Python Code for QSS Chapter 7: Uncertainty

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First Printing

[ ]:
```
import pandas as pd
import numpy as np
from scipy import stats
import matplotlib.pyplot as plt
import seaborn as sns
```

Section 7.1: Estimation

Section 7.1.1: Unbiasedness and Consistency

[ ]:
```
# simulation parameters
n = 100 # sample size
mu0 = 0 # mean of Y_i(0)
sd0 = 1 # standard deviation of Y_i(0)
mu1 = 1 # mean of Y_i(1)
sd1 = 1 # standard deviation of Y_i(1)

# generate a sample
Y0 = stats.norm.rvs(size=n, loc=mu0, scale=sd0)
Y1 = stats.norm.rvs(size=n, loc=mu1, scale=sd1)
tau = Y1 - Y0 # individual treatment effect
# true value of the sample average treatment effect
SATE = tau.mean()
SATE
```

[ ]:
```
0.9834900501046896
```

[ ]:
```
# repeatedly conduct randomized controlled trials
sims = 5000 # repeat 5,000 times, we could do more
diff_means = np.zeros(sims) # container
sample_vector = np.concatenate((np.ones(int(n/2)), np.zeros(int(n/2))))

for i in range(sims):
    # randomize the treatment by sampling of a vector of 0's and 1's
    treat = np.random.choice(sample_vector, size=n, replace=False)
    # difference-in-means
    diff_means[i] = Y1[treat==1].mean() - Y0[treat==0].mean()
```

```python
# estimation of error for SATE
est_error = diff_means - SATE

est_error.mean()
```

[ ]: -0.00035168297815206984

[ ]:
```python
pd.Series(est_error).describe().round(5)
```

[ ]:
```
count    5000.00000
mean       -0.00035
std         0.13922
min        -0.51761
25%        -0.09457
50%        -0.00221
75%         0.09464
max         0.48361
dtype: float64
```

[ ]:
```python
# PATE simulation
PATE = mu1 - mu0
diff_means = np.zeros(sims)

for i in range(sims):
    # generate a sample for each simulation
    Y0 = stats.norm.rvs(size=n, loc=mu0, scale=sd0)
    Y1 = stats.norm.rvs(size=n, loc=mu1, scale=sd1)
    treat = np.random.choice(sample_vector, size=n, replace=False)
    diff_means[i] = Y1[treat==1].mean() - Y0[treat==0].mean()

# estimation error for PATE
est_error = diff_means - PATE

# unbiased
est_error.mean()
```

