A Real 'Pane': Determining the Thermal Conductivity of Glass

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The CENCO thermal conductivity apparatus[1] was employed to measure the thermal conductivity of a piece of glass. By putting a piece of glass in between a vessel maintained at a constant temperature of 90°C and a reciever at room temperature heat was able to flow through the glass from the vessel to the reciever. By measuring the voltage difference between the vessel and reciever, we were able to measure the change in temperature of the reciever over time. Upon further analysis, we found that the thermal conductivity, κ, of the piece of glass has a value of 0.56 ± 0.01 J/s$\cdot$ m K. Researching values of κ for glass yields many results depending on the type of glass. Typical values of κ for glass range from 0.2 to 1.05 J/s$\cdot$ m K and our value falls within this range[2]. Our results confirm the fact that glass is typically a poor conductor of heat, and thus, a good insulator.

I. INTRODUCTION

In 1822, Joseph Fourier published his work on thermal conduction in which he proved that the flow of heat through a material was equal to the product of the material's thermal conductivity, κ, and the negative of the temperature gradient[3]. Since this discovery, materials have been tested in order to determine how well they conduct heat. Metals have proved to be excellent conductors of heat while gases are poor conductors because the bonds among atoms are stronger in metals than in gases and metals have many free electrons[4]. Glass, however, comes in many kinds and typically doesn't have a high value for κ due to its lack of free electrons, thus making it a great insulator. The value of κ for glass ranges between 0.2 and 1.05 joule per second-meter-degree Kelvin (J/s$\cdot$ m K), depending on the type of glass in question[2]. Our goal was to measure the thermal conductivity of a piece of glass by measuring the change in voltage as heat was transfered from the vessel to the reciever by going through the glass (see Fig. 1).

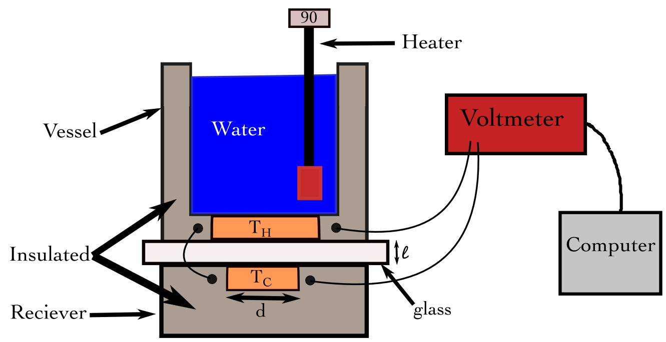

FIG. 1. Schematic of the apparatus used to measure the thermal conductivity of glass. The vessel and source have copper plugs that are insulated. The copper plug in the vessel is heated to the temperature of the water bath, which is set at 90° C, and the copper plug of the reciever is intitially at room temperature, along with the piece of glass. The piece of glass is put on top of the reciever and is put in contact with the copper plug of the vessel, allowing for the flow of heat from the vessel to the reciever. The voltage between the vessel and reciever is inputted to the voltmeter which is then outputed to the computer to be recorded.

II. THEORY

The rate at which heat, Q, flows through a material in time t is proportional to the cross-sectional area A, the temperature difference $(T_H - T_C)$ and is inversely proportional to the thickness l (see Fig.2). This rate can be written as

$$\frac{dQ}{dt} = \frac{\kappa A(T_H - T_C)}{l}, \tag{1}$$

where κ is the coefficient of thermal conductivity.

While this material has heat flowing through it, the temperature of the copper plug in the reciever, T_C, begins to rise. This flow of heat to the reciever can be modeled as

$$\frac{dQ}{dt} = Mc\frac{dT_C}{dt}, \tag{2}$$

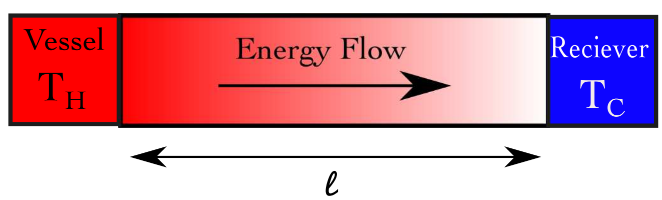

FIG. 2. The physical phenomena of heat flow. The thermal energy flows from the high energy source (T_H) to lower energy (T_C) until an equilibrium is reached. The energy flows over a distance l through the material.

where M and c are the mass and specific heat of the copper plug, respectively.

