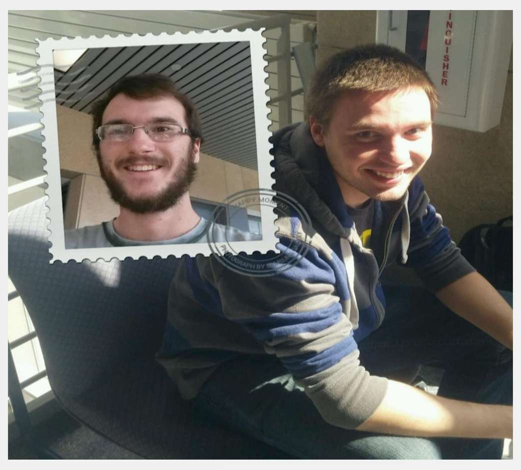
Modeling the Motion of a Spring Pendulum in 2D



Evan Van de Wall and Jeffrey Hejna
Department of Physics and Astronomy
Ithaca College

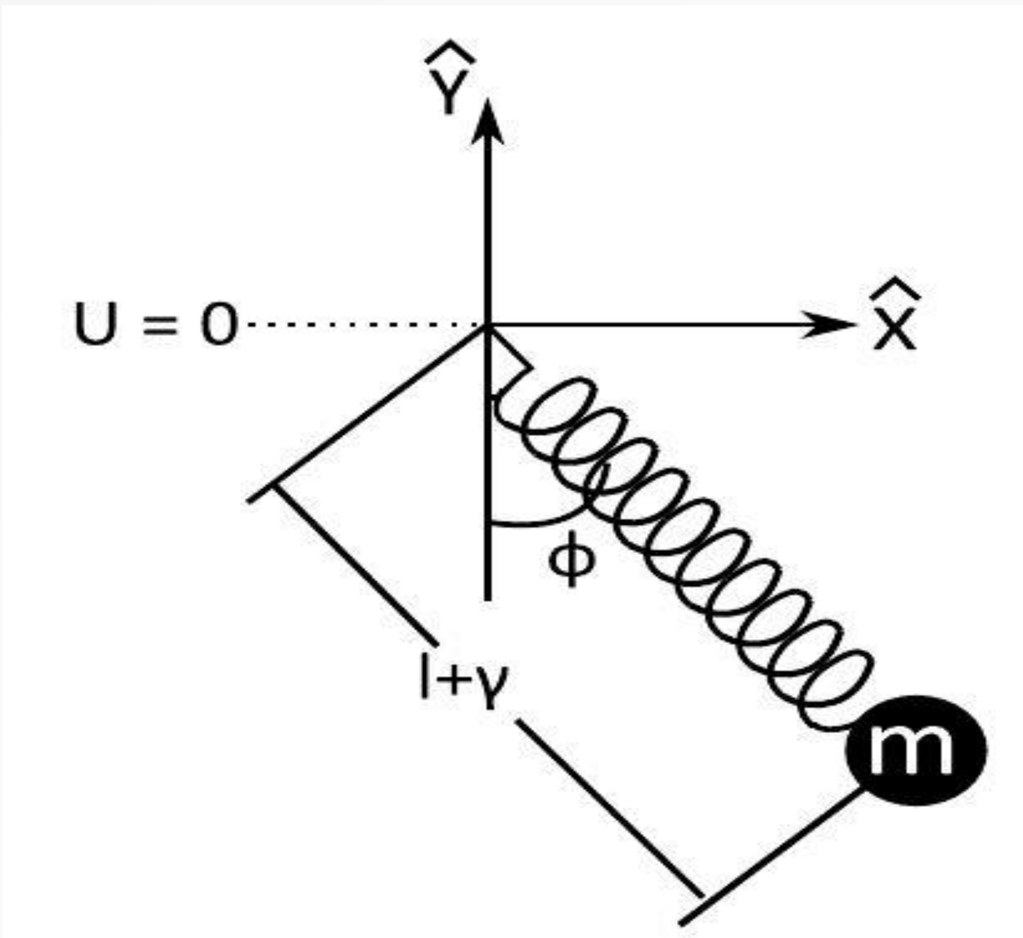
Introduction

A spring pendulum combines the two relatively simple motions of a spring and a pendulum. When these two motions are joined together, the resulting motion becomes much more interesting and increases in complexity.

Our goals were to:

1) Model the motion of a spring pendulum without drag.

2) Model the motion of a spring pendulum with drag.



Equations of Motion Without Drag

We can model a spring pendulum by taking into account the kinetic and potential energies and use a Lagrangian.

For our system:

$$T = \frac{1}{2}m(\dot{\gamma}^2 + (l+\gamma)^2\dot{\varphi}^2)$$

$$U = -mg(l+\gamma)\cos(\varphi) + \frac{1}{2}k\gamma^2$$

Where l is the equilibrium length of the spring and mass, γ is the length the spring has been stretched, φ is the angle from the equilibrium position, and k is the spring constant..

