R05943029 LSV HW1 郭鈞哲

1.

① (XOR) $A \oplus B$

| B \ A | 0 | 1 |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 0 |

(AND) $A \cdot B$

| B \ A | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |

$A \oplus B$ & $A \cdot B \in \mathbb{B} = \{0,1\}$
satisfies closure laws

② Suppose $A, B, C \in \mathbb{B}$.
$A \oplus B = B \oplus A$ by TT
$A \cdot B = B \cdot A$ by TT
satisfies commutative laws

③ $A \oplus (B \cdot C) \neq (A \oplus B) \cdot (A \oplus C)$

| BC \ A | 0 | 1 |
|---|---|---|
| 00 | 0 | 1 |
| 01 | 0 | 1 |
| 11 | 1 | 0 |
| 10 | 0 | 1 |

| BC \ A | 0 | 1 |
|---|---|---|
| 00 | 0 | 1 |
| 01 | 0 | 0 |
| 11 | 1 | 0 |
| 10 | 0 | 0 |

violates Distributed laws

④
$\underline{0} \oplus A = A$
$\underline{1} \cdot A = A$
satisfies

⑤ $a \oplus a' = 1$
$a \cdot a' = 0$ satisfies

無法形成 Boolean Algebra，因為違反 Postulates 3.

2. Suppose $x_1$ and $x_2$ are $a$'s complement and they are different.

$$\Rightarrow \begin{cases} a + x_1 = 1 \dots ① \\ a \cdot x_1 = 0 \dots ② \\ a + x_2 = 1 \dots ③ \\ a \cdot x_2 = 0 \dots ④ \end{cases}$$

$$x_1 = x_1 \cdot 1 \overset{③}{=} x_1 \cdot (a + x_2) \overset{\text{Postulate 3}}{=} x_1 \cdot a + x_1 \cdot x_2 \overset{②}{=} 0 + x_1 \cdot x_2$$
$$\overset{④}{=} (a \cdot x_2) + x_1 \cdot x_2 \overset{\text{Postulate 3}}{=} (a + x_1) \cdot x_2 \overset{①}{=} 1 \cdot x_2 = x_2 \quad (\rightarrow\leftarrow)$$

Therefore, Complement is unique.

3. (a) $a + (a \cdot b) = a \cdot 1 + (a \cdot b) = a \cdot (1 + b) = a \cdot 1 = a$ proved

(b) Use (a) property with Duality, we have $a \cdot (a+b) = a \dots ①$
We want to prove $(a+b) + a'b'$ equals to 1 and $(a+b) \cdot (a'b')$ equals to 0

$$(a+b) + (a'b') = (a+b) \cdot (a + a') + (a'b') = \underline{(a+b)a} + (a+b)a' + a'b' \overset{①}{=} a + aa' + \underline{ba' + a'b'}$$
$$= a + 0 + a'(b + b') = \underline{a} + 0 + \underline{a' \cdot 1} = 1 \quad \text{proved}$$

$$(a+b) \cdot (a'b') = aa'b' + ba'b' = (aa')b' + (bb')a' = 0 + 0 = 0 \quad \text{proved}$$

Therefore, $(a+b)' = a'b'$ because $a'b'$ is $(a+b)$'s complement.

4. $\mathbb{B} = \{0, 1, \alpha', \alpha\}$, $f = \alpha' x + y$

| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x$ | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | $\alpha'$ | $\alpha'$ | $\alpha'$ | $\alpha'$ | $\alpha$ | $\alpha$ | $\alpha$ | $\alpha$ |
| $y$ | 0 | 1 | $\alpha'$ | $\alpha$ | 0 | 1 | $\alpha'$ | $\alpha$ | 0 | 1 | $\alpha'$ | $\alpha$ | 0 | 1 | $\alpha'$ | $\alpha$ |
| $f$ | 0 | 1 | $\alpha'$ | $\alpha$ | $\alpha'$ | 1 | $\alpha'$ | 1 | $\alpha'$ | 1 | $\alpha'$ | 1 | 0 | 1 | $\alpha'$ | $\alpha$ |
| | $y$ | | | | $\alpha' + y$ | | | | $\alpha' + y$ | | | | $y$ | | | |

5. Suppose we have two varibles $a$ and $b \in \mathbb{B}$, and by Boole's expansion, we have $f(a,b) = f(0,b)a' + f(1,b)a = f(0,0)a'b' + f(0,1)a'b + f(1,0)ab' + f(1,1)ab$

we can recursively use Boole's Expansion Thm to get Minterm Canonical Form.

for $n$ variables $x_1, x_2, \dots, x_n \in \mathbb{B}$, by obvious extension, we can have

$$f(x_1, \dots, x_n) = f(0,\dots,0)x_1' \cdots x_n' + \dots + f(1,\dots,1)x_1 \cdots x_n$$

if we make $\left\{\begin{array}{l} x^0 \equiv x',\ x^1 \equiv x,\ X = (x_1,\dots,x_n) \\ A \equiv (a_1,\dots,a_n) \\ X^A = (x_1^{a_1},\dots,x_n^{a_n}) \end{array}\right\}$, we can get

$$f(X) = \sum_{A \in \{0,1\}^n} f(A) \cdot X^A$$ Minterm Canonical Form get

6. By Minterm Thm. each variable can expands 2 term, just use 0 and 1.

$f \in \mathbb{B}$, and $|\mathbb{B}| = m$, output has $m$ possible values

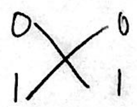
We have $n$ variables.

Then we have $m^{2^n}$ Boolean functions.

7. for $f(x) \in \mathbb{B}$, $\{f(x)\}$ is a Boolean Algebra, $\{f(x)\} = \mathbb{B}_0 \subset \mathbb{B}$,

$\forall x_0 \in \mathbb{B}_0$, $f(x_0) = x_0'$ and, $\forall x_0 \in B \setminus B_0$, $f(x_0) = c$, where $c$ is a constant, and $c \in \mathbb{B}_0$

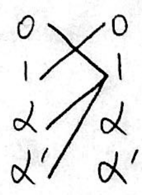
for example ①



$f(0) = 1$

$f(f(0)) = 0$

②



$f(0) = f(\alpha) = f(\alpha') = 1$

$f(1) = 0$

8. (a) We want to express $f_1$ by $\square f_2(y,z)$ $\quad f_1 = x'y'z' + x'yz' + xy'z' + xy'z + xyz$

$f_1 = (x'+x)y'z' + (x)y'z + (x')yz' + (x)yz$

$= (1)y'z' + (x)y'z + (x')yz' + (x)yz = f_2(0,0)y'z' + f_2(0,1)y'z + f_2(1,0)yz' + f_2(1,1)yz$

where $\mathbb{B} = \{0, 1, x, x'\}$, $0+0=0$, $0+1=1$, $1+1=1$, $0+x=x$, $0+x'=x'$, $1+x=1$, $1+x'=1$, $x+x'=1$, $0\cdot1=0$, $0\cdot0=0$, $1\cdot1=1$, $1\cdot x=x$, $1\cdot x'=x'$, $0\cdot x=0$, $0\cdot x'=0$, $x\cdot x'=0$

(b) $f_1 = (x'y'+x'y+xy')z' + (xy'+xy)z = (x'+xy')z' + (x)z$

$= f_3(0)z' + f_3(1)z$, where $\mathbb{B} = \{0, 1, x, x', y, y', x+y, x'+y', x'+y, x+y', x'y', xy, xy', x'y, xy+x'y', (x+y)\cdot(x'+y')\}$