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1. Compare the two function n^2 and $\frac{2^n}{4}$ for various values of n.

   Determine when the second becomes larger then the first.

n	n^2	$2^n/4$
1	1	1/2
2	4	1
3	9	2
…	…	…
7	49	32
8	64	64
9	81	128

A: $\frac{2^n}{4}$ become larger then n^2 when $n > 8$.

3. Define the frequency counts for all statements in the following two program segments.

(a)

```
for (i = 1; i <= n; i++)                n + 1
    for (j = 1; j <= i; j++)            n(n+1)/2 + n
        for (k = 1; k <= j; k++)        n(n+1)(n+2)/6 + n(n+1)/2
            x++;                        n(n+1)(n+2)/6
                                      +
                                      ---------------------------
```

$(n^3 + 6n^2 + 11n + 3)/3$

(b)

```
i = 1;                  1
while (i <= n)          n + 1
{                       0
    x++;                n
    i++;                n
}                       0
                      +
                      ----------
                        3n + 2
```

8. Show that the following equalities are correct.

(a) $5n^2 - 6n = \Theta(n^2)$

$$\because 4n^2 \le 5n^2 - 6n \le 5n^2, n \ge 6$$

$$\therefore 5n^2 - 6n = \Theta(n^2)$$

(d) $\sum_{i=0}^{n} i^2 = \Theta(n^3)$

$$\sum_{i=0}^{n} i^2 = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$$

$$\because \frac{1}{3}n^3 \le \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n \le n^3, n \ge 1$$

$$\therefore \sum_{i=0}^{n} i^2 = \Theta(n^3)$$

9. Show that the following equalities are incorrect.

(c) $\frac{n^2}{\log(n)} = \Theta(n^2)$

Assume there exist c_1, c_2, n_0 such that $c_1 n^2 \le \frac{n^2}{\log(n)} \le c_2 n^2$ for all n, $n \ge n_0$

$$c_1 \le \frac{1}{\log(n)}, n \ge n_0$$

When $n \to \infty$

$$\lim_{n\to\infty} \frac{1}{\log(n)} = 0$$

$$\Rightarrow c_1 \le 0$$

where no such positive constant c_1 can satisfied

$$\therefore \frac{n^2}{\log(n)} \ne \Theta(n^2)$$

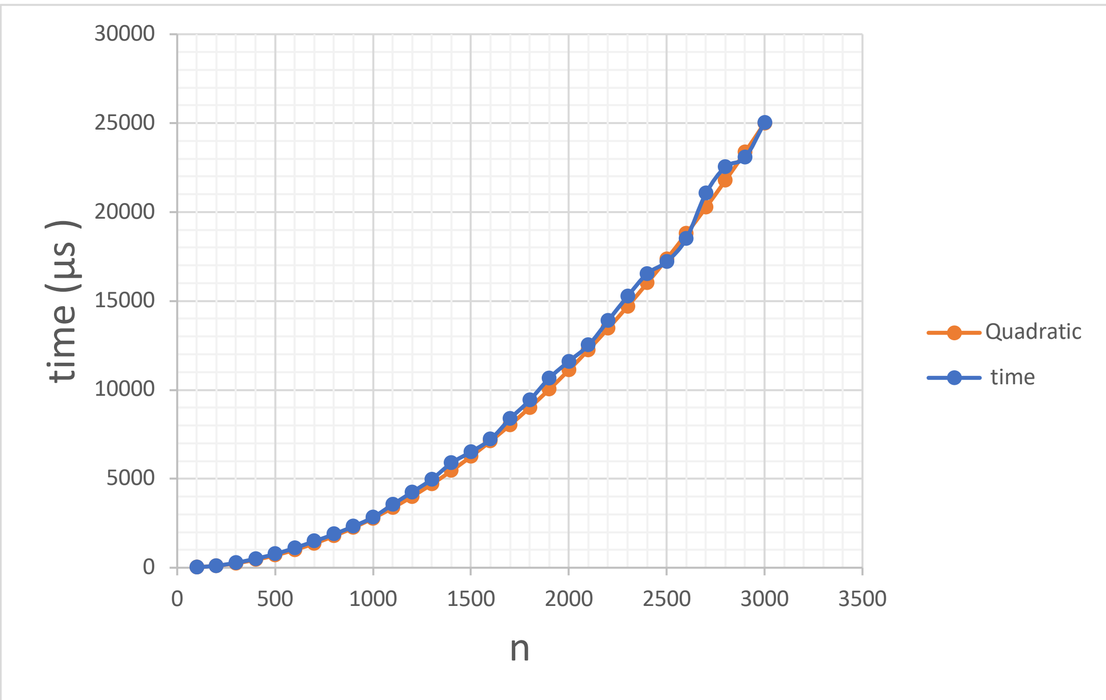
13. Cosider function ***Add*** (Program 1.22).

(a) Obtain run times for n = 100, 200, …, 3000. (take average of 10000 run)

n	time (μs)	n	time (μs)	n	time (μs)
100	27.8507	1100	3552.73	2100	12506.4
200	111.734	1200	4250.77	2200	13879.9
300	276.885	1300	4968.85	2300	15261.1
400	502.863	1400	5880.97	2400	16530.9
500	769.191	1500	6513.54	2500	17210.8
600	1106.45	1600	7212.4	2600	18497.8
700	1487.13	1700	8378.37	2700	21043.6
800	1880.14	1800	9411.55	2800	22543.3
900	2326.48	1900	10655.4	2900	23063.9
1000	2834.85	2000	11602.4	3000	25031.3

(b) Plot the times obtained in part (a).

The orange line is a quadratic function ($x^2/360$)
The blue line is the execution time of function ***ADD***



14. Consider function ***matrix multiplication*** (Program 1.35).

(a) Obtain run times for n = 100, 200, …, 3000. (take average of 100 run)

n	time (ms)	n	time (ms)	n	time (ms)
100	3.13	1100	4733.27	2100	32806.7
200	28.15	1200	6147.23	2200	37777
300	95.51	1300	7783.73	2300	43012.7
400	236.8	1400	9561.44	2400	49043
500	461.84	1500	11822.5	2500	55292
600	752.14	1600	14331.9	2600	62153.7
700	1191.52	1700	17198.3	2700	69435
800	1766.72	1800	20415	2800	77504.3
900	2661.42	1900	24458	2900	86802.7
1000	3562.75	2000	28367.3	3000	95587

(b) Plot the times obtained in part (a).

The orange line is a cubic function ($x^3/282000$)
The blue line is the execution time of function ***matrix multiplication***