# Math 101 HW 31

Jeff Carney

April 26, 2017

## 1

**Q** : Suppose $\{x_n\}$ is a sequence that diverges, but does not diverge to either $+\infty$ or $-\infty$. Either prove that at least one of lim sup $x_n$ and lim inf $x_n$ are real numbers and are defined or provide a counterexample.

Since $\{x_n\}$ does not diverge to either $+\infty$ or $-\infty$ it must be bounded. By lemma 1 in the presentation we know that if a sequence is bounded its lim inf is a real number. Thus, lim inf $x_n$ is a real number.

## 2

**Q** : Suppose $\{x_n\}$ is a sequence that converges. What is the relationship between lim sup $x_n$ and lim inf $x_n$? Be sure to prove this relationship

By lemma 5 in the presentation we know that a sequence converges iff its lim sup equals its lim inf. Thus, lim sup $x_n$ = lim inf $x_n$.

## 3

**Q** : Suppose that $\{x_n\}$ and $\{y_n\}$ are sequences of real numbers such that $\forall n \in \mathbb{N}$ $\{x_n\} \leq \{y_n\}$ . Show that lim sup $x_n \leq$ lim sup $y_n$ and lim inf $x_n \leq$ lim inf $y_n$.

Let $\{a_n\}$ and $\{b_n\}$ be sequences s.t. $\forall n \in \mathbb{N}$ $a_n = \sup\{x_n, x_{n+1}, ...\} \leq b_n = \sup\{y_n, y_{n+1}, ...\}$. Then we have lim sup $x_n = \lim a_n \leq \lim b_n =$ lim sup $y_n$. So lim sup $x_n \leq$ lim sup $y_n$.

Now let $\{c_n\}$ and $\{d_n\}$ be sequences s.t. $\forall n \in \mathbb{N}$ $c_n = \inf\{x_n, x_{n+1}, ...\} \leq d_n = \inf\{y_n, y_{n+1}, ...\}$. Then we have lim inf $x_n = \lim c_n \leq \lim d_n =$ lim inf $y_n$. So lim inf $x_n \leq$ lim inf $y_n$.

$\square$