CSCE 452-500
Project Two Report
10/24/2019

Full Metal Kinematics

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Project Design/Description

The purpose of this project was to implement a three link robotic arm that can rotate around three joints as well as translate the end point in the x and y direction and paint at the location of its end effector in order to demonstrate the application of forward kinematics and inverse kinematics in the field of robotics. This project built upon project one by adding the translational movement of its end effector. As a result, the underlying object-oriented definition of the robot arms is identical to that of project one. The arm is still drawn by using a sequence of transformation matrix multiplication to obtain the endpoints of each joint in the original reference frame. Upon any change to the joint angle, the new transformation matrices are calculated, and the arms are redrawn. Upon clicking the translation buttons, the new (x, y) point is calculated, and the joint angles are developed using inverse kinematics shown in the appendix. Our group chose to change the event-based painting to a toggle-based painting following feedback from project one. Thus, the paint brush can now paint continuous lines.

Our group did not run into any notable challenges through this project. Development of the inverse kinematics was somewhat complicated, but the code application is trivial. There were no challenging design decisions as the project was an extension of project one. Because of this, long, detailed meetings were not necessary.

Installation Instructions

Please run the following command from within the project directory to install the necessary packages. Please note that this command requires both Pip 3 and Python 3 be installed.

```
pip install -r requirements.txt
```

Following this installation, please run the main.py file found in the directory folder, ex:

```
python main.py
```

