

SOME QUESTIONS ON DATUMS AND DATUM TRANSFORMATIONS

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1.1 What are some definitions of datums?

datum - any quantity or set of such quantities that may serve as a referent or basis for calculation of other quantities.

geodetic datum - a set of constants specifying the coordinate system used for geodetic control, i.e. for calculating coordinates of points on the Earth.

At least eight constants are needed to form a complete datum: 3 to specify the location of the origin, 3 to specify the orientation of the coordinate system and two to specify the dimensions of the reference ellipsoid.

(These and other useful definitions are included in the National Geodetic Survey _Geodetic_Glossary_ available from the NGS Info Center and elsewhere. The Info Center can be reached at 301.713.3242 (phone).)

As can be seen from the definition, a coordinate value is defined in terms of the datum in which it is computed.

For a more elaborate (and elegant) explanation, check Dr. Peter Dana's excellent web site at University of Texas.

by: Donald M. Mulcare dmulcar@ibm.net 3/20/98

1.2 What is the difference between NAD83 and NAD83(HARN)?

The High Accuracy Reference Network (HARN) is a superset of points determined by GPS to accuracies of 1ppm or better. Not all states have a HARN. Once a statewide HARN is complete, the statewide control network is adjusted to it.

Some background, at the time of its adoption in 1986, the NAD 83 did not include GPS observations. As GPS allows much greater precision, 10 ppm can be obtained almost trivially, it showed the shortcomings of the NAD 83 system. The NGS in cooperation with individual states started a program of GPS surveys at the 1ppm or better level. The work should be complete by the end of 1997.

Currently, there are two types of NAD 83 points. One is NAD 83 (1986) which reflects the original nationwide adjustment (which does *NOT* include GPS). The other is labeled NAD 83 (####), the #### refers to the date of the adjustment of the statewide network. These designations will change shortly to exclude the date and merely indicate whether the point has been adjusted to the HARN.

by: Donald M. Mulcare dmulcar@ibm.net 2/8/97

We (the National Geodetic Survey) do not have 3-D transformation parameters, per se. Transformations are accomplished using software (called NADCON). This software uses a method in which first and second order geodetic data in the NGS data base is modeled using a minimum curvature algorithm (Briggs, 1974) to produce a grid of values. Simple interpolation techniques are then used to estimate "corrections between NAD 83 (1986) and the 'precise' NAD83 (19xx).

The accuracy of the transformation is estimated to be 5-10 cm generally in the conterminous US.

The software and documentation can be obtained by accessing the NGS Web Site: <http://ngs.noaa.gov>

by: Maralyn Vorhauer maralyn@dancer.ngs.noaa.gov 5/9/97

The US Army Corps of Engineers provides GEOTRANS, a coordinate transformation package, free to any US citizen. GEOTRANS runs on Windows and on Solaris.

In a single step, GEOTRANS converts between any of the following coordinate systems, and between any of over 100 datums with an accuracy.

Geodetic (Latitude, Longitude)
 Geocentric 3D Cartesian
 Mercator Projection
 Transverse Mercator Projection
 Polar Stereographic Projection
 Lambert Conformal Conic Projection
 UTM
 UPS
 MGRS

GEOTRANS comes with a users manual. Tests for developers, documentation, and reusable software modules in either ANSI C and as DLLs are also available.

Please reply to Geospatial Information and Imagery Requirements Branch at rbinfo@tec.army.mil. We will not send the DLLs, ANSI C code or tests unless you request them specifically.

This software is brought to you by the Topographic Engineering Center, a US Army Corps of Engineer Lab (<http://www.tec.army.mil>).by: Dan Specht 25 Sep 1998 with corrections by Ray Caputo 18 July 2002.

1.3 What is the difference between WGS 84 and GRS 80?

Parameter	Notation	Units	GRS80	WGS84
Semimajor axis	a	m	6378137	6378137
Angular velocity	w	rad/s	7292115E-11	7292115E-11
Gravitational Const.	GM	m ³ /s ²	986005E8	3986005E8
Dynamic form Factor				
unnormalized form	J2		108263E-8	
normalized form	C2,0			-484.16685E-6
Seminor axis	b	m	6356752.3141	6356752.3142
Eccentricity squared	e ²		0.00669438002290	0.00669437999013
Flattening	f		0.00335281068118	0.00335281066474
Reciprocal flattening	f ⁻¹		298.257222101	298.257223563
Polar radius of curv	c	m	6399593.6259	6399593.6258

Not much difference!

Ref: North American Datum of 1983
 NOAA Professional Paper NOS 2
 Charles R. Schwarz, Editor
 U.S. Department of Commerce

by:Sam Wormley swormley@cnde.iastate.edu 3/7/97

1.4 How do I convert between NAD83 and WGS 84?

I think the proposition of doing a "datum shift" between NAD83 and WGS84 is a waste of time. The difference between the two is the philosophy of how an inertial datum should be defined, and is ignored by all but a handful of geodesists that are working at extraordinary levels of accuracy.

The ellipsoid of the World Geodetic System of 1984 is defined by four constants, one being the "bar C sub 20" Normalized Second Degree Zonal Harmonic Coefficient of the Gravitational Potential - the "J2" term is computed. The ellipsoid of the Geodetic Reference System of 1980 is defined by four constants, one being the "J2" Dynamical Form Factor - the "bar C sub 20" term is computed.

Geoid models (EGM96 versus GEOID96) will yield meaningful differences in ellipsoid/geoid separations (vertical) for North America, horizontal positions will not reflect anything beyond micro-fractions of a millimeter at best.

The differences between the semi-minor axes are computational (secondary DEPENDENT) variables based on different definitions of the gravity field. They are called "Derived Geometric Constants." The semi-major axis of the WGS84 ellipsoid is known to an accuracy of plus or minus two meters.

Transformations based on a derived difference of a tenth of a millimeter with an constant's accuracy of two meters is, IMHO, silly.

