Python Code for QSS Chapter 6: Probability

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First Printing

[ ]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
from math import comb, exp, factorial, log
```

Section 6.1: Probability

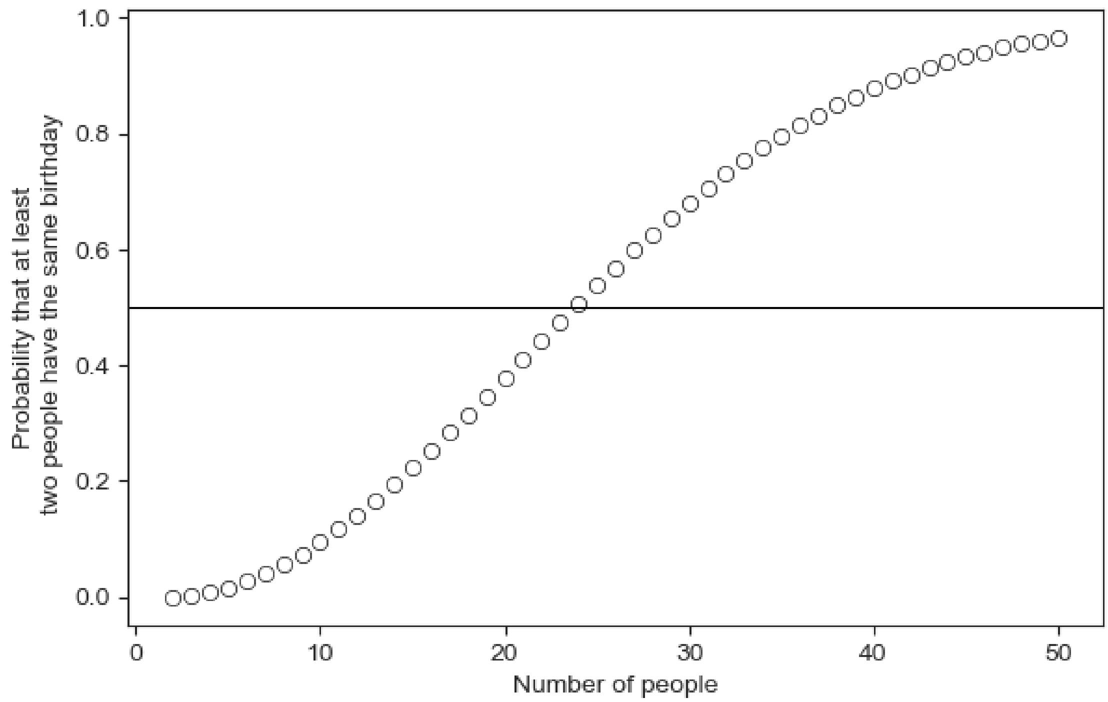
Section 6.1.1: Frequentist versus Bayesian

Section 6.1.2: Definition and Axioms

Section 6.1.3: Permutations

[ ]:
```
def birthday(k):
    logdenom = k * log(365) + log(factorial(365 - k)) # log denominator
    lognumer = log(factorial(365)) # log numerator
    # P(at least two have the same bday) = 1 - P(nobody has the same bday)
    pr = 1 - exp(lognumer - logdenom) # transform back
    return pr

k = pd.Series(np.arange(1, 51))

bday = k.apply(birthday) # apply the function to each element of k

bday.index = k # add labels

sns.set_style('ticks')

sns.relplot(
    x=k, y=bday, color='white', edgecolor='black', height=4, aspect=1.5
).set(ylabel='Probability that at least\n two people have the same birthday',
      xlabel='Number of people').despine(right=False, top=False)

# horizontal line at 0.5
plt.axhline(0.5, color='black', linewidth=0.75)
```

[ ]:
```
<matplotlib.lines.Line2D at 0x29fb5e96ec0>
```



[ ]:
```
bday.loc[20:25]
```

[ ]:
```
20    0.411438
21    0.443688
22    0.475695
23    0.507297
24    0.538344
25    0.568700
dtype: float64
```

Section 6.1.4: Sampling With and Without Replacement

[ ]:
```
k = 23 # number of people
sims = 10000 # number of simulations
event = 0 # initialize counter

for i in range(sims):
    days = np.random.choice(np.arange(1,366), size=k, replace=True)
    days_unique = np.unique(days) # number of unique days
    '''
    if there are duplicates, the number of unique birthdays will be less than
    the number of birthdays, which is 'k'
    '''
    if len(days_unique) < len(days):
```

