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DIGITAL SEA SHELLS

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ABSTRACT

The spiral shells of molluscs may be simulated by suitable mathematical expressions. This paper reviews progress in this field, and describes techniques for the generation of synthetic shells. The method presented is fast enough for convenient interactive use, and is capable of simulating the shell shapes of limpets, bivalves, scaphopods, ammonites and conventionally-coiled gastropods. Novel features include the scope of the model, the addition of sculpture to the basic shell shape, and the incorporation of texture and simple colored patterns.

INTRODUCTION

The shells of many molluscs are elegantly symmetrical and this feature has attracted much interest over the centuries from naturalists, artists and mathematicians. The molluscan spiral was identified in the 17th century as being self-similar, or equiangular, and was found to obey a simple mathematical relationship. Moseley was able to use this property of sea shell shapes to derive expressions for the volume and center of gravity of simple shell forms in 1838[1]. However, these models were apparently never applied in the 19th century due to the labor-intensive nature of the calculations required. This, and other, early work in the field is reviewed in the classic book by D'Arcy Thompson[2].

Mathematical models to describe the shape of mollusc shells have generally been derived from either a consideration of the mechanism of growth of the shells[3-7], or from a consideration of the forces which can act on a shell at certain times in the mollusc's life[8]. In the latter view, it is considered that biological structures will organize themselves into a condition of constant stress over their surfaces [9], or, in the specific case of a mollusc shell, the structures will follow the trajectories of tensile clocksprings[8]. In the case of those models that are based rather on an analysis of the growth, there are a further two divisions: The earlier models [1,3,10], as well as the model presented here, invoke the presence of a static axis of coiling for shells. Others have argued that this axis has no biological significance, and have derived models based more closely on the actual biological growth process[5,7,11,12]. However, it has been argued that there is a correspondence between the parameters of each of the various available mathematical models[8,13], and that this correspondence must also exist with respect to the yet- undiscovered genetic model[10].

The development of graphical representations of shells has closely followed developments in the broader field of computer graphics. Published images have changed over the last 30 years from wireframe shells with no hidden line suppression[3,14], through to ray-traced images produced on graphics supercomputers[15,16]. The associated mathematical expression of the shell shapes has varied independently of graphical representation from simple [3] through to elaborate[15]. Models for mollusc shell shape have been developed or used, for example, as an aid to taxonomy[17], an adjunct to ecological[18] or evolutionary[19] studies, or simply for their own sake[16]. Much attention has recently been paid to the subject of simulating the irregular shape of the ammonite *Nipponites* [11,12,20,21].

It is the purpose of the present work to describe a model and method of shell representation which, while more comprehensive and realistic than many other published schemes, is fast and effective, even on a personal computer. The model, and associated computer program, is used by the author as an interactive aid to studies on the evolution of mollusc shell shape. The model serves not only as an aid in rapidly visualizing various shell shapes, but also as a means by which biologically interesting parameters such as the surface-to-volume ratio can be calculated.

DEVELOPMENT OF THE FIELD

Mapping shells to paper or screen

A synthetic sea shell is a surface, and as such may be defined by some appropriate mathematical expression. The expressions used since Moseley's pioneering work have usually employed the angle of rotation, θ, (or, recently, Φ [8,20,21]) of the aperture of the shell about an imaginary coiling axis as one of the generating parameters. A succession of θ (or Φ) values can produce a spiral, which, if it unwinds around and along a central axis in a self-similar way, may be termed a helicospiral[14]. The spiral defined by the suture line on a shell is an example of a helicospiral. A second parameter is required to generate a surface. This parameter could be considered to generate the aperture at a given value of θ. This second parameter is denoted in the present work as s, following the notation established by Løvtrup and von Sydow[10]. In D'Arcy Thompson's terms, lines drawn at a constant s are called generating spirals, and lines at constant θ called generating curves[2]. Clearly a synthetic shell can be established by plotting generating spirals, generating curves, or both, or by shading the patches defined by intersections of the generating curves and spirals.

