Python Code for QSS Chapter 7: Uncertainty

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First Printing

[ ]:
```python
import pandas as pd
import numpy as np
from scipy import stats
import matplotlib.pyplot as plt
import seaborn as sns
```

Section 7.1: Estimation

Section 7.1.1: Unbiasedness and Consistency

[ ]:
```python
# simulation parameters
n = 100 # sample size
mu0 = 0 # mean of Y_i(0)
sd0 = 1 # standard deviation of Y_i(0)
mu1 = 1 # mean of Y_i(1)
sd1 = 1 # standard deviation of Y_i(1)

# generate a sample
Y0 = stats.norm.rvs(size=n, loc=mu0, scale=sd0)
Y1 = stats.norm.rvs(size=n, loc=mu1, scale=sd1)
tau = Y1 - Y0 # individual treatment effect
# true value of the sample average treatment effect
SATE = tau.mean()
SATE
```

[ ]:
```
1.1921486490063289
```

[ ]:
```python
# repeatedly conduct randomized controlled trials
sims = 5000 # repeat 5,000 times, we could do more
diff_means = np.zeros(sims) # container
sample_vector = np.concatenate((np.ones(int(n/2)), np.zeros(int(n/2))))

for i in range(sims):
    # randomize the treatment by sampling of a vector of 0's and 1's
    treat = np.random.choice(sample_vector, size=n, replace=False)
    # difference-in-means
    diff_means[i] = Y1[treat==1].mean() - Y0[treat==0].mean()
```

```python
# estimation of error for SATE
est_error = diff_means - SATE

est_error.mean()
```

[ ]: 0.00357660333591375

[ ]:
```python
pd.Series(est_error).describe().round(5)
```

[ ]:
```
count    5000.00000
mean        0.00358
std         0.14772
min        -0.55627
25%        -0.09562
50%         0.00149
75%         0.10765
max         0.59405
dtype: float64
```

[ ]:
```python
# PATE simulation
PATE = mu1 - mu0
diff_means = np.zeros(sims)

for i in range(sims):
    # generate a sample for each simulation
    Y0 = stats.norm.rvs(size=n, loc=mu0, scale=sd0)
    Y1 = stats.norm.rvs(size=n, loc=mu1, scale=sd1)
    treat = np.random.choice(sample_vector, size=n, replace=False)
    diff_means[i] = Y1[treat==1].mean() - Y0[treat==0].mean()

# estimation error for PATE
est_error = diff_means - PATE

# unbiased
est_error.mean()
```

