Sections and Chapters

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ENME 625: Multi-Displinary Optimization
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Contents

1 Introduction

This is the first section.

2 Unconstrained MOGA Problems

We used this textbook [**?**]

2.1 ZDT1

The first test problem, denoted as ZDT1 in (insert reference) is shown below.

$$
\begin{aligned}
\text{Minimize} \quad & f_1(\mathbf{x}) = x_1 \\
\text{Minimize} \quad & f_2(\mathbf{x}) = g(x) * h(x) \\
\text{where} \quad & g(x) = 1 + \frac{9}{(n-1)} \sum_{i=2}^{n} x_i \\
& h(x) = 1 - \sqrt{\frac{f_1(x)}{g(x)}} \\
& n = 30 \\
& 0 \leq \mathbf{x} \leq 1
\end{aligned}
$$

The true Pareto frontier for this problem occurs when $x_i = 0$ for i = 2,...,30. Figure 1 shows a sample result from both MATLABs built in MOGA and the MOGA developed in this project. The quality metrics chosen to evaluate this problem are Coverage Difference (CD)and Pareto Spread (OS). Ten runs for each algorithm were performed and the mean and standard deviation of each metric are tabulated in Table 1.

Table 1: Quality Metrics for ZDT1

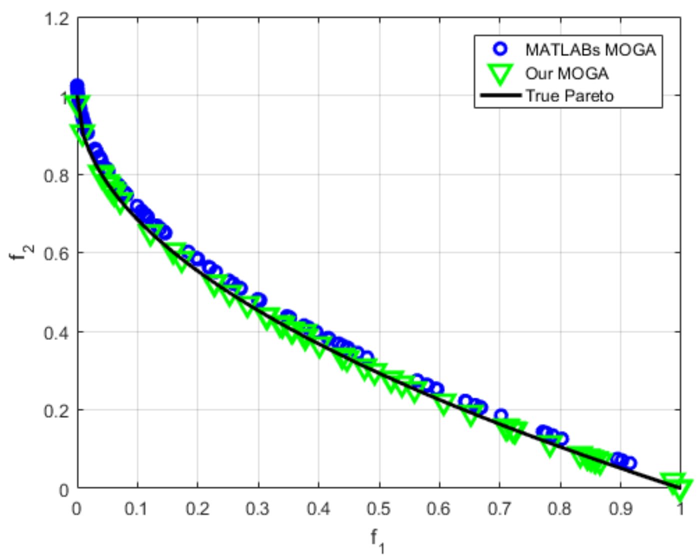
Metric	MATLABs MOGA	Our MOGA
CD	0.3874 (0.0164)	0.3582 (0.0040)
OS	0.9605 (0.1088)	0.9283 (0.0928)

From these metrics, there is certainly a trade-off between MATLABs MOGA and the MOGA developed in this project. The coverage difference of the new MOGA is better in this problem whereas the Pareto spread is improved when using MATLABs MOGA.

2.2 ZDT2

The second test problem, denoted as ZDT2 in (insert reference) is shown below.

Figure 1: Example Pareto Results for ZDT1



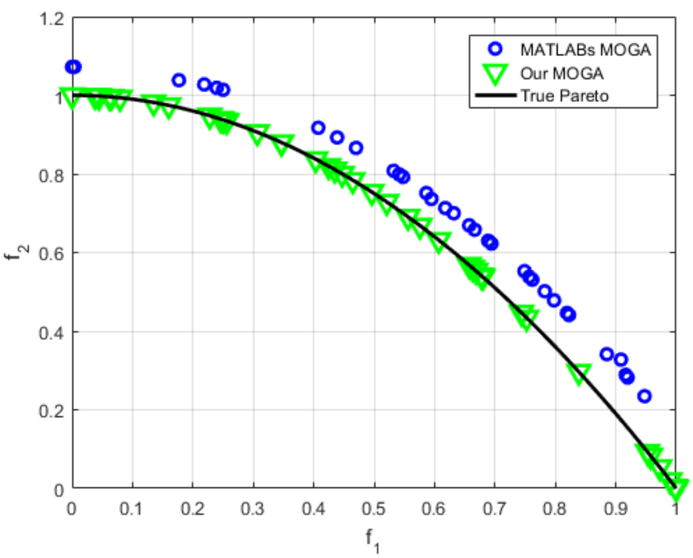
$$\begin{aligned} \text{Minimize} \quad & f_1(\mathbf{x}) = x_1 \\ \text{Minimize} \quad & f_2(\mathbf{x}) = g(x) * h(x) \\ \text{where} \quad & g(x) = 1 + \frac{9}{(n-1)} \sum_{i=2}^{n} x_i \\ & h(x) = 1 - \frac{f_1(x)}{g(x)}^2 \\ & n = 30 \\ & 0 \leq \mathbf{x} \leq 1 \end{aligned}$$

