

Logic Synthesis & Verification, Fall 2017

National Taiwan University

Problem Set 1

Due on 2018/10/15 before lecture

1 [Boolean Algebra Definition]

(10%) Does the tuple $(\{0, 1\}, \oplus, \cdot, 0, 1)$, where \oplus and \cdot stand for Boolean XOR and AND operations, respectively, form a Boolean algebra? Check the five postulates and indicate whether each is satisfied.

2 [Complement Uniqueness]

(10%) Given any Boolean algebra $(\mathbb{B}, \cdot, +, \underline{0}, \underline{1})$, show that for any $a \in \mathbb{B}$, there is a unique $a' \in \mathbb{B}$ such that both equalities $a + a' = \underline{1}$ and $a \cdot a' = \underline{0}$ hold.

3 [Properties of Boolean Algebra]

(20%) Prove the following equalities of Boolean algebra using ONLY the five postulates of Boolean algebra (or other properties that you have proven using only the postulates). Please specify clearly which postulate is applied in each step of your derivation.

(a) $a + (a \cdot b) = a$

(b) $(a + b)' = \underline{a' \cdot b'}$

4 [Boolean formula and Boolean function]

(10%) Given a Boolean algebra $(\mathbb{B}, \cdot, +, \underline{0}, \underline{1})$ with $\mathbb{B} = \{0, 1, \alpha, \alpha'\}$, construct the function table for the two-variable Boolean function $f : \mathbb{B}^2 \rightarrow \mathbb{B}$ corresponding to the Boolean formula $\alpha'x + y$.

5 [Minterm Canonical Form]

(10%) Prove the theorem of minterm canonical form using Boole's expansion theorem.

6 [Number of Boolean Functions]

(10%) How many Boolean functions of n variables are there under a Boolean algebra with $|\mathbb{B}| = m$? Please explain your answer.

2 **Problem Set 1**

7 [Complement and Boolean function]

(10%) For any Boolean algebra $(\mathbb{B}, \cdot, +, \underline{0}, \underline{1})$, the Boolean functions $f : \mathbb{B} \rightarrow \mathbb{B}$ in a certain set satisfy the identity

$$f(f(x)) = (f(x))'$$

for all elements $x \in \mathbb{B}$. Describe this set of functions in simple terms. Explain your method clearly.

8 Alternative Views on Boolean Functions

(20%) Consider a three-variable Boolean function f_1 over $\mathbb{B} = \{0, 1\}$ with

$$f_1 = (1)x'y'z' + (0)x'y'z + (1)x'yz' + (0)x'yz + (1)xy'z' + (1)xy'z + (0)xyz' + (1)xyz.$$

- (a) Define a two-variable Boolean function $f_2(y, z)$ taking values over $\{0, 1, x, x'\}$ as an alternative representation of f_1 . What is the underlying Boolean algebra? (Please specify the five tuple and elaborate the two operators.)
- (b) Define a one-variable Boolean function $f_3(z)$ as an alternative representation of f_1 . What is the underlying Boolean algebra?