

# Testing and Formal Methods Quiz — With Some Answers

September 2014

## Abstract

This Quiz is meant to get an idea of your level of knowledge and skill concerning testing and formal methods. Please view it as an adventure. If you don't know what to answer, try to explain what baffles you. The results of this quiz play absolutely no role in any official assessment scheme. They are purely meant as information that will enable your teacher to fine-tune his course to the audience. Next time, we will start with discussing the quiz questions. Answers in English or Dutch. (This form is in English to cater for non-native speakers of Dutch — if there are any.)

**Question 1** Please give your name, student number, and email address:

**Question 2** What do you expect of this course?

**Question 3** Describe your experience with software testing.

**Question 4** Do you have any experience with formal specification? If 'yes', in what form?

**Question 5** Do you have any experience with logic? If 'yes', in what form?

**Question 6** Do you have experience with functional programming? Which languages? How much?

**Question 7** Assume someone gives you a lists of tests for a software module (say, in the form of a test script). What are methods to check the quality of the test script?

**Question 8** Are you familiar with set theoretic notation? Do you know how to read and use Venn diagrams?

All answers are OK, but if your answer to the first question is ‘no’ you should consult the beginning of [http://en.wikipedia.org/wiki/Set\\_theory](http://en.wikipedia.org/wiki/Set_theory). If your answer to the second question is ‘no’ you should consult [http://en.wikipedia.org/wiki/Venn\\_diagram](http://en.wikipedia.org/wiki/Venn_diagram).

**Question 9** Please paraphrase the following statements:

1.  $A \subseteq B$ ,
2.  $B \in \mathcal{P}(A)$ ,
3.  $B \in 2^A$ ,
4.  $A \cap B \neq \emptyset$ .

Answers:

1.  $A \subseteq B$ :  $A$  is a subset of  $B$ .
2.  $B \in \mathcal{P}(A)$ :  $B$  is an element of the power set of  $B$ , that is to say,  $B$  is a subset of  $A$ .
3.  $B \in 2^A$ :  $B$  is a subset of  $A$ .
4.  $A \cap B \neq \emptyset$ :  $A$  and  $B$  have at least one element in common.

If you did not see that (2) and (3) have the same meaning, or if you had other trouble with this question, Chapter 4 of *The Haskell Road* will set you straight.

**Question 10** Which set is given by the following specification:

$$\{2n + 1 \mid n \in \mathbb{N}\}.$$

Answer: the set of odd natural numbers.

**Question 11** Do you think there is a connection between testing, formal specification, and logic? Explain.

Answer: there are close connections. The course will teach you all about them.

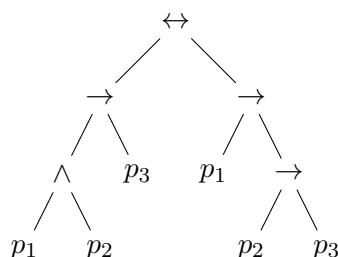
**Question 12** Here is a context free grammar for a formal language. Assume  $p$  ranges over a set of symbols  $\{p_1, p_2, p_3, \dots\}$ .

$$\varphi ::= p \mid (\neg\varphi) \mid (\varphi \wedge \varphi) \mid (\varphi \vee \varphi) \mid (\varphi \rightarrow \varphi) \mid (\varphi \leftrightarrow \varphi)$$

The following expression has type  $\varphi$ . Give a parse tree for it.

$$(((p_1 \wedge p_2) \rightarrow p_3) \leftrightarrow (p_1 \rightarrow (p_2 \rightarrow p_3)))$$

Answer: here is a parse tree:



**Question 13** Three software engineers are discussing the grammar of the previous question. Engineer A says: the grammar generates a finite language, for every expression generated by the grammar has finite length. Engineer B says: the grammar generates an infinite language, for the grammar can generate expressions that are infinitely long. Engineer C says: the grammar generates an infinite language, but all of the expressions that are generated by the grammar have finite length. Which of them is right, and why?

Answer: Engineer C is right. For a language to be infinite means that the language has an infinite number of expressions. This is the case for the example language, for there is no ‘largest’ expression. Still, every expression of the language consists of a finite number of symbols.

**Question 14** Indicate which statement is correct, and why.

1. The language HTML is context-free because there is a specification of HTML in the form of a context-free grammar.
2. The language HTML is context-free because HTML is a programming language, and every programming language is context-free.
3. The language HTML is context-free because HTML files can be processed by the parsing program of a browser, and each language that can be processed mechanically is context-free.

The first statement is correct. The definition of HTML is given in terms of context free grammar rules. The second statement is false because not every programming language has a context free syntax. The third statement is false because there are formal languages that are not context free but still can be parsed by means of a program.

