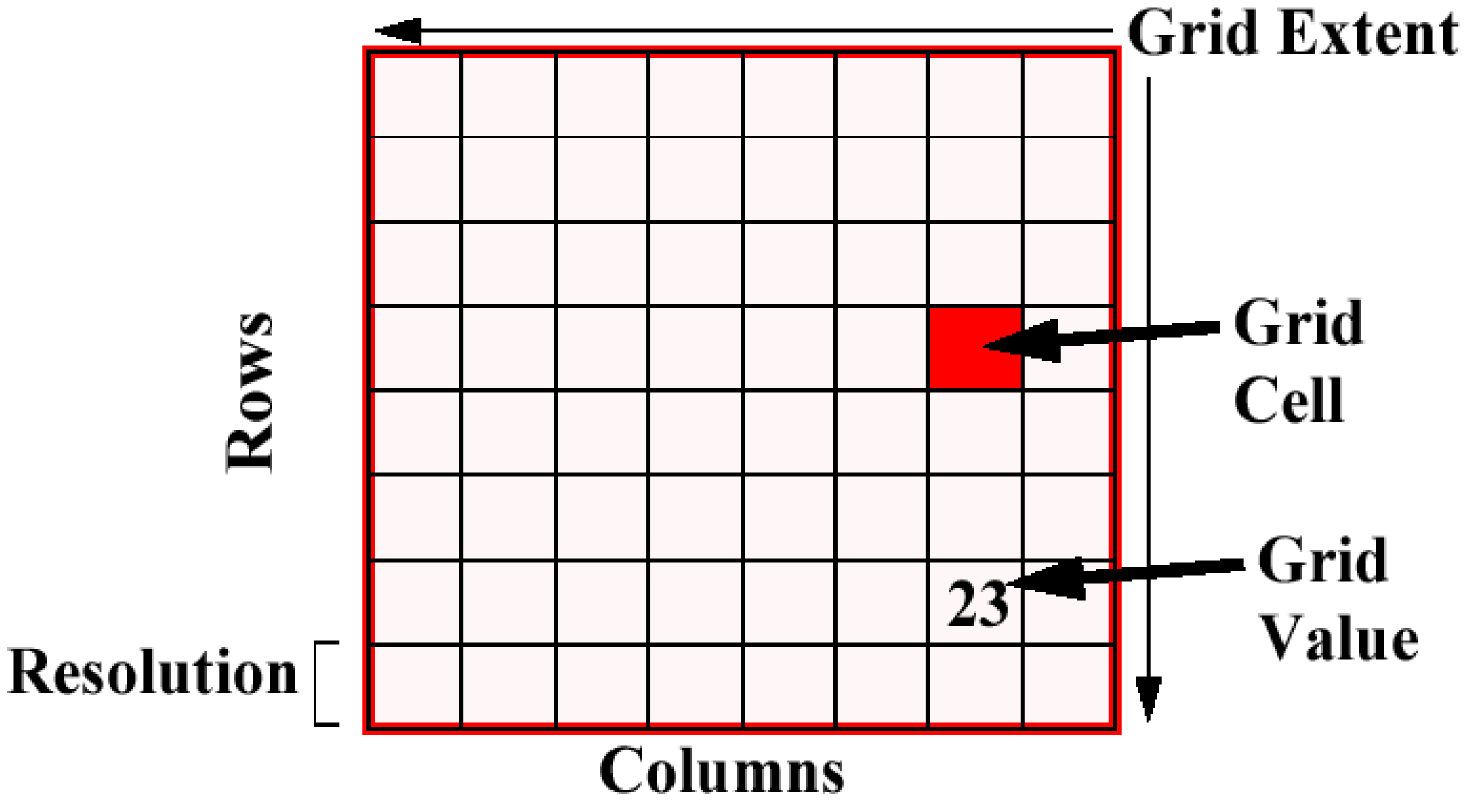
Analytical and Computer Cartography

Lecture 12:
Grids, interpolation and extrapolation

The power of the grid

- Easiest representation of a field variable, measureable at all places
- Can vary spacing to suit task
- Interpolation necessary when data are not at grid intersections, or are irregular or sparse
- Interpolation becomes extrapolation
  - When areas deficient of points are interpolated
  - When interpolation is carried outside the data area

Generic structure for a grid

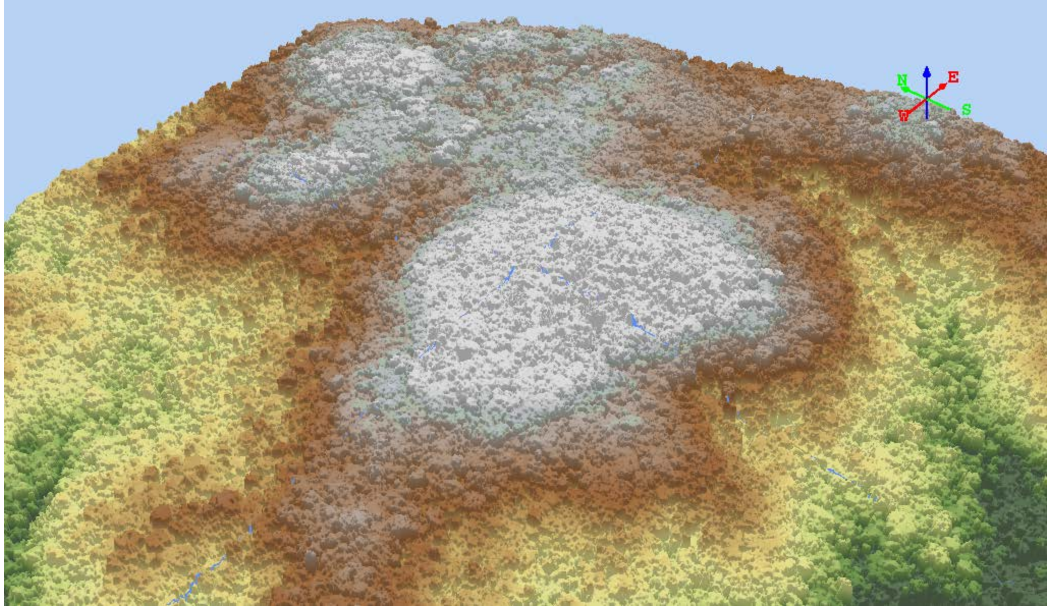


Models for terrain

- Contours: Vector
- Regular point samples
- Irregular point samples
- DEMs
- Surface patches
- TIN
- Voxel
- 3D point cloud

3D Transformations

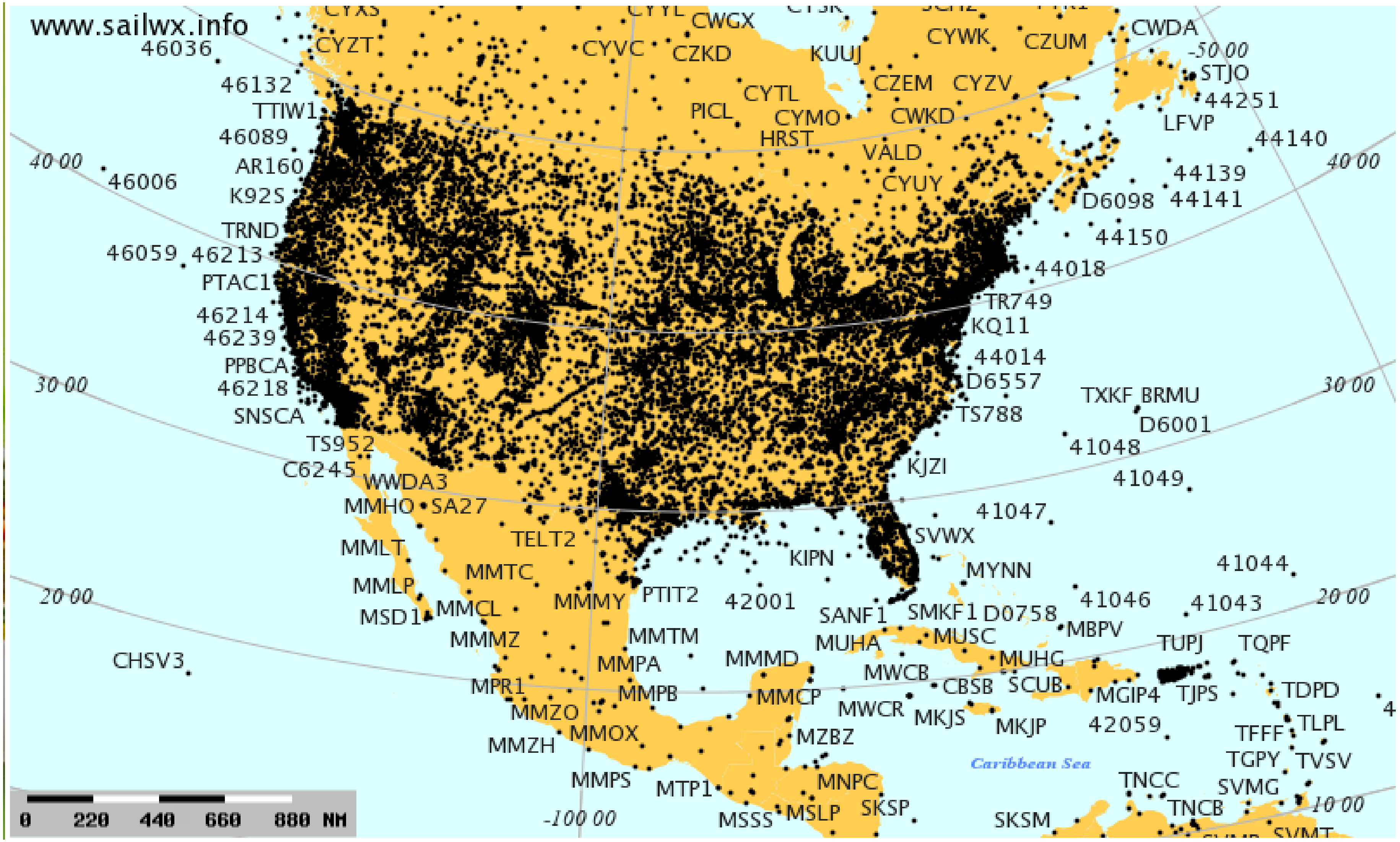
- Not yet considered in a transformational context
- 3D data often for land surface or bottom surface of ocean
- Single valued function
- Need three coordinates to determine location (X, Y, Z) or (λ, Φ, h)



