MATHEMATICS

ENGINEERING

APPLICATION OF DERIVATIVE

BANSAL CLASSES
PRIVATE LIMITED
Ideal for Scholars

APPLICATION OF DERIVATIVE

CONTENT

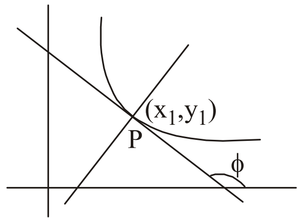
APPLICATION OF DERIVATIVE

TANGENT & NORMAL

Define : $\tan\phi = \left.\dfrac{dy}{dx}\right|_P$

(1) Equation of a tangent at P (x_1, y_1)

$$y - y_1 = \left.\frac{dy}{dx}\right|_{x_1, y_1}(x - x_1)$$



(2) Equation of normal at (x_1, y_1)

$$y - y_1 = -\frac{1}{\left(\dfrac{dy}{dx}\right)_{x_1, y_1}}(x - x_1)\ ,\ \text{if}\ \left.\frac{dy}{dx}\right]_{x_1, y_1}\ \text{exists.}$$

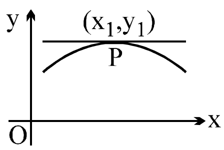
However in some cases $\dfrac{dy}{dx}$ fails to exist but still a tangent can be drawn e.g. case of vertical tangent.

Note that the point (x_1, y_1) must lie on the equation of the curve, the tangent and normal.

Important notes to remember:

(a) If $\left.\dfrac{dy}{dx}\right|_{x_1, y_1} = 0 \Rightarrow$ tangent is parallel to x-axis and converse.

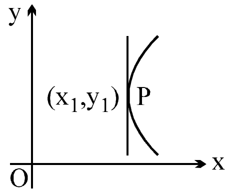
If tangent is parallel to $ax + by + c = 0 \Rightarrow \dfrac{dy}{dx} = -\dfrac{a}{b}$



(b) If $\left.\dfrac{dy}{dx}\right|_{x_1, y_1} \to \infty$ or $\left.\dfrac{dx}{dy}\right|_{x_1, y_1} = 0 \Rightarrow$ tangent is perpendicular to x-axis.

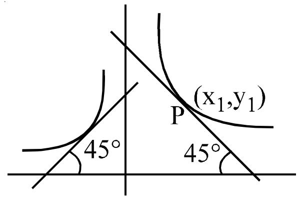
If tangent with a finite slope is perpendicular to $ax + by + c = 0$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{x_1, y_1} \cdot \left(-\frac{a}{b}\right) = -1.$$

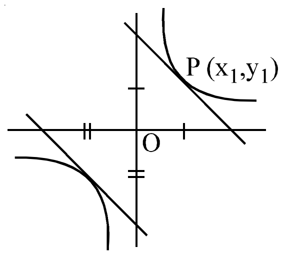


(3) If the tangent at P (x_1, y_1) on the curve is equally inclined to the coordinate axes

$$\Rightarrow \quad \left.\frac{dy}{dx}\right|_{x_1,y_1} = \pm 1.$$



(4) If the tangent makes equal non zero intercept on the coordinate axes then $\left.\frac{dy}{dx}\right|_{x_1,y_1} = -1$



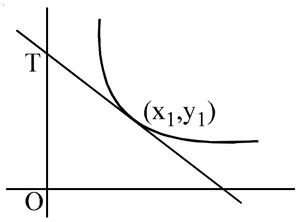
(5) If tangent cuts off from the coordinate axes equal distance from the origin $\Rightarrow \frac{dy}{dx} = \pm 1.$

(6) OT is called the initial ordinate of the tangent

$$Y - y = \frac{dy}{dx}(X - x)$$

put $X = 0$ to get

$\therefore \quad Y = OT = y - x\frac{dy}{dx}$ (It is the y intercept of a tangent at P)



(7) **Concept:** $F(x)=f(x)\cdot g(x)$ are such that $f(x)$ is continuous at $x = a$ and $g(x)$ is differentiable at $x = a$ with $g(a)=0$ then the product function $f(x)\cdot g(x)$ is differentiable at $x = a$.

SOME COMMON PARAMETRIC COORDINATES ON A CURVE:

(a) for $x^{2/3} + y^{2/3} = a^{2/3}$ take parametric coordinate $x = a\cos^3\theta$ & $y = a\sin^3\theta$.

(b) for $\sqrt{x} + \sqrt{y} = \sqrt{a}$ take $x = a\cos^4\theta$ & $y = a\sin^4\theta$.