The first person to actually generate synthetic shell shapes may have been Raup, who, in 1962, used a four parameter model and a plotter to rough out the outline of generating curves of his digital sea shells at 180° intervals[22]. Similar techniques were used by Løvtrup and von Sydow[5,10]. The shell shape itself was then sketched over the outline by hand, or the outlines were used as a cross sectional slice through the shell. Illert[4,15] has simulated shells by marking the position of closely spaced generating curves with dots. Once again, the shell itself seemed to have been drawn by hand, using the generating curves as a guide. Later Raup employed an oscilloscope as an output device[3]. In this scheme, the generating curves were traced out by a glowing dot, and the image recorded with a camera. There was no suppression of hidden lines in this case. Raup's model was driven by 4 parameters, and was able to simulate the shape of many living or fossil molluscs using a circular aperture. The model was well-used as an aid to the understanding of the evolution of ammonites, an extinct type of cephalopod[19]. Others have subsequently used Raup's model and output system, for example Joubert[23]. Alternatively, Raup also made use of a few strategic generating spirals, which were plotted out to yield the outline of a shell, as viewed down the axis of coiling[3,19]. No hidden line suppression was necessary provided that only a subset of the possible shell shapes was drawn. The scheme is suitable for bivalves, brachiopods, and planispiral shells such as ammonites.

More recently, Ackerley has used generating spirals without hidden line suppression to represent molluscs shells[7,24], while Okamoto used closely spaced generating curves with and without hidden line suppression to simulate a wide range of strangely-shaped heteromorph ammonites[11]. Savazzi[14] combined generating spirals and generating curves to produce wire-frame shells, and a similar representation, but with hidden line suppression, has been previously published by the present author[6,13].

Very beautiful shell-like shapes were produced by Pickover[16,21]. He used a necklace of interpenetrating spheres and ray tracing techniques to generate shell shapes. However, Pickover's shapes lack apertures, an essential requirement of a real mollusc shell. Recent publications by Illert[8,15] contain colour figures of shell shapes that have been produced from his model using ray tracing by Bronsvoort *et al.* These unicolour shells are arguably amongst the finest synthetic shells published.

The challenge facing the earlier workers in this field was simply to get the calculations done, for even simple shells. Later the main practical problem seems to have been that of suppressing those generating spirals or curves that should be invisible. Finally, devotees turned towards improving the appearance of their synthetic shells. Surface sculpture or ornamentation may have been first added to these models by Illert[4], who added collabral (axial) ridges to his basic shell shape using a sinusoidal function. Bronsvoort (in Illert[15]) has produced ray-traced images of such shells, and the aperture is both visible and shaded in a relatively realistic manner. In later work by Illert these ridges were modified to produce a discontinuous shell surface with periodic "flares"[15]. This is somewhat like the sculpture on real shells of the *Epitonium* genus, such as the famous precious wentletrap. A more general type of surface sculpture was added by the present author in 1989[6]. This permitted periodic nodules to be added to the shell shape, with the angular separation, amplitude, and size of the nodules controlled by various parameters. Collabral ridges were a subset of the possible forms.

Pickover added colored axial bands[6] to shells that had slightly ribbed surfaces (axial ribs) as a result of his method of interpenetrating spheres. The banding effect was produced by spheres of alternating colors. The texture of the surfaces was varied at will from specular to matte.

THE "16 PARAMETER" MODEL

A expression suitable for synthesizing the shapes of many mollusc shells, ranging from limpets and bivalves through to conventionally coiled gastropods has been previously described by the present author[6,13]. The revised parametric equations for x, y and z are given in an Appendix here as a convenience, as are some sample sets of parameter values. Discussion of the biological significance of the parameters is available in the original publications [6,13].

The method to be described is relatively fast and requires less than 320K of RAM. A typical solid sea-shell is made of 80 000 facets and is produced on a 33 MHz 386 PC at 800x600 pixel by 256 colour SuperVGA resolution in less than 4 minutes. This time includes generation of the coordinates from the model, rotation of the three dimensional image, hidden surface suppression and final scan conversion. Drawing the shell with fewer facets can decrease the total time required to less than 1 minute. Alternatively, the program can write the image out as an Encapsulated PostScript file, for rendering on a colour PostScript printer. This route is, however, significantly slower.

The method used may be summarized as follows:

1. Choose the range of the parameters θ and s, as well as a suitable step size and fill an array with up to 10900 integer (x,y,z) coordinates. This defines a shell with 21800 primary facets since each "quadrilateral" of coordinates defines two triangular facets. 10900 is the maximum number of 3 x 2 byte triplets that can fit in one 64K segment on a PC. Use of an integer array carries the risk of over and underflows, as well as rounding errors, but the increase in speed and efficiency is regarded as worthwhile.

