Forecasting Technology Performance: Initial Tests with Time Series Extrapolation and Patent Data

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1 Overview

This report describes principles of trend extrapolation for a lay audience (without equations) and shows how different kinds of trend extrapolation worked on historical data of technology trends. We ask the question "Which methods would have allowed me to better predict the future, if I had been standing in 1990?" With the presently available data, the simplest models performed best.

This study had 3 tasks:

1. Create statistical software to enable principled extrapolation of technology trends, including forecasts with correctly-stated uncertainty.

2. Test what kinds of trend extrapolation would have best predicted past technology performance data.

3. Test if using patent data could have improved predictions of technology performance data.

The software for task 1 has been written, rigorously tested and made publicly available.[1] The software was used to perform task 2 and 3, the results of which are described here. Task 3 was unlikely to work, given noisy historical data, and in fact it didn't. However, the analysis pipeline that was created is now ready for any cleaner data that may be assembled in the future. This study thus provides the tools and examples for future forecasters to make better predictions of technology.

More technical details about the models, software and data are included at the end of this report. They are presented in a form designed to be useful for those looking to implement these methods, and all code used in this study is available online[2].

2 Forecasting Technology Trends

Consider a technology, such as electronic computers. We care about computers because they provide capabilities (e.g. processing information) given some resources (e.g. time, energy or dollars). Forecasting technologies' capability per unit resource is the object of interest for this study. For example, say we will have $1,000 to buy a computer in 2030; how many instructions per second will that computer be able to calculate? A simple way make a forecast is trend extrapolation: look at historical data, identify a trend, and draw the trend out to the future date. In the case of electronic computers, we can observe historical data from the 1940s until today (Fig. 2.1A). What trend do we identify?

[1] `https://github.com/jeffalstott/pystan_time_series`
[2] `https://github.com/jeffalstott/technologytimeseries_forecasting`

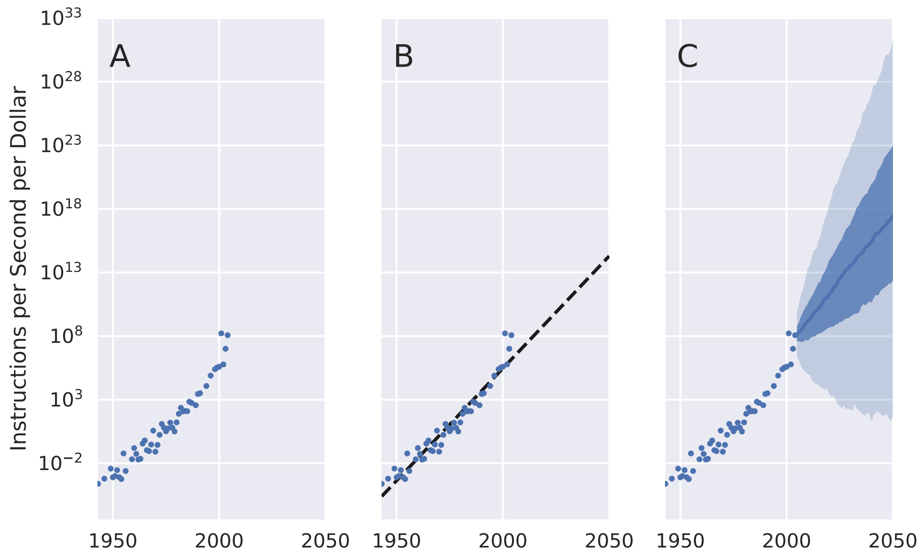


Figure 2.1: **Extrapolating technology trends with different methods.** A) Historical prices of computing power B) Using linear regression to forecast future computing power C) Using Bayesian trend extrapolation methods to make probabilistic forecasts. Light band: 95% confidence interval. Dark band: 50% confidence interval. Line: Median forecast.

2.1 Making uncertain forecasts

The simplest and most common trend extrapolation technique is to literally draw a line through the data points, perhaps through linear regression, and find an answer: $5.8 * 10^{13}$ (Fig. 2.1B). This process is fast, but unfortunately it has several downsides, chief among them that it's wrong. That *exact* predicted performance will virtually certainly not happen. A better forecast would give statements with uncertainties, like this:

- $1.9 * 10^5 - 4.7 * 10^{25}$: 95% probability
- $3.7 * 10^{12} - 6.0 * 10^{19}$: 50% probability

A forecast like the one in Fig. 2.1C not only gives predictions, but uncertainties about those predictions (the broader, lighter band is the 95% confidence interval, and the narrower, darker band is the 50% confidence interval). This is a forecast that can be well-calibrated, if done well - observed events should be in the 95% confidence interval 95% of the time. This is a forecast that is operationally useful, as opposed to a point prediction that is virtually guaranteed to be correct 0% of the time.

Methods to do trend extrapolation with probabilistic forecasts are well-studied and well-used, particularly in finance. As such, people have developed countless possible descriptions for a trend (such trends are often called "models"). For example, perhaps the simplest model is this: "every year the technology improves by a fixed amount, but each year there is some random noise around that improvement." One possible interpretation of such a model is that the fixed improvement rate is due to something innate about the technology itself, while the noise each year is due to other factors like economic conditions, random chance that a certain pair of inventors meet each other in the hallway, etc. A somewhat more complex model could be: "every year the technology improves by a fixed amount, and each year there is some noise around that improvement, but the effects of last year's noise are also still felt a small amount." Such a model would just be adding the concept that the shock of, say, an economic recession could last for more than one year. In both cases, the uncertainty about what the future holds is because of the noise; we might know what the general shape of the noise is like on average, but we don't know what the precise effect of the noise will be each year. The effects of this noise accumulates, which is why our predictions' confidence intervals get wider as we forecast further into the future (Fig. 2.1C).

2.2 Uncertainty in our knowledge: Bayesian statistics

While trend extrapolation techniques have been around for many years, only recently have researchers started to apply these methods to historical technology trends.[3] The most recent research is a good start (and heavily inspired the present study), but there are opportunities for further advancement. One way to better understand historical trends, and thus make even better forecasts, is to understand our uncertainty even better. In the above paragraph, we considered a model in which there's a fixed improvement every year, but it's the random noise that is causing our forecasts to be imprecise. This might be a true story about the world, but it's an incomplete story about our knowledge of that world. A more complete story is that we think the technology improves a fixed amount each year, *but we don't even know for certain what that fixed amount is.* Thus, our forecasts should not only take into account our uncertainty on the future random noise, but should also take into account our uncertainty about the size of the fixed improvement every year[4]. Our forecasts can now combine two sources of uncertainty: the uncertainty of what values random variables will produce each year *and* our uncertainty of what value fixed variables have across all years. The forecasts in Figure 2.1C do this, as will all forecasts in this study. The method for describing our uncertainty of the parts of the model is called Bayesian statistics, and the principle is simple:

1. Start with some prior belief on what the values of the parts of the model could be. In lieu of any prior knowledge, these should be broad, diffuse beliefs, like "Somewhere between zero and a million, but somewhat higher probability around a thousand."

2. Look at historical data, and consider how likely that data would have been had the parts of the model had different values.

[3] J. D. Farmer and F. Lafond. "How Predictable Is Technological Progress?" *Research Policy* 45.3 (Apr. 2016), pp. 647–665
[4] We can even take into account our uncertainty in the average shape of the noise

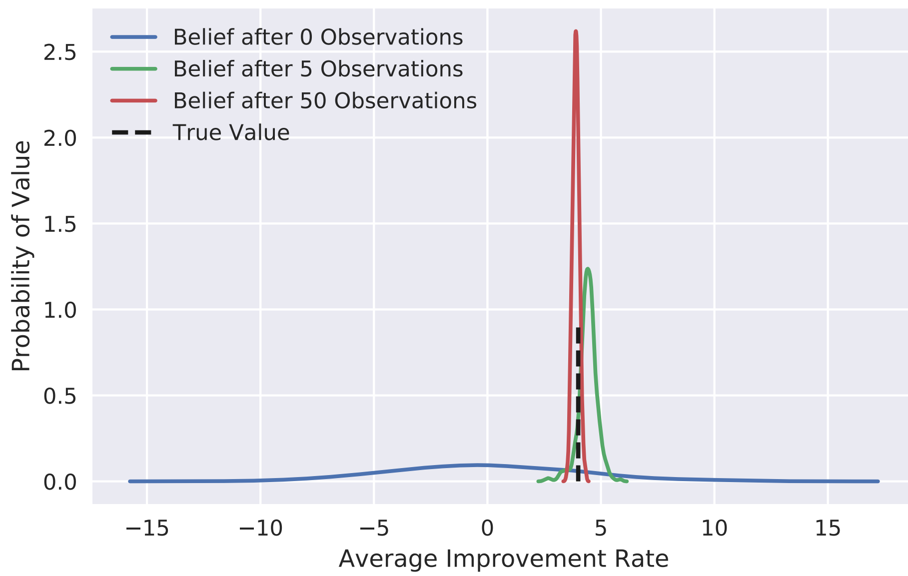


Figure 2.2: **Bayesian inference of an unknown parameter.** A technology trend was simulated with an average yearly improvement rate, but with noise that could raise or lower the improvement each individual year. Before observing any data, we started with a broad prior belief as to what the average improvement rate could be. After observing just 5 years of data, the belief tightened towards the true value, and after 50 years of observation the belief was tightly centered on the true value.

3. Combine parts 1 and 2 to update to the data-driven, "posterior" belief of what the values of the parts of the model are.

This process is shown for some simulated data in Figure 2.2. As we add more data our posterior belief gets tighter and tighter around the true value, but we never have complete certainty in the values of the parts of the model.[5]

2.3 Uncertainty in our models: model comparison

We also have uncertainty not just about the values of parts of a model, but on whether we're using the right model at all. Previously we considered two models: both had noise every year, but in one model the noise had some influence on the next year as well. We are uncertain about which model better describes the data.[6] It's possible to combine both models into a single model, and then make forecasts that incorporate our uncertainty as to which model is correct.[7]

If we want to get more certainty as to which model is correct, we can test them, by seeing which better

[5] A nice property of this method is that if we don't have much data, we're not entirely stuck: we can rely on our prior belief to make predictions. We will see the merits of this later in sections 4.1 and A3

[6] Sometimes we're very confident about what model or description of the data is right, because we know what the underlying mechanisms are in the system we're studying, but even that confidence is effectively built out of observing lots of historical data.

[7] In this specific case, the second model does this automatically: the only difference is in the influence of noise a year later, so if the data suggests that influence is 0, that indicates the first model better describes the data.

predicts future data. This is the central activity of this study: we compared the forecasting power of several kinds of models. Each model's values were determined, with Bayesian statistics, by using historical data up to a given year (specifically, 1990), and then that model was used to forecast "future" data (from 1991 to 2013). Those models that better predicted the future data are more likely to be useful in predicting further future data, including data from new technologies.

It is worth noting that comparing models is no guarantee that any of the models will be any good. Some of the technology trends examined here are forecasted very well by the models tested, but others are far off. This may sometimes be because there was just insufficient data to infer the right values for the model. More often, it's because the model is very wrong. Seeing such discrepancies and proposing new models to try is the job for the scientist. This is what led us to try to expand on simple trend extrapolation models with patent data.[8]

2.4 Evaluating forecasts

Comparing model's forecasts requires scoring how good each forecast was. In an ideal operational context, we would have some operational consequence for more or less accurate forecasts (e.g. dollars saved). This would let us say "A forecast that is off by this much is worth $\$X$, and a forecast that is off by that much is worth $\$Y$." Then for each forecasted data point, we can evaluate the value of the model's prediction and sum the values to cache out the model's accuracy in dollars.

Without an operational context, we will score models' predictions of each forecasted data point using a more generic measure. First, for each forecasted data point we calculate the model's predicted probability density.[9] Then we take the log of that probability density. This log(probability density) is a standard in industry and academia for evaluating predictions of all kinds because it has justifications in information theory, but for present purposes it is sufficient to know that larger numbers are more accurate predictions.

Figure 2.3 shows a set of forecasts and the log(probability densities) of those forecasts. We will summarize how well a model forecasts future data by simply averaging the log(probability density) of all forecasts it makes, across all technologies. Again, in an operational context this would not be appropriate; not all technologies are equally operationally important, so getting the predictions of some technologies better is more important than others. However, in the absence of an operational context to weight the different technologies, we will simply average them.

[8] It didn't work.

[9] For continuous data, the probability for any specific predicted value is 0, but the probability *density* around that value may be a positive number.

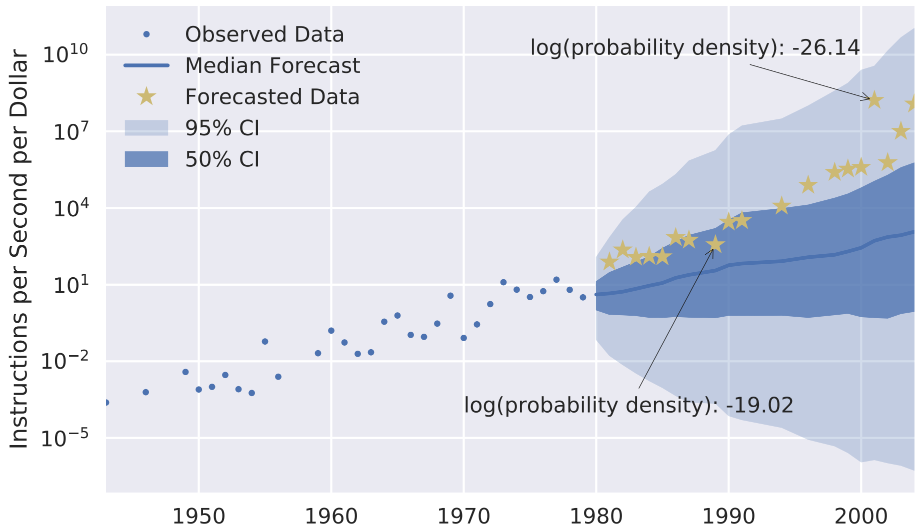


Figure 2.3: **Making and testing forecasts of technology.** Changes in the price of computer power were modeled using historical data up until 1980 (blue dots). This model made probabilistic forecasts of future prices (line and shaded region), which were then tested using subsequent historical data from 1981 onward (yellow stars). A forecast's accuracy for a given year was quantified as the log of the probability density of the true data for that year.

3 Historical Data

The historical technology data used in this study came from previously published articles[10] and is hosted online at `https://github.com/jeffalstott/technologytimeseries`. These data included 130 different trends with a total of 2,204 data points (Fig. 3.1). Descriptions of all of these data are included in the Appendix (section A1.1). Each individual trend was for some technology (e.g. "Batteries" or "Ethylene") and the data was about some dimension of the quality of that technology (e.g. "energy storage per kilogram" or "kilograms per dollar"). A single technology could have multiple trends, with each trend reflecting a different dimension of quality. We separate these trends into two varieties:

1. **Price**: the amount of currency to either purchase or produce a unit or output of the artifact at that moment in history (e.g. "kilograms per dollar" or "computations per dollar"). This number could go lower or higher over time.

2. **Performance**: the amount of functional output that the artifact could produce for some unit of input (e.g. "energy storage per kilogram"). This data only reflects record-breakers, so the number is only able to go higher over time.

[10] Farmer and Lafond, see n. 3
C. L. Magee et al. "Quantitative Empirical Trends in Technical Performance". *Technological Forecasting and Social Change* 104 (Mar. 2016), pp. 237–246
B. Nagy et al. "Statistical Basis for Predicting Technological Progress". *PloS ONE* 8.2 (Jan. 2013), e52669–e52669

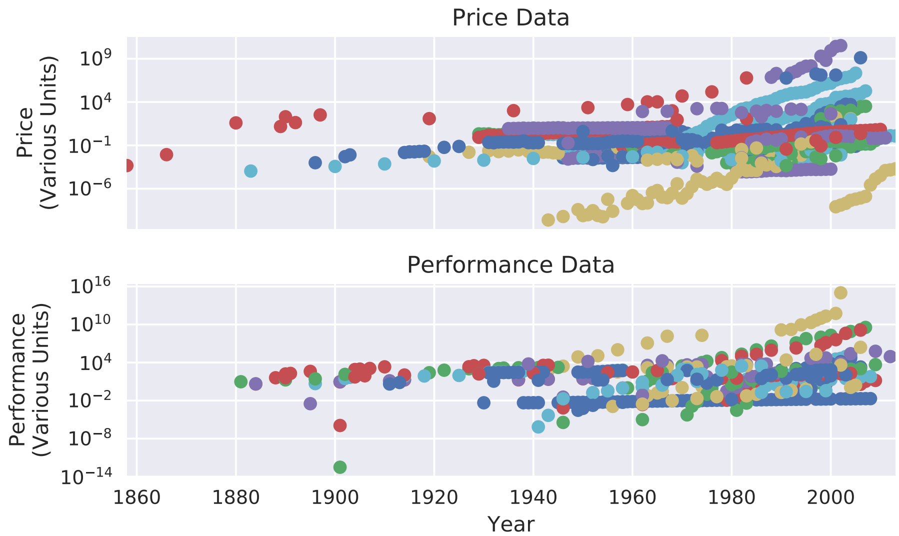


Figure 3.1: **The historical technology data used for creating and testing forecasts.** Top) Price data, in the form of the amount of capability per dollar. Bottom) Performance data, in the form of the amount of capability per some physical resource (e.g. computers' computations per joule, batteries' energy per kilogram, etc.)

Sometimes inventors patent their inventions, which leaves a public record of those inventions that may contain useful information for making forecasts. In this study, most (72) of the technology price and performance trends were accompanied by patent data. For these technologies, previous studies had identified/defined their functional "domains," such as "magnetic information storage" or "genomics."[11] Patents were then found that related to these domains. These patents were identified using a system that begins with domain expert analysis of patent texts but then ends with domains defined through patent metadata. This metadata is curated by the US Patent & Trademark Office for millions of patents, and so allowed the researchers to automatically match large numbers of patents with domains. These domains of patents can then be matched up with technology price and performance trends. Previous research has found that some patent features correlate with technologies' average improvement speed.[12] In the present study, we tested if this data could be used to improve forecasting.

10 of the trends were questionable in some way, generally because it seemed the recorded data didn't actually reflect the stated measure (e.g. wrong order of magnitude) or the measure seemed ill-defined (e.g.

[11] C. L. Benson and C. L. Magee. "A Framework for Analyzing the Underlying Inventions That Drive Technical Improvements in a Specific Technological Field". *Engineering Management Research* 1.1 (Apr. 2012), pp. 2–14
C. L. Benson and C. L. Magee. "Technology Structural Implications from the Extension of a Patent Search Method". *Scientometrics* 102.3 (Dec. 2, 2014), pp. 1965–1985

[12] C. L. Benson and C. L. Magee. "Quantitative Determination of Technological Improvement from Patent Data". *PLoS ONE* 10.4 (Apr. 15, 2015), e0121635
G. Triulzi, J. Alstott, and C. L. Magee. *Predicting Technology Performance Improvement Rates by Mining Patent Data.* SSRN Scholarly Paper ID 2987588. Rochester, NY: Social Science Research Network, June 26, 2017

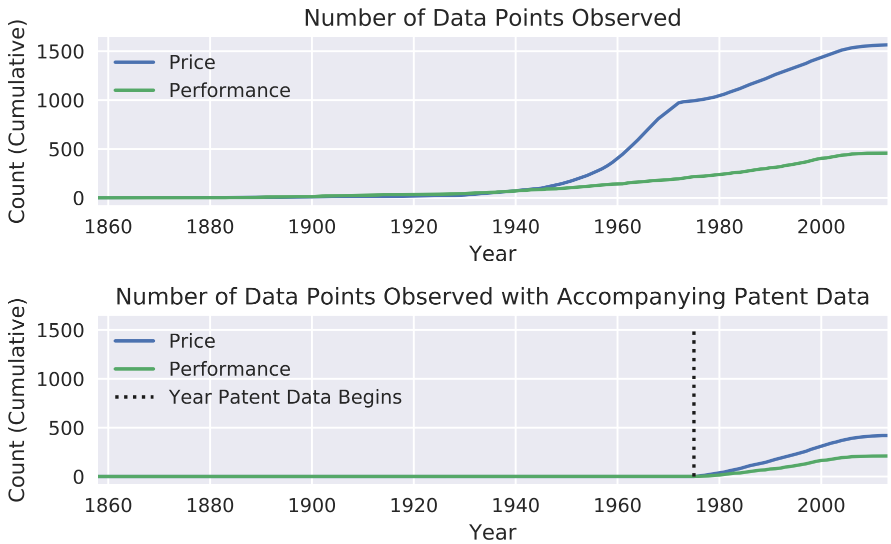


Figure 3.2: **The cumulative amount of historical data that had been observed by any possible training year.** Not all technology trends were paired with patent data, and digital patent records were only available staring in 1975.

one could increase the performance just by having two copies of the artifact next to each other). These trends were removed, leaving 120 trends with 2,022 data points (price: 1,565; performance: 457). These data are the shown in Figure 3.1.

Forecasts were made through hindcasting - only data that appeared before a specific date was used to train the model, and then subsequent data that appeared after that date was forecasted. Thus a relevant question for training the model is not how much data there is in total, but how much existed before a given date. That is the amount of data that could be used to train a model on that date. The cumulative counts of the data are shown in Figure 3.2. Unfortunately, digitized patent data was only available after 1975. As such, technology trend data that had associated patent data only began at that date, and accumulated to a much smaller amount of total data.

Because we tested our forecasts on real historical data data, we needed some real historical data left to test with. In general, the later the date that we selected to train the models with, the less data we would have left to forecast and test on. However, this was not always the case. Because historical technology trends had a variety of start dates, we could only hope to train a model and make forecasts for those technologies that we had already "seen," and so could only include those technologies for forecasting. We required that a technology trend needed 3 observed data points before we would train a model with it and make forecasts. Selecting a later training date, then, would allow more technologies to enter as candidates for forecasting. Thus, we have both a desire to push for earlier training dates to leave subsequent data for forecasting, and

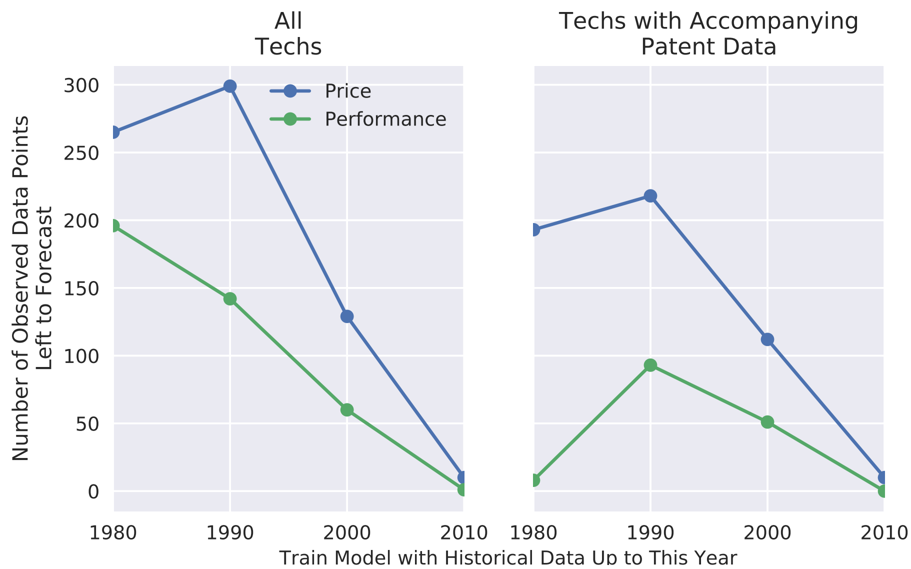


Figure 3.3: **The number of data points available to forecast, given different training years.** Because the various technology trends started and stopped in different years, training in later years would allow more technologies to have had a trend. However, training in later years would leave fewer subsequent data points to forecast. In all conditions the observed data points were spread out across about 20 unique technologies. The training year used for this study was 1990.

a desire to push for later training dates to allow more trends to be included for forecasting. Figure 3.3 shows the net effects of these two forces: the total number of data points that could be forecasted, given different training dates. In all cases, the observed data points are spread across about 20 technology trends. We selected 1990 as our training year to maximize the number of data points left to forecast and test the models.

Figure 3.4 shows the final data that was used for training the models and for testing forecasts. When we tested adding patent data to improve forecasts, only a portion of this data was used, as only a portion of it had been connected to patent domains.

4 Comparing Forecast Accuracy

We began with perhaps the simplest possible trend extrapolation model, which we used as an example in section 2.1: "every year the technology improves by a fixed amount, but each year there is some random noise around that improvement." The sizes of the fixed improvement and of the noise are the two parameters that the model must estimate from historical data and are what the models use to make forecasts. The previously shown Figure 2.3 is an example forecast from this model, and all forecasts for all technologies are shown in section A1.2.

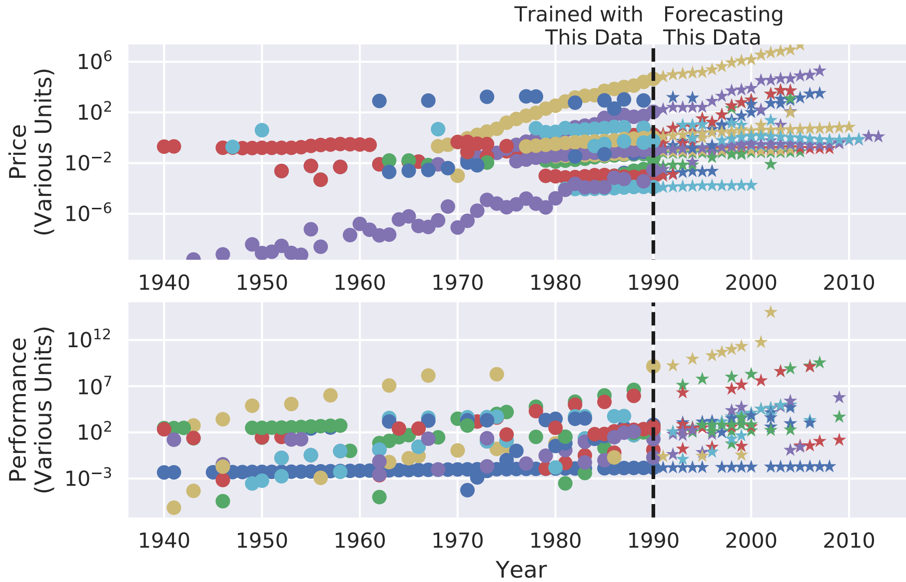


Figure 3.4: **The technology data used to train and test the forecasting models.**

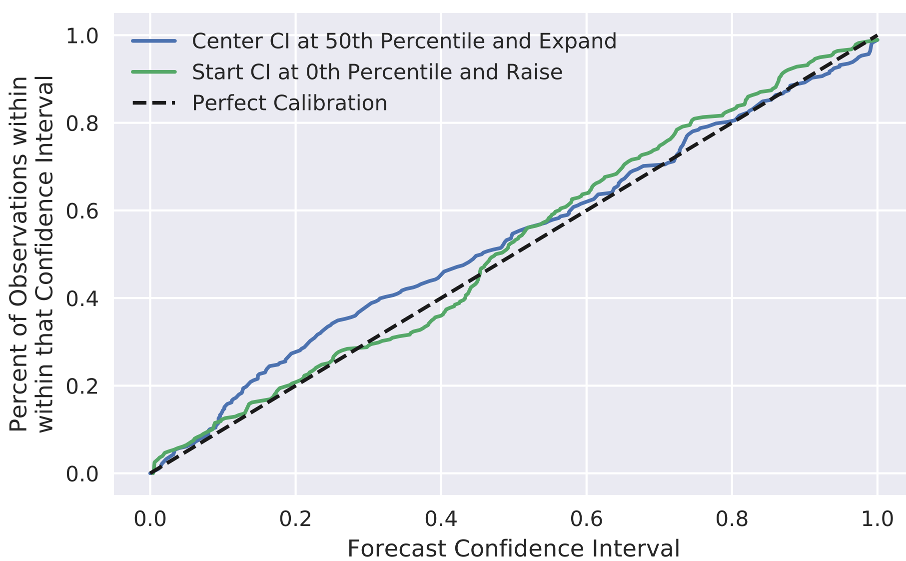


Figure 4.1: **Predictive models of technology price were well-calibrated.** For a perfectly-calibrated model, a forecast that a future data point will happen within a confidence interval of 95% will later see data points falling within that range 95% of the time. This is true for all confidence interval widths (dashed line). The predictive models of technology price closely followed this line using two different ways of defining the confidence interval. Blue line: centering the confidence interval at the median (50th percentile) predicted value and then expanding the width to incorporate higher and lower possible values. Green line: starting the confidence interval at the lowest (0th percentile) possible value and raising the confidence interval to consider larger possible values.

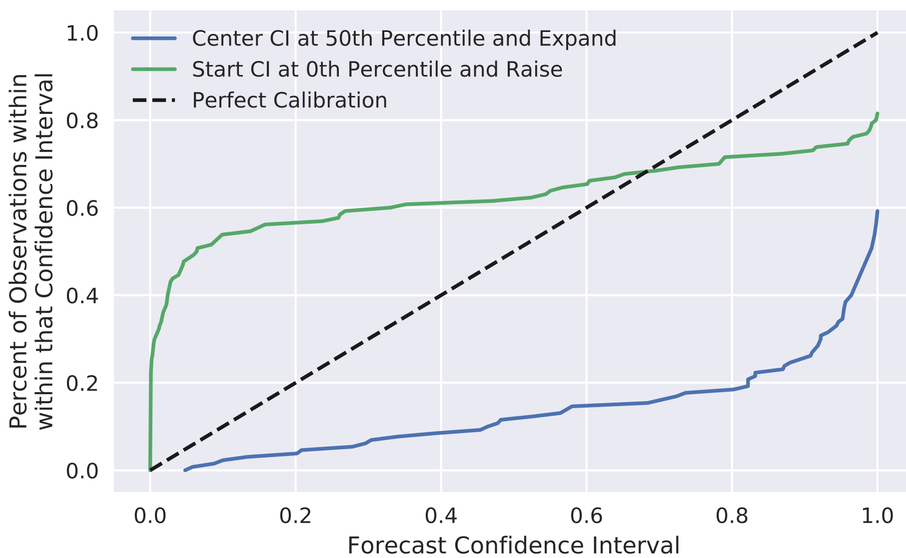


Figure 4.2: **Predictive models of technology performance were terribly calibrated.** As Figure 4.1, except for technology performance data.

The models of price were well-calibrated (Fig. 4.1). Of the 278 data points tested, the true data points occurred within the forecasts' 95% confidence interval 94% of the time, and within the 50% confidence interval 55% of the time. This is a marked advance in forecasting over simply drawing a line through the data points, as it means that we have statements of uncertainty about our forecasts that are explicit, quantitative, and (most usefully) rather close to true. Making forecasts that are correct about their uncertainty is useful, but it is even better if those forecasts can be more accurate; we want the same well-calibrated confidence intervals, but we want those confidence intervals to be tighter. We will return to this goal below.

Unfortunately, the models of performance were poorly calibrated (Fig. 4.2). Of the 130 data points tested, the true data points occurred within the forecasts' 95% confidence interval just 35% of the time, and within the 50% confidence interval just 12% of the time. These mistakes show the models of performance are overconfident (their confidence intervals are too narrow). This fact is likely because the models are misspecified; they simply don't describe what's happening in the data well. Consider trying to use a straight line to model data that is actually a parabola (a U-shaped set of points). The model wouldn't describe the data well at all, and would yield terrible prediction. Better forecasts might be made with models that better describe the data.

In pursuit of more accurate forecasting, we trained 6 different kinds of more complex models and compared the accuracy of their forecasts to those of the simplest model described above. These models added additional complexity like "the size of the improvement in this year depends somewhat on the size of the improvement last year," or even "the size of the improvement in this year depends somewhat on the size of the improvement

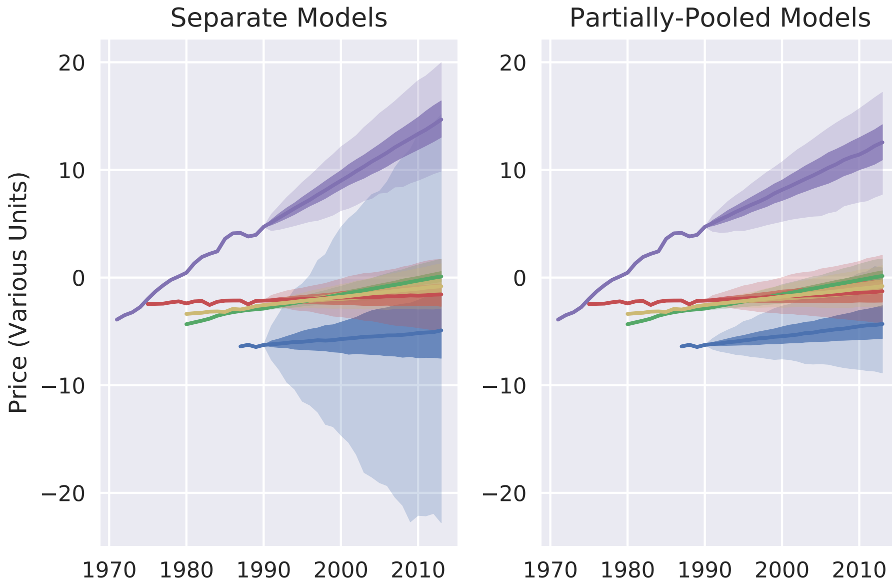


Figure 4.3: **Combining inference of multiple technologies' trend extrapolation models can tighten forecasts.** Left) When modeling technologies separately, there was not much data for the price of natural gas-fired combined cycle gas turbines (blue). Accordingly, this left great uncertainty as to the year improvement rate or the variability around that rate, which led to very broad predictions. Right) By partially pooling the inference of all the technologies' trends, other technologies' behavior can inform our models of trends with little data. This allows for tighter forecasts.

5 years ago." Interestingly, these more complex models did not systematically raise the forecasting accuracy beyond the simplest model we started with (Figs. A2.1, A2.2). For each model some trends were forecasted slightly better, and others slightly worse, but the collective improvement was at best nil.

4.1 Partial Pooling

Another possible way to help statistical inference, and thus forecasting accuracy, is to recognize that each of these individual technology trends is not completely isolated. Instead, what we learn from one trend could help us when evaluating another. This could be particularly helpful if one technology doesn't have much data. We might not be able to very confidently infer what its yearly improvement rate is, but we might know from looking at other technologies what a *typical* improvement rate is, and that can help inform our guess as to what this *particular* technology's improvement rate is. Figure 4.3 shows an example of this. On the left, we see data and forecasts for several technology trends, but one of them (blue) has very little data and so has great uncertainty in its forecasts (wide confidence intervals). On the right, we modify these models so that they assume that part of the information about each trend is unique to that trend but part is drawn from a shared pool of information that is true about all trends; this model structure is called "partial

Figure 4.4: **Partial pooling was mildly helpful for several technologies with less data, but hurt forecasting for one.** Trends visualized are for price data. The technology whose forecasts were notably hurt was hard drive prices (which had only 3 data points to train with).

pooling." Partial pooling can thus decrease our uncertainty for forecasts, and we see this on the right with the forecasts for the blue trend being much tighter.

Partial pooling is not guaranteed to be helpful, because there's no guarantee that different trends are actually similar in any way. By using partial pooling, we're asserting that knowing something about how nuclear power improves should inform us about how hybrid corn improves. While this is likely true in a very general sense, if we read too much into how nuclear power improves, we may overapply that knowledge to hybrid corn and miss out on the properties that are different for corn. This happened when we used partial pooling to model price data. In general, partial pooling's effects on forecasts were either negligible or slight improvements, particularly for those technologies with little historical data to train on (Fig. 4.4). But the overapplication happened with hard drive prices, which had only 3 data points to train on. The model drew the wrong inferences from the other technologies and was overconfident in its predictions, making the forecasts worse. In aggregate, using partial pooling was not wildly helpful for price nor performance forecasting (Figs. A2.3 and A2.4).

