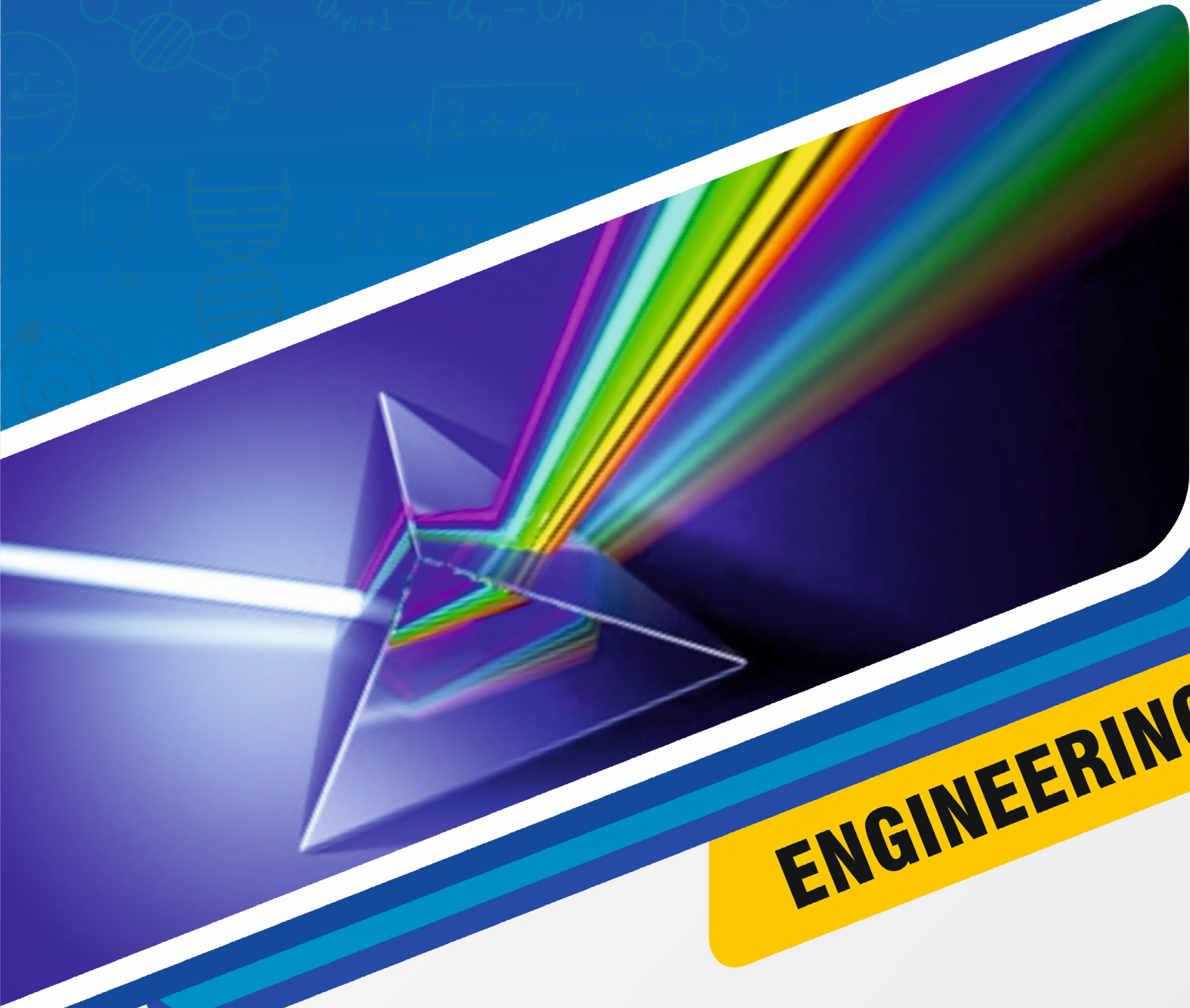
PHYSICS



ENGINEERING

CENTRE OF MASS, MOMENTUM & COLLISION

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CENTRE OF MASS,
MOMENTUM & COLLISION

