Math 101 HW 9

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Q : Let $\{x_n\}$and $\{y_n\}$be sequences, and let $x_n \to l$. Suppose that there exists an $M \in \mathbb{N}$ such that for all $n > M$, $y_n = x_n$. Prove that $y_n \to l$.

Since $x_n \to l$, $\forall \varepsilon > 0$, $\exists N \in \mathbb{N}$ s.t. if $n > N$, then $|x_n - l| < \varepsilon$. Either $M > N$ or $N \geq M$.

Case 1: Assume $M > N$

Since $\forall \varepsilon > 0$ and $\forall n > N$ $|x_n - l| < \varepsilon$, we know that $\forall n > M$ $|x_n - l| < \varepsilon$. But $\forall n > M$, $y_n = x_n$, so $\forall n > M$ $|y_n - l| < \varepsilon$. $\therefore y_n \to l$.

Case 2: Assume $N \geq M$

Since $\forall \varepsilon > 0$ and $\forall n > N$ $|x_n - l| < \varepsilon$ and $N \geq M$, then $\forall n > N$, $y_n = x_n$. Thus, $\forall \varepsilon > 0$ and $\forall n > N$, $|y_n - l| < \varepsilon$. $\therefore y_n \to l$

2

Q : Suppose that $\{x_n\}$is a sequence of integers which converges to l. Prove that there exists an $N \in \mathbb{N}$ s.t. for all $n > N$, $x_n = l$.

$\forall \varepsilon > 0$, $\exists N \in \mathbb{N}$ s.t. if $n > N$, then $|x_n - l| < \varepsilon$. Let $\varepsilon < \frac{1}{2}$. $\exists N \in \mathbb{N}$ s.t. if $n > N$, then $|x_n - l| < \varepsilon \Rightarrow -\varepsilon < x_n - l < \varepsilon \Rightarrow l - \varepsilon < x_n < l + \varepsilon$. $x_n \in (l - \varepsilon, l + \varepsilon)$. Thus, there is at least one integer in the interval $(l - \varepsilon, l + \varepsilon)$.

But because $\varepsilon < \frac{1}{2}$ we know that $(l - \varepsilon, l + \varepsilon)$ contains at most one integer. Let $z \in \mathbb{Z}$ s.t. $z \in (l - \varepsilon, l + \varepsilon)$. $z, x_n \in (l - \varepsilon, l + \varepsilon)$ but because this interval contains only one integer we know that $x_n = z$. Thus, $x_n \to z$. We are given that $x_n \to l$ so by uniqueness of limits we know that $z = l = x_n$

□

3

> **Q** : Suppose $z_n \to l$ and $l \neq 0$. Prove that there is an $N \in \mathbb{N}$ such that if $n > N$ then $|z_n| > \frac{|l|}{2}$.

$\forall \varepsilon > 0$, $\exists N \in \mathbb{N}$ s.t. if $n > N$, then $|z_n - l| < \varepsilon$. Let $\varepsilon = \frac{|l|}{2}$. Then $\exists N \in \mathbb{N}$ s.t. if $n > N$, then $|z_n - l| < \varepsilon$. Let $n > N$.

$$|z_n - l| < \epsilon$$

$$|z_n - l| \leq |z_n| - |l|$$

$$-|z_n - l| \geq |l| - |z_n|$$

$$|l| - |z_n| \leq -|z_n - l| \leq |z_n - l| < \varepsilon$$

$$|l| - |z_n| < \varepsilon$$

$$-|z_n| < -|l| + \varepsilon$$

$$|z_n| > |l| - \varepsilon$$

$$|z_n| > |l| - \frac{|l|}{2}$$

$$|z_n| > \frac{|l|}{2}$$

4

> **Q** : Let $k \in \mathbb{N}$ and suppose that $\{x_n\}$and $\{y_n\}$are sequences such that $x_n \to l$ and for all n, $y_n = x_{n+k}$. Prove that $y_n \to l$.

Since $x_n \to l$ we know that $\forall \varepsilon > 0$ $\exists N \in \mathbb{N}$ s.t. if $n > N$, then $|x_n - l| < \varepsilon$.$\Rightarrow$ $\forall \varepsilon > 0$ if $n + k > N$, then $|x_{n+k} - l| < \varepsilon \Rightarrow |y_n - l| < \varepsilon$. Thus, $y_n \to l$.