Python Code for QSS Chapter 7: Uncertainty

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First Printing

[ ]:
```
import pandas as pd
import numpy as np
from scipy import stats
import matplotlib.pyplot as plt
import seaborn as sns
```

Section 7.1: Estimation

Section 7.1.1: Unbiasedness and Consistency

[ ]:
```
# simulation parameters
n = 100 # sample size
mu0 = 0 # mean of Y_i(0)
sd0 = 1 # standard deviation of Y_i(0)
mu1 = 1 # mean of Y_i(1)
sd1 = 1 # standard deviation of Y_i(1)

# generate a sample
Y0 = stats.norm.rvs(size=n, loc=mu0, scale=sd0)
Y1 = stats.norm.rvs(size=n, loc=mu1, scale=sd1)
tau = Y1 - Y0 # individual treatment effect
# true value of the sample average treatment effect
SATE = tau.mean()
SATE
```

[ ]:
```
1.1697124670570664
```

[ ]:
```
# repeatedly conduct randomized controlled trials
sims = 5000 # repeat 5,000 times, we could do more
diff_means = np.zeros(sims) # container
sample_vector = np.concatenate((np.ones(int(n/2)), np.zeros(int(n/2))))

for i in range(sims):
    # randomize the treatment by sampling of a vector of 0's and 1's
    treat = np.random.choice(sample_vector, size=n, replace=False)
    # difference-in-means
    diff_means[i] = Y1[treat==1].mean() - Y0[treat==0].mean()
```

```python
# estimation of error for SATE
est_error = diff_means - SATE

est_error.mean()
```

[ ]: 0.001367231467261098

[ ]:
```python
pd.Series(est_error).describe().round(5)
```

[ ]:
```
count    5000.00000
mean        0.00137
std         0.13285
min        -0.47264
25%        -0.09028
50%         0.00054
75%         0.09551
max         0.44256
dtype: float64
```

[ ]:
```python
# PATE simulation
PATE = mu1 - mu0
diff_means = np.zeros(sims)

for i in range(sims):
    # generate a sample for each simulation
    Y0 = stats.norm.rvs(size=n, loc=mu0, scale=sd0)
    Y1 = stats.norm.rvs(size=n, loc=mu1, scale=sd1)
    treat = np.random.choice(sample_vector, size=n, replace=False)
    diff_means[i] = Y1[treat==1].mean() - Y0[treat==0].mean()

# estimation error for PATE
est_error = diff_means - PATE

# unbiased
est_error.mean()
```

[ ]: 0.001799964703385013

[ ]:
```python
pd.Series(est_error).describe().round(5)
```

[ ]:
```
count    5000.00000
mean        0.00180
std         0.19858
min        -0.68073
25%        -0.13276
50%         0.00138
75%         0.13692
max         0.89020
```

