Python Code for QSS Chapter 6: Probability

Kosuke Imai, Python code by Jeff Allen

First Printing

[ ]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
from math import comb, exp, factorial, log
```

Section 6.1: Probability

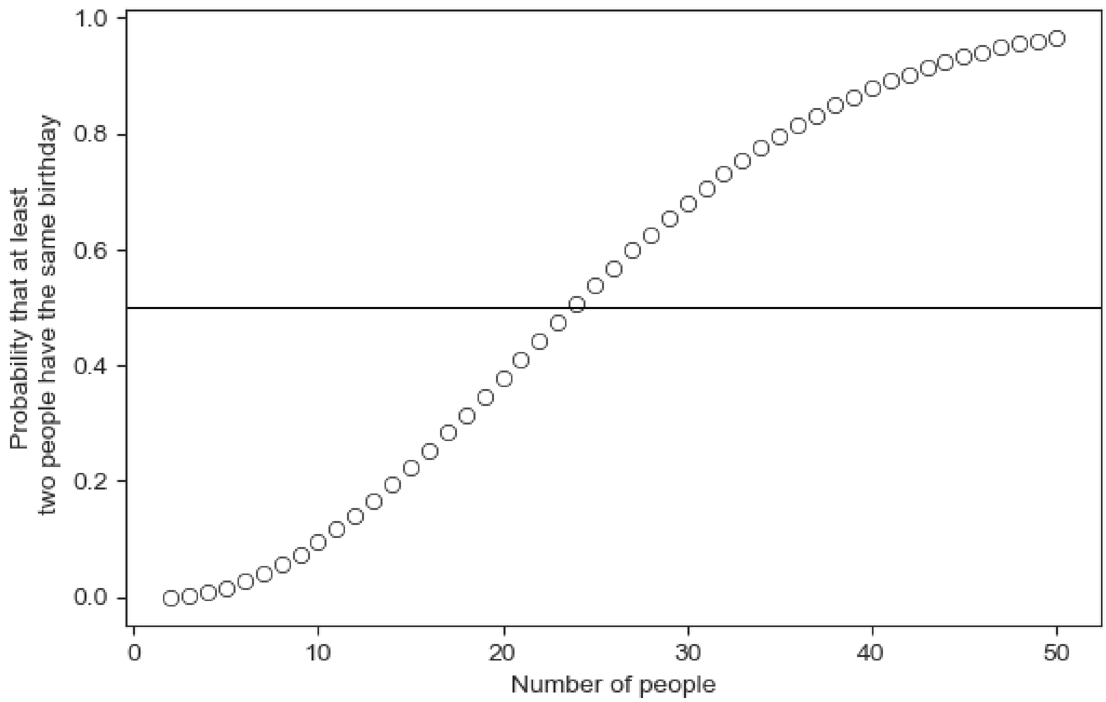
Section 6.1.1: Frequentist versus Bayesian

Section 6.1.2: Definition and Axioms

Section 6.1.3: Permutations

[ ]:
```
def birthday(k):
    logdenom = k * log(365) + log(factorial(365 - k)) # log denominator
    lognumer = log(factorial(365)) # log numerator
    # P(at least two have the same bday) = 1 - P(nobody has the same bday)
    pr = 1 - exp(lognumer - logdenom) # transform back
    return pr

k = pd.Series(np.arange(1, 51))

bday = k.apply(birthday) # apply the function to each element of k

bday.index = k # add labels

sns.set_style('ticks')

sns.relplot(
    x=k, y=bday, color='white', edgecolor='black', height=4, aspect=1.5
).set(ylabel='Probability that at least\n two people have the same birthday',
      xlabel='Number of people').despine(right=False, top=False)

# horizontal line at 0.5
plt.axhline(0.5, color='black', linewidth=0.75)
```

[ ]:
```
<matplotlib.lines.Line2D at 0x2bd5dc7eef0>
```




```
[ ]: bday.loc[20:25]
```

```
[ ]: 20    0.411438
     21    0.443688
     22    0.475695
     23    0.507297
     24    0.538344
     25    0.568700
     dtype: float64
```

Section 6.1.4: Sampling With and Without Replacement

```
[ ]: k = 23 # number of people
     sims = 10000 # number of simulations
     event = 0 # initialize counter

     for i in range(sims):
         days = np.random.choice(np.arange(1,366), size=k, replace=True)
         days_unique = np.unique(days) # number of unique days
         '''
         if there are duplicates, the number of unique birthdays will be less than
         the number of birthdays, which is 'k'
         '''
         if len(days_unique) < len(days):
```

```
        event += 1

answer = event / sims
answer
```

[ ]: 0.506

Section 6.1.5: Combinations

[ ]:
```
comb(84, 6)
```

[ ]: 406481544

Section 6.2: Conditional Probability

Section 6.2.1: Conditional, Marginal, and Joint Probabilities

[ ]:
```
FLVoters = pd.read_csv('FLVoters.csv')

FLVoters.shape # before removal of missing data
```

[ ]: (10000, 6)

[ ]:
```
FLVoters.info() # there is one missing surname
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 10000 entries, 0 to 9999
Data columns (total 6 columns):
 #   Column   Non-Null Count  Dtype
---  ------   --------------  -----
 0   surname  9999 non-null   object
 1   county   10000 non-null  int64
 2   VTD      10000 non-null  int64
 3   age      9992 non-null   float64
 4   gender   9992 non-null   object
 5   race     9126 non-null   object
dtypes: float64(1), int64(2), object(3)
memory usage: 468.9+ KB
```

[ ]:
```
# print the record with the missing surname
FLVoters[FLVoters['surname'].isnull()]
```

[ ]:

	surname	county	VTD	age	gender	race
349	NaN	5	14	70.0	f	white

Looking at the raw data, it turns out that one voter's surname is Null. Pandas treated the name as missing. We need to override this behavior and treat Ms. Null's name as a string.

[ ]:
```
FLVoters.head() # the surnames are in all caps
```