"Therefore, as long as the preceding is recognized, it can be stated that WGS84 and NAD83 are based on the same ellipsoid."

by: Clifford J. Mugnier cjmce@uno.edu 2/7/98

If you examine DMA Technical Report 8350.2, DOD WGS 84: Its Definition and Relationships with Local Geodetic Systems, dated 30 Sep 1987 you will note that the transformation parameters between NAD 83 and WGS 84 (deltaX, deltaY and deltaZ) are all zero (0). There is a difference in the flattening of -0.00000016. (See page 7-24).

The classified portions of the document are blank in the version available from either the National Geodetic Survey (NGS) Info Center at 301.713.3242 or from the commercial firm NAVTECH at 1.800.NAV.0885 (and elsewhere)

Complicating the statement that more most users NAD 83 and WGS 84 are equivalent is the decision by NIMA (formerly DMA) to redefine WGS84 to make it and keep it in line with the ITRF. The information below was extracted from an NGS summary of precise satellite ephemeris reference frames.

(Codes are used in NGS software.)

13. WGS 84 (G730) Epoch 1994.0 Used by DMA from 0000 UTC January 2, 1994, until 2400 UTC September 28, 1996 (Realignment of WGS 84 to ITRF 91 using GPS at GPS Week 730). [Blue Book code 13]
14. WGS 84 (G730) Epoch 1994.0 (Broadcast GPS Ephemeris) Used by DoD USAF from June 29, 1994, until January 28, 1997. [Blue Book code 14]
15. ITRF94 (epoch 1996.0) Used by NGS from 0000 UTC June 30, 1996, until 2400 February 28, 1998. [Blue Book code 15]
16. WGS 84 (G873) Epoch 1997.0 Used by NIMA (formerly DMA) from 0000 UTC September 29, 1996, until the present (realignment of WGS 84 [G730] to ITRF 94). [Blue Book code 16]
17. WGS 84 (G873) Epoch 1997.0 (Broadcast GPS Ephemeris) Used by DoD USAF from January 29, 1997, until the present. [Blue Book code 17]

The Naval Surface Warfare Center in Dahlgren, Va had a technical report detailing the most recent WGS84 realignment on their web site. I don't have the web site address handy. If you are unable to find it yourself, please advise and I'll forward the link.

by: Donald M. Mulcare email: dmulcar@ibm.net 3/12/98

1.5 What is the difference between NAD 83 and the ITRF?

Much of the information below was based on a draft version of a paper relating NAD 83 to the ITRF written by Stephen Frakes of the National Geodetic Survey.

NAD 83 was implemented in 1986 with an adjustment combining terrestrial data with space-based data such as 3-D positions from satellite Doppler and 3D vectors from VLBI. The global parameters used in NAD83, X,Y,Z shifts and X, Y rotations plus scale were fixed to a priori values based on Doppler. The Z rotation was defined by astronomical azimuths. VLBI rotations and scale were solved as a means to compare systematic differences between the VLBI and NAD 83 coordinate systems. The NAD 83 adjustment did not include GPS. Positions determined in this adjustment are labeled NAD 83 (1986). The adoption of GPS revealed inadequacies in the system which prompted upgrade surveys (described below).

If your question is how the US National Geodetic Survey Continuously Operating Reference Stations (CORS) station positions indicated as "NAD 83 Precise" were derived, they were determined by a seven-parameter transformation which relates the NAD 83 system to the ITRF.

There are however, another version of NAD 83 coordinates which are part of an upgrade program. In 1988 the NGS began to upgrade the NAD 83 system by performing high accuracy GPS surveys. In order to establish control stations for these upgrade surveys the following steps were performed:

1. A set of transformation parameters were derived between the NAD 83 VLBI stations and coordinates from the 1989 solution of the ITRF (ITRF 89) for a set of a dozen common stations.
2. The scale of ITRF89 was accepted
3. The derived transformation parameters, translations and rotations, but not the scale were applied to the complete set of ITRF89 positions to get NAD83 values with corrected scale.

The NGS datasheets show the station position with its datum. States with a high accuracy reference network (HARN) will show the datum as NAD 83 (####) the #### indicating the date of the adjustment. A NAD 83 (1986) position is also provided on the data sheet. NAD 83 (1986) refers to the original NAD adjustment.

by: Donald M. Mulcare dmulcar@ibm.net 8/20/97

For a discussion on the NAD83 and ITRF transformation look at

<ftp://cors.ngs.noaa.gov/coord/derivation/derivation.txt>

But as this following note shows, the transformation parameters are not universally applicable.

Fellow Canadians,

Please note that the NAD83-ITRF94 transformation defined by the above US NGS document does not apply to NAD83 in Canada. In Canada the transformation at epoch E (years) from ITRF94 geocentric coordinates (Xi, Yi, Zi) to NAD83(CSRS) coordinates (Xn, Yn, Zn) is defined by (converted to same notation as NGS):

$$X_n = T_x + (1 + S) * X_i + R_z * Y_i - R_y * Z_i$$

$$Y_n = T_y - R_z * X_i + (1 + S) * Y_i + R_x * Z_i$$

$$Z_n = T_z + R_y * X_i - R_x * Y_i + (1 + S) * Z_i$$

where

$$T_x = 0.9392 \text{ m} - 0.0004 \text{ m/y} * (E - 1996.0)$$

$$T_y = -1.9762 \text{ m} + 0.0004 \text{ m/y} * (E - 1996.0)$$

$$T_z = -0.5386 \text{ m} - 0.0008 \text{ m/y} * (E - 1996.0)$$

$$R_x = 13431\text{e-11 rad} + 25\text{e-11 rad/y} * (E - 1996.0)$$

$$R_y = 4497\text{e-11 rad} - 369\text{e-11 rad/y} * (E - 1996.0)$$

$$R_z = 5118\text{e-11 rad} - 11\text{e-11 rad/y} * (E - 1996.0)$$

$$S = 0.0049 \text{ ppm}$$

The Canadian-US differences between the two transformations are:

$$\Delta T_x = -0.0346 \text{ m} - 0.0004 \text{ m/y} * (E - 1994.0)$$

$$\Delta T_y = -0.0309 \text{ m} + 0.0004 \text{ m/y} * (E - 1994.0)$$

$$\Delta T_z = 0.0100 \text{ m} - 0.0008 \text{ m/y} * (E - 1994.0)$$

$$\Delta S = 0.0049 \text{ ppm}$$

You can judge for yourself if these differences are significant for your applications. For more information and transformations from other realizations of ITRF, contact the Geodetic Survey Division, Geomatics Canada <mailto:information@geod.NRCan.gc.ca>.

by: Mike Craymer craymer@nrcan.gc.ca 5/8/97

1.6 From UTM, can I convert to any datum I want?

Nope. You have to first go to Geodetic Coordinates, perform a Datum Shift in Geocentric Coordinates with either a 3-parameter or a 7-parameter transformation, then transform back to Geodetic Coordinates, then to whatever Grid system you want on the new Datum.