[ ]: -0.0004819361838640656

[ ]:
```python
pd.Series(est_error).describe().round(5)
```

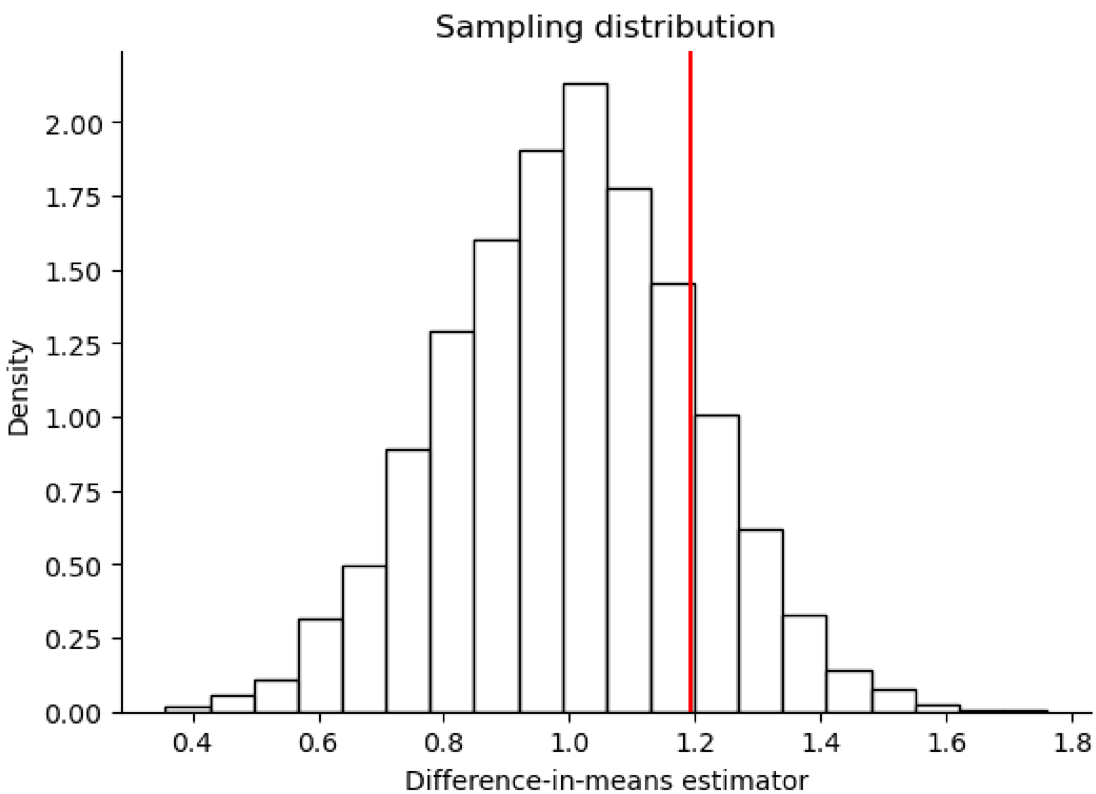
[ ]:
```
count    5000.00000
mean       -0.00048
std         0.19611
min        -0.64294
25%        -0.13539
50%         0.00253
75%         0.13312
max         0.76021
```

dtype: float64

Section 7.1.2: Standard Error

[ ]:
```
sns.displot(
    diff_means, stat='density', color='white', edgecolor='black',
    height=4, aspect=1.5, bins=20
).set(title='Sampling distribution', xlabel='Difference-in-means estimator')

plt.axvline(SATE, color='red') # true value of SATE
```

[ ]: <matplotlib.lines.Line2D at 0x28259406b00>



[ ]:
```
diff_means.std(ddof=1)
```

[ ]: 0.19610767928628656

[ ]:
```
np.sqrt(((diff_means - SATE)**2).mean())
```

[ ]: 0.2748764678567342

[ ]:
```
# PATE simulation with standard error
sims = 5000
diff_means = np.zeros(sims)
se = np.zeros(sims)

for i in range(sims):
    # generate a sample for each simulation
    Y0 = stats.norm.rvs(size=n, loc=mu0, scale=sd0)
    Y1 = stats.norm.rvs(size=n, loc=mu1, scale=sd1)
    # randomize treatment by sampling the vector of 0's and 1's created above
    treat = np.random.choice(sample_vector, size=n, replace=False)
    diff_means[i] = Y1[treat==1].mean() - Y0[treat==0].mean()
    se[i] = (np.sqrt(Y1[treat==1].var(ddof=1) / (n/2) +
                     Y0[treat==0].var(ddof=1) / (n/2)))

diff_means.std(ddof=1)
```

[ ]: 0.2014251388718649

[ ]:
```
se.mean()
```

[ ]: 0.1993032046456901

Section 7.1.3: Confidence Intervals

[ ]:
```
n = 1000 # sample size
x_bar = 0.6 # point estimate
s_e = np.sqrt(x_bar * (1-x_bar) / n) # standard error

# 99% confidence intervals; display as a tuple
((x_bar - stats.norm.ppf(0.995) * s_e).round(5),
 (x_bar + stats.norm.ppf(0.995) * s_e).round(5))
```

[ ]: (0.5601, 0.6399)

[ ]:
```
# 95% confidence intervals
((x_bar - stats.norm.ppf(0.975) * s_e).round(5),
 (x_bar + stats.norm.ppf(0.975) * s_e).round(5))
```

[ ]: (0.56964, 0.63036)

[ ]:
```
# 90% confidence intervals
((x_bar - stats.norm.ppf(0.95) * s_e).round(5),
 (x_bar + stats.norm.ppf(0.95) * s_e).round(5))
```

[ ]: (0.57452, 0.62548)

