Fringe Frenzy: Finding the Wavelength of Lasers and the Index of Refraction of Air

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The Michelson Interferometer apparatus and a red and green laser were used to determine the wavelength of these laser beams. By adding a glass cell into the apparatus and using the red laser, we were able to determine the index of refraction of air. By shining a laser light through a beam splitter and having these split beams meet back again by use of mirrors, we were able to produce an interference pattern that we could manipulate by moving the movable mirror. Our data allows us to examine the wave nature of light and how these light waves interfere with each other to produce fringes. Our results indicate that by counting the number of fringes that pass while moving a mirror, the wavelength of a beam can be determined. Also, by changing the pressure within the glass cell and counting the number of fringes that pass gives us the index of refraction of air. After analyzing the data, we find that the red laser and green laser have a wavelength of 670 ± 40 nm and 553 ± 8 nm, respectively. The accepted value of the red laser, 632 nm, is within our margin of error and differs from our calculated value by 6%. The accepted value of the green laser, 543 nm, is just outside of our margin of error and differs from our calculated value by only 2%. We also find that the index of refraction of air, n, has a value of 1.00 ± 0.05. The accepted value of n, 1.000235 at 76 cm Hg and 15°C, is within our margin of error and differs from our calculated value by less than 0.1%. These results give insight on the physical nature of light waves and how their interference patterns allow us to determine their characteristic wavelength and how these waves propagate through mediums.

I. INTRODUCTION

The goals of the experiments were finding the wavelength of two different lasers and finding the index of refraction of air by using a Michelson Interferometer (see Fig. 1). By counting the number of fringes from the interference pattern that passed by on the viewing screen as we turned the micrometer knob, we were able to calculate the wavelength of the two lasers which have accepted values of 632 nm and 543 nm. I will refer to the lasers with accepted wavelengths of 632 nm and 543 nm as the red and green laser, respectively. By adding a vaccuum chamber into our apparatus and pumping the air out and then counting the number of fringes that passed while the air was flowing out we were able to calculate the index of refraction of air which has an accepted value of 1.0002760 in dry air at $15°C$.

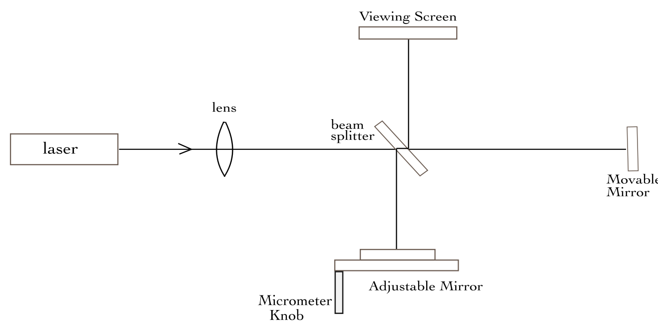

FIG. 1. Schematic of the Michelson Interferometer used to measure the wavelength of the laser beam. The lens is used to spread the beam in order to produce the interference pattern produced on the screen. The beam splitter splits the original beam in half and the mirrors are used to reflect these beams back in which they end up meeting again when heading toward the viewing screen. The movable mirror is controlled by the micrometer knob and by moving the mirror, the interference pattern changes.

II. THEORY

A. Finding λ

The Michelson inteferometer is set up in such a way that the beam of light from the laser strikes the beamsplitter and 50% is reflected while the other 50% is transmitted (See Fig. 1). The beam that is transmitted is bounced back by the movable mirror and is once again reflected toward the viewing screen and transmitted by the beam splitter back toward the laser. Similarly, the beam that was originally reflected by the beam splitter is bounced back by the adjustable mirror and is once again reflected toward the laser and transmitted by the beam splitter toward the viewing screen. Thus, we have two portions of the original beam heading toward the viewing screen. When a lens is placed between the laser source and the beam splitter, the beam spreads out and an interference pattern can be seen on the viewing screen (see Fig. 2).

The pattern of dark and bright fringes are due to the fact that each portion of the beam has a certain distance to travel to meet up again (the point after the beam splitter where they head toward the viewing screen). When they meet at that point, they may be in phase (color fringes) or out of phase (space between colored fringes). This difference is due to the fact that light can be modeled as waves and these waves obey superpostion. Thus, if two waves of the same amplitude interfere with each other, they will add together. Depending on if they are

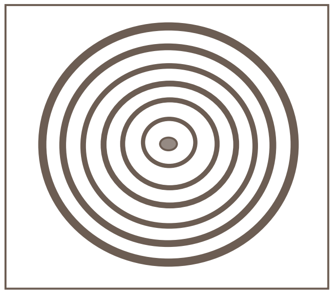

FIG. 2. The interference pattern that is seen on the viewing screen once everything is aligned correctly in the Michelson Interferometer apparatus. The black fringes represent waves that are interefering constructively and would be whatever color the laser's light happens to be. The white spaces between the black fringes represent waves that are interefering destructively.

