Chapter 4: Artificial Neural Networks

CS 536: Machine Learning

Littman (Wu, TA)

Administration

First assignment

Run algorithms on ebay data prepared by Yihua

Artificial Neural Networks

[Read Ch. 4]

[Review exercises 4.1, 4.2, 4.5, 4.9, 4.11]

- Threshold units
- Gradient descent
- Multilayer networks
- Backpropagation
- Hidden layer representations
- Example: Face Recognition
- Advanced topics

Connectionist Models

Consider humans:

- Neuron switching time ~ .001 second
- Number of neurons ~ 10^{10}
- Connections per neuron ~ 10^{4-5}
- Scene recognition time ~ .1 second
- 100 inference steps doesn't seem like enough

→ much parallel computation

Artificial Networks

Properties of artificial neural nets (ANNs):

- Many neuron-like threshold switching units
- Many weighted interconnections among units
- Highly parallel, distributed process
- Emphasis on tuning weights automatically

When to Consider ANNs

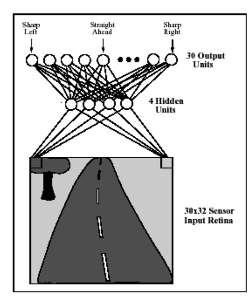
- Input is high-dimensional discrete or real-valued (e.g. raw sensor input)
- Output is discrete or real valued
- Output is a vector of values
- Possibly noisy data
- Form of target function is unknown
- Human readability of result is unimportant

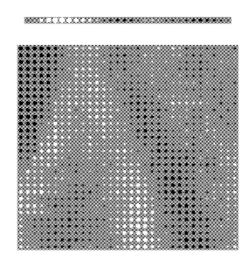
ANNs: Example Uses

Examples:

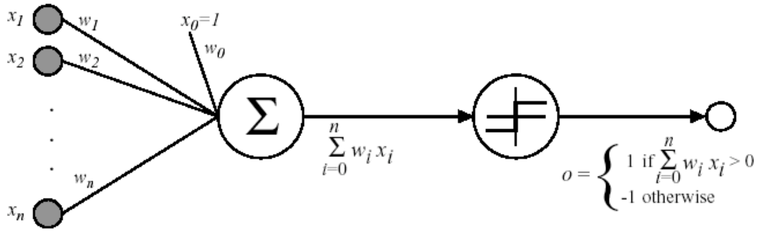
- Speech phoneme recognition [Waibel]
- Image classification [Kanade, Baluja, Rowley]
- Financial prediction
- Backgammon [Tesauro]

ALVINN drives on highways





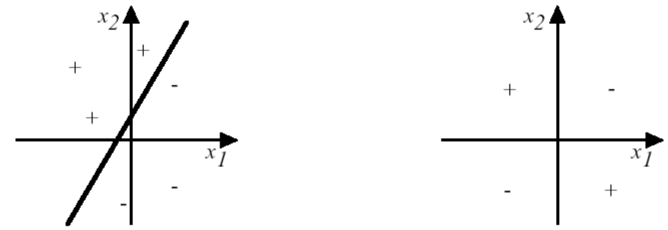
Perceptron



$$o(x_1, \ldots, x_n) = \begin{cases} 1 & \text{if } w_0 + w_1 x_1 + \cdots + w_n x_n > 0 \\ -1 & \text{otherwise.} \end{cases}$$

Or, more succinctly: $o(\boldsymbol{x}) = \text{sgn}(\boldsymbol{w} \cdot \boldsymbol{x})$

Perceptron Decision Surface



A single unit can represent some useful functions

- What weights represent
  g(x1, x2) = AND(x1, x2)? Majority, Or

But some functions not representable

- e.g., not linearly separable
- Therefore, we'll want networks of these...

Perceptron training rule

$$w_i \leftarrow w_i + \Delta w_i$$

where

$$\Delta w_i = \eta\, (t - o)\, x_i$$

Where:

- $t = c(\boldsymbol{x})$ is target value
- o is perceptron output
- η is small constant (e.g., .1) called the *learning rate* (or *step size*)

Perceptron training rule

Can prove it will converge

- If training data is linearly separable
- and η sufficiently small

Gradient Descent

To understand, consider simpler *linear unit*, where

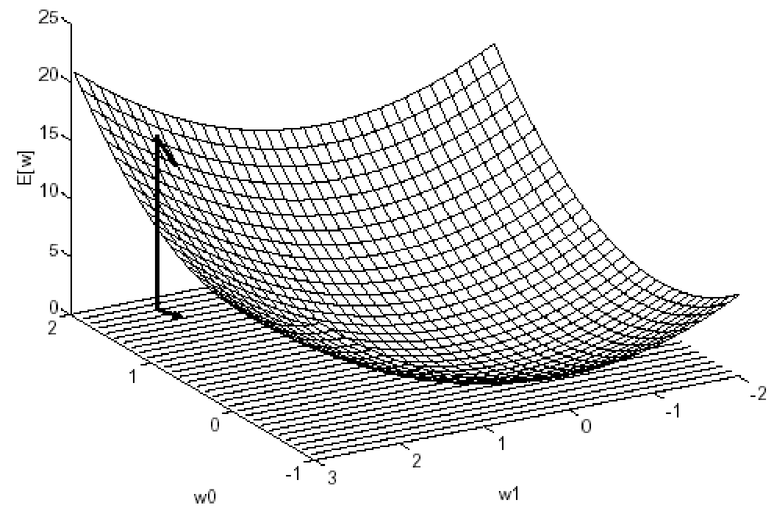
$$o = w_0 + w_1 x_1 + \ldots + w_n x_n$$