[ ]:
```
# empty container matrices for 2 sets of confidence intervals
ci95 = np.zeros(sims*2).reshape(sims, 2)
ci90 = np.zeros(sims*2).reshape(sims, 2)

# 95 percent confidence intervals
ci95[:,0] = diff_means - stats.norm.ppf(0.975) * se # lower limit
ci95[:,1] = diff_means + stats.norm.ppf(0.975) * se # upper limit

# 90 percent confidence intervals
ci90[:,0] = diff_means - stats.norm.ppf(0.95) * se # lower limit
ci90[:,1] = diff_means + stats.norm.ppf(0.95) * se # upper limit

# coverage rate for 95% confidence interval
((ci95[:,0] <= 1) & (ci95[:,1] >= 1)).mean()
```

[ ]: 0.9468

[ ]:
```
# coverage rate for 90% confidence interval
((ci90[:,0] <= 1) & (ci90[:,1] >= 1)).mean()
```

[ ]: 0.897

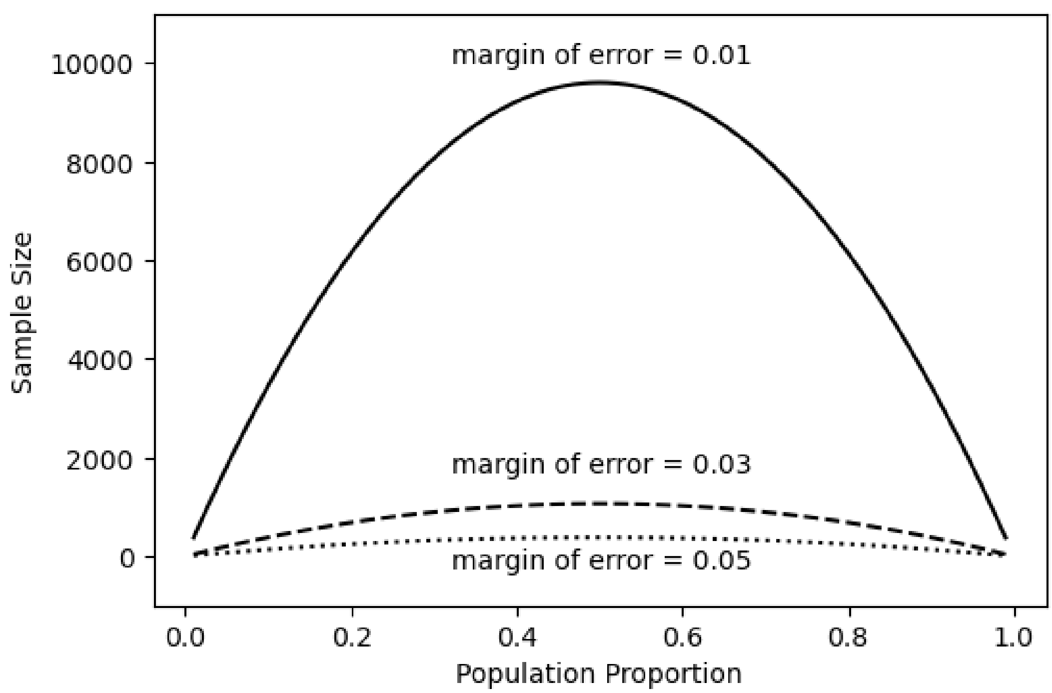
[ ]:
```
p = 0.6 # true parameter value
n = np.array([50, 100, 1000]) # 3 sample sizes to be examined
alpha = 0.05
sims = 5000 # number of simulations
results = np.zeros(len(n)) # a container for results

for i in range(len(n)):
    ci_results = np.zeros(sims) # a container for whether CI contains truth
    # loop for repeated hypothetical survey sampling
    for j in range(sims):
        data = stats.binom.rvs(n=1, p=p, size=n[i]) # simple random sampling
        x_bar = data.mean() # sample proportion as an estimate
        s_e = np.sqrt(x_bar * (1-x_bar) / n[i]) # standard errors
        ci_lower = x_bar - stats.norm.ppf(1-alpha/2) * s_e
        ci_upper = x_bar + stats.norm.ppf(1-alpha/2) * s_e
        ci_results[j] = (p >= ci_lower) & (p <= ci_upper)
    # proportion of CIs that contain the true value
    results[i] = ci_results.mean()

results
```