4.2 Adding Patent Data

We tested if using information from patents could improve forecasts for those technology trends that had accompanying patent data. Patents are rich with metadata, such as the citations they make to other patents. These metadata can be used to calculate multitudes of properties of patents, but these five have

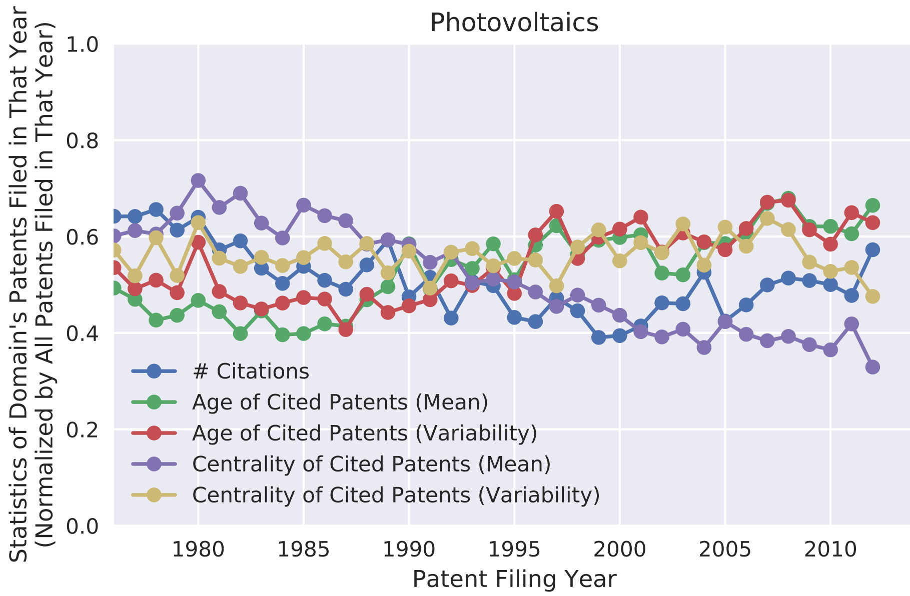


Figure 4.5: **Several features of patents associated with photovoltaics, by patent filing year.** Each feature is expressed as a percentile score, compared to all patents in that filing year, and average across the patents relevant to photovoltaics. Data for other patent domains are shown in section A1.3. These were the predictors used to attempt to improve technology forecasts.

been previously found to either correlate with technology improvement rates or patent quality:

- the number of citations the patents made
- the average age of the patents' cited patents
- the variability of the age of the patents' cited patents
- the average centrality[13] of the patents' cited patents
- the variability of the centrality of the patents' cited patents

An example of these data for the domain of photovoltaics is shown in Figure 4.5, and figures for all domains are in section A1.3. The method for calculating these predictors was as follows: First, the candidate predictors were calculated for each individual patent. Then, each patent's predictor values were compared to the values of all patents filed in the same year, then expressed as a percentile. Finally, the normalized predictor values for all patents in a domain were averaged together, for each filing year. This year-by-year, normalized predictor data was the data used to try to improve technology performance and price forecasting.

[13]The centrality of a patent is how central it is in the *entire* patent citation network, incorporating all observed years of data. It is a measure of how many other patents are reachable from the target patent by going through its citations, then through its cited patents citations, and so on.

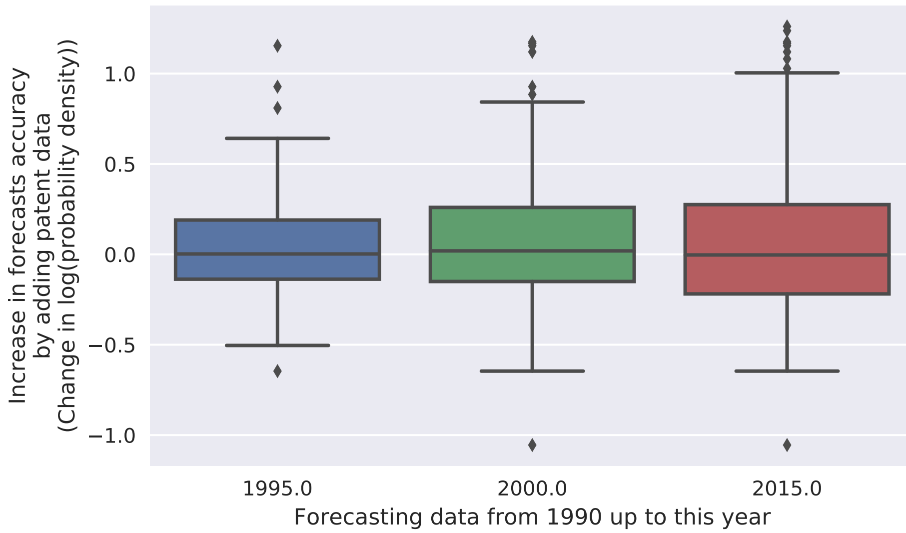


Figure 4.6: **Adding patent data did not improve forecasts of technology price.** Box plots: The difference in all forecasted data point's log(probability density) after adding patent data to the model vs. not including patent data. Regardless of whether one considers just data that was 5 years into the future (blue), 10 years (green) or all possible data (red), the forecasts were not consistently better. This visualization shows the results for using only one predictor: the average centrality of the patents cited. However, the forecasting accuracies are qualitatively similar with other predictors (see Figs. A2.5-A2.14).

Because patent data was only available from 1975 onward, these models were only trained with patent and technology data from 1975 onward (up to 1990, for a total of 15 years of possible data).

Again, the simplest model was "every year the technology improves by a fixed amount, but each year there is some random noise around that improvement." We added patent data to this by adding "the patent predictor from a previous year may also have some influence." Unfortunately, not one of the patent predictors systematically improved forecasting above the simplest model (Figs. 4.6, 4.7). We tried adding influence from just the immediately previous year, 5 years previous, or both, but none of these modifications markedly altered the forecasting accuracy of the models (Figs. A2.5-A2.14).

5 What Went Right, What Went Wrong

About half of the tasks went right and about half went wrong. "Right" here means that we produced a forecasting system that does the desired task well. "Wrong" here means that we produced results that went counter to a hypothesis. However, both positive and negative results are still results, and both can be learned from.

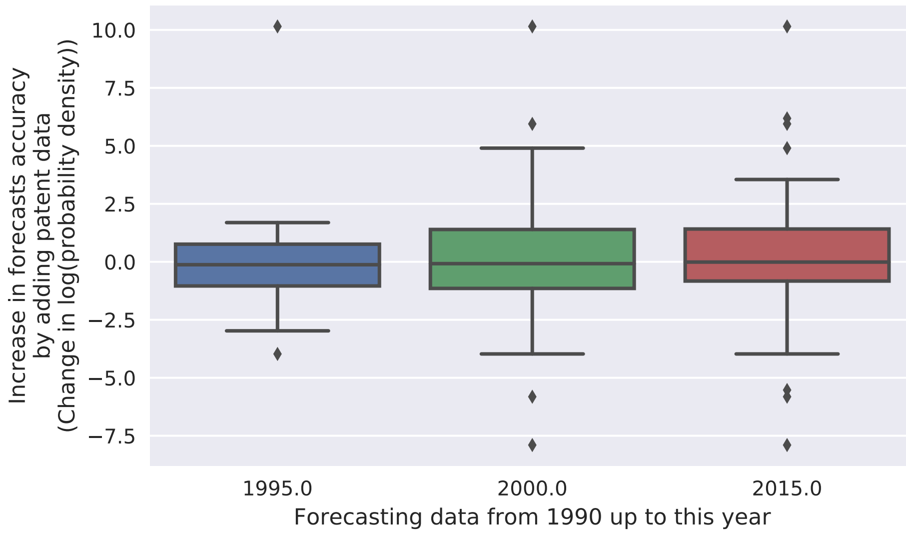


Figure 4.7: **Adding patent data did not improve forecasts of technology performance.** As Figure 4.6, but for technology performance trends instead of price trends.

5.1 Price was well-specified, while Performance was not

Forecasting of price data worked very well, in that well-calibrated forecasts were made across many disparate technologies. Previous research[14] pioneered using the same trend extrapolation methods on part of this data, but using non-Bayesian methods. Those methods produced confidence intervals that were less well-calibrated than the forecasts of the present study, and that is likely due to an advantage of Bayesian prediction: uncertainty in our parameters. The forecasts here took into account our uncertainty in the models' parameters, and so had wider confidence intervals.[15] Interestingly, that previous research used slightly more complicated models to fit the data, but here we find that they conferred no advantage in forecasting accuracy.

Forecasting of performance data, however, was rubbish. Some of this could be due to prosaic reasons: there was less performance data than price data, there were more gaps with missing years of data than the price data, and much of the data came from a pieced-together collection that may have been more prone to errors. But that shouldn't have led to forecasting this bad. Instead, what was probably most important is that the models were misspecified.

One obvious way that the models were misspecified is that this data is clearly more of a sequence of rough steps than a smooth progression. The models we created didn't allow for the possibility of long periods of no improvement, followed by a jump. This led the real data to be persistently lagging under or leaping

[14] Farmer and Lafond, see n. 3

[15] Put technically: The forecasted data points were modeled as normally-distributed random variables. In a Bayesian framework, estimating a normally-distributed random variable with a scale that is also an inferred parameter yields a posterior distribution that has heavier-than-normal tails. In a frequentist framework the scale is just taken as the best-fit value, and the forecasts are normally distributed.

over the models' forecasts, which creates the terrible prediction. It's worth noting that the *reality* may not have such rough steps. With more complete sampling, the year-by-year changes in artifact performance would still be steps but could be more smooth. However, what we are trying to model here is not just the performance of artifacts in the world, but also the rate at which observations are recorded into our data set. Adding a simple step function into the models would not be difficult and is low-hanging fruit for making improvements. There is, however, a wide world of more complex models for record-breakers[16]; testing this wider set of models would be a more substantive project.

5.2 Patents: *This* data is not a predictor

Using patent data to improve technology forecasting was a task that, if it had succeeded, would have been a huge advance. For years it has been an open research question as to how to match up technologies (artifacts that people are using) with patents (legal documents filed to protect property rights on a particular way of doing things). While in principle any given product may employ techniques that the producing company has patented, in practice companies do not advertise or even record virtually any of those patent-product pairings. What's more, precise patent-product matching is often not the right description of reality. Instead, there is a cluster of patents created by an ecosystem of inventors that contains the collective knowledge of an area of technology, and this ecosystem sometimes also yields quantifiably better artifacts. How to identify such ecosystems of patents has been an area of active research.

This study used patents that were previously identified with the specific aim of matching them to historical performance and price data.[17] That line of research found that these patents had citation-based features that correlated strongly with technology's average improvement rates.[18] However, this data has not borne more accurate forecasts than trend extrapolation alone.[19]

Because matching patents and artifacts is so complex, the current results are not a definitive conclusion that patents cannot be used to improve technology forecasts. There are many ways in which the methods could be revised, including how technologies are matched with patent domains (e.g. perhaps they are a weighted sum of multiple domains) and how patent domains are defined (e.g. natural language processing may better extract important developments from patents than citation data). Given continued advances in

[16] A. Berdahl et al. "On the Records" (May 11, 2017). arXiv: `1705.04353 [physics]`

[17] Benson and Magee, see n. 11
Benson and Magee, "Technology Structural Implications from the Extension of a Patent Search Method", see n. 11

[18] Benson and Magee, "Quantitative Determination of Technological Improvement from Patent Data", see n. 12
Triulzi, Alstott, and Magee, see n. 12

[19] If one does not have data for a historical trend, then this particular patent data may still be useful for making estimates of which technologies are improving faster. These researchers are pursuing exactly this strategy: "System and Method for Quantifying and Presenting Information Representative of Technological Improvements in a Target Technological Domain Based on Patent Metrics". US20160358077 A1. C. L. Benson and C. L. Magee. International Classification G06F17/30, G06N5/02; Cooperative Classification G06F17/30424, G06N5/02. Dec. 8, 2016

large-scale data collection and processing, it's a reasonable prediction that technical data like patents will eventually be used to make forecasts that are more accurate than simple trend extrapolation.

Appendix

A1 Technology Data and Forecasts

A1.1 Metadata on Historical Trends

The following pages include metadata on the technology trend data used in this study. The data were collected by contacting the authors of the original journal articles in which these data were published, and the metadata are reproduced nearly exactly as they were received from the researchers. As such, they are rough and require interpretation, which is exactly the point: we did not want to overinterpret the data by simplifying or polishing it. The only modification to the data is that the data from the Farmer & Lafond paper is of the form "lower is better," with units to match. For this study we simply inverted that data so "higher is better," and thus the units are the reverse of those shown here.

Some data were identified as questionable, for reasons given previously, and are labeled as such here. Those data were not used in this study.

Technology Trend Name	Units	Original Article	Type	Number Data Points	First Year	Last Year	Domain (for Patents)
AcrylicFiber	1966 USD/lbs	Farmer_Lafond	Price	13	1960	1972	
Acrylonitrile	1966 USD/lbs	Farmer_Lafond	Price	14	1959	1972	
Aluminum	1966 USD/lbs	Farmer_Lafond	Price	17	1956	1972	
Ammonia	1966 USD/lbs	Farmer_Lafond	Price	13	1960	1972	
Aniline	1966 USD/lbs	Farmer_Lafond	Price	12	1961	1972	
Automotive (US)	Gallons/Mile	Farmer_Lafond	Performance_Questionable	21	1985	2005	COMB_ENGINE
Beer (Japan)	1955 Yen	Farmer_Lafond	Price	18	1951	1968	
Benzene	1958 USD	Farmer_Lafond	Price	17	1952	1968	
BisphenolA	1966 USD/lbs	Farmer_Lafond	Price	14	1959	1972	
Caprolactam	1966 USD/lbs	Farmer_Lafond	Price	11	1962	1972	
CarbonBlack	1966 USD/lbs	Farmer_Lafond	Price	9	1964	1972	
CarbonDisulfide	1966 USD/lbs	Farmer_Lafond	Price	10	1963	1972	
CCGT Power	1990 USD/kW	Farmer_Lafond	Price	10	1987	1996	
Concentrating Solar	US cents/kWh	Farmer_Lafond	Price	26	1980	2005	
Corn (US)	acres/1000 bushels	Farmer_Lafond	Price	34	1975	2008	HYBRID_CORN
Crude Oil	1958 USD	Farmer_Lafond	Price	23	1946	1968	
Cyclohexane	1966 USD/lbs	Farmer_Lafond	Price	17	1956	1972	
DNA Sequencing	2013 USD/human-size genome	Farmer_Lafond	Price	13	2001	2013	GENOME
DRAM	2005 USD/thousand bits	Farmer_Lafond	Price	37	1971	2007	MAGNETIC_INFO_STORAGE
Electric Range	1958 USD	Farmer_Lafond	Price	22	1946	1967	
Ethanol (Brazil)	2002 USD/GJ	Farmer_Lafond	Price	25	1980	2004	
Ethanolamine	1966 USD/lbs	Farmer_Lafond	Price	18	1955	1972	
Ethylene	1966 USD/lbs	Farmer_Lafond	Price	13	1960	1972	
Formaldehyde	1966 USD/lbs	Farmer_Lafond	Price	11	1962	1972	
Free Standing Gas Range	1958 USD	Farmer_Lafond	Price	22	1946	1967	
Geothermal Electricity	2005 US cents/kWh	Farmer_Lafond	Price	26	1980	2005	
Hard Disk Drive	2005 USD/megabyte	Farmer_Lafond	Price	20	1988	2007	MAGNETIC_INFO_STORAGE
HydrofluoricAcid	1966 USD/lbs	Farmer_Lafond	Price	11	1962	1972	
IsopropylAlcohol	1966 USD/lbs	Farmer_Lafond	Price	9	1964	1972	
Laser Diode	Yen	Farmer_Lafond	Price	13	1982	1994	
Low Density Polyethylene	1958 USD/pound	Farmer_Lafond	Price	17	1952	1968	
Magnesium	1966 USD/lbs	Farmer_Lafond	Price	19	1954	1972	
MaleicAnhydride	1966 USD/lbs	Farmer_Lafond	Price	14	1959	1972	
Methanol	1966 USD/lbs	Farmer_Lafond	Price	16	1957	1972	
Milk (US)	Heads/Mil.lbs	Farmer_Lafond	Performance	65	1930	2008	
Monochrome Television	1958 USD per unit	Farmer_Lafond	Price	22	1947	1968	
Motor Gasoline	1958 USD/Gallon	Farmer_Lafond	Price	23	1946	1968	
NeopreneRubber	1966 USD/lbs	Farmer_Lafond	Price	13	1960	1972	
Nuclear Electricity	2004 USD/Watt	Farmer_Lafond	Price	20	1970	1989	
Onshore Gas Pipeline	dollar/mile-inch	Farmer_Lafond	Price	14	1979	1992	
Paraxylene	1958 USD	Farmer_Lafond	Price	12	1957	1968	
Pentaerythritol	1966 USD/lbs	Farmer_Lafond	Price	21	1952	1972	
Phenol	1966 USD/lbs	Farmer_Lafond	Price	14	1959	1972	
Photovoltaics	2013 USD/Wp	Farmer_Lafond	Price	34	1980	2013	SOLAR_PV
PhthalicAnhydride	1966 USD/lbs	Farmer_Lafond	Price	18	1955	1972	
PolyesterFiber	1966 USD/lbs	Farmer_Lafond	Price	13	1960	1972	
PolyethyleneHD	1966 USD/lbs	Farmer_Lafond	Price	15	1958	1972	
PolyethyleneLD	1966 USD/lbs	Farmer_Lafond	Price	15	1958	1972	
Polypropylene	1958 USD/pound	Farmer_Lafond	Price	10	1959	1968	
Polystyrene	1958 USD/pound	Farmer_Lafond	Price	26	1943	1968	
Polyvinylchloride	1958 USD/pound	Farmer_Lafond	Price	23	1946	1968	
Primary Aluminum	1958 USD/pound	Farmer_Lafond	Price	40	1929	1968	
Primary Magnesium	1958 USD/pound	Farmer_Lafond	Price	40	1929	1968	
Refined Cane Sugar	1958 USD	Farmer_Lafond	Price	34	1935	1968	
Sodium	1966 USD/lbs	Farmer_Lafond	Price	16	1957	1972	

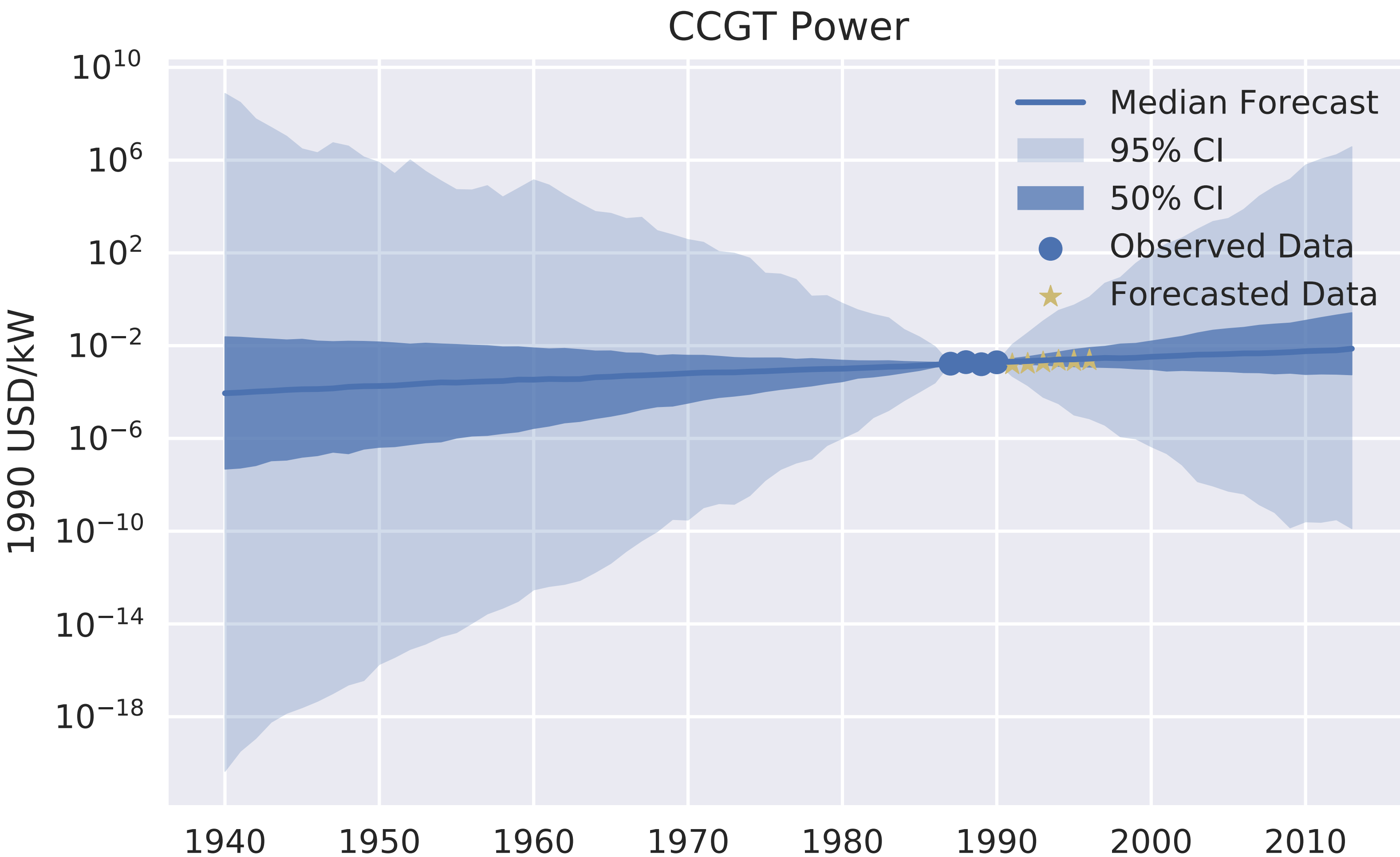
SodiumChlorate	1966 USD/lbs	Farmer_Lafond	Price	15	1958	1972	
SodiumHydrosulfite	1966 USD/lbs	Farmer_Lafond	Price	9	1964	1972	
Sorbitol	1966 USD/lbs	Farmer_Lafond	Price	8	1965	1972	
Styrene	1966 USD/lbs	Farmer_Lafond	Price	15	1958	1972	
Titanium Sponge	1958 USD/lbs	Farmer_Lafond	Price	19	1950	1968	
Titanium Dioxide	1966 USD/lbs	Farmer_Lafond	Price	9	1964	1972	
Transistor	2005 USD	Farmer_Lafond	Price	38	1968	2005	ELECTRIC_COMPUTATION
Urea	1966 USD/lbs	Farmer_Lafond	Price	12	1961	1972	
VinylAcetate	1966 USD/lbs	Farmer_Lafond	Price	13	1960	1972	
VinylChloride	1966 USD/lbs	Farmer_Lafond	Price	11	1962	1972	
Wind Turbine (Denmark)	DKK/kW	Farmer_Lafond	Price	20	1981	2000	WIND
integrated_circuit_memory_transistors_per_die	#/die	Magee_et_al	Performance	20	1959	2007	IC
integrated_circuit_microprocessor_transistors_per_die	#/die	Magee_et_al	Performance	12	1972	2006	IC
02A_magnetic_memory_tape_mbits_per_$	tape mbits/$	Magee_et_al	Price	14	1952	2004	MAGNETIC_INFO_STORAGE
02A_magnetic_memory_tape_mbits_per_cc	mbits/cc	Magee_et_al	Performance	13	1952	2004	MAGNETIC_INFO_STORAGE
02B_magnetic_memory_harddisk_mbits_per_$	mbits/$	Magee_et_al	Price	19	1956	2004	MAGNETIC_INFO_STORAGE
02B_magnetic_memory_harddisk_mbits_per_cc	mbit/cc	Magee_et_al	Performance	24	1956	2003	MAGNETIC_INFO_STORAGE
02C_magnetic_memory_tape_harddisk_mbits_per_$	tape & harddisk mbits/$	Magee_et_al	Price	27	1952	2004	MAGNETIC_INFO_STORAGE
02C_magnetic_memory_tape_harddisk_mbits_per_cc	tape & hard disk combined mbits/cc	Magee_et_al	Performance	25	1952	2004	MAGNETIC_INFO_STORAGE
optical_memory_per_$	optical mbits/$	Magee_et_al	Price	4	1998	2004	OPTICAL_INFO_STORAGE
optical_memory_per_cc	Mbits/cc	Magee_et_al	Performance	11	1981	2004	OPTICAL_INFO_STORAGE
electrical_info_transmission_kbps_per_$	band per cost per length (kbps / Million $ / Km)	Magee_et_al	Price	10	1858	1983	ELECTRIC_TELECOM
electrical_info_transmission_kbps	bandwidth kbps	Magee_et_al	Performance_Questionable	12	1858	1983	ELECTRIC_TELECOM
optical_telcom_bandwidth_kbps_per_$_km	Band per cost per length (kbps / Million $ / Km)	Magee_et_al	Price	12	1988	2002	OPTICAL_TELECOM
optical_telecom_bandwidth_kbps	Bandwidth kbps	Magee_et_al	Performance_Questionable	12	1988	2002	OPTICAL_TELECOM
Wireless_Telecomunication_coverage_bps_per_m2	Coverage Density (bps per Sqmts, Bandwith = 10 Mhz)	Magee_et_al	Performance	9	1901	2009	WIRELESS_TELECOM
Wireless_Telecomunication_spectral_efficiency_bps_per_hz	Spectral Efficiency (bps/Hz)	Magee_et_al	Performance	15	1901	2009	WIRELESS_TELECOM
Wireless_Telecomunication_throughput_kbps	Throughput (Kbps)	Magee_et_al	Performance	13	1895	2009	WIRELESS_TELECOM
computation_electronic_CPS	CPS	Magee_et_al	Performance	17	1943	2007	ELECTRIC_COMPUTATION
electronic_computation_mips_per_$	MIPS/Cost	Magee_et_al	Price	51	1943	2004	ELECTRIC_COMPUTATION
electronic_computation_mips	MIPS	Magee_et_al	Performance_Questionable	55	1943	2004	ELECTRIC_COMPUTATION
camera_sensitivity_mv_micro_sqm	mv/?m^2	Magee_et_al	Performance	11	1987	2008	CAMERA
MRI_mm_sec_$_resolution_cost	1/(res.sec.$)	Magee_et_al	Price_Questionable	6	1980	2006	MRI
MRI_mm_sec_resolution	1/(resol*scan time)	Magee_et_al	Performance	10	1949	2006	MRI
CTscan_mm_scantime_resolution	1/(resol*scan time)	Magee_et_al	Performance	13	1971	2006	CT
Genome_sequencing_base_pairs	BP/$	Magee_et_al	Price	8	1970	2004	GENOME
batteries_wh_$	Wh/$	Magee_et_al	Price	22	1950	2005	BATTERIES
batteries_wh_kg_energy_density	wh/kg	Magee_et_al	Performance	19	1884	2004	BATTERIES
batteries_wh_lit_energy_density	wh/lit	Magee_et_al	Performance	21	1884	2003	BATTERIES
capacitors_kwhr_$	Wh/$	Magee_et_al	Price	7	1985	2005	CAPACITOR
capacitors_kwhr_kg	Wh/kg	Magee_et_al	Performance	11	1962	2005	CAPACITOR
capacitors_kwhr_lit	Wh/Liter	Magee_et_al	Performance	7	1941	2000	CAPACITOR
Flywheel_kwh_kg	kwh/kg	Magee_et_al	Performance	7	1975	2003	FLYWHEEL
AC_electricity_transmission_powered_distance_per_cost_wkm_$	W?Km/$, Powered distance per cost	Magee_et_al	Price	9	1889	1983	ELECTRO_POWERTRANS
AC_electricity_transmission_powered_distance_wkm	W?Km, Powered distance	Magee_et_al	Performance_Questionable	9	1889	1983	ELECTRO_POWERTRANS
DC_electricity_transmission_powered_distance_per_cost_wkm_$	W?Km/$, Powered distance per cost	Magee_et_al	Price	13	1962	2000	ELECTRO_POWERTRANS
DC_electricity_transmission_powered_distance_wkm	W?Km, Powered distance	Magee_et_al	Performance_Questionable	15	1962	2000	ELECTRO_POWERTRANS
combustion_air_piston_engine_w_per_$	W/$	Magee_et_al	Price	13	1919	1945	COMB_ENGINE
combustion_air_piston_engine_w_per_kg	W/kg	Magee_et_al	Performance	8	1919	1945	COMB_ENGINE
combustion_air_piston_engine_w_per_lit	w/lit	Magee_et_al	Performance	13	1888	1945	COMB_ENGINE
combustion_air_turbine_engine_w_per_$	W/$	Magee_et_al	Price	10	1963	2002	COMB_ENGINE
combustion_air_turbine_engine_w_per_kg	W/kg	Magee_et_al	Performance	10	1963	2002	COMB_ENGINE
combustion_air_turbine_engine_w_per_lit	W/lit	Magee_et_al	Performance	11	1955	2002	COMB_ENGINE
combustion_pass_car_engine_w_per_$	W/$	Magee_et_al	Price	56	1896	1994	COMB_ENGINE
combustion_pass_car_engine_w_per_kg	W/kg	Magee_et_al	Performance	17	1896	1994	COMB_ENGINE
combustion_pass_car_piston_engine_w_per_lit	W/Liter	Magee_et_al	Performance	27	1931	2001	COMB_ENGINE

electricmotor_w_per_kg_11.14.2013	W/kg	Magee_et_al	Performance	11	1881	1993	ELECTRIC_MOTOR
electricmotor_w_per_lit_11.14.2013	W/Liter	Magee_et_al	Performance	9	1890	1995	ELECTRIC_MOTOR
solar_PV_Wpeak_$	Watts per dollar	Magee_et_al	Price	35	1968	2009	SOLAR_PV
solar_PV_kwh_module$2011	module kwhr/2011$	Magee_et_al	Price	34	1977	2010	SOLAR_PV
wind_turbine_W_$	W/$	Magee_et_al	Price	22	1947	2011	WIND
FuelCell_kW_$	kW/$	Magee_et_al	Price	9	1963	1996	FUELCELL
Artificial_Illumination_Incandescent_1000lumenhrs_per_$	1000 lumenhour/$	Magee_et_al	Price	11	1883	1990	INCANDESCENT
Artificial_Illumination_LED_lumen_$	lumen/$	Magee_et_al	Price_Questionable	13	1972	2009	LED
Artificial_Illumination_LED_lumen_lamp	lumen/lamp	Magee_et_al	Performance_Questionable	17	1967	2010	LED
aeroplane_transport_pass_mile_hr	Pass-miles/hr	Magee_et_al	Performance_Questionable	12	1926	1975	AIRCRAFT
Milling_yearlyaverage_hp.accuracy	average HP/total averge accuracy	Magee_et_al	Performance	5	1939	2012	MILLING
3dprintingSLA_speed_buildvolume_layerthickness_cost	Speed * build volume/(layer thickness *cost)	Magee_et_al	Price	5	1991	2006	TRID_PRINTING
3dprintingSLA_speed_buildvolume_layerthickness_machinesize_cost	Speed * build volume/(layer thickness *machine size*cost)	Magee_et_al	Price	5	1991	2006	TRID_PRINTING
3dprintingSLA_speed_layerthickness_cost	Speed/(layer thickness*cost)	Magee_et_al	Price	5	1991	2006	TRID_PRINTING
3dprintingSLA_speed_layerthickness	Speed/layer thickness	Magee_et_al	Performance	3	1991	2006	TRID_PRINTING
photolithography_arealthroughput_accuracy_cost	areal Throughput/(resolution*cost)	Magee_et_al	Price	4	1969	1986	PHOTOLITHOGRAPHY
photolithography_arealthroughput_accuracy	areal Throughput/accuracy	Magee_et_al	Performance	7	1962	1986	PHOTOLITHOGRAPHY
photolithography_arealthroughput_cost	areal throughput/cost	Magee_et_al	Price	4	1969	1986	PHOTOLITHOGRAPHY
superconductivity_critical_temperature	deg K	Magee_et_al	Performance	13	1911	1995	SUPERCONDUCTOR

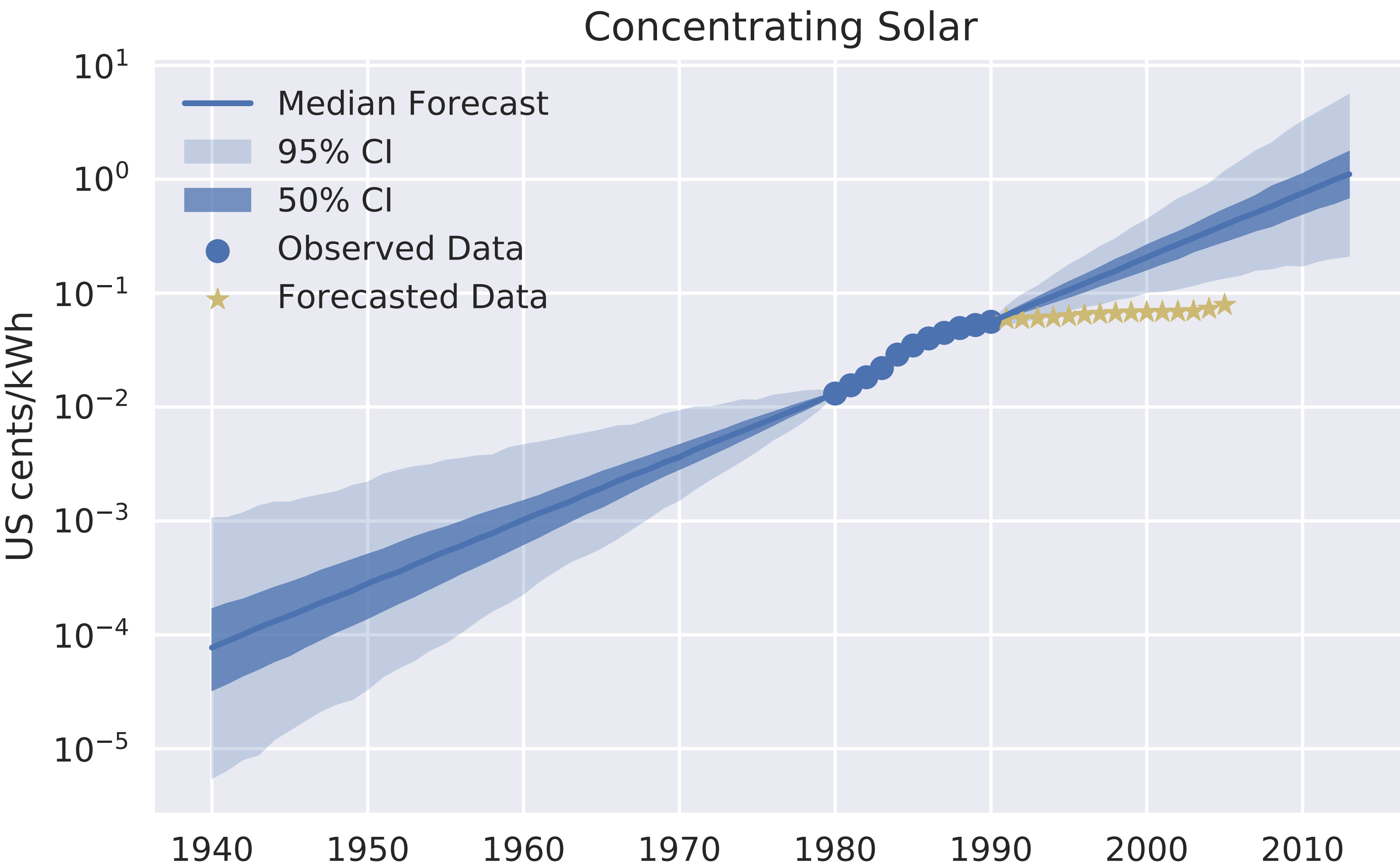
A1.2 All Technology Forecasts

The following pages show the trends and forecasts for all technologies in the study (first price trends, then performance trends). Note that in price trends the technology can get both better and worse, and so the forecasts reflect this, while in performance trends the technology is defined as only able to improve with time, and so the forecasts reflect that. Additionally, some of these technology trends had missing data (i.e. years in which there were no observed or recorded data points). The software developed for this study automatically makes estimates on what the missing data was, just as it does for the forecasts (which is just "missing" future data!). Such missing data could include data before the data point; there's no mathematical difference between making forecasts into the future vs. backcasts into the past, and so these are automatically generated by the software.