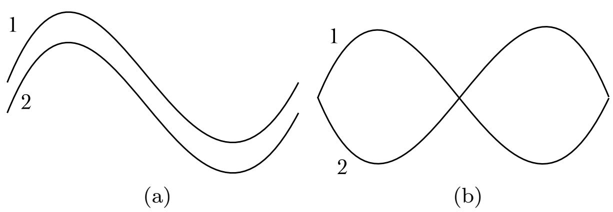

FIG. 3. (a) represents two waves that are interfering constructively while (b) represents two waves interfering destructively. These waves obey superposition in that they will add together and (a) will produce the fringes with the laser's color while (b) will produce the dark fringes between the colored fringes.

in phase or out of phase will determine whether these waves add constructively, thus producing colored fringes, or destructively, which would result in the space between the color fringes. Fig. 3 represents what waves look like when they are in phase and are constructively interfering, or out of phase and destructively interfering.

We are able to move the movable mirror by turning the mircometer knob. This changes the interference pattern. By moving the mirror 1/4 of the laser's wavelength toward the beam splitter, the distance that portion of the beam has to travel is reduced by 1/2 the wavelength (1/4 on the way to the mirror and 1/4 on the way back to the beam splitter). This action results in the waves that were once out of phase now being in phase and vice versa. This changes the fringe pattern in that the bright fringes will become dark and the dark fringes will become bright ones. By moving the mirror another 1/4 of the laser's wavelength, the fringes will change once again to their original state before the mirror was moved at all. Thus, by moving the mirror a distance d, and counting N, the number of of times the fringe pattern returns to its original state, the wavelength of the laser can be determined as:

$$\lambda = \frac{2d}{N}, \tag{1}$$

where the factor of 2 is added because d/N correlates to 1/2 of the wavelength. By solving Eqn. 1 for N we obtain:

$$d = \frac{\lambda}{2}N. \tag{2}$$

Thus, by graphing d vs. N, a line with a slope of $\lambda/2$ can be obtained. By multiplying this slope by 2, the wavelength of the laser can be determined.

B. Finding index of refraction of air

The wavelength of laser light depends on the index of refraction, n, of the medium it is traveling through. This can be written as:

$$\lambda = \frac{\lambda_o}{n}, \tag{3}$$

where λ_o is the wavelength in a vacuum. The number of wavelengths within a glass cell of length d is given by $2d/\lambda$, where the factor of 2 is added because the beam travels through the glass cell twice, which is located between the movable mirror and the beam splitter (see Fig.4). Therefore, by combining $2d/\lambda$ and Eqn. 3, it can be concluded that there are $2dn/\lambda_o$ wavelengths within the glass cell.

As the pressure changes within the cell, so does the index of refraction. Since there are $2dn/\lambda_o$ wavelengths within the cell at a given n, if n changes, the number of wavelengths also changes. Since each fringe correspondes to a wavelength, counting the number of fringes, N, that go by yields the number of changes of wavelengths within the cell. This can be written as:

$$N = \frac{2d}{\lambda_o}(n_f - n_i), \tag{4}$$

where n_i and n_f are the initial and final indexes of refraction in the cell, respectively. By plotting N vs. d we can find the least squares line where the slope is equivalent to $\lambda/2$.

Since the change in n is caused by the change in pressure, Eqn. 4 can be rewritten as:

$$N = \frac{2d}{\lambda_o}\frac{dn}{dp}\Delta p, \tag{5}$$

where Δp is the change in pressure and dn/dp is the change in the index of refraction as the pressure changes. By counting the number of fringes that pass from several changes of pressure it is possible to graph N vs. Δp from Eqn. 5. The slope of this line, q, would be:

$$q = \frac{2d}{\lambda_o}\frac{dn}{dp}. \tag{6}$$

Solving Eqn. 6 for dn yields:

$$\int_1^n dn = q\frac{\lambda_o}{2d}\int_0^{p_{atm}} dp, \tag{7}$$

where the limits of dn are from a vacuum to some index, n, and the limits of dp are from a vacuum to atmospheric pressure. After integrating Eqn. 7 and solving for n, the following formula is obtained:

$$n = 1 + q\frac{\lambda_o}{2d}p_{atm}. \tag{8}$$

Although λ_o corresponds to the wavelength in a vacuum, the difference between λ and λ_o for the laser is negligible and n can be calculated using λ, q, and p_{atm}, which has a value of 76 cm Hg.