Interface Screenshots

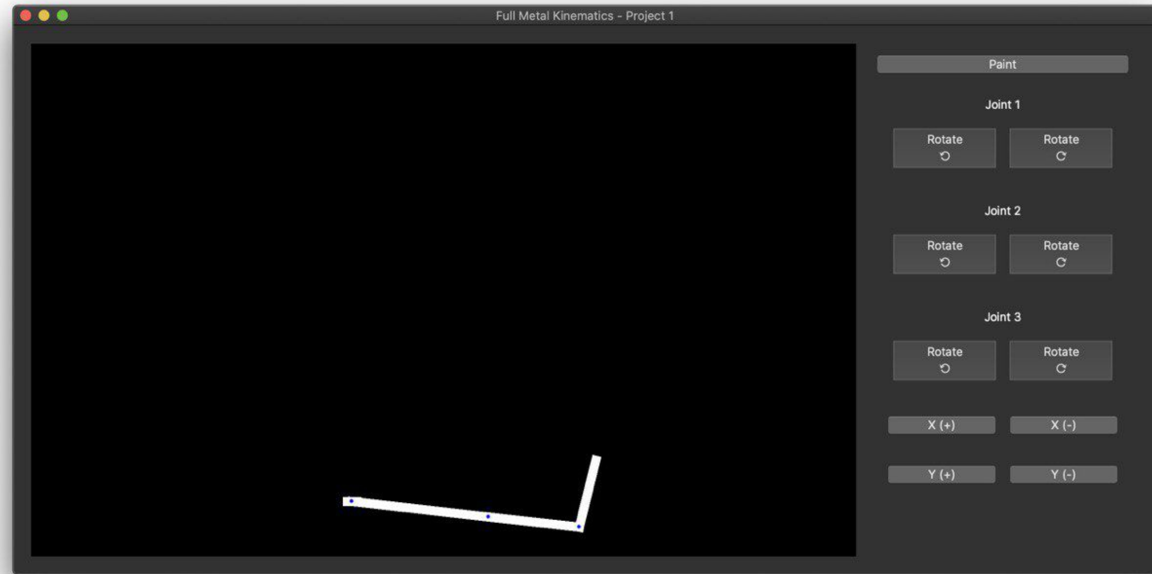


Figure 1: Demonstration of arm orientation in case II on Mac OS

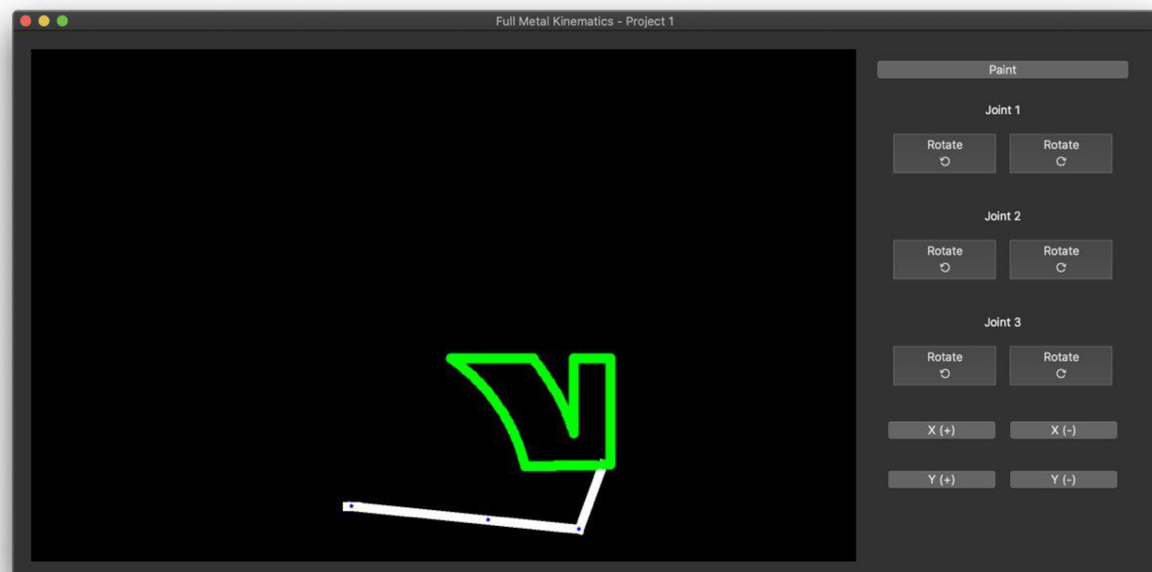


Figure 2: Demonstration of robot arm drawing capability on Mac OS

Task Management

Thomas McDonald:	Meeting Minutes, GUI modification
Jon Williamson:	Mathematical Description and Solution, Report
Jeff Hykin:	Website development
Alejandro Londono:	Code implementation
Mitchell Eldridge:	Code implementation

Meeting Logs

Project Two, Meeting One Minutes
10/21/2019

These minutes are a paraphrased summary of the meeting conversation.

1. Discuss What Has Been Done

   Jeff: I have developed the skeleton for Project 2 on the website.

   Jon: I have derived all of the math we need to solve this problem.

2. Discuss Next Steps

   Thomas: Let's discuss what is left.

   Mitchell: We can base this project on the last one and just add the new capabilities.

   Alejandro: We can implement equations in code tomorrow for the next meeting. Created repo for this project?

   Thomas: Sounds good. I will go ahead and add the new buttons before tomorrow.

Project Two, Meeting TwoMinutes
10/22/2019

These minutes are a paraphrased summary of the meeting conversation.