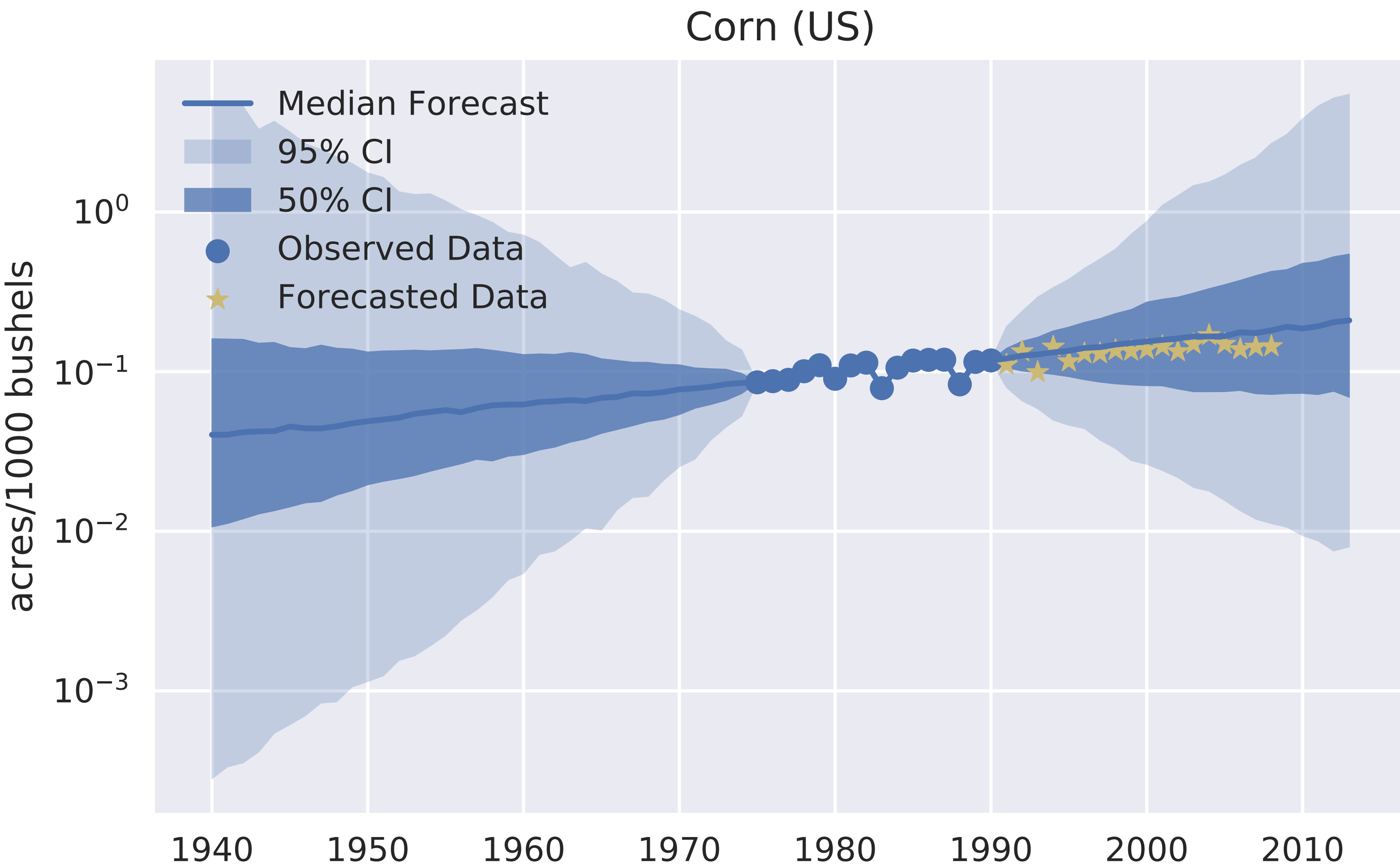
As described in the previous section, the technology names and units are shown exactly as they were received from the original researchers. Note that, as described previously, for the data from Farmer & Lafond the units are of the form "lower is better," and so to match with the rest of the data these were inverted. Thus the units are the reverse of those shown for these graphs.

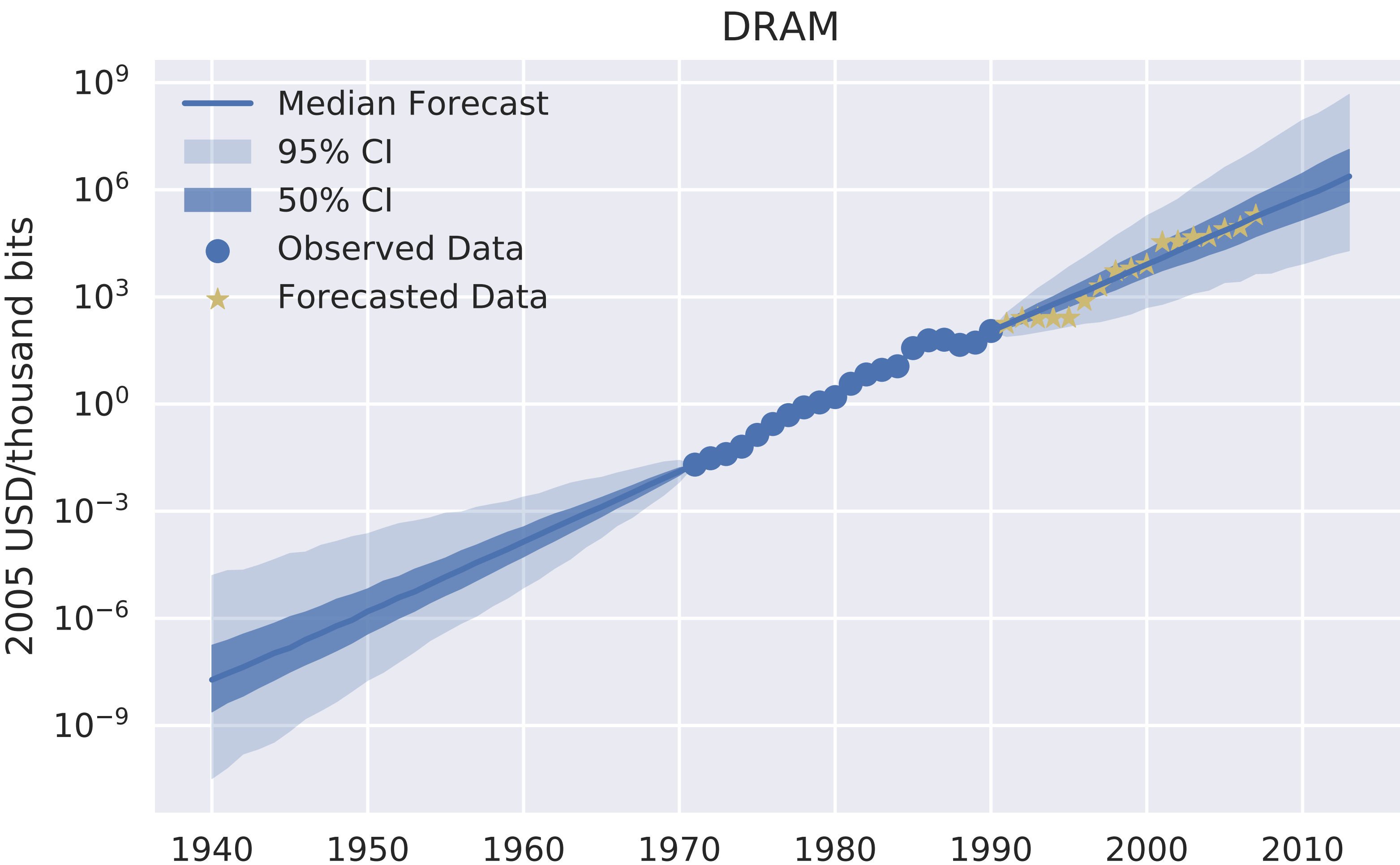


CCGT Power
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
10^{10}
10^{6}
10^{2}
10^{-2}
10^{-6}
10^{-10}
10^{-14}
10^{-18}
1990 USD/kW
1940
1950
1960
1970
1980
1990
2000
2010

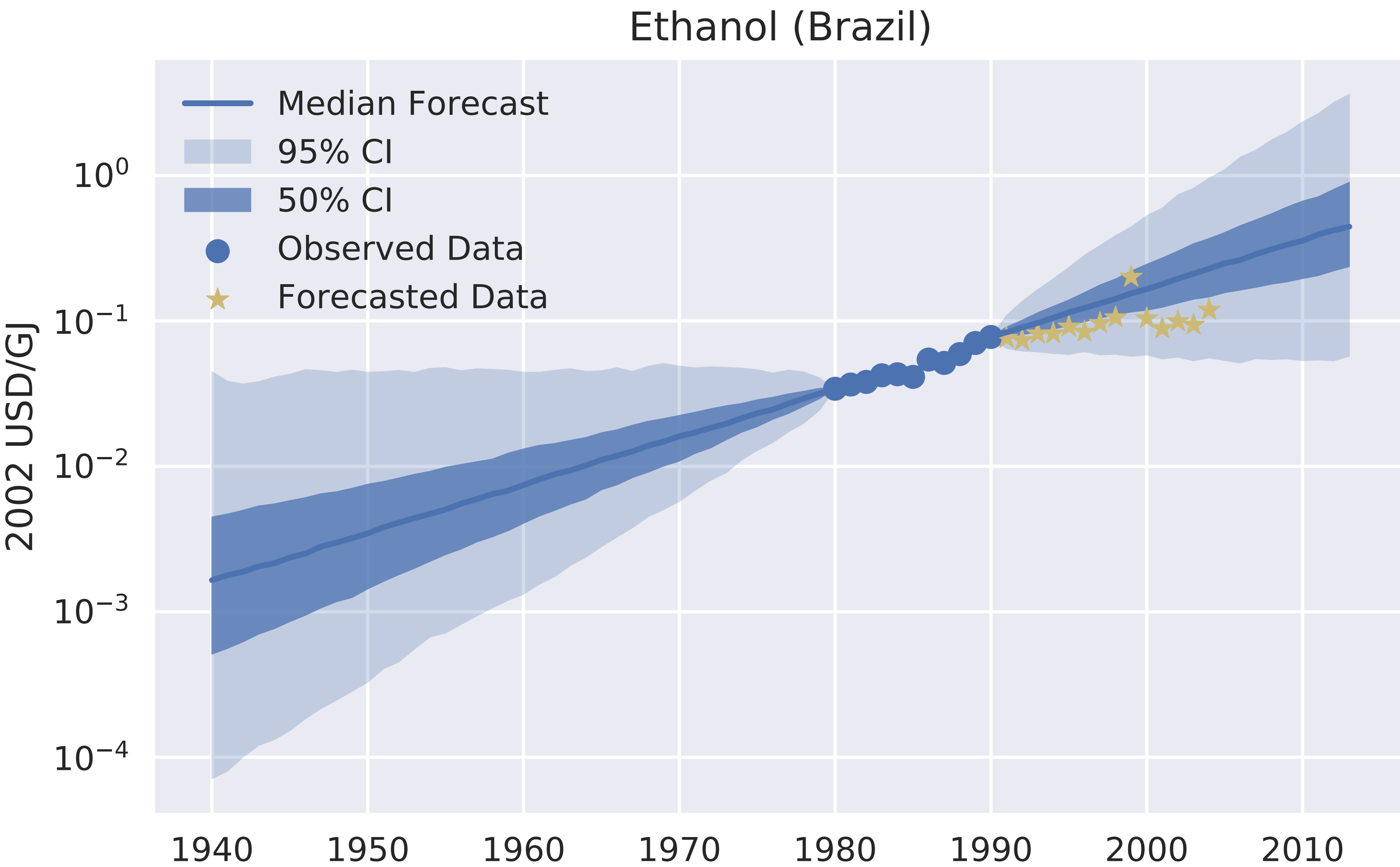


Concentrating Solar
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
US cents/kWh
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
1940
1950
1960
1970
1980
1990
2000
2010

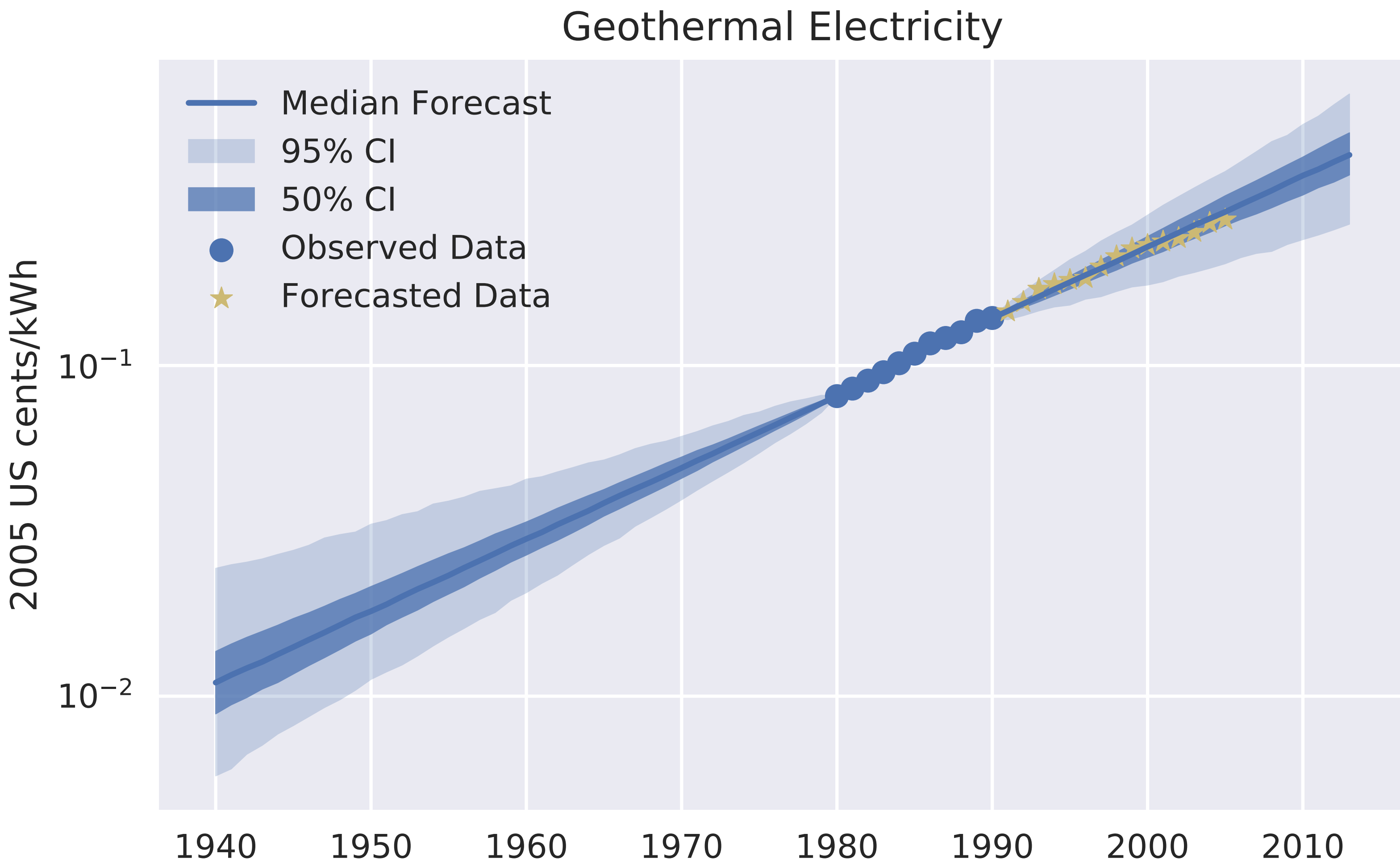

Corn (US)
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
acres/1000 bushels
10^{0}
10^{-1}
10^{-2}
10^{-3}
1940
1950
1960
1970
1980
1990
2000
2010



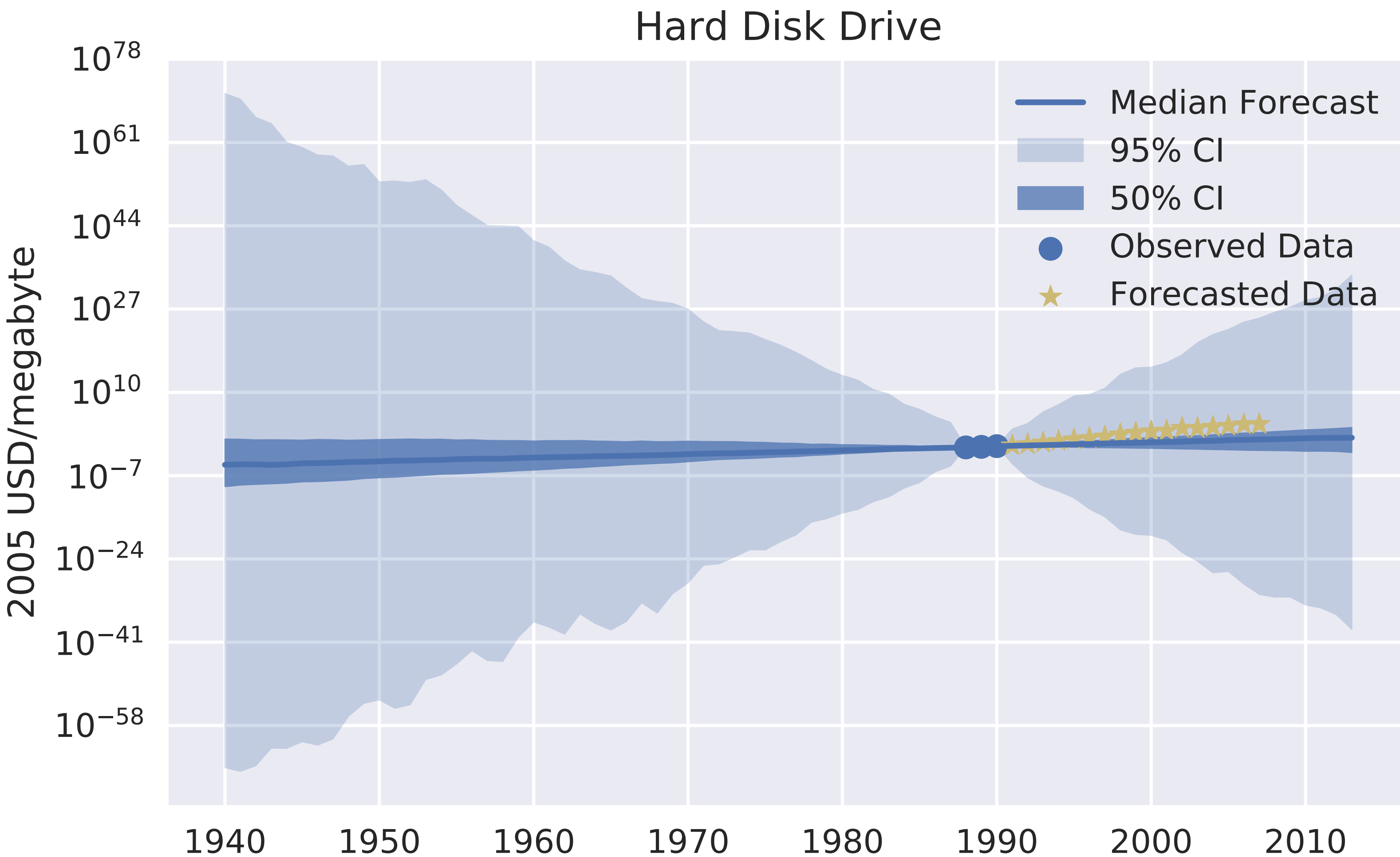
DRAM
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
2005 USD/thousand bits
10^9
10^6
10^3
10^0
10^{-3}
10^{-6}
10^{-9}
1940
1950
1960
1970
1980
1990
2000
2010



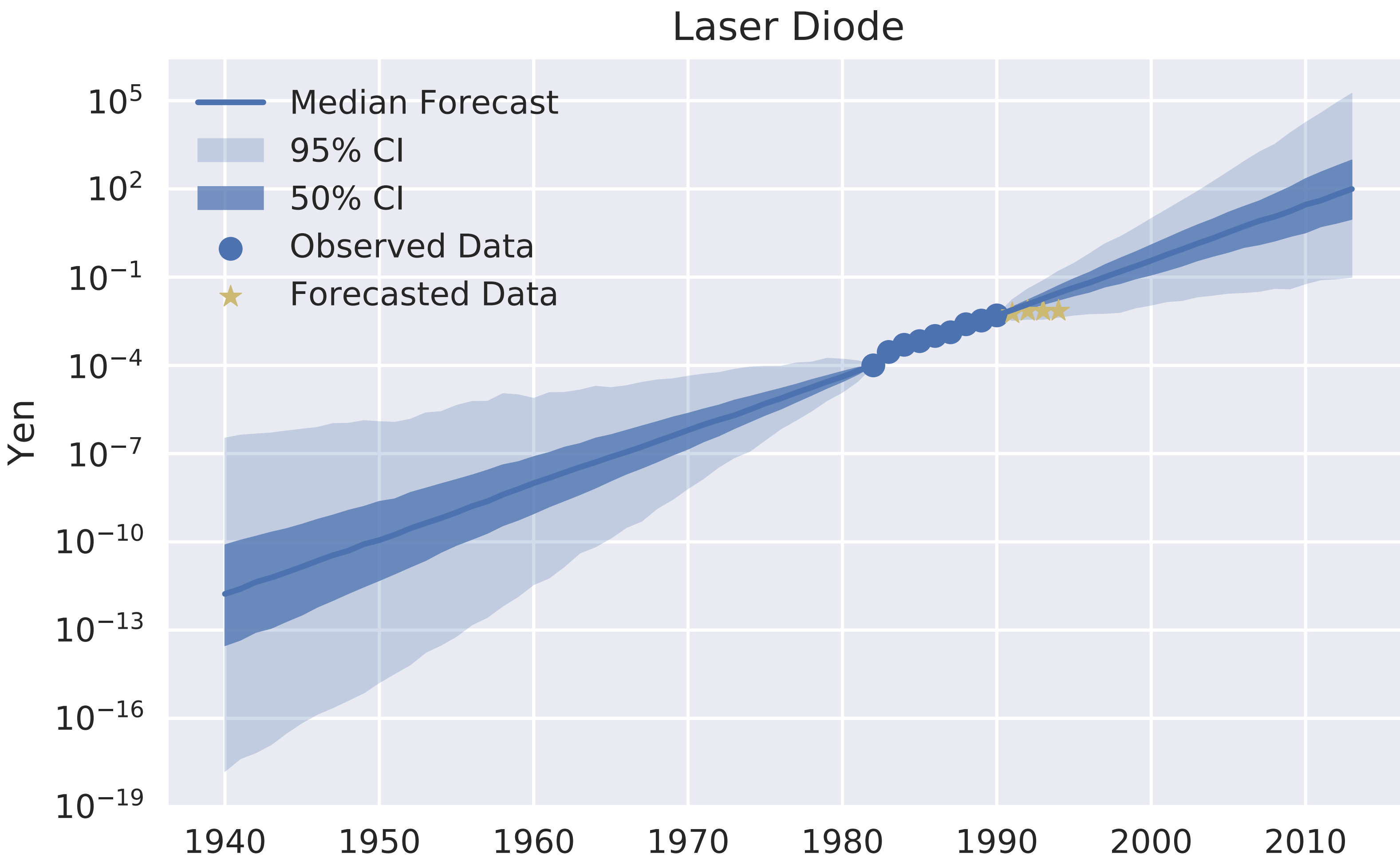
Ethanol (Brazil)
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
2002 USD/GJ
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
1940
1950
1960
1970
1980
1990
2000
2010



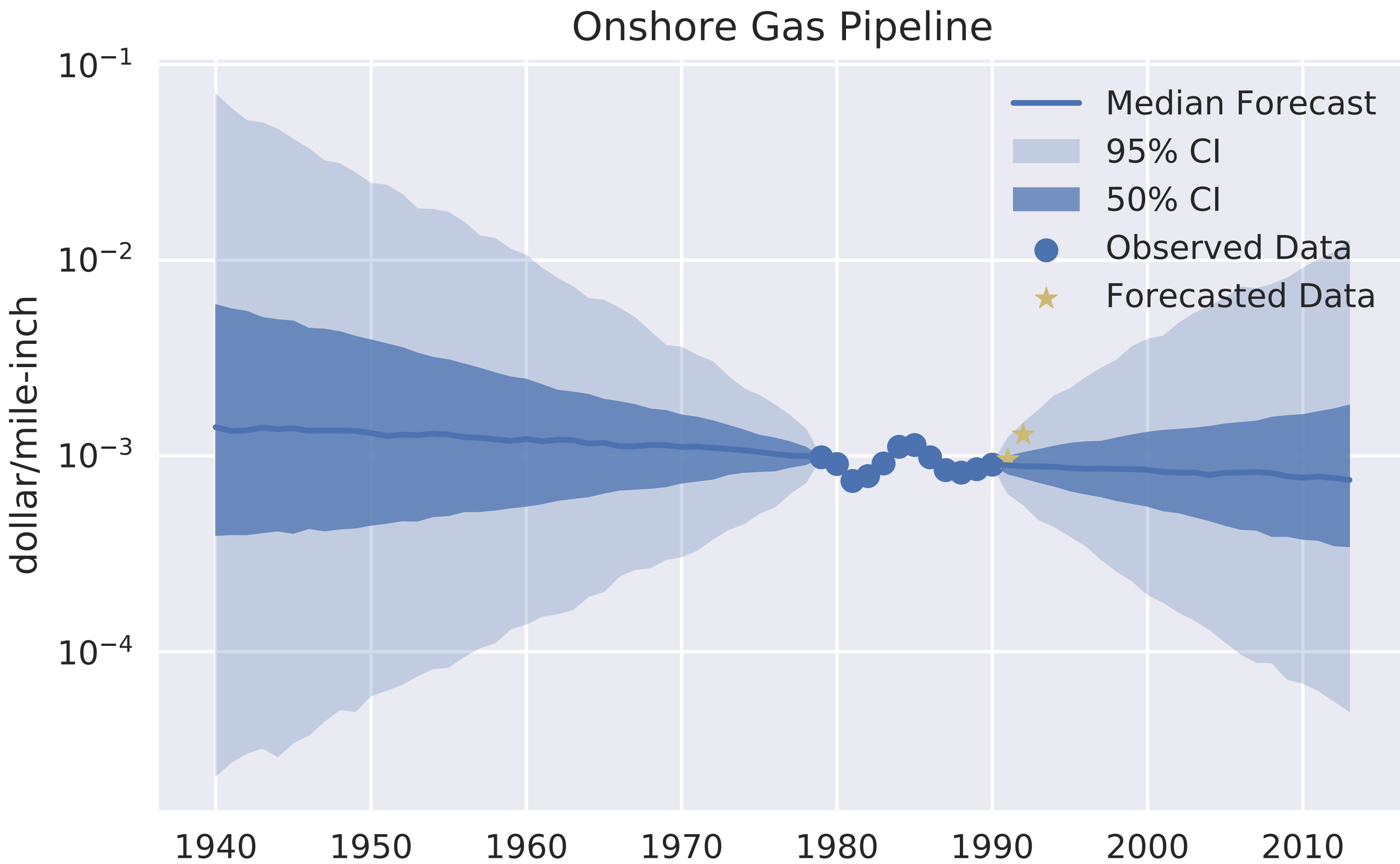
Geothermal Electricity
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
2005 US cents/kWh
10^{-1}
10^{-2}
1940
1950
1960
1970
1980
1990
2000
2010



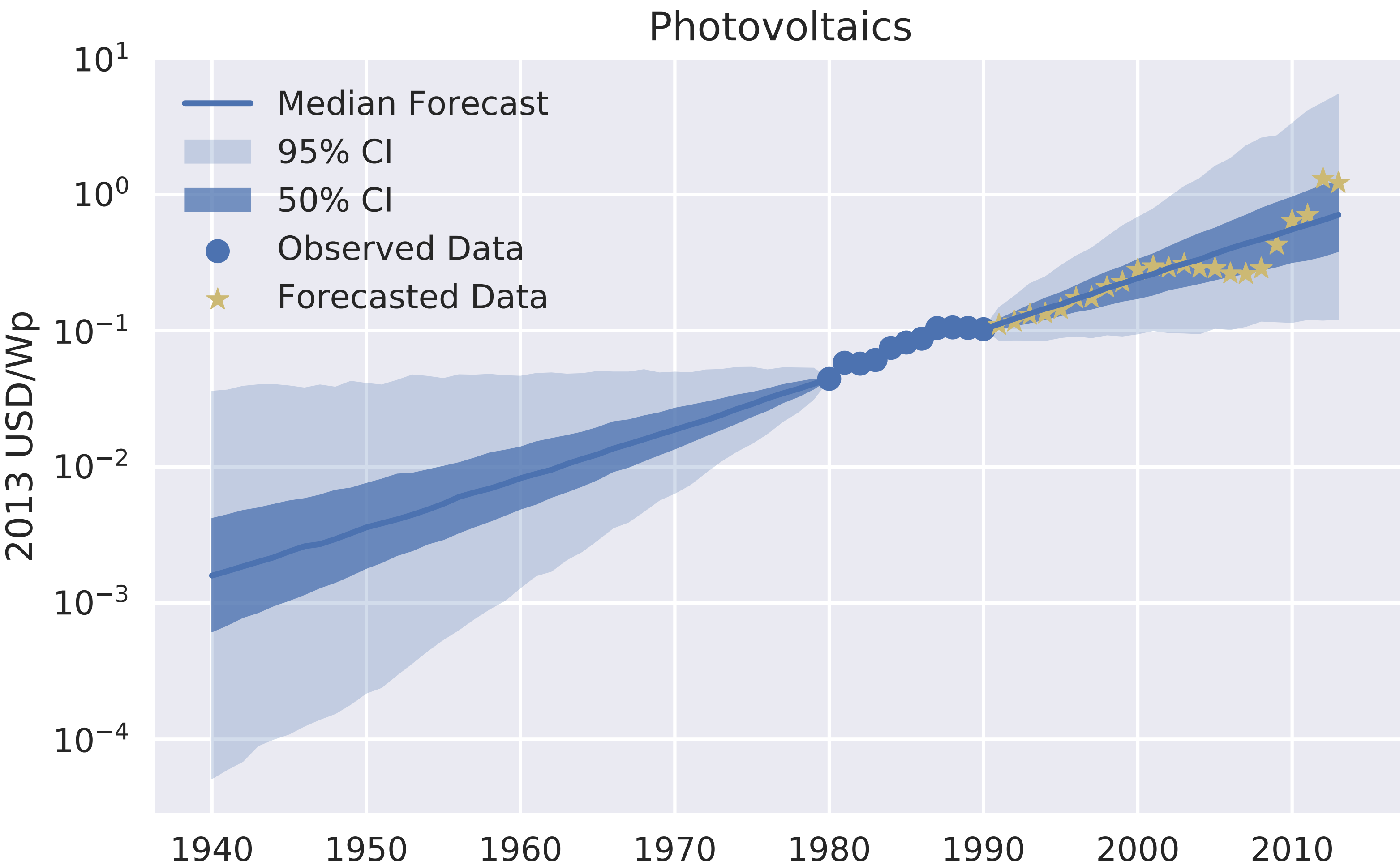
Hard Disk Drive
2005 USD/megabyte
10^{78}
10^{61}
10^{44}
10^{27}
10^{10}
10^{-7}
10^{-24}
10^{-41}
10^{-58}
1940
1950
1960
1970
1980
1990
2000
2010
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data

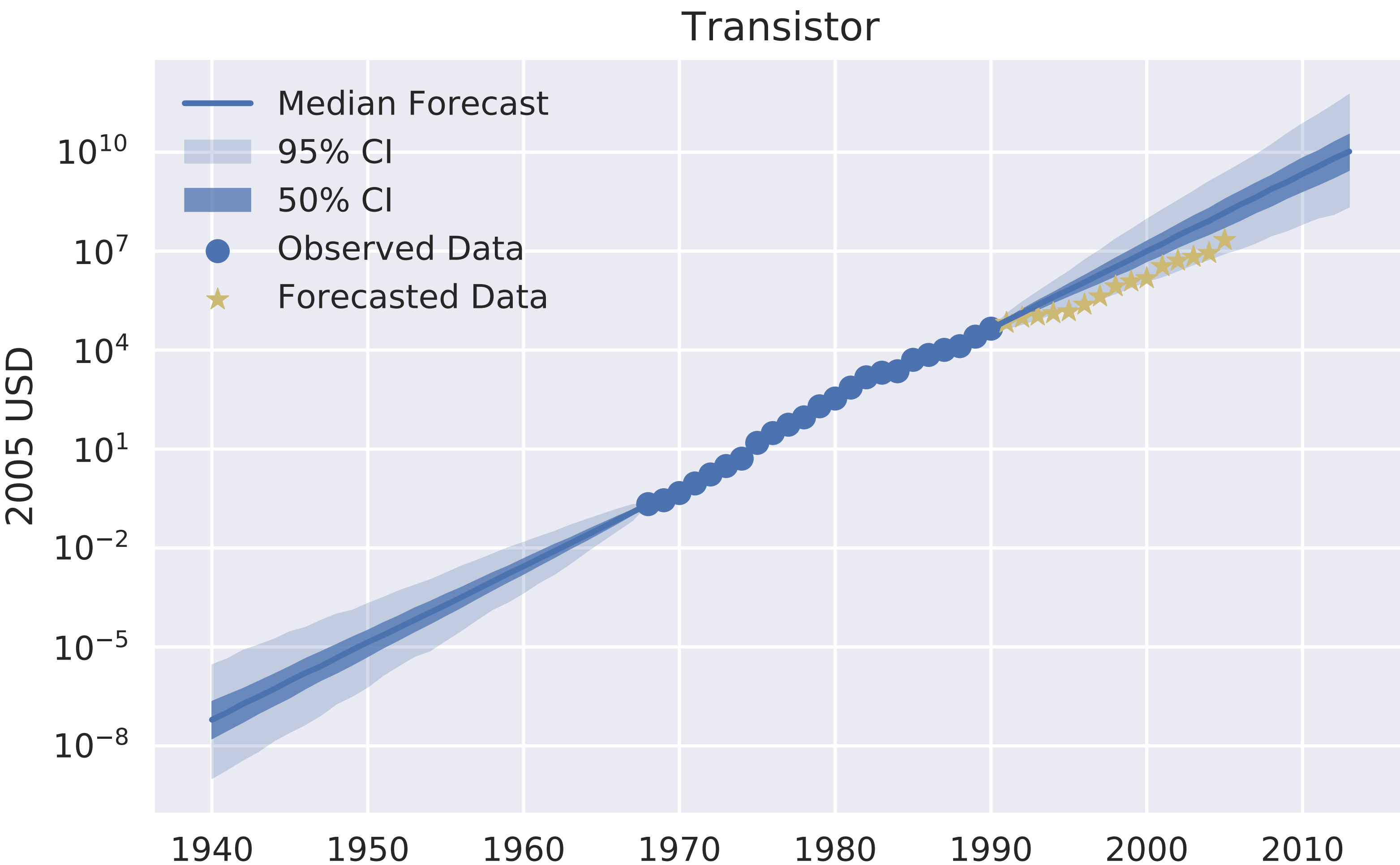


Laser Diode
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
Yen
10^{5}
10^{2}
10^{-1}
10^{-4}
10^{-7}
10^{-10}
10^{-13}
10^{-16}
10^{-19}
1940
1950
1960
1970
1980
1990
2000
2010