[ ]: array([0.941 , 0.9546, 0.9498])

Section 7.1.4: Margin of Error and Sample Size Calculation in Polls

[ ]:
```
MoE = np.array([0.01, 0.03, 0.05]) # the desired margin of error
p = np.arange(0.01, 1, 0.01)
n = 1.96**2 * p * (1-p) / MoE[0]**2
n2 = 1.96**2 * p * (1-p) / MoE[1]**2
n3 = 1.96**2 * p * (1-p) / MoE[2]**2

fig, ax = plt.subplots(figsize=(6,4))

sns.lineplot(x=p, y=n, ax=ax, color='black').set(
    ylim=(-1000, 11000), xlabel='Population Proportion', ylabel='Sample Size'
)

sns.lineplot(x=p, y=n2, ax=ax, color='black', linestyle='--')

sns.lineplot(x=p, y=n3, ax=ax, color='black', linestyle=':')

# Add text labels
ax.text(0.32, 10000, 'margin of error = 0.01', fontsize=10)
ax.text(0.32, 1700, 'margin of error = 0.03', fontsize=10)
ax.text(0.32, -250, 'margin of error = 0.05', fontsize=10)
```

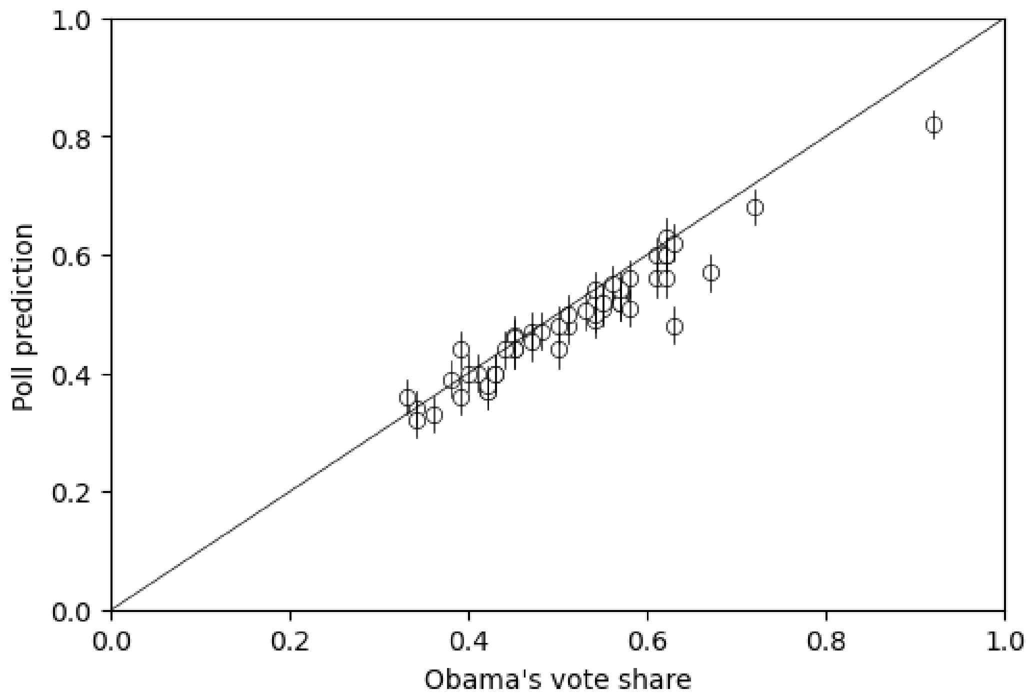
[ ]:
```
Text(0.32, -250, 'margin of error = 0.05')
```