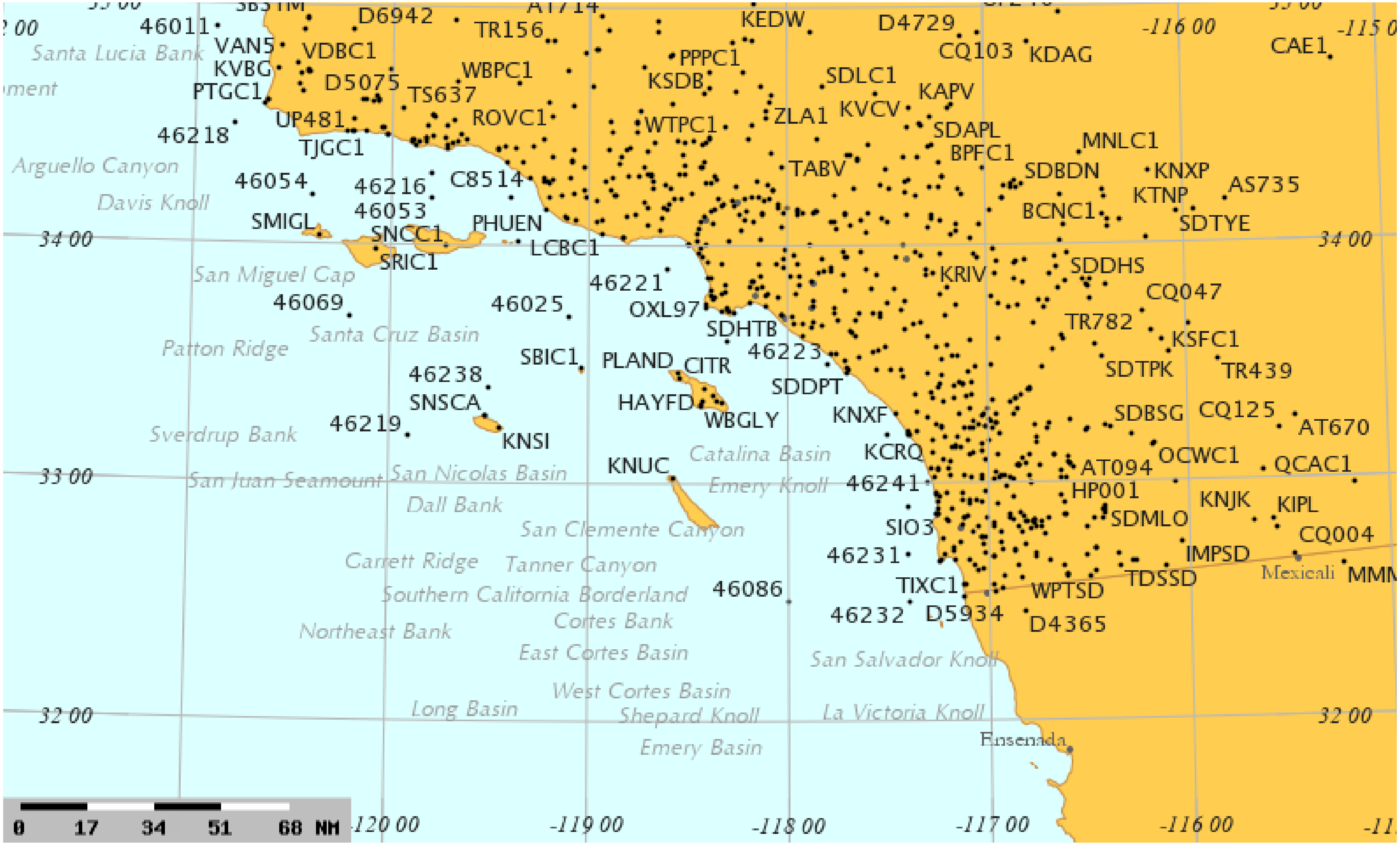
Terrain (surface) analysis

- Part of analytical cartography concerned with analysis of field’s upper surface is *terrain analysis*
- Includes terrain representation and symbolization issues as they relate to terrain data
- Points, TIN and grids are used to store terrain
- How do we do transformations?
- In all cases we must fill in data where none exists in space = interpolation
- Often due to map projection
- True even when data are dense, such as with LiDAR

The interpolation problem



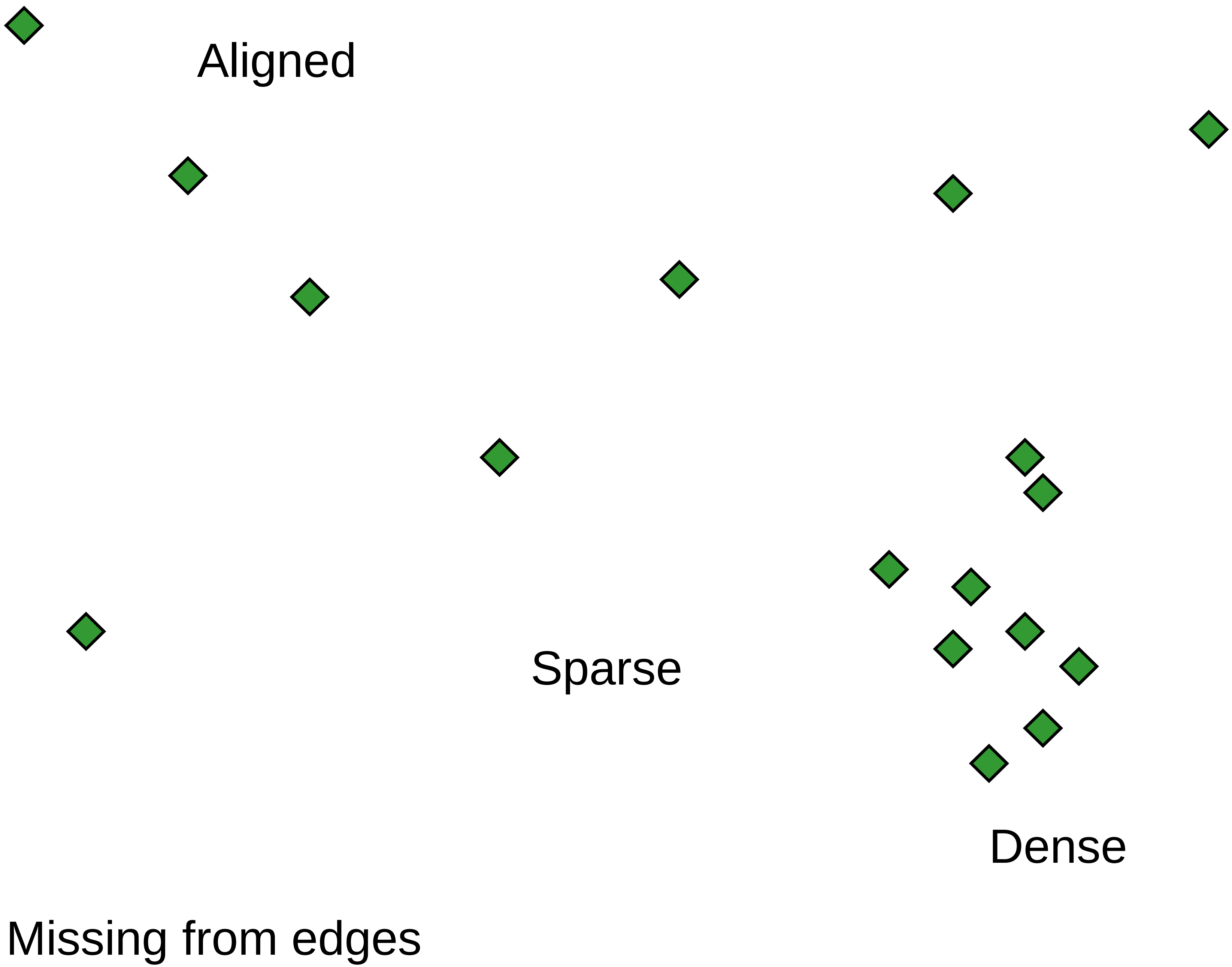
Southern California



Santa Barbara County

Interpolation to a Grid

- Given a set of point elevations (x, y, z) generate a new set of points at the nodes of a regular grid so that the interpolated surface is a reasonable representation of the surface sampled by the points.
- Imposes a model of the true surface on the sample
- "Model" is a mathematical model of the neighborhood relationship
- Influence of a single point = f(1/d)
- Can be constrained to fit all points
- Should contain *z* extremes, and local extrema
- Most models are algorithmic local operators
- Work cell-to-cell. Operative cell = kernel


Aligned
Sparse
Dense
Missing from edges

Weighting Methods

- Impose z = f (1/d)
- Computationally rather intensive
- e.g. 200 x 200 cells 1000 points = 40 x 10^6 distance calculations
- If all points are used and sorted by distance, called "brute force" method
- Possible to use sorted search and tiling (Hodgson, ERDAS)
- Distance can be weighted and powered by n = friction of distance
- Can be refined with break lines
- Use cos (angle) to prevent shadowing

Inverse Distance Weighting

-

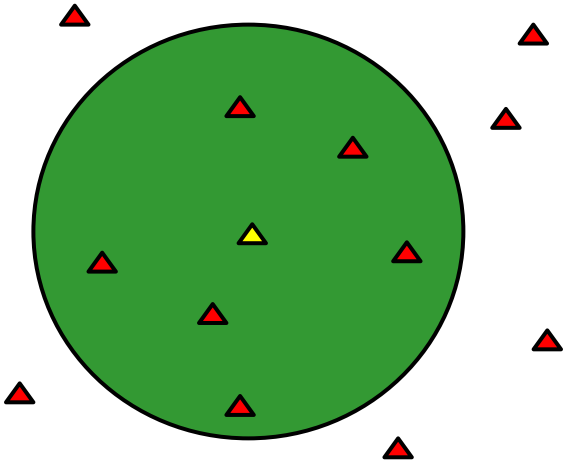
$$Z_{i,j} = \frac{\sum_{p=1}^{R} z_p d_p^{-n}}{\sum_{p=1}^{R} d_p^{-n}}$$