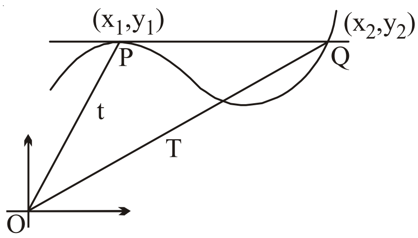
(c) $\frac{x^n}{a^n} + \frac{y^n}{b^n} = 1$ taken $x = a(\sin\theta)^{2/n}$ & $y = b(\sin\theta)^{2/n}$.

(d) for $c^2(x^2 + y^2) = x^2y^2$ take $x = c\sec\theta$ and $y = c\,\mathrm{cosec}\,\theta$.

(e) for $y^2 = x^3$, take $x = t^2$ and $y = t^3$.

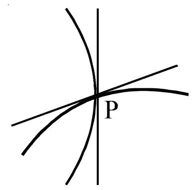
Note: The tangent at P meeting the curve again at Q.

$$\Rightarrow \left.\frac{dy}{dx}\right|_P = \frac{y_2 - y_1}{x_2 - x_1}$$

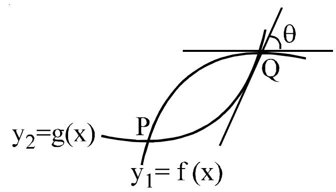


ANGLE OF INTERSECTION OF TWO CURVES :

Definition :



The angle of intersection of two curves at a point P is defined as the angle between the two tangents to the curve at their point of intersection.



If the curves are orthogonal then

$$\left(\frac{dy_1}{dx}\right)\left(\frac{dy_2}{dx}\right) = -1 \text{ everywhere whereever they intersect.}$$

$$\text{If } \left(\frac{dy_1}{dx}\right)_P \left(\frac{dy_2}{dx}\right)_P = -1 \text{ but } \left(\frac{dy_1}{dx}\right)_Q \left(\frac{dy_2}{dx}\right)_Q \neq -1$$

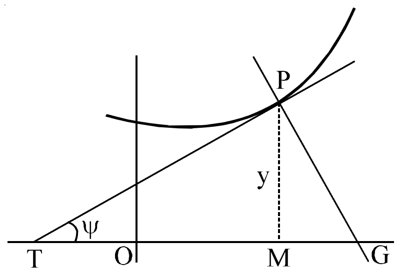
then the two curves are orthogonal at P but not at Q hence they are not orthogonal.

e.g. $y^2 = 4ax$ & $y = e^{-x/2a}$; $xy = a^2$ & $x^2 - y^2 = b^2$ and $y = ax$ & $x^2 + y^2 = c^2$ are orthogonal but $y^2 = 4ax$ and $x^2 = 4by$ are not orthogonal.

Note : If the curves touch at $P(x_1, y_1)$ then $\theta = 0$ hence $f'(x_1) = g'(x_1)$

LENGTH OF TANGENT, NORMAL, SUBTANGENT AND SUBNORMAL:

(i) **Tangent :**



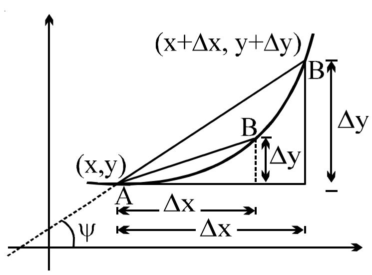
$$PT = MP \operatorname{cosec} \psi = y\sqrt{1+\cot^2\psi} = \left| \frac{y\sqrt{1+\left(\frac{dy}{dx}\right)^2}}{\frac{dy}{dx}} \right|$$

(ii) **Subtangent :** $TM = MP \cot \psi = \left| \frac{y}{(dy/dx)} \right|$

(iii) **Normal :** $GP = MP \sec \psi = y\sqrt{1+\tan^2\psi} = \left| y\sqrt{1+\left(\frac{dy}{dx}\right)^2} \right|$

(iv) **Subnormal :** $MG = MP \tan y = \left| y\left(\frac{dy}{dx}\right) \right|$

APPROXIMATION AND DIFFERENTIALS :



For the figure it is clear that if Δx and Δy are sufficiently small quantities then

$$\frac{\Delta y}{\Delta x} = \tan \psi \cong \frac{dy}{dx} = f'(x)$$

Hence approximate change in the value of y, called its differential is given by

$$\Delta y = f'(x) \cdot \Delta x \qquad ....(1)$$