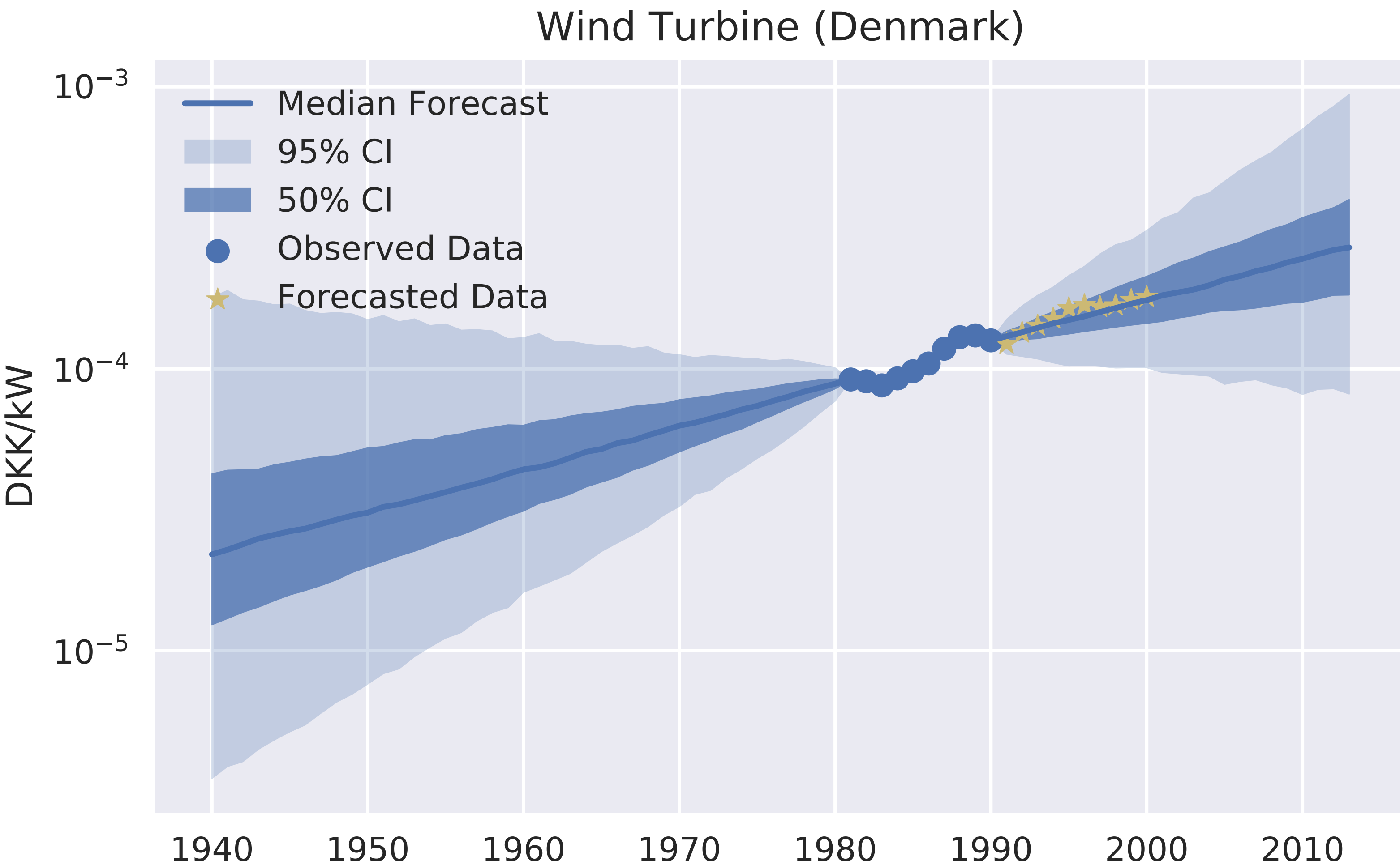


Onshore Gas Pipeline
dollar/mile-inch
10^{-1}
10^{-2}
10^{-3}
10^{-4}
1940
1950
1960
1970
1980
1990
2000
2010
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data

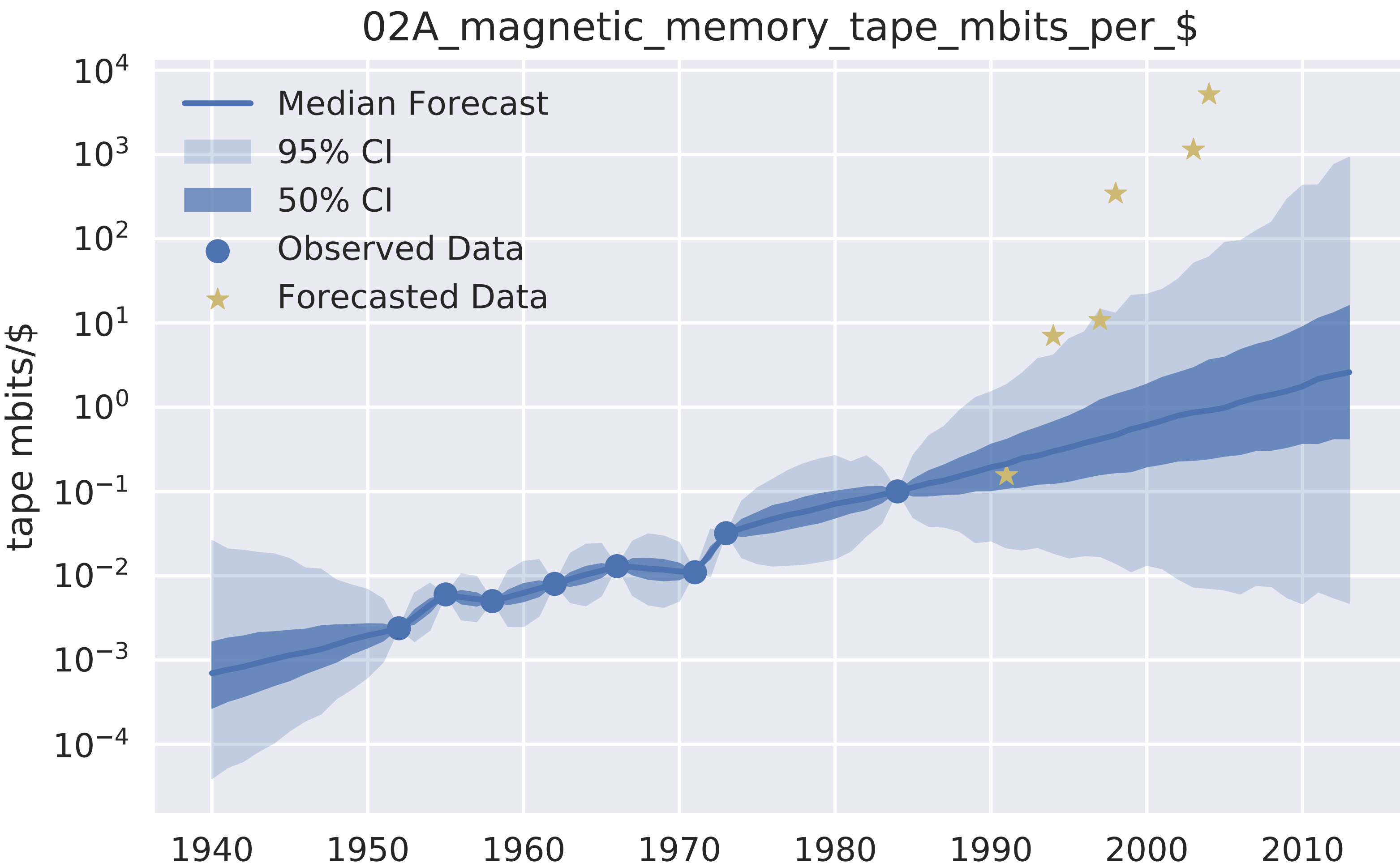

Photovoltaics
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
2013 USD/Wp
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
1940
1950
1960
1970
1980
1990
2000
2010

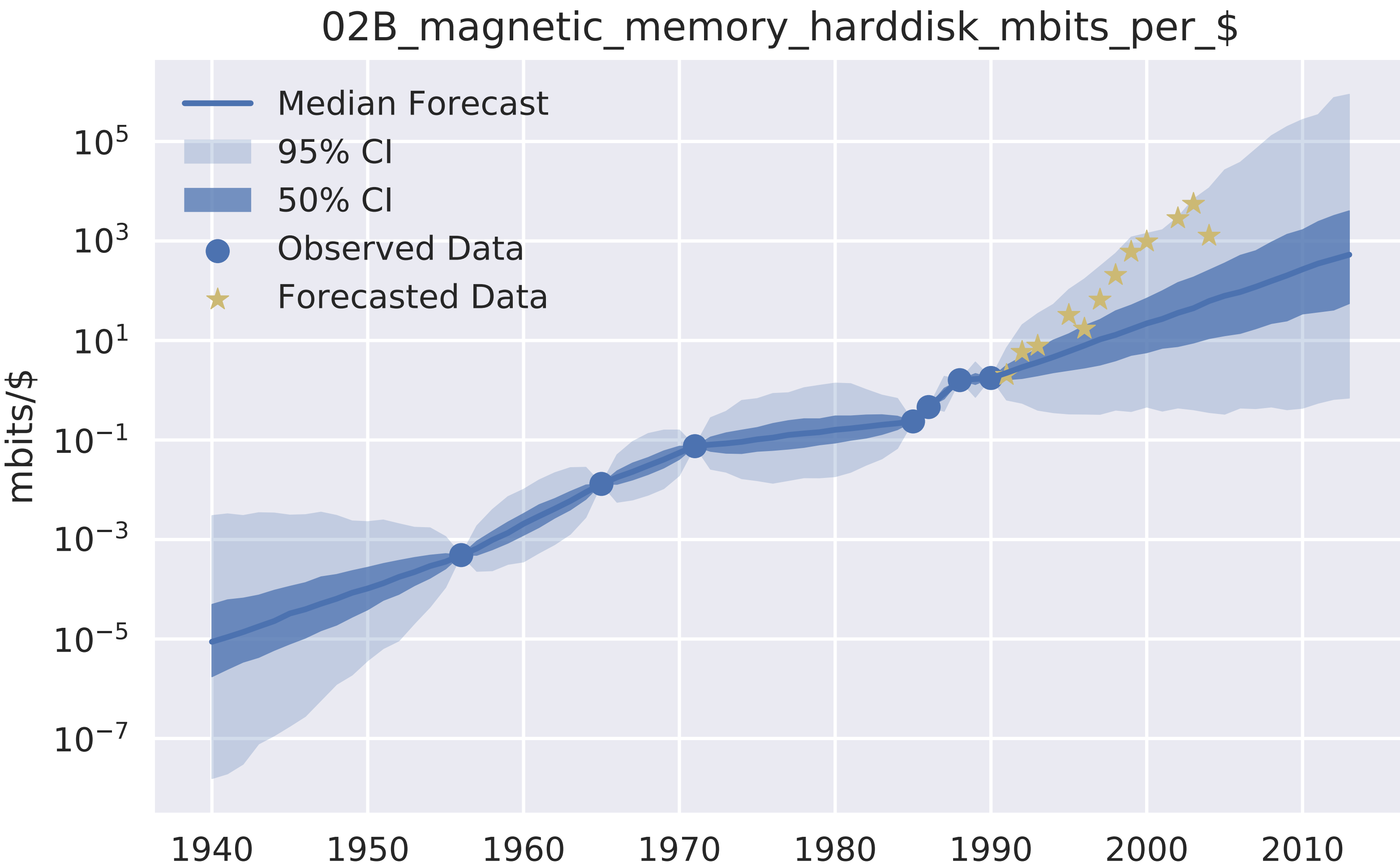


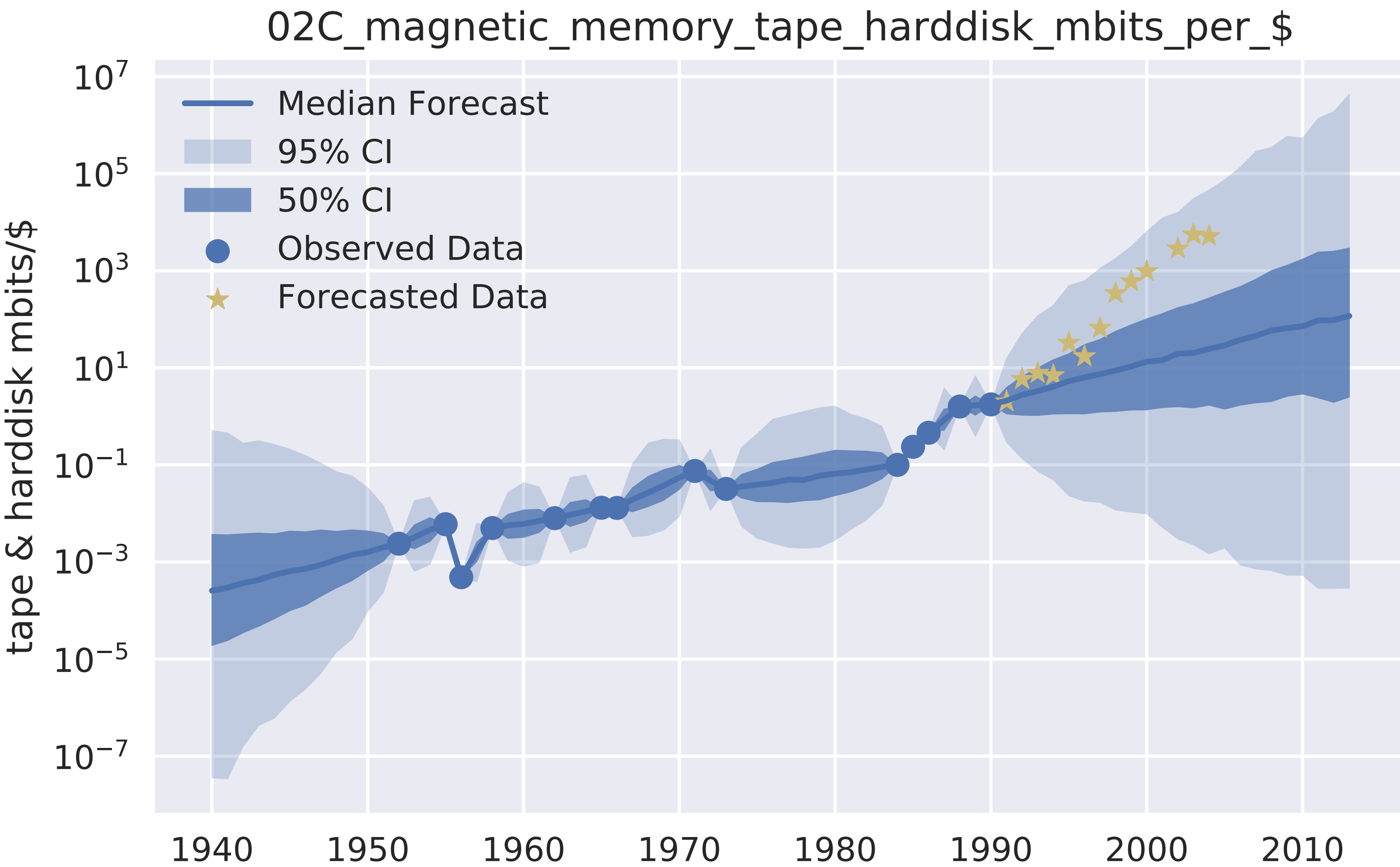
Transistor
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
2005 USD
10^{10}
10^{7}
10^{4}
10^{1}
10^{-2}
10^{-5}
10^{-8}
1940
1950
1960
1970
1980
1990
2000
2010

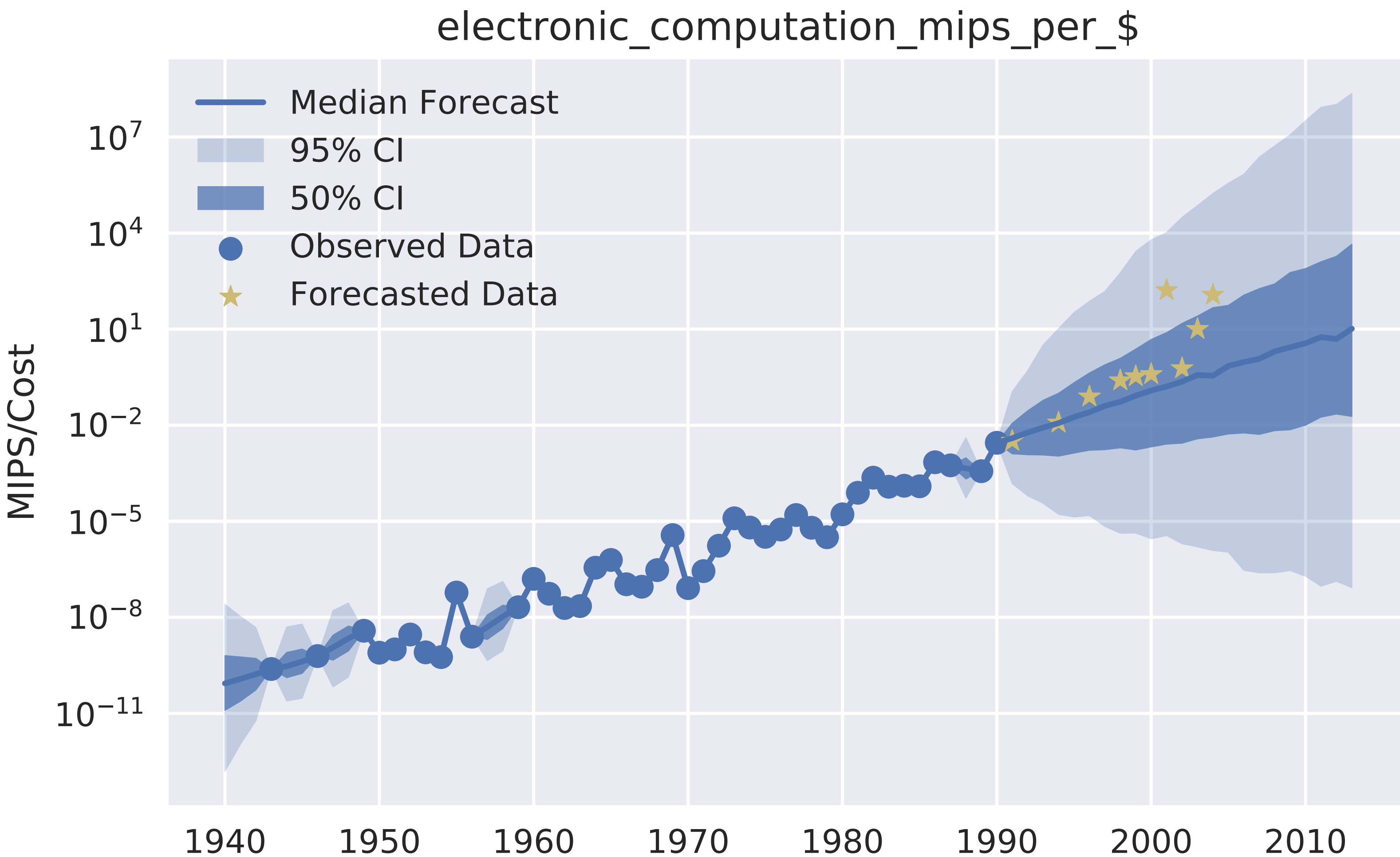


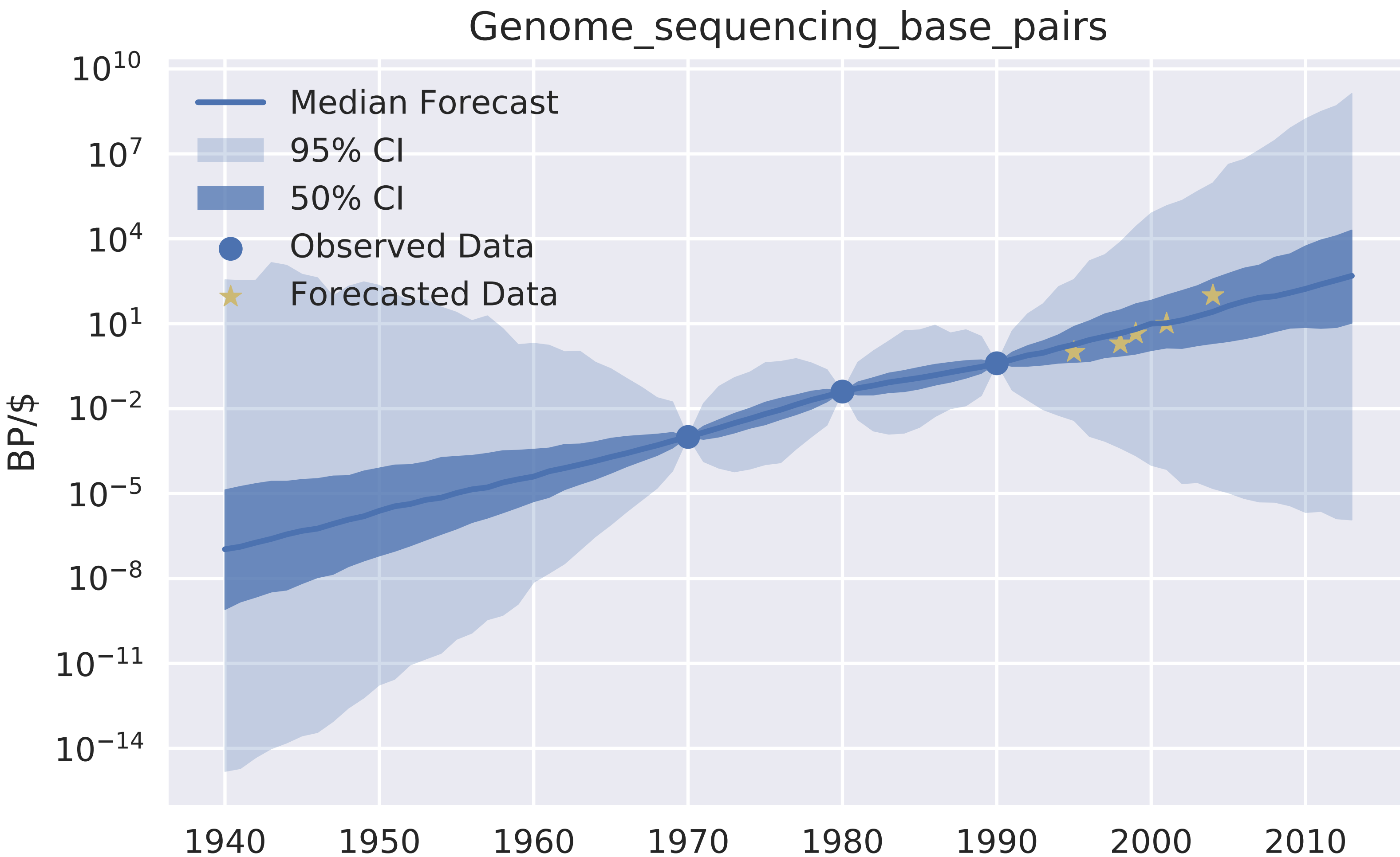
Wind Turbine (Denmark)
10^{-3}
10^{-4}
10^{-5}
DKK/kW
1940
1950
1960
1970
1980
1990
2000
2010
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data


02A_magnetic_memory_tape_mbits_per_$
tape mbits/$
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
1940
1950
1960
1970
1980
1990
2000
2010

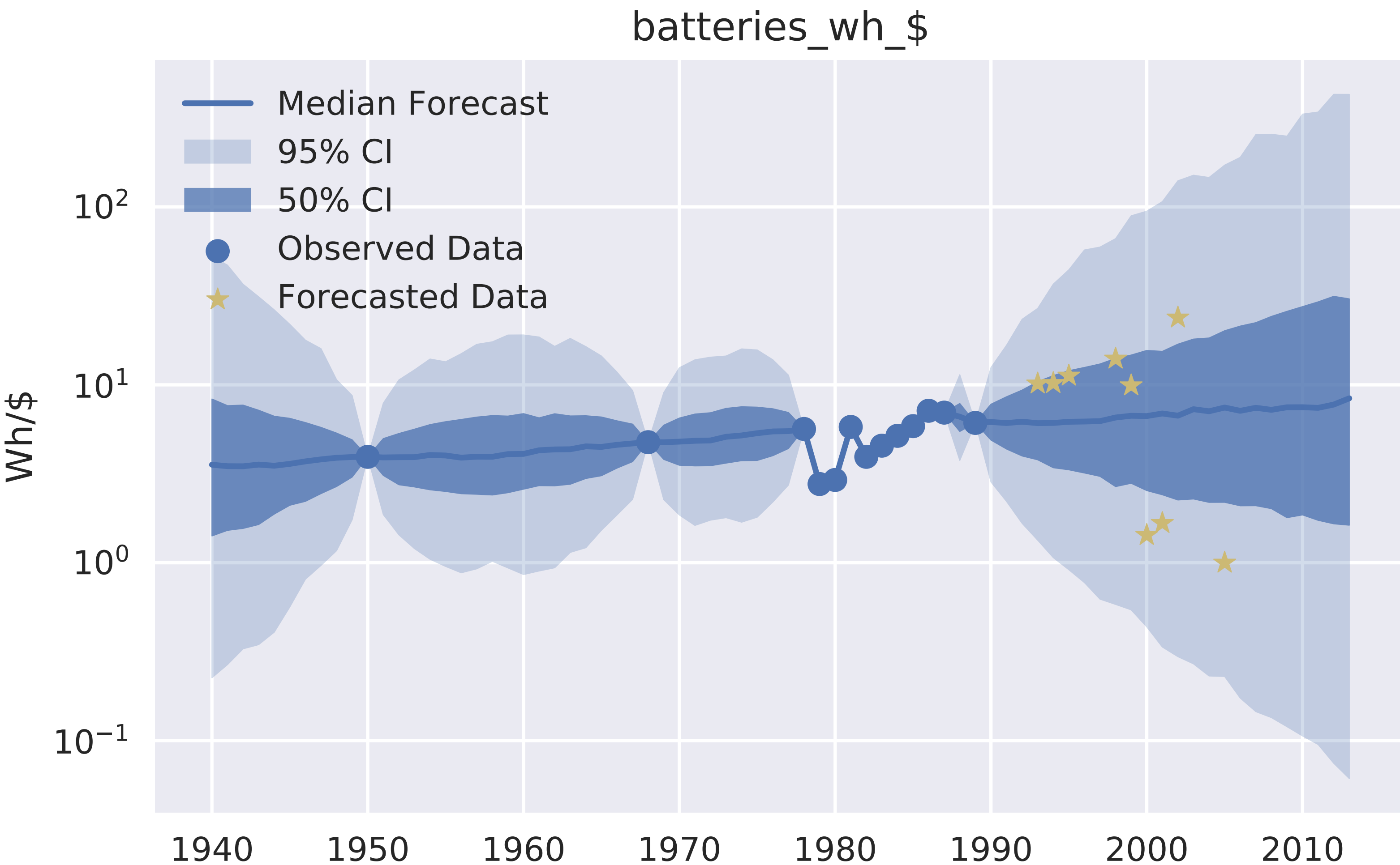

02B_magnetic_memory_harddisk_mbits_per_$
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
mbits/$
10^5
10^3
10^1
10^{-1}
10^{-3}
10^{-5}
10^{-7}
1940
1950
1960
1970
1980
1990
2000
2010

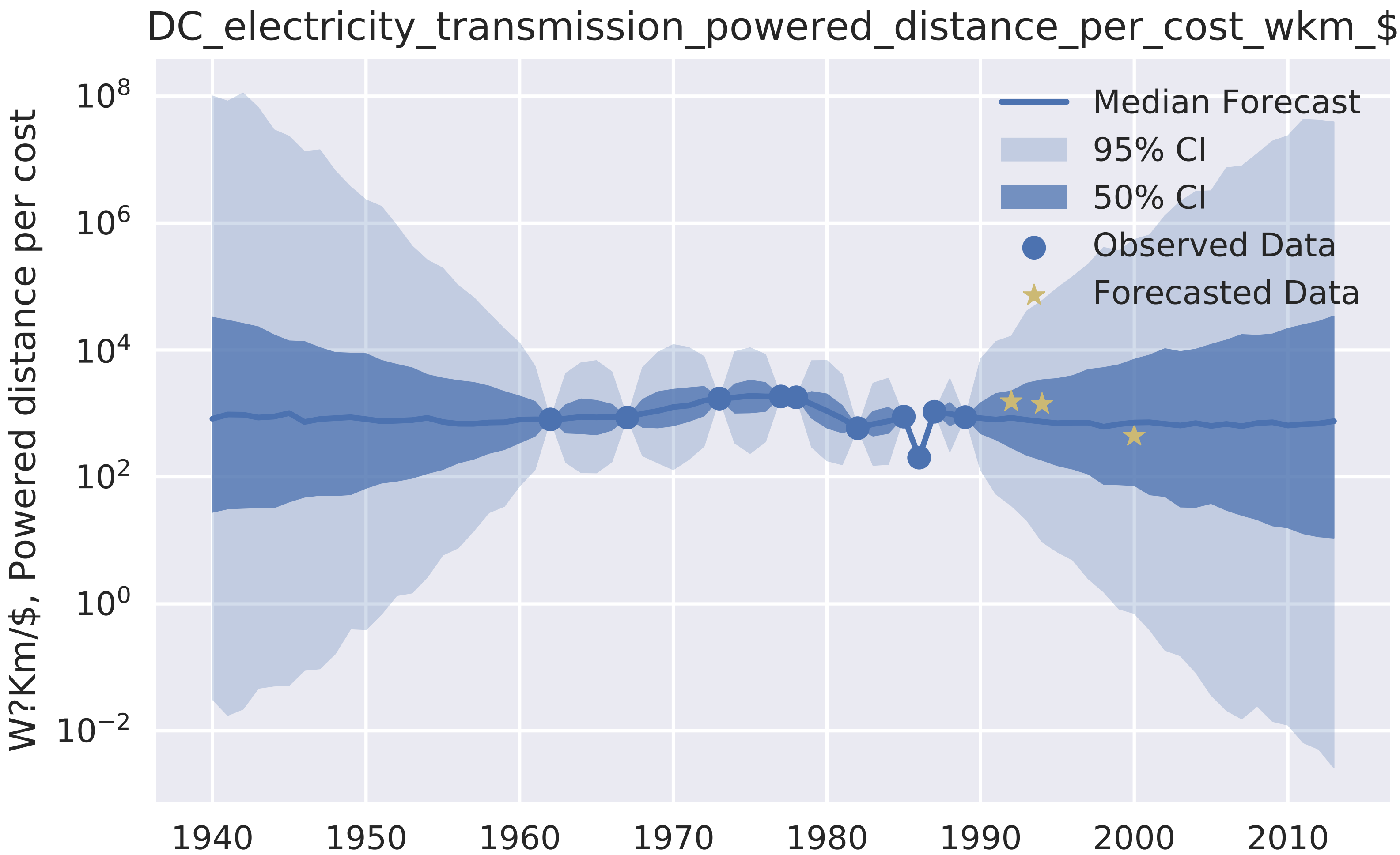

02C_magnetic_memory_tape_harddisk_mbits_per_$
tape & harddisk mbits/$
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
10^{7}
10^{5}
10^{3}
10^{1}
10^{-1}
10^{-3}
10^{-5}
10^{-7}
1940
1950
1960
1970
1980
1990
2000
2010


electronic_computation_mips_per_$
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
MIPS/Cost
10^{7}
10^{4}
10^{1}
10^{-2}
10^{-5}
10^{-8}
10^{-11}
1940
1950
1960
1970
1980
1990
2000
2010

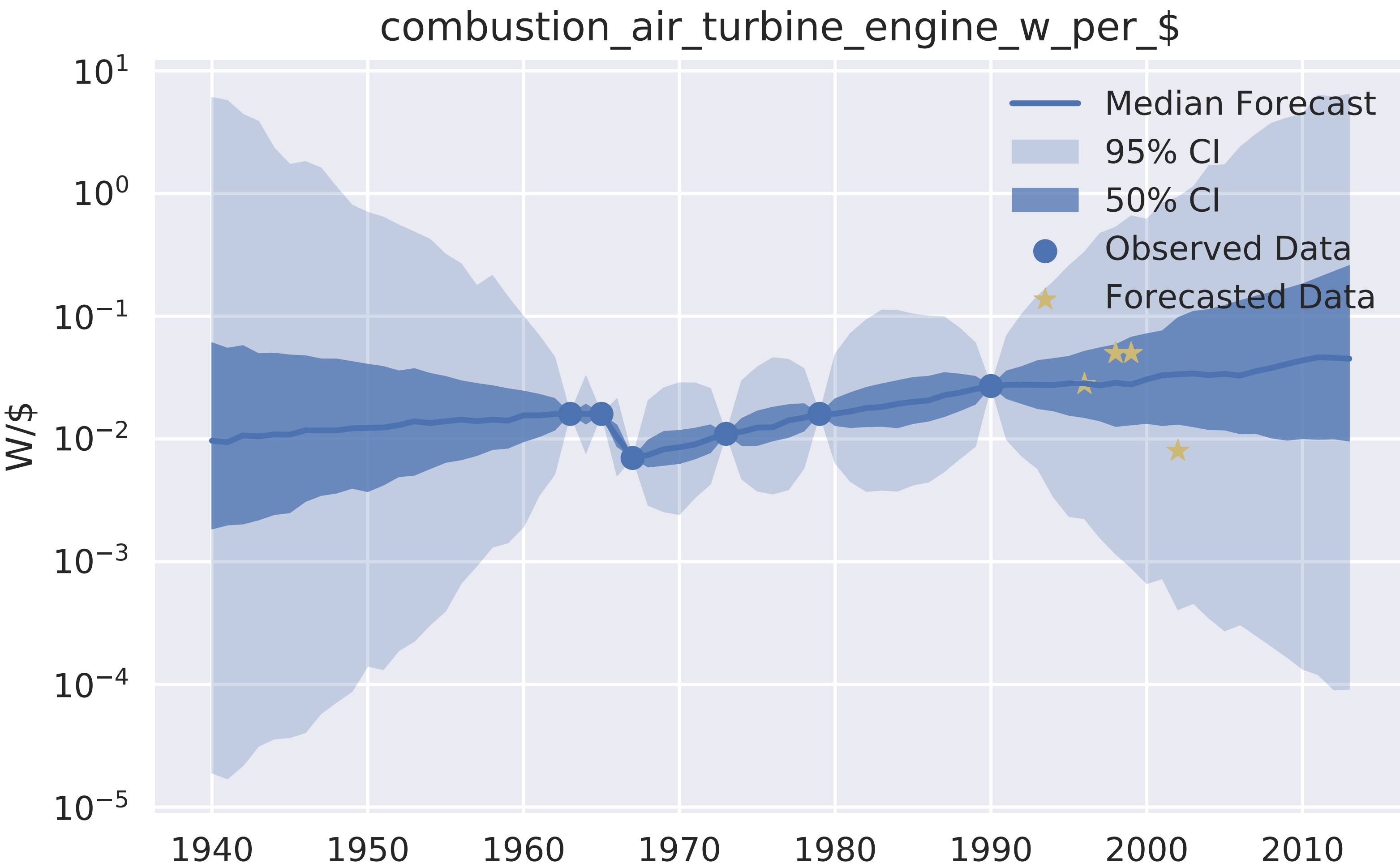


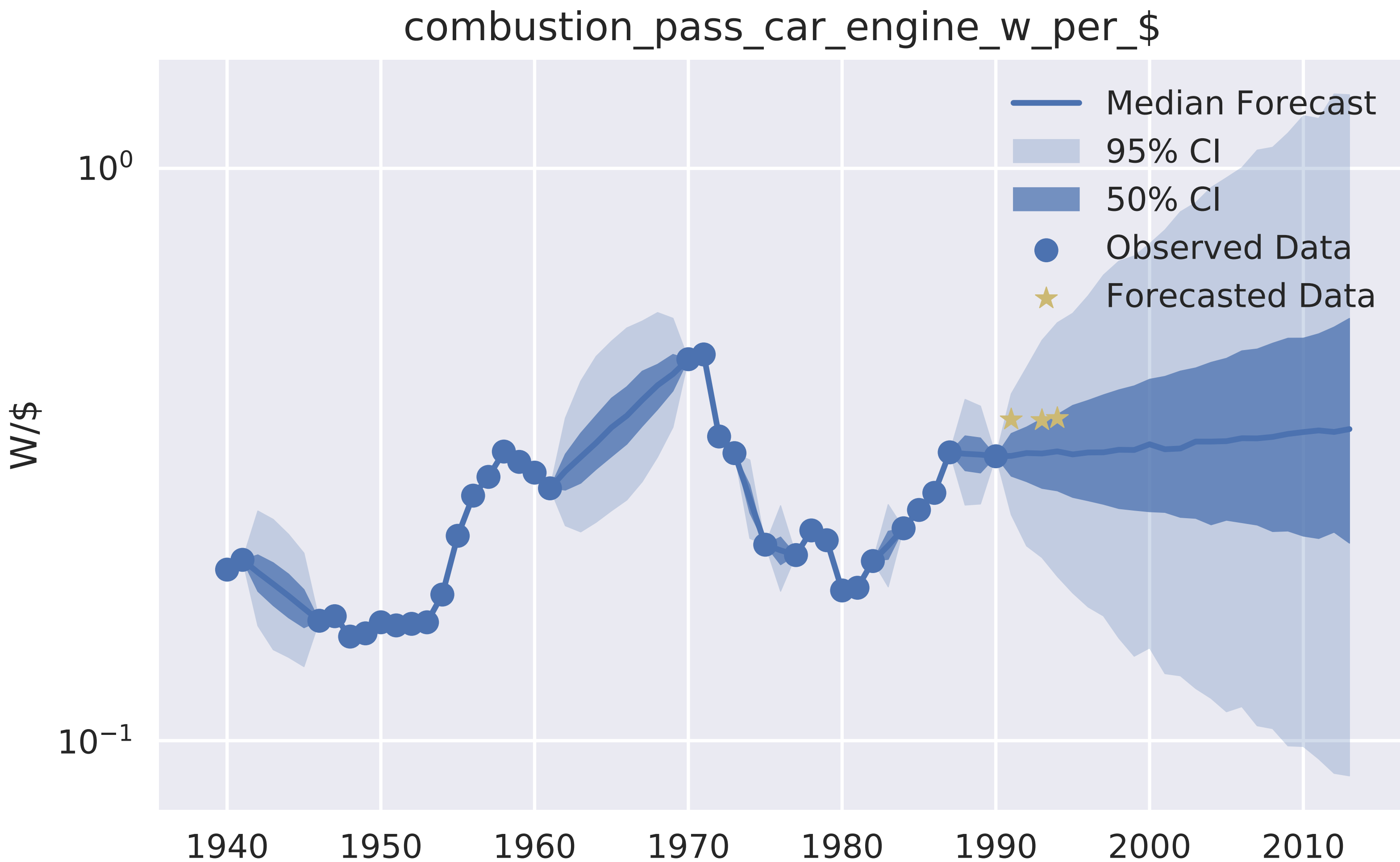
Genome_sequencing_base_pairs
BP/$
10^{10}
10^{7}
10^{4}
10^{1}
10^{-2}
10^{-5}
10^{-8}
10^{-11}
10^{-14}
1940
1950
1960
1970
1980
1990
2000
2010
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data


batteries_wh_$
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
10^2
10^1
10^0
10^{-1}
Wh/$
1940
1950
1960
1970
1980
1990
2000
2010