```
[ ]:    surname  county  VTD   age gender   race
     0   PIEDRA     115   66  58.0      f  white
     1    LYNCH     115   13  51.0      m  white
     2  CHESTER     115  103  63.0      m    NaN
     3  LATHROP     115   80  54.0      m  white
     4   HUMMEL     115    8  77.0      f  white
```

```
[ ]: FLVoters['surname'] = np.where(
         FLVoters['surname'].isnull(), 'NULL', FLVoters['surname'])

     FLVoters = FLVoters.dropna()

     FLVoters.shape # after removal of missing data
```

```
[ ]: (9113, 6)
```

```
[ ]: margin_race = FLVoters['race'].value_counts(normalize=True).sort_index()

     margin_race
```

```
[ ]: race
     asian       0.019203
     black       0.131022
     hispanic    0.130802
     native      0.003182
     other       0.034017
     white       0.681773
     Name: proportion, dtype: float64
```

```
[ ]: margin_gender = FLVoters['gender'].value_counts(normalize=True)

     margin_gender
```

```
[ ]: gender
     f    0.535828
     m    0.464172
     Name: proportion, dtype: float64
```

```
[ ]: FLVoters['race'][FLVoters.gender == 'f'].value_counts(
         normalize=True).sort_index()
```

```
[ ]: race
     asian       0.016998
     black       0.138849
     hispanic    0.136392
     native      0.003481
     other       0.032357
     white       0.671923
```

```
Name: proportion, dtype: float64
```

```
[ ]: joint_p = pd.crosstab(FLVoters.race, FLVoters.gender, normalize=True)

     joint_p
```

```
[ ]: gender           f         m
     race
     asian     0.009108  0.010095
     black     0.074399  0.056622
     hispanic  0.073082  0.057720
     native    0.001865  0.001317
     other     0.017338  0.016679
     white     0.360035  0.321738
```

To obtain the row sums in pandas, we specify `axis='columns'` in the `.sum()` method. This may seem counterintuitive, but the logic is that we need to collapse the columns to calculate the sum of each row.

```
[ ]: # row sums
     joint_p.sum(axis='columns')
```

```
[ ]: race
     asian       0.019203
     black       0.131022
     hispanic    0.130802
     native      0.003182
     other       0.034017
     white       0.681773
     dtype: float64
```

```
[ ]: # column sums
     joint_p.sum(axis='rows')
```

```
[ ]: gender
     f    0.535828
     m    0.464172
     dtype: float64
```

```
[ ]: # Develop age group categories; start with a list of n-1 conditions
     conditions = [
           (FLVoters.age <= 20)
         , (FLVoters.age > 20) & (FLVoters.age <= 40)
         , (FLVoters.age > 40) & (FLVoters.age <= 60)
     ]

     choices  = [1, 2, 3]
```

```python
# Assign 4 to voters older than 60
FLVoters["age_group"] = np.select(conditions, choices, 4)

joint3 = pd.crosstab([FLVoters.race, FLVoters.age_group], FLVoters.gender,
                     normalize=True)

# print the first 8 rows
joint3.head(8)
```

[ ]:
```
gender                    f         m
race  age_group
asian 1           0.000110  0.000219
      2           0.002634  0.002853
      3           0.004170  0.005157
      4           0.002195  0.001865
black 1           0.001646  0.001646
      2           0.028092  0.022825
      3           0.025787  0.018984
      4           0.018874  0.013168
```

[ ]:
```python
# marginal probabilities for age groups
margin_age = FLVoters['age_group'].value_counts(normalize=True).sort_index()

margin_age
```

[ ]:
```
age_group
1    0.017667
2    0.270932
3    0.360474
4    0.350927
Name: proportion, dtype: float64
```

[ ]:
```python
# take a look at the joint3 index for a few observations
joint3.index[:3]
```

[ ]:
```
MultiIndex([('asian', 1),
            ('asian', 2),
            ('asian', 3)],
           names=['race', 'age_group'])
```

[ ]:
```python
# select elements from a multi-index using .loc and tuples
joint3.loc[('asian', 3), 'f']
```

[ ]:
```
0.004169867222648963
```

[ ]:
```python
# P(black and female | above 60)
joint3.loc[('black', 4), 'f'] / margin_age[4]
```

[ ]: 0.05378361475922452

[ ]:
```
# two-way joint probability table for age group and gender
joint2 = pd.crosstab(FLVoters['age_group'], FLVoters['gender'],
                     normalize=True)