[ ]: 0.00040107622939003568

[ ]:
```python
pd.Series(est_error).describe().round(5)
```

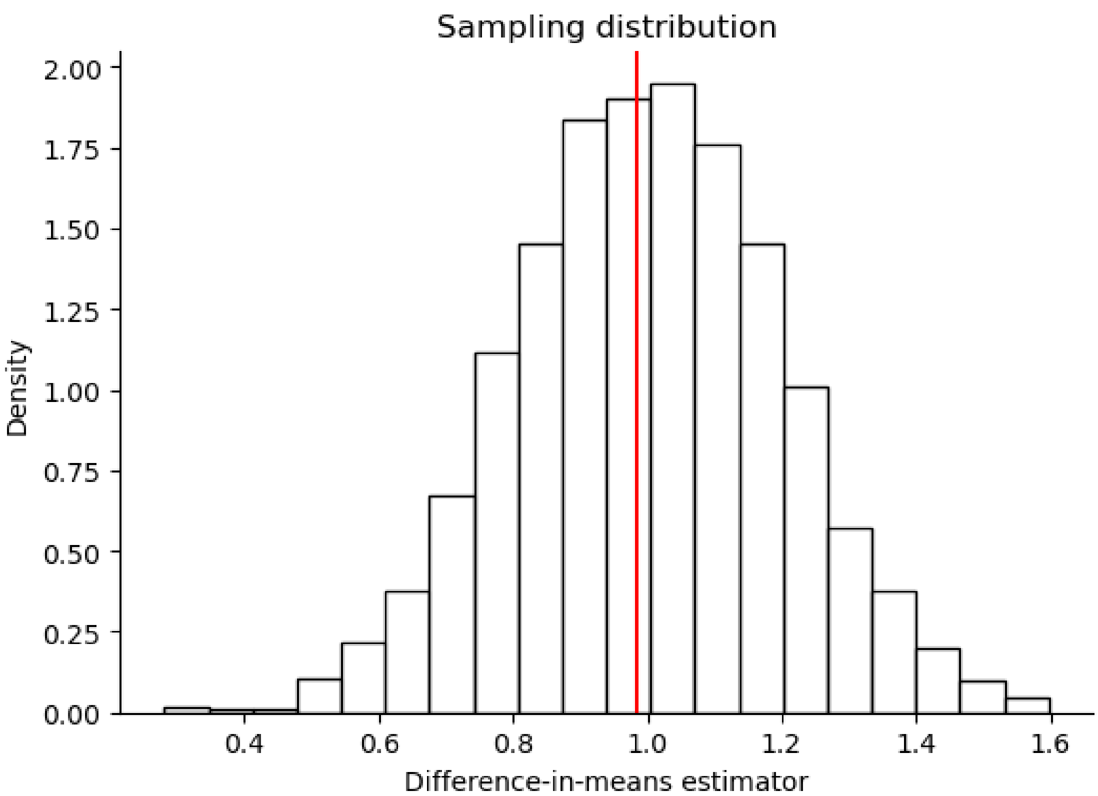
[ ]:
```
count    5000.00000
mean        0.00040
std         0.19712
min        -0.71942
25%        -0.13428
50%         0.00031
75%         0.13493
max         0.59762
```

dtype: float64

Section 7.1.2: Standard Error

[ ]:
```
sns.displot(
    diff_means, stat='density', color='white', edgecolor='black',
    height=4, aspect=1.5, bins=20
).set(title='Sampling distribution', xlabel='Difference-in-means estimator')

plt.axvline(SATE, color='red') # true value of SATE
```

[ ]: <matplotlib.lines.Line2D at 0x2605dab2200>



[ ]:
```
diff_means.std(ddof=1)
```

[ ]: 0.1971197024641733

[ ]:
```
np.sqrt(((diff_means - SATE)**2).mean())
```

[ ]: 0.19782413570829674

```
[ ]: # PATE simulation with standard error
sims = 5000
diff_means = np.zeros(sims)
se = np.zeros(sims)

for i in range(sims):
    # generate a sample for each simulation
    Y0 = stats.norm.rvs(size=n, loc=mu0, scale=sd0)
    Y1 = stats.norm.rvs(size=n, loc=mu1, scale=sd1)
    # randomize treatment by sampling the vector of 0's and 1's created above
    treat = np.random.choice(sample_vector, size=n, replace=False)
    diff_means[i] = Y1[treat==1].mean() - Y0[treat==0].mean()
    se[i] = (np.sqrt(Y1[treat==1].var(ddof=1) / (n/2) +
                     Y0[treat==0].var(ddof=1) / (n/2)))

diff_means.std(ddof=1)
```

```
[ ]: 0.20396202223618143
```

```
[ ]: se.mean()
```

```
[ ]: 0.1993245289521019
```

Section 7.1.3: Confidence Intervals

```
[ ]: n = 1000 # sample size
x_bar = 0.6 # point estimate
s_e = np.sqrt(x_bar * (1-x_bar) / n) # standard error

# 99% confidence intervals; display as a tuple
((x_bar - stats.norm.ppf(0.995) * s_e).round(5),
 (x_bar + stats.norm.ppf(0.995) * s_e).round(5))
```

```
[ ]: (0.5601, 0.6399)
```

```
[ ]: # 95% confidence intervals
((x_bar - stats.norm.ppf(0.975) * s_e).round(5),
 (x_bar + stats.norm.ppf(0.975) * s_e).round(5))
```

```
[ ]: (0.56964, 0.63036)
```

```
[ ]: # 90% confidence intervals
((x_bar - stats.norm.ppf(0.95) * s_e).round(5),
 (x_bar + stats.norm.ppf(0.95) * s_e).round(5))
```

```
[ ]: (0.57452, 0.62548)
```