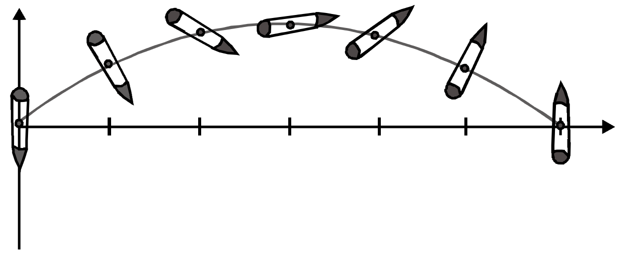
CONTENT

Centre of Mass, Momentum and Collision

INTRODUCTION

When we have dealt with an extended body (that is a body that has size), we assumed that it can be approximated to be a point particle or that it underwent only translational motion. Real "extended" bodies, however, can undergo rotational and other types of motion as well. For example, if you flip a pen in air, you will find that its motion is indeed very complex as every part of the pen moves in a different way. Therefore, a pen can not be represented as a particle, but as a system of particles. However, if you closely look, you will find that one of the special points of the pen moves in a simple parabolic path, as if pen's entire mass is concentrated there. That point is called the 'center of mass' of the pen. Thus, precisely speaking centre of mass is the location where the entire mass of system of particles is assumed to be concentrated.

Centre of mass is an imaginary point, which *may or may not* be located on the system.

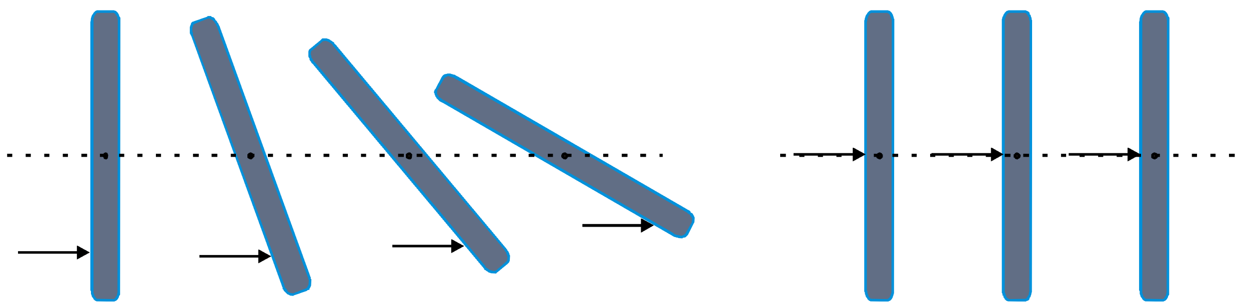


To locate, centre of mass, of a body, balance the body (let us say pen) on outstretched finger. The point on the axis, above your finger is centre of mass of the pen.

When a force is applied on a body apart from the magnitude & direction of $\vec{F}$ the motion of the body also depends upon the point of application.

Thus, $\vec{F} = m\vec{a}$ is not valid for all particles but for a special point i.e. centre of mass, $\vec{a}_{cm} = \dfrac{\vec{F}}{m}$.

Also, if the net force applied is along a line passing through the centre of mass of the body, all the particles of the body move with same velocity and acceleration.



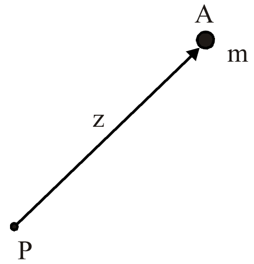
Position of centre of mass

First of all we find the position of Centre of mass of a system of particles. Just to make the subject easy we classify a system of particles in three groups :

1. System of two particles
2. System of a large number of particles and
3. Continuous bodies.

Now, lest us take them separately.

Mass Moment

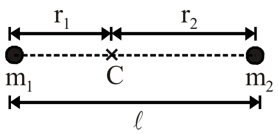


It is defined as the product of mass of the particle and distance of the particle from the point about which mass moment is taken. It is a vector quantity and its direction is directed from the point about which it is taken to the particle, as shown in figure, the mass moment of particle A (mass = m) about the point P is given by z.

There is an important property of centre of mass associated with the mass moments of the components of the system which forms the basis of analytical determination of centre of mass of a system. The property is *"The summation of mass moments of all the components of a system about its centre of mass is always equal to zero"*. This statement is an experimentally verified property which does not require any analytical proof. It can be used as a universal property in all types of systems.

1. Position of Centre of mass of two particles

Consider the situation shown in figure. Two masses m_1 & m_2 are separated by a distance l, let C be the centre of mass of the system at a distance r_1 from m_1 and $(l–r_1)$ from m_2. According to the property of mass moments about centre of mass of system of two particles *The summation of mass moments of all the components of a system about its centre of mass is always equal to zero* we have



$$m_1\vec{r}_1 + m_2\vec{r}_2 = \vec{0}$$

in scalar form, $-m_1r_1 + m_2r_2 = 0$ (as r_1 is towards left we consider it –ve)

$$m_1r_1 = m_2r_2 \qquad ....(i)$$

$$\frac{r_1}{r_2} = \frac{m_2}{m_1}$$

$$\frac{r_1}{r_1+r_2} = \frac{m_2}{m_1+m_2} \Rightarrow \frac{r_1}{\ell} = \frac{m_2}{m_1+m_2}$$

$$r_1 = \frac{m_2\ell}{m_1+m_2}$$

From equation (i). The distance of centre of mass from any of the particle (r) is inversely proportional to the mass of the particle (m).