The true Pareto frontier for this problem, similar to ZDT1, occurs when x_i = 0 for i = 2,...,30. Figure 2 shows a sample result from both MATLABs built in MOGA and the MOGA developed in this project. Table 2 summarizes the mean quality metrics for each algorithm for ten runs.

Table 2: Quality Metrics for ZDT2

Metric	MATLABs MOGA	Our MOGA
CD	0.7832 (0.0821)	0.6971 (0.0094)
OS	0.8781 (0.0946)	1.0086 (0.0278)

Figure 2: Example Pareto Results for ZDT2



2.3 ZDT3

The third test problem, denoted as ZDT3 in (insert reference) is shown below.

$$
\begin{aligned}
\text{Minimize} \quad & f_1(\mathbf{x}) = x_1 \\
\text{Minimize} \quad & f_2(\mathbf{x}) = g(x) * h(x) \\
\text{where} \quad & g(x) = 1 + \frac{9}{(n-1)} \sum_{i=2}^{n} x_i \\
& h(x) = 1 - \sqrt{\frac{f_1(x)}{g(x)}} - \frac{f_1(x)}{g(x)} \sin(10\pi f_1) \\
& n = 30 \\
& 0 \leq \mathbf{x} \leq 1
\end{aligned}
$$

The true Pareto frontier for this problem again occurs when $x_i = 0$ for i = 2,...,30. Figure 3 shows a sample result from both MATLABs built in MOGA and the MOGA developed in this project. Table 3 summarizes the mean quality metrics for each algorithm for ten runs.

Overall, MATLABs MOGA outperforms the MOGA developed in this project in both coverage difference and Pareto spread. A paired t-test shows that the difference in the means for coverage difference is statistically significant (p¡0.05), while the difference is not statistically significant in Pareto Spread (p=0.21).

Figure 3: Example Pareto Results for ZDT3

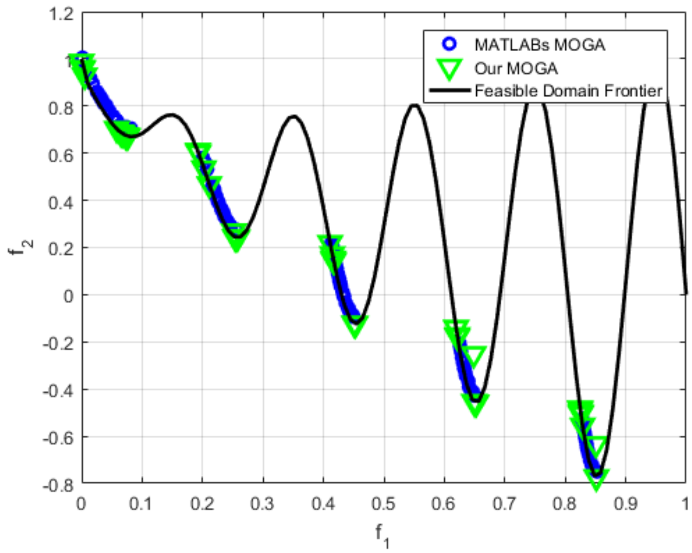


Table 3: Quality Metrics for ZDT3

Metric	MATLABs MOGA	Our MOGA
CD	0.7407 (0.0050)	0.7554 (0.0031)
OS	0.8565 (0.0098)	0.8489 (0.0180)