Z=height

D=distance

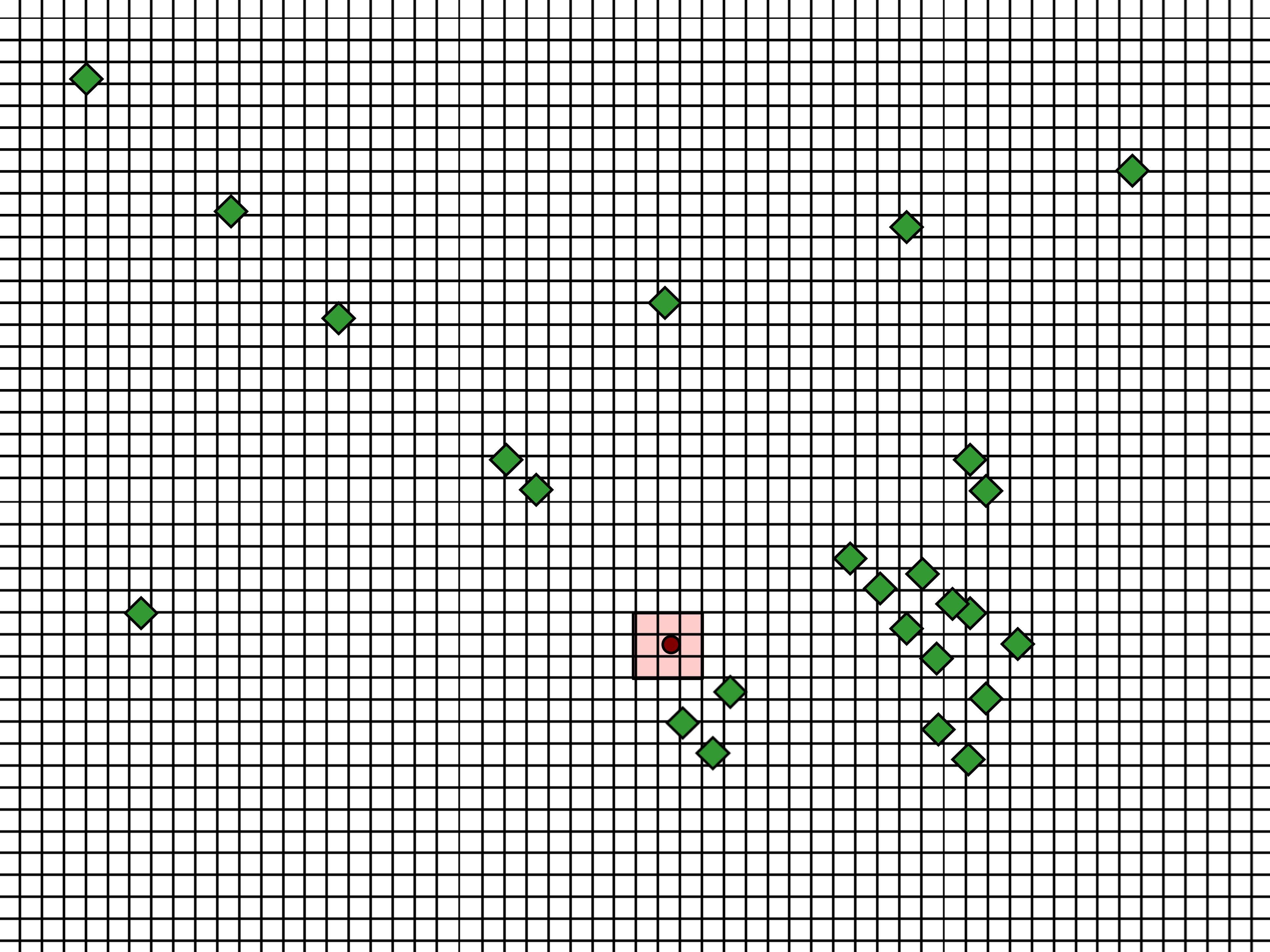
P=1….R

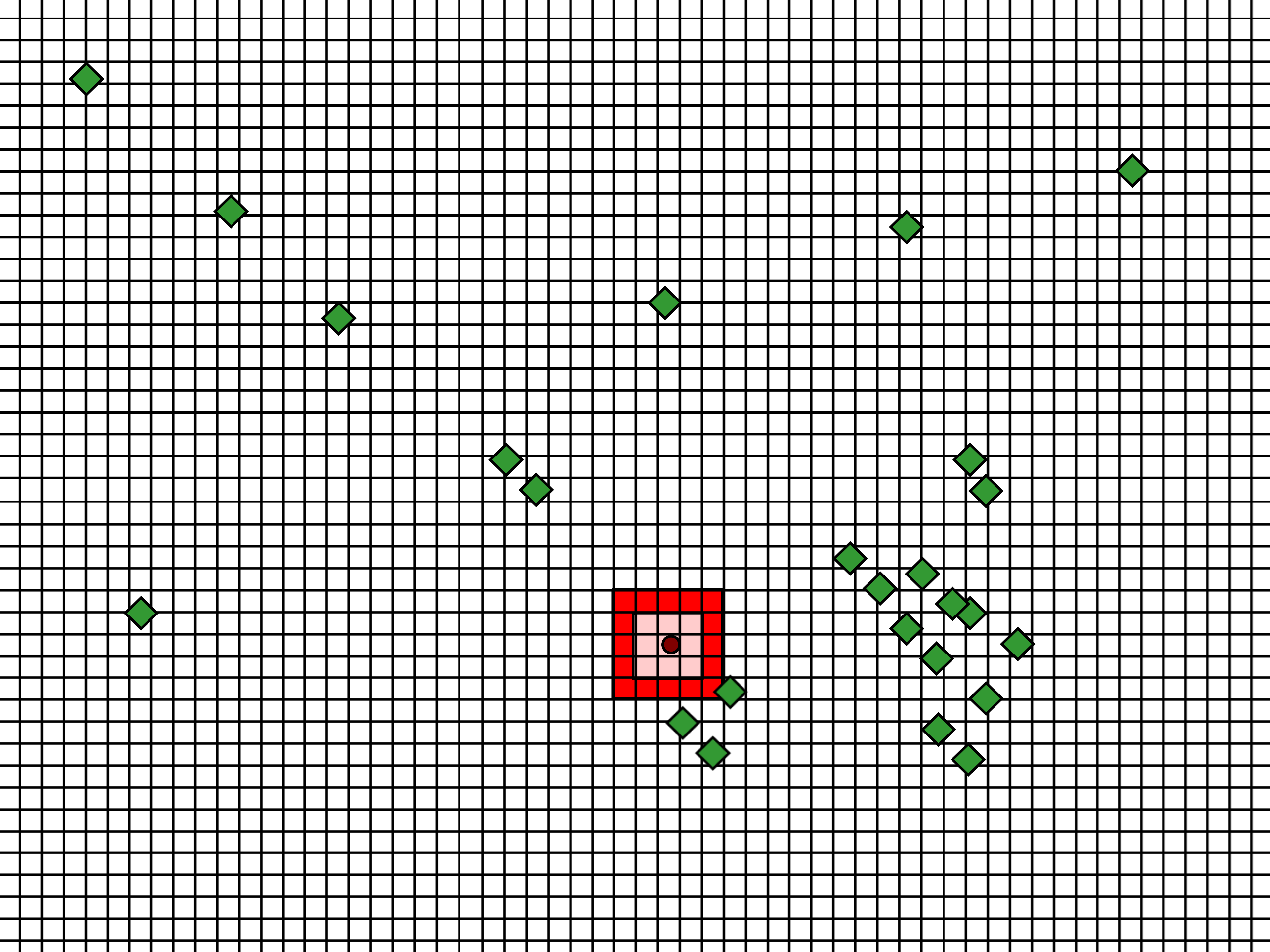
n= ?

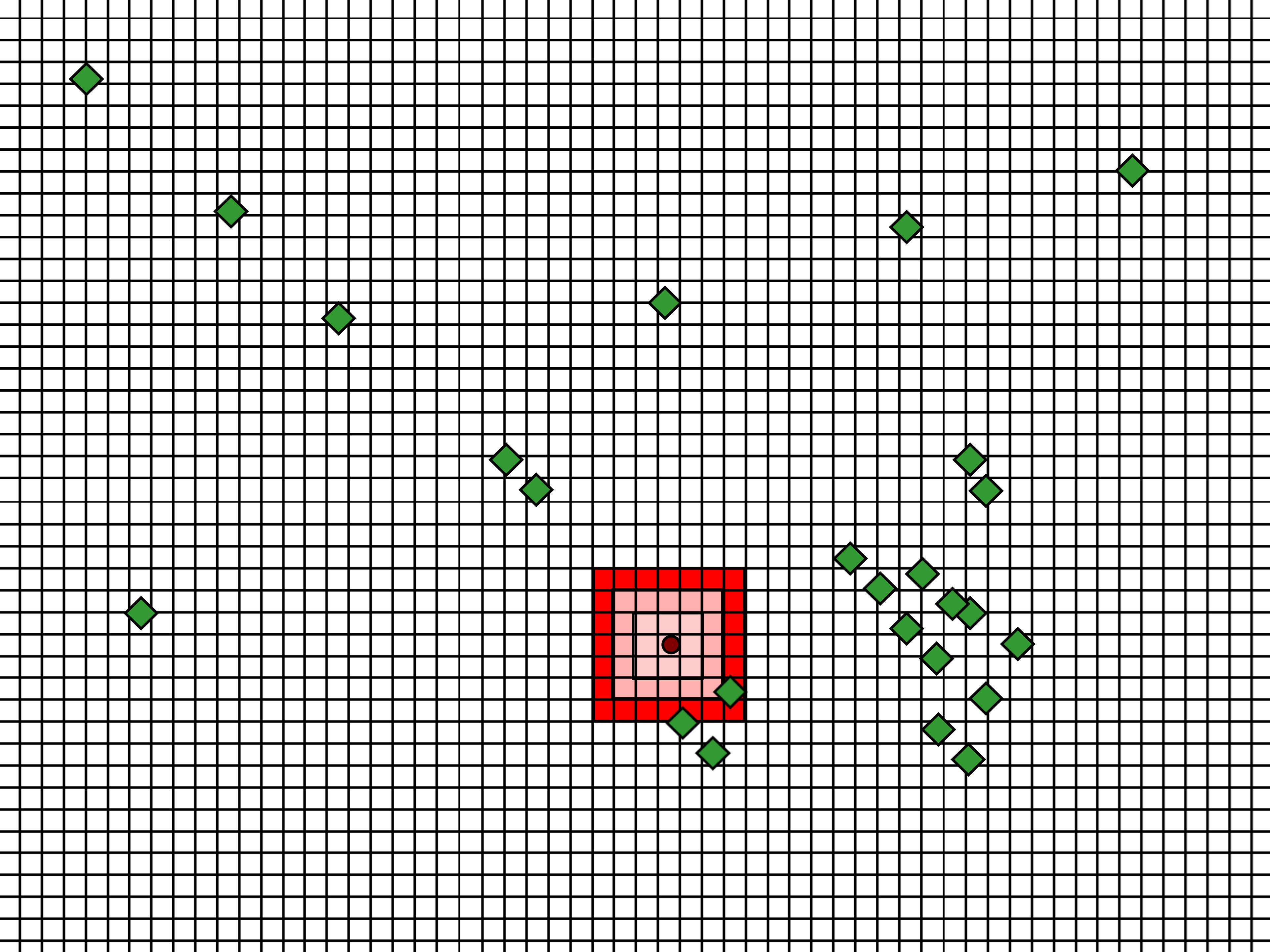


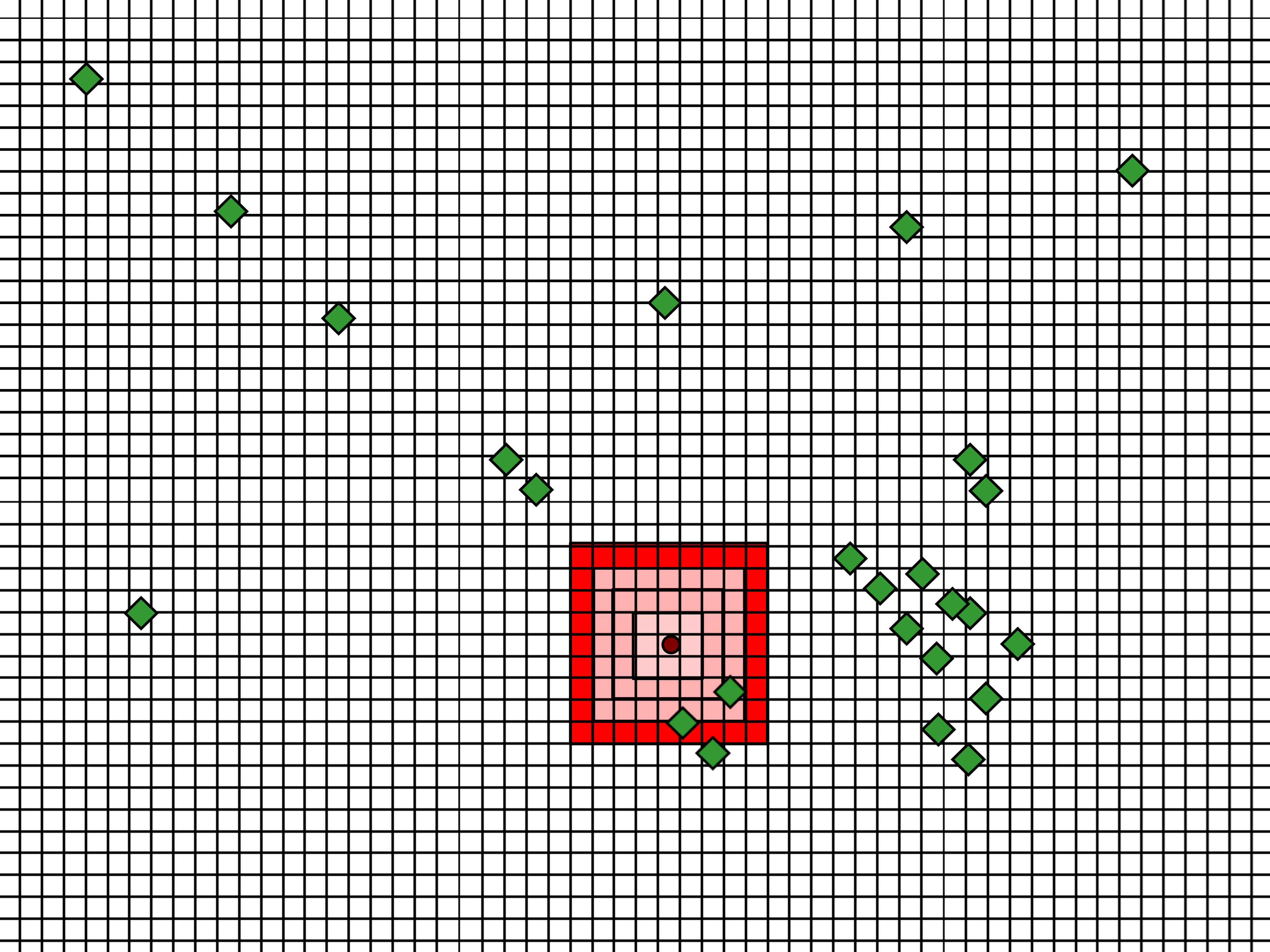
Inverse Distance Weighting: Clarke Algorithm

