Math 101 Homework 7

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1

Let S be a non-empty set of integers which is bounded above. So $\exists b \in \mathbb{R}$ s.t. b = lub(S). Let b = lub(S). $b - 1 < b \Rightarrow b - 1$ is not an upper bound for S. So $\exists x \in$ S s.t. $x > b - 1$. Let $x \in$ S where $x > b - 1$ and let $s \in$ S. Since b = lub(S), $s \leq b$. Also $x > b - 1$ so $x + 1 > b$.

$$s \leq b < x + 1$$

$$s < x + 1$$

$$s - (x + 1) < 0 \quad , s \in \mathbb{Z}, x + 1 \in \mathbb{Z}$$

Two distinct integers differ by at least 1. Since s and $x + 1$ are distinct integers and $s < x + 1$:

$$s - (x + 1) \leq -1$$

$$s - x - 1 \leq -1$$

$$s - x \leq 0$$

$$s \leq x$$

Therefore x is an upper bound for S and $x \in$ S. $\Rightarrow x$ is the largest element of S.

2

The LUB Axium states that every non-empty set of reals which is bounded above has a lub. Let A $\subseteq \mathbb{R}$ where A $\neq \emptyset$ and A is bounded below. By the LUB Axiom, A has a lub. Let p = lub(A). Let B = $\{-a|a \in$A$\}$. B is bounded below and glb(B) = -p, as we proved in Homework 4. Thus, the LUB Axiom implies the GLB Axiom.

3

Let A be a non-empty set of reals that is bounded above. Assume that a and b are both least upper bounds for A. Now assume a $\neq$ b. WLOG a $<$ b. By definition of a lub, if b is a lub then any number smaller than b is not an upper bound. But a $<$ b and a is an upper bound $\Rightarrow\Leftarrow$. Thus if a and b are lubs then a = b.

4

Let $a, b \in \mathbb{R}$ s.t. $a \neq b$. WLOG $a < b$. So$\sqrt{2} + a <\sqrt{2} + b$. Note that by the density of the rationals, there exists a rational number between$\sqrt{2} + a$ and $\sqrt{2} + b$. Let c be a rational number s.t. $\sqrt{2} + a < c <\sqrt{2} + b$. From this we know that $a < c -\sqrt{2} < b$. $c -\sqrt{2}$ is irrational, thus between any two distinct reals there is an irrational number.