Wearables: other functional approaches

ENAR 2021

Overview

- Multi-level functional methods
- Multi-level methods for generalized (e.g binary) functional curves
- Registration of generalized (e.g. binary) functional curves

Multi-level functional principal component analysis (MFPCA)

NIMH Family Study of Affective Spectrum Disorders

- 350 participants; ages from 10 to 84
- 5 diagnosis groups: BPI, BPII, MDD, Other and Control

Data:

- 2 weeks of follow-up measurements
- minute-by-minute activity counts
- 4945 person-days; 7,120,800 data points

Goal:

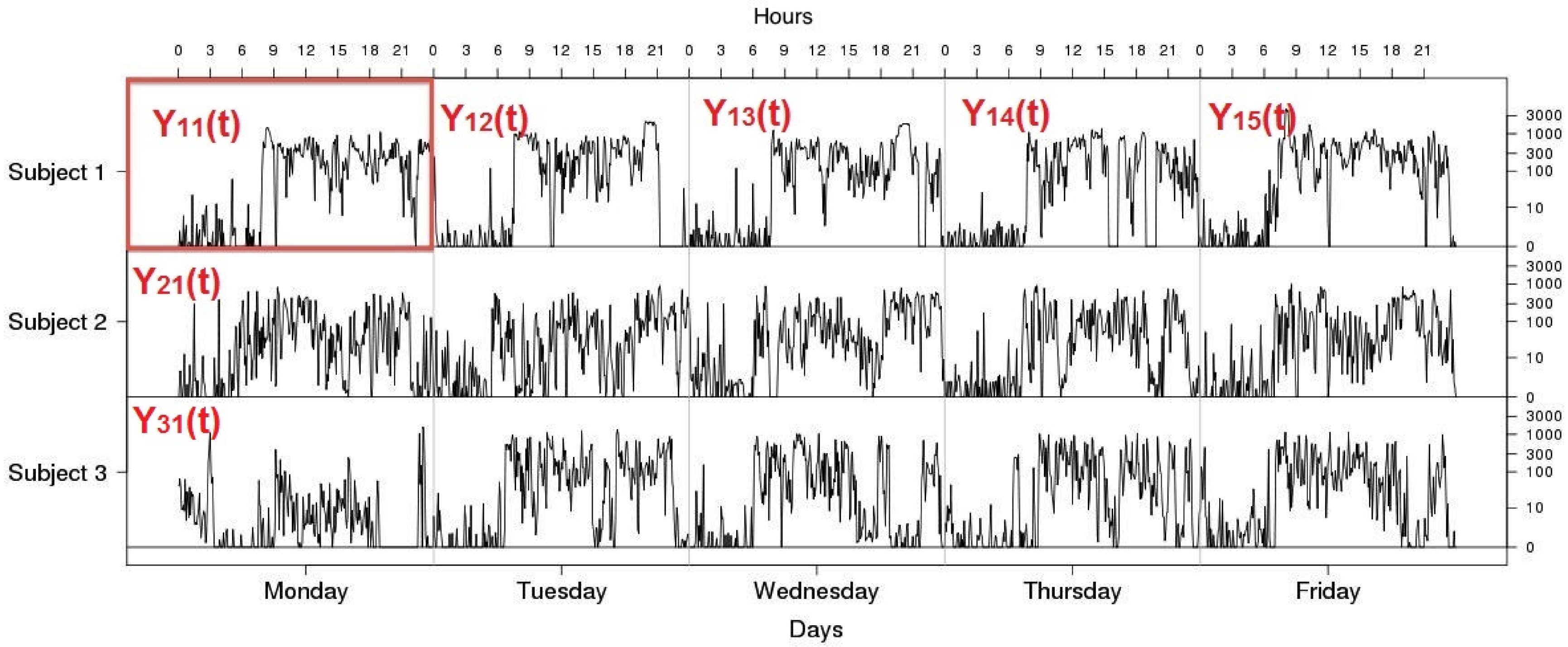
Extract representative patterns that comprise daily activity