**Question 15**  $A$  is a set with  $n$  elements, where  $n$  is some natural number. How many elements does the power set of  $A$  have?

Answer: The power set of  $A$  has  $2^n$  elements.

This generated many wrong answers, such as  $n^2$ ,  $n^n$ ,  $n!$ .

If you had this one wrong, work out your answer for small  $n$  and compare. You can use Haskell as a tool for this:

```
Prelude> map (\ n -> 2^n) [0..10]
[1,2,4,8,16,32,64,128,256,512,1024]
Prelude> map (\ n -> n^2) [0..10]
[0,1,4,9,16,25,36,49,64,81,100]
Prelude> map (\ n -> n^n) [0..10]
[1,1,4,27,256,3125,46656,823543,16777216,387420489,10000000000]
Prelude> map (\n -> product [1..n]) [0..10]
[1,1,2,6,24,120,720,5040,40320,362880,3628800]
```

**Question 16**  $\mathbb{N}$  is the set of natural numbers. How many elements does the power set of  $\mathbb{N}$  have? The same number of elements as  $\mathbb{N}$ ? More elements? But what does that mean?

Answer: The power set of  $\mathbb{N}$  has more elements than the set of natural numbers: there is no bijection between  $\mathbb{N}$  and  $\mathcal{P}(\mathbb{N})$ , but there is an injection from  $\mathbb{N}$  to  $\mathcal{P}(\mathbb{N})$ , for example  $\lambda n.\{n\}$  is such an injection.

If you don't understand why this means that  $\mathbb{N}$  and  $\mathcal{P}(\mathbb{N})$  have different sizes (or: different cardinalities), please look up the lemma *Uncountable set* on Wikipedia.

**Question 17** A relation is set of pairs over some universe  $U$ . A relation  $R$  is transitive if it holds for all  $x, y, z$  that if  $(x, y) \in R$  and  $(y, z) \in R$ , then also  $(x, z) \in R$ . To say that the relation of 'friendship' is transitive boils down to saying that it holds for anyone that the friends of their friends are their friends.

Which of the following relations are transitive?

1.  $\{(1, 2), (2, 3), (3, 4)\}$ .
2.  $\{(1, 2), (2, 3), (3, 4), (1, 3), (2, 4)\}$ .
3.  $\{(1, 2), (2, 3), (3, 4), (1, 3), (2, 4), (1, 4)\}$ .
4.  $\{(1, 2), (2, 1)\}$ .
5.  $\{(1, 1), (2, 2)\}$ .

Answer:

1.  $\{(1, 2), (2, 3), (3, 4)\}$ . Not transitive.

2.  $\{(1, 2), (2, 3), (3, 4), (1, 3), (2, 4)\}$ . Not transitive.
3.  $\{(1, 2), (2, 3), (3, 4), (1, 3), (2, 4), (1, 4)\}$ . Transitive.
4.  $\{(1, 2), (2, 1)\}$ . Not transitive.
5.  $\{(1, 1), (2, 2)\}$ . Transitive.

**Question 18** Give the transitive closures of each of the relations in question 17:

1.  $\{(1, 2), (2, 3), (3, 4)\}$ .
2.  $\{(1, 2), (2, 3), (3, 4), (1, 3), (2, 4)\}$ .
3.  $\{(1, 2), (2, 3), (3, 4), (1, 3), (2, 4), (1, 4)\}$ .
4.  $\{(1, 2), (2, 1)\}$ .
5.  $\{(1, 1), (2, 2)\}$ .

Answer:

1.  $\{(1, 2), (2, 3), (3, 4)\}$  has transitive closure  $\{(1, 2), (2, 3), (3, 4), (1, 3), (2, 4), (1, 4)\}$ .
2.  $\{(1, 2), (2, 3), (3, 4), (1, 3), (2, 4)\}$  has transitive closure  $\{(1, 2), (2, 3), (3, 4), (1, 3), (2, 4), (1, 4)\}$ .
3.  $\{(1, 2), (2, 3), (3, 4), (1, 3), (2, 4), (1, 4)\}$ . has transitive closure  $\{(1, 2), (2, 3), (3, 4), (1, 3), (2, 4), (1, 4)\}$ .
4.  $\{(1, 2), (2, 1)\}$  has transitive closure  $\{(1, 1), (1, 2), (2, 1), (2, 2)\}$ .
5.  $\{(1, 1), (2, 2)\}$  has transitive closure  $\{(1, 1), (2, 2)\}$ .

Notice that if a relation is transitive then its transitive closure is itself, for in this case it is not necessary to add pairs to the relation in order to make it transitive.