[ ]:
```
# empty container matrices for 2 sets of confidence intervals
ci95 = np.zeros(sims*2).reshape(sims, 2)
ci90 = np.zeros(sims*2).reshape(sims, 2)

# 95 percent confidence intervals
ci95[:,0] = diff_means - stats.norm.ppf(0.975) * se # lower limit
ci95[:,1] = diff_means + stats.norm.ppf(0.975) * se # upper limit

# 90 percent confidence intervals
ci90[:,0] = diff_means - stats.norm.ppf(0.95) * se # lower limit
ci90[:,1] = diff_means + stats.norm.ppf(0.95) * se # upper limit

# coverage rate for 95% confidence interval
((ci95[:,0] <= 1) & (ci95[:,1] >= 1)).mean()
```

[ ]: 0.9462

[ ]:
```
# coverage rate for 90% confidence interval
((ci90[:,0] <= 1) & (ci90[:,1] >= 1)).mean()
```

[ ]: 0.8886

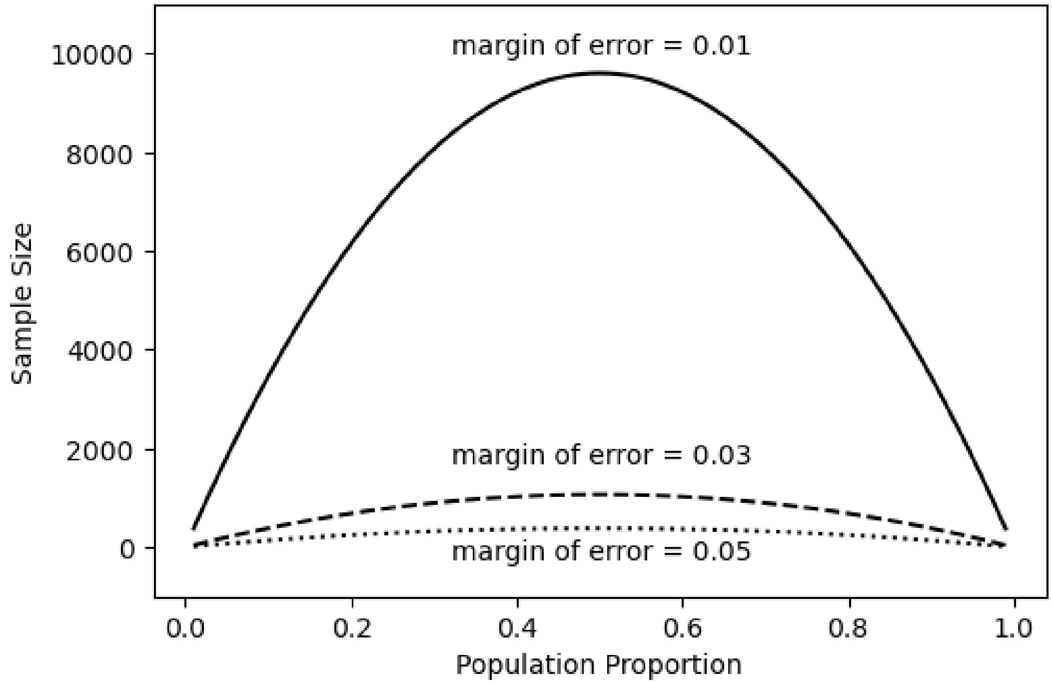
[ ]:
```
p = 0.6 # true parameter value
n = np.array([50, 100, 1000]) # 3 sample sizes to be examined
alpha = 0.05
sims = 5000 # number of simulations
results = np.zeros(len(n)) # a container for results

for i in range(len(n)):
    ci_results = np.zeros(sims) # a container for whether CI contains truth
    # loop for repeated hypothetical survey sampling
    for j in range(sims):
        data = stats.binom.rvs(n=1, p=p, size=n[i]) # simple random sampling
        x_bar = data.mean() # sample proportion as an estimate
        s_e = np.sqrt(x_bar * (1-x_bar) / n[i]) # standard errors
        ci_lower = x_bar - stats.norm.ppf(1-alpha/2) * s_e
        ci_upper = x_bar + stats.norm.ppf(1-alpha/2) * s_e
        ci_results[j] = (p >= ci_lower) & (p <= ci_upper)
    # proportion of CIs that contain the true value
    results[i] = ci_results.mean()

results
```

