Math 101 HW 13

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February 22, 2017

9.10

(a)

Q : Show that if $\lim s_n = +\infty$ and $k > 0$, then $\lim(ks_n) = +\infty$.

Let $\lim\ s_n = +\infty$ and $k, M > 0$. Thus, $\exists N \in \mathbb{N}$ s.t. if $n > N$ then $s_n > \frac{M}{k}$. Let $n > N$. Thus, $ks_n > k\frac{M}{k} = M \Rightarrow ks_n > M$. $\therefore \lim(ks_n) = +\infty$.

(b)

Q : Show $\lim(s_n) = +\infty$ if and only if $\lim(-s_n) =$ -∞.

($\Rightarrow$)
Assume that $\lim(s_n) = +\infty$. Thus, $\forall M > 0\ \exists N \in \mathbb{N}$ s.t. if $n > N$ then $s_n > M$. Let $M > 0$ and $n > N$. Then $s_n > M$. So, $-s_n < -M$. $\therefore \lim(-s_n) = -\infty$.

(c)

Q : Show that if $\lim(s_n) = +\infty$ and $k < 0$, then $\lim(ks_n) = -\infty$.

Let $\lim\ s_n = +\infty$ and $M > 0$ and $k < 0$. Thus, $\exists N \in \mathbb{N}$ s.t. if $n > N$ then $s_n > \frac{M}{-k}$. Let $n > N$. Thus, $ks_n < k\frac{M}{-k} = -M \Rightarrow ks_n < -M$. $\therefore \lim(ks_n) = -\infty$.

1

Q : Suppose that $s_n \to \infty$ and $t_n \to a$ with $a < 0$. Prove that $s_n t_n \to -\infty$

Let $M < 0$. Then $\frac{2M}{a} > 0$. Since $s_n \to \infty$, $\exists N_1 \in \mathbb{N}$ s.t. if $n > N_1$ $s_n > \frac{2M}{a}$. Let $\varepsilon = \frac{-a}{2}$. Since $t_n \to a$, $\exists N_2 \in \mathbb{N}$ s.t. if $n > N_2$ then $|t_n - a| < \varepsilon$. Thus $a + \frac{a}{2} = a - \varepsilon < t_n < a + \varepsilon < \frac{a}{2} \Rightarrow t_n < \frac{a}{2}$. Let $n > \max\{N_1, N_2\}$. Then $s_n t_n < s_n \frac{a}{2} < \frac{2M}{a}\frac{a}{2} = M \Rightarrow s_n t_n < M$. $\therefore s_n t_n \to -\infty$.

3

Q : Suppose that $s_n \to -\infty$ and $k < 0$. Prove that $ks_n \to \infty$.

Since $s_n \to -\infty$ $\forall M < 0$ $\exists N_1 \in \mathbb{N}$ s.t. if $n > N_1$ then $s_n < M$. Thus, $\forall M > 0$ and $\forall k < 0$ $\exists N \in \mathbb{N}$ s.t. if $n > N$ then $s_n < \frac{M}{k}$. Let $k < 0$ and $M > 0$. Then $s_n < \frac{M}{k} \Rightarrow ks_n > k\frac{M}{k} = M$. Hence, $ks_n > M$. $\therefore ks_n \to \infty$.