III. EXPERIMENT

A. Finding λ for Laser X and Y

Setting up the Michelson Interferometer required some time to make sure everything was aligned properly. This meant that the mirrors needed to be perpendicular to the beams of light and that the beams struck the center of each mirror. The beam splitter also needed to be at a 45° angle with respect to the laser (see Fig. 1).

We used an electronic counter as our viewing screen. The counter counted the number of fringes that passed upon turning the micrometer knob. In order to test whether this counter was reliable, we manually counted different amounts of fringes and compared the micrometer readings to those collected from using the counter. We concluded that the counter was reliable and used it for the rest of the experiments. There were times where the counter failed to work but we were able to fix the problem and obtain consistent data.

Another problem we faced was mechanical backlash. Mechanical backlash refers to a small amount of give before the mirror begins to move after reversing the direction of the micrometer knob. This is caused by the slight gaps between the gears which results in turning the micrometer knob but the mirror doesn't move. This would result in a larger reading on the micrometer knob than how far the mirror actually moved. Backlash is reduced by making sure to turn the micrometer knob by one full counterclockwise turn before counting fringes. For each measurement we started at the 75 µm mark on the micrometer knob and turned the knob counterclockwise and recorded the displacement. We then turned the knob clockwise until we were 20 µm past the 75 µm mark. We then turned it counterclockwise until we were again at the 75 µm mark before begining more measurements. This procedure was followed for all measurements made.

We decided to count 25 fringes first and repeated this 5 times in order to obtain an accurate average of the distance needed to move the mirror to obtain this number of fringes. Similarly, we recorded 5 measurements for counts of 30, 35, 40, 45, and 50 fringes.

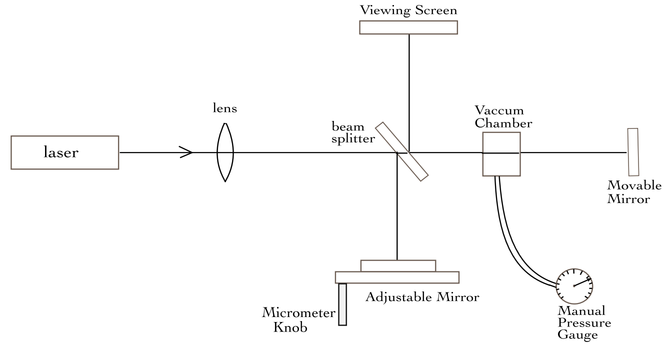

FIG. 4. Schematic of the Michelson Interferometer with a glass cell used to measure the index of refraction of air. The glass cell in placed between the beam splitter and the movable mirror and the cell is conected to a manual pressure gauge that can pump the air out of it. By changing the pressure, the index of refraction within the cell changes and the fringe pattern on the viewing screen changes. Measuring the number of fringes that pass for specific changes of pressures allows us to find the index of refraction of air.

B. Finding index of refraction of air with He-Ne Laser

In order to find the index of refraction of air, we added a glass cell with a length, d, of 4.4 ± 0.1 cm between the movable mirror and the beam splitter (see Fig.4). The measurement of the cell was determined by using a ruler and the error was estimated based on the tick marks on the ruler. We attached a vacuum pump with a pressure gauge in order to obtain different pressures within the cell. I pumped the air out of the cell until we got to an appropriate value and then I slowly let the air in until the pressure gauge got to a value of 10 cm Hg. During this time my partner counted the number of fringes that passed. We recorded data for 6 different pressure changes and repeated each individual pressure change 5 times in order to obtain an average number of fringe counts.

IV. RESULTS

A. λ of Red Laser

Upon graphing the number of fringes, N, versus the distance the mirror moved, d, for the red laser, we obtain Fig. 5. The best fit line was obtained by least squares fitting and its slope and its error corresponds to half the wavelength. Multiplying this slope value by 2 yields a value of 670 ± 40 nm for λ. The accepted value for the wavelength of this laser is 632 nm which correlates to a

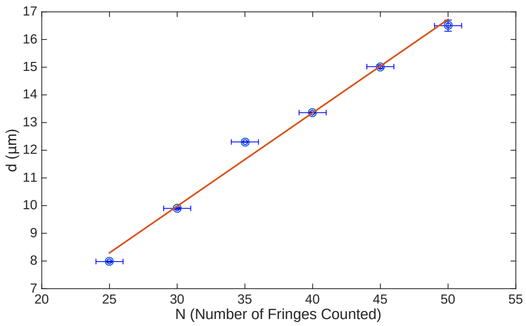

FIG. 5. Upon graphing the distance the movable mirror moved vs. the number of fringes that passed for the red laser, we obtain a line with a slope of 0.34 ± 0.02 μm/count. This value corresponds to half of the wavelength of the laser light. Multiplying this slope by 2 gives us a value of λ of 670 ± 40 nm.