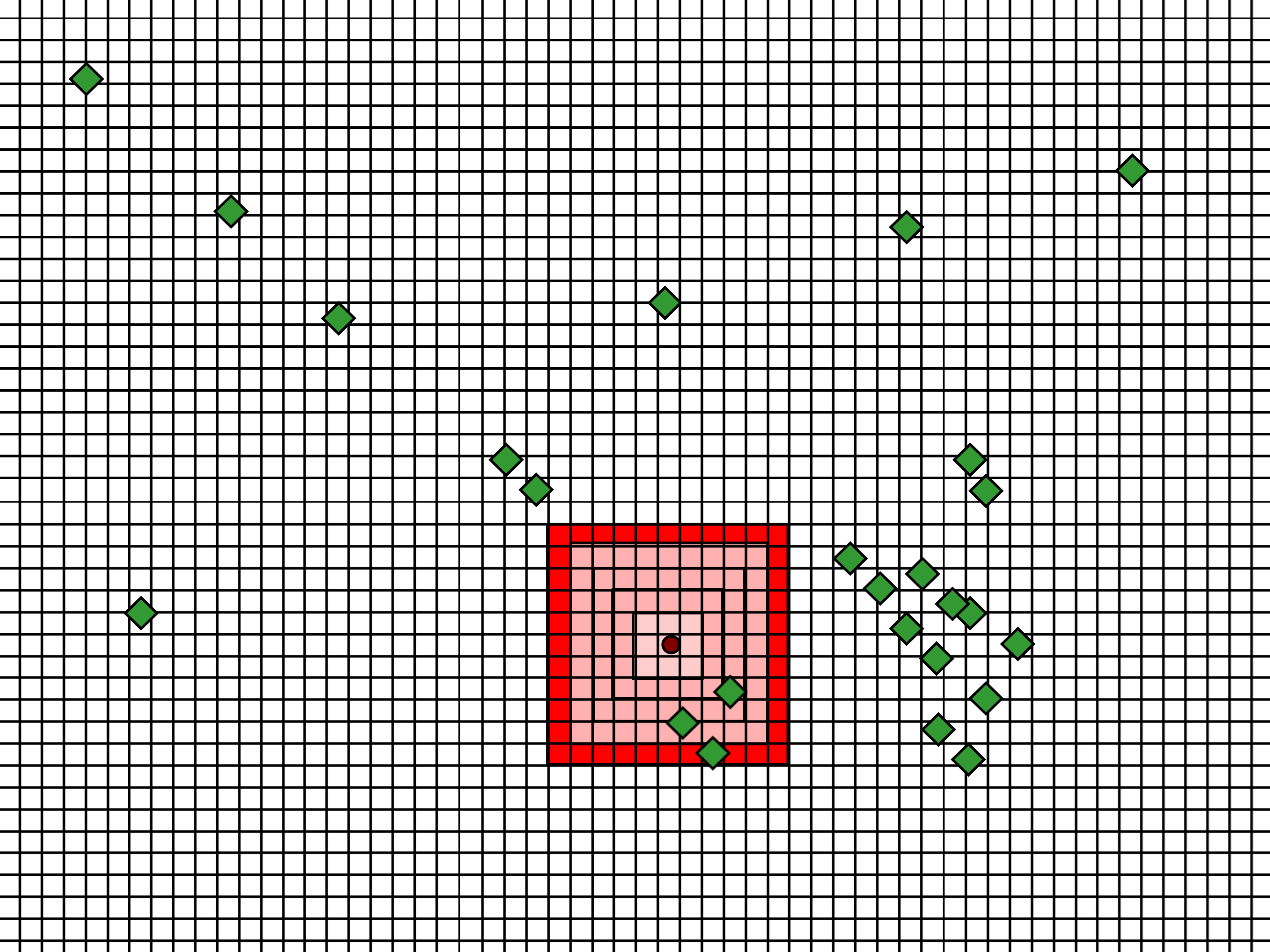
- inverse_d
- Assigns points to cells
- Averages multiples
- For all unfilled cells
  - search outward using an increasingly large odd square neighborhood (3 x 3), (5 x 5) etc.
  - Only need to search next row/col in each direction
  - until at least npts are found
  - apply inverse distance weighting
- Has been parallelized (and found highly efficient) by Armstrong





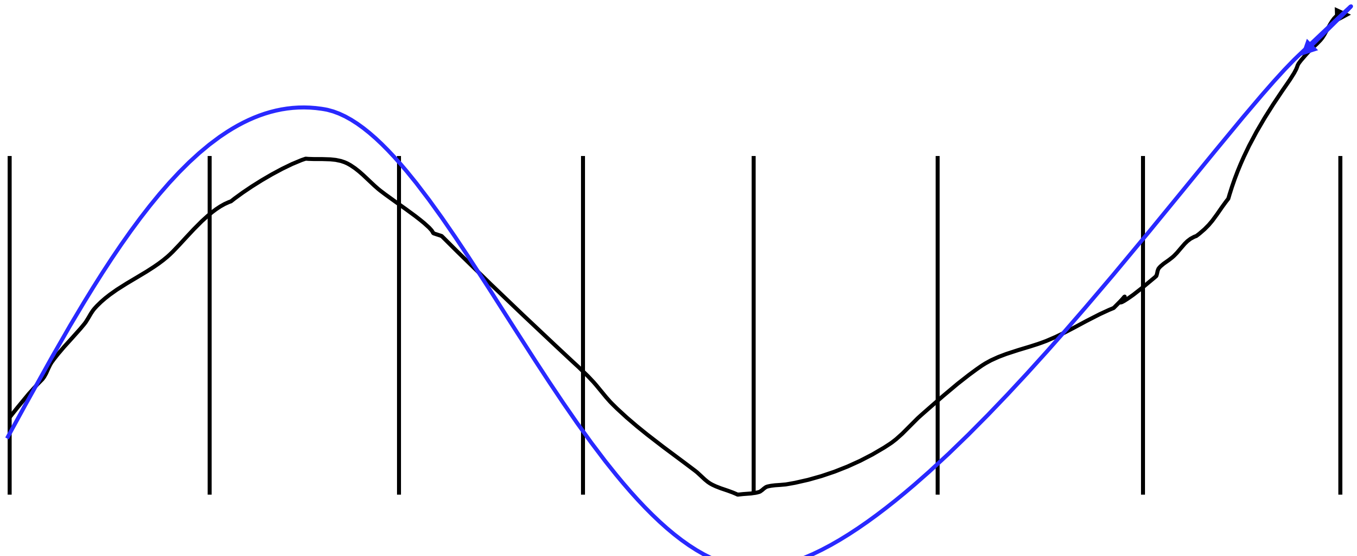






Trend Projection Methods

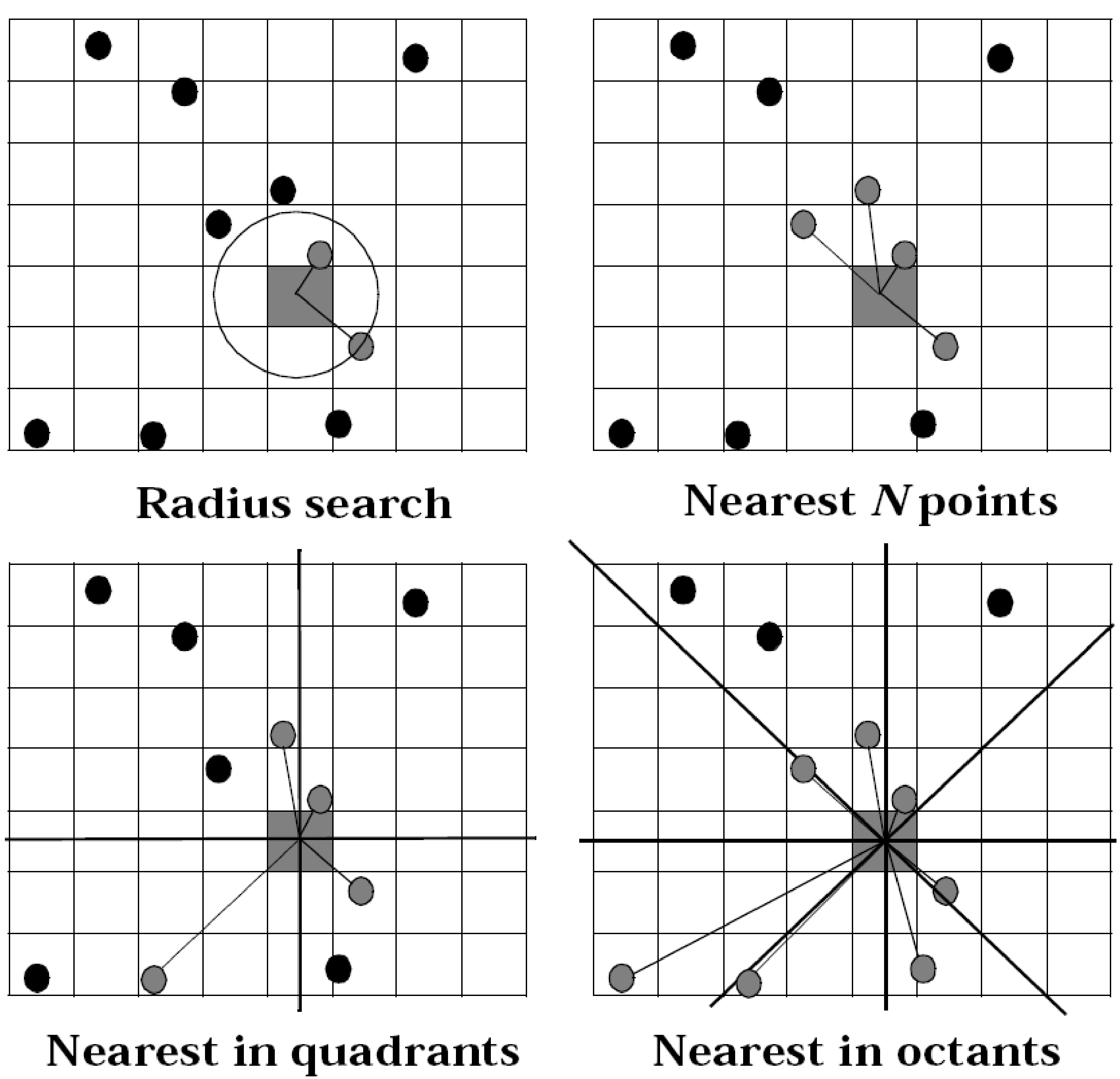
- Way to overcome high/low constraint
- Assumes that sampling missed extrema
- Locally fits trend, trend surface or bicubic spline
- Least squares solution
- Useful when data are sparse, texture required



Search Patterns

- Many possible ways to define interpolated "region" R
- Can use # points or distance
- Problems in
  - Sparse areas
  - Dense areas
  - Edges
- Bias can be reduced by changing search strategy