2.4 OSY

This test problem, denoted as OSY in (insert reference) is shown below.

$$
\begin{aligned}
\text{Minimize} \quad & f_1(\mathbf{x}) = -(25(x_1-2)^2 + (x_2-2)^2 + (x_3-1)^2 + (x_4-4)^2 + (x_5-1)^2) \\
\text{Minimize} \quad & f_2(\mathbf{x}) = \sum_{i=1}^{6} x_i^2 \\
\text{Subject to} \quad & g_1(x) = 1 - \frac{x_1+x_2}{2} \le 0 \\
& g_2(x) = \frac{x_1+x_2}{6} - 1 \le 0 \\
& g_3(x) = \frac{x_2-x_1}{2} - 1 \le 0 \\
& g_4(x) = \frac{x_1-3x_2}{2} - 1 \le 0 \\
& g_5(x) = \frac{(x_3-3)^2+x_4}{4} - 1 \le 0 \\
& g_6(x) = 1 - \frac{(x_5-3)^2+x_6}{4} \le 0 \\
& \quad 0 \le x_1, x_2, x_6 \le 10 \\
& \quad 1 \le x_3, x_5 \le 5 \\
& \quad 0 \le x_4 \le 6
\end{aligned}
$$

The true Pareto frontier for this problem again occurs when $x_i = 0$ for i = 2,...,30. Figure **??** shows a sample result from both MATLABs built in MOGA and the MOGA developed in this project. Table 4 summarizes the mean quality metrics for each algorithm for ten runs.

Table 4: Quality Metrics for OSY

Metric	MATLABs MOGA	Our MOGA
CD		
OS		

3 Constrained MOGA Problems

This is a reference to Azarms constraint paper [**?**].

3.1 TNK

This test problem, denoted as TNK in (insert reference) is shown below with the true Pareto frontier shown in Figure 4.

$$
\begin{array}{ll}
\text{Minimize} & f_1(\mathbf{x}) = x_1 \\
\text{Minimize} & f_2(\mathbf{x}) = x_2 \\
\text{Subject to} & g_1(x) = -x_1^2 - x_2^2 + 1 + 0.1\cos(16\arctan(\dfrac{x_1}{x_2})) \leq 0 \\
 & g_2(x) = (x_1 - 0.5)^2 + (x_2 - 0.5)^2 - 0.5 \leq 0 \\
 & \quad 0 \leq x_1, x_2 \leq \pi
\end{array}
$$

Figure 4: True Pareto Frontier for TNK

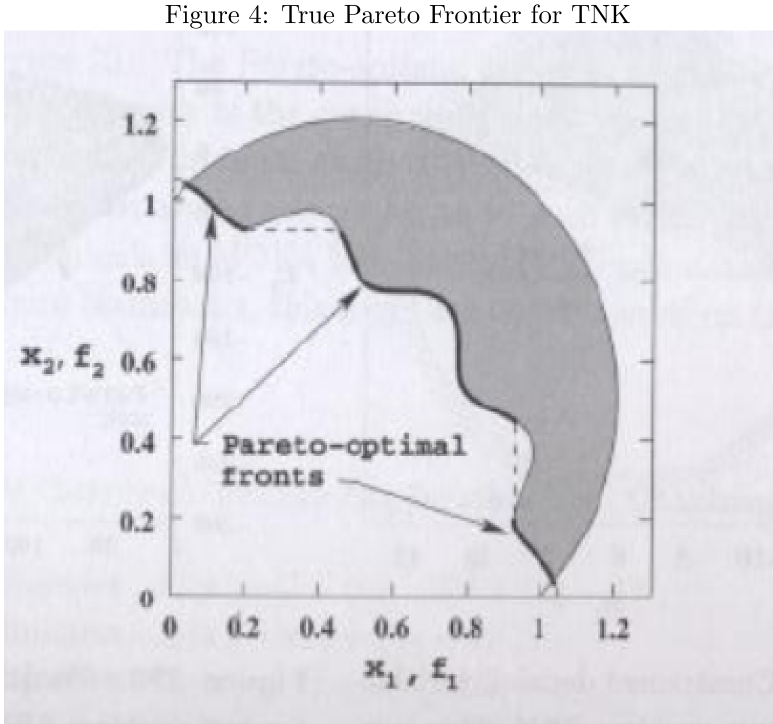


Figure 5 shows a sample result from both MATLABs built in MOGA and the MOGA developed in this project. Table 5 summarizes the mean quality metrics for each algorithm for ten runs.