Quantify multilevel variations after adjusting age and disease effects

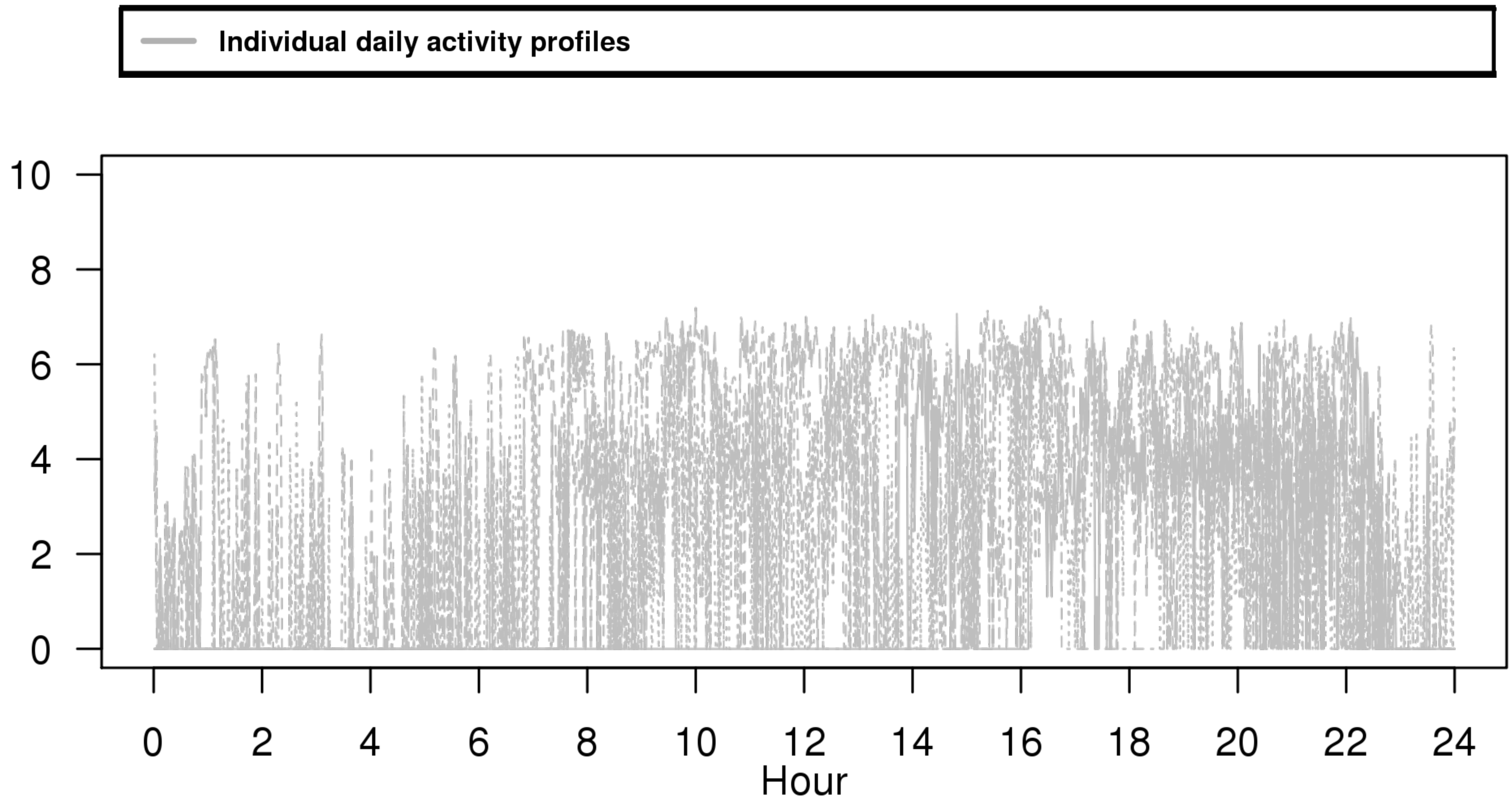
- Subject heterogeneity
- Day-to-day variation
- Age and disease effects in mean and variance components

Daily Activity Profiles

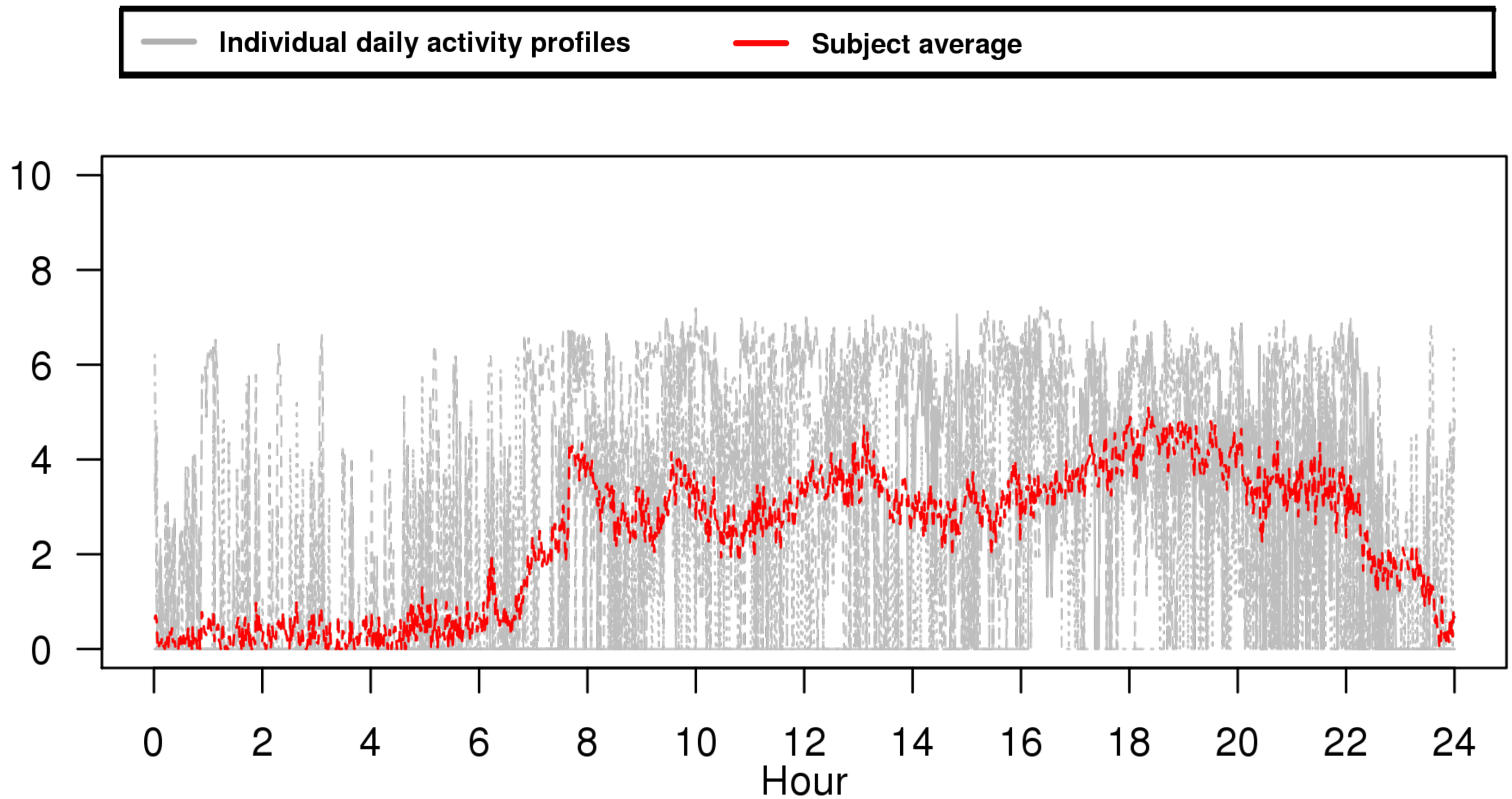
$$Y_{ij}(t),\ i = 1, 2, \cdots, I; j = 1, 2, \cdots, J_i$$