1. Discuss What Has Been Done

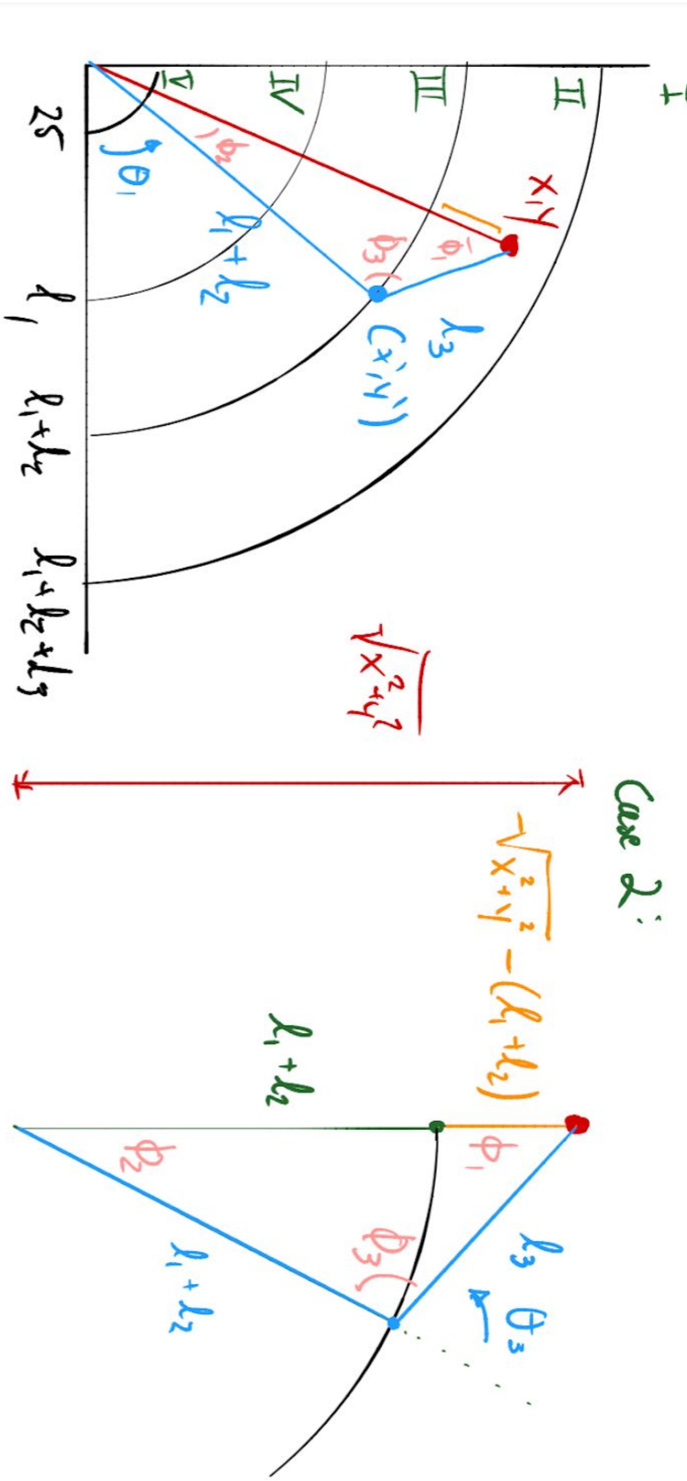
   Thomas: I implemented the buttons in the GUI.

Jeff: I continued to work on the website and framed this project..

2. Next Steps

    Group: [Begins to Code]

Appendix



Law of Cosines:

$$\left(\sqrt{x^2+y^2}\right)^2 = l_3^2 + (l_1+l_2)^2 - 2(l_3)(l_1+l_2)\cos(\phi_3)$$

$$\phi_3 = \cos^{-1}\left(\frac{l_3^2 + (l_1+l_2)^2 - (x^2+y^2)}{2(l_3)(l_1+l_2)}\right) = \cos^{-1}\left(\frac{l_1^2 + l_2^2 + l_3^2 + 2l_1l_2 - x^2 - y^2}{2(l_3)(l_1+l_2)}\right)$$

$$\pi - \phi_3 = \theta_3 \therefore$$

$$\theta_3 = \pi - \cos^{-1}\left(\frac{l_3^2 + (l_1+l_2)^2 - (x^2+y^2)}{2(l_3)(l_1+l_2)}\right)$$

Similarly,

$$l_3^2 = \left(\sqrt{x^2+y^2}\right)^2 + (l_1+l_2)^2 - 2\left(\sqrt{x^2+y^2}\right)(l_1+l_2)\cos(\phi_2)$$

$$\phi_2 = \cos^{-1}\left(\frac{x^2+y^2+(l_1+l_2)^2-l_3^2}{2\left(\sqrt{x^2+y^2}\right)(l_1+l_2)}\right) = \cos^{-1}\left(\frac{x^2+y^2+l_1^2+l_2^2-l_3^2+2l_1l_2}{2(l_1+l_2)\sqrt{x^2+y^2}}\right)$$