Search patterns



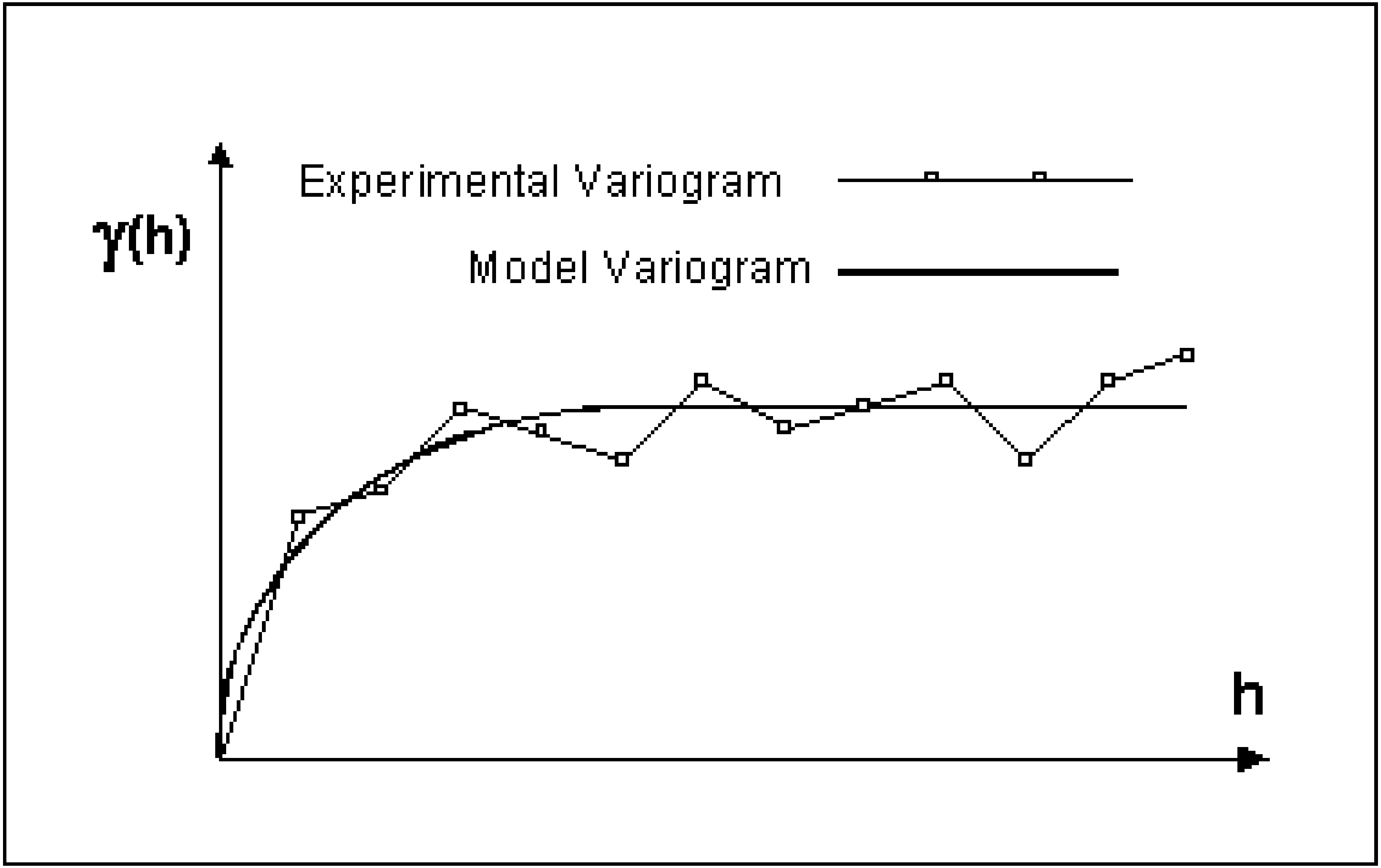
e.g. Quicklook gridding

- Assign points to cells
- Assign cells average of 4 neighboring cells
- Keep doubling cell size until all cells filled
- Looks “blocky” but works quickly
- Unbiased

Kriging

- "Optimal interpolation method" by D.G. Krige
- Origin in geology (geostatistics, gold mining)
- Spatial variation = f(drift, random-correlated, random noise)
- To use Kriging
  - Model and extract drift
  - Compute variogram
  - Model variogram
  - Compute expected variance at d, and so best estimate of local mean
- Several alternative methods
- Universal Kriging best when local trends are well defined
- Kriging produces best estimate and estimate of variance at all places on map

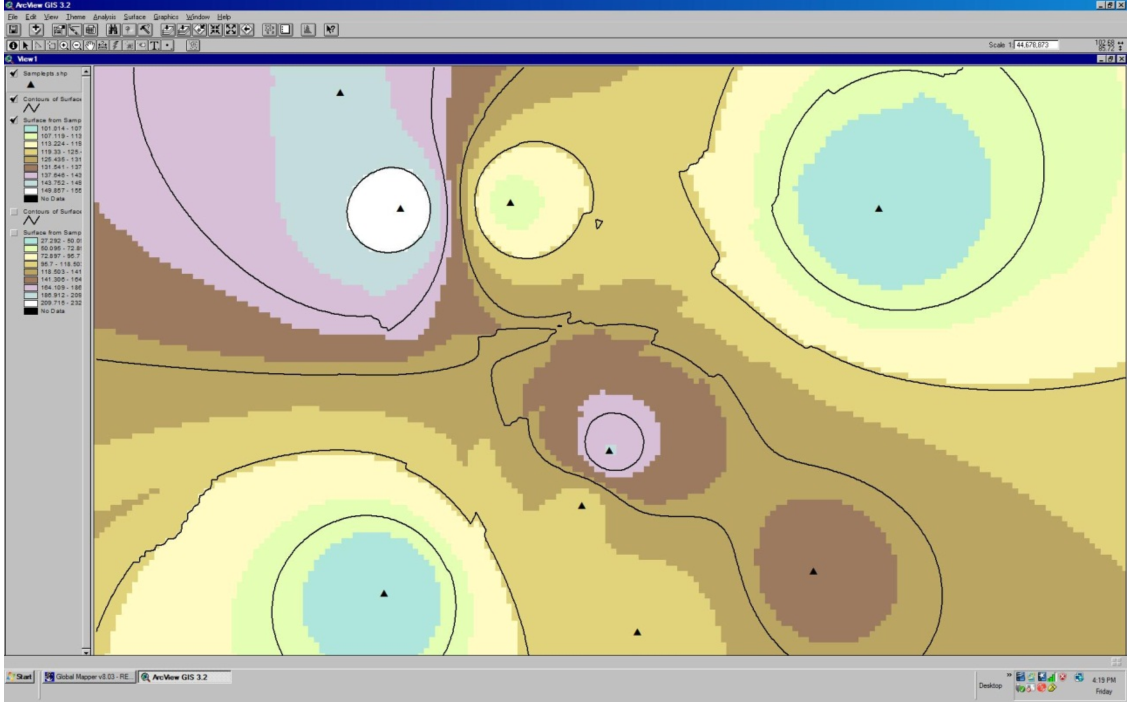
Variogram

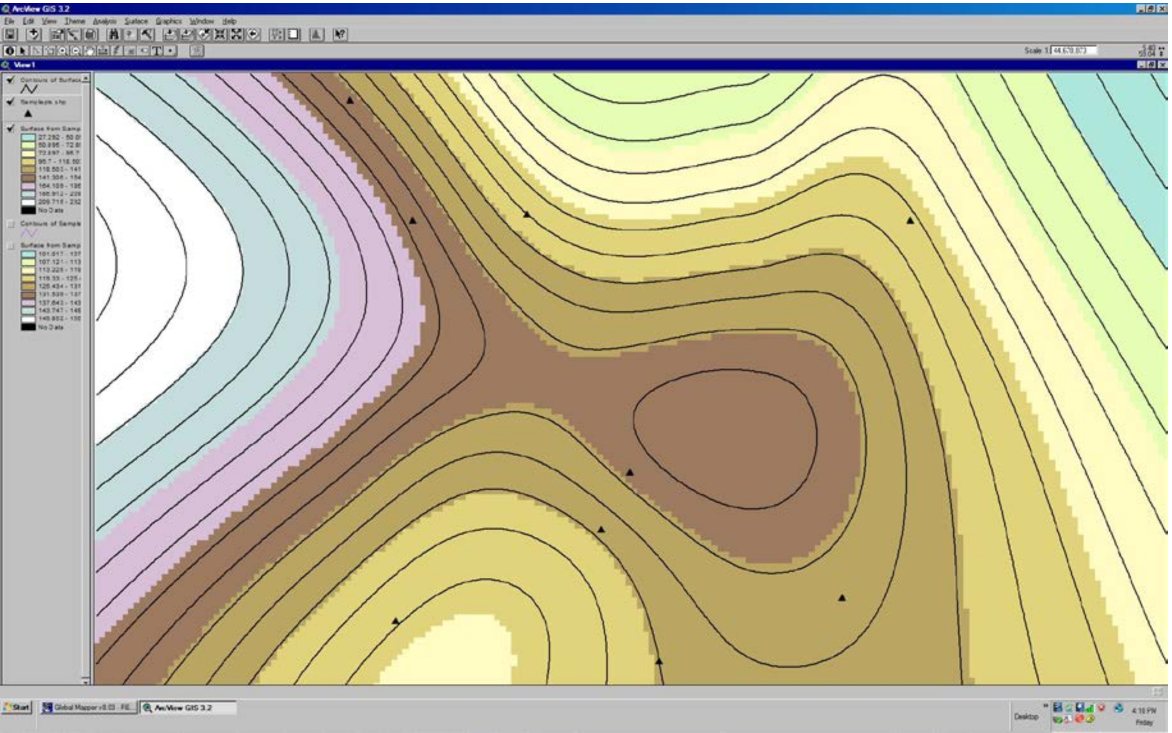


Alternative Methods

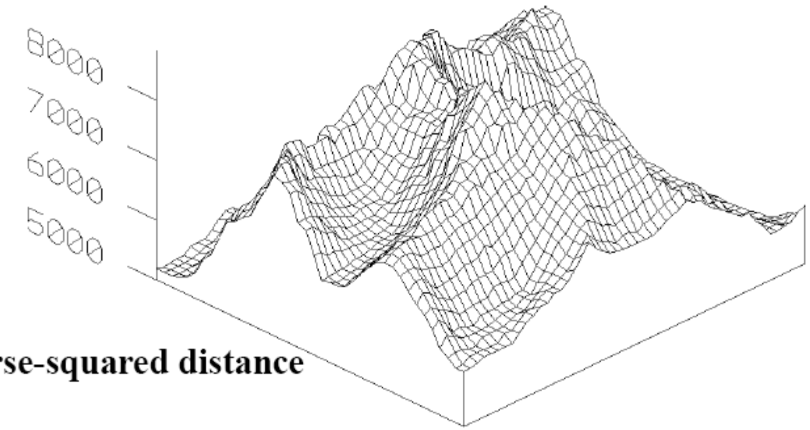
- Many ways to make the point-to-grid interpolation
- Invertibility?
- Can results be compared and tested analytically
- Use portion of points and test results with remainder
- Examing spatial distribution of difference between methods
- Best results are obtained when field is sampled with knowledge of the terrain structure and the method to be used

Algorithm matters: IDW (5) vs. Splines (12, 0.1)