joint2
```

[ ]:
```
gender            f         m
age_group
1          0.009657  0.008011
2          0.143092  0.127839
3          0.189839  0.170635
4          0.193240  0.157687
```

[ ]:
```
# P(above 60 and female)
joint2.loc[4, 'f']
```

[ ]: 0.1932404257653901

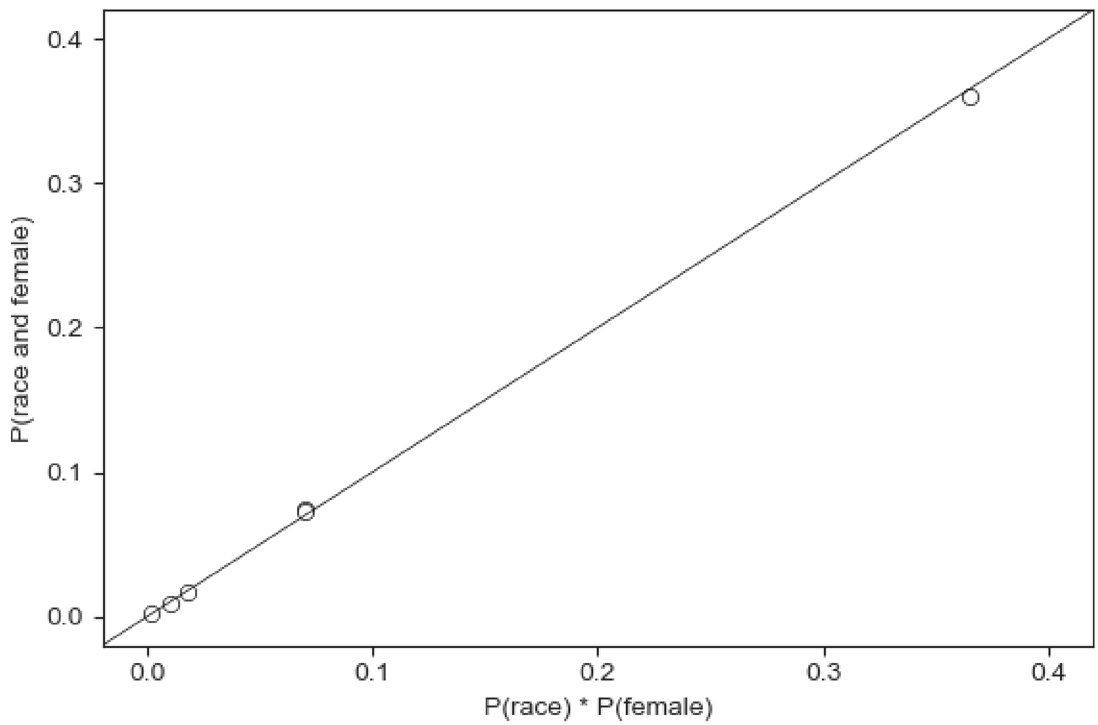
[ ]:
```
# P(black | female and above 60)
joint3.loc[('black', 4), 'f'] / joint2.loc[4, 'f']
```

[ ]: 0.097671777399205

Section 6.2.2: Independence

[ ]:
```
# store plotting parameters
lims = (-0.02, 0.42)
ticks = [0, .1, .2, .3, .4]

sns.relplot(
    x=margin_race * margin_gender['f'], y=joint_p['f'],
    color='white', edgecolor='black', height=4, aspect=1.5
).set(xlabel='P(race) * P(female)', ylabel='P(race and female)',
      xlim=lims, ylim=lims, xticks=ticks, yticks=ticks).despine(
          right=False, top=False)

plt.gca().axline((0, 0), slope=1, color='black', linewidth=0.5)
```

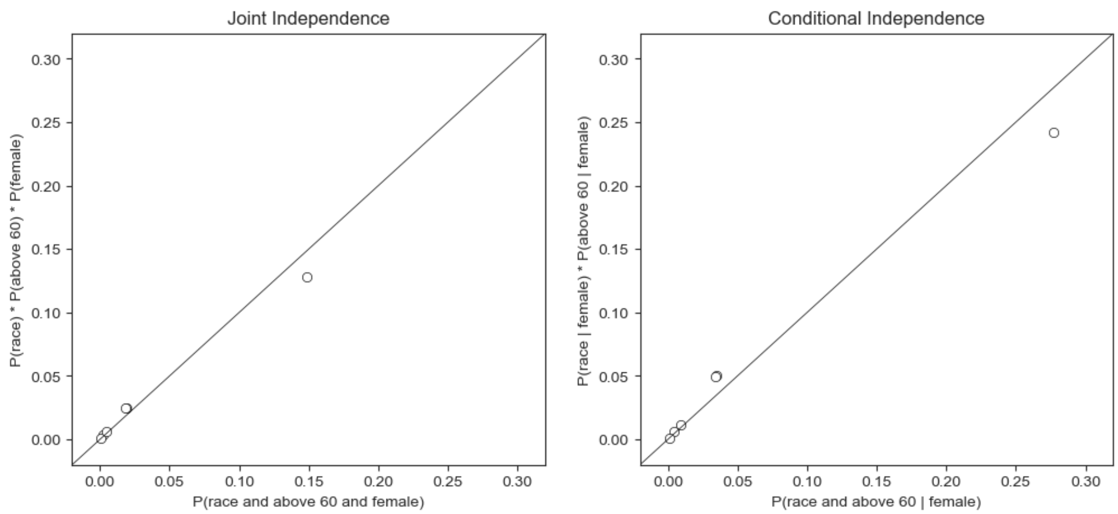
[ ]: <matplotlib.lines._AxLine at 0x2bd6089e470>



[ ]:
```
# subplots for joint and conditional independence
fig, axs = plt.subplots(1, 2, figsize=(12, 5))

lims = (-0.02, 0.32)

# joint independence
sns.scatterplot(
    x=joint3.loc[(slice(None), 4), 'f'].droplevel('age_group'),
    y=margin_race * margin_age[4] * margin_gender['f'],
    color='white', edgecolor='black', ax=axs[0]
).set(xlabel='P(race and above 60 and female)',
      ylabel='P(race) * P(above 60) * P(female)',
      title='Joint Independence', xlim=lims, ylim=lims)

axs[0].axline((0, 0), slope=1, color='black', linewidth=0.5)

# conditional independence given female
sns.scatterplot(
    x=(joint3.loc[(slice(None), 4), 'f'] /
       margin_gender['f']).droplevel('age_group'),
    y=((joint_p['f'] / margin_gender['f']) *
       (joint2.loc[4, 'f'] / margin_gender['f'])),
    color='white', edgecolor='black', ax=axs[1]
```