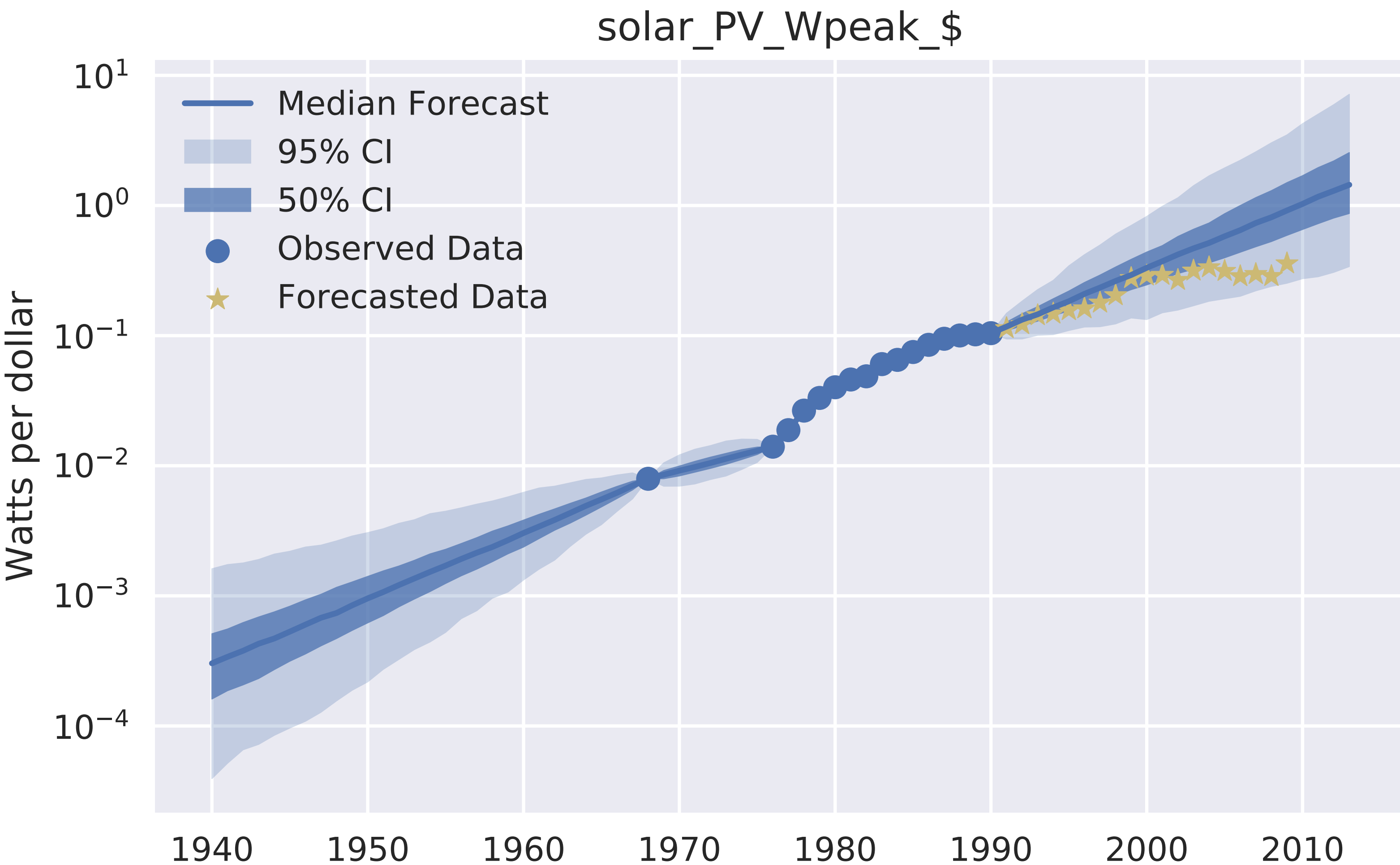


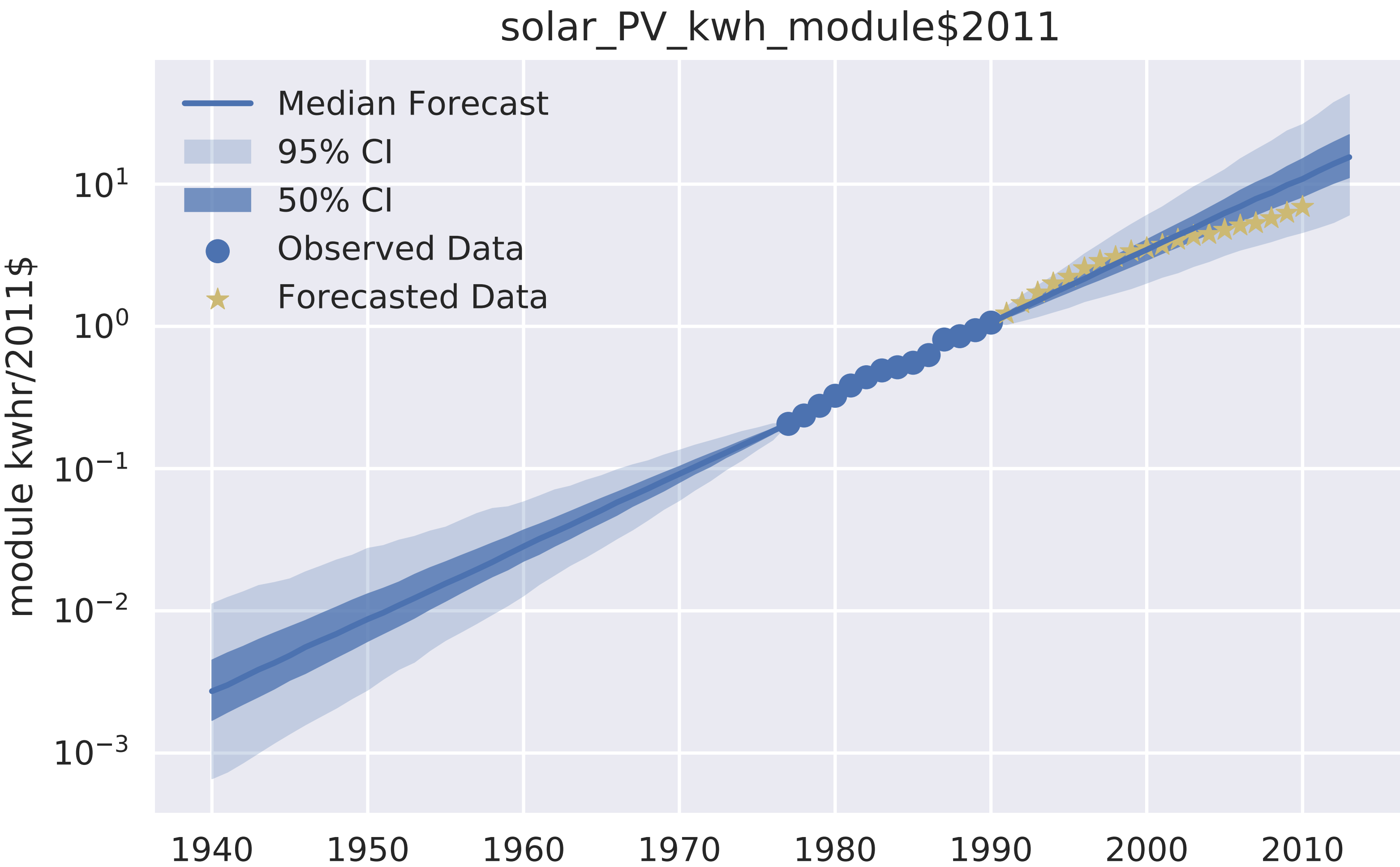
DC_electricity_transmission_powered_distance_per_cost_wkm_$
W?Km/$, Powered distance per cost
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
10^8
10^6
10^4
10^2
10^0
10^{-2}
1940
1950
1960
1970
1980
1990
2000
2010

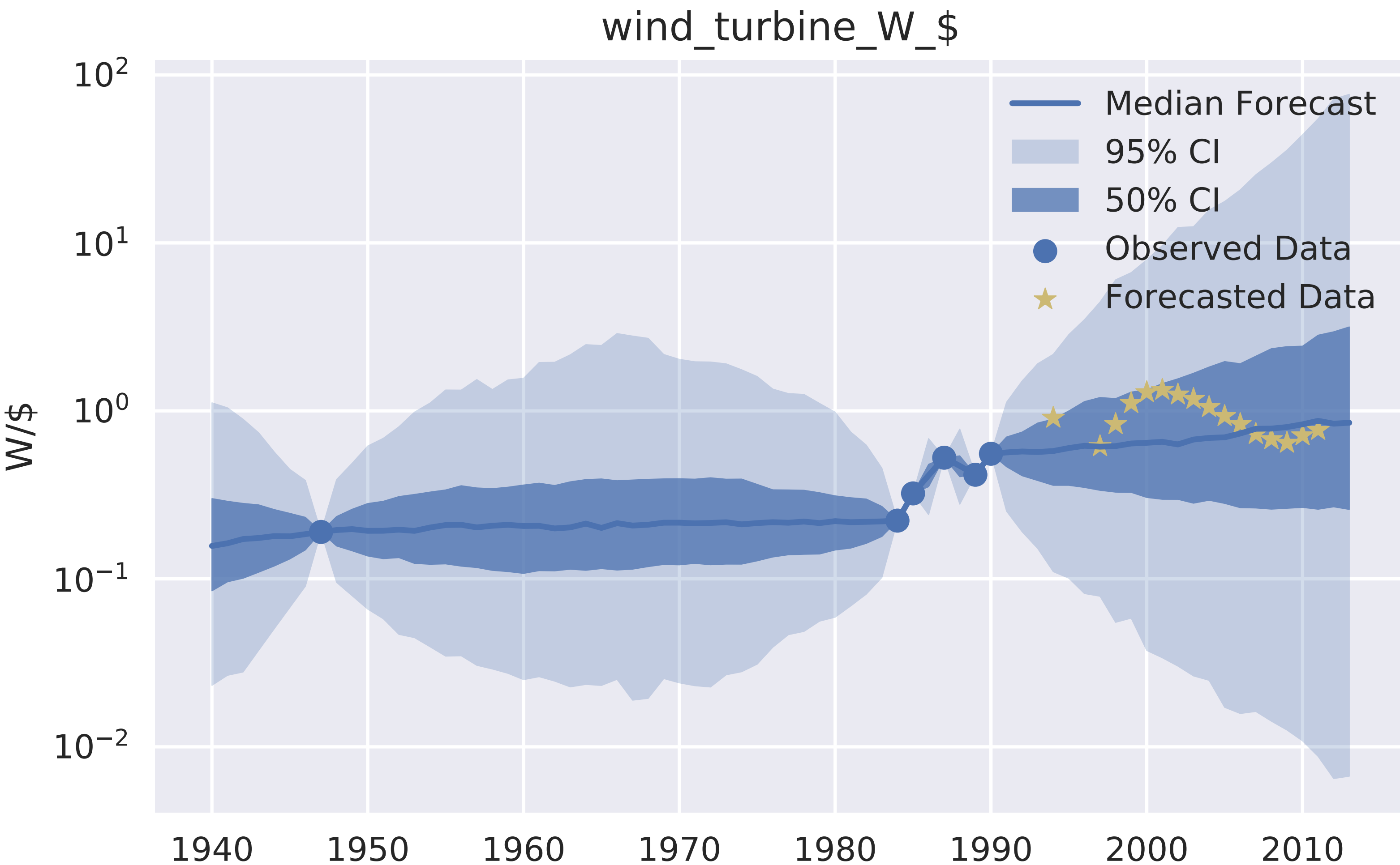

combustion_air_turbine_engine_w_per_$
W/$
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
1940
1950
1960
1970
1980
1990
2000
2010
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data



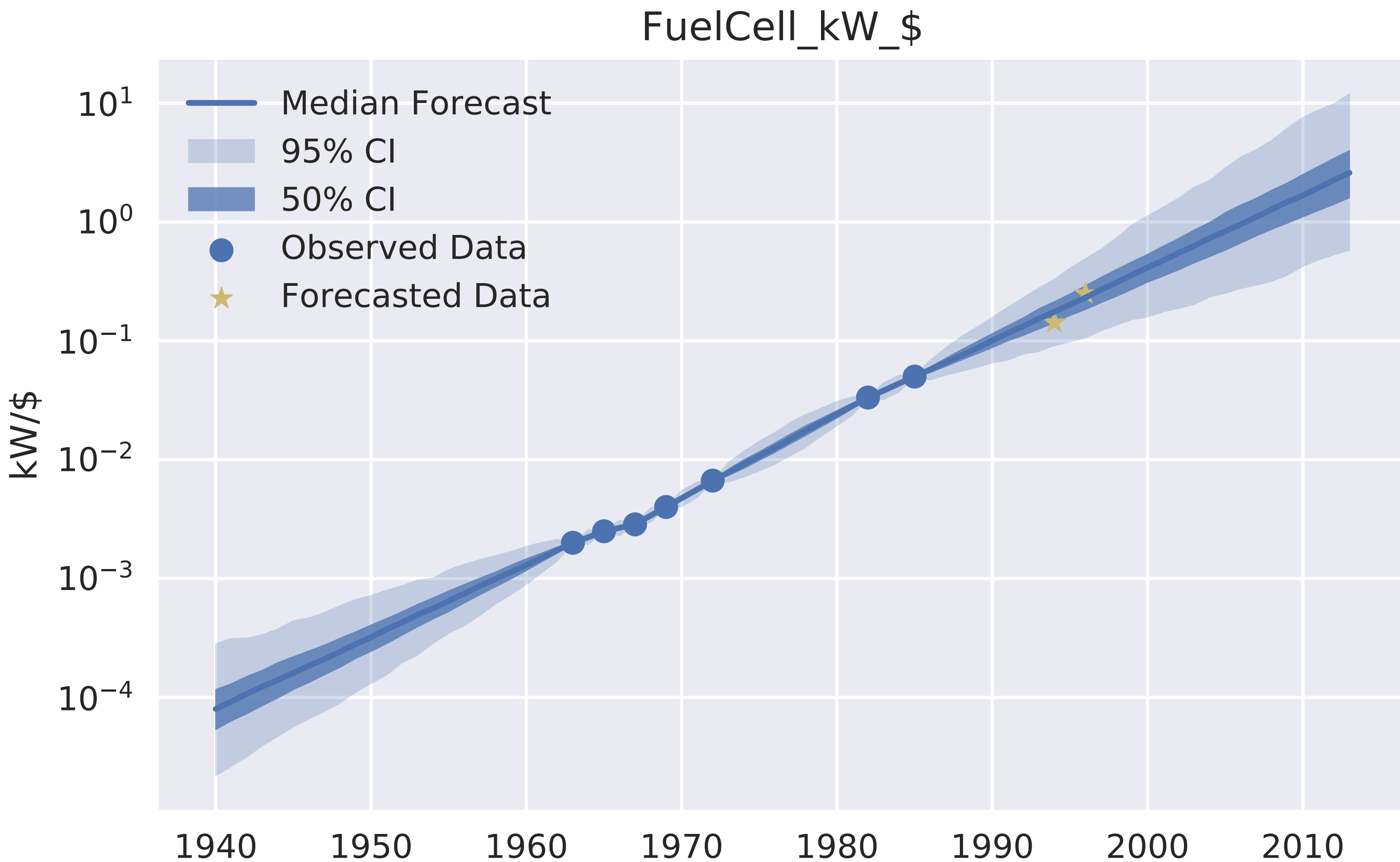
combustion_pass_car_engine_w_per_$
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
W/$
10^0
10^{-1}
1940
1950
1960
1970
1980
1990
2000
2010

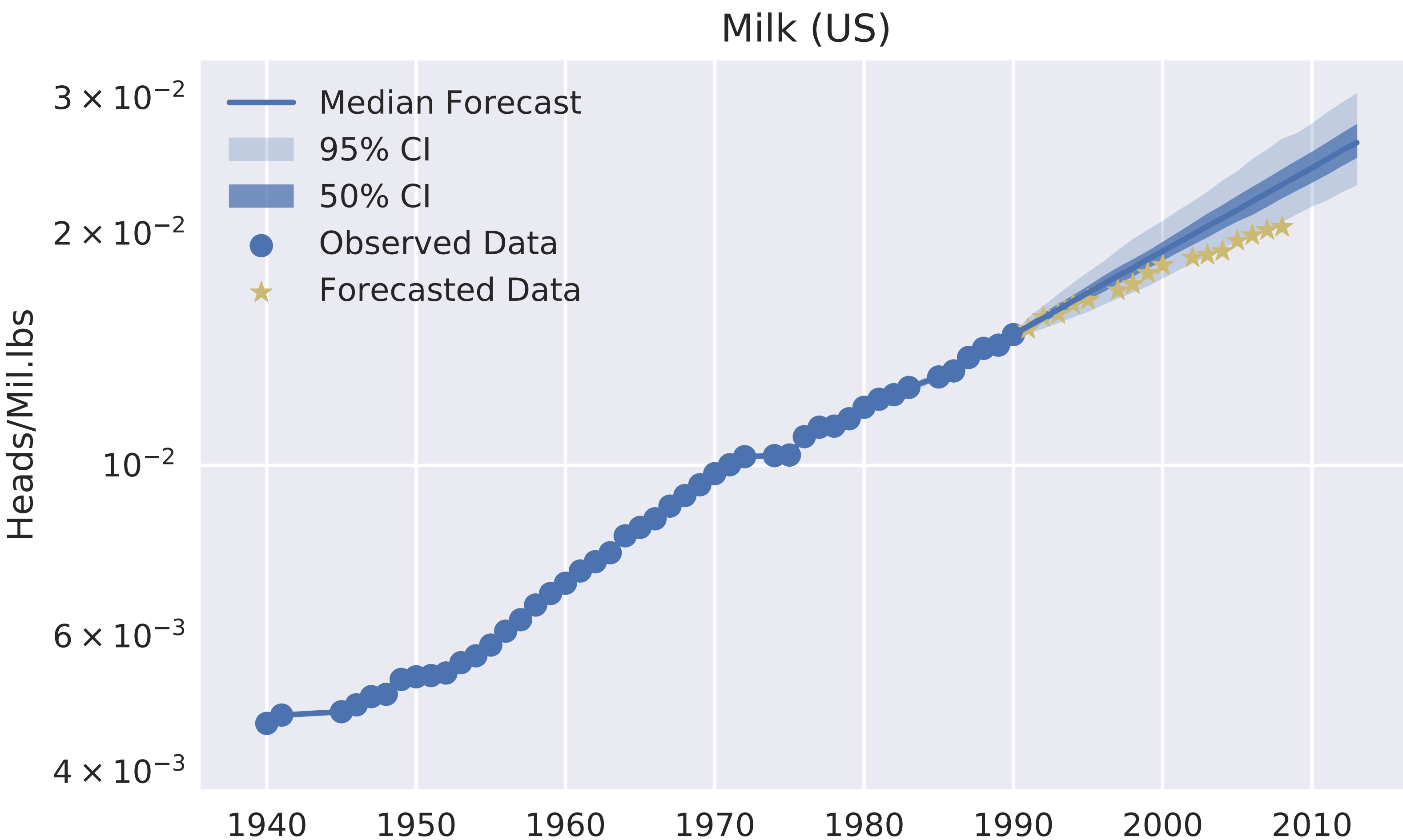

solar_PV_Wpeak_$
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
Watts per dollar
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
1940
1950
1960
1970
1980
1990
2000
2010

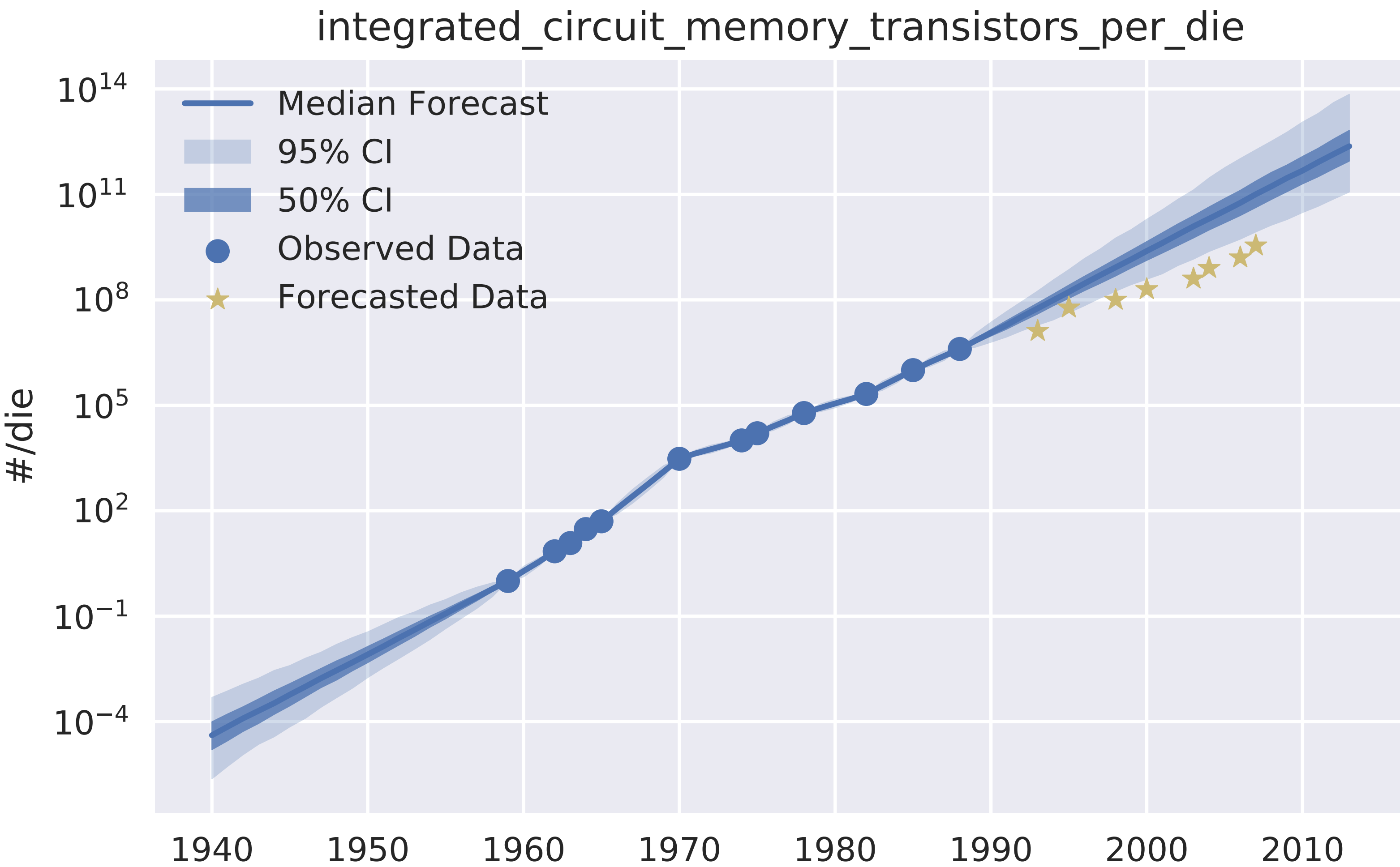

solar_PV_kwh_module$2011
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
module kwhr/2011$
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
1940
1950
1960
1970
1980
1990
2000
2010

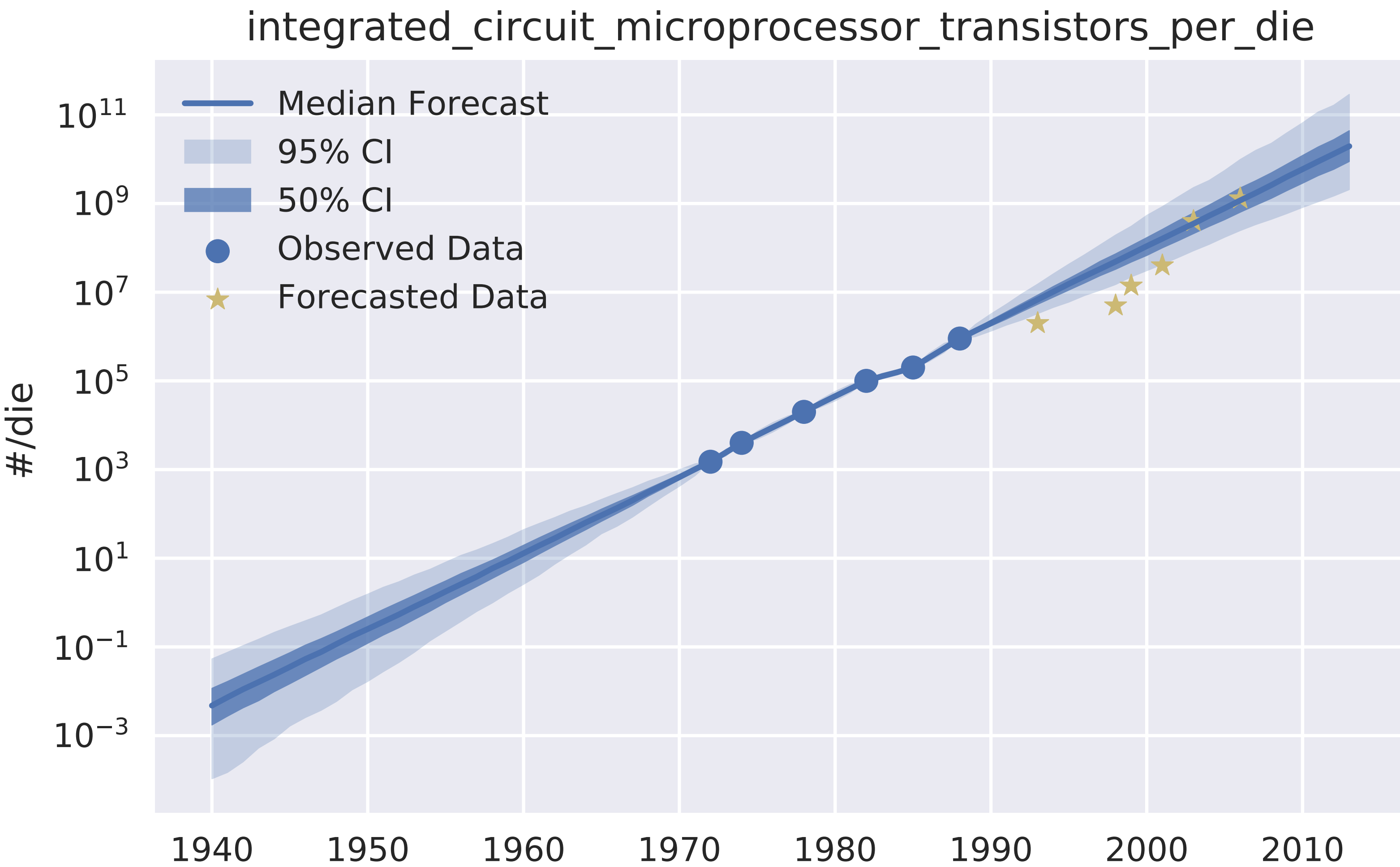


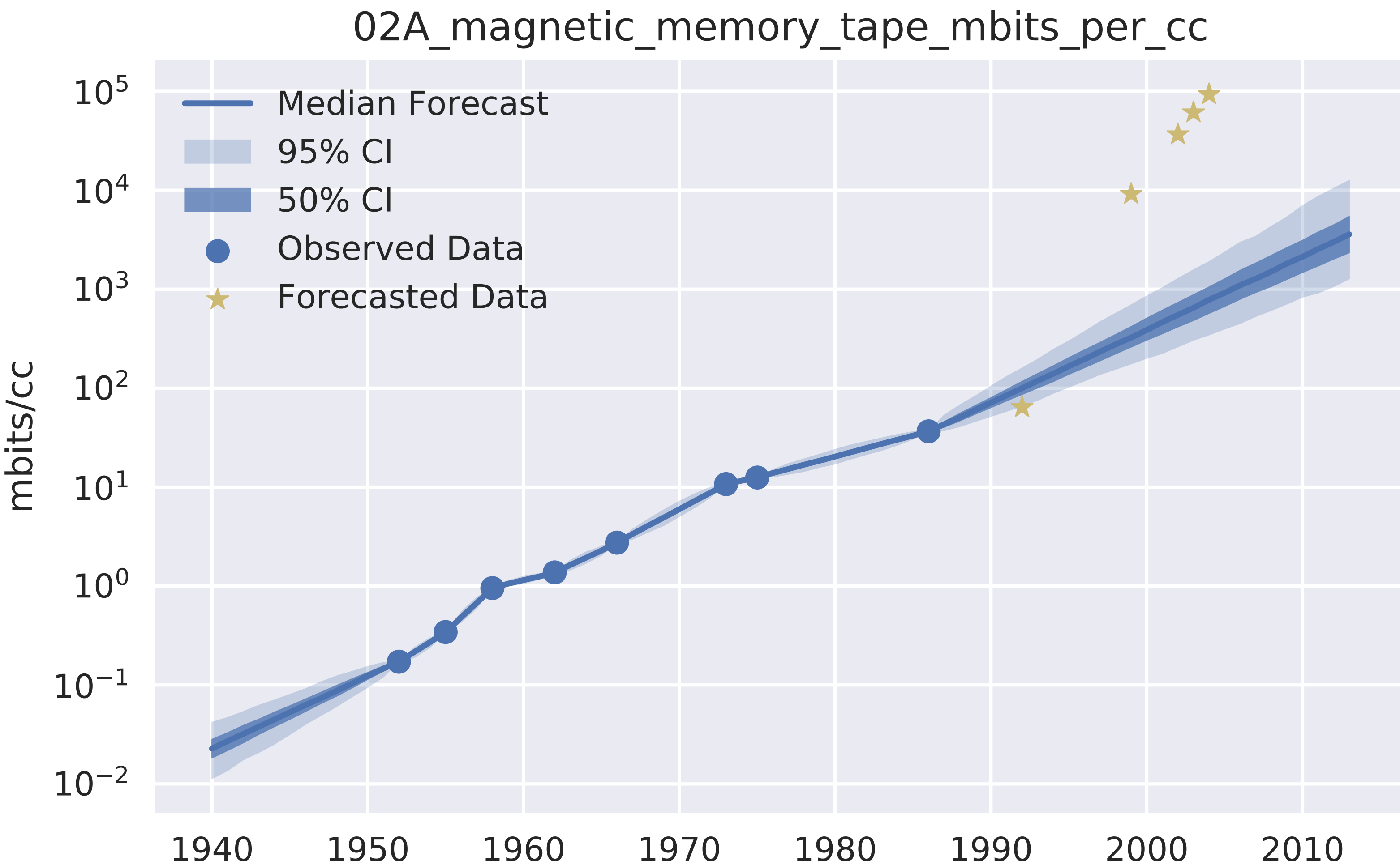
wind_turbine_W_$
W/$
10^2
10^1
10^0
10^{-1}
10^{-2}
1940
1950
1960
1970
1980
1990
2000
2010
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data


FuelCell_kW_$
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
kW/$
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
1940
1950
1960
1970
1980
1990
2000
2010

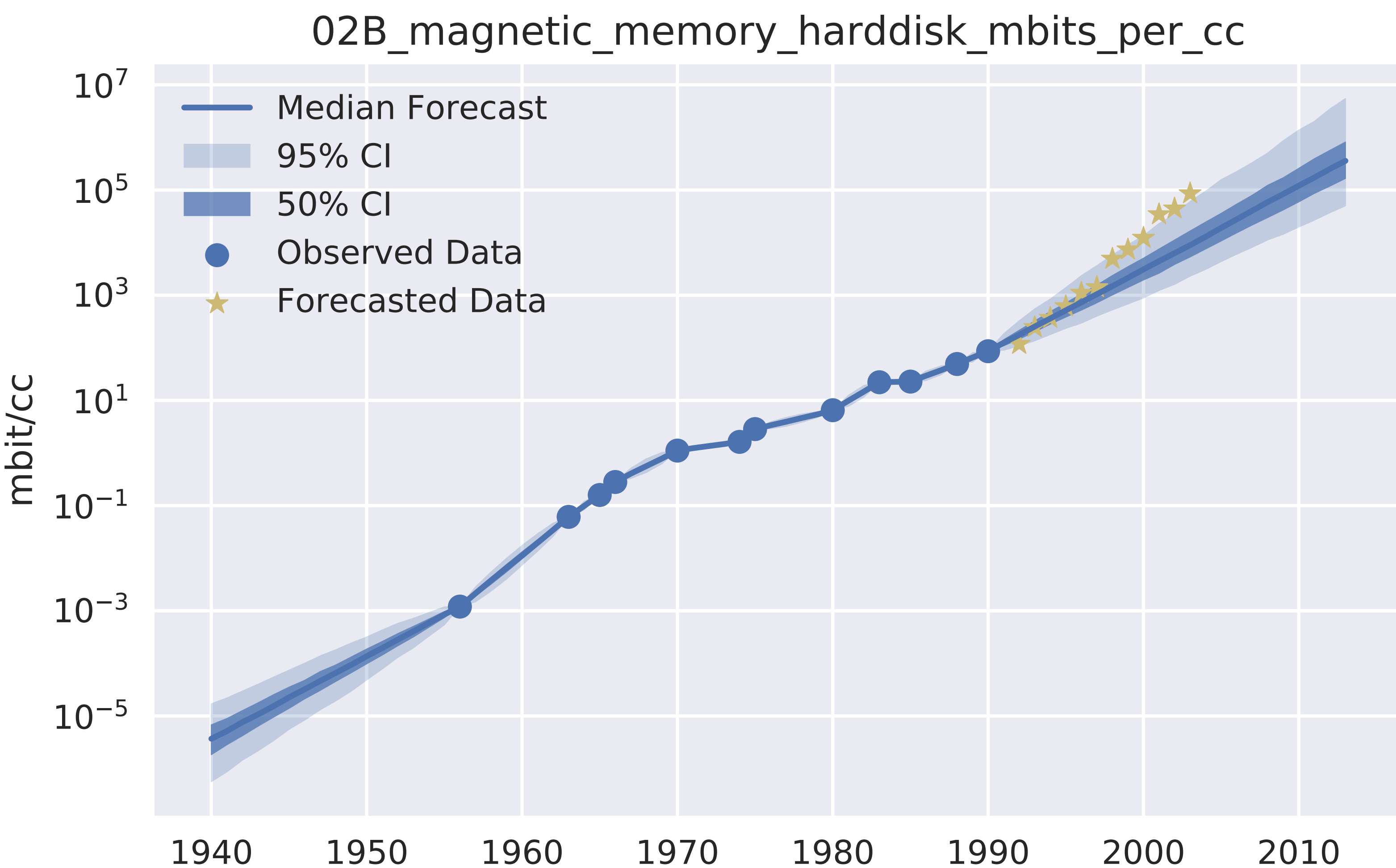

Milk (US)
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
Heads/Mil.lbs
3×10^{-2}
2×10^{-2}
10^{-2}
6×10^{-3}
4×10^{-3}
1940
1950
1960
1970
1980
1990
2000
2010