2. Scan convert the primary facets onto the screen using the depth-sort algorithm[22]. Sorting is accomplished in one pass using the hash sort method and a 128K integer hash array.

OR

3. If an improved quality is required then, starting from the back, recover the θ and s values for each primary facet, split it into four secondary facets, sort these four facets, and plot as before. Real numbers can be used when drawing with secondary facets since the data are discarded after use. This improves the quality of the shading.

4. Shading of the exterior surfaces is achieved by calculating the inclination from a imaginary light source perpendicular to the screen using a modified version of Phong's law. Shading of the interior surfaces is achieved by an empirical blend of depth cuing by reduced intensity and Lambert's law[25].

5. Surface texture is added, if desired, to objects drawn with secondary facets by superimposing a controlled, random deviation from the correct position of each coordinate, by a process akin to bump mapping[25].

The scheme described above works well with simulated shells on account of their geometry but, as is well-known, is not completely general in applicability. One weakness is the scheme's inability to correctly image interpenetrating facets, but these should not occur in correctly generated shells.

CONCHOLOGICAL EXHIBITION

The process of shell synthesis is illustrated in Figure 1, which shows a *Planorbis* shell at two stages in the image creation process, another *Planorbis* shell with rather exotic coloring, and the same shell shape with a bump mapped surface.

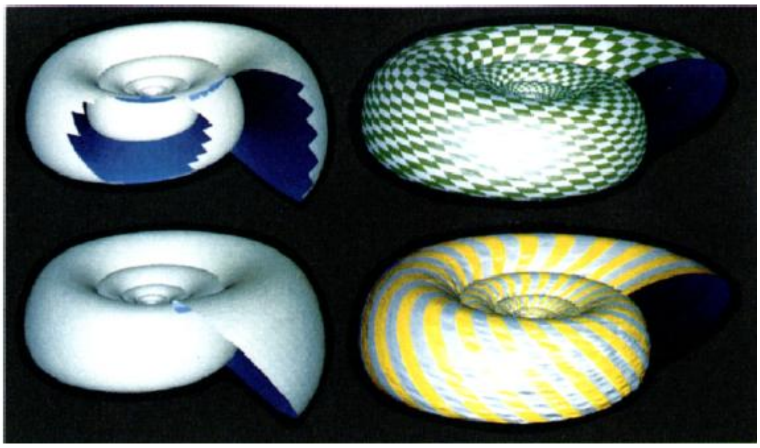

Figure 1. Simulation of a *Planorbis* shell showing cutaway (top left), completed shell (bottom left), fantastic color scheme (top right), and bump-mapped surface (bottom right).

A particular convenience of the method is the ability to rapidly display shells in various orientations. Figure 2 shows interior and exterior views of a valve of the bivalve *Codakia*, as well as images of a hypothetical gastropod from the front and from underneath.

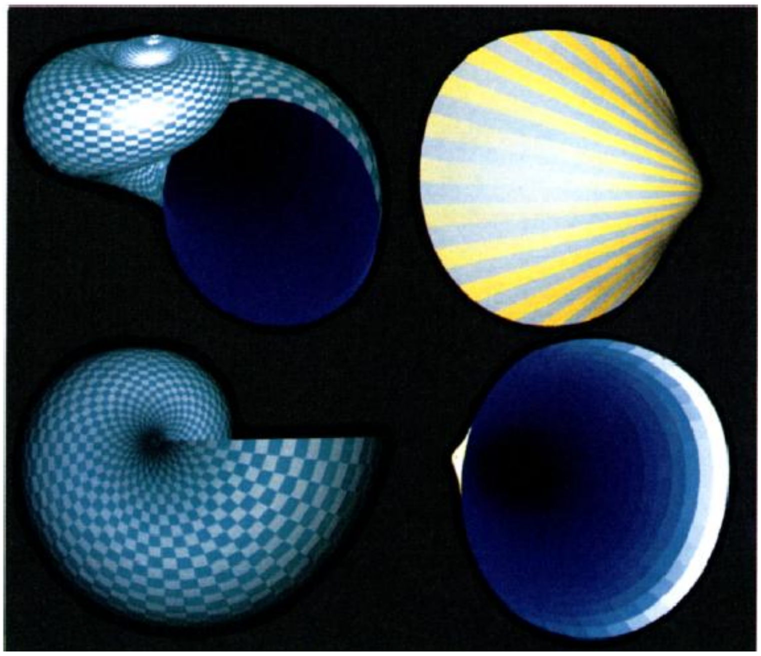

Figure 2. Simulation of a *Natica* shell (left) and that of *Codakia*, a bivalve (right).