$$r \propto \frac{1}{m}$$

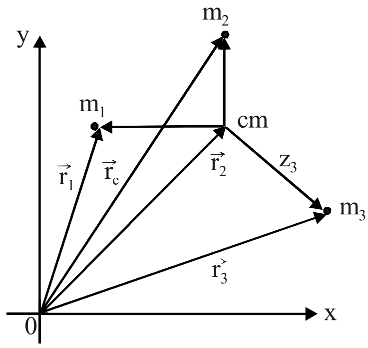
$r_1 = r_2 = \dfrac{d}{2}$ if $m_1 = m_2$, i.e. Centre of mass lies midway between the two particle of equal masses.

Similarly, $r_1 > r_2$ if $m_1 < m_2$ and $r_1 < r_2$ if $m_1 > m_2$ i.e. Centre of mass is nearer to the particle having larger mass.

2. Definition of Centre of mass for point particles :

Consider the situation shown in figure. There are three masses in a coordinate system with respective coordinates (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3). The position vectors of these masses with respect of origin can be given as

$$\vec{r}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k} \qquad \vec{r}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k} \qquad \vec{r}_3 = x_3\hat{i} + y_3\hat{j} + z_3\hat{k}$$



In this system, we will now locate the position of centre of mass. Let the coordinates of centre of mass be (x_c, y_c, z_c) and so the position vector will be

$$\vec{r}_c = x_c\hat{i} + y_c\hat{j} + z_c\hat{k}$$

The mass moments of the masses m_1, m_2 and m_3 about the centre of mass can be given as

$$\vec{z}_1 = m_1\,\vec{r}_{1/c} = m_1\,.\,(\vec{r}_1 - \vec{r}_c) \qquad \vec{z}_2 = m_2\,\vec{r}_{2/c} = m_2\,.\,(\vec{r}_2 - \vec{r}_c) \qquad \vec{z}_3 = m_3\,\vec{r}_{3/c} = m_3\,.\,(\vec{r}_3 - \vec{r}_c)$$

According to the property of mass moments, (the summation of mass moments of all the components of a system about its centre of mass is always equal to zero) we have

$$\vec{z}_1 + \vec{z}_2 + \vec{z}_3 = 0$$

$$m_1\,.\,(\vec{r}_1 - \vec{r}_c) + m_2\,.\,(\vec{r}_2 - \vec{r}_c) + m_3\,.\,(\vec{r}_3 - \vec{r}_c) = \vec{0}$$

On solving we get

$$\vec{r}_c = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3}{m_1 + m_2 + m_3} \qquad \text{....(i)}$$

This relation can also be generalized for n mass system. Now by substituting the vector in terms of unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$ and comparing the coefficients of $\hat{i}$, $\hat{j}$ and $\hat{k}$ we get

$$x_c\hat{i} + y_c\hat{j} + z_c\hat{k} = \frac{m_1(x_1\hat{i} + y_1\hat{j} + z_1\hat{k}) + m_2(x_2\hat{i} + y_2\hat{j} + z_2\hat{k}) + m_3(x_3\hat{i} + y_3\hat{j} + z_3\hat{k})}{m_1 + m_2 + m_3}$$

$$x_c = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} \qquad \text{....(ii)}$$

$$y_c = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} \qquad \text{....(iii)}$$

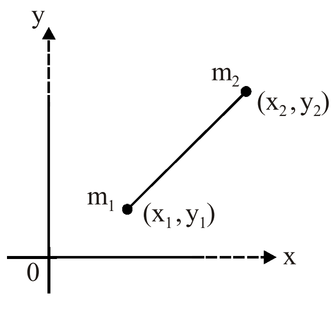
$$z_c = \frac{m_1z_1 + m_2z_2 + m_3z_3}{m_1 + m_2 + m_3} \qquad \text{....(iv)}$$

Equation (ii), (iii), (iv) can also be extended to n - objects system.

$$\vec{r}_{C.M.} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + ...m_n\vec{r}_n}{m_1 + m_2 + m_3 + ...m_n} = \frac{\Sigma m_i\vec{r}_i}{\Sigma m_i}$$

Thus $x_{cm} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + ...}{m_1 + m_2 + m_3 + ...}$

For a two body system this equation reduces to



$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2}$$

Note: *Centre of mass divides two point masses in inverse ratio of their masses*