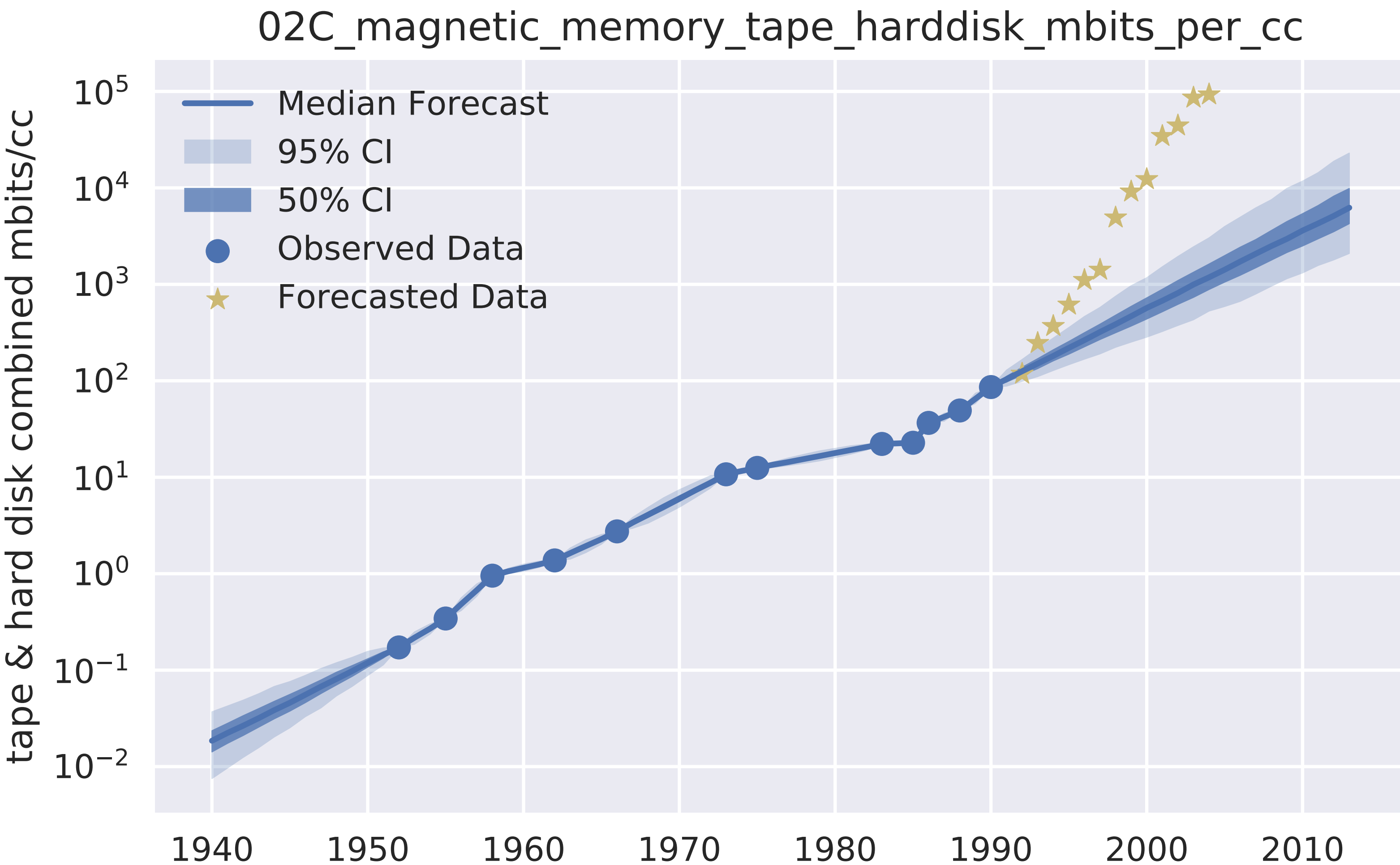

integrated_circuit_memory_transistors_per_die
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
#/die
10^{14}
10^{11}
10^{8}
10^{5}
10^{2}
10^{-1}
10^{-4}
1940
1950
1960
1970
1980
1990
2000
2010


integrated_circuit_microprocessor_transistors_per_die
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
#/die
10^{11}
10^{9}
10^{7}
10^{5}
10^{3}
10^{1}
10^{-1}
10^{-3}
1940
1950
1960
1970
1980
1990
2000
2010

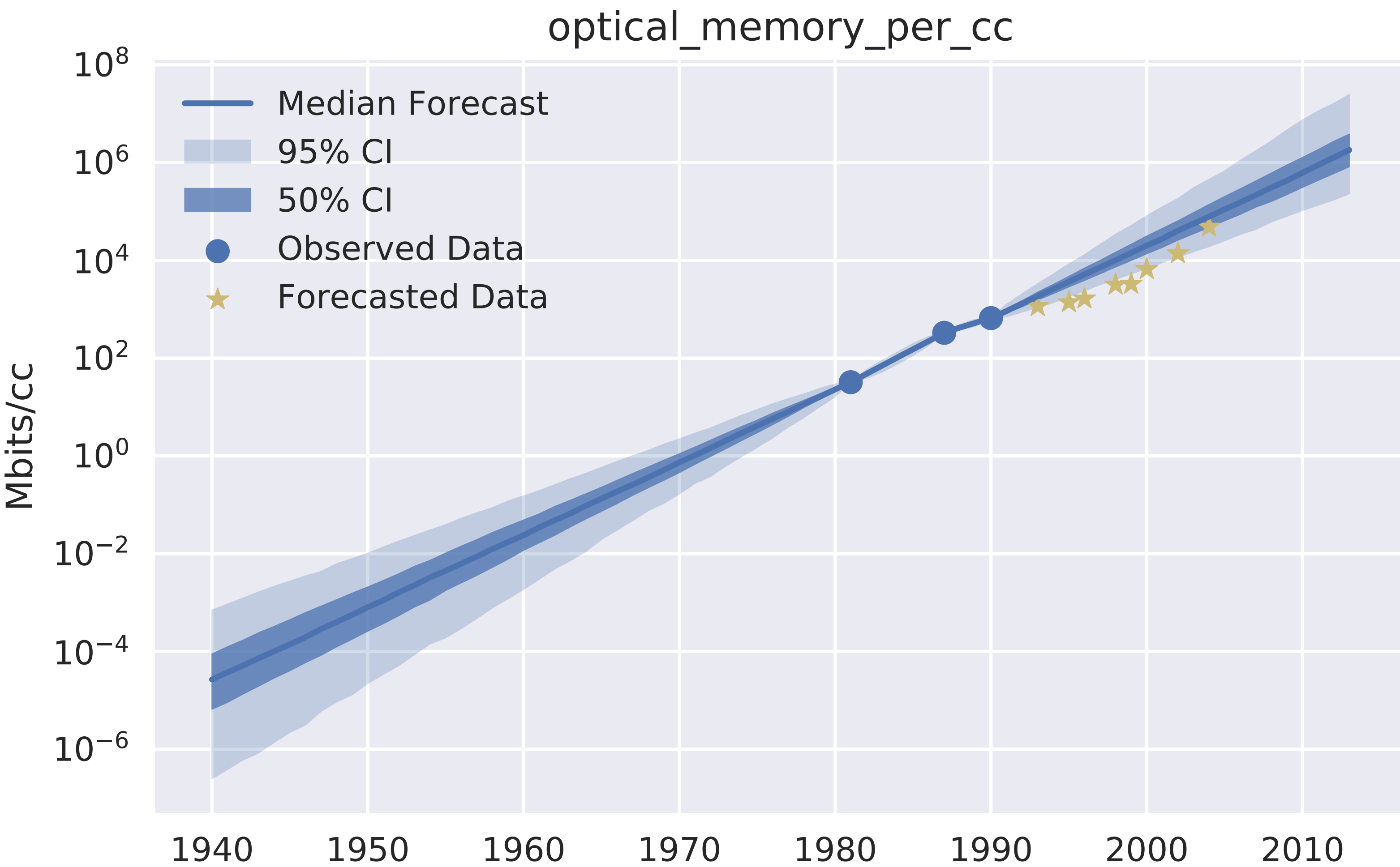


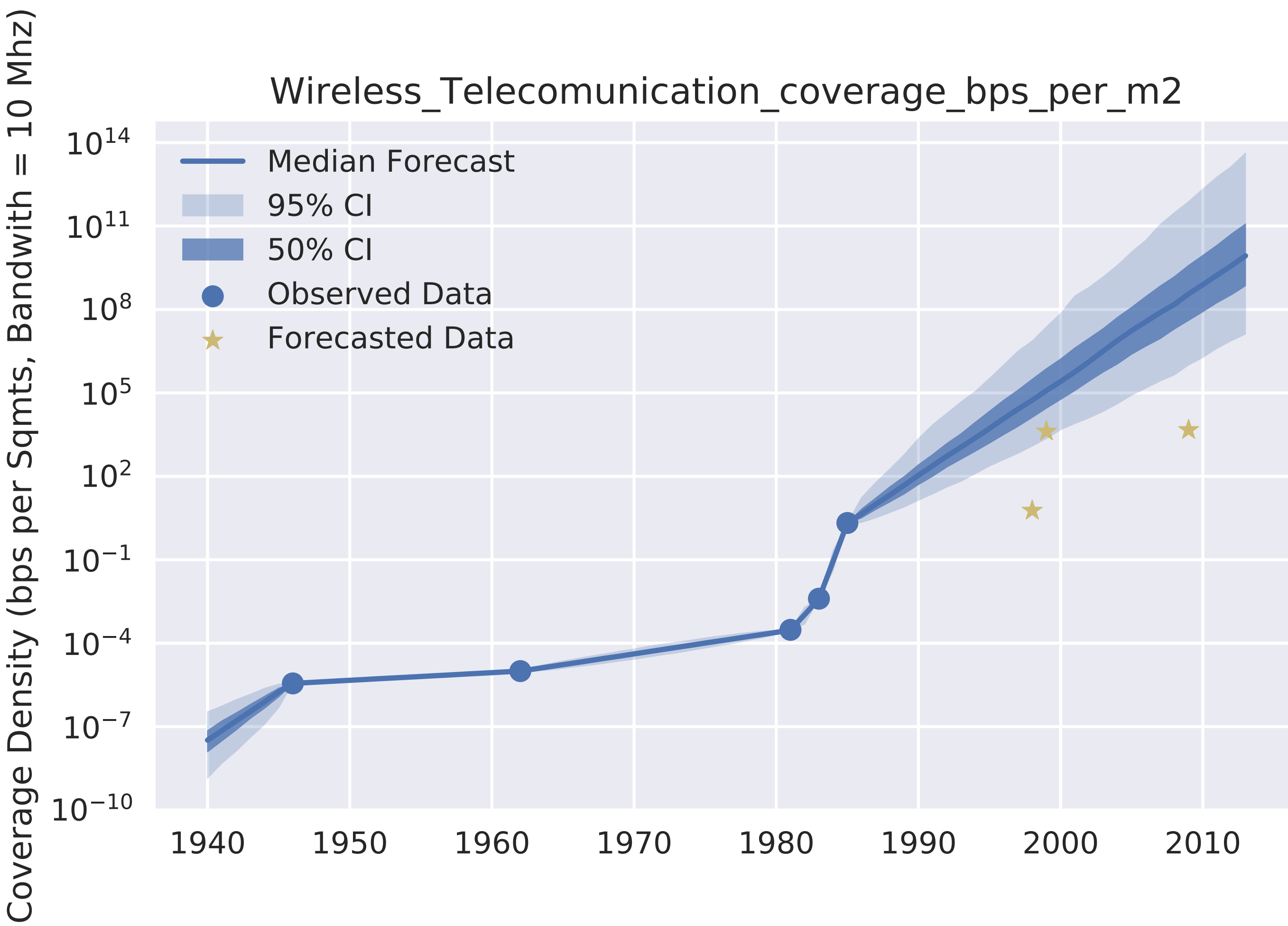
02A_magnetic_memory_tape_mbits_per_cc
mbits/cc
10^5
10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
1940
1950
1960
1970
1980
1990
2000
2010
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data


02B_magnetic_memory_harddisk_mbits_per_cc
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
mbit/cc
10^{7}
10^{5}
10^{3}
10^{1}
10^{-1}
10^{-3}
10^{-5}
1940
1950
1960
1970
1980
1990
2000
2010

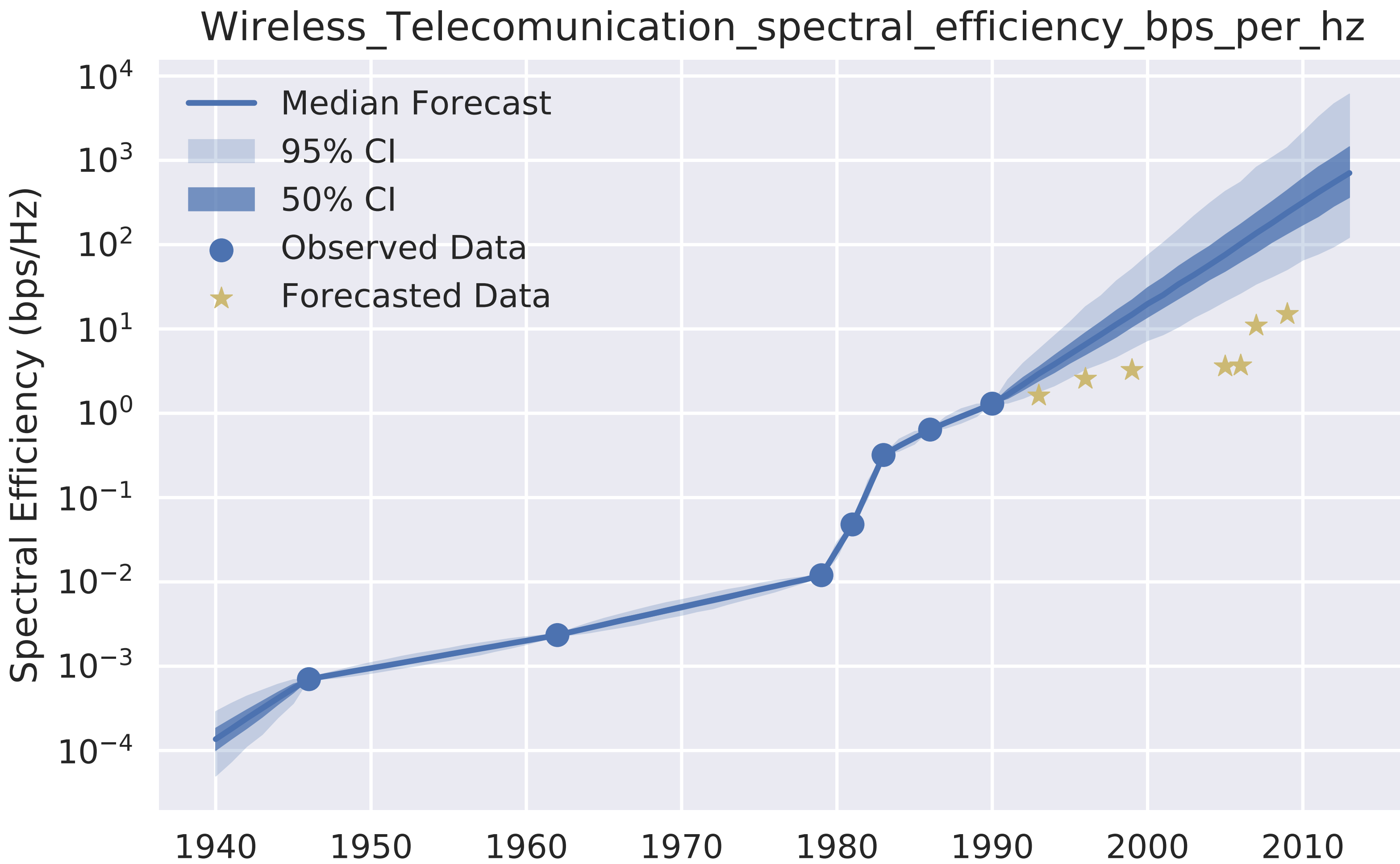


02C_magnetic_memory_tape_harddisk_mbits_per_cc
tape & hard disk combined mbits/cc
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
10^5
10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
1940
1950
1960
1970
1980
1990
2000
2010

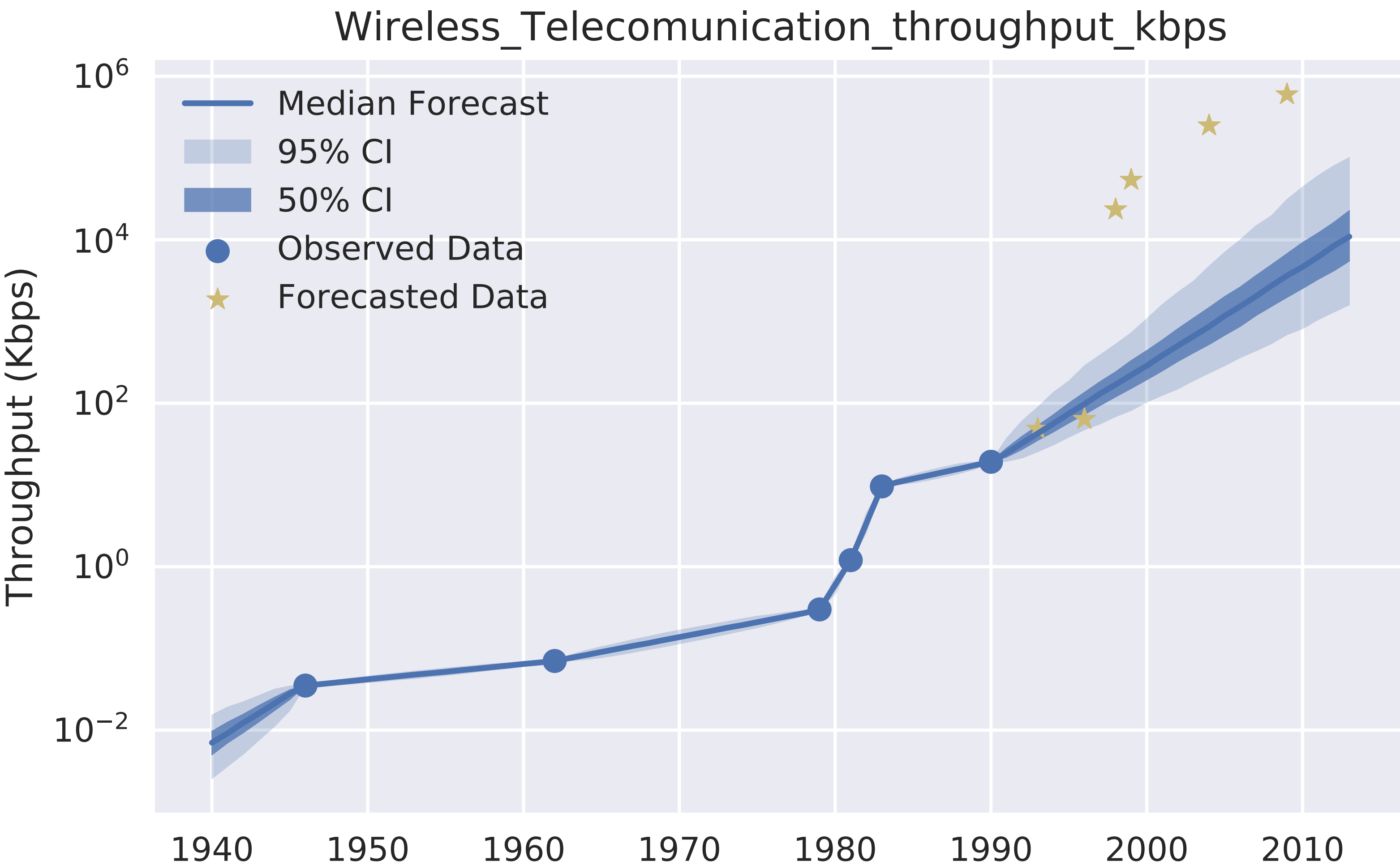

optical_memory_per_cc
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
Mbits/cc
10^{8}
10^{6}
10^{4}
10^{2}
10^{0}
10^{-2}
10^{-4}
10^{-6}
1940
1950
1960
1970
1980
1990
2000
2010



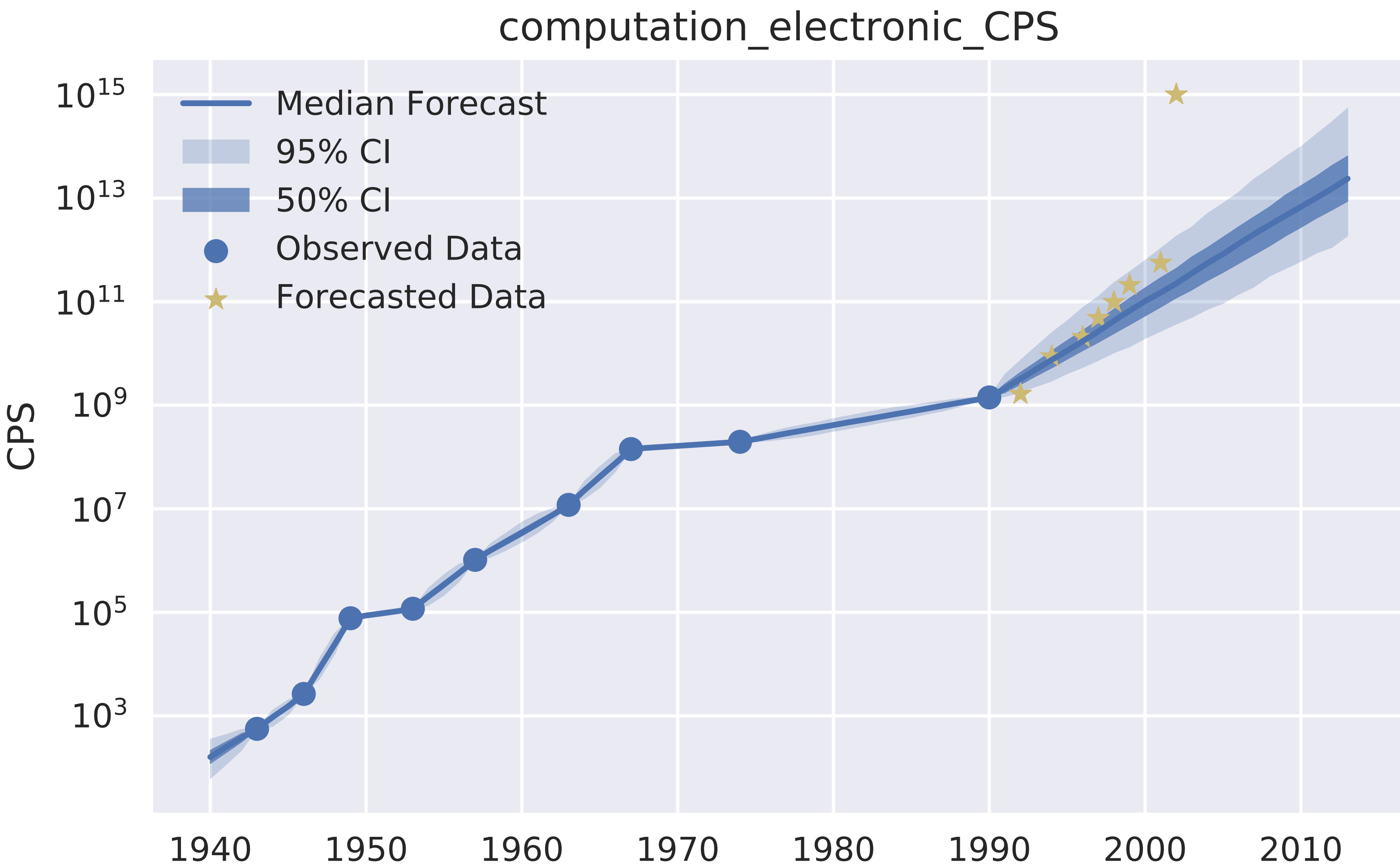
Wireless_Telecomunication_coverage_bps_per_m2
Coverage Density (bps per Sqmts, Bandwith = 10 Mhz)
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
10^{14}
10^{11}
10^{8}
10^{5}
10^{2}
10^{-1}
10^{-4}
10^{-7}
10^{-10}
1940
1950
1960
1970
1980
1990
2000
2010

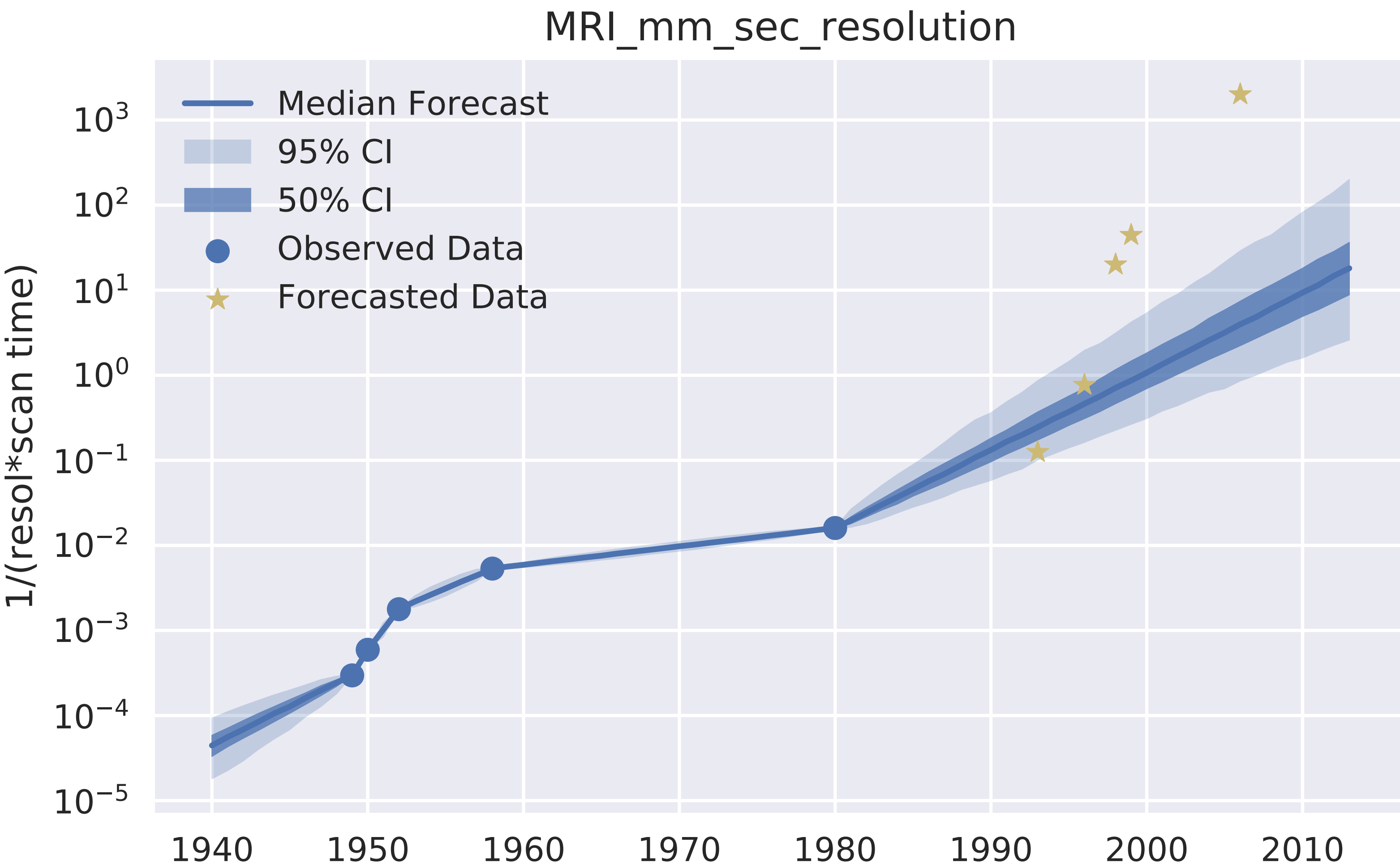


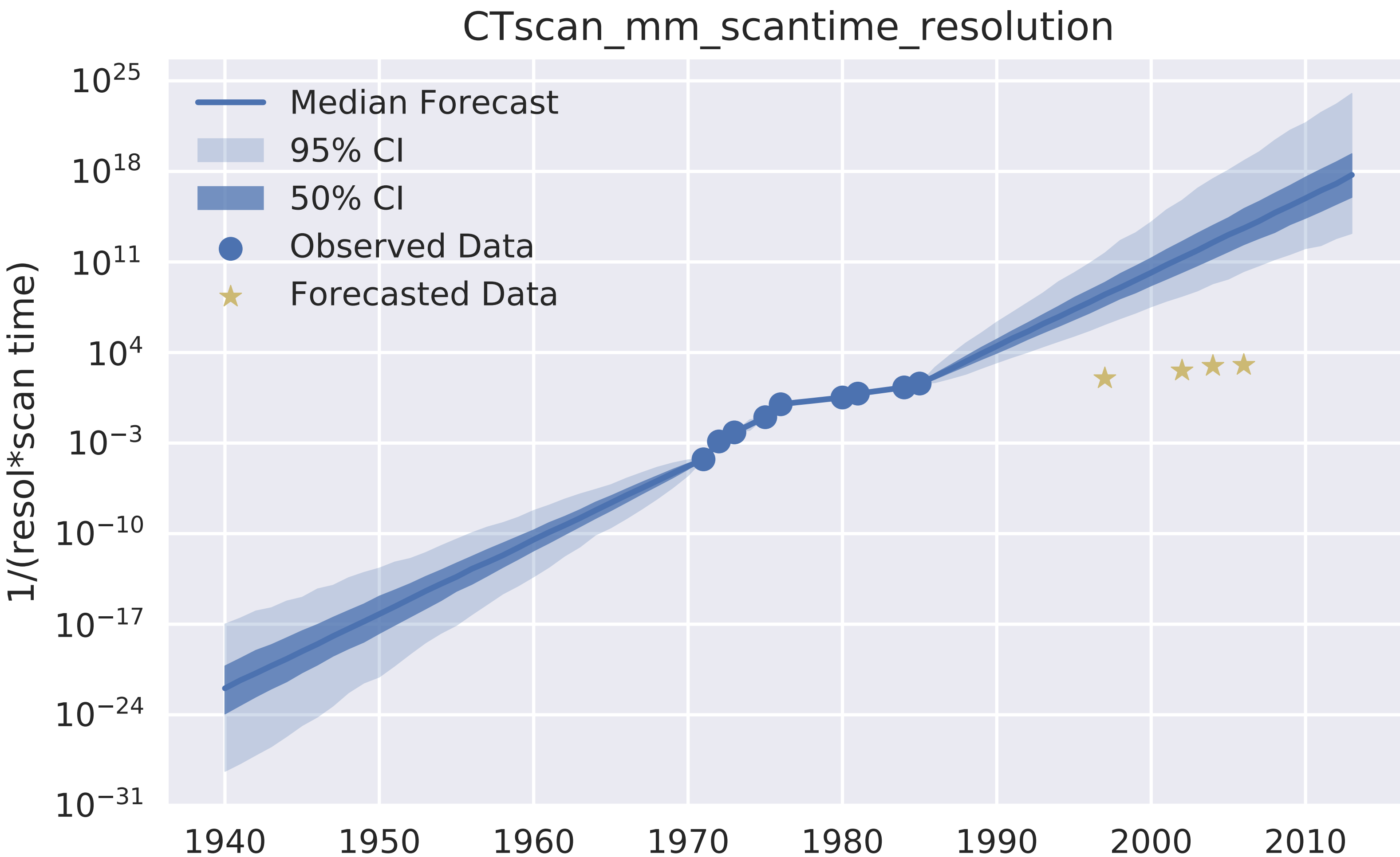
Wireless_Telecomunication_spectral_efficiency_bps_per_hz
Spectral Efficiency (bps/Hz)
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
1940
1950
1960
1970
1980
1990
2000
2010



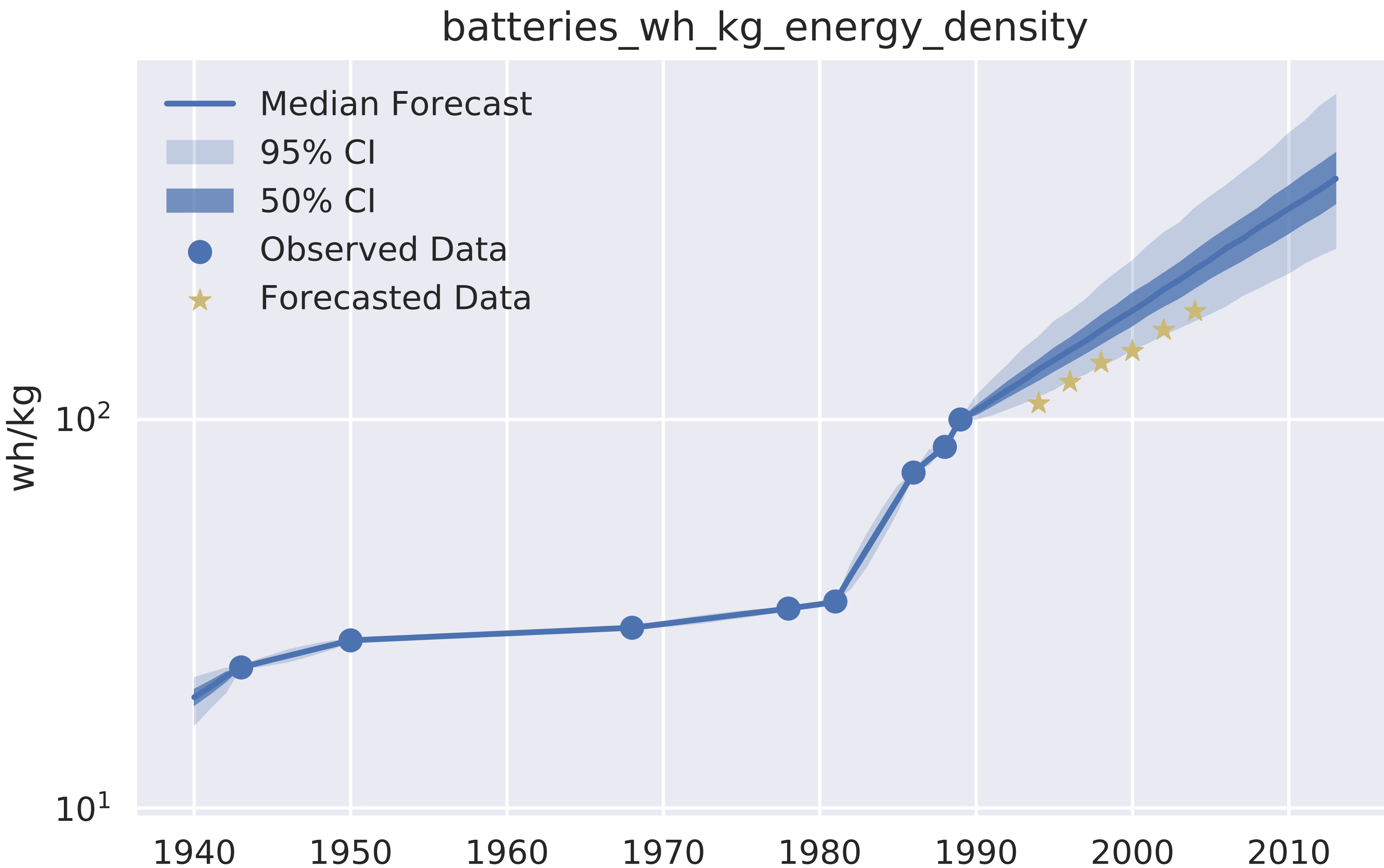
Wireless_Telecomunication_throughput_kbps
Throughput (Kbps)
10^6
10^4
10^2
10^0
10^{-2}
1940
1950
1960
1970
1980
1990
2000
2010
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data


computation_electronic_CPS
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
CPS
10^{15}
10^{13}
10^{11}
10^{9}
10^{7}
10^{5}
10^{3}
1940
1950
1960
1970
1980
1990
2000
2010