Comparing the estimated Pareto frontiers to the true Pareto frontier, is is clear that our MOGA outperforms MATLAB's MOGA. The Pareto spread in our MOGA is significantly higher however the coverage difference is higher in MATLABs MOGA (p¡0.05).

Figure 5: Example Pareto Results for TNK

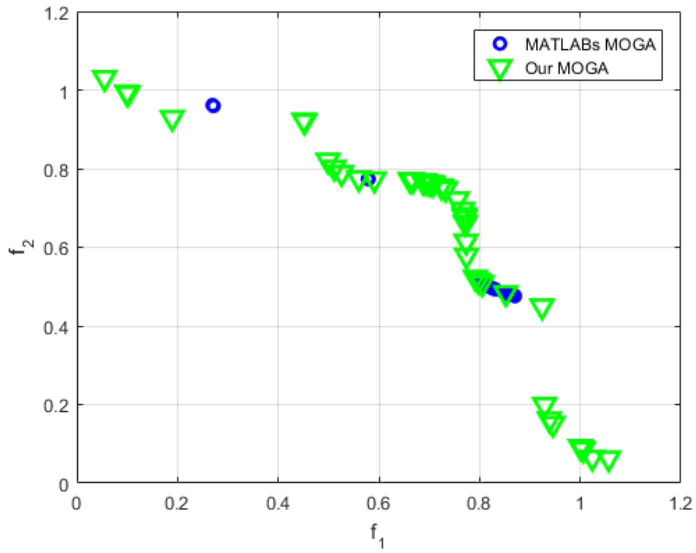


Table 5: Quality Metrics for TNK

Metric	MATLABs MOGA	Our MOGA
CD	0.8581 (0.0480)	0.7792 (0.0036)
OS	0.4177 (0.3216)	0.9763 (0.0178)

3.2 CTP

This test problem, denoted as CTP in (insert reference) is shown below and the true Pareto frontier is shown in Figure 6.

$$
\begin{aligned}
\text{Minimize} \quad & f_1(\mathbf{x}) = x_1 \\
\text{Minimize} \quad & f_2(\mathbf{x}) = g(x)(1 - \sqrt{\frac{f_1(x)}{g(x)}} \\
\text{Subject to} \quad & g_1(x) = a|\sin(b\pi(\sin(\theta)(f_2(x) - e) + \cos(\theta) f_1(x))^c)|^d \\
& \quad - \cos(\theta)(f_2(x) - e) - \sin(\theta) f_1(x) \leq 0 \\
\text{where} \quad & \theta = -0.2\pi, a = 0.2, b = 10, c = 1, d = 6, e = 1 \\
& g(x) = |1 + (\sum_{i=2}^{10} x_i)^{0.25}| \\
& 0 \leq x_1 \leq 1 \\
& -5 \leq x_i \leq 5, i = 2, ..., 10
\end{aligned}
$$