[ ]: array([0.9368, 0.9422, 0.9498])

Section 7.1.4: Margin of Error and Sample Size Calculation in Polls

[ ]:
```
MoE = np.array([0.01, 0.03, 0.05]) # the desired margin of error
p = np.arange(0.01, 1, 0.01)
n = 1.96**2 * p * (1-p) / MoE[0]**2
n2 = 1.96**2 * p * (1-p) / MoE[1]**2
n3 = 1.96**2 * p * (1-p) / MoE[2]**2

fig, ax = plt.subplots(figsize=(6,4))

sns.lineplot(x=p, y=n, ax=ax, color='black').set(
    ylim=(-1000, 11000), xlabel='Population Proportion', ylabel='Sample Size'
)

sns.lineplot(x=p, y=n2, ax=ax, color='black', linestyle='--')

sns.lineplot(x=p, y=n3, ax=ax, color='black', linestyle=':')

# Add text labels
ax.text(0.32, 10000, 'margin of error = 0.01', fontsize=10)
ax.text(0.32, 1700, 'margin of error = 0.03', fontsize=10)
ax.text(0.32, -250, 'margin of error = 0.05', fontsize=10)
```

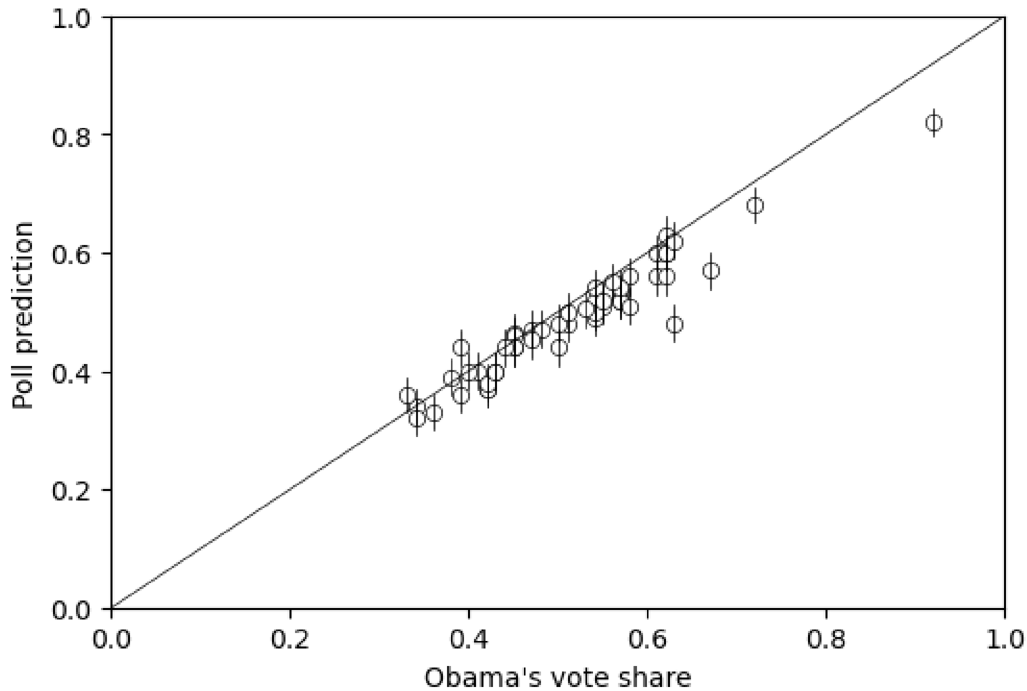
[ ]:
```
Text(0.32, -250, 'margin of error = 0.05')
```