```
        event += 1

answer = event / sims
answer
```

[ ]: 0.5081

Section 6.1.5: Combinations

[ ]:
```
comb(84, 6)
```

[ ]: 406481544

Section 6.2: Conditional Probability

Section 6.2.1: Conditional, Marginal, and Joint Probabilities

[ ]:
```
FLVoters = pd.read_csv('FLVoters.csv')

FLVoters.shape # before removal of missing data
```

[ ]: (10000, 6)

[ ]:
```
FLVoters.info() # there is one missing surname
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 10000 entries, 0 to 9999
Data columns (total 6 columns):
 #   Column   Non-Null Count  Dtype
---  ------   --------------  -----
 0   surname  9999 non-null   object
 1   county   10000 non-null  int64
 2   VTD      10000 non-null  int64
 3   age      9992 non-null   float64
 4   gender   9992 non-null   object
 5   race     9126 non-null   object
dtypes: float64(1), int64(2), object(3)
memory usage: 468.9+ KB
```

[ ]:
```
# print the record with the missing surname
FLVoters[FLVoters['surname'].isnull()]
```

[ ]:

	surname	county	VTD	age	gender	race
349	NaN	5	14	70.0	f	white

Looking at the raw data, it turns out that one voter's surname is Null. Pandas treated the name as missing. We need to override this behavior and treat Ms. Null's name as a string.

[ ]:
```
FLVoters.head() # the surnames are in all caps
```

```
[ ]:   surname  county  VTD   age gender   race
    0   PIEDRA     115   66  58.0      f  white
    1    LYNCH     115   13  51.0      m  white
    2  CHESTER     115  103  63.0      m    NaN
    3  LATHROP     115   80  54.0      m  white
    4   HUMMEL     115    8  77.0      f  white
```

```
[ ]: FLVoters['surname'] = np.where(
         FLVoters['surname'].isnull(), 'NULL', FLVoters['surname'])

     FLVoters = FLVoters.dropna()

     FLVoters.shape # after removal of missing data
```

```
[ ]: (9113, 6)
```

```
[ ]: margin_race = FLVoters['race'].value_counts(normalize=True).sort_index()

     margin_race
```

```
[ ]: race
     asian       0.019203
     black       0.131022
     hispanic    0.130802
     native      0.003182
     other       0.034017
     white       0.681773
     Name: proportion, dtype: float64
```

```
[ ]: margin_gender = FLVoters['gender'].value_counts(normalize=True)

     margin_gender
```

```
[ ]: gender
     f    0.535828
     m    0.464172
     Name: proportion, dtype: float64
```

```
[ ]: FLVoters['race'][FLVoters.gender == 'f'].value_counts(
         normalize=True).sort_index()
```

```
[ ]: race
     asian       0.016998
     black       0.138849
     hispanic    0.136392
     native      0.003481
     other       0.032357
     white       0.671923
```

```
Name: proportion, dtype: float64
```

[ ]:
```python
joint_p = pd.crosstab(FLVoters.race, FLVoters.gender, normalize=True)

joint_p
```

[ ]:
```
gender           f         m
race
asian     0.009108  0.010095
black     0.074399  0.056622
hispanic  0.073082  0.057720
native    0.001865  0.001317
other     0.017338  0.016679
white     0.360035  0.321738
```

To obtain the row sums in pandas, we specify `axis='columns'` in the `.sum()` method. This may seem counterintuitive, but the logic is that we need to collapse the columns to calculate the sum of each row.

[ ]:
```python
# row sums
joint_p.sum(axis='columns')
```

[ ]:
```
race
asian       0.019203
black       0.131022
hispanic    0.130802
native      0.003182
other       0.034017
white       0.681773
dtype: float64
```

[ ]:
```python
# column sums
joint_p.sum(axis='rows')
```

[ ]:
```
gender
f    0.535828
m    0.464172
dtype: float64
```

[ ]:
```python
# Develop age group categories; start with a list of n-1 conditions
conditions = [
      (FLVoters.age <= 20)
    , (FLVoters.age > 20) & (FLVoters.age <= 40)
    , (FLVoters.age > 40) & (FLVoters.age <= 60)
]

choices  = [1, 2, 3]
```

```python
# Assign 4 to voters older than 60
FLVoters["age_group"] = np.select(conditions, choices, 4)

joint3 = pd.crosstab([FLVoters.race, FLVoters.age_group], FLVoters.gender,
                     normalize=True)

