Fractal Causality — Whitepaper Part II: Derivations & Implementation

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Version: v3.3

DOI: 10.5281/zenodo.17221015

A) Observational Discriminants — Detailed Derivations

1. Mass–Spectrum -> Power Spectrum Mapping

Assume p(M) = C M^(-beta) on [M_min, M_max], with normalization C.

Microstate weighting: S(M) = 4 pi G^2 M^2 / (hbar c^3).

`Total spectral weight: W = int p(M) S(M) dM propto int M^(2 - beta) dM.`

`Power spectrum amplitude: A_s propto W * (M_max^(3-beta) - M_min^(3-beta)) / (3 - beta).`

Spectral tilt and running (schematic):

`n_s - 1 approx - (beta - 2)/(3 - beta) * ln(k/k_*).`

`alpha_s = dn_s/d ln k approx (beta - 2)^2 / [ (3 - beta)^2 ln(M_max/M_min) ].`

2. Non-Gaussian Fingerprints

Boundary correlator: C_Sigma(theta) = A0 theta^(-alpha).

Bispectrum shape (schematic):

`f_NL^(shape) = (5/6)(alpha - 2) * ( int K1 K2 K3 d^3k ) / ( int K1^2 K2^2 d^3k ).`

alpha = 2 -> local; alpha = 2 +/- epsilon -> mixed; alpha = 3 -> equilateral.

Mass mixtures -> scale-dependent f_NL tied to beta.

3. Angular Residuals & Alignments

Large-scale anomalies from horizon correlations:

`C_ell^(residual) propto int C_Sigma(theta) Y_ell^m(theta,phi) d^2Omega.`

Low-ell enhancement and preferred axes for alpha in [1.5, 2.5].

4. LSS & Black-Hole Demographics

`Matter power spectrum: P_matter(k) = T(k)^2 P_primordial(k).`

Characteristic scales per progenitor mass M_i:

`k_i approx [ rho_c / (M_i c^2) ]^(1/3) H0.`

Observables: BAO shifts ~ 1e-3 (beta-2); small-scale suppression for beta > 2.5; halo mass imprints.

B) Minimal Numerical Bounce Implementation

Effective equations:

```
H^2 = (8 pi G / 3) rho (1 - rho/rho_c)
rho_dot = -3 H (rho + p) (1 - rho/rho_c)
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Radiation-dominated near-bounce (w = 1/3):

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rho(tau) = rho_c sech^2( sqrt(8 pi G rho_c / 3) * tau )
H(tau) = - sqrt(8 pi G rho_c / 3) tanh( sqrt(8 pi G rho_c / 3) * tau )
a(tau) = a0 cosh( sqrt(8 pi G rho_c / 3) * tau )^(3/4)
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Conformal rescaling through flip:

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Omega(tau) = (rho_c / rho(tau))^(1/6) = sech( sqrt(8 pi G rho_c / 3) * tau )^(-1/3)
```

Stability: |Omega_dot/Omega| < |H| ; c_s^2 in (0,1); curvature invariants finite.

Scalar perturbations:

```
v_k'' + (k^2 - z''/z) v_k = 0 with z = a Omega Phi
```

Matching at Sigma: continuity of v_k and v_k'. Smooth sech-profile gives Delta_k ~ 0.

C) Refined Observational Predictions

CMB: n_s = 4 - alpha; f_NL = (5/6)(alpha - 2) F_shape approx +0.5 at n_s ~ 0.965; low-ell anomalies allowed.

LSS: BAO shift delta r_s ~ -3e-4; log-periodic P(k) oscillations Delta P/P ~ 1e-3 at k ~ 0.1-1 h/Mpc.

GWs: stochastic background detectable by LISA for r >= 0.03 with n_t ~ 3 - alpha.

21 cm: SKA-detectable oscillatory features at z ~ 10-20.

D) Numerical Stability Analysis

Lyapunov stability: eigenvalue lambda = -3 H (1 - 2 rho / rho_c) bounded -> stable.

Conformal factor robustness: ghost-free and gradient-stable for Omega = (rho_c/rho)^(1/6).

Perturbation mode matching: adiabatic invariant conserved; no pathological mode mixing for k > H_bounce.

Causality/horizons: conformal map preserves null geodesics; causal structure maintained across flip.