[ ]:
```python
# election and polling results, by state
pres08 = pd.read_csv('pres08.csv')
polls08 = pd.read_csv('polls08.csv')

# convert to a date object
polls08['middate'] = pd.to_datetime(polls08['middate'])

# number of days to the election
from datetime import datetime
election_day = datetime.strptime('2008-11-04', '%Y-%m-%d')
polls08['days_to_election'] = (election_day - polls08['middate']).dt.days

# extract unique state names which the loop will iterate through
st_names = polls08['state'].unique()

# create an empty 51 X 3 placeholder Data Frame
poll_pred = pd.DataFrame(np.zeros(51*3).reshape(51, 3), index=st_names)

# loop across the 50 states plus DC
for i in range(len(st_names)):
    # subset the ith state
    state_data = polls08[polls08['state']==st_names[i]]
    # further subset the latest polls within the state
    latest = (state_data['days_to_election']==
              state_data['days_to_election'].min())
    # compute the mean of the latest polls and store it
    poll_pred.iloc[i, 0] = state_data['Obama'][latest].mean() / 100

# upper and lower confidence limits
n = 1000 # sample size
alpha = 0.05
se = np.sqrt(poll_pred.iloc[:,0] * (1-poll_pred.iloc[:,0]) / n) # standard error
poll_pred.iloc[:,1] = poll_pred.iloc[:,0] - stats.norm.ppf(1-alpha/2) * se
poll_pred.iloc[:,2] = poll_pred.iloc[:,0] + stats.norm.ppf(1-alpha/2) * se
```

[ ]:
```python
# plot the results
fig, ax = plt.subplots(figsize=(6,4))

sns.scatterplot(
    x = pres08['Obama'] / 100, y = poll_pred.iloc[:,0].reset_index(drop=True),
    ax=ax, color='white', edgecolor='black'
).set(xlabel="Obama's vote share", ylabel='Poll prediction',
      xlim=(0, 1), ylim=(0, 1))

ax.axline((0, 0), slope=1, color='black', linewidth=0.5)

# adding 95% confidence intervals for each state
```

```
for i in range(len(st_names)):
    ax.plot(
        [pres08['Obama'][i] / 100] * 2,
        [poll_pred.iloc[i,1], poll_pred.iloc[i,2]],
        color='black', linewidth=0.5
    )
```



[ ]:
```
# proportion of confidence intervals that contain the election day outcome
# reset index: can only compare identically-labeled Series objects
((poll_pred.iloc[:,1].reset_index(drop=True) <= pres08['Obama'] / 100 ) &
 (poll_pred.iloc[:,2].reset_index(drop=True) >= pres08['Obama'] / 100)).mean()
```

[ ]: 0.5882352941176471

[ ]:
```
# bias
bias=(poll_pred.iloc[:,0].reset_index(drop=True) - pres08['Obama']/100).mean()
bias
```

[ ]: -0.026797385620915028

[ ]:
```
# bias corrected estimate
poll_bias = poll_pred.iloc[:,0] - bias

# bias corrected standard error
```

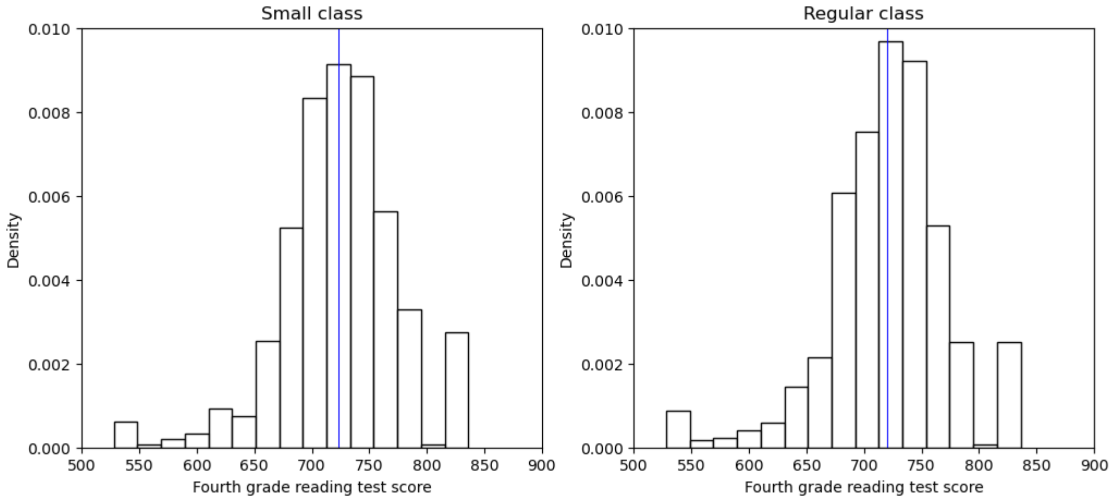
```
se_bias = np.sqrt(poll_bias * (1-poll_bias) / n)

# bias corrected confidence intervals
ci_bias_lower = poll_bias - stats.norm.ppf(1-alpha/2) * se_bias
ci_bias_upper = poll_bias + stats.norm.ppf(1-alpha/2) * se_bias

# proportion of bias corrected CIs that contain election day outcome
((ci_bias_lower.reset_index(drop=True) <= pres08['Obama'] / 100) &
 (ci_bias_upper.reset_index(drop=True) >= pres08['Obama'] / 100)).mean()
```