```
).set(xlabel='P(race and above 60 | female)',
      ylabel='P(race | female) * P(above 60 | female)',
      title='Conditional Independence', xlim=lims, ylim=lims)

axs[1].axline((0, 0), slope=1, color='black', linewidth=0.5)
```

[ ]: <matplotlib.lines._AxLine at 0x2bd6093fe50>



[ ]:
```
# Monty Hall problem
sims = 1000
doors = np.array(['goat', 'goat', 'car'])
# Store empty vector of strings with same dtype as doors
result_switch = np.empty(sims, dtype=doors.dtype)
result_noswitch = np.empty(sims, dtype=doors.dtype)

for i in range(sims):
    # randomly choose the initial door
    first = np.random.choice(np.arange(0,3))
    result_noswitch[i] = doors[first]
    remain = np.delete(doors, first) # remaining two doors
    if doors[first] == 'car': # two goats left
        monty = np.random.choice(np.arange(0,2))
    else: # one goat and one car left
        monty = np.arange(0,2)[remain=='goat']
    result_switch[i] = np.delete(remain, monty)[0]

(result_noswitch == 'car').mean()
```

[ ]: 0.327

[ ]:
```python
(result_switch == 'car').mean()
```

[ ]: 0.673

Section 6.2.3: Bayes' Rule

Section 6.2.4: Predicting Race Using Surname and Residence Location

[ ]:
```python
cnames = pd.read_csv('names.csv')

cnames.info() # one surname is missing
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 151671 entries, 0 to 151670
Data columns (total 7 columns):
 #   Column       Non-Null Count   Dtype
---  ------       --------------   -----
 0   surname      151670 non-null  object
 1   count        151671 non-null  int64
 2   pctwhite     151671 non-null  float64
 3   pctblack     151671 non-null  float64
 4   pctapi       151671 non-null  float64
 5   pcthispanic  151671 non-null  float64
 6   pctothers    151671 non-null  float64
dtypes: float64(5), int64(1), object(1)
memory usage: 8.1+ MB
```

[ ]:
```python
# As with FLVoters, ensure the surname "NULL" is treated as a string
cnames['surname'] = np.where(
    cnames['surname'].isnull(), 'NULL', cnames['surname'])

cnames.shape
```

[ ]: (151671, 7)

[ ]:
```python
# merge the two data frames (inner join)
FLVoters = pd.merge(FLVoters, cnames, on='surname')

FLVoters.shape
```

[ ]: (8022, 13)

[ ]:
```python
# store relevant variables
vars = ["pctwhite", "pctblack", "pctapi", "pcthispanic", "pctothers"]

# Whites
whites = FLVoters.loc[FLVoters.race == 'white'].copy()
(whites[vars].max(axis='columns') == whites['pctwhite']).mean()
```

[ ]: 0.950218023255814

[ ]:
```
# Blacks
blacks = FLVoters.loc[FLVoters.race == 'black'].copy()
(blacks[vars].max(axis='columns') == blacks['pctblack']).mean()
```

[ ]: 0.16048237476808905

[ ]:
```
# Hispanics
hispanics = FLVoters.loc[FLVoters.race == 'hispanic'].copy()
(hispanics[vars].max(axis='columns') == hispanics['pcthispanic']).mean()
```

[ ]: 0.8465298142717498

[ ]:
```
# Asian
asians = FLVoters.loc[FLVoters.race == 'asian'].copy()
(asians[vars].max(axis='columns') == asians['pctapi']).mean()
```

[ ]: 0.5642857142857143

[ ]:
```
# White false discovery rate
1 - (FLVoters['race'][FLVoters[vars].max(axis='columns') ==
                      FLVoters['pctwhite']] == "white").mean()
```

[ ]: 0.19736034376918354

[ ]:
```
# Black false discovery rate
1 - (FLVoters['race'][FLVoters[vars].max(axis='columns') ==
                      FLVoters['pctblack']] == "black").mean()
```

[ ]: 0.3294573643410853

[ ]:
```
# Hispanic false discovery rate
1 - (FLVoters['race'][FLVoters[vars].max(axis='columns') ==
                      FLVoters['pcthispanic']] == "hispanic").mean()
```

[ ]: 0.22747546833184662

[ ]:
```
# Asian false discovery rate
1 - (FLVoters['race'][FLVoters[vars].max(axis='columns') ==
                      FLVoters['pctapi']] == "asian").mean()
```

[ ]: 0.3416666666666667

[ ]:
```
FLCensus = pd.read_csv('FLCensusVTD.csv')

# compute proportions by applying np.average to each column with pop weight
census_race = ['white', 'black', 'api', 'hispanic', 'others']
```

```python
race_prop = FLCensus[census_race].apply(
    lambda x: np.average(x, weights=FLCensus['total.pop']))

race_prop # race proportions in Florida
```

[ ]:
```
white       0.578934
black       0.151644
api         0.024197
hispanic    0.224655
others      0.020570
dtype: float64
```

[ ]:
```python
# store total count from original cnames data
total_count = cnames['count'].sum()

# P(surname | race) = P(race | surname) * P(surname) / P(race) in Florida
FLVoters['name_white'] = (
       (FLVoters['pctwhite'] / 100) * (FLVoters['count'] / total_count) /
       race_prop['white'])

FLVoters['name_black'] = (
       (FLVoters['pctblack'] / 100) * (FLVoters['count'] / total_count) /
       race_prop['black'])

FLVoters['name_hispanic'] = (
       (FLVoters['pcthispanic'] / 100) * (FLVoters['count'] / total_count) /
       race_prop['hispanic'])

FLVoters['name_asian'] = (
       (FLVoters['pctapi'] / 100) * (FLVoters['count'] / total_count) /
       race_prop['api'])

FLVoters['name_others'] = (
       (FLVoters['pctothers'] / 100) * (FLVoters['count'] / total_count) /
       race_prop['others'])
```

[ ]:
```python
# merge FLVoters with FLCensus by county and VTD using left join
FLVoters = pd.merge(FLVoters, FLCensus, on=['county', 'VTD'], how='left')