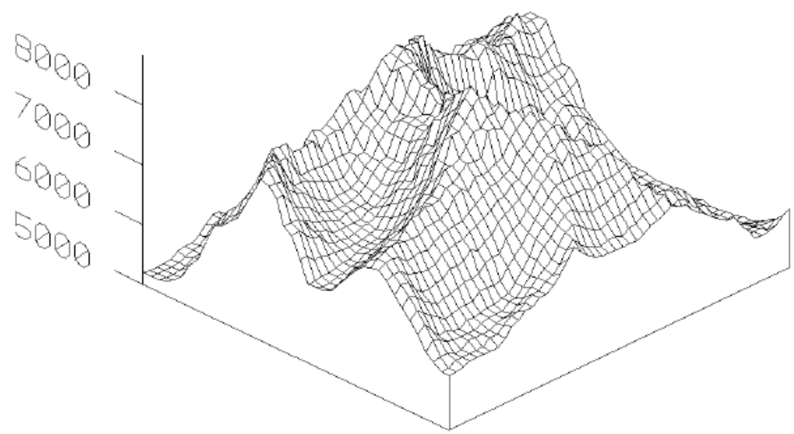




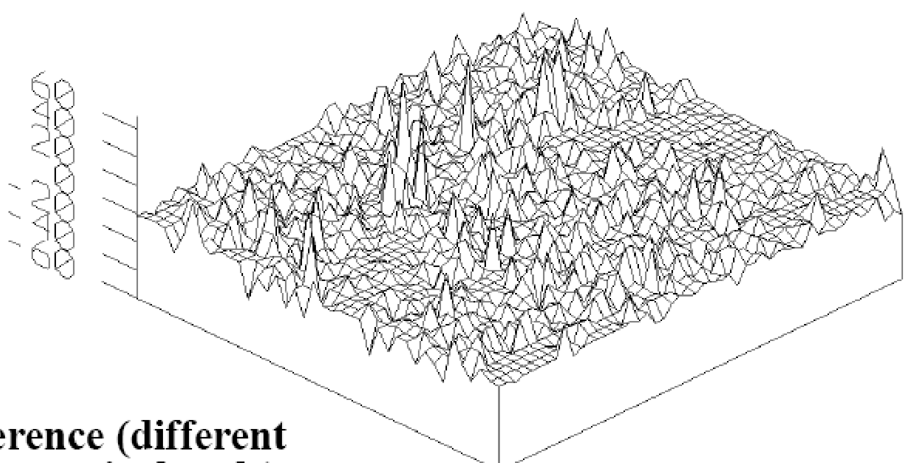
Error analysis



Inverse-squared distance



Kriging

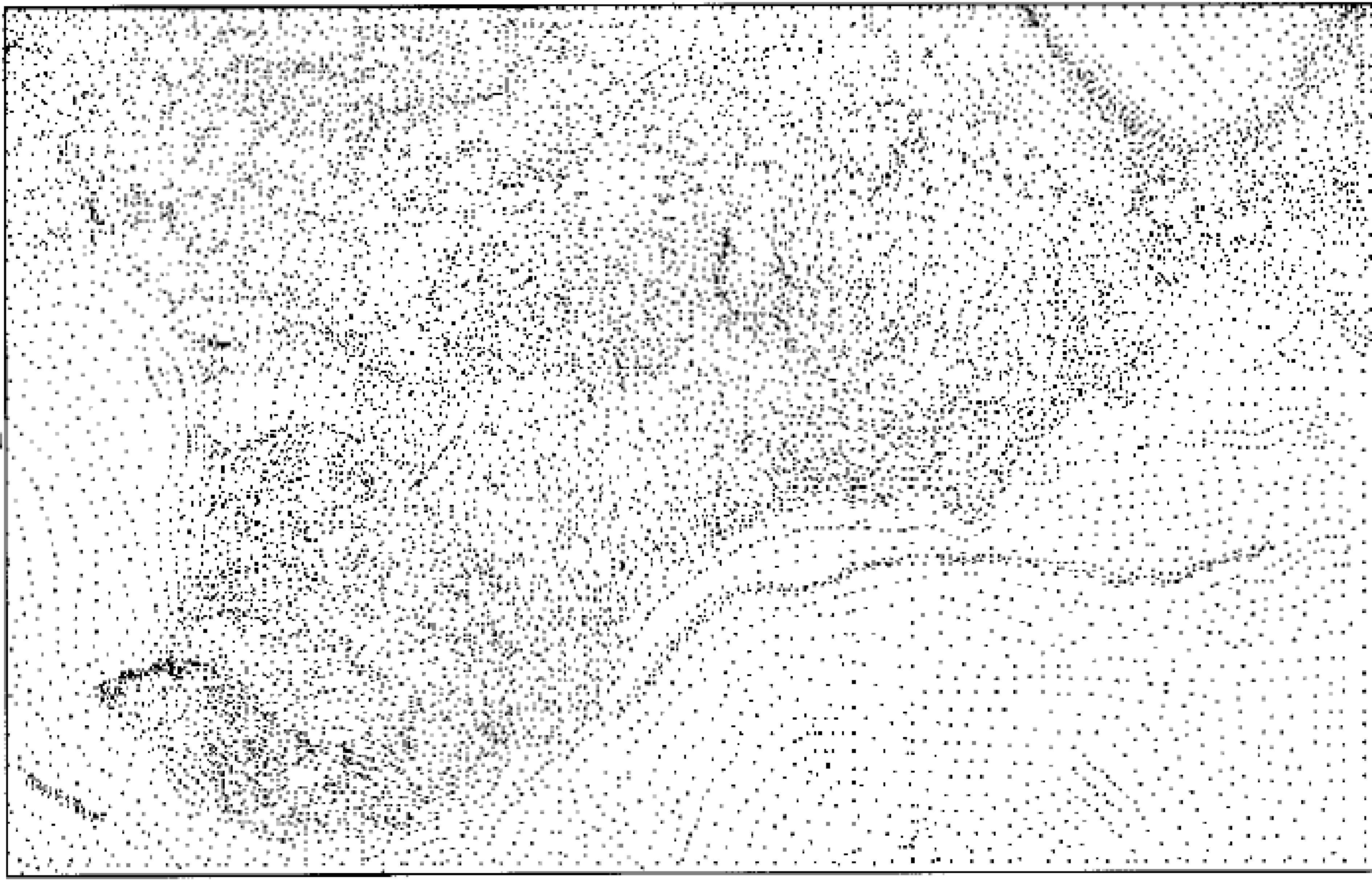


Difference (different vertical scale)

Surface measurement

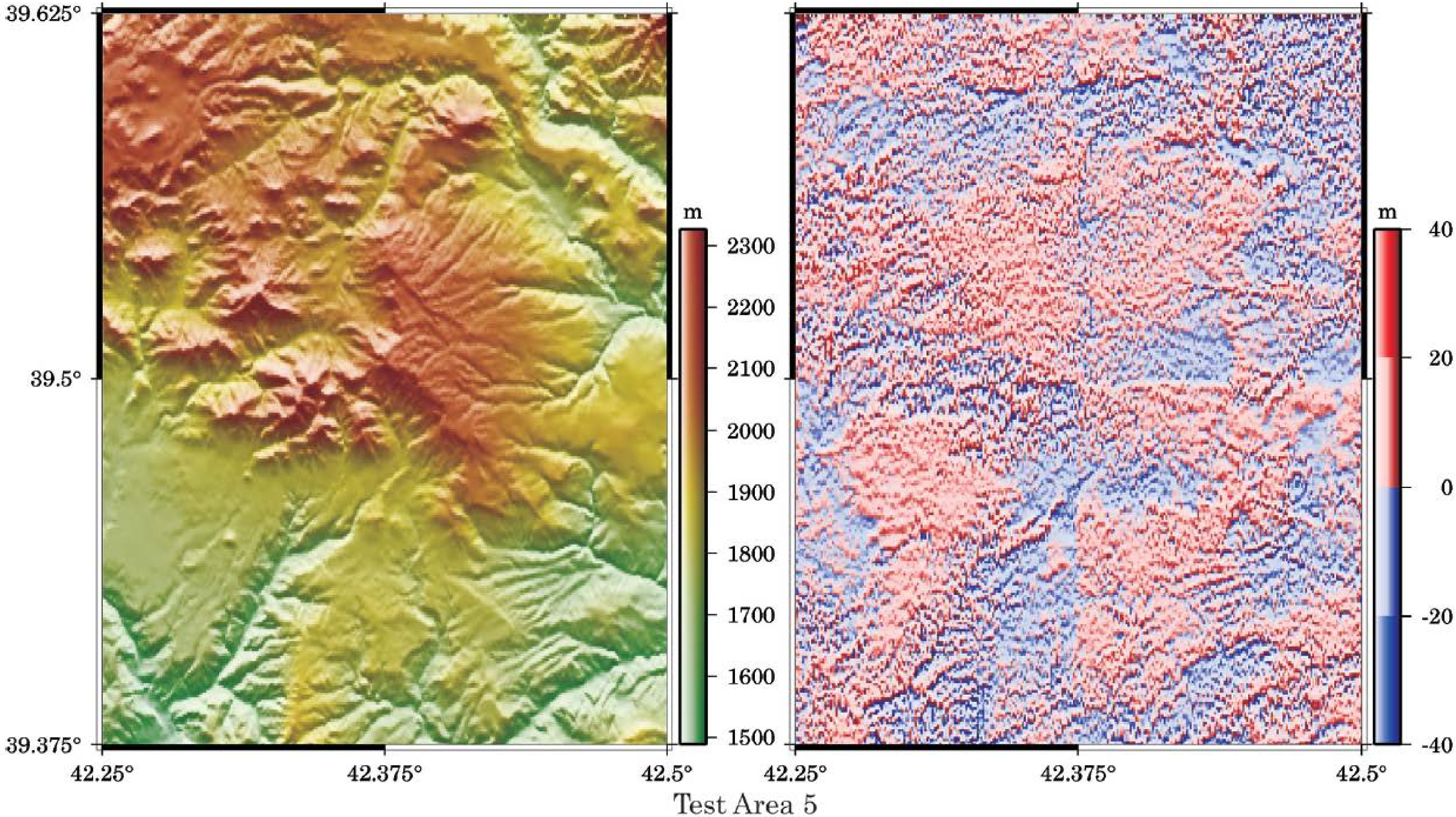
- Can create DEMs directly from stereo imagery
- LIDAR creates a dense point cloud that must be sampled
- Traditional surveying creates points, often along profiles
- GPS gives points (but note z st.dev.)
- Can digitize lines from maps: Usually contours!
- Tiling errors possible
- Averaging in blocks common