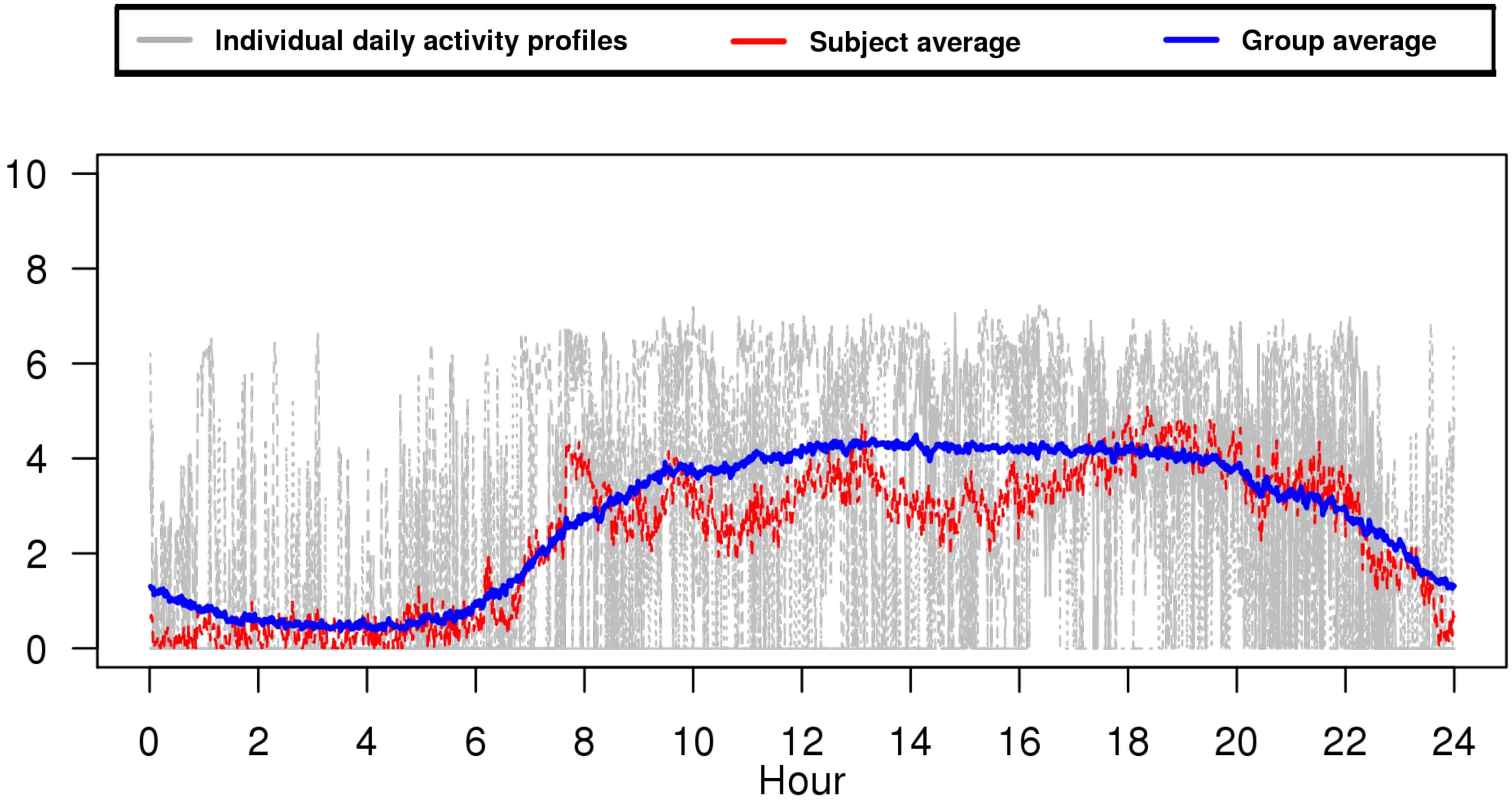
Control and age 67 (> 60)