# P(surname | residence) = sum_race P(surname | race) P(race | residence)
FLVoters['name_residence'] = (
    FLVoters['name_white'] * FLVoters['white'] +
    FLVoters['name_black'] * FLVoters['black'] +
    FLVoters['name_hispanic'] * FLVoters['hispanic'] +
    FLVoters['name_asian'] * FLVoters['api'] +
    FLVoters['name_others'] * FLVoters['others'])
```

[ ]:
```python
'''
P(race | surname, residence) = P(surname | race) * P(race | residence) /
P(surname | residence)
'''

FLVoters['pre_white'] = (FLVoters.name_white * FLVoters.white /
                         FLVoters.name_residence)

FLVoters['pre_black'] = (FLVoters.name_black * FLVoters.black /
                         FLVoters.name_residence)

FLVoters['pre_hispanic'] = (FLVoters.name_hispanic * FLVoters.hispanic /
                            FLVoters.name_residence)

FLVoters['pre_asian'] = (FLVoters.name_asian * FLVoters.api /
                         FLVoters.name_residence)

FLVoters['pre_others'] = (1 - FLVoters.pre_white - FLVoters.pre_black -
                          FLVoters.pre_hispanic - FLVoters.pre_asian)
```

[ ]:
```python
# relevant variables
vars1 = ['pre_white', 'pre_black', 'pre_hispanic', 'pre_asian', 'pre_others']

# Whites
whites = FLVoters.loc[FLVoters.race == 'white'].copy()
(whites[vars1].max(axis='columns') == whites['pre_white']).mean()
```

[ ]: 0.9418604651162791

[ ]:
```python
# Blacks
blacks = FLVoters.loc[FLVoters.race == 'black'].copy()
(blacks[vars1].max(axis='columns') == blacks['pre_black']).mean()
```

[ ]: 0.62708719851577

[ ]:
```python
# Hispanics
hispanics = FLVoters.loc[FLVoters.race == 'hispanic'].copy()
(hispanics[vars1].max(axis='columns') == hispanics['pre_hispanic']).mean()
```

[ ]: 0.8572825024437928

[ ]:
```python
# Asians
asians = FLVoters.loc[FLVoters.race == 'asian'].copy()
(asians[vars1].max(axis='columns') == asians['pre_asian']).mean()
```

[ ]: 0.6071428571428571

[ ]:
```
# proportion of blacks among those with surname "White"
cnames['pctblack'][cnames.surname == "WHITE"]
```

[ ]:
```
19    27.38
Name: pctblack, dtype: float64
```

[ ]:
```
# predicted probability of being black given residence location
FLVoters['pre_black'][FLVoters.surname == "WHITE"].describe()
```

[ ]:
```
count    24.000000
mean      0.250711
std       0.293894
min       0.004588
25%       0.072232
50%       0.159496
75%       0.293640
max       0.981864
Name: pre_black, dtype: float64
```

[ ]:
```
# Whites
1 - (FLVoters['race'][FLVoters[vars1].max(axis='columns') ==
                      FLVoters['pre_white']] == "white").mean()
```

[ ]:
```
0.12239715591670897
```

[ ]:
```
# Blacks
1 - (FLVoters['race'][FLVoters[vars1].max(axis='columns') ==
                      FLVoters['pre_black']] == "black").mean()
```

[ ]:
```
0.22029988465974626
```

[ ]:
```
# Hispanics
1 - (FLVoters['race'][FLVoters[vars1].max(axis='columns') ==
                      FLVoters['pre_hispanic']] == "hispanic").mean()
```

[ ]:
```
0.21133093525179858
```

[ ]:
```
# Asians
1 - (FLVoters['race'][FLVoters[vars1].max(axis='columns') ==
                      FLVoters['pre_asian']] == "asian").mean()
```

[ ]:
```
0.3307086614173228
```

Section 6.3: Random Variables and Probability Distributions

Section 6.3.1: Random Variables

Section 6.3.2: Bernoulli and Uniform Distributions

[ ]:
```
from scipy import stats

# uniform PDF: x = 0.5, interval = [0,1]
stats.uniform.pdf(x=0.5, loc=0, scale=1) # loc = a, scale = b-a
```

[ ]:
```
1.0
```

[ ]:
```
# uniform CDF: x = 1, interval = [-2, 2]
a = -2
b = 2
stats.uniform.cdf(x=1, loc=a, scale=b-a)
```

[ ]:
```
0.75
```

[ ]:
```
sims = 1000
p = 0.5 # success probabilities
x = stats.uniform.rvs(size=sims, loc=0, scale=1)

type(x) # a numpy array
```

[ ]:
```
numpy.ndarray
```

[ ]:
```
x[:6]
```

[ ]:
```
array([0.95269   , 0.88899348, 0.30649262, 0.64109809, 0.93214079,
       0.6156068 ])
```

[ ]:
```
y = (x <= p).astype(int)
y[:6]
```

[ ]:
```
array([0, 0, 1, 0, 0, 0])
```

[ ]:
```
y.mean() # close to success probability p, proportion of 1's vs. 0's
```

[ ]:
```
0.501
```

Section 6.3.3: Binomial Distribution

[ ]:
```
# PMF: k = 2, n = 3, p = 0.5
stats.binom.pmf(k=2, n=3, p=0.5)
```

[ ]:
```
0.3750000000000001
```

[ ]:
```python
# CDF: k = 1, n = 3, p = 0.5
stats.binom.cdf(k=1, n=3, p=0.5)
```

[ ]: 0.5

[ ]:
```python
# number of voters who turn out
voters = np.array([1000, 10000, 100000])

stats.binom.pmf(voters/2, n=voters, p=0.5)
```

[ ]: array([0.02522502, 0.00797865, 0.00252313])

Section 6.3.4: Normal Distribution

[ ]:
```python
# plus minus 1 standard deviation from the mean
stats.norm.cdf(1) - stats.norm.cdf(-1)
```

[ ]: 0.6826894921370859

[ ]:
```python
# plus minus 2 standard deviations from the mean
stats.norm.cdf(2) - stats.norm.cdf(-2)
```

[ ]: 0.9544997361036416

[ ]:
```python
mu = 5
sigma = 2

