Logistic Regression – Interpretation Cheat Sheet

Summary

Logistic regression is a statistical model used to predict the probability of an outcome. *Example*: Predict if a customer will default on their credit card loan (Y=1) or not (Y=0) based on income, remaining balance, and student status.

Probability

$$\pi = P(Y=1|X) = \frac{1}{1+exp(-(\beta_0+\beta_1 X_1+\ldots+\beta_p X_p))}$$

π is the probability of an outcome ($Y=1$) given covariates $X_1,\ldots,X_p$ and coefficients $\beta_0,\beta_1,\ldots,\beta_p$. *Example*: Probability of loan default as a function of income, credit balance, and student status. Unfortunately, coefficients can't be interpreted on the level of probabilities, but we have to use (log) odds.

Odds

$$Odds = \frac{\pi}{1-\pi} = exp(\beta_0+\beta_1 X_1+\ldots+\beta_p X_p)$$

Expected no. of $Y=1$ per $Y=0$. *Example*: Odds of 1.5 means 1.5 loan defaults per non-defaults.

Log Odds

$$ln\left(\frac{\pi}{1-\pi}\right) = \beta_0+\beta_1 X_1+\ldots+\beta_p X_p$$

Logarithm of odds. Also called logits. All following interpretation templates work for log odds when replacing $exp(\beta_j)$ with β_j, "odds" with "log odds", and removing "multiplicatively".

Example: loan default

covariate	β	$exp(\beta)$	95% CI of β	p-value
Intercept	-11	0.0	[-14.4;-8.4]	0.00
student	-1.8	0.16	[-3.4; -0.3]	0.02
balance	0.62	1.86	[0.48; 0.79]	0.00
income	-0.25	0.78	[-5.0; 4.6]	0.92

- student "yes" or "no"; categorical; reference: "no"
- balance: outstanding credit card balance in \$100s
- income: yearly income in \$100 000
- CI stands for confidence interval of β

Intercept

$exp(\beta_0)$ is the odds of $Y=1$ given all continuous covariates are 0 and all categorical covariates are set to the reference category. *Example*: The odds of loan default is $exp(-11) \approx 0.000017$ given student=no, balance=0 and income=0.

Continuous Coefficient

Template: A one-unit increase in X_j changes the odds for $Y=1$ multiplicatively by $exp(\beta_j)$, holding all other covariates constant. *Example*: A \$100 increase in credit balance increases the odds for loan default 1.86-fold, holding student status and income constant.

- Positive $\beta_j \rightarrow$ increasing X_j increases odds.
- Negative $\beta_j \rightarrow$ increasing X_j decreases odds.

Categorical Coefficient

Template: Changing X_j from [*reference*] to [*other category*] changes the odds for $Y=1$ multiplicatively by $exp(\beta_j)$, holding all other covariates constant. *Example*: Changing student status from "no" to "yes" decreases the odds of loan default by a factor of 0.16, holding all other covariates constant.

p-value

The p-value for a coefficient β_j is the probability to observe this value or a more extreme one under the Null Hypothesis of $\beta_j = 0$ (meaning that the covariate doesn't affect the odds of the outcome).
If $p < \alpha$, a pre-defined threshold (often $\alpha = 0.05$), the coefficient is significantly different from 0. *Example*: The effect of student status is significantly different from 0 ($p = 0.02$).

Interpretation of Confidence Interval

- The confidence interval measures the uncertainty of a coefficient.
- If you were to repeat the analysis 100 times with new samples and compute a 95% confidence interval each time, you would expect 95/100 of the intervals to contain the true coefficient.
- If the confidence interval doesn't cover 0, it's equivalent to rejecting the Null Hypothesis to the chosen α-level.

(Log) Odds Ratio

$$OR = \frac{Odds_1}{Odds_2} \quad \text{and} \quad logOR = \frac{ln(Odds_1)}{ln(Odds_2)}$$

(Log) odds ratios compare (log) odds of two data points.
Coefficient interpretations are based on (log) odds ratios, as they compare data points with X_j+1 and with X_j: $OR = Odds(X_j+1)/Odds(X_j) = exp(\beta_j)$. For categorical covariates the ratio reflects the change from reference to another category. I recommend just using the provided templates and not getting a headache from the (log) odds ratios.