Figure 6: True Pareto Frontier for CTP

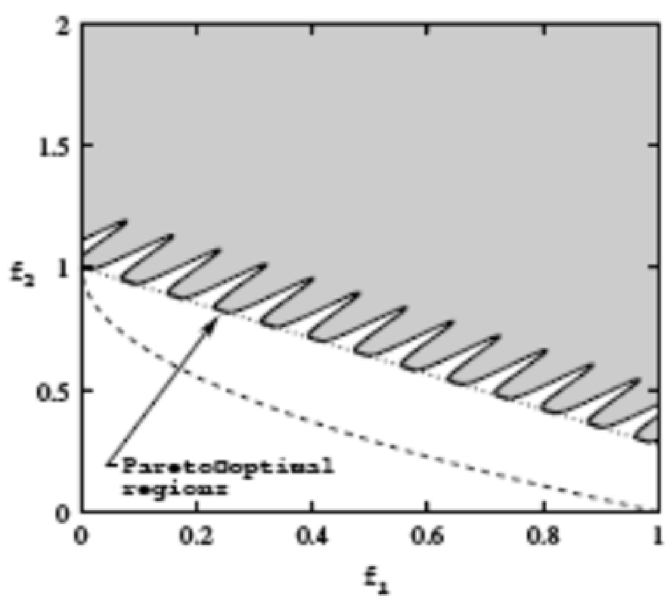


Figure 7 shows a sample result from both MATLABs built in MOGA and the MOGA developed in this project. Table 6 summarizes the mean quality metrics for each algorithm for ten runs.

Figure 7: Example Pareto Results for CTP

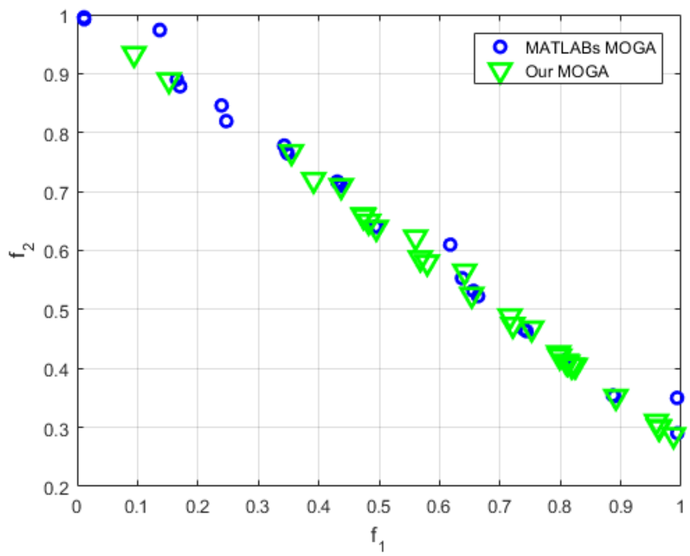


Table 6: Quality Metrics for CTP

Metric	MATLABs MOGA	Our MOGA
CD	0.6802 (0.0067)	0.6802 (0.0092)
OS	0.7901 (0.0734)	0.5959 (0.1324)

The mean values for coverage difference are exactly the same for both algorithms. In fact a paired t-test also shows the means are not statistically difference(p¿0.05). For Pareto spread, MATLAB's MOGA performs the highest.