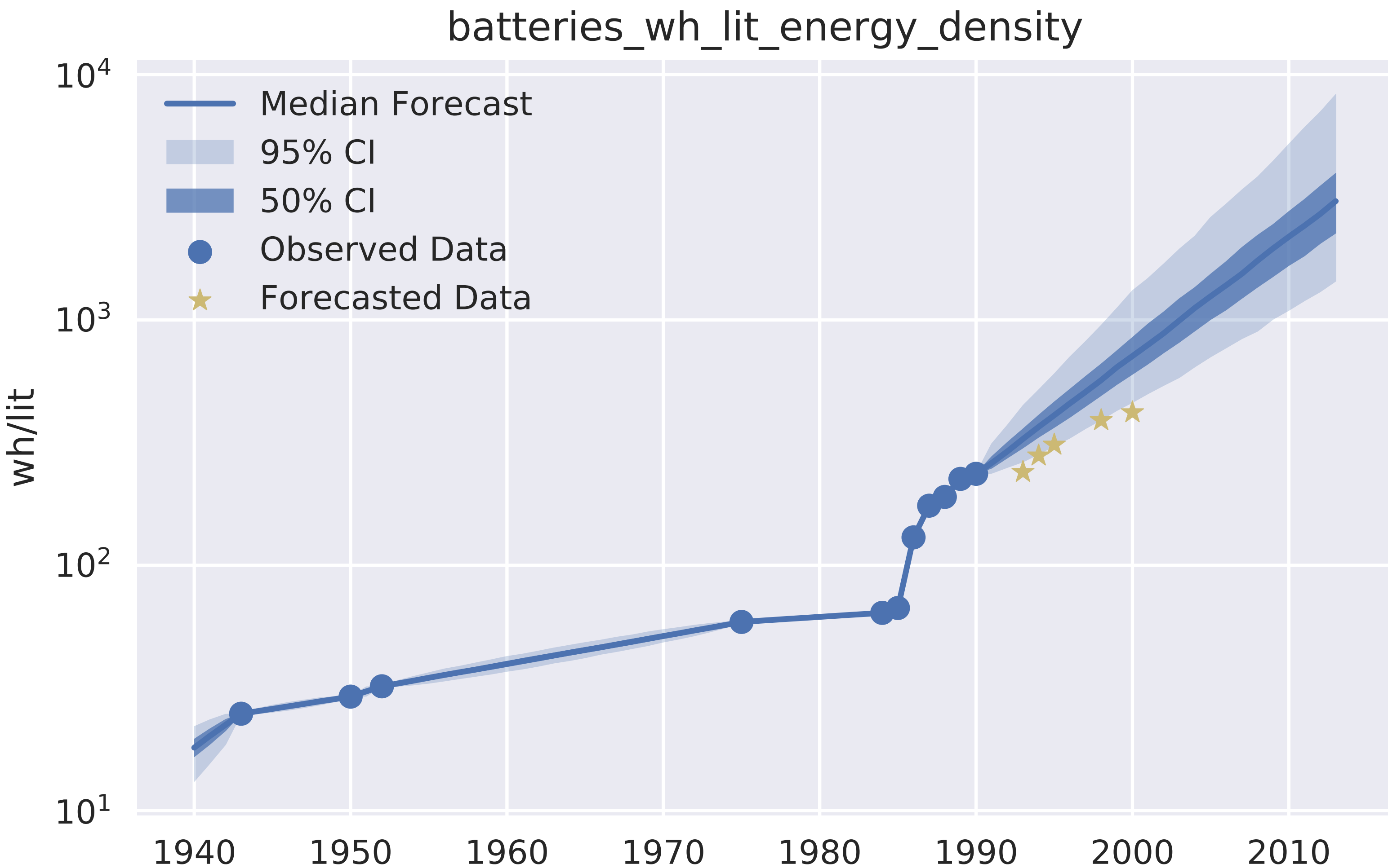

MRI_mm_sec_resolution
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
1/(resol*scan time)
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
1940
1950
1960
1970
1980
1990
2000
2010

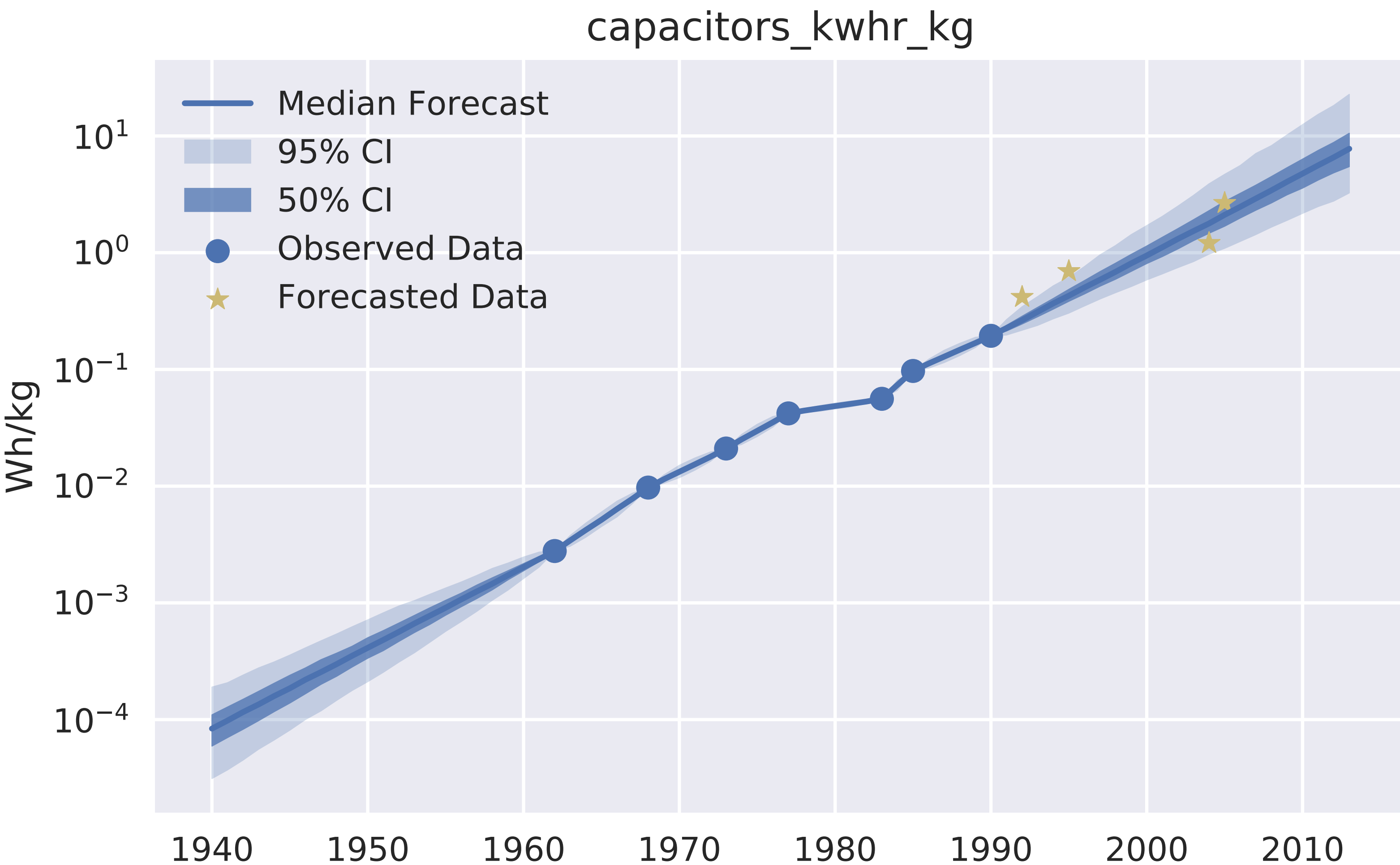


CTscan_mm_scantime_resolution
1/(resol*scan time)
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
10^{25}
10^{18}
10^{11}
10^{4}
10^{-3}
10^{-10}
10^{-17}
10^{-24}
10^{-31}
1940
1950
1960
1970
1980
1990
2000
2010

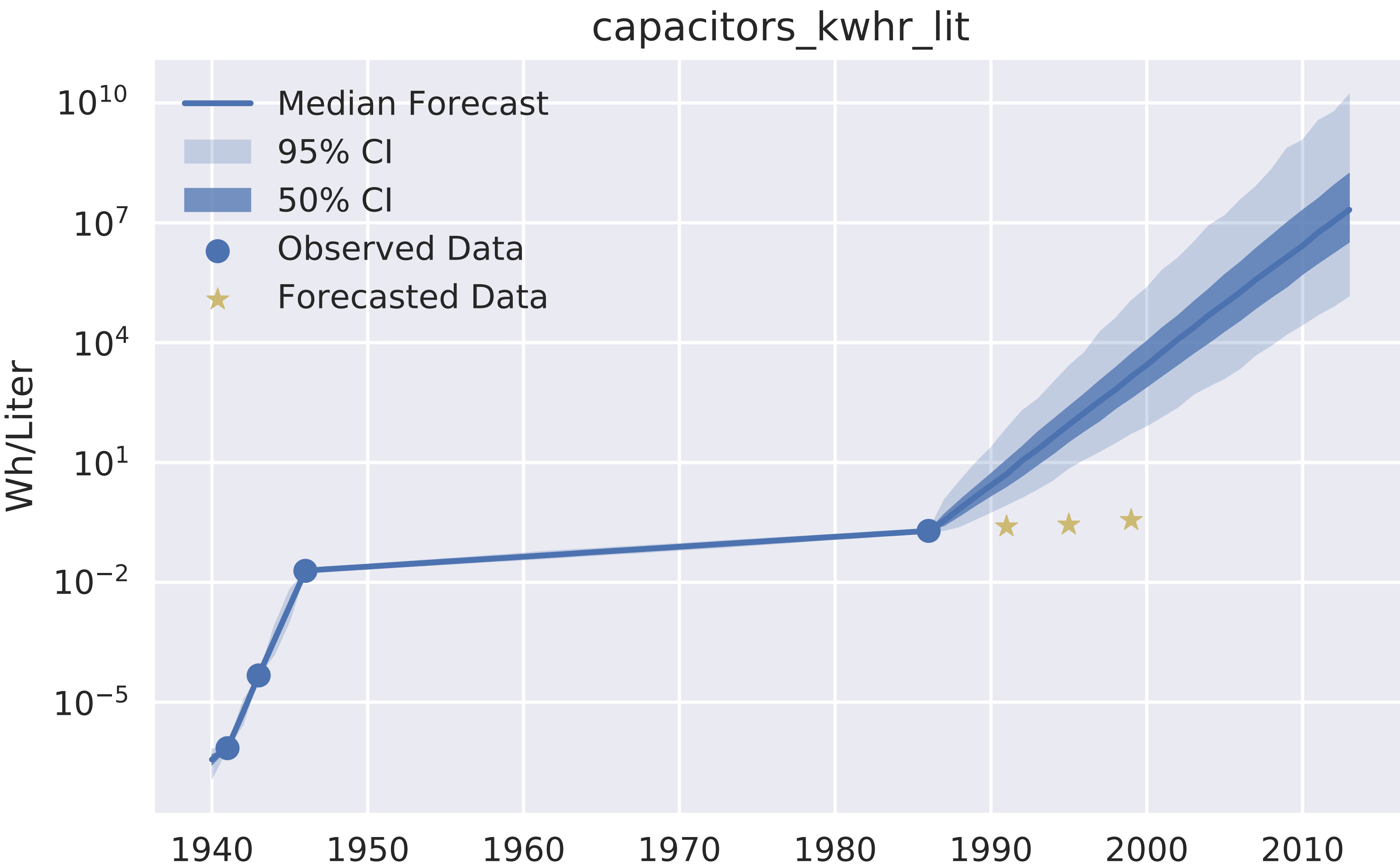


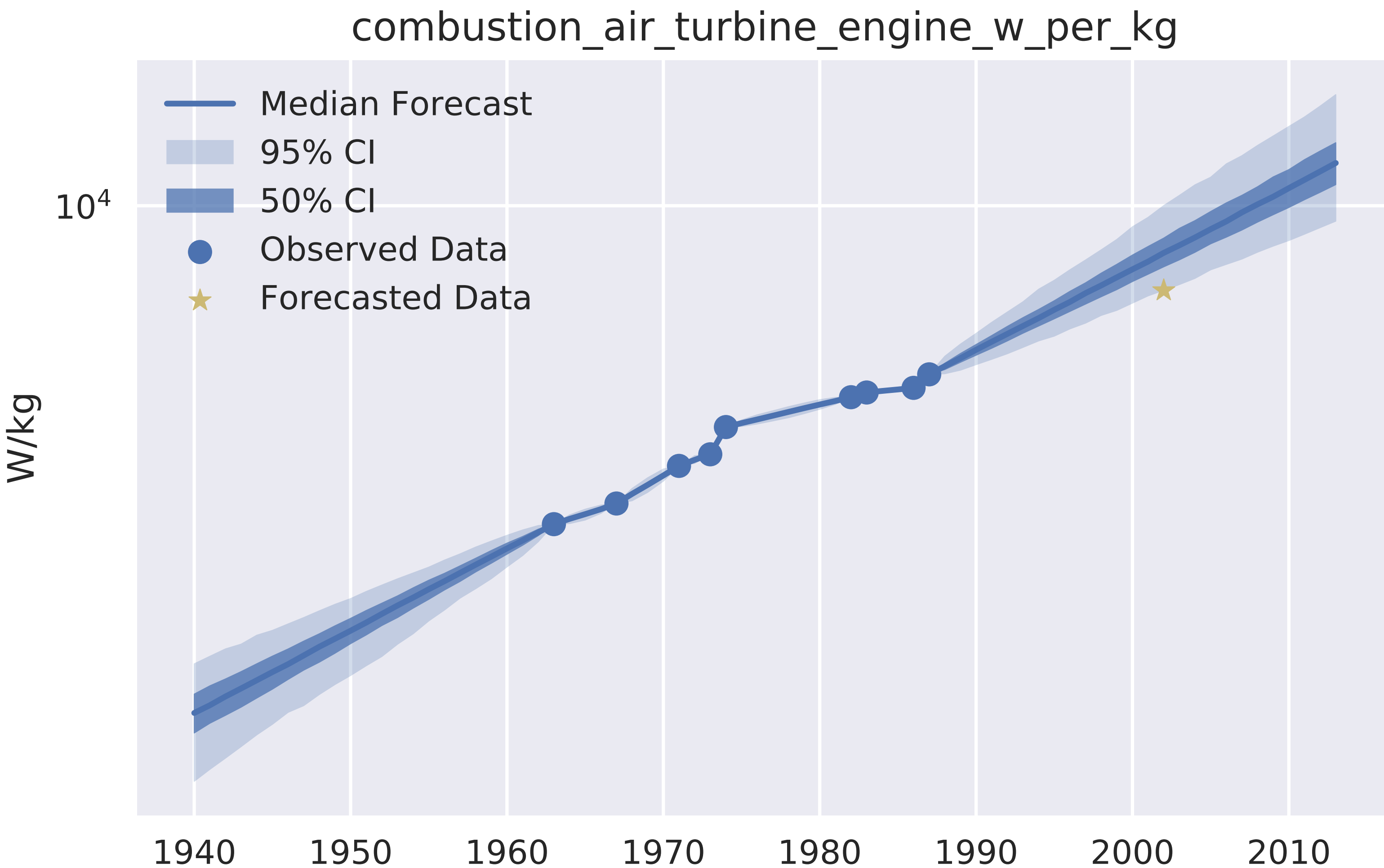
batteries_wh_kg_energy_density
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
wh/kg
10^2
10^1
1940
1950
1960
1970
1980
1990
2000
2010


batteries_wh_lit_energy_density
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
wh/lit
10^4
10^3
10^2
10^1
1940
1950
1960
1970
1980
1990
2000
2010

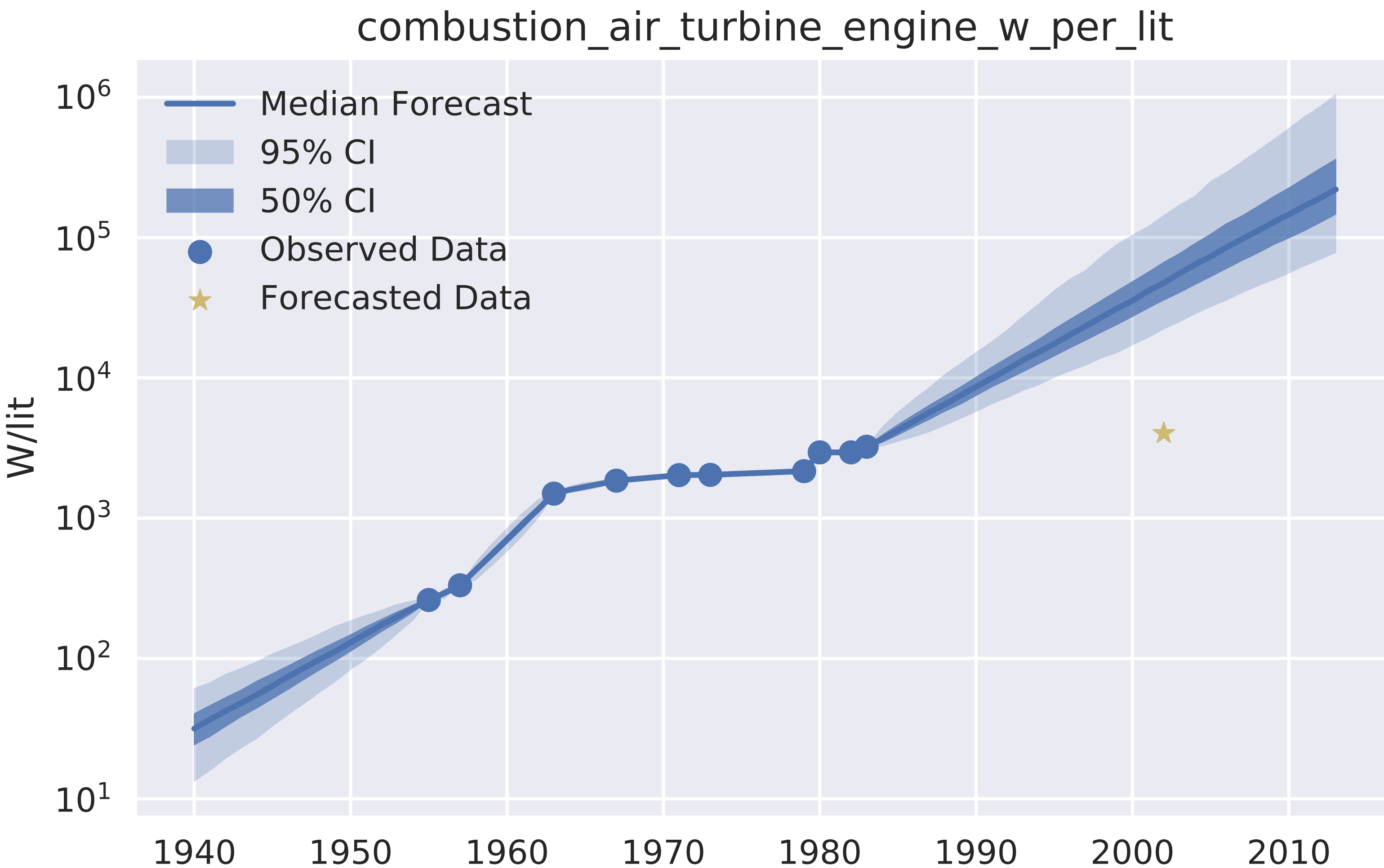


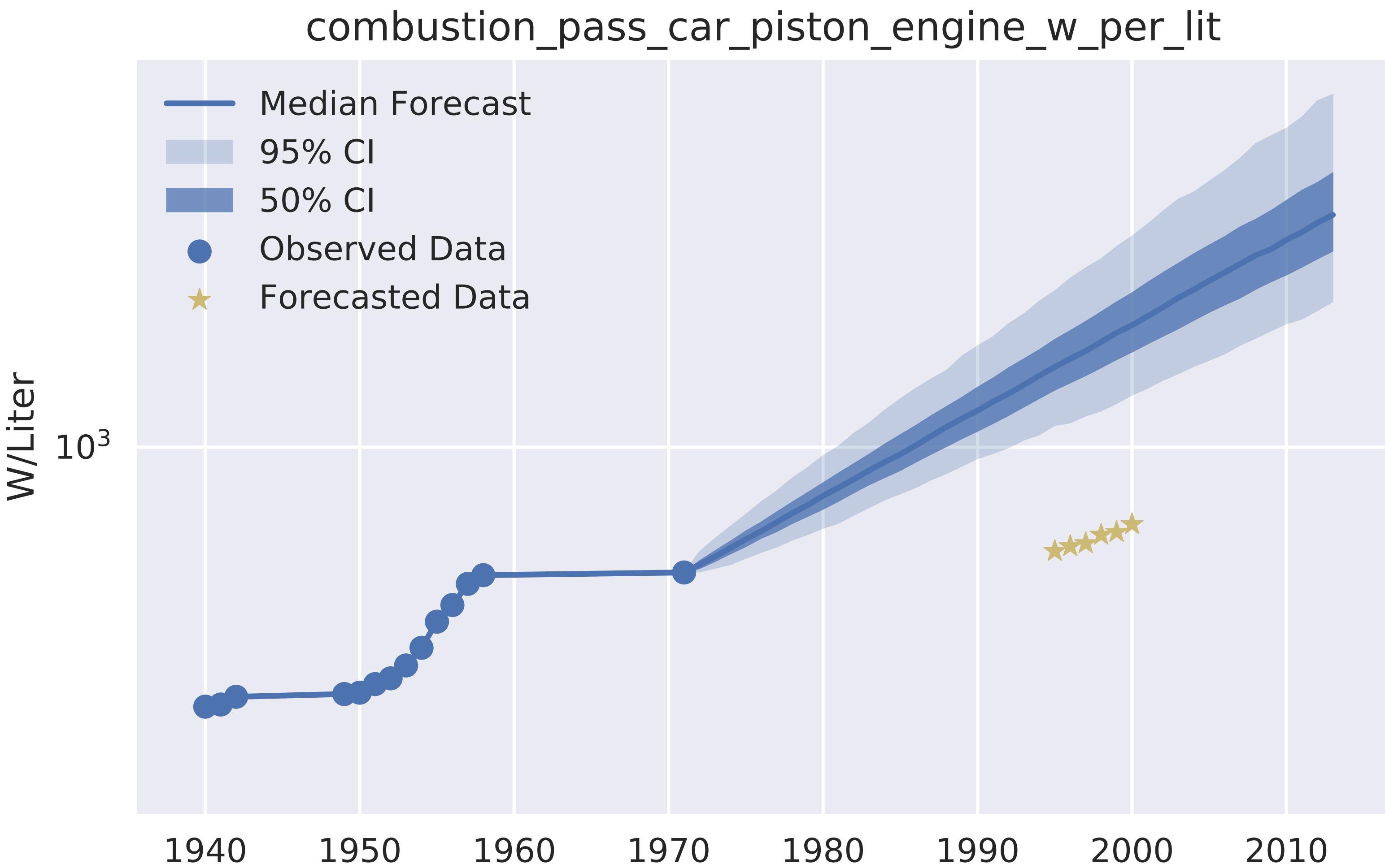
capacitors_kwhr_kg
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
Wh/kg
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
1940
1950
1960
1970
1980
1990
2000
2010

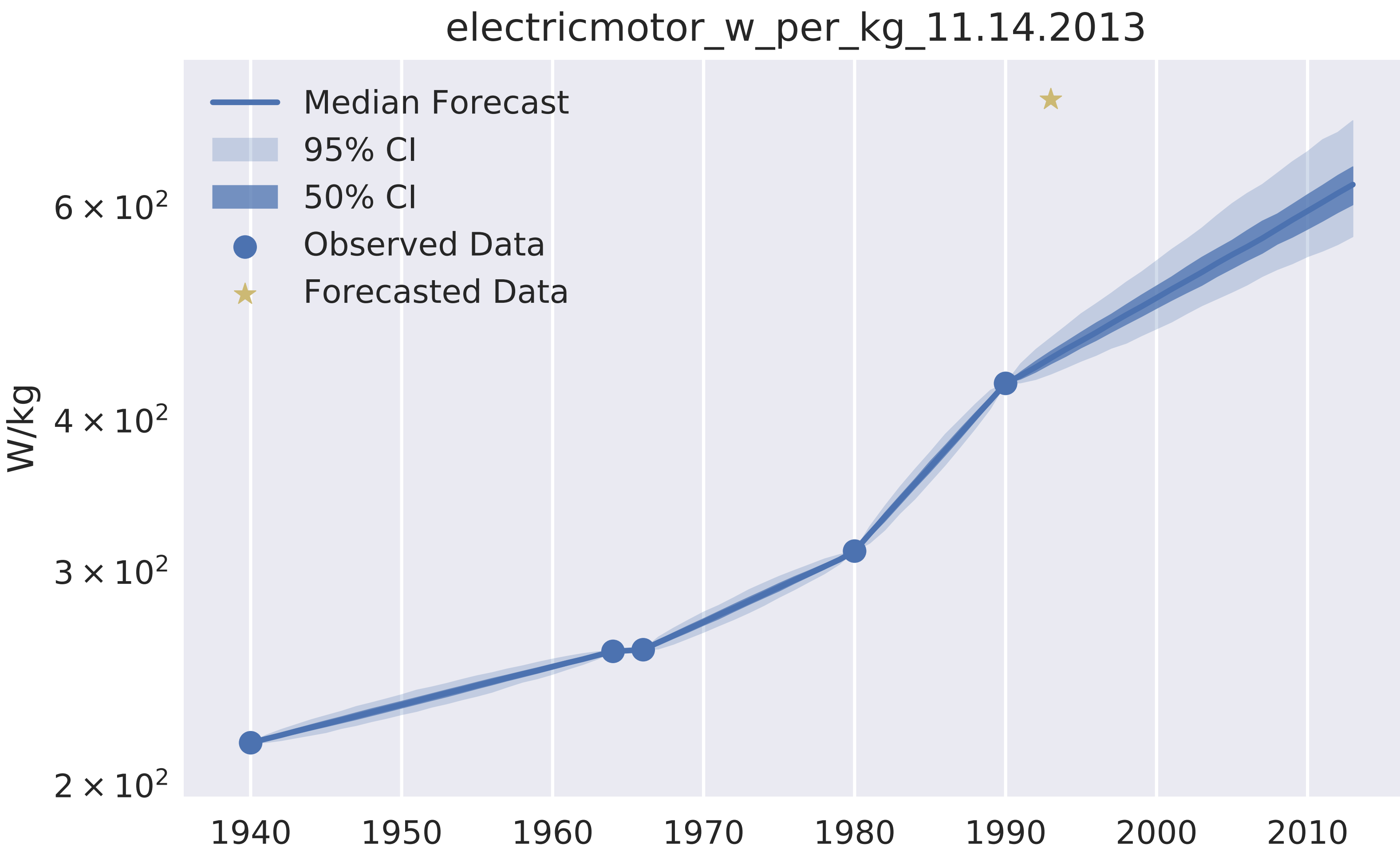

capacitors_kwhr_lit
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
Wh/Liter
10^{10}
10^{7}
10^{4}
10^{1}
10^{-2}
10^{-5}
1940
1950
1960
1970
1980
1990
2000
2010



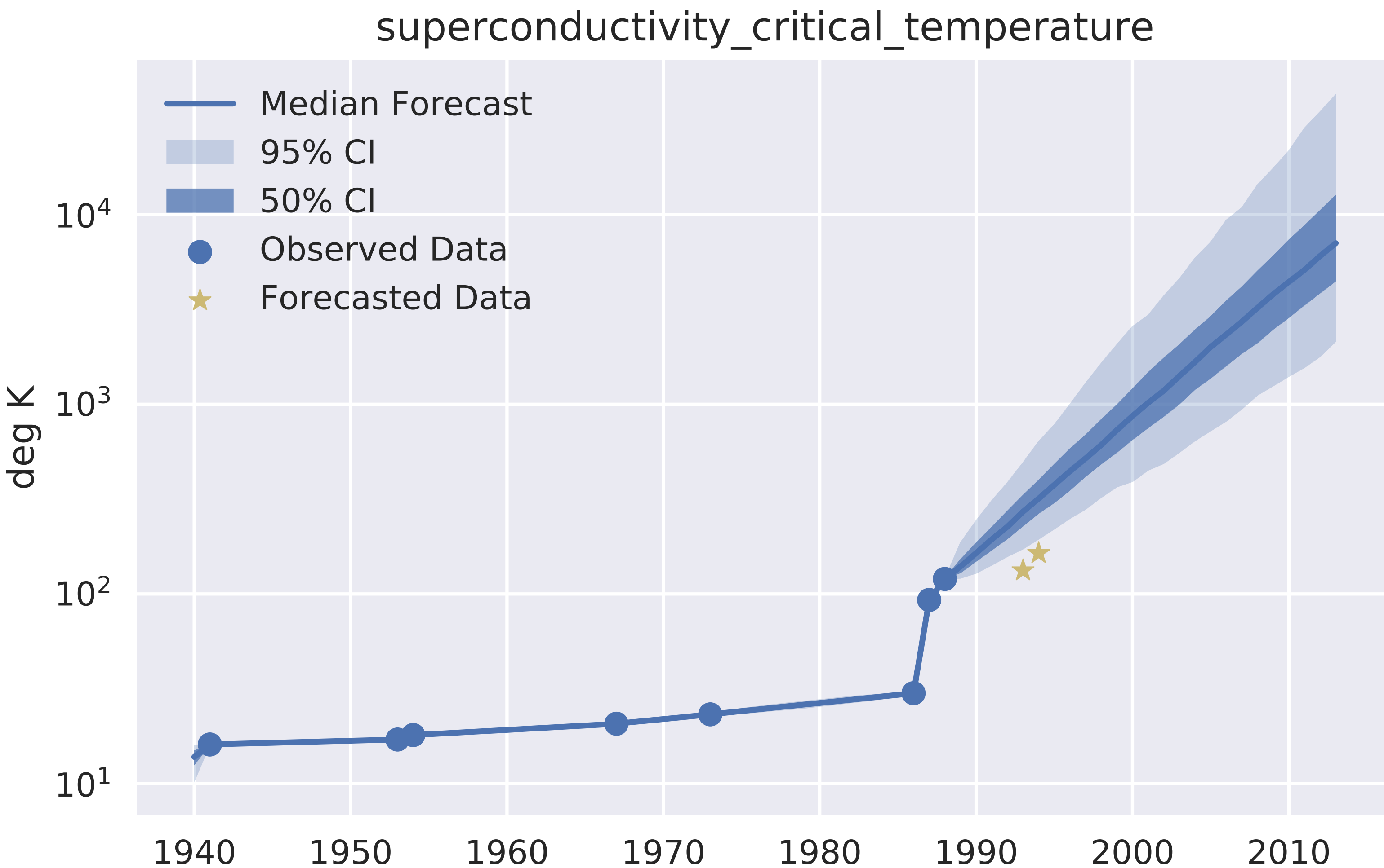
combustion_air_turbine_engine_w_per_kg
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
10^4
W/kg
1940
1950
1960
1970
1980
1990
2000
2010


combustion_air_turbine_engine_w_per_lit
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
W/lit
10^6
10^5
10^4
10^3
10^2
10^1
1940
1950
1960
1970
1980
1990
2000
2010


combustion_pass_car_piston_engine_w_per_lit
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
W/Liter
10^3
1940
1950
1960
1970
1980
1990
2000
2010

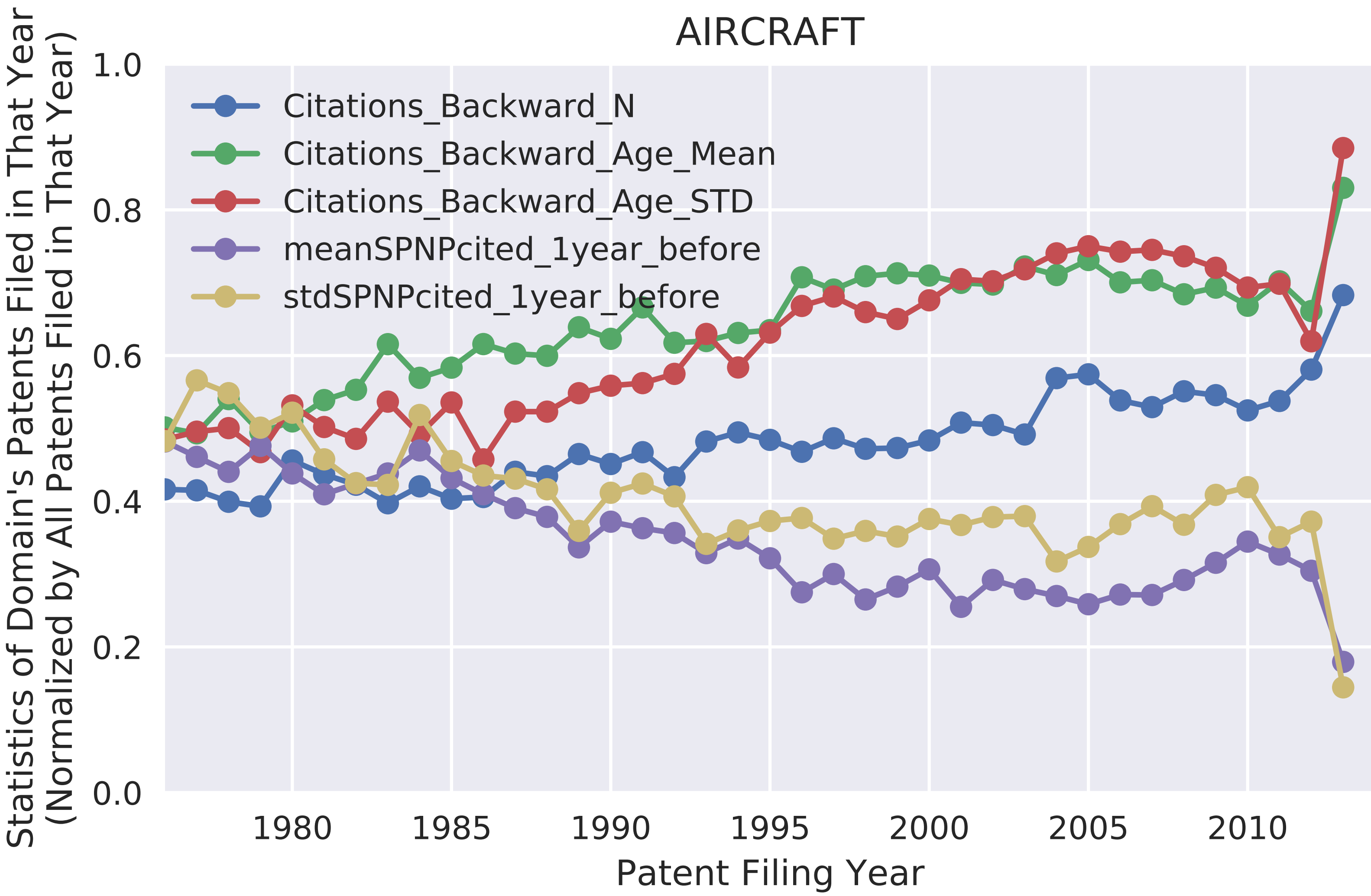


electricmotor_w_per_kg_11.14.2013
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
W/kg
6×10^2
4×10^2
3×10^2
2×10^2
1940
1950
1960
1970
1980
1990
2000
2010


superconductivity_critical_temperature
Median Forecast
95% CI
50% CI
Observed Data
Forecasted Data
deg K
10^4
10^3
10^2
10^1
1940
1950
1960
1970
1980
1990
2000
2010

