Math101 Homework 0

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Original statement: $\exists l$ s.t. $\forall \epsilon > 0, \exists$ N $\in N$ s.t. if $n >$ N then $|n - l| < \epsilon$
Negation: Not $\exists l$ s.t. $\forall \epsilon > 0, \exists$ N $\in N$ s.t. if $n >$ N then $|n - l| < \epsilon$
$\Rightarrow \forall l$ not $\forall \epsilon > 0, \exists$ N $\in N$ s.t. if $n >$ N then $|n - l| < \epsilon$
$\Rightarrow \forall l \; \exists \epsilon > 0$, not $\exists$ N $\in N$ s.t. if $n >$ N then $|n - l| < \epsilon$
$\Rightarrow \forall l \; \exists \epsilon > 0 \; \forall$ N $\in N$ not $\forall n >$ N then $|n - l| < \epsilon$
$\Rightarrow \forall l \; \exists \epsilon > 0 \; \forall$ N $\in N \; \exists n >$ N s.t. $|n - l| \geq \epsilon$

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Base case: $n_0 = 2$
$p(n_0) =$ the product of two odd numbers is odd
$p(n) =$ the product of n odd numbers is odd (where $n \geq 2$)

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a) $\forall x \in R, \exists n \in N s.t. n > x$
True, both R and N are infinte sets thus for any given $x \in R$ there must be an integer in N greater than x.
b) $\exists n \in N s.t. \forall x \in R \; n > x$
False, both sets are infite; further $N \subseteq R$. Thus if $n \in N$, then $n \in R$. Therefore $\exists x \in R$ s.t. $x \geq n$.

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Two Rational Numbers:
Let $\frac{a}{b} \in Q$ and $\frac{c}{d} \in Q$ where $b \neq 0$ and $d \neq 0$, and $a, b, c, d \in Z$

Multiplication:
$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ and given Z is closed under multiplaction $\Rightarrow ac, bd \in Z \Rightarrow \frac{ac}{bd} \in Q$.
Thus the product of two rational numbers is a rational number.

Addition:
$\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad+bc}{bd}$
Given that Z is closed under multiplication $ad, bc, bd \in Z$. And given Z is closed under addition $\Rightarrow ad + bc \in Z$. Therefore, since $bd, ad + bc \in Z \Rightarrow \frac{ad+bc}{bd} \in Q$.
Thus, the sum of two rational numbers is a rational number.

A Rational Number and An Irrational Number:
Let $\frac{a}{b} \in Q$, where $b \neq 0$, $a \neq 0$, and $a, b \in Z$ and Let $c \in R - Q$

Multiplication:
$\frac{a}{b} \cdot c = \frac{ac}{b}$. Assume that $ac \in Z$, then $\frac{ac}{a} \in Q$.
$\frac{ac}{a} = \frac{c}{1}$ but $c \notin Z$. Thus, $\frac{ac}{a} \notin Q$ and given that $a \neq 0$ then $ac \notin Z \Rightarrow \frac{ac}{b} \notin Q$
So the product of a rational number and an irrational number is an irrational number.

Addition:
$\frac{a}{b} + c = \frac{a}{b} + \frac{bc}{b} = \frac{a+bc}{b}$ but note that $bc \notin Z \Rightarrow a + bc \notin Z \Rightarrow \frac{a+bc}{b} \notin Q$
Thus, the sum of a rational number and an irrational number is an irrational number.

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$\boxed{\text{WTS: That } g \circ f \text{ is one-to-one and onto}}$

Onto:
Let $x \in R$ be any real number. We are given that g is onto $\Rightarrow \exists y \in R$ s.t. $g(y) = x$.
We are given that f is onto $\Rightarrow \exists z \in R$ s.t. $f(z) = y$.
$g(f(z)) = g(y) = x, \forall x \in R \Rightarrow g \circ f$ is onto.

One-to-One:
Let $a, b \in R$ and assume $g(f(a)) = g(f(b))$
We are given that g is one-to-one $\Rightarrow f(a) = f(b)$
We are given that f is one-to-one as well $\Rightarrow a = b$
Thus for any $a, b \in R$ if $g(f(a)) = g(f(b))$ then $a = b \Rightarrow g \circ f$ is one-to-one