# plus minus 1 standard deviation from the mean
(stats.norm.cdf(mu + sigma, loc=mu, scale=sigma) -
 stats.norm.cdf(mu - sigma, loc=mu, scale=sigma))
```

[ ]: 0.6826894921370859

[ ]:
```python
# plus minus 2 standard deviations from the mean
(stats.norm.cdf(mu + 2*sigma, loc=mu, scale=sigma) -
 stats.norm.cdf(mu - 2*sigma, loc=mu, scale=sigma))
```

[ ]: 0.9544997361036416

[ ]:
```python
# Replicate model from 4.2.5
pres08 = pd.read_csv('pres08.csv')

# import pres12 from the PREDICTION folder
pres12 = pd.read_csv('../PREDICTION/pres12.csv')

# merge the two elections by state
pres = pd.merge(pres08, pres12, on='state')
```

```python
# Use the scipy zscore function to standardize Obama's vote share
# Set ddof=1 to ensure the standard deviation denominator is n-1
pres['Obama2008_z'] = stats.zscore(pres['Obama_x'], ddof=1)
pres['Obama2012_z'] = stats.zscore(pres['Obama_y'], ddof=1)
```

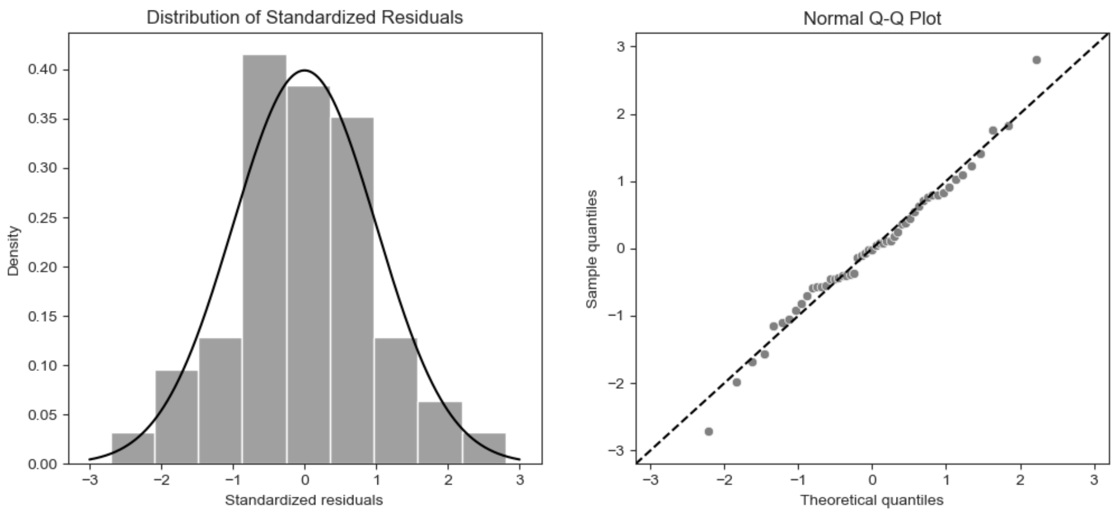
Note that in chapter 4, we built a function to calculate the z-score using the pandas **.std()** *method. The default ddof=1 for the pandas method. By contrast, the default ddof=0 for the numpy* **std** *function and the scipy* **zscore** *function.*

[ ]:
```python
import statsmodels.formula.api as smf

fit1 = smf.ols('Obama2012_z ~ -1 + Obama2008_z', data=pres).fit()

e = fit1.resid

# z-score of residuals
e_zscore = stats.zscore(e, ddof=1)

# alternatively, we can divide the residuals by the standard deviation
e_zscore = e / np.std(e, ddof=1)
```

[ ]:
```python
# Plot a histogram and Q-Q plot of the standardized residuals

## First, calculate some inputs for the plots
x = np.arange(-3, 3.01, 0.01)
x_pdf = stats.norm.pdf(x) # PDF of x

## Find quantiles for Q-Q plot using scipy.stats.probplot
quantiles = stats.probplot(e_zscore)
osm = quantiles[0][0] # ordered statistic medians (theoretical quantiles)
osr = quantiles[0][1] # ordered statistic ranks (sample quantiles)

fig, axs = plt.subplots(1, 2, figsize=(12, 5))

# Histogram of residuals
sns.histplot(e_zscore, stat='density', color='gray', ax=axs[0]).set(
    xlabel='Standardized residuals',
    title='Distribution of Standardized Residuals')

# Overlay the normal density
sns.lineplot(x=x, y=x_pdf, color='black', ax=axs[0])

# Q-Q plot
sns.scatterplot(x=osm, y=osr, color='gray', ax=axs[1]).set(
    xlabel='Theoretical quantiles', ylabel='Sample quantiles',
    title='Normal Q-Q Plot', xlim=(-3.2, 3.2), ylim=(-3.2, 3.2))