Control and age 67 ($>$ 60)



Control and age > 60



Without loss of generality, assume $Y_{ij}(t) = Y'_{ij}(t) - \mu(t, v_i)$

- Decomposability and additivity

$$Y_{ij}(t) = X_i(t) + U_{ij}(t)$$

- Identifiability

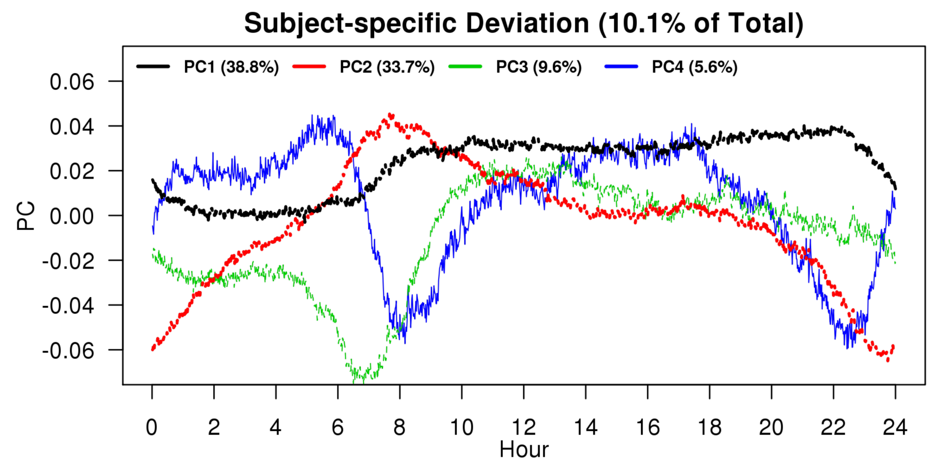
  Latent processes are mean zero and mutually uncorrelated.

- Data correlation captured by covariance operators

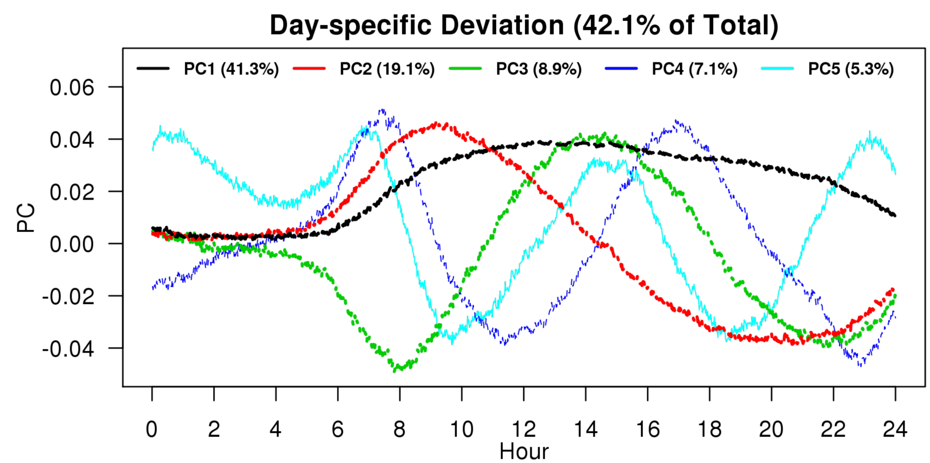
$$E\{Y_{ij}(t) - Y_{kl}(t)\}\{(Y_{ij}(s) - Y_{kl}(s)\}^T = \begin{cases} 2K_U(t,s) & i = k, j \neq l \\ 2\{K_X(t,s) + K_U(t,s)\} & i \neq k \end{cases}$$

  where $K_X(t,s) := \mathrm{Cov}\{X(t), X(s)\}$, similar definitions for K_U.

- Subject heterogeneity accounts for 10.1% of total variability
- The first 4 principal components explain 87.8% of the subject heterogeneity



- Day-specific deviation and random noise along the curve together accounts for the remaining 89.9% of total variability
- The first 5 principal components explain 81.8% of day-to-day variation

