

Binary class:

For dimensions 4 - 8, I use the greedy algorithm to estimate the minimum points required to memorize. The number of required points may be larger than the actual minimum number. Therefore, I cannot show the higher the dimension, the n / n_{avg} approaches to 2. However, I can show the limit of 2 predictions per parameter.

```
> python3 binary_d_2.py
class 2: d=2, n=4, avg points needed to memorize: n_avg=2.625, n/n_avg=1.5238095238095237
class 2: d=3, n=8, avg points needed to memorize: n_avg=4.4609375, n/n_avg=1.7933450087565674
100%|
class 2: d=4, n=16, avg points needed to memorize: n_avg=8.734375, n/n_avg=1.8318425760286225
100%|
class 2: d=5, n=32, avg points needed to memorize: n_avg=17.953125, n/n_avg=1.7824194952132288
100%|
class 2: d=6, n=64, avg points needed to memorize: n_avg=37.875, n/n_avg=1.6897689768976898
100%|
class 2: d=8, n=256, avg points needed to memorize: n_avg=159.0625, n/n_avg=1.6094302554027504
```

3 classes:

For dimensions 4 - 8, I use the greedy algorithm to estimate the minimum points required to memorize as the binary one. The n / n_{avg} could not exceed 1.5.

```
class 3: d=2, n=4, avg points needed to memorize: n_avg=3.0, n/n_avg=1.3333333333333333
class 3: d=3, n=8, avg points needed to memorize: n_avg=5.744398719707362, n/n_avg=1.3926609886173684
100%|
class 3: d=4, n=16, avg points needed to memorize: n_avg=11.640625, n/n_avg=1.374496644295302
100%|
class 3: d=5, n=32, avg points needed to memorize: n_avg=23.109375, n/n_avg=1.3847194050033806
100%|
class 3: d=6, n=64, avg points needed to memorize: n_avg=47.21875, n/n_avg=1.3553937789543349
100%|
class 3: d=8, n=256, avg points needed to memorize: n_avg=197.609375, n/n_avg=1.295485095279513
```

4 classes:

For dimensions 3 - 8, I use the greedy algorithm to estimate the minimum points required to memorize. The n / n_{avg} could not exceed 1.333.

```
class 4: d=2, n=4, avg points needed to memorize: n_avg=3.34375, n/n_avg=1.1962616822429906
100%|
class 4: d=3, n=8, avg points needed to memorize: n_avg=6.515625, n/n_avg=1.2278177458033572
100%|
class 4: d=4, n=16, avg points needed to memorize: n_avg=12.46875, n/n_avg=1.2832080200501252
100%|
class 4: d=5, n=32, avg points needed to memorize: n_avg=25.078125, n/n_avg=1.27601246105919
100%|
class 4: d=6, n=64, avg points needed to memorize: n_avg=51.671875, n/n_avg=1.2385848200786211
100%|
class 4: d=8, n=256, avg points needed to memorize: n_avg=211.984375, n/n_avg=1.2076361760153314
```