Let's learn w_i's to minimize squared error

$$E[\mathbf{w}] \equiv 1/2 \sum_{d \text{ in } D} (t_d - o_d)^2$$

Where D is set of training examples

Error Surface



Gradient Descent

Gradient

$\nabla E[\mathbf{w}] = [\partial E/\partial w_0, \partial E/\partial w_1, \ldots, \partial E/\partial w_n]$

Training rule:

$$\Delta \mathbf{w} = -\eta \nabla E[\mathbf{w}]$$

in other words:

$$\Delta w_i = -\eta\, \partial E/\partial w_i$$

Gradient of Error

$\partial E/\partial w_i$

$$= \partial/\partial w_i \; 1/2 \sum_d (t_d - o_d)^2$$

$$= 1/2 \sum_d \partial/\partial w_i (t_d - o_d)^2$$

$$= 1/2 \sum_d 2 (t_d - o_d)\, \partial/\partial w_i (t_d - o_d)$$

$$= \sum_d (t_d - o_d)\, \partial/\partial w_i (t_d - \mathbf{w}\, \mathbf{x}_d)$$

$$= \sum_d (t_d - o_d)(-x_{i,d})$$

Gradient Descent Code

GRADIENT-DESCENT(training examples, η)

*Each training example is a pair of the form <**x**, t>, where **x** is the vector of input values, and t is the target output value. η is the learning rate (e.g., .05).*

- Initialize each w_i to some small random value
- Until the termination condition is met, Do
  - Initialize each Δw_i to zero.
  - For each <***x**, t*> in training examples, Do
    - Input the instance ***x*** to the unit and compute the output *o*
    - For each linear unit weight w_i, Do

      $$\Delta w_i \leftarrow \Delta w_i + \eta (t-o) x_i$$
  - For each linear unit weight w_i, Do

    $$w_i \leftarrow w_i + \Delta w_i$$

Summary

Perceptron training rule will succeed if

- Training examples are linearly separable
- Sufficiently small learning rate η

Linear unit training uses gradient descent

- Guaranteed to converge to hypothesis with minimum squared error
- Given sufficiently small learning rate η
- Even when training data contains noise
- Even when training data not *H* separable

Stochastic Gradient Descent

Batch mode Gradient Descent:

Do until satisfied

1. Compute the gradient $\nabla E_D[\boldsymbol{w}]$
2. $\boldsymbol{w} \leftarrow \boldsymbol{w} - \nabla E_D[\boldsymbol{w}]$

Incremental mode Gradient Descent:

Do until satisfied

- For each training example *d* in *D*
  1. Compute the gradient $\nabla E_d[\boldsymbol{w}]$
  2. $\boldsymbol{w} \leftarrow \boldsymbol{w} - \nabla E_d[\boldsymbol{w}]$

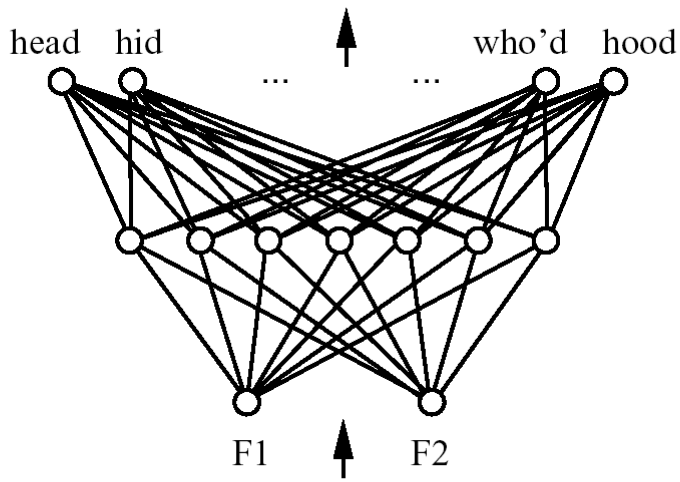
More Stochastic Grad. Desc.