There are a number of "free" packages that can do this for you without the need to bother with the details if your accuracy requirements will tolerate a black-box transform. Common packages are NADCON from the National Geodetic Survey and CorpsCon from the Topographic Engineering Center IF YOU ARE WORKING IN THE UNITED STATES. Foreign datum shift stuff is available from the NIMA site under the general topic of "MUSE," the specific package is MADTRAN. Note that the NIMA package is for modest accuracy requirements, it generally is not suitable for scales larger than 1:50,000. The NADCON/CorpsCon packages are suitable for larger scale applications but still fall short for geodetic survey accuracy. You, the user, need to access the accuracy requirements for your application. If you do not know, then you need to get help before you proceed further.

This stuff gets deep quick. I would recommend a bit of research before "jumping into" Datum Shifts. I publish a (nearly) monthly column in "Photogrammetric Engineering and Remote Sensing" on "Grids and Datums." Each installment is a different continent and country.

by: Clifford J. Mugnier cjmce@uno.edu 2/25/98

1.7 How do I compute the coordinate based on the other datum? Do I have to convert to ECEF first?

You're going the right way! ECEF is essential. Convert the lat/long to ECEF (using the correct parameters for the spheroid they are on), then apply the dX,Y,Z, scale factor (and rotations if you have them). Getting the signs right here is vital! In some conversions, reversing the signs can put you +1 km from the correct position. Use the resulting ECEF value to compute lat & long on the new spheroid, once again using the correct parameters.

To give you an example, let us assume we have a point at 25d 00m 00s South Latitude, 141d 00m 00s East Longitude with a spheroidal height of 65.459m. This is on the Australian Geodetic Datum 1984 (AGD84) (6378160, 1/298.25) We want to convert to WGS84 (6378137.00, 1/298.257223563) The datum shift FROM ADG84 TO WGS84 is dX -116.00 dY -50.47 dZ141.690. Scale & rotations have been ignored.

The ECEF pos is then applying dXYZ gives

-4495085.606 -116.00 -4495201.606

3640048.551 -50.47 3639998.081

-2679111.4.. 141.69 -2678969.710

This gives a WGS84 position of

24d 59m 55.0101s

141d 00m 04.0020s

81.417m (spheroidal ht.) (this might be slightly off)

For anyone trying to repeat this, please don't email me if its slightly out!!! This is simply a fast and dirty approximation done with software designed for a 7 parameter transform. :-)

For anyone who wants to look seriously at transforms, I can recommend Ausligs calc page (URL below)

<http://www.auslig.gov.au/geodesy/calcs.htm>

There are a number of Excel spreadsheets to perform the above calcs etc.

by: Doug Hall dwh@fl.net.au 3/8/98

1.8 What is the difference in elevation when switching between WGS 84 and NAD 27?

Maps using the NAD 27 datum also generally use the National Geodetic Vertical Datum of 1929. Elevations are measured against Mean Sea Level (MSL). The two datums (horizontal and vertical) are, essentially independent of one another.

Maps using the more recent NAD 83 datum (very similar to WGS 84.. within a meter or two) will still have essentially the same elevations above MSL.

It is interesting how the GPS (based on WGS 84) works. Four or more satellites can be used to determine three position dimensions and time. Position dimensions are computed by the receiver in Earth-Centered, Earth-Fixed X, Y, Z (ECEF XYZ) coordinates. When more than "the best four" satellites are used, this is called an over-determined solution.

Most receivers compute (and store) the geodetic (in WGS 84) coordinates of latitude, longitude and height above the ellipsoid (HAE). WGS 84 coordinates are transformed and displayed in whatever datum you choose. Always choose the datum of the map you are working with.

The height displayed on most consumer handheld GPS receivers is orthometric height, the height above mean sea level (MSL). It is straight forward to approximate MSL world wide by interpolation of the GEOID model (table) and making the simple calculation:

$$\text{Orthometric Height (MSL)} = \text{HAE} - \text{Geoid Undulation}$$

This is what your GPS receiver does. The height accuracy is generally half as accurate as the horizontal position accuracy, and often thought of little use, but can be very useful once differential corrections are applied. See: <http://www.cnde.iastate.edu/staff/swormley/gps/dgps.html> for more about differential corrections.

REFERENCES

Brown R G, Hwang P Y C, Introduction To Random Signals And Applied Kalman Filtering, John Wiley & Sons, Inc., New York, 1992

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Dana P H, Geodetic Datum Overview, <http://www.utexas.edu/depts/grg/gcraft/notes/datum/datum.html>, Department of Geography, University of Texas at Austin 1995, 1996, 1997

GeoExplorer Operation Manual, Trimble Part No. 21281-00

by: Sam Wormley swormley@cnde.iastate.edu 5/28/97

1.9 Conventional transformations between datums use only a relative offset between different frames of reference, but no relative rotation. Is this really enough?

The relative offsets in terms of a 3-Parameter Datum Shift is because the single board computer (sbc) in the old Magnavox 1502 Transit Satellite Doppler Receivers could only accommodate that simple model in 1972 when they were put into production. When the WGS72 Datum was declassified in 1976, the 1502's were declassified and sold to the public. THAT's when people started to learn what Datums were all about. Considering that before then, the fanciest (absolute) positioning hardware was a First-Order Astronomical Theodolite with a radio chronometer and impersonal micrometer that after two evenings of observations got you a position to +/- 100 meters.