# print the first 8 rows
joint3.head(8)
```

[ ]:
```
gender                 f         m
race  age_group
asian 1          0.000110  0.000219
      2          0.002634  0.002853
      3          0.004170  0.005157
      4          0.002195  0.001865
black 1          0.001646  0.001646
      2          0.028092  0.022825
      3          0.025787  0.018984
      4          0.018874  0.013168
```

[ ]:
```python
# marginal probabilities for age groups
margin_age = FLVoters['age_group'].value_counts(normalize=True).sort_index()

margin_age
```

[ ]:
```
age_group
1    0.017667
2    0.270932
3    0.360474
4    0.350927
Name: proportion, dtype: float64
```

[ ]:
```python
# take a look at the joint3 index for a few observations
joint3.index[:3]
```

[ ]:
```
MultiIndex([('asian', 1),
            ('asian', 2),
            ('asian', 3)],
           names=['race', 'age_group'])
```

[ ]:
```python
# select elements from a multi-index using .loc and tuples
joint3.loc[('asian', 3), 'f']
```

[ ]:
```
0.004169867222648963
```

[ ]:
```python
# P(black and female | above 60)
joint3.loc[('black', 4), 'f'] / margin_age[4]
```

[ ]: 0.05378361475922452

[ ]:
```python
# two-way joint probability table for age group and gender
joint2 = pd.crosstab(FLVoters['age_group'], FLVoters['gender'],
                     normalize=True)

joint2
```

[ ]:
```
gender            f         m
age_group
1          0.009657  0.008011
2          0.143092  0.127839
3          0.189839  0.170635
4          0.193240  0.157687
```

[ ]:
```python
# P(above 60 and female)
joint2.loc[4, 'f']
```

[ ]: 0.1932404257653901

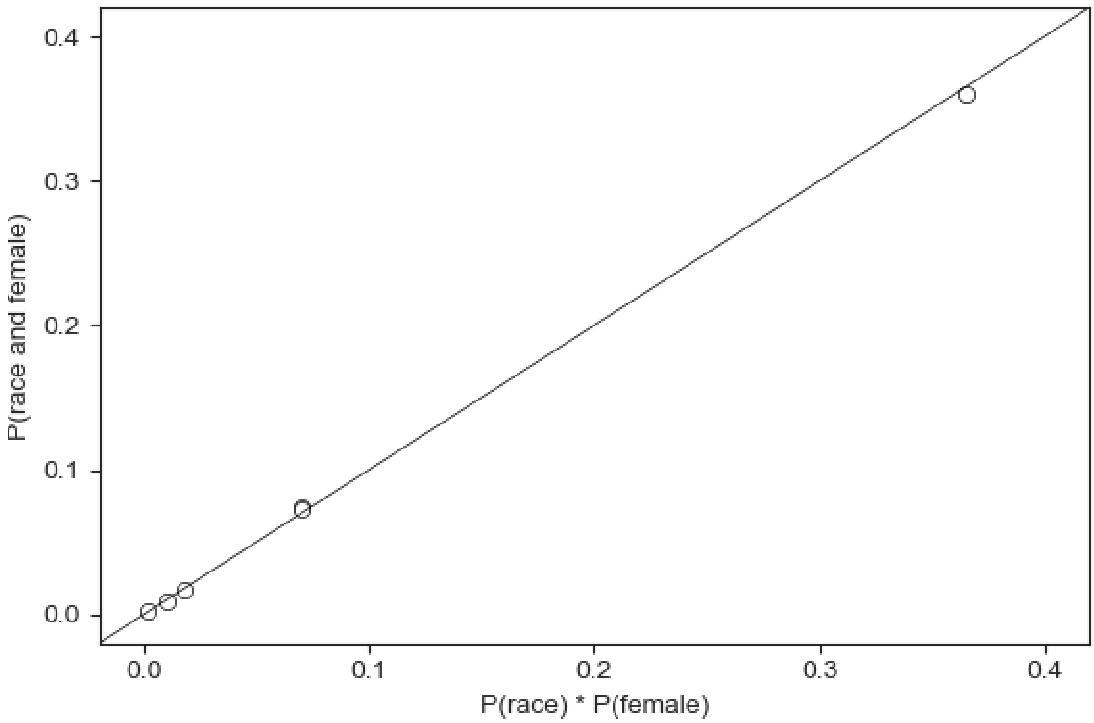
[ ]:
```python
# P(black | female and above 60)
joint3.loc[('black', 4), 'f'] / joint2.loc[4, 'f']
```

[ ]: 0.097671777399205

Section 6.2.2: Independence

[ ]:
```python
# store plotting parameters
lims = (-0.02, 0.42)
ticks = [0, .1, .2, .3, .4]

sns.relplot(
    x=margin_race * margin_gender['f'], y=joint_p['f'],
    color='white', edgecolor='black', height=4, aspect=1.5
).set(xlabel='P(race) * P(female)', ylabel='P(race and female)',
      xlim=lims, ylim=lims, xticks=ticks, yticks=ticks).despine(
          right=False, top=False)