$$E_D[\boldsymbol{w}] \equiv 1/2 \sum_{d \text{ in } D} (t_d - o_d)^2$$

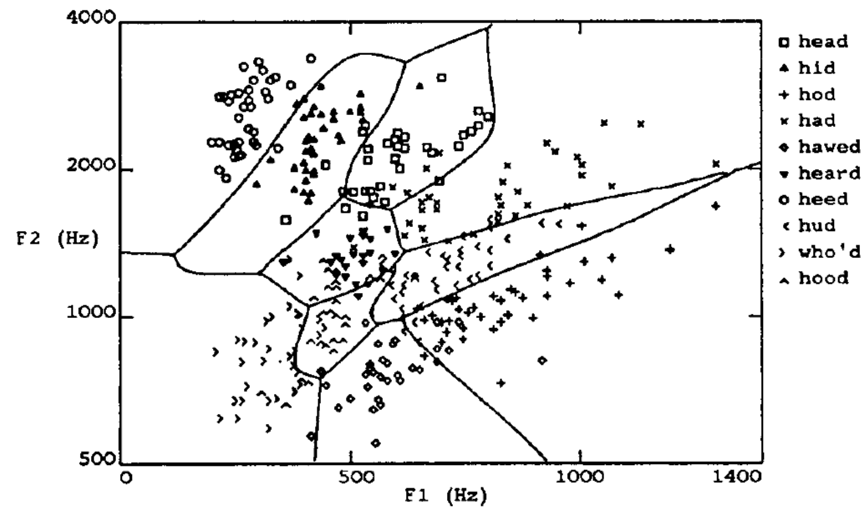
$$E_d[\boldsymbol{w}] \equiv 1/2 (t_d - o_d)^2$$

Incremental Gradient Descent can approximate *Batch Gradient Descent* arbitrarily closely if η set small enough

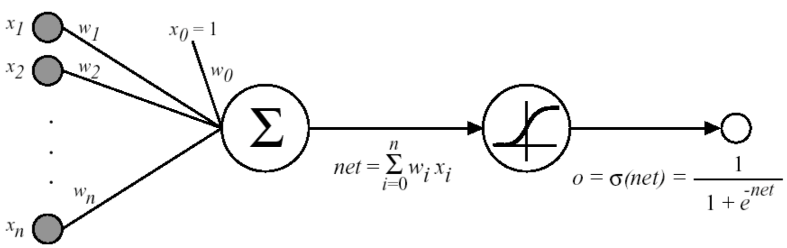
Multilayer Networks



Decision Boundaries



Sigmoid Unit



σ(x) is the sigmoid (s-like) function

$1/(1 + e^{-x})$

Derivatives of Sigmoids

Nice property:

$d\,\sigma(x)/dx = \sigma(x)\,(1-\sigma(x))$

We can derive gradient decent rules to train

- One sigmoid unit
- *Multilayer networks* of sigmoid units → *Backpropagation*

Error Gradient for Sigmoid

$\partial E/\partial w_i$

$= \partial/\partial w_i \; 1/2 \; \Sigma_d \; (t_d - o_d)^2$

$= 1/2 \; \Sigma_d \; \partial/\partial w_i (t_d - o_d)^2$

$= 1/2 \; \Sigma_d \; 2 \, (t_d - o_d) \; \partial/\partial w_i (t_d - o_d)$

$= \Sigma_d \; (t_d - o_d) \; (-\partial o_d/\partial w_i)$

$= - \Sigma_d \; (t_d - o_d) \; (\partial o_d/\partial net_d \, \partial net_d/\partial w_i)$

Even more…

But we know:

$\partial o_d/\partial net_d$

$= \partial\sigma(net_d)/\partial net_d = o_d \, (1 - o_d)$

$\partial net_d/\partial w_i = \partial(\boldsymbol{w} \cdot \boldsymbol{x}_d) \, /\partial w_i = x_{i,d}$

So:

$\partial E/\partial w_i = - \Sigma_d \; (t_d - o_d) o_d \, (1 - o_d) \; x_{i,d}$

Backpropagation Algorithm

Initialize all weights to small random numbers.

Until satisfied, Do

- For each training example, Do
  1. Input the training example to the network and compute the network outputs
  2. For each output unit k

     $\delta_k = o_k(1 - o_k)(t_k - o_k)$
  3. For each hidden unit h

     $\delta_h = o_h(1 - o_h) \, \Sigma_{k \text{ in } outputs} \; w_{h,k} \, \delta_d$
  4. Update each network weight $w_{i,j}$

     $w_{i,j} \leftarrow w_{i,j} + \Delta w_{i,j}$ where $\Delta w_{i,j} = \eta \, \delta_j x_{i,j}$

More on Backpropagation

- Gradient descent over entire *network* weight vector
- Easily generalized to arbitrary directed graphs
- Will find a local, not necessarily global error minimum
  - In practice, often works well (can run multiple times)

More more

- Often include weight *momentum* α

  $$\Delta w_{i,j}(\boldsymbol{n}) = \eta\, \delta_j x_{i,j} + \alpha\, \Delta w_{i,j}(\boldsymbol{n-1})$$

- Minimizes error over training examples
  - Will it generalize well to subsequent examples?
- Training can take thousands of iterations $\rightarrow$ slow!
- Using network after training is very fast