



The Limit Cycle of a simple pendulum (Red curve)

# THE LYAPUNOV EXPONENT

**The Lyapunov exponent "l" is a quantitative measure of the exponential divergence of a chaotic system from a non chaotic trajectory.**

**If the intial conditions of the system vary by a small amount 's' at time t=0, then the separation of the two different paths of the system after a time 't' is given by:**

**S(t)~S.e^lt**

**The chaos becomes appreciable for a time t >> 1/l. As time progresses, the separation becomes comparable to the dimensions of the phase space- at which point the future path of the system is essentially a random one.**

# CHAOS!

## Towards determinism:

If the lyapunov exponent is negative, then s(t) measures the rate at which a chaotic system approaches an idealized deterministic system.

S(t) ~ S.e^[-l]t

## Towards Randomness:

If the lyapunov exponent is  positive, then s(t) measures the rate at which a deterministic system moves towards a chaotic trajectory.

S(t) ~ S.e^lt

# THANKS!

**You can download this presentation as a PDF at:**

**bit.ly/ycmchaos**

# RESOURCES USED:

**Video: “the butterfly effect” :**
**https://youtu.be/WepOorvo2I4**

**“The chaos book” by chaos book.org :**
**http://goo.gl/Hm7Nbf**

**Online reference manual:**
**http://goo.gl/YAtBz1**