A collection of simulations of the shape (but not the colors) of several gastropod shells without surface sculpture is shown in Figure 3. Shells range from the limpet *Helcion* through to two variations of *Conus*. Mollusc shells with surface sculpture are shown in Figure 4, which includes the volute *Lyria*, a fossil ammonite and some totally imaginary shells.

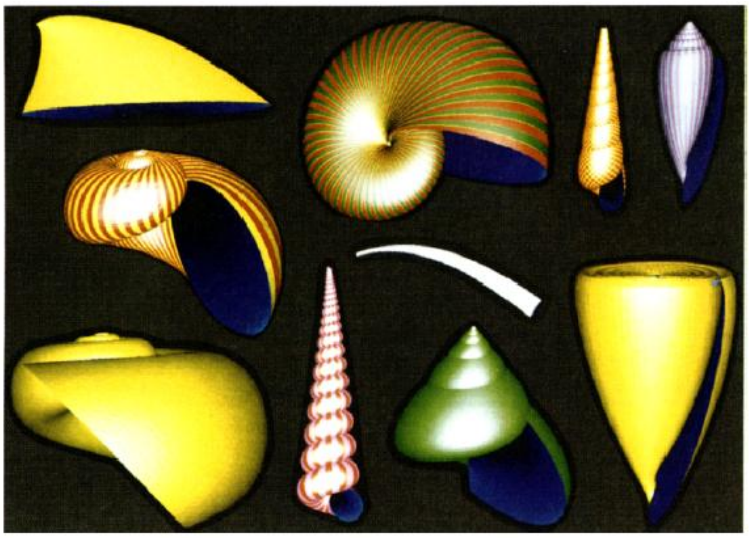

Figure 3. A variety of simulated shells without sculpture or knobs. Examples include the limpet *Helcion* (top left), two *Conus* (right), a *Nautilus* (top centre) and a *Turritella* (bottom, pink).

Figure 4. Simulated shells with surface sculpture including the volute *Lyria* (top left) and an ammonite (bottom right). The other examples are not based on particular shells.

CONCLUSIONS

The simulation of mollusc shells brings together a blend of techniques and technologies. The mathematical models used are empirical, although attempts have been made to interpret the model parameters in biological terms. Although considerable progress has been made over the last three decades, there remain many more interesting challenges. These include the more realistic representation of the patterns on real shells, and the synthesis of shells, such as *Murex*, with less regular shapes. Development of the topic will no doubt continue to closely mirror advances in the general field of computer graphics.

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APPENDIX

List of symbols and definition of parameters

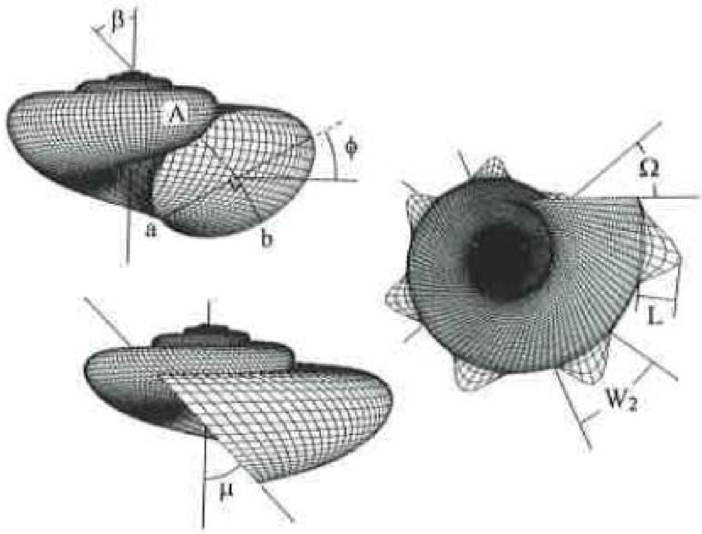

Figure 5. Definition of some of the model's parameters.

