Math 101 HW 10

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Please grade 8.1c, 8.3, and 8.5

8.1c

Q : Prove that $\lim \frac{2n-1}{3n+2} = \frac{2}{3}$

Let $\varepsilon > 0$ and $N = \frac{-7}{9\varepsilon} - \frac{2}{3}$. Then $n > N$ implies $n > \frac{-7}{9\varepsilon} - \frac{2}{3}$, hence $3n > \frac{-7}{3\varepsilon} - 2$, hence $3n + 2 > \frac{-7}{3\varepsilon}$, hence $\frac{-7}{3(3n+2)} < \varepsilon$. $\frac{2n-1}{3n+2} - \frac{2}{3} = \frac{-7}{3(3(n+2)} \Rightarrow \frac{2n-1}{3n+2} - \frac{2}{3} < \epsilon$. Thus, $\lim \frac{2n-1}{3n+2} = \frac{2}{3}$.

$\square$

8.3

Q : Let (s_n) be a sequence of nonnegative real numbers, and suppose $\lim s_n = 0$. Prove $\lim \sqrt{s_n} = 0$.

Assume that $\lim \sqrt{s_n} \neq 0$. Then by the multiplication rule $\lim \sqrt{s_n}\sqrt{s_n} \neq 0$. But $\sqrt{s_n}\sqrt{s_n} = s_n$ and $\lim s_n = 0 \Rightarrow\Leftarrow$. Thus, $\lim \sqrt{s_n} = 0$.

$\square$

8.4

> **Q** : Let (t_n) be a bounded sequence, i.e., there exists M such that $|t_n| \leq M$ for all n, and let (s_n) be a sequence such that $\lim s_n = 0$. Prove $\lim(s_n t_n) = 0$.

8.5

(a)

> **Q** : Consider three sequences (a_n), (b_n), and (s_n) s.t. $a_n \leq s_n \leq b_n$ for all n and $\lim a_n = \lim b_n = s$. Prove $\lim s_n = s$.

Since $\lim a_n = s$, $\forall \varepsilon > 0$, $\exists N_1 \in \mathbb{N}$ s.t. if $n > N_1$, then $|a_n - s| < \varepsilon \Rightarrow -\varepsilon < a_n - s < \varepsilon \Rightarrow s - \varepsilon < a_n < s + \varepsilon$. Since $\lim b_n = s$, $\forall \varepsilon > 0$, $\exists N_2 \in \mathbb{N}$ s.t. if $n > N_e$, then $|b_n - s| < \varepsilon \Rightarrow -\varepsilon < b_n - s < \varepsilon \Rightarrow s - \varepsilon < b_n < s + \varepsilon$. Let $\varepsilon > 0$ and $N = \max\{N_1, N_2\}$ and $n > N$, then $s - \varepsilon < a_n \leq b_n < s + \varepsilon$. Since $\forall n a_n \leq s_n \leq b_n$, $s - \varepsilon < a_n \leq s_n \leq b_n < s + \varepsilon$. Thus, $s - \varepsilon < s_n < s + \varepsilon \Rightarrow -\varepsilon < s_n - s < \varepsilon \Rightarrow |s_n - s| < \varepsilon$. $\therefore \lim s_n = s$.

$\square$

(b)

> **Q** : Suppose (s_n) and (t_n) are sequences such that $|s_n| \leq t_n$ for all n and $\lim t_n = 0$. Prove $\lim s_n = 0$.

By statement of the problem, $\forall \varepsilon > 0$, $\exists N \in \mathbb{N}$ s.t. if $n > N$, then $|t_n| < \varepsilon$. Let $\varepsilon > 0$ and $n > N$. We have that $|s_n| \leq t_n \leq |t_n| \Rightarrow |s_n| \leq |t_n| < \varepsilon \Rightarrow |t_n| < \varepsilon$. $\therefore \lim s_n = 0$.

$\square$