CS3400 Machine Learning Problem Set 1: Linear Algebra Review Name:

1) For each of the following, indicate if it is a vector or matrix and its dimensions:

1. $u \in \mathbb{R}^5$
2. $A \in \mathbb{R}^{3\times4}$
3. A^T where $A \in \mathbb{C}^{4\times8}$

2) Given the values of the entries indicated below.

$$U = \begin{bmatrix} 1 & 3 & 5 \\ 7 & 11 & 13 \\ 17 & 19 & 23 \end{bmatrix}$$

$$v = \begin{bmatrix} 2 \\ 4 \\ 6 \\ 8 \\ 10 \end{bmatrix}$$

1. $U_{1,1}$
2. $U_{2,3}$
3. v_4
4. The diagonal elements of U.

3) Linear algebra operations require that the shapes of the matrices and/or vectors match up. For each operation below, indicate if it is valid. If it is valid, give the dimensions of the resulting object. Note that $N \times 1$ and $1 \times N$ are used to indicate column and row vectors, respectively.

1. $u \cdot v$ where $u, v \in \mathbb{R}^{5\times1}$
2. uv
3. $u^T v$
4. uv^T
5. $u + v$
6. UV where $U \in \mathbb{R}^{5\times6}, V \in \mathbb{R}^{6\times7}$
7. $U^T V$
8. UV^T

4) Perform the following linear algebra operations and write the result.

$$u = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

$$v = \begin{bmatrix} 7 \\ 11 \\ 13 \end{bmatrix}$$

1. $u \cdot v$
2. uv^T
3. $u^T v$
4. UV where $U = \begin{bmatrix} 1 & 3 & 5 \\ 7 & 11 & 13 \\ 17 & 19 & 23 \end{bmatrix}$ and $V = \begin{bmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \\ 14 & 16 & 18 \end{bmatrix}$

5) Give the vectors β and x that make the following equations equivalent.

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$$

$$y = x^T \beta$$

6) Norms and distance.

1. Write the squared norm $\|v\|^2$ of a vector v in terms of a dot product.
2. Convert the equation for the Euclidean distance between two vectors u and v into vector notation using vector arithmetic and norms.

$$d(u, v) = \sqrt{(u_1 - v_1)^2 + (u_2 - v_2)^2 + \cdots + (u_n - v_n)^2}$$