Angular parameters

α	: equiangular angle of spiral
β	: angle between z-axis and line from aperture local origin to xyz origin.
ϕ	: tilt of ellipse major axis from horizontal plane
Ω	: amount of azimuthal rotation of aperture
μ	: amount of 'leaning over' of aperture
s_{min}	: angle at which aperture generating curve begins
s_{max}	: angle at which aperture generating curve ends
P	: position of nodule in terms of the angle, s
W_1	: width of nodule in s-direction
W_2	: width of nodule in θ-direction

Linear dimensions

A	: distance from main origin to local origin of aperture at $\theta = 0$.
a	: major radius (long axis) of ellipse at $\theta = 0$
b	: minor radius (short axis) of ellipse at $\theta = 0$
R	: length of aperture-generating vector
L	: height of nodule at $\theta = 0$

Other

x, y, z	: Cartesian coordinates
r, θ	: polar coordinates
r_0	: radius of spiral at $\theta = 0$
N	: number of nodules per whorl
D	: sense of coiling, 1 = dextral, -1 = sinistral
k	: amplitude of the surface ornamentation

Parametric equations for mollusc shell shape

A family of shapes that, to a first order approximation, includes most real and fossil mollusc shells shapes as a subset is given by:

$$x = D.[A.\sin(\beta).\cos(\theta) + R.\cos(s+\phi).\cos(\theta+\Omega) - R.\sin(\mu).\sin(s+\phi).\sin(\theta)].e^{\theta.\cot(\alpha)} \quad (1)$$

$$y = [-A.\sin(\beta).\sin(\theta) - R.\cos(s+\phi).\sin(\theta+\Omega) - R.\sin(\mu).\sin(s+\phi).\cos(\theta)].e^{\theta.\cot(\alpha)} \quad (2)$$

$$z = [-A.\cos(\beta) + R.\sin(s+\phi).\cos(\mu)].e^{\theta.\cot(\alpha)} \quad (3)$$

where R is the function of θ and s that generates the generating curve. If R_e is an ellipse, and a surface sculpture term, k, is superimposed, then

$$R = f(\theta,s) = R_e + k \quad (4)$$

and R_e is given by:

$$R_e = [a^{-2}.\cos^2(s) + b^{-2}.\sin^2(s)]^{-0.5} \quad (5)$$

and surface sculpture, k, by

$$k = L.e^{-[2(s-P)/W_1]^2}.e^{-[2.g(\theta)/W_2]^2} \quad (9)$$

Alternatively, R may be any other function of s, including, for example, a cardiod:

$$R = a'(1+\cos(s)) \quad (10)$$

or the relation

$$R = [a^2.\cos^2(s) + b^2.\sin^2(s)]^{0.5} \quad (11)$$

Equation (11) generates shells with interesting "waisted" apertures[20].

The function, $g(\theta)$, yields a number which varies periodically as θ increases and was expressed as:

$$g(\theta) = 360/N * (\theta *N/360 – \mathrm{round}(\theta *N/360)) \quad (12)$$

The function 'round' rounds up or down according to the normal rules.

The shell shape is generated by varying the parameters s and θ between suitable limits.

Table 1. Parameter values required to generate some of the shells shown in the Figures. Note: D is 1 in every case.

Mollusc	α, °	β, °	ϕ, °	μ,	Ω, °	s_{min}	s_{max}	A	a	b	P	W_1	W_2	N	L
Natalina	80	40	55	10	30	-270	62	25	12	16	0	0	0	0	0
Lyria	83.9	-19	45	1	-2	-51	9	50	40	14	0	6	27	8	4
Turritella	88.9	4	55	1	-2	-267	39	22.2	1.3	1.5	0	0	0	0	0
Oxystele	84.9	7	-36	1	-2	-70	70	47	40	19	0	0	0	0	0
Planorbis	84	85	10	45	5	-150	130	45	20	30	0	0	0	0	0
Ammonite	83	90	1	1	1	-170	170	2.5	1.0	0.9	10	100	20	15	0.5
Conus	87	7	78	0	0	-180	2	7	4.3	1.0	0	0	0	0	0
Helcion	18	90	0	0	-40	-180	180	450	400	310	0	0	0	0	0
Nautilus	80	90	0	0	0	-163	163	2	2	1.5	0	0	0	0	0
Codakia	37	90	0	1	1	-180	180	10500	10000	10500	0	0	0	0	0