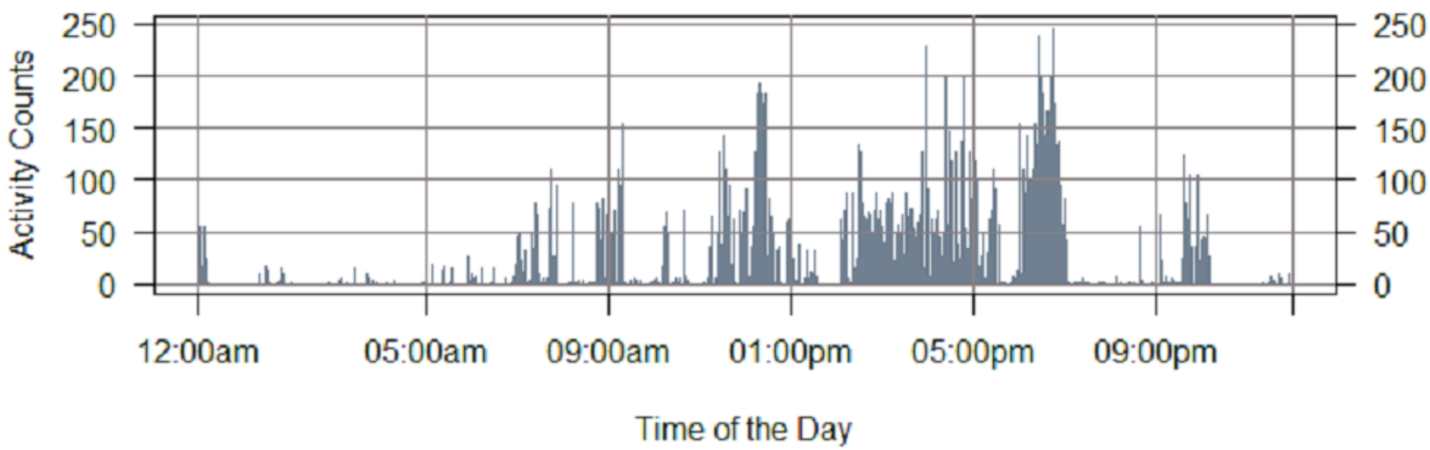


References

- Di, C.Z., Crainiceanu, C.M., Caffo, B.S. and Punjabi, N.M., 2009. Multilevel functional principal component analysis. *The annals of applied statistics*, *3*(1), p.458.
- Zipunnikov V., Caffo B.S., Yousem D.M, Davatzikos C., Schwartz B.S., Crainiceanu C. (2011),Multilevel Functional Principal Component Analysis for High-Dimensional Data. Journal of Computational and Graphical Statistics, 20(4), pp. 852-873
- Shou, H., Zipunnikov, V., Crainiceanu C., Greven, S. (2015) Structured Functional Principal Component Analysis
  Biometrics, 71 (1), pp. 247-757

Multi-level Generalized Function-on-Scalar Regression and Principal Component Analysis

- Binary time series $Y_{ij}(t)$: activity/inactivity
- Covariates $x_{ij,k}$: age, gender, BMI, etc

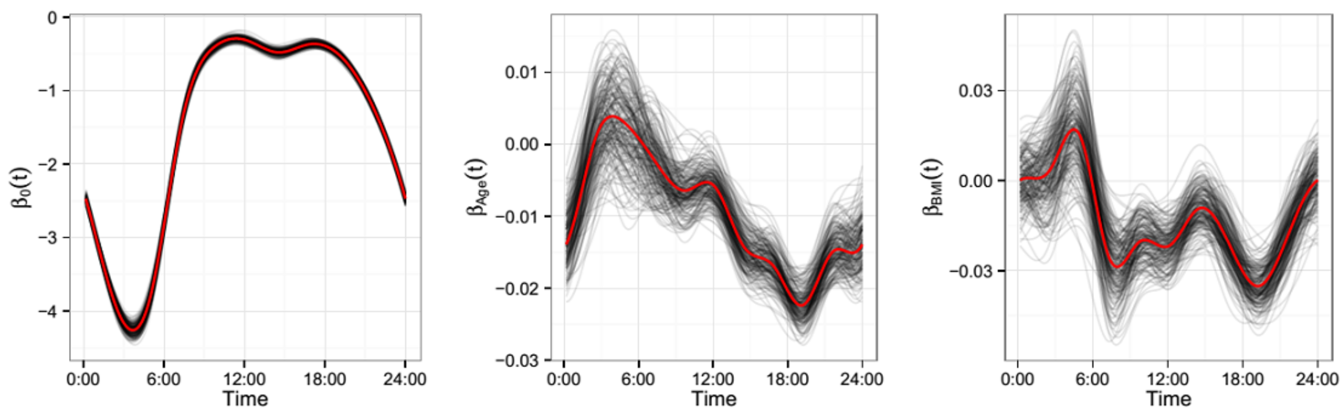


- Generalized Multilevel Functional-on-Scalar Regression

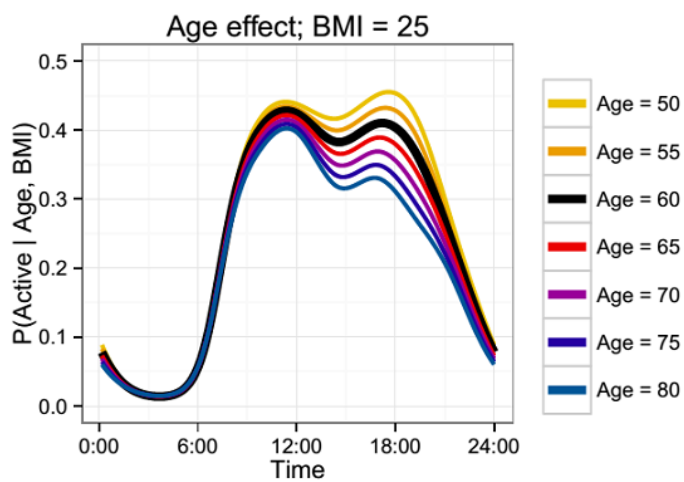
  - $Y_{ij}(t)$ is a generalized response curve
  - $Y_{ij}(t)$ comes from an exponential family