Our Lagrangian becomes:

$$L = \frac{1}{2}m(\dot{\gamma}^2 + (l+\gamma)^2\dot{\varphi}^2) + mg(l+\gamma)\cos(\varphi) - \frac{1}{2}k\gamma^2$$

Simplifying gives us the following equations:

$$\ddot{\gamma} = m(l+\gamma)\dot{\varphi}^2 + g cos(\varphi) - \frac{k}{m}\gamma$$

$$\ddot{\varphi} = \frac{-g sin(\varphi)}{(l+\gamma)} - \frac{2\dot{\gamma}\dot{\varphi}}{(l+\gamma)}$$

Equations of Motion With Drag

To account for linear drag, additional terms need to be added to our previous equations.

Our new equations are:

$$\ddot{\gamma} = m(l+\gamma)\dot{\varphi}^2 + g cos(\varphi) - \frac{k}{m}\gamma - \frac{b}{m}\dot{\gamma}$$

$$\ddot{\varphi} = \frac{-g sin(\varphi)}{(l+\gamma)} - \frac{2\dot{\gamma}\dot{\varphi}}{(l+\gamma)} - \frac{b}{m}(l+\gamma)\dot{\varphi}$$

Where b is the linear drag coefficient.

Results: Without Drag

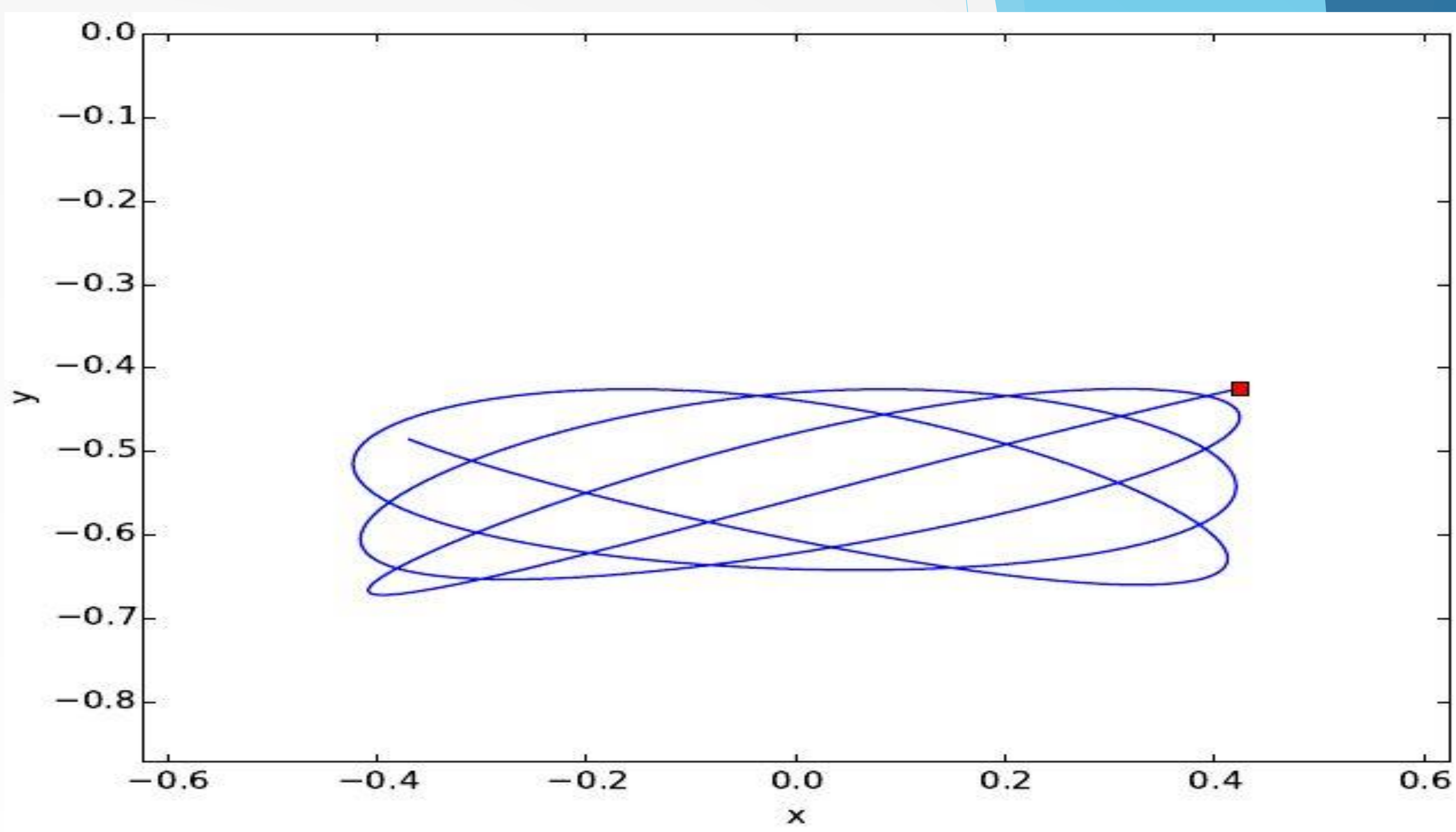


Figure 1. This graph shows the movement of a mass of 0.295 kg during a five second interval with an equilibrium spring length of 0.16m and a spring constant of 7.22505 N/m. When no drag is present, the Y versus X motion of the mass is periodic and doesn't decay in amplitude.