Florinsky: Analysis of DEM error

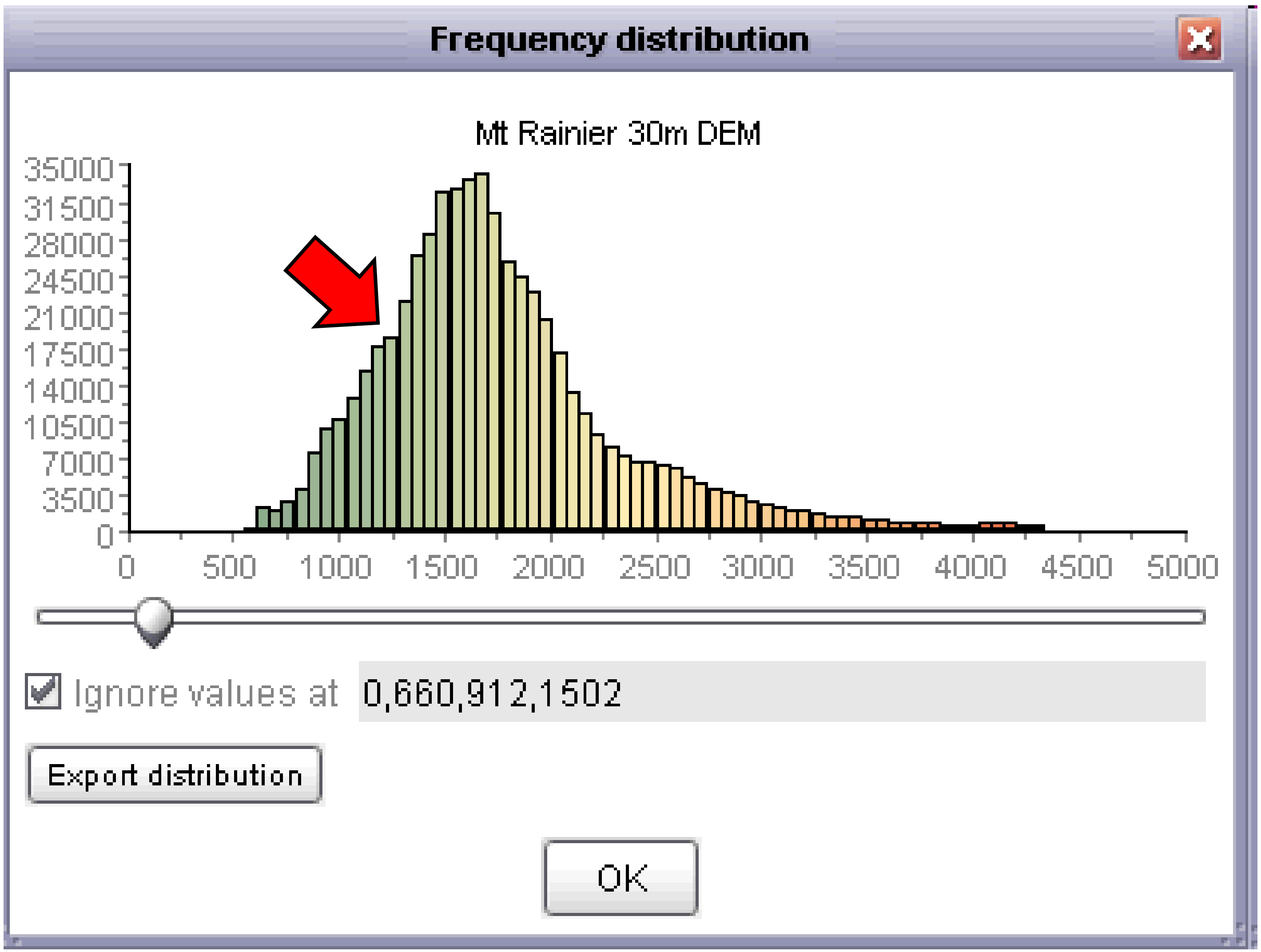


Impact on aspect

Tiling error

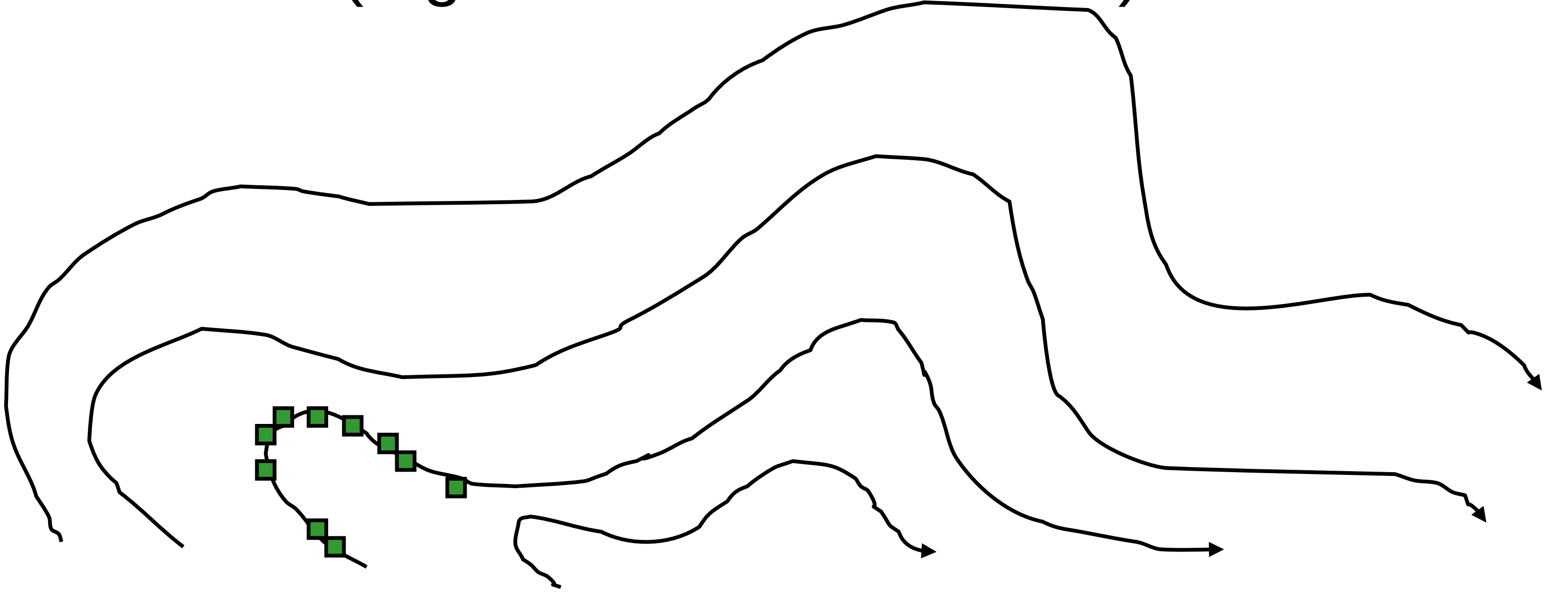


Wedding cake effect: Visible in height histogram

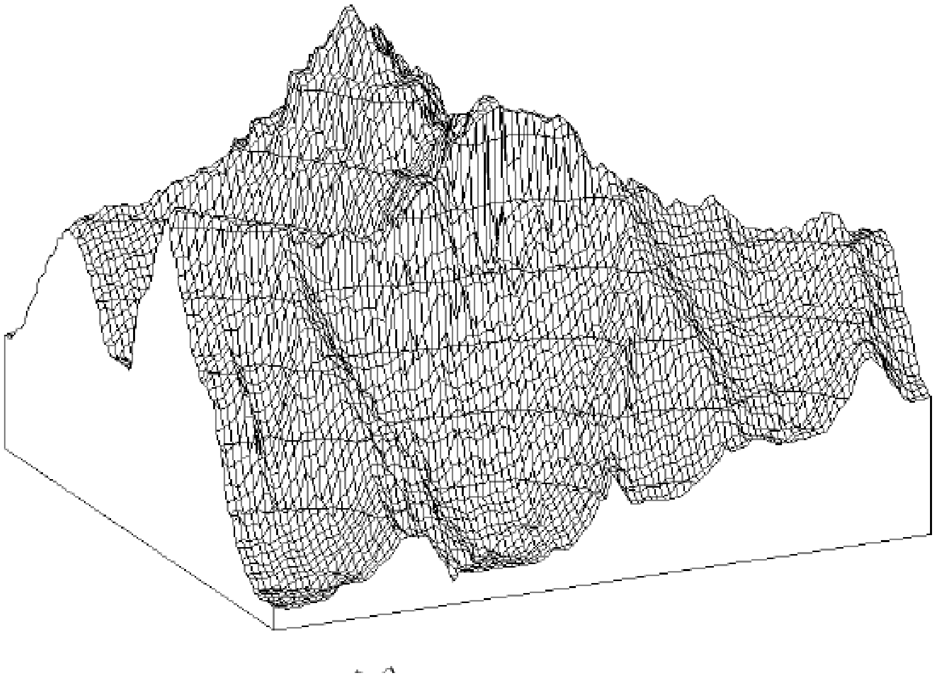


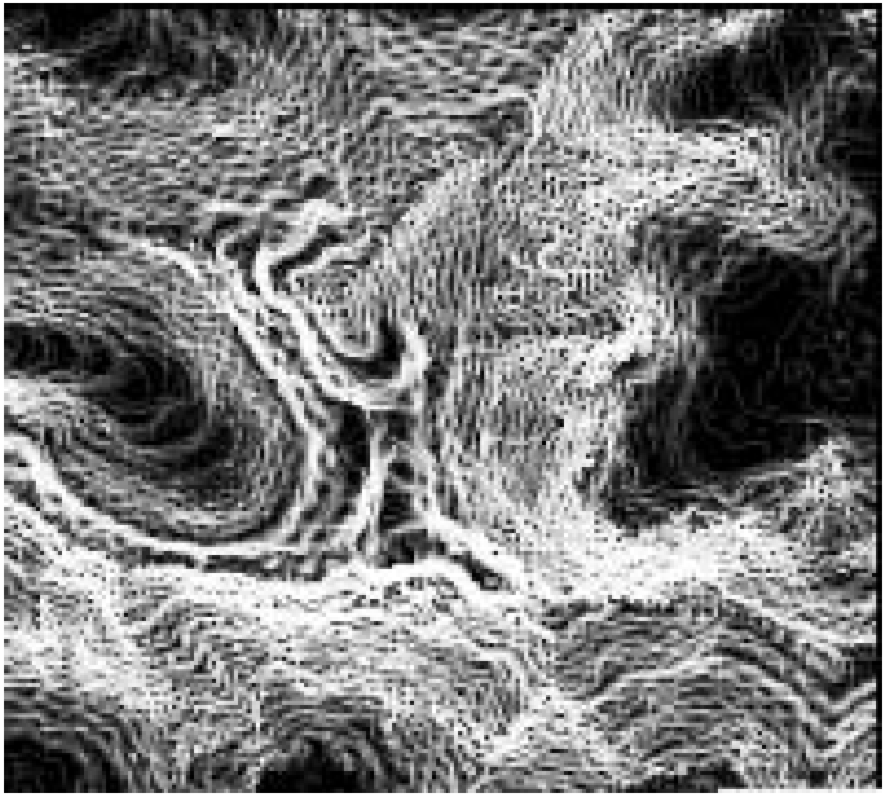
Surface-Specific Point Sampling

- Terrain "Skeleton“
- Wedding Cake effect
- Specific problem when grids are made from contours (e.g. 3 arc second DEMs)

