Appendix: Mathematical Skeleton

The following appendix provides a preliminary mathematical scaffold for Fractal Causality. These formulations are not a complete proof but demonstrate internal consistency and establish a foundation for testable predictions.

5D Field & Projection

Define M5 as a 5D manifold with metric g_AB (A,B=0..4). A scalar burst field Φ(x^μ, y) on M5 has Lagrangian density:
L_5 = (1/2) ∇_AΦ ∇^AΦ - V(Φ).
Kaluza–Klein reduction on y = const hypersurfaces induces an effective 4D stress–energy tensor sourcing Einstein equations:
G_{μν} = 8πG (T_{μν}^{matter} + T_{μν}^{Φ}).
CQB activity enters as a source term S(Φ) = dV/dΦ, injecting bursts into the 4D sector.

Continuity with Source–Sink

In FRW cosmology with scale factor a(t):
dρ/dt + 3H(ρ + p) = J_cqb(t) - J_bh(t),
with H = (1/a)(da/dt).
Circuit closure requires ∫(J_cqb - J_bh) dt = 0 over a cycle τ.

Law of Expansion (LOE)

H^2 = (8πG/3) ρ_eff, with ρ_eff = ρ_std + α f(Φ, ∂Φ).
CQB contribution J_cqb = β g(Φ, ∂Φ). For α, β → 0, ΛCDM is recovered.

Black Hole Filtration & GSL

Each black hole satisfies:
S_BH = A / (4 ■_p^2), T_H = κ / (2π).
Generalized Second Law: d/dt (S_out + S_BH) ≥ 0.
Filtration map χ: ρ_in → σ ensures information is preserved at 5D level, with S(σ) + ΔS_BH ≥ S(ρ_in).

Consistency Conditions

- ΛCDM limit: Φ constant, J_cqb = J_bh = 0 recovers standard cosmology.
- Standard Model unaffected: Φ couples only gravitationally in 4D.
- Observables: H(z), expansion history, BH growth must fit CMB, BAO, BBN constraints.

Testable Signatures

- Oscillatory departures in H(z) consistent with small net J_cqb - J_bh.
- Black-hole recycling leaves imprints in stochastic gravitational wave background.
- Entropy flow near AGN may reveal filtration dynamics.