[ ]:
```python
# election and polling results, by state
pres08 = pd.read_csv('pres08.csv')
polls08 = pd.read_csv('polls08.csv')

# convert to a date object
polls08['middate'] = pd.to_datetime(polls08['middate'])

# number of days to the election
from datetime import datetime
election_day = datetime.strptime('2008-11-04', '%Y-%m-%d')
polls08['days_to_election'] = (election_day - polls08['middate']).dt.days

# extract unique state names which the loop will iterate through
st_names = polls08['state'].unique()

# create an empty 51 X 3 placeholder Data Frame
poll_pred = pd.DataFrame(np.zeros(51*3).reshape(51, 3), index=st_names)

# loop across the 50 states plus DC
for i in range(len(st_names)):
    # subset the ith state
    state_data = polls08[polls08['state']==st_names[i]]
    # further subset the latest polls within the state
    latest = (state_data['days_to_election']==
              state_data['days_to_election'].min())
    # compute the mean of the latest polls and store it
    poll_pred.iloc[i, 0] = state_data['Obama'][latest].mean() / 100

# upper and lower confidence limits
n = 1000 # sample size
alpha = 0.05
se = np.sqrt(poll_pred.iloc[:,0] * (1-poll_pred.iloc[:,0]) / n) # standard error
poll_pred.iloc[:,1] = poll_pred.iloc[:,0] - stats.norm.ppf(1-alpha/2) * se
poll_pred.iloc[:,2] = poll_pred.iloc[:,0] + stats.norm.ppf(1-alpha/2) * se
```

[ ]:
```python
# plot the results
fig, ax = plt.subplots(figsize=(6,4))

sns.scatterplot(
    x = pres08['Obama'] / 100, y = poll_pred.iloc[:,0].reset_index(drop=True),
    ax=ax, color='white', edgecolor='black'
).set(xlabel="Obama's vote share", ylabel='Poll prediction',
      xlim=(0, 1), ylim=(0, 1))

ax.axline((0, 0), slope=1, color='black', linewidth=0.5)

# adding 95% confidence intervals for each state
```

```
for i in range(len(st_names)):
    ax.plot(
        [pres08['Obama'][i] / 100] * 2,
        [poll_pred.iloc[i,1], poll_pred.iloc[i,2]],
        color='black', linewidth=0.5
    )
```



[ ]:
```
# proportion of confidence intervals that contain the election day outcome
# reset index: can only compare identically-labeled Series objects
((poll_pred.iloc[:,1].reset_index(drop=True) <= pres08['Obama'] / 100 ) &
 (poll_pred.iloc[:,2].reset_index(drop=True) >= pres08['Obama'] / 100)).mean()
```

[ ]: 0.5882352941176471

[ ]:
```
# bias
bias=(poll_pred.iloc[:,0].reset_index(drop=True) - pres08['Obama']/100).mean()
bias
```

[ ]: -0.026797385620915028

[ ]:
```
# bias corrected estimate
poll_bias = poll_pred.iloc[:,0] - bias

# bias corrected standard error
```

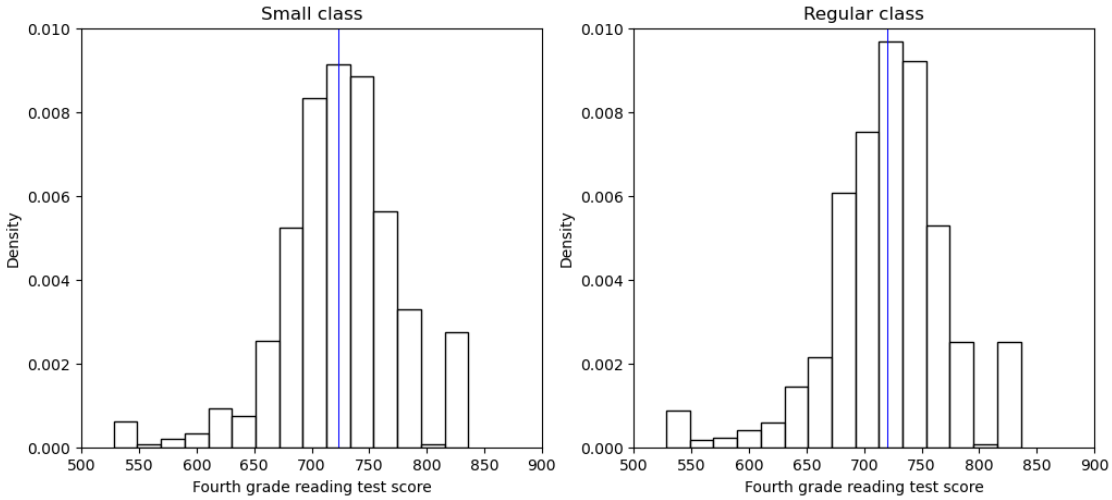
```python
se_bias = np.sqrt(poll_bias * (1-poll_bias) / n)

# bias corrected confidence intervals
ci_bias_lower = poll_bias - stats.norm.ppf(1-alpha/2) * se_bias
ci_bias_upper = poll_bias + stats.norm.ppf(1-alpha/2) * se_bias

# proportion of bias corrected CIs that contain election day outcome
((ci_bias_lower.reset_index(drop=True) <= pres08['Obama'] / 100) &
 (ci_bias_upper.reset_index(drop=True) >= pres08['Obama'] / 100)).mean()
```

