Math 101 HW 14

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9.11

(a)

> **Q** : Show that if $\lim\, s_n = +\infty$ and $\text{glb}\{t_n : n \in \mathbb{N}\} > -\infty$, then $\lim(s_n + t_n) = +\infty$.

Let $M > 0$. Since $\lim\, s_n = +\infty$, $\exists N \in \mathbb{N}$ s.t. if $n > N$ then $s_n > M -$ glb$\{t_n : n \in \mathbb{N}\}$. Let $n > N$. We have that $M \leq M -$ glb$\{t_n : n \in \mathbb{N}\} + t_n < s_n + t_n$. Hence, $M < s_n + t_n$. $\therefore \lim(s_n + t_n) = +\infty$.

(b)

> **Q** : Show that if $\lim\, s_n = +\infty$ and $\lim\, t_n > -\infty$, then $\lim(s_n + t_n) = +\infty$.

Let $M > 0$. Since $\lim\, t_n > -\infty$ $\exists M_1 > -\infty$ s.t. $\exists N_1 \in \mathbb{N}$ s.t. if $n > N_1$ then $t_n > M_1$. Since $\lim\, s_n = +\infty$ then $\exists N_2 \in \mathbb{N}$ s.t. if $n > N_2$ then $s_n > M - M_1$. Let $n > \max\{N_1, N_2\}$. Then $M = M - t_n + t_n \leq M - M_1 + t_n < s_n + t_n$. Hence $M < s_n + t_n$. $\therefore \lim(s_n + t_n) = +\infty$.
(c)

> **Q** : Show that if $\lim\, s_n = +\infty$ and if $\{t_n\}$ is a bounded sequence, then $\lim(s_n + t_n) = +\infty$.

Let $M > 0$. Since $\{t_n\}$ is bounded, $\exists M_1 > 0$ s.t. $\forall n \in \mathbb{N}$, $|t_n| \leq M_1 \Rightarrow -M_1 \leq t_n \leq M_1$. Since $\lim s_n = +\infty$, $\exists N \in \mathbb{N}$ s.t. if $n > N$ then $M - M_1 < s_n$. Let $n > N$. Then $M = M - t_n + t_n \leq M - M_1 + t_n < s_n + t_n$. Hence $M < s_n + t_n$. $\therefore \lim(s_n + t_n) = +\infty$.

1

Q : Let $x_n \to l$. Let $\{y_n\}$ be a sequence obtained by rearranging the order of the terms of $\{x_n\}$. Prove that $y_n \to l$

Since $x_n \to l$ we have $\forall \varepsilon > 0$ $\exists N \in \mathbb{N}$ s.t. if $n > N$ then $|x_n - l| < \varepsilon$. Let $\varepsilon > 0$. Thus there are an infinite number of terms in the sequence $\{x_n\}$ for which $|x_n - l| < \varepsilon$. Just as important, there are a finite number of terms in the sequence $\{x_n\}$ for which $|x_n - l| < \varepsilon$ is not true. So if $\{y_n\}$ is a sequence obtained by rearranging the terms of $\{x_n\}$ then there are an finite number of terms in the sequence $\{y_n\}$ for which $|y_n - l| < \varepsilon$ is not true and there are an infinite number of terms in the sequence $\{y_n\}$ for which $|y_n - l| < \varepsilon$ is not true. Since the head of a sequence is finite then there must be a tail in $\{y_n\}$ for which $|y_n - l| < \varepsilon$ is true. Thus $\exists N_1 \in \mathbb{N}$ s.t. if $n > N_1$ then $|y_n - l| < \varepsilon \Rightarrow y_n \to l$.

3

Q : Suppose that S is a set of real numbers which is not bounded above. Prove that there exists a sequence $\{x_n\}$ contained in S, such that $x_n \to \infty$

Since S is not bounded above $\forall M > 0$ $\exists s \in S$ s.t. $s > M$. In other words if we create a sequence with all the terms of S and call it $\{x_n\}$, then $\forall M > 0$ $\{x_n\}$ will have a tail greater than M. But this is precisely the definition of diverging to infinity. Thus $x_n \to \infty$.