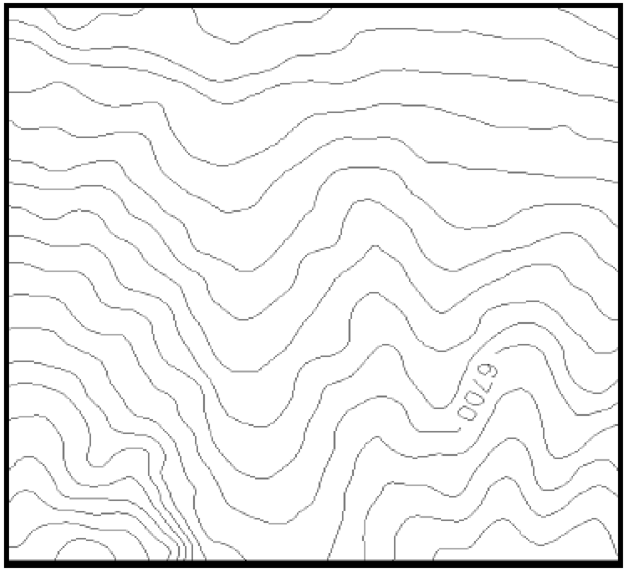


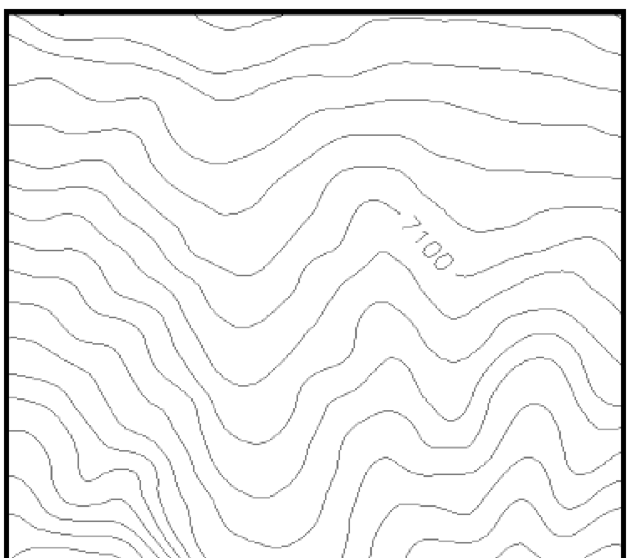
Wedding cake effect





Terrain filtering





Original Data

125	124	123	121
121	125	125	126
118	120	128	129

Hanning filter

1/16	1/8	1/16
1/8	1/4	1/8
1/16	1/8	1/16

Filter × Original

7.81	15.5	7.69
15.1	31.2	15.6
7.38	15.0	8.0

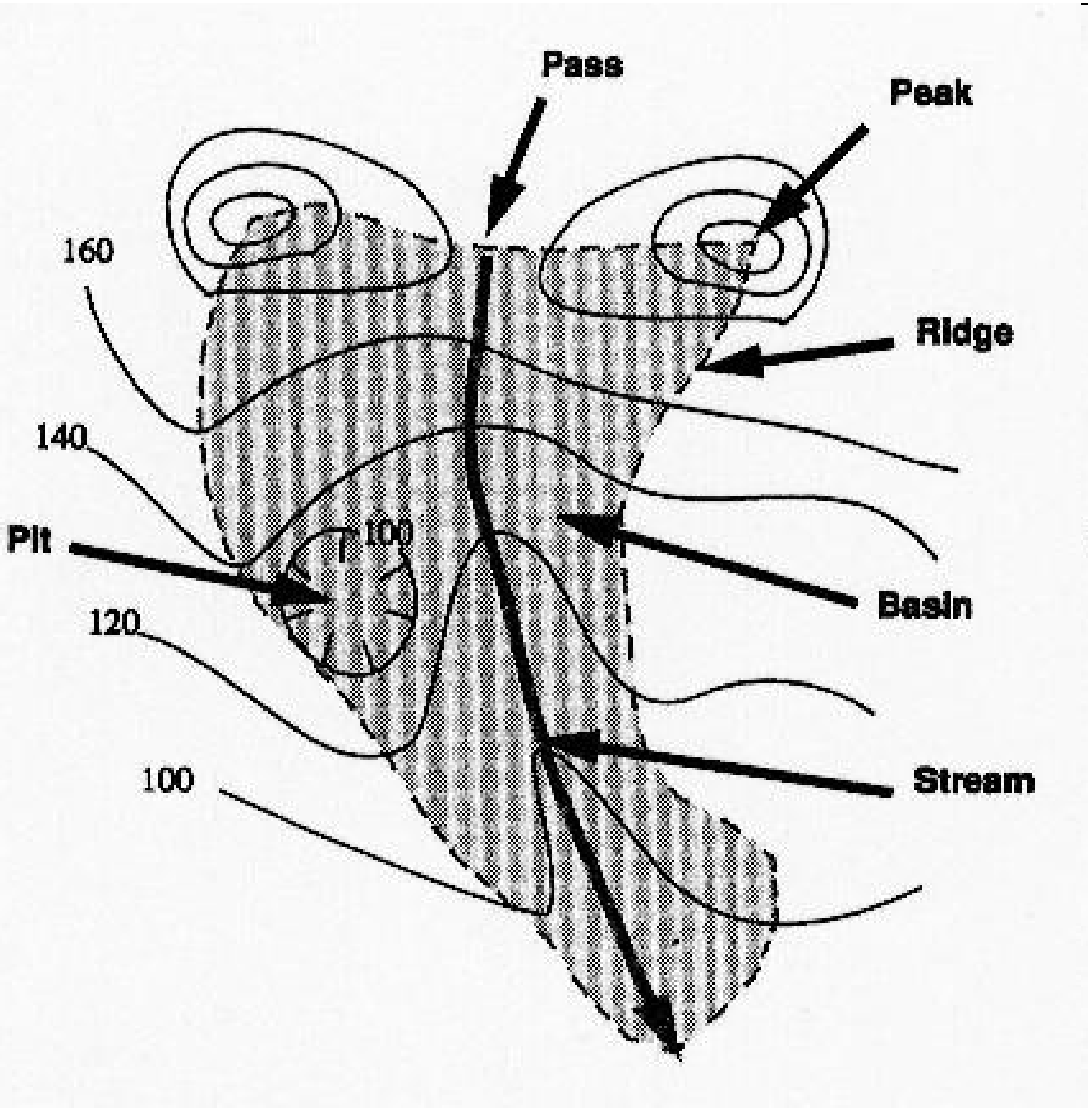
Summation for kernel

*	*	*	*
*	123		
*			

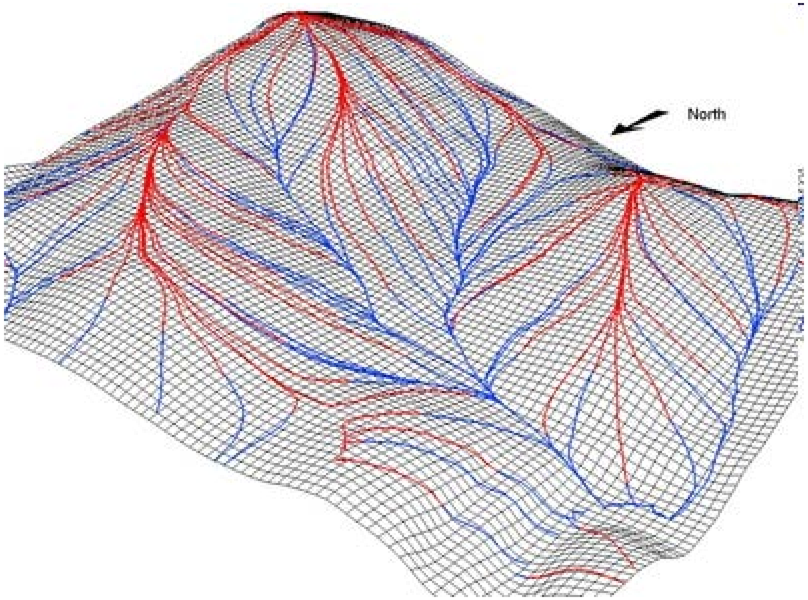
Terrain Analysis

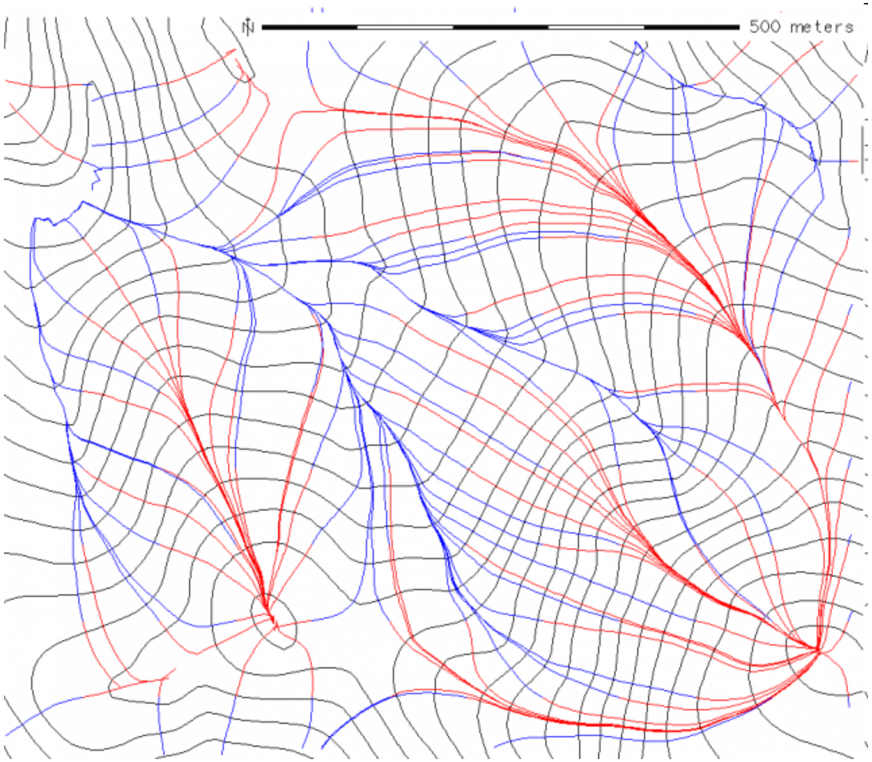
- Surface network extraction
- Surface network character, e.g. Strahler order
- Profile and Line-of-Sight
- Intervisibility and Viewshed
- Terrain modeling
- Vizualization

Warntz Network



Terrain "skeleton" Warntz network Surface network





Visualization

Back to the Future; J. Mower 2009

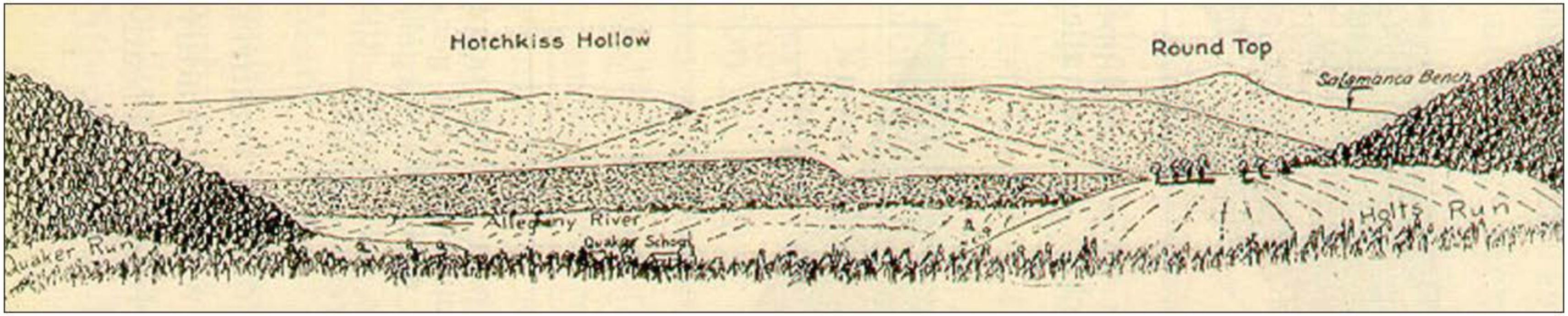


Figure 7. Lobeck's drawing of the view from a point within Allegany State Park in New York, near Quaker Run, and looking toward Hotchkiss Hollow on the west side of the Allegany River. Printed with permission of the New York State Museum, Albany New York, 12203.

Figure 13. The composite image, including emissive white surface shading, silhouette lines, crease lines, and slope lines. Silhouettes are rendered in 3-point width; crease and slope lines are rendered in 1-point width.

Summary

- Analytical cartography also deals with surfaces or fields
- Terrain is the classic example, giving terrain analysis
- Surfaces can be transformed into other surfaces, e.g. slope, aspect
- Surfaces can be transformed into lines and networks, e.g. Warntz network, contours
- Interpolation is almost always necessary, often point to surface
- Interpolation results are highly sensitive to method
- Terrain representation and visualization also important