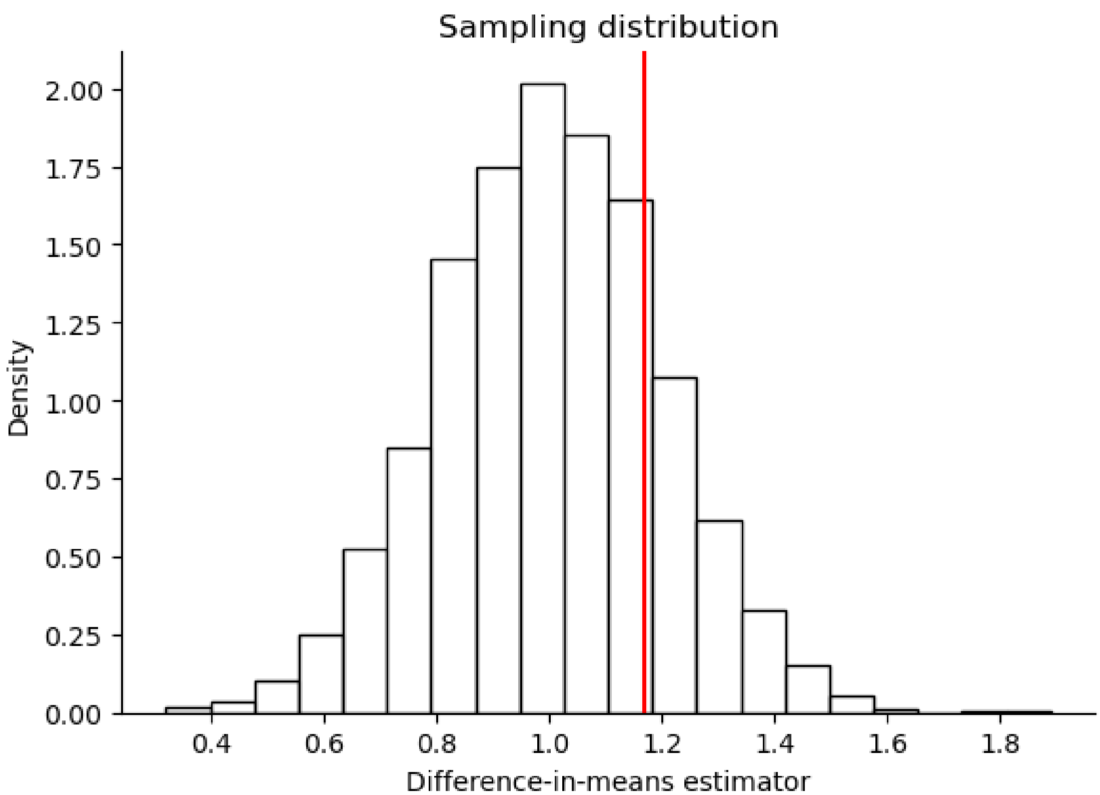
```
dtype: float64
```

Section 7.1.2: Standard Error

[ ]:
```
sns.displot(
    diff_means, stat='density', color='white', edgecolor='black',
    height=4, aspect=1.5, bins=20
).set(title='Sampling distribution', xlabel='Difference-in-means estimator')

plt.axvline(SATE, color='red') # true value of SATE
```

[ ]:
```
<matplotlib.lines.Line2D at 0x15c74806440>
```



[ ]:
```
diff_means.std(ddof=1)
```

[ ]:
```
0.1985836723621929
```

[ ]:
```
np.sqrt(((diff_means - SATE)**2).mean())
```

[ ]:
```
0.26004268165157823
```

[ ]:
```
# PATE simulation with standard error
sims = 5000
diff_means = np.zeros(sims)
se = np.zeros(sims)

for i in range(sims):
    # generate a sample for each simulation
    Y0 = stats.norm.rvs(size=n, loc=mu0, scale=sd0)
    Y1 = stats.norm.rvs(size=n, loc=mu1, scale=sd1)
    # randomize treatment by sampling the vector of 0's and 1's created above
    treat = np.random.choice(sample_vector, size=n, replace=False)
    diff_means[i] = Y1[treat==1].mean() - Y0[treat==0].mean()
    se[i] = (np.sqrt(Y1[treat==1].var(ddof=1) / (n/2) +
                     Y0[treat==0].var(ddof=1) / (n/2)))

diff_means.std(ddof=1)
```

[ ]:
```
0.2006386254978984
```

[ ]:
```
se.mean()
```

[ ]:
```
0.1995025026886699
```

Section 7.1.3: Confidence Intervals

[ ]:
```
n = 1000 # sample size
x_bar = 0.6 # point estimate
s_e = np.sqrt(x_bar * (1-x_bar) / n) # standard error

# 99% confidence intervals; display as a tuple
((x_bar - stats.norm.ppf(0.995) * s_e).round(5),
 (x_bar + stats.norm.ppf(0.995) * s_e).round(5))
```

[ ]:
```
(0.5601, 0.6399)
```

[ ]:
```
# 95% confidence intervals
((x_bar - stats.norm.ppf(0.975) * s_e).round(5),
 (x_bar + stats.norm.ppf(0.975) * s_e).round(5))
```

[ ]:
```
(0.56964, 0.63036)
```

[ ]:
```
# 90% confidence intervals
((x_bar - stats.norm.ppf(0.95) * s_e).round(5),
 (x_bar + stats.norm.ppf(0.95) * s_e).round(5))
```

[ ]:
```
(0.57452, 0.62548)
```

```
[ ]: # empty container matrices for 2 sets of confidence intervals
ci95 = np.zeros(sims*2).reshape(sims, 2)
ci90 = np.zeros(sims*2).reshape(sims, 2)

# 95 percent confidence intervals
ci95[:,0] = diff_means - stats.norm.ppf(0.975) * se # lower limit
ci95[:,1] = diff_means + stats.norm.ppf(0.975) * se # upper limit

# 90 percent confidence intervals
ci90[:,0] = diff_means - stats.norm.ppf(0.95) * se # lower limit
ci90[:,1] = diff_means + stats.norm.ppf(0.95) * se # upper limit

# coverage rate for 95% confidence interval
((ci95[:,0] <= 1) & (ci95[:,1] >= 1)).mean()
```

```
[ ]: 0.9448
```

```
[ ]: # coverage rate for 90% confidence interval
((ci90[:,0] <= 1) & (ci90[:,1] >= 1)).mean()
```

```
[ ]: 0.8954
```

```
[ ]: p = 0.6 # true parameter value
n = np.array([50, 100, 1000]) # 3 sample sizes to be examined
alpha = 0.05
sims = 5000 # number of simulations
results = np.zeros(len(n)) # a container for results

for i in range(len(n)):
    ci_results = np.zeros(sims) # a container for whether CI contains truth
    # loop for repeated hypothetical survey sampling
    for j in range(sims):
        data = stats.binom.rvs(n=1, p=p, size=n[i]) # simple random sampling
        x_bar = data.mean() # sample proportion as an estimate
        s_e = np.sqrt(x_bar * (1-x_bar) / n[i]) # standard errors
        ci_lower = x_bar - stats.norm.ppf(1-alpha/2) * s_e
        ci_upper = x_bar + stats.norm.ppf(1-alpha/2) * s_e
        ci_results[j] = (p >= ci_lower) & (p <= ci_upper)
    # proportion of CIs that contain the true value
    results[i] = ci_results.mean()

results
```

```
[ ]: array([0.9394, 0.9422, 0.9492])
```

Section 7.1.4: Margin of Error and Sample Size Calculation in Polls

In Progress