[ ]: 0.7647058823529411

Section 7.1.5: Analysis of Randomized Controlled Trials

[ ]:
```python
STAR = pd.read_csv('STAR.csv')

fig, axs = plt.subplots(1, 2, figsize=(12,5))

sns.histplot(
    STAR['g4reading'][STAR.classtype==1], stat = 'density', ax=axs[0],
    color='white', edgecolor='black', bins=15
).set(ylim=(0, 0.01), xlim=(500, 900), title='Small class',
      xlabel='Fourth grade reading test score')

axs[0].axvline(STAR['g4reading'][STAR.classtype==1].mean(),
               color='blue', linewidth=0.75)

sns.histplot(
    STAR['g4reading'][STAR.classtype==2], stat = 'density', ax=axs[1],
    color='white', edgecolor='black', bins=15
).set(ylim=(0, 0.01), xlim=(500, 900), title='Regular class',
      xlabel='Fourth grade reading test score')

axs[1].axvline(STAR['g4reading'][STAR.classtype==2].mean(),
                color='blue', linewidth=0.75)
```

[ ]: <matplotlib.lines.Line2D at 0x2605e403eb0>



[ ]:
```
# estimate and standard error for small class size
n_small = (STAR['classtype']==1 & STAR['g4reading'].notnull()).sum()
est_small = STAR['g4reading'][STAR.classtype==1].mean()
se_small = STAR['g4reading'][STAR.classtype==1].std() / np.sqrt(n_small)
est_small, se_small
```

[ ]: (723.3911845730028, 1.9130122952458233)

[ ]:
```
# estimate and standard error for regular class size
n_regular = ((STAR['classtype']==2) &
             (STAR['classtype'].notnull()) &
             (STAR['g4reading'].notnull())).sum()
est_regular = STAR['g4reading'][STAR.classtype==2].mean()
se_regular = STAR['g4reading'][STAR.classtype==2].std() / np.sqrt(n_regular)
est_regular, se_regular
```

[ ]: (719.88995215311, 1.8388496908502467)

[ ]:
```
alpha = 0.05

# 95% confidence intervals for small class size
ci_small = (est_small - stats.norm.ppf(1-alpha/2) * se_small,
            est_small + stats.norm.ppf(1-alpha/2) * se_small)
ci_small
```

[ ]: (719.6417493723386, 727.1406197736669)

[ ]:
```
# 95% confidence intervals for regular class size
ci_regular = (est_regular - stats.norm.ppf(1-alpha/2) * se_regular,
              est_regular + stats.norm.ppf(1-alpha/2) * se_regular)
```

```
ci_regular
```

[ ]: (716.2858729860609, 723.4940313201591)

[ ]:
```
# difference in means estimator
ate_est = est_small - est_regular
ate_est
```

[ ]: 3.5012324198927445

[ ]:
```
# standard error and 95% confidence interval
ate_se = np.sqrt(se_small**2 + se_regular**2)
ate_se
```

