Math 101 HW 32

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1

Q : Suppose that $f_n : [0,1] \to \mathbb{R}$ by $f_n(x) = x^n$. Find the function f that f_n converges to.

Let $f(x) = \begin{cases} 0 & \text{if } 0 \leq x < 1 \\ 1 & \text{if } x = 1 \end{cases}$
If $x \in [0,1)$ then $\lim_{x\to\infty} x^n = 0$ and if $x = 1$ then $\lim_{x\to\infty} x^n = 1$. Thus, f_n converges pointwise to f.

2

Q : Let f_n be the sequence of functions on $(0,\infty)$ defined by $f_n(x) = \frac{nx}{1+n^2x^2}$. Does this function converge pointwise or unifomly, neither pointwise or uniformly, or both pointwise and uniformly?

$f_n(x) = \frac{nx}{1+n^2x^2} = \frac{x}{\frac{1}{n}+nx^2}$. So $\lim_{x\to\infty} f_n(x) = \lim_{x\to\infty} \frac{x}{\frac{1}{n}+nx^2} = 0$. Thus f_n converges pointwise to $f(x) = 0$.

3

Q : Let $g_n(x) = \frac{1}{n(1+x^2)}$. Does this function converge uniformly on $\mathbb{R}$.

Let $\varepsilon > 0$. Let $N = \frac{1}{\varepsilon}$. Now let $n > N$ and let $x \in \mathbb{R}$. Claim: $g_n(x) \to g$ uniformly where $g(x) = 0$.

$$|g_n(x) - g(x)| = |\frac{1}{n(1+x^2)} - 0| = |\frac{1}{n(1+x^2)}| \leq \frac{1}{n} < \frac{1}{N} < \varepsilon$$

Thus g_n converges to g uniformly where $g(x) = 0$.