# 45-degree line
```

```
axs[1].axline((0, 0), slope=1, color='black', linestyle='--')
```

[ ]: <matplotlib.lines._AxLine at 0x2bd61af1c60>



Note that we could have used ***probplot*** *to create a Q-Q plot directly by passing a plot or an axis to the* ***plot*** *argument. However, obtaining the quantiles enables us to customize the plot a bit more.*

[ ]:
```
# e is a pandas series; we can use the pandas .std() method
e_sd = e.std()
e_sd
```

[ ]: 0.1812238619213575

[ ]:
```
CA_2008 = pres['Obama2008_z'][pres['state'] == 'CA']
CA_2008
```

[ ]:
```
4    0.872063
Name: Obama2008_z, dtype: float64
```

[ ]:
```
# CA_2008 is a series with index 4; extract the value using .iloc
CA_mean2012 = fit1.params * CA_2008.iloc[0]
CA_mean2012
```

[ ]:
```
Obama2008_z    0.857623
dtype: float64
```

[ ]:
```
# area to the right; greater than CA_2008
1 - stats.norm.cdf(CA_2008, loc=CA_mean2012, scale=e_sd)
```

[ ]: array([0.46824629])

[ ]:
```
TX_2008 = pres['Obama2008_z'][pres['state'] == 'TX']
TX_mean2012 = fit1.params * TX_2008.iloc[0]
TX_mean2012
```

[ ]:
```
Obama2008_z   -0.656754
dtype: float64
```

[ ]:
```
1 - stats.norm.cdf(TX_2008, loc=TX_mean2012, scale=e_sd)
```

[ ]:
```
array([0.52432713])
```

Section 6.3.5: Expectation and Variance

[ ]:
```
# theoretical variance: p was set to 0.5 earlier
p * (1-p)
```

[ ]:
```
0.25
```

[ ]:
```
# sample variance using 'y' generated earlier through simulation
y.var(ddof=1)
```

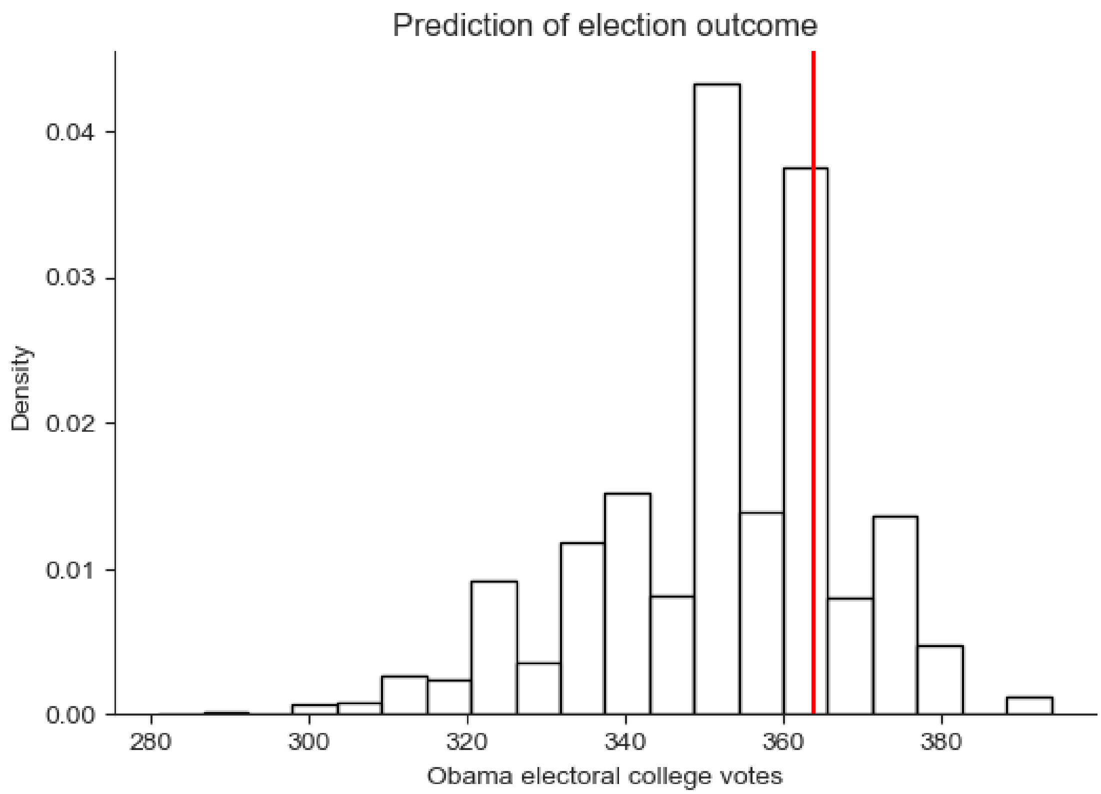
[ ]:
```
0.2502492492492492
```

Section 6.3.6: Predicting Election Outcomes with Uncertainty

[ ]:
```
# two party vote share
pres08['p'] = pres08['Obama'] / (pres08['Obama'] + pres08['McCain'])

n_states = pres08.shape[0]
n = 1000
sims = 10000

# Obama's electoral votes
Obama_ev = np.zeros(sims)

for i in range(sims):
    # samples number of votes for Obama in each state
    draws = stats.binom.rvs(1000, p=pres08.p, size=n_states)
    # sums state's electoral college votes if Obama wins the majority
    Obama_ev[i] = pres08.EV[draws > n/2].sum()
```

[ ]:
```
sns.displot(
    Obama_ev, stat='density', color='white', edgecolor='black',
    height=4, aspect=1.5, bins=20,
).set(title='Prediction of election outcome',
      xlabel='Obama electoral college votes')

plt.axvline(364, color='red') # actual result
```

[ ]: <matplotlib.lines.Line2D at 0x2bd6203bee0>



[ ]:
```
pd.Series(Obama_ev).describe().round(2)
```

[ ]:
```
count    10000.00
mean       352.25
std         16.36
min        281.00
25%        341.00
50%        353.00
75%        364.00
max        394.00
dtype: float64
```

[ ]:
```
Obama_ev.mean()
```

[ ]: 352.2486

[ ]:
```
# probability of binomial random variable taking greater than n/2 votes
(pres08['EV'] * (1 - stats.binom.cdf(n/2, n=n, p=pres08.p))).sum()
```

[ ]: 352.138751890897

[ ]:
```
# approximate variance using Monte Carlo draws
Obama_ev.var(ddof=1)
```

[ ]: 267.78417645764574

[ ]:
```
# theoretical variance
pres08['pb'] = (1 - stats.binom.cdf(n/2, n=n, p=pres08.p))