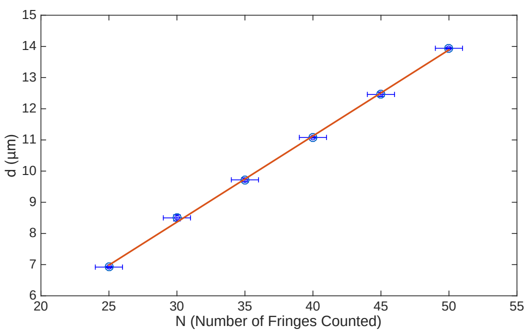

FIG. 6. Upon graphing the distance the movable mirror moved vs. the number of fringes that passed for the green laser, we obtain a line with a slope of 0.276 ± 0.004 μm/count. This value corresponds to half of the wavelength of the laser light. Multiplying this slope by 2 gives us a value of λ of 553 ± 8 nm.

6% error and our value with its error overlaps this value.

B. λ of Green Laser

Similarly for the green laser, we graphed the number of fringes, N, versus the distance the mirror moved, d, and obtained Fig. 6. Upon calculating the slope of the least squares line we obtained a wavelength of 553 ± 8 nm. The accepted value for the wavelength of this laser is 543 nm which correlates to a 2% error and our value with its error is just outside this accepted value.

C. Index of Refraction of Air

Upon graphing the number of fringes counted, N, versus the change in pressure, Δp we obtain Fig. 7. Using a least squares line we obtained a value of its slope, q, and its error δq of 0.43 ± 0.02 counts/cm Hg. Plugging in our values of λ, d, p, and q into Eqn. 8 we obtain a value of 1.00 ± 0.05 for the index of refraction of air, where the error was determined by:

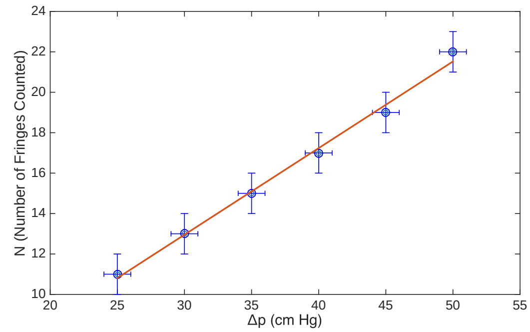

FIG. 7. Upon graphing the number of fringes counted vs. the change in pressure of the cell we obtain a least squares line with a slope of 0.43 ± 0.02 counts/cm Hg. Multiplying this value by $p\lambda_o/2d$ and adding 1 yields the index of refraction , n, of air which we calculated to be $1.00 \pm .05$.

$$\delta n = n\sqrt{\left(\frac{\delta q}{q}\right)^2 + \left(\frac{\delta \lambda_o}{\lambda_o}\right)^2 + \left(\frac{\delta d}{d}\right)^2 + \left(\frac{\delta p}{p}\right)^2} \quad (9)$$

An accepted value of n for dry air at 15°C is 1.0002760 and our calculated value corresponds to less than 0.1% error.

V. CONCLUSIONS

We have successfully observed the interference of laser light through the use of the Michelson Interferometer. By manipulating the distance of the mirror we were able to change the interference pattern. Counting the number of fringes that passed as we moved the mirror allowed us to determine the wavelength of both the red and green lasers, which we calculated to be 670 ± 40 nm and 553 ± 8 nm, respectively. The accepted value for the wavelength of the red laser is 632 nm which corresponds to a 6% error and the accepted value for the wavelength of the green laser is 543 nm which corresponds to a 2% error. These results reveal that the interference pattern contains a lot of information about the physical nature of the light that it comes from, specifically, its characteristic wavelength.

By adding a glass cell and changing the pressure within that cell, we were able to change the index of refraction within the cell and successfully calculate the index of refraction of air. By counting the number of fringes that passed as we changed the pressure within the cell, we obtained a linear relationship between these quantities and calculated the index of refraction, n, to be 1.00 ± 0.05. The accepted value for n in dry air at 15° C and 76 cm Hg is 1.0002760 which corresponds to less than 0.1% error. These results reveal more informtaion about the physical nature of light in that the index of refraction of a medium, like air, affects how light propagates through that medium.

The results from finding the wavelength of the lasers could be improved in that multiple counters could be used to check for consistency that they are functioning properly and yielding the correct number of counts. Also, a different laser with a smaller wavelength (He-Cd) could be used to see if this method can obtain its wavelength. Using the green laser to determine the index of refraction of air could also be taken on in order to determine if similar results are obtained.

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