$$\tan^{-1}\left(\frac{y}{x}\right) = \phi_2 + \theta_1 \therefore$$

$$\theta_1 = \tan^{-1}\left(\frac{y}{x}\right) - \cos^{-1}\left(\frac{x^2+y^2+(l_1+l_2)^2-l_3^2}{2\left(\sqrt{x^2+y^2}\right)(l_1+l_2)}\right)$$

$$\theta_2 = 0$$

Case II Summary, defined by $l_1 + l_2 \le \sqrt{x^2+y^2} \le l_1 + l_2 + l_3$

$$\theta_1 = \tan^{-1}\left(\frac{y}{x}\right) - \cos^{-1}\left(\frac{x^2+y^2+(l_1+l_2)^2-l_3^2}{2\left(\sqrt{x^2+y^2}\right)(l_1+l_2)}\right)$$

$$\theta_2 = 0$$

$$\theta_3 = \pi - \cos^{-1}\left(\frac{l_3^2+(l_1+l_2)^2-(x^2+y^2)}{2(l_3)(l_1+l_2)}\right)$$

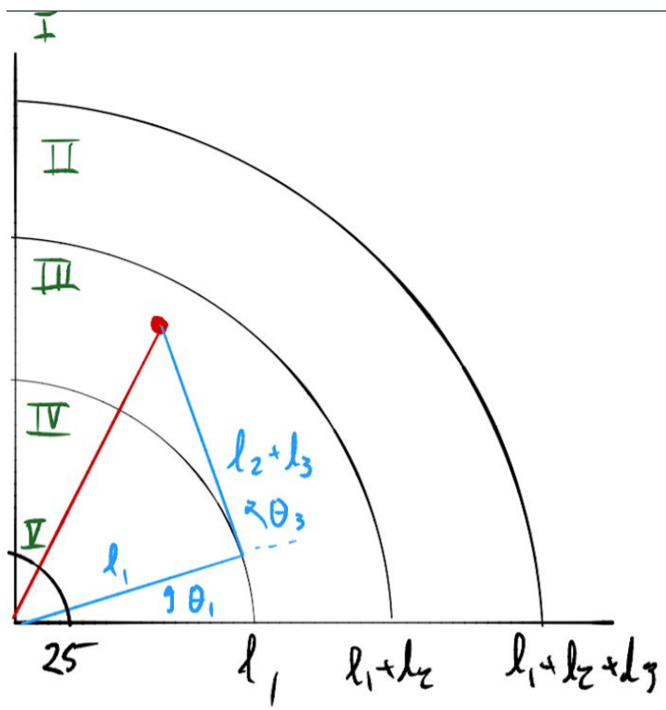
Case I Summary, defined by $\sqrt{x^2+y^2} > l_1 + l_2 + l_3$

No Solution, but closest is

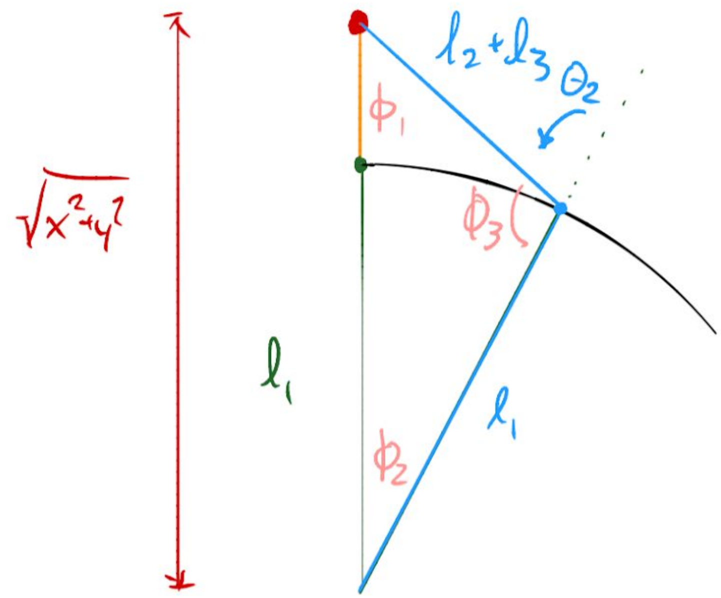
$$\theta_1 = \tan^{-1}\left(\frac{y'}{x}\right)$$

$$\theta_2 = 0$$

$$\theta_3 = 0$$



Case III :