It is enough for small regions and countries if you're delivering 155 mm Howitzer Cannon fire or sending Cruise missiles with High Explosive Warheads. It is not generally suitable for geodetic applications and many GIS applications if you need an accuracy better than +/- 25 meters.

The "free" stuff from NIMA is intended for destroying large geographic areas. Oil companies use it for rough reconnaissance positioning. Ignorant technicians misuse it for precise mapping.

What's enough? I like the 7-parameter Bursa-Wolfe Datum Shift and I like the 7-parameter Molodensky Datum Shift. Those accommodate rotations in all three geocentric axes as well as an overall scale factor. That is usually enough for most small countries.

by: Clifford J. Mugnier cjmce@uno.edu 10/08/98

1.10 Is a geoid, such as GEOID96 or the geopotential model EGM96, the same as a vertical datum, like NGVD 1929 or NAVD 1988?

EGM96 is the latest geoid for the world. GEOID96 is the latest geoid specifically for the United States of America. It is a smidgen different, but worth it if you're diddling with vertical stuff from GPS observations.

Those geoids are NOT the same as NAVD88, certainly NOT NGVD29.

The latest thing is using GPS geodetic-quality receivers and GEOID96 to obtain reliable vertical in the U.S. However, it does require a number of very specific procedures to obtain reliable results within a few centimeters.

I am referring a copy of this to Mr. Dave Doyle of the National Geodetic Survey in Silver Spring, Maryland. Mr. Doyle can help you better than I can, and he can also refer you to specialists within NGS regarding specifications of hardware, software options, publications, etc.

by: Clifford J. Mugnier cjmce@uno.edu 10/14/98

Neither NAVD 88 or especially NGVD29 is exactly correlated to the geoid, although NAVD 88 is a much better approximation. In addition the concept of "heights above MSL" is really only applicable to small islands, since mean sea level is not a level surface. Consequently most countries actually refer to heights above (or below) some defined vertical datum (e.g., NAVD 88). Cliff's point about GEOID96 and EGM96 are also right on. EGM96 models the relationship between the WGS 84 ellipsoid and the global geoid to approximately +/- 1m. I personally believe this is a little optimistic, but it's what the National Imagery and Mapping Agency (NIMA) claims. GEOID96 relates the ellipsoid and geoid in the conterminous U.S. to approximately 10 cm. Depending on the requirements for accuracy this may be sufficient. If your requirements are in the 2-5 cm range then I would recommend you download NOS NGS-58 - Guidelines for Establishing GPS-Derived Ellipsoid Heights from our web site at http://sinbad.ngs.noaa.gov/PUBS_LIB/pub_index.html.

by: Dave Doyle daved@ngs.noaa.gov 14 Oct 1998

2.1 What is the best way to calculate the great circle distance (ignoring elevation differences) between 2 points?

If the distance is less than about 20 km and the locations of the 2 points in Cartesian coordinates are X1,Y1 and X2,Y2 then the Pythagorean Theorem

$$d = \sqrt{(X2-X1)^2 + (Y2-Y1)^2}$$

will result in an error of: less than 30 meters for latitudes less than 70 degrees; less than 20 meters for latitudes less than 50 degrees; less than 9 meters for latitudes less than 30 degrees.

Otherwise, presuming a spherical Earth with radius R (see below), and the locations of the 2 points in spherical coordinates (longitude and latitude) are lon1,lat1 and lon2,lat2 then Sinnott's Formula (from R.W. Sinnott, "Virtues of the Haversine", Sky and Telescope, vol. 68, no. 2, 1984, p. 159):

$$d_{lon} = lon2 - lon1$$

$$dlat = lat2 - lat1$$

$$a = \sin^2(dlat/2) + \cos(lat1) \cos(lat2) \sin^2(dlon/2)$$

$$c = 2 \arcsin(\sqrt{a})$$

$$d = R c$$

will give mathematically and computationally exact results. The intermediate result c is the great circle distance in radians.

The Pythagorean flat-Earth approximation assumes not only that great circles are negligibly different from straight lines, but that the parallels of latitude are negligibly different from great circles. Close to the poles, the parallels of latitude are not only shorter than great circles, but indispensably curved, as well. Taking this into account leads to the use of polar coordinates and the planar law of cosines for computing short distances near the poles: The Polar Coordinate Flat-Earth Formula

$$a = \pi/2 - lat1$$

$$b = \pi/2 - lat2$$

$$d = R \sqrt{a^2 + b^2 - 2 a b \cos(lat2 - lat1)}$$

will give smaller errors than the Pythagorean Theorem for high latitudes, but even at 88 degrees the error can be as large as 20 meters when the distance between the points is 20 km.

An UNRELIABLE way to calculate distance on a spherical Earth is the Law of Cosines for Spherical Trigonometry

**** NOT RECOMMENDED ****

$$a = \sin(lat1) \sin(lat2) + \cos(lat1) \cos(lat2) \cos(lon2 - lon1)$$

$$c = \arccos(a)$$

$$d = R c$$

although this formula is mathematically exact, it is unreliable for small distances because the inverse cosine is ill-conditioned. Sinnott (in the article cited above) offers the following table to illustrate the point:

$\cos(5 \text{ degrees}) = 0.996194698$
 $\cos(1 \text{ degree}) = 0.999847695$
 $\cos(1 \text{ minute}) = 0.999999577$
 $\cos(1 \text{ second}) = 0.999999999882$
 $\cos(0.05 \text{ sec}) = 0.9999999999971$

A computer carrying seven significant figures cannot distinguish the cosines of any distances smaller than about one minute of arc.

Q5.1a: What value should I use for the radius of the Earth, R ?

The shape of the Earth is well approximated by an oblate spheroid with a polar radius of 6357 km and an equatorial radius of 6378 km. Any value in that range will do, such as

$$R = 6378 - 21 \sin(lat) \text{ See the WARNING below!}$$

where lat is the latitude in which the bulk of your calculations occur, PROVIDED a spherical approximation is satisfactory.