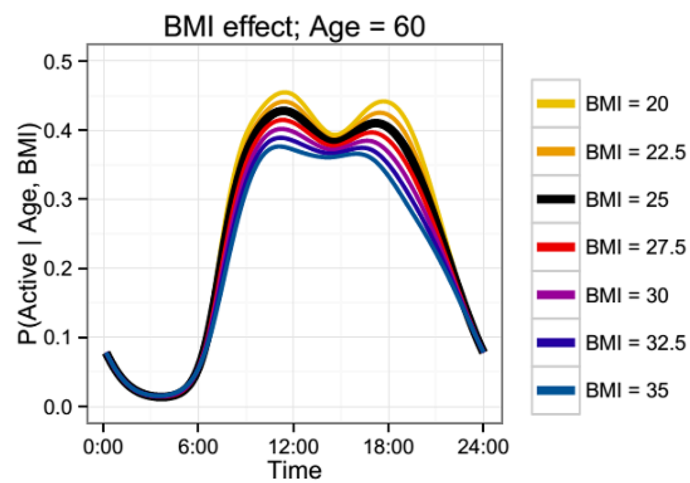
$$
\begin{aligned}
\mathrm{E}[Y_{ij}(t)|b_i(t), v_{ij}(t)] &= \mu_{ij}(t) \\
g(\mu_{ij}(t)) &= \beta_0(t) + \sum_{k=1}^{p} x_{ij,k}\beta_k(t) + b_i(t) + v_{ij}(t) \\
&\approx \beta_0(t) + \sum_{k=1}^{p} x_{ij,k}\beta_k(t) + \sum_{k=1}^{K^{(1)}} c_{ik}^{(1)}\psi_k^{(1)}(t) + \sum_{k=1}^{K^{(2)}} c_{ijk}^{(2)}\psi_k^{(2)}(t).
\end{aligned}
$$

- Generalized Multilevel Functional-on-Scalar Regression
  - Estimated functional mean, bmi, and age effects



- Estimated functional mean, bmi, and age effects



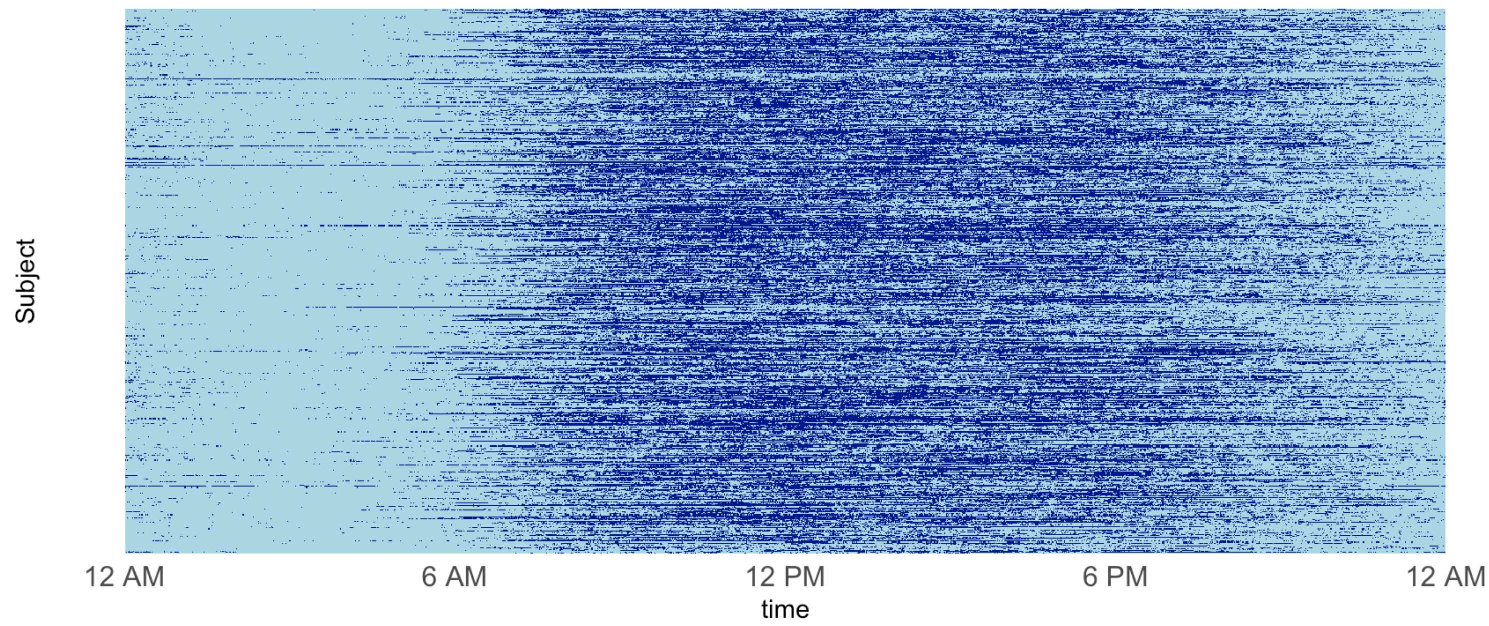


References:

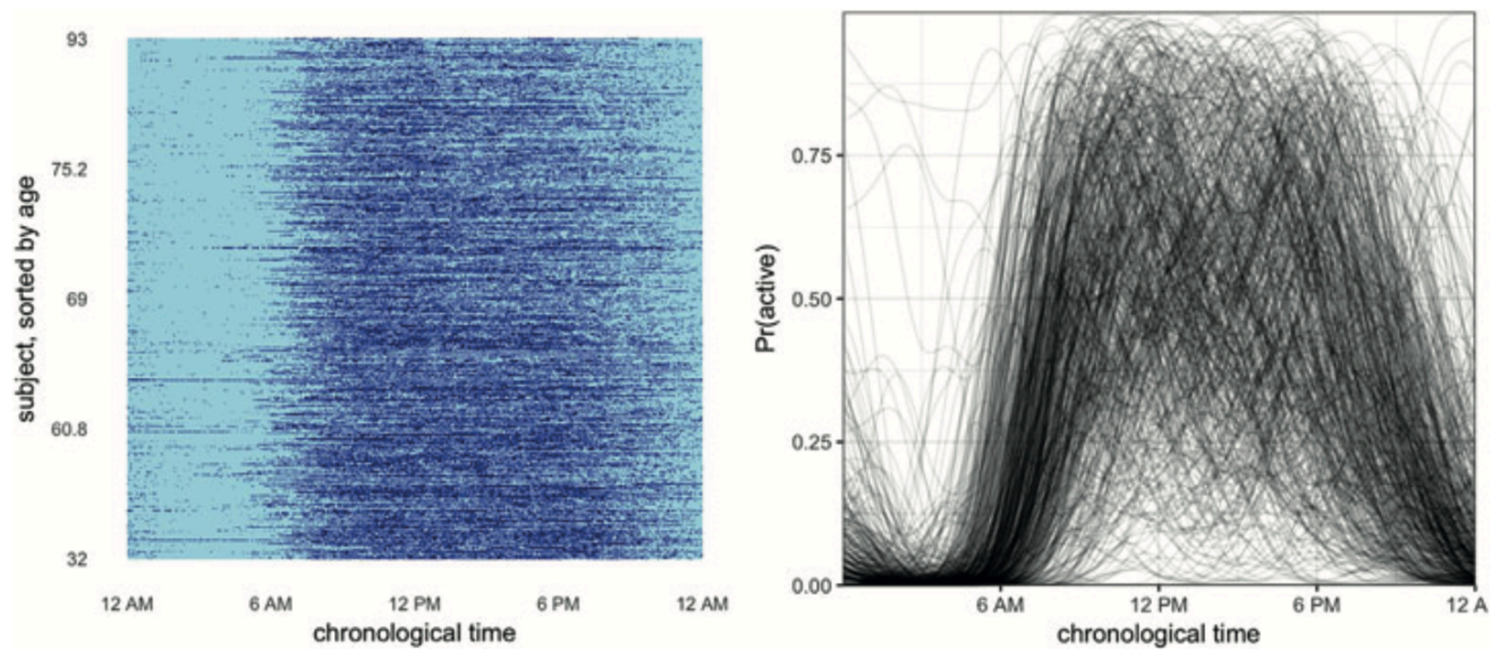
- Goldsmith, J., Zipunnikov, V, Schrack, J., Generalized Multilevel Function-on-Scalar Regression and Principal Component Analysis Biometrics, 71 (2), pp. 344-353