Law of Cosines:

$$\left(\sqrt{x^2+y^2}\right)^2 = (l_2+l_3)^2 + l_1^2 - 2(l_2+l_3)(l_1)\cos(\phi_3)$$

$$\phi_3 = \cos^{-1}\left(\frac{(l_2+l_3)^2 + l_1^2 - (x^2+y^2)}{2(l_2+l_3)(l_1)}\right)$$

$$\theta_2 = \pi - \phi_3 \therefore$$

$$\theta_2 = \pi - \cos^{-1}\left(\frac{(l_2+l_3)^2 + l_1^2 - (x^2+y^2)}{2(l_2+l_3)(l_1)}\right)$$

Similarly,

$$(l_2+l_3)^2 = l_1^2 + \left(\sqrt{x^2+y^2}\right)^2 - 2(l_1)\left(\sqrt{x^2+y^2}\right)\cos(\phi_2)$$

$$\phi_2 = \cos^{-1}\left(\frac{l_1^2+x^2+y^2-(l_2+l_3)^2}{2(l_1)\left(\sqrt{x^2+y^2}\right)}\right)$$

$$\tan^{-1}\left(\frac{y}{x}\right) = \theta_1 + \phi_2 \therefore$$

$$\theta_1 = \tan^{-1}\left(\frac{y}{x}\right) - \cos^{-1}\left(\frac{l_1^2+x^2+y^2-(l_2+l_3)^2}{2(l_1)\left(\sqrt{x^2+y^2}\right)}\right)$$

$$\theta_3 = 0$$

Case III Summary, defined by $l_1 \le \sqrt{x^2+y^2} < l_1 + l_2$

$$\boxed{\begin{aligned}\theta_1 &= \tan^{-1}\left(\frac{y}{x}\right) - \cos^{-1}\left(\frac{l_1^2+x^2+y^2-(l_2+l_3)^2}{2(l_1)(\sqrt{x^2+y^2})}\right)\\ \theta_2 &= \pi - \cos^{-1}\left(\frac{(l_2+l_3)^2+l_1^2-(x^2+y^2)}{2(l_2+l_3)(l_1)}\right)\\ \theta_3 &= 0\end{aligned}}$$

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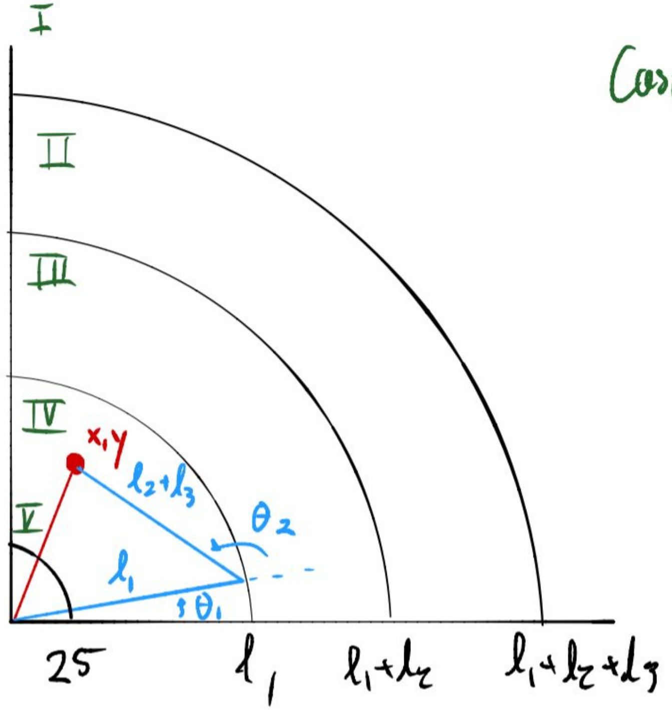
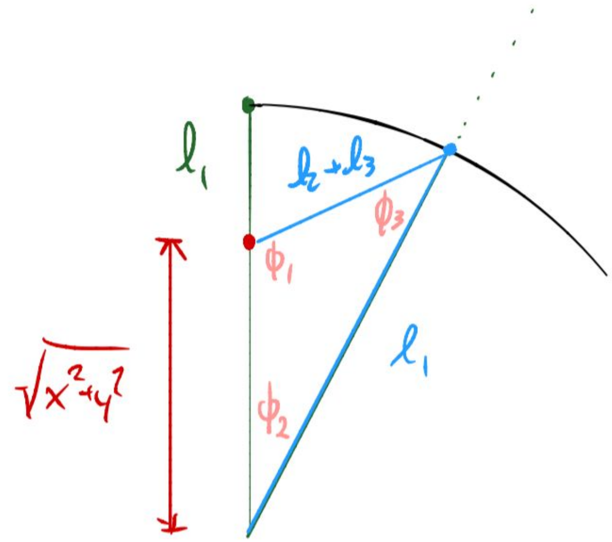
Case IV:





Because $l_2+l_3 > l_1$, entire sub quadrant is reachable. It is then clear that this is the same as case III. The lower bound comes from domain restriction on $\cos^{-1}(x)$ from θ_2:

$$\left(\frac{(l_2+l_3)^2+l_1^2-(x^2+y^2)}{2(l_2+l_3)(l_1)}\right) \leq 1 \quad \begin{cases} l_1 = 150 \\ l_2 = 100 \\ l_3 = 75 \end{cases} \rightarrow \begin{array}{l} x^2+y^2 < 625 \\ \sqrt{x^2+y^2} < 25 \end{array}$$

Summary Case $\underline{IV}$: $25 \le \sqrt{x^2+y^2} < l_1$

$$\boxed{\begin{aligned} \theta_1 &= \tan^{-1}\left(\frac{y}{x}\right) - \cos^{-1}\left(\frac{l_1^2+x^2+y^2-(l_2+l_3)^2}{2(l_1)(\sqrt{x^2+y^2})}\right) \\ \theta_2 &= \pi - \cos^{-1}\left(\frac{(l_2+l_3)^2+l_1^2-(x^2+y^2)}{2(l_2+l_3)(l_1)}\right) \\ \theta_3 &= 0 \end{aligned}}$$

Same as case $\underline{III}$, combining the domains makes $25 \le \sqrt{x^2+y^2} < l_1 + l_2$