WARNING: The above formula for R gives but a rough approximation to the radius of curvature. According to Snyder ("Map Projections - A Working Manual", by John P. Snyder, U.S. Geological Survey Professional Paper 1395, United States Government Printing Office, Washington DC, 1987, p24), the radius of curvature varies with direction and latitude; in the plane of the meridian it is given by

$$R' = a (1 - e^2) / (1 - e^2 \sin^2(lat))^{3/2}$$

where a is the equatorial radius; b is the polar radius; and e is the eccentricity of the ellipsoid $= (1 - b^2/a^2)^{1/2}$.

When is it NOT okay to assume the Earth is a sphere?

A quick test is: Compute the values of R produced by the above equation when you use the highest and lowest latitudes that occur in your analysis. Use these two values in your analysis. If the different results are different enough to cause you to change your action (or your recommendation, or your interpretation of the implication of the results, etc.), then assuming the Earth is spherical is NOT okay.

For most purposes, it is quite satisfactory to treat the Earth as a sphere. If necessary, an oblate spheroid can provide a better approximation.

The shape the Earth would assume if it were all measured at mean sea level is called the geoid. The geoid varies no more than about a hundred meters above or below a well-fitting ellipsoid, a variation far less than the ellipsoid varies from the sphere. Terrain relief is reported relative to the geoid. (Paraphrased from p. 11 of the book by Snyder cited above.) Distances on the surface of the geoid are not particularly meaningful.

Distances on the surface of the terrain, whether geodesic, on roads, cross-country, or straight-line, depend on relief and on the status of engineering projects. Hence, computation is idiosyncratic and not well suited to simple approximations.

by: Seymore Dupa grumpy@en.com 5/13/97

Here are some notes on the subject which may help. In summary, spherical earth model, easy. Ellipsoid model, you need an embedded programme such as the USGS "Inverse".

DISTANCE CALCULATIONS

ELLIPSOID EARTH MODELS. Accurate distance calculations using ellipsoid earth models such as WGS84 are complex because of the double-curvature of the surface of an ellipsoid. You cannot even take the local earth radius for a 'simple' distance calculation, because it is the radius of curvature which matters, and not the distance from the centre of the earth. Because of the double-curvature of the ellipse surface, the local radius of curvature differs between E-W and N-S (and any other direction), so you see the problem! A sphere is much easier. However, PC-based programmes are available for distances on the WGS84 ellipsoid such as the US National Geodetic Survey (NGS) 'inverse' programme.

And another freeware programme from Vic Fraenckel, available via ftp from: [ftp.wizvax.net/pub/personal/victorf](ftp:wizvax.net/pub/personal/victorf) Grab the files INVERSE.ZIP and DIRECT.ZIP

GEODETIC DATUM. Accuracy also depends on all lat/long figures being to the same Geodetic Datum before calculations are performed. For short distances using the same type of map for deriving co-ordinates, this is generally fulfilled, but for long distances the Geodetic Datums for the co-ordinates will be different, and for accuracy should be converted to a common datum such as WGS84 by a programme such as the US DMA's MADTRAN (Map Datum TRANSformation) software, available through dma_info_admin@dma.gov

USEFUL WWW SITES:

Geodetic Datums:

<http://www.utexas.edu/depts/grg/gcraft/notes/datum/datum.html>

Coordinate Systems:

<http://www.utexas.edu/depts/grg/gcraft/notes/coordsys/coordsys.html>

[Map Projections:](#)

—

href="http://www.utexas.edu/depts/grg/gcraft/notes/mapproj/mapproj.html">http://www.utexas.edu/depts/grg/gcraft/notes/mapproj/mapproj.html

SPHERICAL EARTH MODELS. For less accurate results but for ease of calculation, a constant earth radius is sometimes assumed. For instance, the Federation Aeronautique Internationale (FAI) in Paris uses a radius of exactly 6371 kilometres. This is approximately the radius of a sphere which has the same volume as the WGS84 ellipsoid. A calculation method is given in one of their documents:

EXTRACT FROM FAI SPORTING CODE SECTION 3 - GLIDERS AND MOTOR GLIDERS

2.2.13 Each degree of angle at the earth's centre, extended upwards to the surface of the Earth, is equivalent to a distance of 1/360 th of the circumference of the 'FAI Sphere'.

Taking Pi as 3.141 592 654, and the FAI radius of 6371 km, each degree is equivalent to the following distance in kilometres:

$$\begin{aligned} 2 \text{ Pi R} / 360 &= (2 \times 3.141592654 \times 6371) / 360 \\ &= 111.192\ 926\ 645 \text{ km per degree subtended at the Earth's centre.} \end{aligned}$$

For an angle X subtended at the Earth's centre between two positions A and B on the Earth's surface which are defined in degrees (and decimal degrees) of Latitude and Longitude, the formula (assuming a spherical Earth) is:

$$\text{Cos X} = (\text{CosLatA} \times \text{CosLatB} \times \text{Cos}(\text{Long A} - \text{Long B})) + (\text{Sin LatA} \times \text{SinLat B})$$

A computer or calculating device should be used which is capable of working to at least 10 significant figures before distances based on the above formula will be considered.

The above formula uses cosines of very small angles (figures close to 1); it may be converted to the one that follows which uses Sines (figures close to zero) which will generally produce a more accurate result when a calculator is used which has a floating point system:

Sin X/2 = Sq Root of all of the following expression:

$$(\text{CosLatA} \times \text{CosLatB} \times \text{Sin}^2((\text{LongA} - \text{LongB}) / 2) + \text{Sin}^2((\text{LatA} - \text{LatB}) / 2))$$

(Note that the symbol ^ denotes an index function eg ^2 is squared)

A computer or calculating device should be used which is capable of working to at least 7 significant figures before distances based on the above formula will be considered.

Spreadsheet programmes generally work to about 15 significant places and are to be preferred to hand-held calculators when accuracy is required.