Setting Eqns. 1 and 2 equal to each other yields

$$\frac{\kappa A(T_H - T_C)}{l} = Mc\frac{dT_C}{dt}. \tag{3}$$

Since the temperature difference between the vessel and reciever is measured by a thermocouple that is attached to a voltmeter we can assume the voltage difference V is proportional to the product of the thermocouple's characteristics (a constant), C, and the temperature difference $(T_H - T_C)$[5]. This can be written as

$$V = C(T_H - T_C). \tag{4}$$

Integrating Eqn. 4 and knowing that T_H is constant, we obtain

$$\frac{dV}{dt} = -C\frac{dT_C}{dt}. \tag{5}$$

By substituting Eqns. 4 and 5 into Eqn. 3 and solving for dt, we obtain

$$dt = -\frac{lMc}{\kappa A}\frac{dV}{V}. \tag{6}$$

By integrating Eqn. 6, we obtain

$$t = -\frac{lMc}{\kappa A}\ln V + k_1, \tag{7}$$

where k_1 is a constant of integration. Knowing that the initial conditions are $V = V_0$ at time $t = 0$, then Eqn. 7 becomes

$$k_1 = \frac{lMc}{\kappa A}\ln V_0. \tag{8}$$

Plugging in Eqn. 8 into Eqn. 7 yields

$$t = -\frac{lMc}{\kappa A}(\ln V - \ln V_0). \tag{9}$$

Thus, if we plot time versus the natural log of the voltage, we can obtain a line with a slope m_1, of

$$m_1 = \frac{\kappa A}{lMc}. \tag{10}$$

Solving for κ from Eqn. 10 gives us

$$\kappa = -\frac{m_1 lMc}{A}. \tag{11}$$

However, we must address the fact that the copper plug does in fact lose heat to the air, since this is not a completely isolated system. Eqn. 11 does, however, allow us to make preliminary calculations which will be discussed in the results.

If we model the change in temperature of the copper plug in the reciever, knowing that heat is lost to the air, we obtain

$$\frac{dT_C}{dt} = m_1(T_H - TC) - m_2(T_C - T_{air}), \tag{12}$$

where T_{air} is the temperature of the air and is assumed to be constant. Rewriting Eqn.12 gives us

$$\frac{dT_C}{dt} = (m_1 + m_2)(T_H - T_C) - m_2(T_H - T_{air}). \tag{13}$$

By substituting Eqns. 4 and 5 into Eqn. 13 and rearranging, we obtain

$$\frac{dV}{V - \frac{m_2 V_0}{m_1 + m_2}} = -(m_1 + m_2)dt. \tag{14}$$

By integrating Eqn. 14 and rearranging, we get

$$\ln\left[V - \frac{m_2 V_0}{m_1 + m_2}\right] = -(m_1 + m_2)t + k_2, \tag{15}$$

where k_2 is the constant of integration. By solving for k_2 when $V = V_0$ and $t = 0$ we get

$$k_2 = \ln\left[\frac{m_1 V_0}{m_1 + m_2}\right]. \tag{16}$$

Plugging in Eqn. 16 into Eqn. 15 and getting rid of the natural logarithms gives us

$$V - \frac{m_2 V_0}{m_1 + m_2} = \frac{m_1 V_0}{m_1 + m_2}e^{-(m_1 + m_2)t}. \tag{17}$$

As t gets large, the term on the right in Eqn. 17 goes to zero and the voltage, V, approaches the steady voltage V_s. Thus, V_s can be written as

$$V_s = \frac{m_2 V_0}{m_1 + m_2}. \tag{18}$$

Substituting Eqn. 18 into Eqn. 15 yields

$$\ln(V - V_s) - \ln[m_1 V_0/(m_1 + m_2)] = -(m_1 + m_2)t \tag{19}$$

By plotting $\ln(V - V_s)$ versus t, we obtain a line with a slope of $(m_1 + m_2)$. Thus, we obtain a value for m_1 of

$$m_1 = (1 - V_s/V_0)(m_1 + m_2). \tag{20}$$

Thus, Eqn. 20 can be calculated with known quanitities and after plugging m_1 into Eqn. 11, we are able to determine the thermal conductivity[1].