References

4 Appendix

Matlab Code

```matlab
clear all;
% close all;
clc;
warning off

global alpha sigma epsilon Mmoga

prompt = 'Which Test Problem Do You Want To Run? \n 1 - ZDT1\n 2 - ZDT2 \n 3 - ZDT3 \n 4 - OSY \n 5 - TNK \n 6 - CTP \n';
prob = input(prompt);
prompt2 = 'How Many Chromosomes? Suggest 10-20 times # variables: ';
nChrome = input(prompt2);
prompt3 = 'How Many Runs? Suggest >40: ';
nRun = input(prompt3);
prompt4 = 'What value for alpha? ';
alpha = input(prompt4);
prompt5 = 'What value for sigma? (Nominal 0.158) ';
sigma = input(prompt5);
prompt6 = 'What value for epsilon? (Nominal 0.22) ';
epsilon = input(prompt6);

prompt7 = 'Save Data? \n 1 - YES \n 2 - NO \n';
save_figure = input(prompt7);

% for test = 1:10
%%
switch prob
    case 1
        problem_function = @(X) ZDT1(X);
        nvar = 30; LB = zeros(1,nvar); UB = ones(1,nvar);
        problem_constraints = [];
        A = []; b = []; Aeq = []; beq = [];
    case 2
        problem_function = @(X) ZDT2(X);
        nvar = 30; LB = zeros(1,nvar); UB = ones(1,nvar);
        problem_constraints = [];
```

```matlab
            A = []; b = []; Aeq = []; beq = [];
        case 3
            problem_function = @(X) ZDT3(X);
            nvar = 30; LB = zeros(1,nvar); UB = ones(1,nvar);
            problem_constraints = [];
            A = []; b = []; Aeq = []; beq = [];
        case 4
            problem_function = @(X) OSY(X);
            nvar = 6; LB = [0,0,1,0,1,0]; UB = [10,10,5,6,5,10];
            A = [-1 -1 0 0 0 0;1 1 0 0 0 0;-1 1 0 0 0 0;1 -3 0 0 0 0]; b = [-2;6;2;2];
            Aeq = []; beq = [];
            problem_constraints = @OSYcon; % Only used in matlab test
        case 5
            problem_function = @(X) TNK(X);
            nvar = 2; LB = [0,0]; UB=[pi,pi];
            A = []; b = []; Aeq = []; beq = [];
            problem_constraints = @TNKcon; % Only used in matlab test

        case 6
            problem_function = @(X) CTP(X);
            nvar = 10; LB = -5*ones(1,10); UB = 5*ones(1,10); LB(1,1) = 0; UB(1,1) = 1;
            A = []; b = []; Aeq = []; beq = [];
            problem_constraints = @CTPcon; % Only used in matlab test
        case 7
            DP=1;
            problem_function = @(X) TNK_Robust(X,DP);
            nvar = 2; LB = [0,0]; UB=[pi,pi];
            A = []; b = []; Aeq = []; beq = [];
        otherwise
            problem_function = @(X) 0;
end


%% Matlab's MOGA

options = optimoptions('gamultiobj','PopulationSize',nChrome,'CrossoverFcn', @crossoverscattered,'Display','final','PlotFcn', { @gaplotpareto },'ParetoFraction', 0.9);