To convert from km/metres to other units:

1 International Nautical Mile = 1852 metres exactly

1 International Statute Mile = 1609.344 metres exactly

= 5280 International Feet exactly

1 International Foot = 0.3048 metres exactly

1 US Survey Foot = 1200 / 3937 metres exactly

1 Metre = 3.280 839 895 01 International Feet

= 3.280 833 333 33 US Survey feet

(Source, US DMA TR 8350.2 Second Edition)

Finally, some detail about the WGS 84 ellipsoid. More is in publications such as the US DMA Technical Report "DMA TR 8350.2" entitled "World Geodetic System 1984, its definition and relationship with local geodetic systems". This is a half-inch thick public-domain publication and may be obtained from the US Geodetic Survey (USGS) at:

USGS Information Services,

PO Box 25286,

Mail Stop 306,

Denver Federal Center,

Denver, Colorado 80225, USA

The price in 1995 was 37.5 dollars and included a diskette of the MADTRAN programme (non-copyright) for converting lat/longs between over 100 different Geodetic Datums.

WGS84 Ellipsoid - General

$a = 6378,137.0$ metres (major semi-axis, ie equatorial radius, by definition)

$b = 6356,752.314$ m (minor semi-axis, ie polar radius)

Ellipsoid flattening ratio $1/f = 298.257\ 223\ 563$ where f is flattening, by definition.

Flattening $f = 0.00\ 335\ 281\ 0665$)

Flattening distance ($a \times f$) is 21.38468575 kilometres

so Polar radius b is $(a - a/f) = 6356,752.314$ m (minor semi-axis)

e is eccentricity, where $e^2 = 2f - f^2$, so $e = 0.08181919$

(The symbol $^$ denotes a power, so 2 is squared, and so forth)

Radius at given geocentric latitude (Distance from Earth centre to Point concerned)

$$r = 1 / (\cos GDlat \times (\sqrt{(1/a^2) + (\tan GDlat^2/b^2)}))$$

or

$$r = \sqrt{[(\cos^2 GD lat / a^2) + (\sin^2 GD Lat / b^2)]}$$

Calculations. Geocentric (GC) latitude is the angle subtended at the earth's centre, whereas Geodetic (GD) latitude is with reference to the local vertical at the point on the ellipse. These two latitudes are the same for a sphere but different for an ellipsoid.

GD Latitude is used for mapping, ie the lat lines on maps are GD lat, derived from measurements of local vertical and compensated for gravitational anomalies so as to be a true vertical to the Geodetic Datum used for the map (eg the WGS 84 ellipsoid). The conversion is:

$$GC\ Lat = \arctan \left(\left(\frac{b^2}{a^2} \right) \tan GD\ Lat \right)$$

where a and b are the equatorial and polar radii of the ellipse concerned.

by: Newport-Peace np@spsys.demon.co.uk

```

===== General Equations =====

denom1 = (cos(lon1)*cos(lat1)*cos(lon1)*cos(lat1) + cos(lat1)*sin(lon1)*cos(lat1)*sin(lon1) + sin(lat1)*(1.0-e2)*sin(lat1)*(1.0-e2))
denom2 = (cos(lon2)*cos(lat2)*cos(lon2)*cos(lat2) + cos(lat2)*sin(lon2)*cos(lat2)*sin(lon2) + sin(lat2)*(1.0-e2)*sin(lat2)*(1.0-e2))
>
angle = cos(lon1)*cos(lat1)*cos(lon2)*cos(lat2) + cos(lat1)*sin(lon1)*cos(lat2)*sin(lon2) + sin(lat1)*sin(lat2)*(1.0-e2)*(1.0-e2)
angle = acos(angle / (sqrt(denom1)*sqrt(denom2)))

/* approximate radius is sqrt(MN) */

distance = fabs(angle * sqrt( ((M1+M2)/2.0)*((N1+N2)/2.0) ));

e2 = 1st Eccentricity Squared = (a^2 - b^2) / (a^2)

N1 = RADIUS_OF_CURVATURE_OF_PRIME_VERTICAL of first point
N2 = RADIUS_OF_CURVATURE_OF_PRIME_VERTICAL of second point
M1 = RADIUS_OF_CURVATURE_OF_MERIDIAN of first point
M2 = RADIUS_OF_CURVATURE_OF_MERIDIAN of second point

To work on a sphere, set a=b, that is, e2=0.0

For WGS84, you could use
a = 6378137.0000000000 meters
b = 6356752.3142451793 meters
e2 = 0.0066943799901413165

```

by: Danny Alberts dha@mbi.com 2/25/97

Here's a snippet from a message originally posted by David Haycraft...

Function GreatCircleArcLength(ByVal lng1,lat1,lng2,lat2 as float) as float

' Calculate length of great circle arc through points (lng1,lat1) and

(lng2,lat2).

' Input units are decimal degrees.

' Output unit is km.

DIM cosZ as float

cosZ=sin(DEG_2_RAD*lat1)*sin(DEG_2_RAD*lat2)

+cos(DEG_2_RAD*lat1)*cos(DEG_2_RAD*lat2)*cos(DEG_2_RAD*(lng2-lng1))

GreatCircleArcLength=6371.1*aCos(cosZ)

End Function

by: Walter Dnes walter.dnes@ec.gc.ca

2.2 How do you calculate latitude/longitude of a point given latitude/longitude and distance/bearing from another point?

You will find distance formula at this place: THE MATH FORUM

Subject: Distance Between Two Points on the Earth

Address: <http://forum.swarthmore.edu/dr.math/problems/coopersmith.6.21.96.html>

Be careful. If you calculate the formula with ACCESS from Microsoft, for example, the functions Cos (Cosine) and Sin (Sine) return the result from radian angle, not degree. So you must translate your degree value before by multiplied it with the value of PI and divide it by 180. The complete formula in kilometers is:

$$\begin{aligned} & \text{Atn}(-(\text{Cos}(\text{A.lat} * 3.14159265358979 / 180) * \text{Cos}(\text{B.lat} * 3.14159265358979 / 180) * \text{Cos} \\ & ((\text{B.lon} - \text{A.lon}) * 3.14159265358979 / 180) + \text{Sin}(\text{A.lat} * 3.14159265358979 / 180) * \text{Sin} \\ & (\text{B.lat} * 3.14159265358979 / 180)) / \text{Sqr}(-(\text{Cos}(\text{A.lat} * 3.14159265358979 / 180) * \text{Cos}(\text{B} \\ & .\text{lat} * 3.14159265358979 / 180) * \text{Cos}((\text{B.lon} - \text{A.lon}) * 3.14159265358979 / 180) + \text{Sin}(\text{A} \\ & .\text{lat} * 3.14159265358979 / 180) * \text{Sin}(\text{B.lat} * 3.14159265358979 / 180)) * (\text{Cos}(\text{A.lat} * 3.1 \\ & 4159265358979 / 180) * \text{Cos}(\text{B.lat} * 3.14159265358979 / 180) * \text{Cos}((\text{B.lon} - \text{A.lon}) * 3.14 \\ & 159265358979 / 180) + \text{Sin}(\text{A.lat} * 3.14159265358979 / 180) * \text{Sin}(\text{B.lat} * 3.14159265358 \\ & 979 / 180)) + 1)) * 2 * \text{Atn}(1)) * 6371 \text{ AS KILOMETERS} \\ & \dots \text{ or } \dots * 3959 \text{ AS MILES} \end{aligned}$$

where A.lat and A.lon are Latitude and Longitude of point A, B.lat and B.lon are Latitude and Longitude of point B.

by: michel-saou michel-saou@msn.com 3/4/97

3.1 What is the Lambert Orthomorphic Projection?

Mathematically, the Lambert Conical Orthomorphic projection is synonymous with the Lambert Conformal Conic. There is NO DIFFERENCE. However, the term, "orthomorphic," is classical British and implies a considerably different problem to the unfortunate user of the aforementioned software package.

For the ellipsoidal case,

The "American Definition" defines a secant zone by two standard parallels, and that is the way the General Cartographic Transformation Program (GCTP) is programmed and is used by many near-illiterate (geodetically unwashed) software houses world wide, particularly the large dunder-headed GIS houses in North America. Works great in the U.S. for the State Plane Coordinate Systems on NAD 1927 or NAD 1983 !!! Does not do diddly elsewhere ...

The "British Definition" defines a secant zone by a Latitude of Origin and a Scale Factor at Origin. This is a very different way of doing things and applies, for the most part, to every Lambert Zone in the entire world outside of the United States and its possessions. If the brilliant software gurus used GCTP as the transformation engine for your application package, too bad for you. Get in line and maybe someone will be willing to develop a "work around" for your project because you pay enough to warrant VIP treatment. (Their "work-arounds" are the joke of the industry!)

Both definitions yield EXACTLY the same projection and Grid System if you know what you're doing.

Now, for the Island of Borneo, there are two British Lamberts. One is defined for Papua, New Guinea on the AS120 Tokang, Balik Papan Datum. It is a true Lambert Conical Orthomorphic. (The Latitude of Origin is below the equator, and probably is listed in the marginal information on your paper map.) The other is defined for Indonesia's portion of the Island and uses Paga Hill 1939 Datum. For that particular Lambert Conical Orthomorphic the Latitude of Origin is defined at the equator (zero degrees) - at which point a Lambert instantaneously converts into a Normal Mercator! Therefore, if you're diddling with Indonesian Borneo, GCTP will accommodate a Mercator. If, on the other hand you're diddling with Papua, New Guinea Borneo; you're probably in deep doo-doo.

by: Clifford J. Mugnier cjmce@uno.edu 19 Feb 1998

3.2 What is the Gauss Krueger Projection?

Gauss Krueger Coordinate System

There is a site in Geneva for the European Petroleum Studies Group (EPSG) that gives free data out on many Grid Systems in Europe. Note that Switzerland does not use a Transverse Mercator; it uses a Rosenmund Oblique Cylindrical that just by chance is already programmed in PROJ4.

European use of the Transverse Mercator has been 100% Gauss-Kruger (u with an umlaut on Kruger) since the Second World War. Prior to WWII, there's some pretty weird stuff (in Europe) that is currently found only in the various colonies settled by the European Nations.

The various formulae that comprise the family of Transverse Mercator truncations are rarely found in English. Most of the stuff is German and Eastern European languages; occasionally French, Spanish, Italian and Portuguese. A little bit is in Mandarin.

by: Clifford J. Mugnier cjmce@uno.edu 2/25/98

3.3 Are there any programs to convert ECEF X,Y,Z to UTM X,Y?

I don't know are there available simple spreadsheet for converting ECEF X,Y,Z to UTM X,Y over WGS84 ellipsoid but using 2 programs it possible:

a) ECEF X,Y,Z to Lat, Long is available to download as MS Excel spreadsheet at: <http://www.auslig.gov.au/geodesy/calcs.htm>

Lat, Long to UTM is online available at: <http://www.geod.emr.ca/html-public/GSDapps/English/gsrug-gtou.html> (Geodetic Survey - Software and Related Data Products Online Demonstration GSRUG Geographic to Universal Transverse Mercator (UTM)) or download MS Excel Spreadsheet at: <http://www.auslig.gov.au/geodesy/calcs.htm> but You also can download Madtran.exe (DOS) from: <ftp://ftp.tapr.org/tapr/SIG/aprssiig/files/dosstuff/>

NB! <http://www.auslig.gov.au/geodesy/calcs.htm> all spreadsheets are based at Australian ellipsoid AGD66 or GDA94, you should just change ellipsoid parameters to WGS-84 ones:

Spheroid: WGS-84

Equatorial Radius (semi-major axis): 6378137.0000 m

Flattening: 1/298.257223563

Polar Radius (semi-minor axis): 6356752.3142 m

Eccentricity: 0.00669437999013

by: Lauri Kreen lauri@eans.ee 05/27/98

4.1 What is the datum/projection of Curacao?

The Datum and Grid on Curacao was established in 1951 and uses a local Transverse Mercator. The origin of the datum is "DP 8" ("Dienst Punt 8" or Survey Point 8). The stuff I have there for WGS84 datum shifts looks really, really good. (sub-centimeter accuracy)

by: Cliff Mugnier cjmce@jazz.ucc.uno.edu 15 Sep 1998

The Datum and Grid on Curacao was established in 1951 and uses a local Transverse Mercator. The origin of the datum is "DP 8" ("Dienst Punt 8" or Survey Point 8).