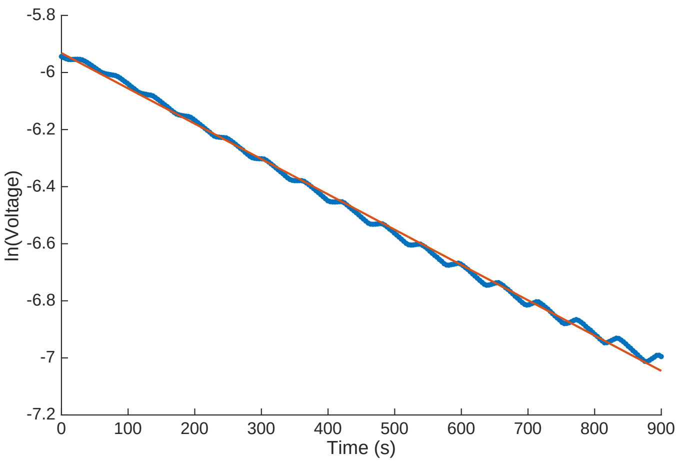

FIG. 3. Graph of the natural log of the voltage versus time. This was the data from the first 15 minutes only. Using least squares fitting, the slope,m_1, was found to be $(-1.200 \pm 0.004) \times 10^{-3}$ s^{-1}. By using this value, we were able to calculate a preliminary value for κ of 0.45 J/s· m K.

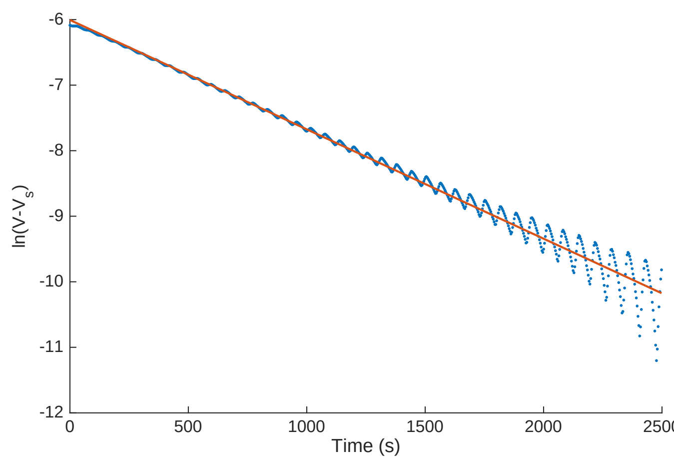

FIG. 4. Graph of the natural log of the difference between the voltage and the steady voltage versus time. This was the data from the full 42 minutes. Using least squares fitting, the slope, $(m_1 + m_2)$, was found to be $(-1.700 \pm 0.007) \times 10^{-3}$ s^{-1}. By using this value, we were able to calculate the final value for κ of 0.56 ± 0.01 J/s· m K.

III. EXPERIMENT

The apparatus was set up and the heater was placed in the water to raise its temperature to 90° C while the reciever and glass were set away from the vessel in order to ensure that they stayed at room temperature. The vessel and reciever were connected by a wire and then they both had wires plugged into the input of a voltmeter. This voltmeter was then connected to a computer (see Fig. 1). Once the water in the vessel was at a steady 90° C, we put the glass on top of the reciever, put them under the vessel, and lowered the vessel so that the copper plug of the vessel was in contact with the glass. We put some masses on top of the vessel to prevent air from being between the glass and copper plug. We also used aluminum foil to cover the hole on top of the vessel to keep the temperature as constant as possible. A program in Labview was ran in order to record the voltage readings every three seconds and the program was ran for an hour[6]. Only the data from the first 42 minutes was used due to the fact that the data past this point resulted in errors in our calculations when using the natural log.