[ ]: 0.7647058823529411

Section 7.1.5: Analysis of Randomized Controlled Trials

[ ]:
```
STAR = pd.read_csv('STAR.csv')

fig, axs = plt.subplots(1, 2, figsize=(12,5))

sns.histplot(
    STAR['g4reading'][STAR.classtype==1], stat = 'density', ax=axs[0],
    color='white', edgecolor='black', bins=15
).set(ylim=(0, 0.01), xlim=(500, 900), title='Small class',
      xlabel='Fourth grade reading test score')

axs[0].axvline(STAR['g4reading'][STAR.classtype==1].mean(),
               color='blue', linewidth=0.75)

sns.histplot(
    STAR['g4reading'][STAR.classtype==2], stat = 'density', ax=axs[1],
    color='white', edgecolor='black', bins=15
).set(ylim=(0, 0.01), xlim=(500, 900), title='Regular class',
      xlabel='Fourth grade reading test score')

axs[1].axvline(STAR['g4reading'][STAR.classtype==2].mean(),
                color='blue', linewidth=0.75)
```

[ ]: <matplotlib.lines.Line2D at 0x2825ae82020>




```
[ ]: # estimate and standard error for small class size
     n_small = (STAR['classtype']==1 & STAR['g4reading'].notnull()).sum()
     est_small = STAR['g4reading'][STAR.classtype==1].mean()
     se_small = STAR['g4reading'][STAR.classtype==1].std() / np.sqrt(n_small)
     est_small, se_small
```

```
[ ]: (723.3911845730028, 1.9130122952458233)
```

```
[ ]: # estimate and standard error for regular class size
     n_regular = ((STAR['classtype']==2) &
                  (STAR['classtype'].notnull()) &
                  (STAR['g4reading'].notnull())).sum()
     est_regular = STAR['g4reading'][STAR.classtype==2].mean()
     se_regular = STAR['g4reading'][STAR.classtype==2].std() / np.sqrt(n_regular)
     est_regular, se_regular
```

```
[ ]: (719.88995215311, 1.8388496908502467)
```

```
[ ]: alpha = 0.05

     # 95% confidence intervals for small class size
     ci_small = (est_small - stats.norm.ppf(1-alpha/2) * se_small,
                 est_small + stats.norm.ppf(1-alpha/2) * se_small)
     ci_small
```

```
[ ]: (719.6417493723386, 727.1406197736669)
```

```
[ ]: # 95% confidence intervals for regular class size
     ci_regular = (est_regular - stats.norm.ppf(1-alpha/2) * se_regular,
                   est_regular + stats.norm.ppf(1-alpha/2) * se_regular)
```

```
ci_regular
```

[ ]: (716.2858729860609, 723.4940313201591)

```
# difference in means estimator
ate_est = est_small - est_regular
ate_est
```

[ ]: 3.5012324198927445

```
# standard error and 95% confidence interval
ate_se = np.sqrt(se_small**2 + se_regular**2)
ate_se
```

[ ]: 2.653485298112982

```
ate_ci = (ate_est - stats.norm.ppf(1-alpha/2) * ate_se,
          ate_est + stats.norm.ppf(1-alpha/2) * ate_se)
ate_ci
```

[ ]: (-1.699503197915229, 8.701968037700718)

Section 7.1.6: Analysis Based on Student's t-Distribution

In Progress