Back in a Flash: Mechanically Measuring the Speed of Light

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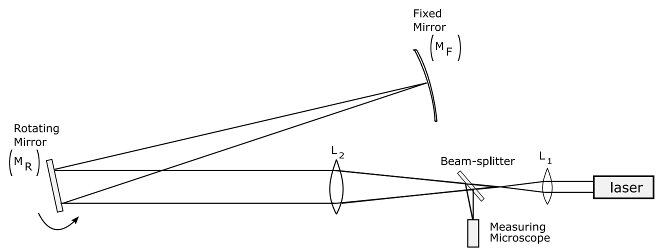

FIG. 1. Schematic of the Foucault Method used to measure the speed of light. Lenses L_1 and L_2 are used to focus the beam from the 0.5 mW He-Ne Laser and have focal lengths of 48 mm and 252 mm respectively. The rotating mirror is allowed to rotate in the clockwise and counterclockwise direction at rotational speeds of 750 and 1500 revolutions per second. We are able to calculate the speed of light by measuring the displacement of the point image from when the rotating mirror is still and when it rotates at various speeds. This measurement is done by use of a measuring microscope.

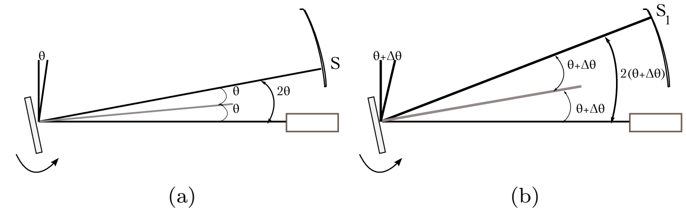

FIG. 2. The angle at which the beam is deflected by the rotating mirror depends on the angle of the rotating mirror with respect to the beam. When the mirror is at an angle of *theta*, (a), the reflected angle and the angle of incidence both have a value of *theta* and the resulting beam is reflected at an angle of 2θ and arrives at point S. Similarly with (b), when the angle rotates and additional $\Delta\theta$, the angle of incidence and reflected angle are both $(\theta + \Delta\theta)$ and so the beam is reflected at an angle of $2(\theta + \Delta\theta)$ and arrives at the point S_1.

I. INTRODUCTION

The goal of this experiment was to determine the speed of light by measuring the displacement of a beam image. We used the Foucault method which involves firing a laser toward a rotating mirror and measuring the change in position of the returned pulses in hopes of accurately measuring the speed of light, which has an accepted value of 2.99792458×10^8 m/s.

II. THEORY

The Foucault method involves the use of a laser, a measuring mircoscope, beam splitter, two lenses, a rotating mirror, and a fixed mirror (see Fig. 3). Lens L_1 is placed such that the beam from the laser is focused to a point **s** and lens L_2 is positioned such that this point image is reflected from the rotating mirror and is focused onto the fixed mirror. When the rotating mirror is stationary the angle of incedence and the reflected angle together make 2θ (see Fig. 2(a)) and the resulting image location, S, can be written as:

$$S = 2D\theta \tag{1}$$

where D is the distance between the two mirrors (see Fig. 3). The two distances, D, in Fig. 3 are equal due to the fact that virtual images are a reflection of the real image so their apparent distance from the mirror will be the same as the distance between the real image and the mirror.

When the mirror rotates, the total angle changes to $2(\theta + \Delta\theta)$ because the incident angle and the reflected angle are both $\theta + \Delta\theta$ (see Fig. 2(b)). The resulting location of the image, S_1 can be written as:

$$S_1 = 2D(\theta + \Delta\theta). \tag{2}$$

With the reflected image from the fixed mirror, we obtain a virtual image with a height of ΔS, a distance $D+B$ away from L_2 where ΔS is defined as the displacement of the image points:

$$\Delta S = S_1 - S. \tag{3}$$

Plugging Eqns. 1 and 2 into Eqn. 3 yields:

$$\Delta S = 2D\Delta\theta. \tag{4}$$

The height of the virtual image, ΔS, will be focused by lens L_2 in the plane of point s and the resulting displacement of the point image, Δs will have a value of $(-i/o)\Delta S$ due to the thin lens theory. $-i$ corresponds to the distance of the lens from the image (distance A on Fig. 3) and o corresponds to the distance of the lens from the virtual image (distance B+D on Fig. 3). The point image displacement that will be measured, $\Delta s'$ is equal to Δs because of the use of the beam splitter. Therefore, we can state:

$$\Delta s' = \frac{A}{D+B}\, 2D\Delta\theta. \tag{5}$$

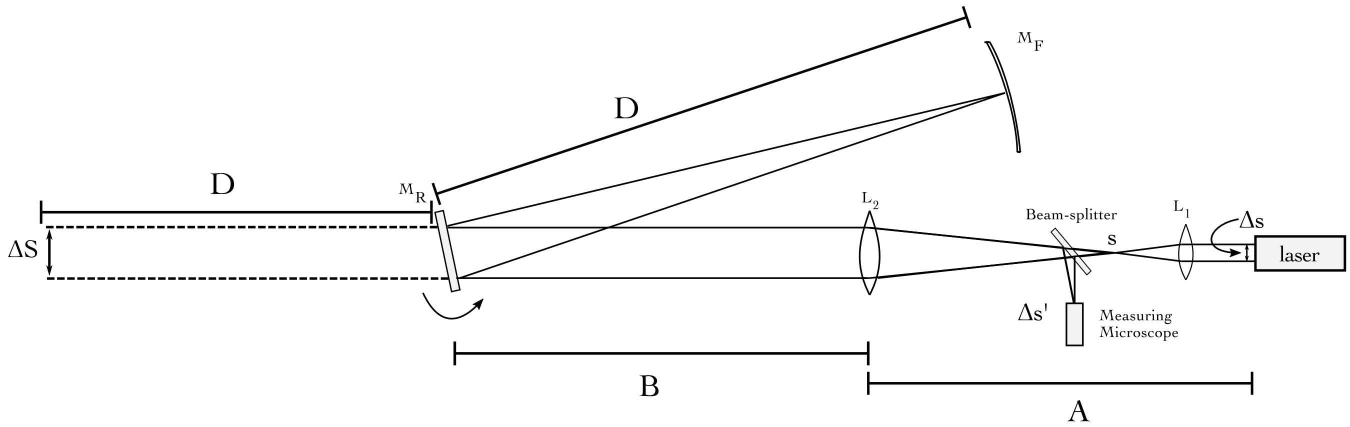

FIG. 3. The Foucault method to obtain the speed of light. The virtual image produced has a height of ΔS and appears to have come from a distance of D+B away from L_2. The distance A corresponds to the distance between the two lenses minus the focal length of L_1. By using the thin lens theory, the height of ΔS can be transformed to $\Delta s'$ which is measured through the measuring microscope. This displacement of the beam, along with the distances the beam travels and the rotational speed of the mirror, can be used to calculate the speed of light.

The angle $\Delta\theta$ depends on the rotational speed of the rotating mirror as well as the time it takes for the beam of light to travel from M_R to M_F. The time taken can be written as 2D/c and the rotational speed is given by ω, which is measured in revolutions/second. Using this information, the change in angle can be written as:

$$\Delta\theta = \frac{2D\omega}{c}. \tag{6}$$

Plugging Eqn. 6 into Eqn. 5 and taking into account that ω should be multiplied by 2π to obtain units of radians/second, we get:

$$\Delta s' = \frac{8\pi AD^2}{c(D+B)}\omega. \tag{7}$$

Therefore, by plotting $\Delta s'$ versus ω, it is possible to obtain the slope, m, which has a value of:

$$m = \frac{8\pi AD^2}{c(D+B)}. \tag{8}$$

By multiplying the inverse of Eqn. 8 by $8\pi AD^2/(D+B)$ we obtain the following equation for c:

$$c = \frac{8\pi AD^2}{m(D+B)} \tag{9}$$

III. EXPERIMENT

After setting up the experiment (see Fig. 1) and aligning the laser such that the laser was confined to a fine point at the center of each mirror, we began taking measurements. We first took a measurement of the point image when the mirror was not rotating by aligning the point image to the center of the cross hairs in the microscope. This was accomplished by turning the adjusting knob on the microscope and recording the micrometer reading. We then set the rotating mirror to 750 rev/s in the clockwise direction and adjusted the knob until the point image was once again in the center of the cross hairs and then recorded the reading. We continued this methodology for the mirror rotating at 1500 rev/s in the same direction as well as 750 rev/s and 1500 rev/s in the counterclockwise direction. By subtracting the measurement made while the mirror was still from the measurements made when the mirror was rotating, we obtained the values of $\Delta s'$, which is the displacement of the image point. This displacement of the point image allows us to measure the speed of light in that the rapid rotation of the mirror reveals the delay of the beam to return to its original location.

Distance	m
A	0.2680 ± 0.0007
B	0.474 ± 0.004
D	9.83 ± 0.05

TABLE I. The distances, in meters, denoted in Fig. 3 with their uncertainties. Note that the fractional uncertainties for m are all on the order of 1 percent.

IV. RESULTS

The distances A and B were measured by using the markings along the optics bench while the distance D was measured by use of a tape measure. These measurements and their error can be seen in Table I. Our error in

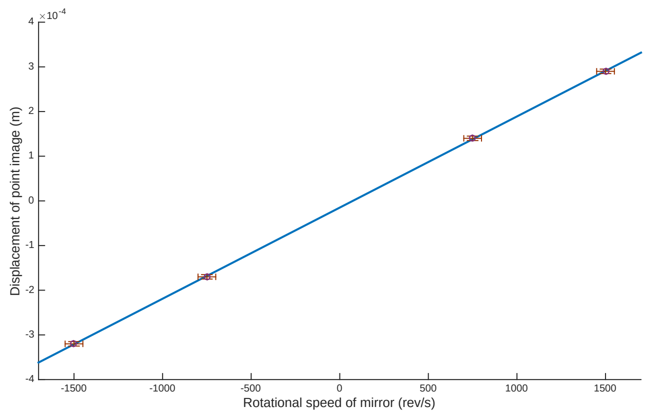

FIG. 4. Upon graphing the displacement of the image point against the corresponding rotational speeds we obtain a line with a slope of $(2.0 \pm 0.1) \times 10^{-}7$ m s/rev. Positive rotational speeds correspond to the mirror rotating in the counterclockwise direction while negative rotational speeds correspond to the mirror rotating in the clockwise direction. By taking the inverse of this slope and multiplying it by $8\pi AD^2/(D+B)$, we obtain a value of the speed of light, c, of $(3.1 \pm 0.2) \times 10^8$ m/s.

our measurements of $\Delta s'$ was less than 0.5% due to the precision of our measuring microscope. Since we could not directly measure ω, we assumed that its error would be on the degree of 5%, giving us an approximate error of ± 50 rev/s for each ω.

The results from calculating $\Delta s'$ for each rotational speed were plotted against their corresponding rotational speed (see Fig. 4). By using the min/max method, we were able to calculate the slope and its error to be $(2.0 \pm 0.1) \times 10^{-7}$ m s/rev. Knowing the error in all of our measurements, we were able to calculate the error in the speed of light, c, using the following formula:

$$\delta c = 8\pi \left[\left(\frac{D^2 \delta A}{m(B+D)} \right)^2 + \left(\frac{AD^2 \delta B}{m(B+D)^2} \right)^2 + \left(\left(\frac{-AD^2}{m(B+D)^2} + \frac{2AD}{m(B+D)} \right) \delta D \right)^2 + \left(\frac{AD^2 \delta m}{m^2(B+D)} \right)^2 \right]^{1/2} \tag{10}$$

Therefore, by using Eqns. 9 and 10 we obtained a value of c of $(3.1 \pm 0.2) \times 10^8$ m/s. This correlates to a 6.5% error and overlaps with the accepted value of 2.99792458 $\times 10^8$ m/s.

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