Registration of binary (0/1) actigraphy profiles

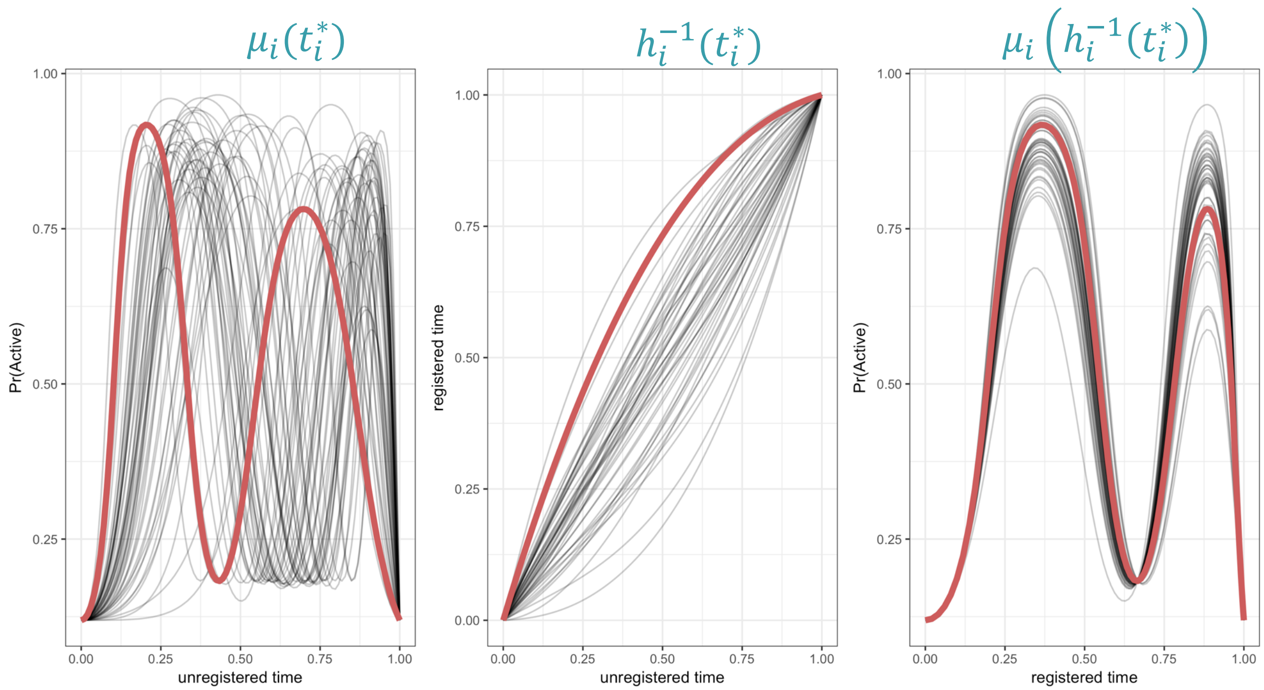
Binary "activity"



Binary "activity"



Registration



Exponential family registration

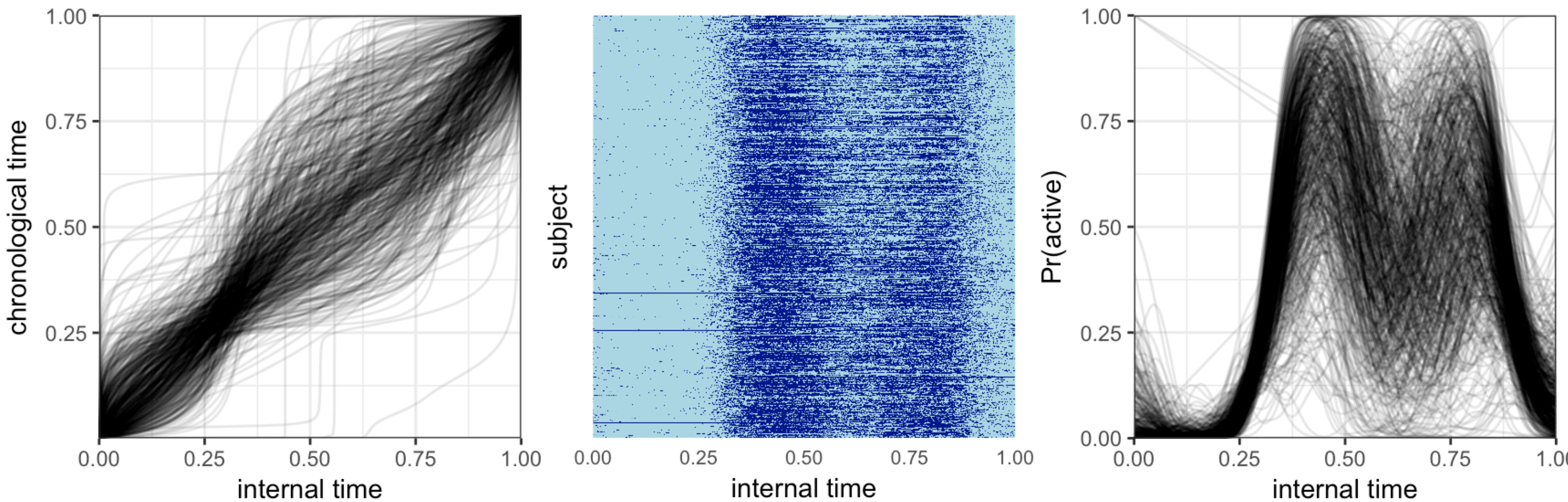
- Step 1
  - Underlying probability of being active is registration 'template'
  - Estimate template, $\mu_i(t)$, conditional on current estimate of time $\hat{t} = \hat{h}_i^{-1}(t_i^*)$

- Step 2
  - Inverse warping functions $h_i^{-1}(t_i^*)$ map from observed to registered time
  - Estimate inverse warping function, $h_i^{-1}(t_i^*) = t$
  - Warping step is MLE conditional on template estimate $\hat{\boldsymbol{\mu}}_i(t)$

- Iterate between step (1) and step (2) until alignment

- Focus on computational efficiency

BLSA – post-registration



References:

- Wrobel, J., Zipunnikov, V., Schrack, J. and Goldsmith, J., 2019. Registration for exponential family functional data. *Biometrics*, *75*(1), pp.48-57.