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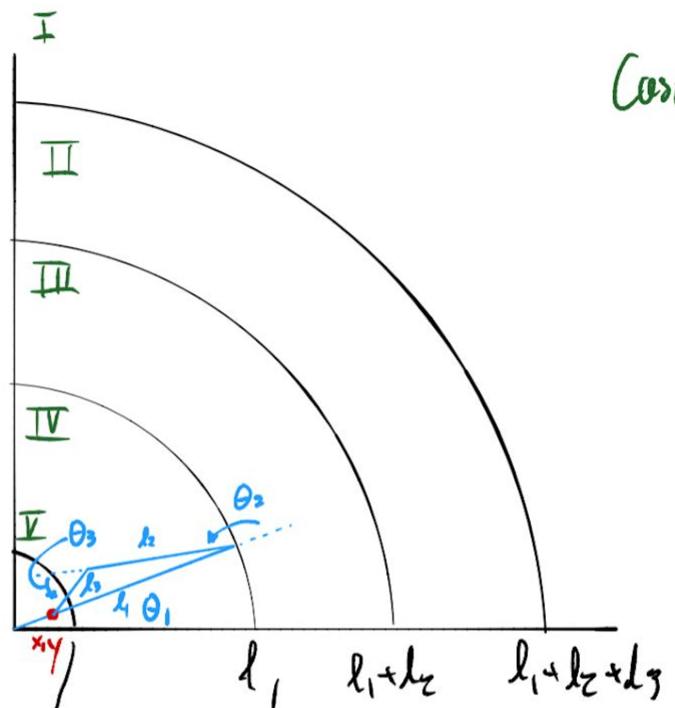
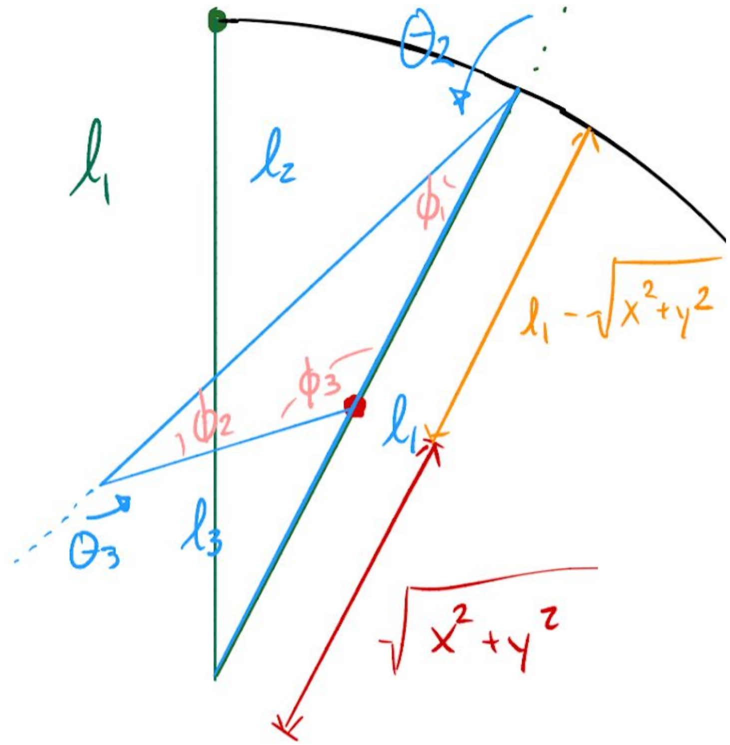
Case V





$\left\{\left[(l_2+l_3)^2 + l_1^2 - 2l_1(l_2+l_3)\right]^{\frac{1}{2}} = 25\right.$

Law of cosines:

$$l_3^2 = l_2^2 + \left(l_1 - \sqrt{x^2+y^2}\right)^2 - 2(l_2)\left(l_1 - \sqrt{x^2+y^2}\right)\cos(\phi_1)$$

$$\phi_1 = \cos^{-1}\left(\frac{l_2^2 + \left(l_1 - \sqrt{x^2+y^2}\right)^2 - l_3^2}{2(l_2)\left(l_1 - \sqrt{x^2+y^2}\right)}\right)$$

while, $\theta_2 = \pi - \phi_2$

Similarly, $\left(l_1 - \sqrt{x^2+y^2}\right)^2 = l_2^2 + l_3^2 - 2(l_2)(l_3)\cos\angle\phi_2$

$$\phi_2 = \cos^{-1}\left(\frac{l_2^2 + l_3^2 - \left(l_1 - \sqrt{x^2+y^2}\right)^2}{2(l_2)(l_3)}\right)$$

while, $\theta_3 = \pi - \phi_2$

Finally, $\theta_1 = \tan^{-1}\left(\frac{y}{x}\right)$

Summary Case $\underline{V}$: $\sqrt{x^2+y^2} < \left[(l_2+l_3)^2 + l_1^2 - 2l_1(l_2+l_3)\right]^{\frac{1}{2}} = 25$

$$\boxed{\begin{aligned}
\theta_1 &= \tan^{-1}\left(\frac{y}{x}\right)\\
\theta_2 &= \pi - \cos^{-1}\left(\frac{l_2^2 + \left(l_1 - \sqrt{x^2+y^2}\right)^2 - l_3^2}{2(l_2)\left(l_1 - \sqrt{x^2+y^2}\right)}\right)\\
\theta_3 &= \pi - \cos^{-1}\left(\frac{l_2^2 + l_3^2 - \left(l_1 - \sqrt{x^2+y^2}\right)^2}{2(l_2)(l_3)}\right)
\end{aligned}}$$