```

```
Mmoga = 1;
[Xmoga,Fmoga] = gamultiobj(problem_function,nvar,A,b,Aeq,beq,LB,UB,problem_constraints,options);
Mmoga = 0;
figure
plot(Fmoga(:,1),Fmoga(:,2),'bo','LineWidth',2);
hold on

%% Our MOGA
Pareto = [];
options = optimoptions(@ga,'PopulationSize',nChrome,'UseVectorized',true,'CrossoverFraction', 0.90);
optF =[];
for gen = 1:nRun
    gen
    Obj_fcn = @(X) fitFCN5(X,problem_function);
    [X,fval,exitflag,output] = ga(Obj_fcn,nvar,A,b,Aeq,beq,LB,UB,[],options);
    [optF(gen,:)] = problem_function(X);
    optX(gen,:) = X;
end
nfunc = optF(1,end-3);

 P = paretoset(optF(:,1:nfunc));
 m = 1;
    for k = 1:length(P)
        if P(k) == 1
            Pareto(m,:) = optF(k,1:2); m = m+1;
        end
    end

% figure
hold on;
plot(Pareto(:,1),Pareto(:,2),'gv','LineWidth',2,'MarkerSize',10)
plot(optF(:,1),optF(:,2),'r*','LineWidth',2)
hold on; grid on; legend('MATLABs MOGA','Our MOGA')
xlabel('f_1'); ylabel('f_2')


handle = gcf;
if save_figure == 1
    %Save the figures
%     dir_val = pwd;
    saveFigure(handle,['prob',num2str(prob),'_nChr',num2str(nChrome),'_nRun',num2str(nRun)]);
```

```
    %Save the .mat file
    problem.prob = prob; problem.nChrome = nChrome; problem.nRun = nRun;
    problem.alpha = alpha; problem.sigma = sigma; problem.epsilon = epsilon;
    problem.optF = optF; problem.Pareto= Pareto; problem.Fmoga = Fmoga;
    results_and_params{prob,1} = problem;
    save(['results_and_params',num2str(prob),'_nChr',num2str(nChrome),'_nRun',num2str(nRun),'_nTest',num2str(test)]);
end

% end
```

```
function [ fit ] = fitFCN5(X, ZD_func)
%NSGA algorithm. Use Approach 1 for sorting

global alpha sigma epsilon
func = ZD_func(X);

%% Find Dominate Points

% Modify existing code to find Pareto points to find dominant layers

% func = [existing_points;ZD_func(X)];

XOLin = X;
UNCT = func(1,end);
nfunc = func(1,end-3);
nconstr = func(1,end-2);
g = func(:,nfunc+1:nfunc+nconstr);
nconstr_eq = func(1,end-1);
h = func(:,nfunc+nconstr+1:nfunc+nconstr+nconstr_eq);
func = func(:,1:nfunc);

[M,~] = size(X);




if nconstr == 0 && nconstr_eq ==0
    nc_col = nfunc + 3;
    init_fit_col = nfunc + 4;
```

```
    sim_col = nfunc+5;
    indecies = [1:M]';
    func = [func,indecies]; %We need to know indecies later so this should save time
    P_temp = func;
    level = 0;
    level_col = nfunc +2;
    func(:,level_col) = 0;
    while ~isempty(P_temp)
        level = level+1;% increment the level value
        if length(P_temp(:,1))== 1
            %If there is only one value left at the end, assign this to a level
            func(P_temp(:,nfunc+1),level_col) = level;
            break
        end
        place = paretoset(P_temp(:,1:nfunc)); % get all the indecies in the lowest layer
        for k = 1:length(place)
            if place(k) == 1
                current_level_indecies = P_temp(k,nfunc+1); %map them from
                func(current_level_indecies, level_col) = level; % assiged from prtp
            end
        end

        P_temp(place, :) = [];
    end

    numLayer = level;

    %Make sure all individuals have a layer number
    flag = 0;
    for k = 1:M
       if func(level_col)==0
           func(k,level_col) = numLayer+1;
           flag = 1;
       end
    end
    if flag == 1, numLayer = numLayer+1; end

    %% Similarity
    %Assess similarity layer-by-layer, assess in objective space.
    var_rem = 0;
```

```
    F_min = M+epsilon;
    for k = 1:numLayer
        Fitness = []; incl = [];
        incl = find(func(:,level_col)==k); % incl = include
        Fitness = func(incl,1:nfunc);
        var_rem = var_rem+length(Fitness(:,1));
        if(isempty(Fitness) == 0)
            if length(incl) == 1

                F_int = F_min-epsilon;
                Fit_share = F_int;
                func(incl,sim_col+1) = F_int;
                func(incl,sim_col) = F_int;
                func(incl,nc_col) = 1;
            else
                for m = 1:nfunc
                    maxF(m) = max(Fitness(:,m));
                    minF(m) = min(Fitness(:,m));
                    func(m,init_fit_col) = minF(m);
                end
                d = []; similar = []; sh = [];
                for i = 1:length(incl)
                    F_int = F_min-epsilon;
                    for j = 1:length(incl)
                        for p = 1:nfunc
                            similar(p) = ((Fitness(i,p)-Fitness(j,p))/(maxF(p)-minF(p)))^2;
                        end
                        d(i,j) = sqrt(sum(similar));