With Drag:

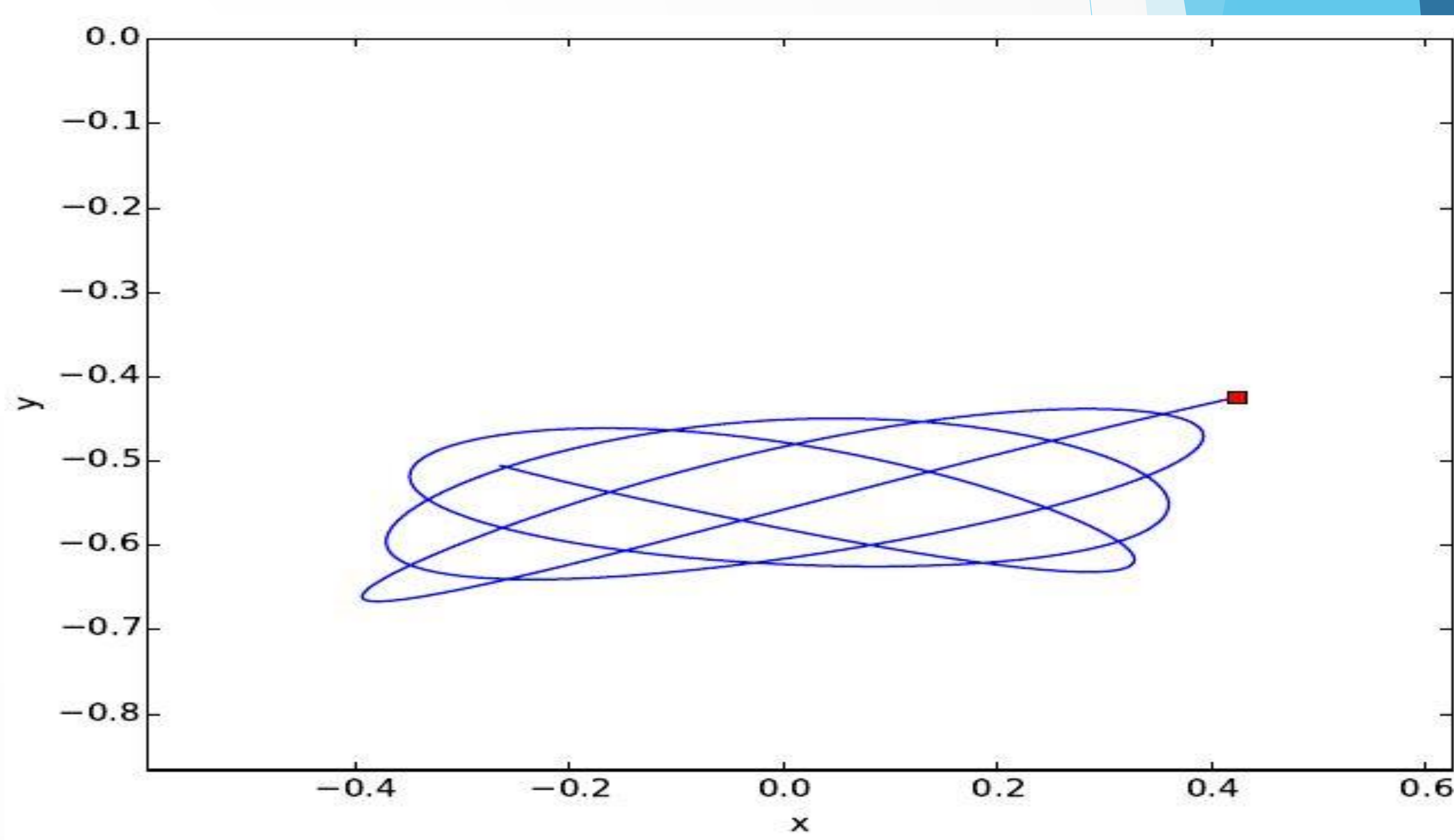


Figure 2. By adding in a linear drag coefficient of 0.05 to our previous conditions, we obtain a Y versus X graph that shows a slight decay in amplitude of the motion.

Conclusions

By utilizing the energies of our system, we were able to use a Lagrangian to find the equations of motion for a mass on a spring pendulum. Python was successfully able to solve the differential equations and produce a graph of the motion. Without drag, the motion was periodic and the amplitudes remained the same throughout time. When drag is added, the loss of energy from the motion is seen in the decrease in amplitudes in the X and Y directions.

Acknowledgments

Ithaca College Department of Physics and Astronomy

References

Taylor, John Robert. *Classical mechanics*. University Science Books, 2005.