[ ]: 2.653485298112982

[ ]:
```
ate_ci = (ate_est - stats.norm.ppf(1-alpha/2) * ate_se,
          ate_est + stats.norm.ppf(1-alpha/2) * ate_se)
ate_ci
```

[ ]: (-1.699503197915229, 8.701968037700718)

Section 7.1.6: Analysis Based on Student's t-Distribution

[ ]:
```
# 95% CI for small class
(est_small - stats.t.ppf(0.975, df=n_small-1) * se_small,
 est_small + stats.t.ppf(0.975, df=n_small-1) * se_small)
```

[ ]: (719.635479522832, 727.1468896231735)

[ ]:
```
# 95% CI based on the central limit theorem
ci_small
```

[ ]: (719.6417493723386, 727.1406197736669)

[ ]:
```
# 95% CI for regular class
(est_regular - stats.t.ppf(0.975, df=n_regular-1) * se_regular,
 est_regular + stats.t.ppf(0.975, df=n_regular-1) * se_regular)
```

[ ]: (716.2806412822123, 723.4992630240077)

[ ]:
```
# 95% CI based on the central limit theorem
ci_regular
```

[ ]: (716.2858729860609, 723.4940313201591)

[ ]:
```
test_result = stats.ttest_ind(
    STAR['g4reading'][STAR.classtype==1],
```

```
    STAR['g4reading'][STAR.classtype==2],
    # override default equal_var=True; False is Welch t-test
    equal_var=False,
    # override default nan_policy='propogate'
    nan_policy='omit')

# extract results from the test_result object
test_result.pvalue.round(5)
```

[ ]: 0.1872

[ ]:
```
# store results for printing
t_stat = test_result.statistic.round(4)
p_value = test_result.pvalue.round(5)
df = test_result.df.round(1)
ci = test_result.confidence_interval(confidence_level=0.95)

print(f"""Welch Two Sample t-test
t-stat: {t_stat}
p-value: {p_value}
df: {df}
95% confidence interval: ({ci[0].round(5)}, {ci[1].round(5)})""")
```

```
Welch Two Sample t-test
t-stat: 1.3195
p-value: 0.1872
df: 1541.2
95% confidence interval: (-1.70359, 8.70606)
```

Section 7.2: Hypothesis Testing

Section 7.2.1: Tea-Testing Experiment

[ ]:
```
from math import comb

# truth: enumerate the number of assignment combinations
true = np.array(
    [comb(4,0) * comb(4,4),
     comb(4,1) * comb(4,3),
     comb(4,2) * comb(4,2),
     comb(4,3) * comb(4,1),
     comb(4,4) * comb(4,0)]
)

true
```

[ ]: array([1, 16, 36, 16, 1])

[ ]:
```python
# compute probability: divide it by the total number of events
true = pd.Series(true / true.sum(), index=[0,2,4,6,8])

true
```

[ ]:
```
0    0.014286
2    0.228571
4    0.514286
6    0.228571
8    0.014286
dtype: float64
```

[ ]:
```python
# simulations
sims=1000
# lady's guess: M stands for 'Milk first', T stands for 'Tea first'
guess=np.array(['M', 'T', 'T', 'M', 'M', 'T', 'T', 'M'])
sample_vector=np.array(['T']*4 + ['M']*4)
correct=pd.Series(np.zeros(sims)) # place holder for number of correct guesses

for i in range(sims):
    # randomize which cups get Milk/Tea first
    cups=np.random.choice(sample_vector, size=len(sample_vector), replace=False)
    correct[i]=(guess==cups).sum() # number of correct guesses

# estimated probability for each number of correct guesse
correct.value_counts(normalize=True).sort_index()
```

[ ]:
```
0.0    0.013
2.0    0.208
4.0    0.539
6.0    0.228
8.0    0.012
Name: proportion, dtype: float64
```

[ ]:
```python
# comparison with analytical answers; the differences are small
correct.value_counts(normalize=True).sort_index() - true
```

[ ]:
```
0.0   -0.001286
2.0   -0.020571
4.0    0.024714
6.0   -0.000571
8.0   -0.002286
dtype: float64
```

Section 7.2.2: The General Framework

In Progress