IV. RESULTS

The natural log of the data collected during the first 15 minutes are represented in Fig. 3. These data were used for a preliminary calculation and upon calculating a best fit line for these data, we obtained a value of m_1 of $(-1.200 \pm 0.004) \times 10^{-3}$ s^{-1}. The mass, M, of the copper plug has a value 340 ± 1 g, where the error was determined by estimation. The width of the glass, l, was measured by use of a caliper and was determined to have a value of 5.50 ± 0.01 mm. The surface area, A, of the copper plug was calculated by measuring the diameter with a ruler and then using the formula, $A = \pi(d/2)^2$. By error propagation, we found that the surface area has a value of $(1.893 \pm 0.008) \times 10^{-3}$ m^2. Using these values, we made a preliminary calculation of κ which has a value of 0.45 J/s· m K. We did not include error because this was a preliminary calculation. This result did, however, let us know that we were in the correct range of 0.2 - 1.05 J/s· m K for κ.

The natural log of the difference between the data collected and the steady voltage during the full 42 minute period can be seen in Fig. 4. The steady voltage, V_s, was determined by statistical analysis of the raw data toward the end of the experiment in which we obtained a value of 0.347 ± 0.001 mV. The initial voltage, V_0, was used from the raw data and has a value of 2.62156 ± 0.00001 mV. Upon calculating the least squares line for the data in Fig. 4, we obtained a value for the slope, $(m_1 + m_2)$, of $(-1.700 \pm 0.007) \times 10^{-3}$ s^{-1}. Using Eqn. 20, Eqn. 11, and the known quanitities, we obtained a value for κ of 0.56 ± 0.01 J/s $\cdot$ m K. The error of κ was calculated using the following formula:

$$\delta\kappa = \kappa\sqrt{\left(\frac{\delta m_1}{m_1}\right)^2 + \left(\frac{\delta l}{l}\right)^2 + \left(\frac{\delta M}{M}\right)^2 + \left(\frac{\delta A}{A}\right)^2} \quad (21)$$

V. CONCLUSIONS

We were successfully able to obtain a reasonable value for the thermal conductivity of a peice of glass. Our value for κ of 0.56 ± 0.01 J/s $\cdot$ m K is within the range of values of κ for different kinds of glass (see Fig. 5). Our result had an uncertainty of less than 2% and our result also

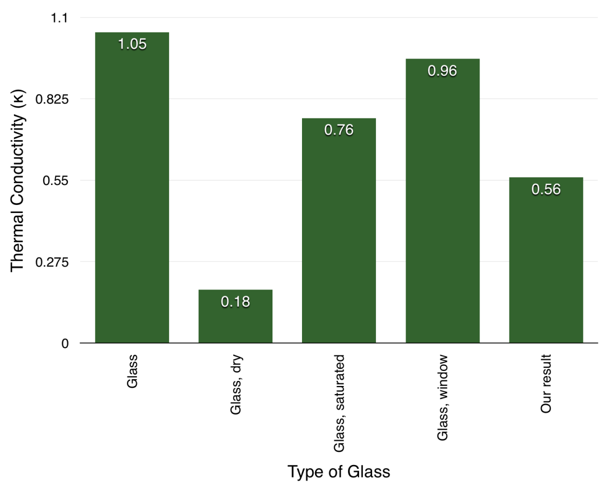

FIG. 5. A bar graph of values for the thermal conductivity of various types of glass. These values range from 0.2 to 1.05 J/s$\cdot$ m K^2. Our value for κ has a value of 0.56 ± 0.01 J/s $\cdot$ m K, which falls within this range of values.

solidifies the fact that glass is a poor conductor of heat but is a great insultator. Future work could involve using different types of metals instead of glass because metals are typically very good conductors so the data would be drastically different than those from using glass. Other future work could include using various types of glass and seeing if the values in Fig. 5 hold up.

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