                        if d(i,j)<=sigma
                            sh(i,j) = 1-(d(i,j)/sigma)^alpha;
                        else sh(i,j) = 0;
                        end
                    end
                    nc(i) = sum(sh(i,:));
                    Fit_share(i) = F_int/nc(i);
                    func(incl(i),nc_col) = nc(i);
                    func(incl(i),sim_col) = Fit_share(i);
                    func(incl(i),sim_col+1) = F_int;

                end
            end
            F_min = min(Fit_share);
```

```
        end

    end

    %Since a greater fitness value is a larger number, we use the inverse
    fit = -func(:,sim_col);

%% Constraint Handling
else
    Cmax = 1.2; Cmin = 0.8; r = 0.8*M;
    CF2 = 0.015;
    CF1 = 0.005;
    rank = zeros(1,M);

    % Assign moderate rank to all feasible solutions
     for k = 1:M
        flag = 0; flag_lin = 0;
        for p = 1:nconstr
            if g(k,p)>0, flag = 1;
            end
            if nconstr_eq ~= 0
                if h(k,p)~=0, flag_lin = 1;
                end
            end
        end
        if flag == 0 && flag_lin == 0
            rank(k) = 0.5*M;
        end
     end

     %Evaluates feasible solutions with uncertaintly applied in problems with
    %uncertainy
     if UNCT == 1
         for k = 1:M
             if rank(k) == 0.5*M
                 options = optimoptions(@ga,'PopulationSize',10,'UseVectorized',true);
                 lb = -2; ub = 2;
                 fitnessfn = @(DP) -TNK_NEGCN2(XOLin(k,:),DP);
                 [DP,fval] = ga(fitnessfn,2,[],[],[],[],lb,ub,[],options);
                 Constval = fval;
```

```
                if Constval > 0
                    rank(k) = 0;
                else
                    rank(k) = 0.5*M;
                end
            else
                rank(k) = 0;
            end
        end
    end

    % Collect together feasible population
    feas_pop = []; infeas_pop = []; m = 1;
    for k = 1:M
        if rank(k) ~= 0
            if isempty(h)
                feas_pop = [feas_pop;func(k,:),g(k,:)];
            else
                feas_pop = [feas_pop;func(k,:),g(k,:),h(k,:)];
            end
        else
             if isempty(h)
                 infeas_pop = [infeas_pop;func(k,:),g(k,:)];
             else
                 infeas_pop = [infeas_pop;func(k,:),g(k,:),h(k,:)];
             end
             loc(m) = k; m = m+1; %keep track of which solutions were infeasible
        end
    end

    % Identify noninferior points
    if ~isempty(feas_pop)
         place = paretoset(feas_pop(:,1:nfunc));
         m = 1;
         for k = 1:length(place)
             if place(k) == 1
                 rank(k) = 1; m = m+1; %Assign noninferior points along with constraint values
             end
         end
    end
```

```
    % Evaluate rank for infeasible individuals
    if ~isempty(infeas_pop)
        g = infeas_pop(:,nfunc+1:nconstr+nfunc);
        h = infeas_pop(:,nconstr+nfunc+1:end);

        for k = 1:length(g(:,1))
            for p = 1:nconstr
                if g(k,p)<=0
                    feas_g(k,p) = 0; delta_g(k,p) = 0;
                else
                    feas_g(k,p) = g(k,p); delta_g(k,p) = 1;
                end
            end
            if nconstr_eq == 0
                feas_h = zeros(length(g(:,1)),1);
                delta_h = zeros(length(g(:,1)),1);
            else
                for n = 1:nconstr_eq
                    feas_h(k,n) = abs(h(k,n));
                end
                if h(k,p)==0, delta_h(k,p) = 0;
                else delta_h(k,p) = 1;
                end
            end
        end
        num1 = sum(feas_g,2)+sum(feas_h,2);
        denom1 = (sum(sum(feas_g))+sum(sum(feas_h)))/M;
        J = nconstr; K = nconstr_eq;
        num2 = (sum(delta_g,2)+sum(delta_h,2));
        denom2 = (J+K);

        factor1 = CF1.*(num1./denom1);
        factor2 = CF2.*(num2./denom2);



        for k = 1:length(g(:,1))
            if (factor1(k)> mean(factor1)) && (factor2(k) < mean(factor2))
                w1 = 0.75; w2 = 0.25;
            elseif (factor1(k) < mean(factor1)) && (factor2(k) > mean(factor2))
                w1 = 0.25; w2 = 0.75;
            else
                w1 = 0.5; w2 = 0.5;
```

```
                end
                fit_constr(k) = -((Cmax-(Cmax-Cmin)*(r-1)/(M-1))-(w1.*factor1(k)+w2.*factor2(k)));
            end

            for k = 1:length(loc)
                rank(loc(k)) = fit_constr(k);
            end
        end
        fit = rank;


end
end
```