plt.gca().axline((0, 0), slope=1, color='black', linewidth=0.5)
```

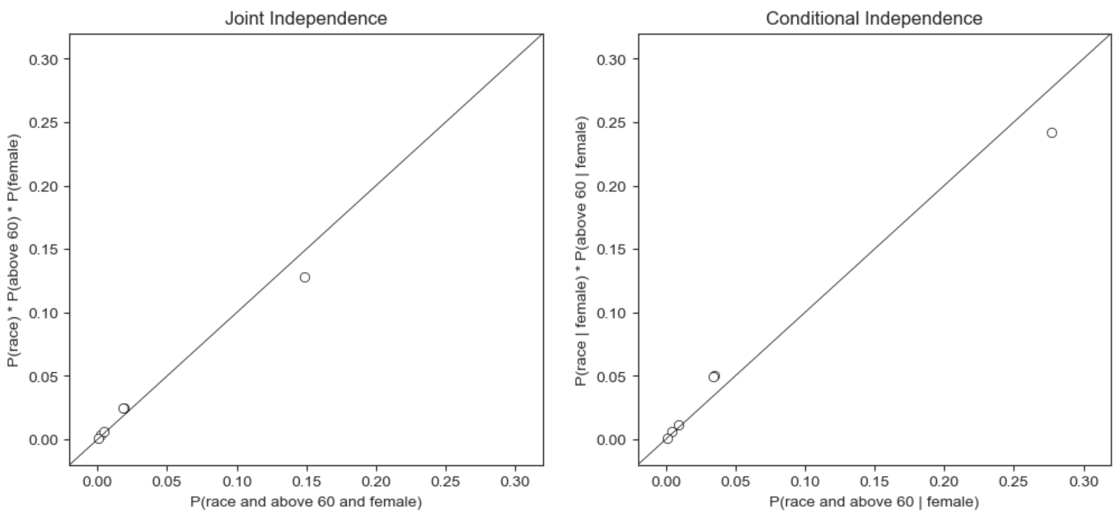
[ ]: <matplotlib.lines._AxLine at 0x29fb8cdef50>



[ ]:
```
# subplots for joint and conditional independence
fig, axs = plt.subplots(1, 2, figsize=(12, 5))

lims = (-0.02, 0.32)

# joint independence
sns.scatterplot(
    x=joint3.loc[(slice(None), 4), 'f'].droplevel('age_group'),
    y=margin_race * margin_age[4] * margin_gender['f'],
    color='white', edgecolor='black', ax=axs[0]
).set(xlabel='P(race and above 60 and female)',
      ylabel='P(race) * P(above 60) * P(female)',
      title='Joint Independence', xlim=lims, ylim=lims)

axs[0].axline((0, 0), slope=1, color='black', linewidth=0.5)

# conditional independence given female
sns.scatterplot(
    x=(joint3.loc[(slice(None), 4), 'f'] /
       margin_gender['f']).droplevel('age_group'),
    y=((joint_p['f'] / margin_gender['f']) *
       (joint2.loc[4, 'f'] / margin_gender['f'])),
    color='white', edgecolor='black', ax=axs[1]
```

```
).set(xlabel='P(race and above 60 | female)',
      ylabel='P(race | female) * P(above 60 | female)',
      title='Conditional Independence', xlim=lims, ylim=lims)

axs[1].axline((0, 0), slope=1, color='black', linewidth=0.5)
```

[ ]: <matplotlib.lines._AxLine at 0x29fb8b5f130>




```
# Monty Hall problem
sims = 1000
doors = np.array(['goat', 'goat', 'car'])
# Store empty vector of strings with same dtype as doors
result_switch = np.empty(sims, dtype=doors.dtype)
result_noswitch = np.empty(sims, dtype=doors.dtype)

for i in range(sims):
    # randomly choose the initial door
    first = np.random.choice(np.arange(0,3))
    result_noswitch[i] = doors[first]
    remain = np.delete(doors, first) # remaining two doors
    if doors[first] == 'car': # two goats left
        monty = np.random.choice(np.arange(0,2))
    else: # one goat and one car left
        monty = np.arange(0,2)[remain=='goat']
    result_switch[i] = np.delete(remain, monty)[0]

(result_noswitch == 'car').mean()
```

[ ]: 0.369

[ ]:
```
(result_switch == 'car').mean()
```

[ ]: 0.631

Section 6.2.3: Bayes' Rule

Section 6.2.4: Predicting Race Using Surname and Residence Location

In Progress