V = (pres08['pb'] * (1 - pres08['pb']) * pres08['EV'] ** 2).sum()
V
```

[ ]: 268.8008377634136

[ ]:
```
# approximate standard deviation using Monte Carlo draws
Obama_ev.std(ddof=1)
```

[ ]: 16.36411245554264

[ ]:
```
# theoretical standard deviation
np.sqrt(V)
```

[ ]: 16.395146774683464

Section 6.4: Large Sample Theorems

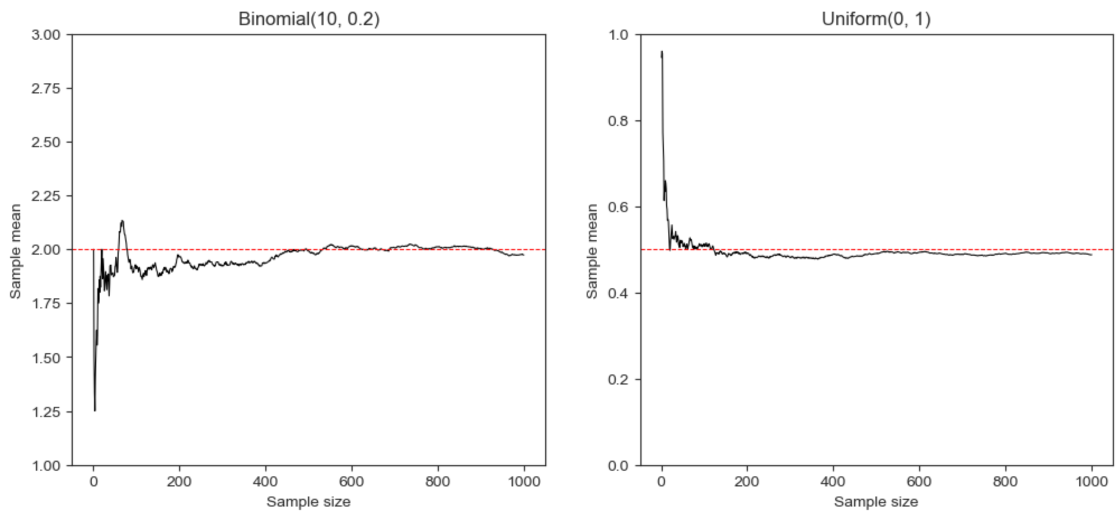
Section 6.4.1: The Law of Large Numbers

[ ]:
```
sims = 1000

x_binom = stats.binom.rvs(n=10, p=0.2, size=sims)

# computing sample mean with varying sample size
mean_binom = x_binom.cumsum() / np.arange(1, sims+1)

# default uniform.rvs is uniform(0, 1)
x_unif = stats.uniform.rvs(size=sims)
mean_unif = x_unif.cumsum() / np.arange(1, sims+1)
```

[ ]:
```
fig, axs = plt.subplots(1, 2, figsize=(12, 5))

# plot for binomial
sns.lineplot(
    x=np.arange(1, sims+1), y=mean_binom, ax=axs[0],
    color='black', linewidth=0.75
).set(title='Binomial(10, 0.2)', xlabel='Sample size', ylabel='Sample mean',
      ylim=(1,3))

axs[0].axhline(2, color='red', linestyle='--', linewidth=0.75)
```

```
# plot for uniform
sns.lineplot(
    x=np.arange(1, sims+1), y=mean_unif, ax=axs[1],
    color='black', linewidth=0.75
).set(title='Uniform(0, 1)', xlabel='Sample size', ylabel='Sample mean',
      ylim=(0,1))

axs[1].axhline(0.5, color='red', linestyle='--', linewidth=0.75)
```

[ ]: <matplotlib.lines.Line2D at 0x2bd621cca60>



Section 6.4.2: The Central Limit Theorem

[ ]:
```
# sims = number of simulations

n_samp = 1000

z_binom=np.zeros(sims)
z_unif=np.zeros(sims)

for i in range(sims):
    x = stats.binom.rvs(n=10, p=0.2, size=n_samp)
    z_binom[i] = (x.mean() - 2) / np.sqrt(1.6 / n_samp)
    x = stats.uniform.rvs(size=n_samp, loc=0, scale=1)
    z_unif[i] = (x.mean() - 0.5) / np.sqrt(1 / (12 * n_samp))

# store the standard normal PDF
x = np.arange(-3, 3.01, 0.01)
```

```python
x_pdf = stats.norm.pdf(x) # PDF of x
```

[ ]:
```python
fig, axs = plt.subplots(1, 2, figsize=(12, 5))

sns.histplot(
    z_binom, stat='density', bins=40, color='white', edgecolor='black',
    ax=axs[0]
).set(xlabel='z-score', title='Binomial(0.2, 10)',
      xlim=(-4, 4), ylim=(0, 0.6))

# Overlay the normal density
sns.lineplot(x=x, y=x_pdf, color='black', linewidth=0.75, ax=axs[0])

sns.histplot(
    z_unif, stat='density', bins=40, color='white', edgecolor='black',
    ax=axs[1]
).set(xlabel='z-score', title='Uniform(0, 1)',
      xlim=(-4, 4), ylim=(0, 0.6))

sns.lineplot(x=x, y=x_pdf, color='black', linewidth=0.75, ax=axs[1])
```

[ ]:
```
<Axes: title={'center': 'Uniform(0, 1)'}, xlabel='z-score', ylabel='Density'>
```