To transform FROM "Curacao Datum of 1951" TO "NAD83" (aprox. Equivalent to WGS84):

dX = -265.766 meters +/- 0.03 meters

dY = +109.445 meters +/- 0.05 meters

dZ = -360.686 meters +/- 0.10 meters

This is based on my analysis of three First-Order Points, the Variance of Unit Weight was 1.02 (that's good).

Therefore,

-----Local Datum-----

DP35 X = 51,372.28 meters

Y = 59,011.79 meters

Z = 9.40 meters

Lat = 12° 11' 24.584" North

Long = 68° 55' 25.002" West of Greenwich

Gauss-Kruger Transverse Mercator Latitude at Origin: 12° 31' 12.360" North

Central Meridian: 69° 59' 34.586" West of Greenwich

False Easting = 10,000.000 meters

False Northing = 15,000.000 meters

Scale Factor at Origin = 1.0

Ellipsoid = International 1909 (Hayford), where a = 6,378,388.0 meters 1/f = 297. -----NAD83----- DP35

Lat = 12° 11' 13.24352" North

Long = 68° 55' 31.91039" West of Greenwich Height = -12.905 meters <

Ellipsoid

by: Clifford J. Mugnier cjmce@uno.edu

4.2 What is the datum/projection of St. Lucia, West Indies?

The projection and datum of St. Lucia, West Indies: The current classical geodetic datum for your country is "DOS 3 Lighthouse 1955" established by the Directorate of Overseas Surveys. (They are now part of the Ordnance Survey of Great Britain.) I do not have the coordinates of the origin or the azimuth of orientation, but the ellipsoid is the "Clarke 1880, (Modified)" where $a = 6,378,249.145$ meters, $1/f = 293.465$. The associated Grid is the British West Indies "BWI Grid" that is based on the Gauss-Kruger Transverse Mercator projection with no special truncations. The Central Meridian is 62W, False Easting is 400 Km, scale factor at origin is reduced to 1/2000. Eastings for your country are always greater than 500 Km., the Northings are always greater than 1,500 Km. (That's a lot of digits for your island!!)

The "Vieux Fort" is an old Astro Datum known to exist, I think it's now obsolete. It's probably "T 1 1949", which is a Tidal bench Mark in that town.

The 1:50,000 mapping of your country is on the North American Datum of 1927 and the Grid is UTM. (That's for NATO use and is ignored by the civilians.) Isn't it comforting to know that they're prepared to "call in" accurate artillery fire? Modern civilization ...

There are four collocated "A" Order GPS points on St. Lucia, two are in your city of Castries. One is known by Mr. Paul Boland (LS 61 G), the other is on Mons Fortune (DCS 33). The orientation to WGS84 is particularly good, but no current mapping or local surveys are known to use it. The Ordnance Survey is rumored to be contemplating changing the West Indies to WGS84, but that will probably be implemented over a very long period of time. When your current 1:2,500 mapping becomes hopelessly obsolete, then maybe.

By: Clifford J. Mugnier cjmce@uno.edu 10 Feb 1998

4.3 What is the datum/projection of Yemen?

In regard to sunny Yemen, there are a number of datums found there. The Aden Datum of 1925 uses the Aden Lambert Zone (Clarke 1880). The Sanaa Datum uses the Yemen Azimuthal Equidistant Grid System (International), and datums with unknown grids exist as:

Kamaran Datum of 1926-27, Ras Karma Datum, Socotra, Socotra Datum of 1957, and Socotra Datum of 1964-65. These weird local datums are all on islands, and are probably Astro stations based on the Clarke 1880 ellipsoid or the International ellipsoid - particularly for the more recent ones.

UTM for the area is based on the European Datum of 1950 (International Ellipsoid). Some UTM may be on Clarke 1880, but if so, it's probably stuff that is over 30 years old. Rumors exist that some stuff was produced on WGS72, but if true, was probably only extremely large scale seismic stuff for geophysical exploration. That stuff would probably only show shot points ...

For 1:100,000 mapping, the British have produced several series of maps through several different agencies. The Russians have also, using the Krassovsky ellipsoid which presumably is referenced to the Pulkovo System of 1942 Datum. The Russian stuff, then, is probably based on the standard Russia Belts which are identical to UTM except that the scale factor at origin is NOT 0.9996 but rather 1.0

by: Cliff Mugnier cjmce@uno.edu 28 Aug 1998

4.4 What is the datum/projection of Puerto Rico?

The Puerto Rico Datum of 1901 is based on its origin at Cardona Island Lighthouse with a defining azimuth to Ponce Southwest Base. The Datum is referenced to the same ellipsoid as NAD 1927 which is the Clarke 1866. The island is now on the NAD83, and the Grid parameters have changed slightly.

Information on ALL of the LEGAL Grid Systems of the United States, its Possessions and its Territories are available free from the U.S. National Geodetic Survey. Try (<http://www.ngs.noaa.gov>). There is software available for FREE there as well as U.S. Geological Survey sites and also from the U.S. Army Topographic Engineering Center (TEC) at Ft. Belvoir, VA.

Everything on this topic for Puerto Rico is available from the U.S. Government for free. The man who knows the most on this official topic is Mr. Dave Doyle DaveD@NGS.noaa.gov .

by: Clifford J. Mugnier cjmce@uno.edu 10/07/98

4.5 What is the datum/projection of Isreal?

The Palestine Datum of 1928 is referenced to the Clarke 1880 ellipsoid (modified). Three grids are found on this datum in Israel, one is the Palestine 1923 Civil Grid (Cassini-Soldner projection) another EXTREMELY SIMILAR is the Palestine 1942 Belt (Gauss-Kruger Transverse Mercator) - the northings are identical, BUT the eastings change with distance from the Central Meridian. The Levant Lambert Zone (1920) uses the French Army Truncated Cubic, also on the Palestine 1928 Datum. There exists some more modern military mapping that is UTM, European Datum 1950 referenced to the International ellipsoid, but I was under the impression that such 1:50,000 scale stuff was quite military-sensitive.