A1.3 All Patent Domains

These candidate predictors were calculated for each filing year for all patents filed in that year and expressed as a percentile, relative to the other patents in that filing year. Then for each domain the value of that predictor for each year was taken to be the average of the domain's patents filed in that year. This was the data used to try to improve technology performance and price forecasting. Because patent data was only available from 1975 onward, these models were only trained with patent and technology data from 1975 onward (up to 1990, for a total of 15 years of possible data).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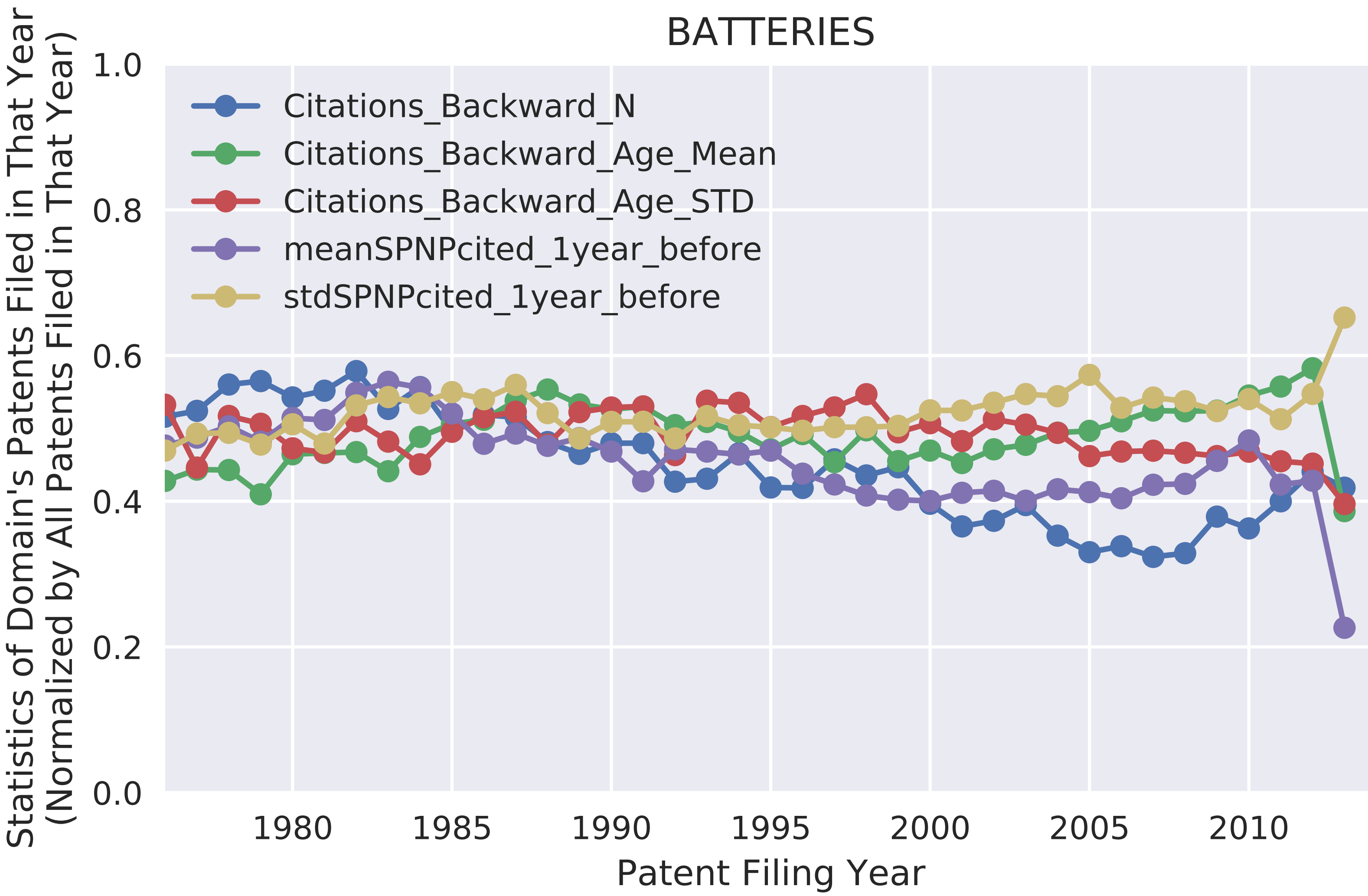
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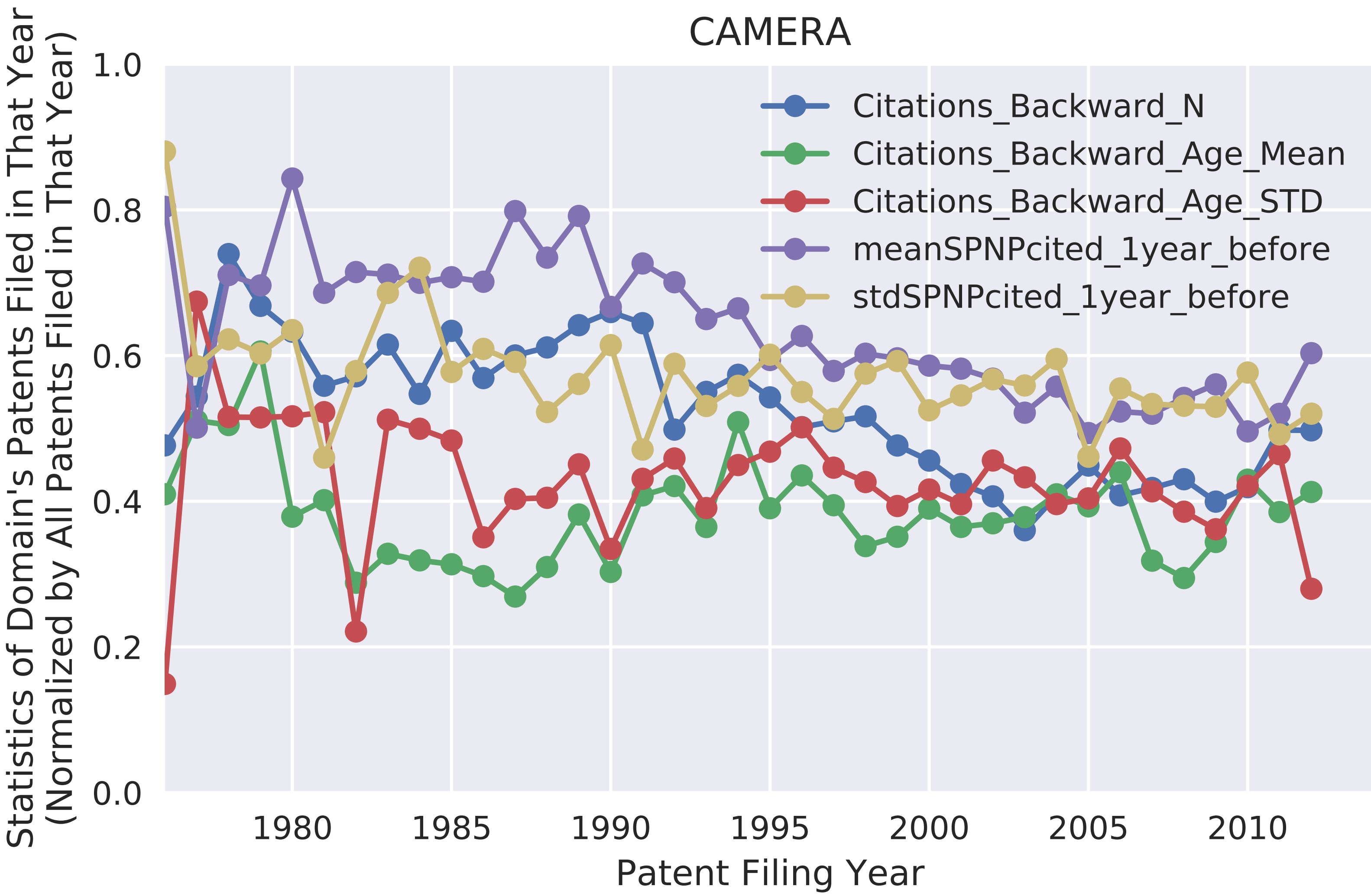
Patent Filing Year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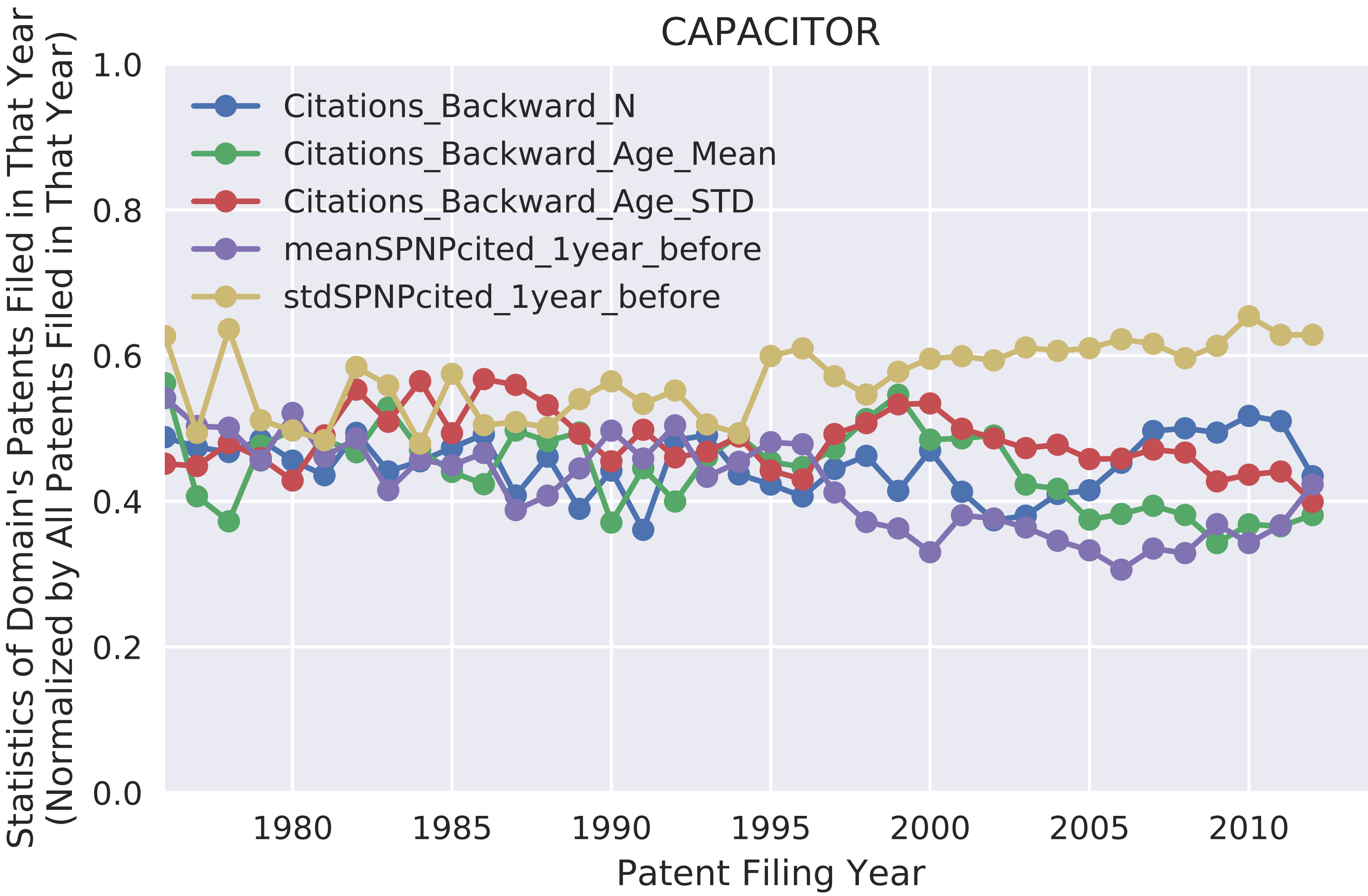

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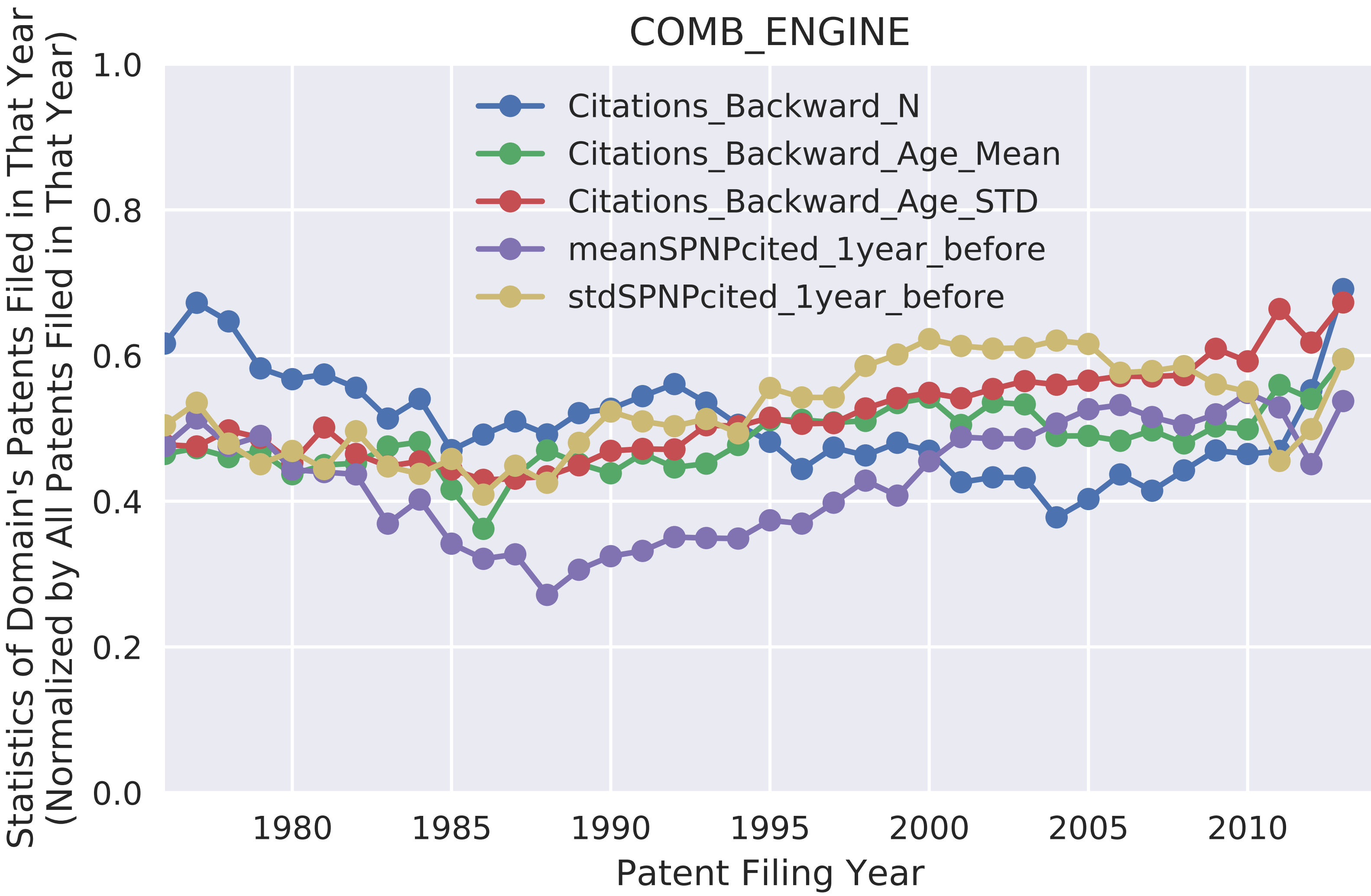

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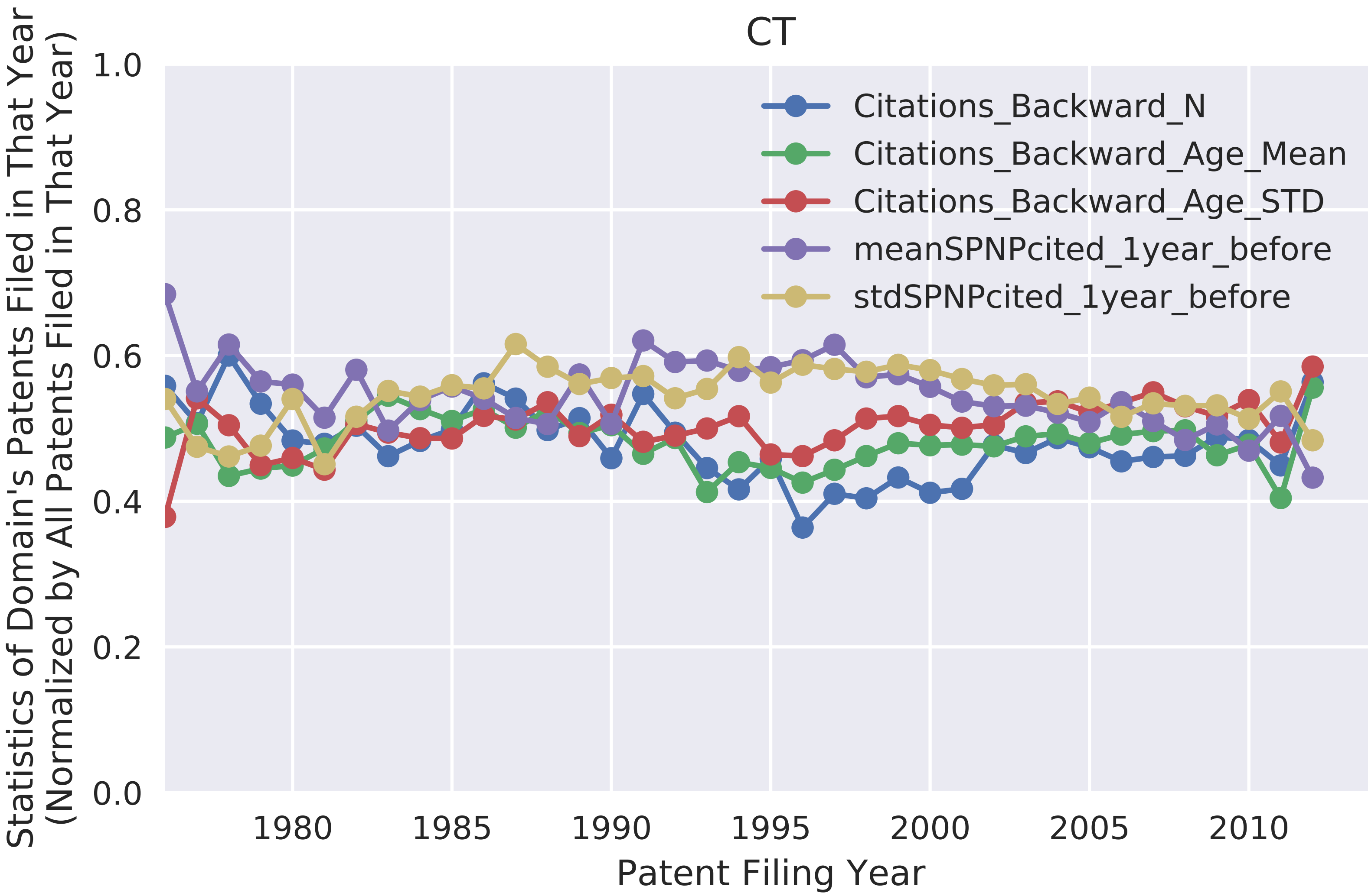
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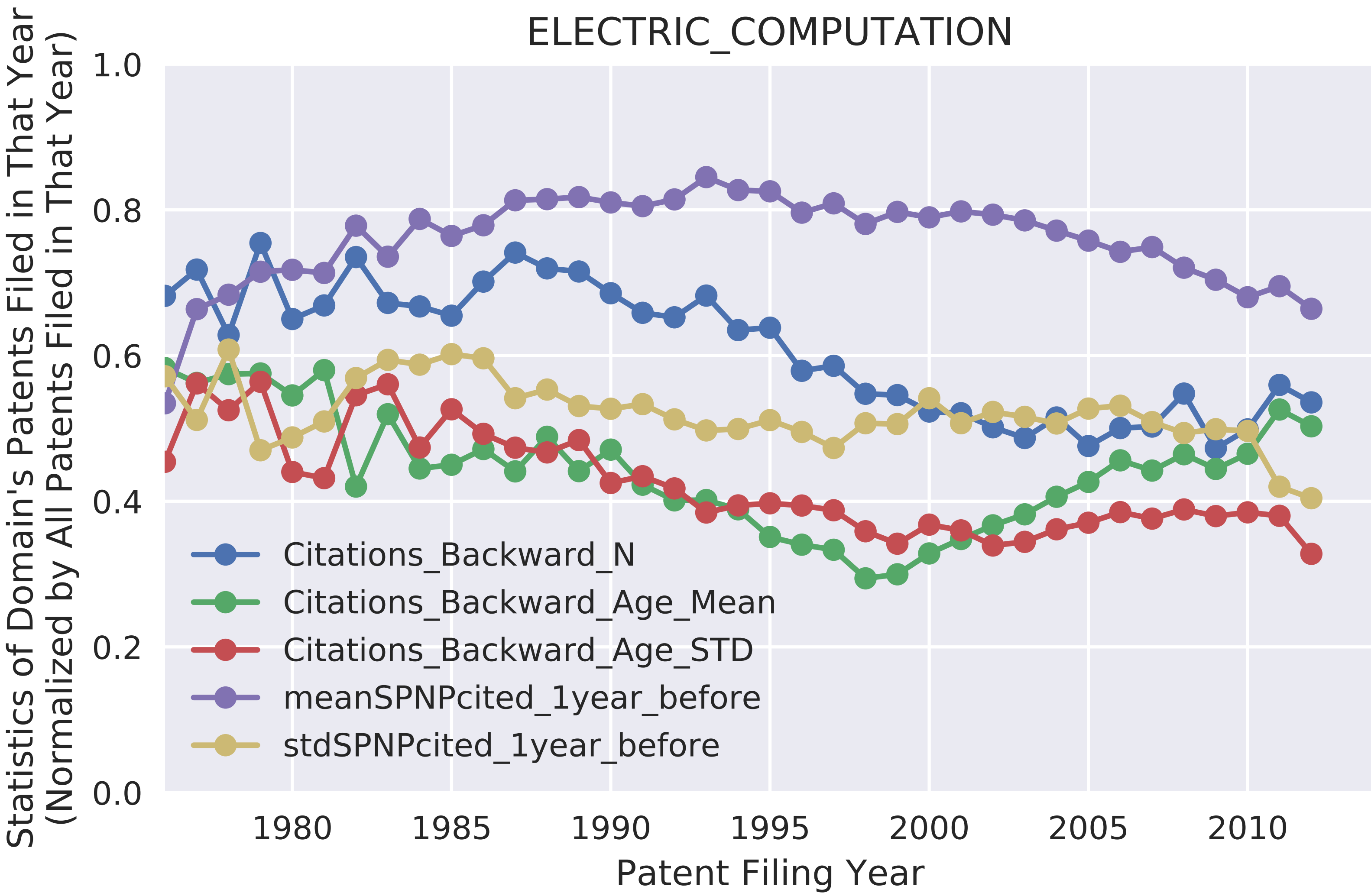
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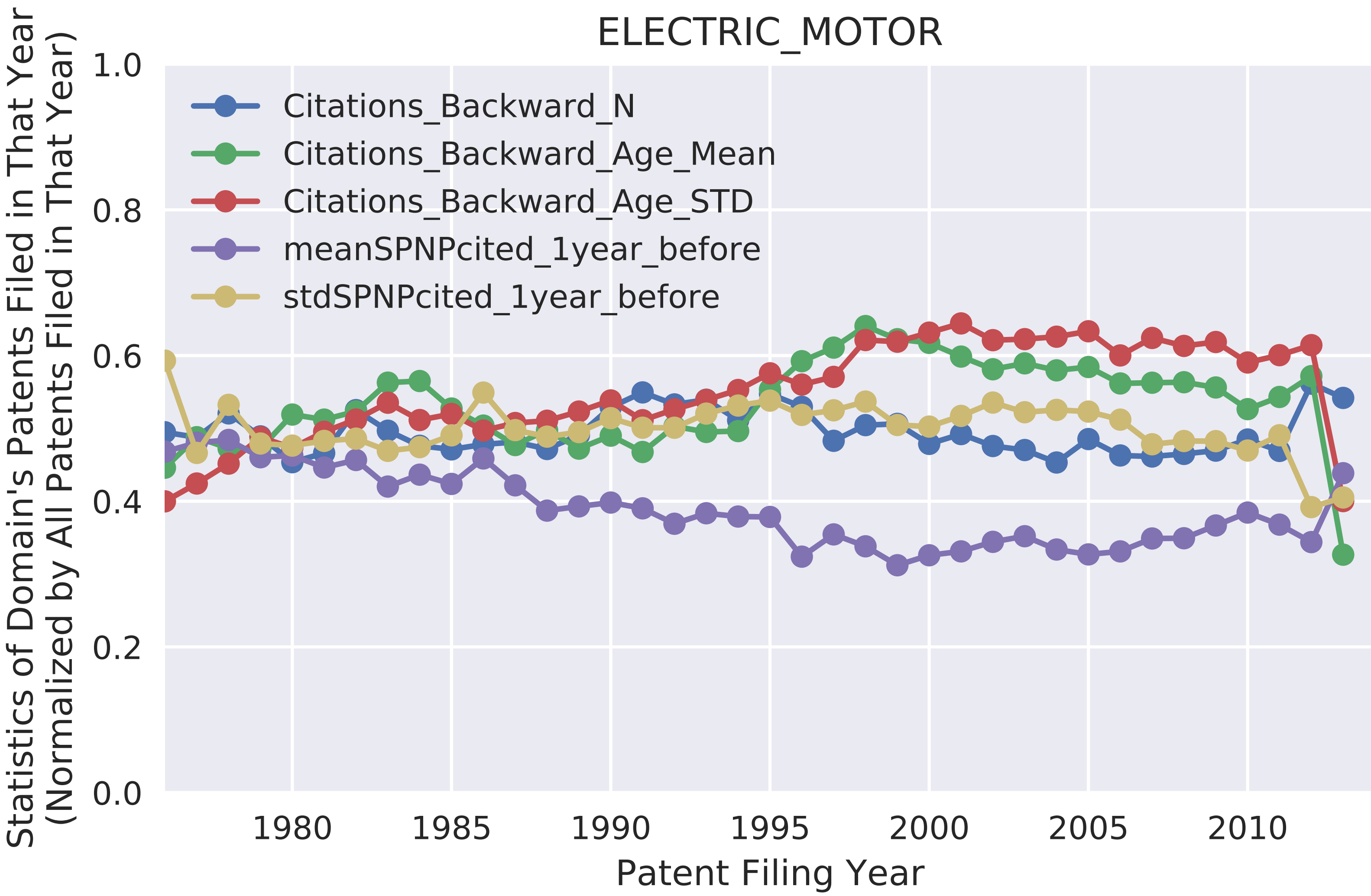
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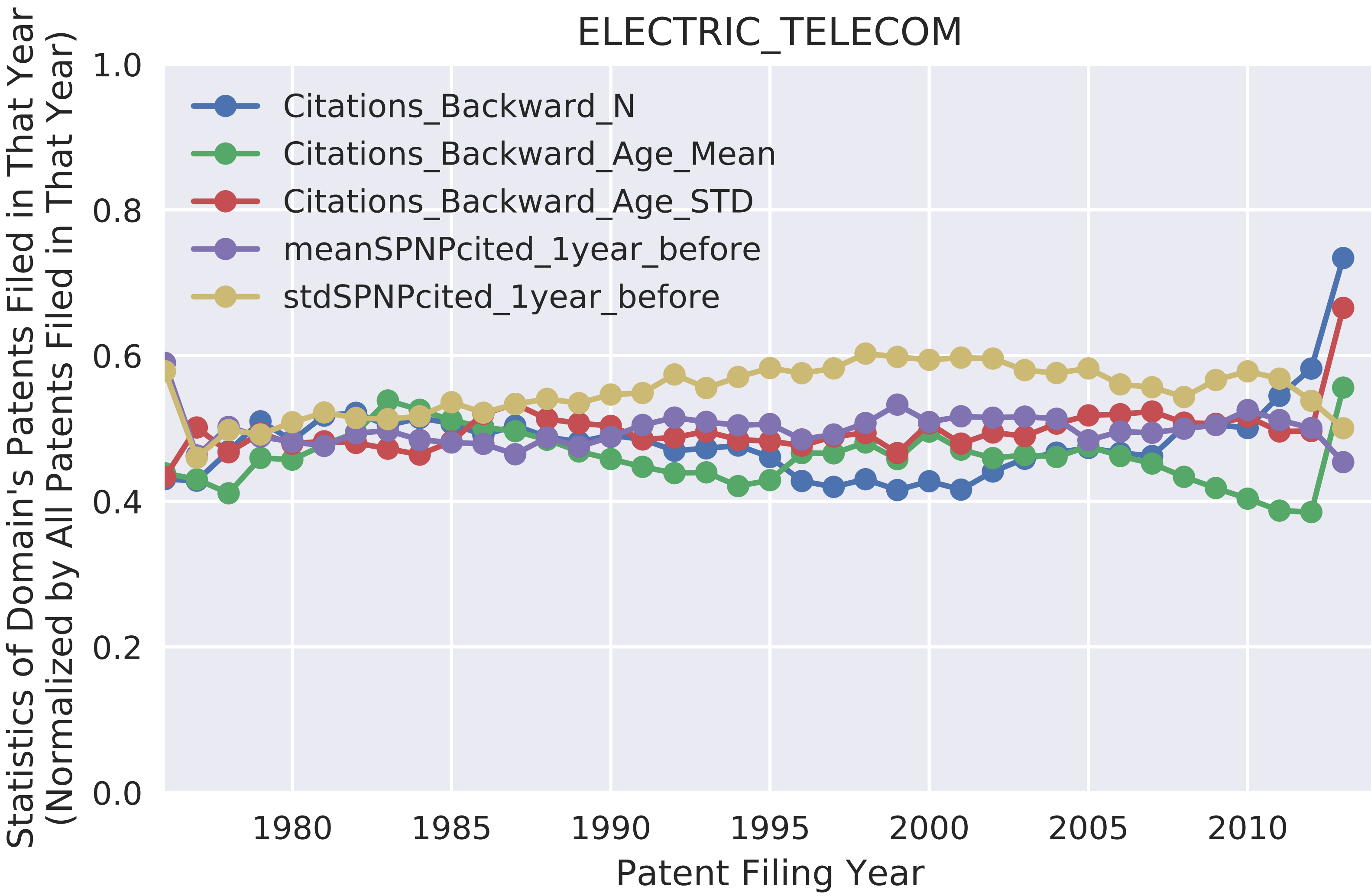
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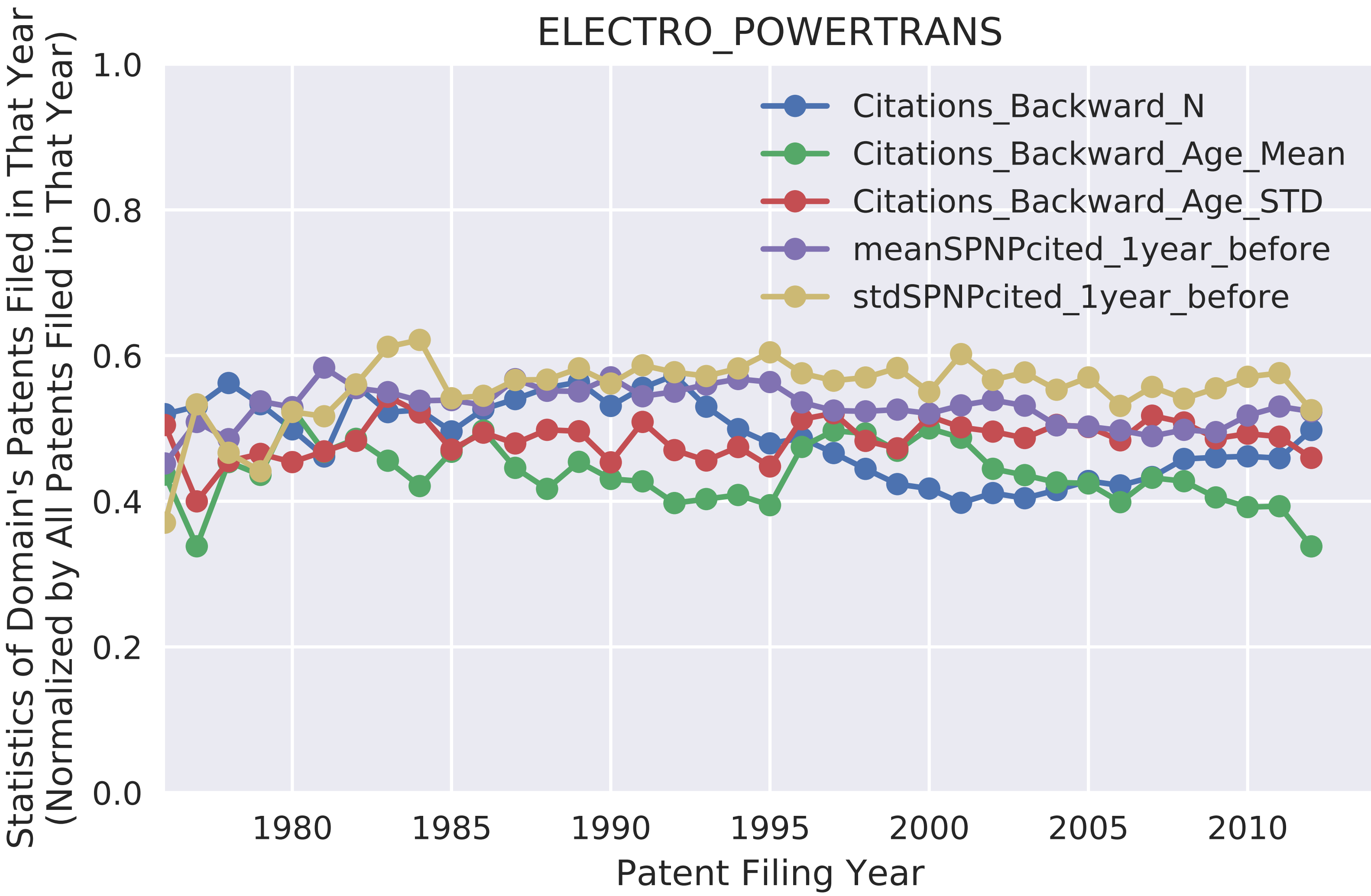

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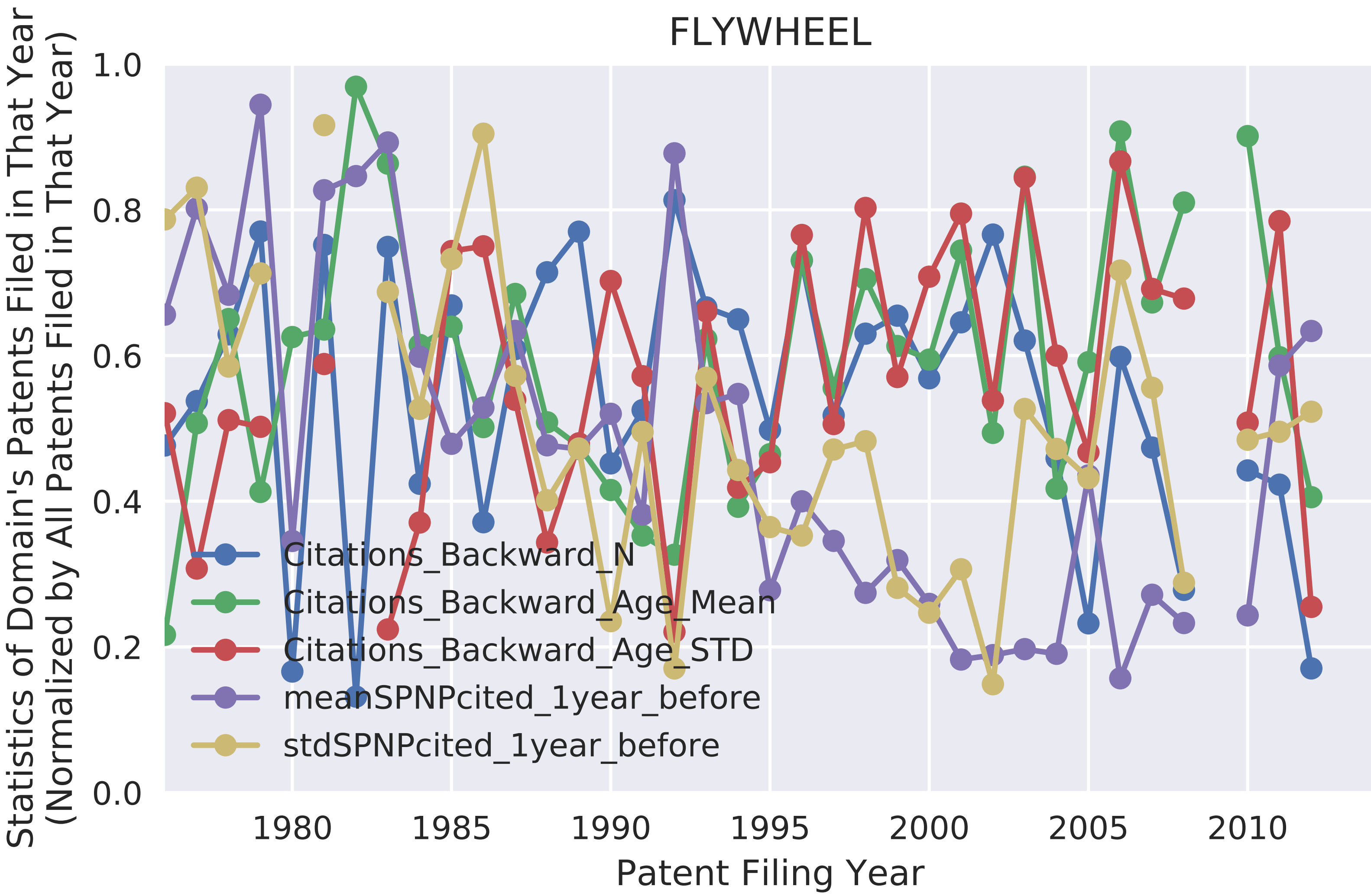
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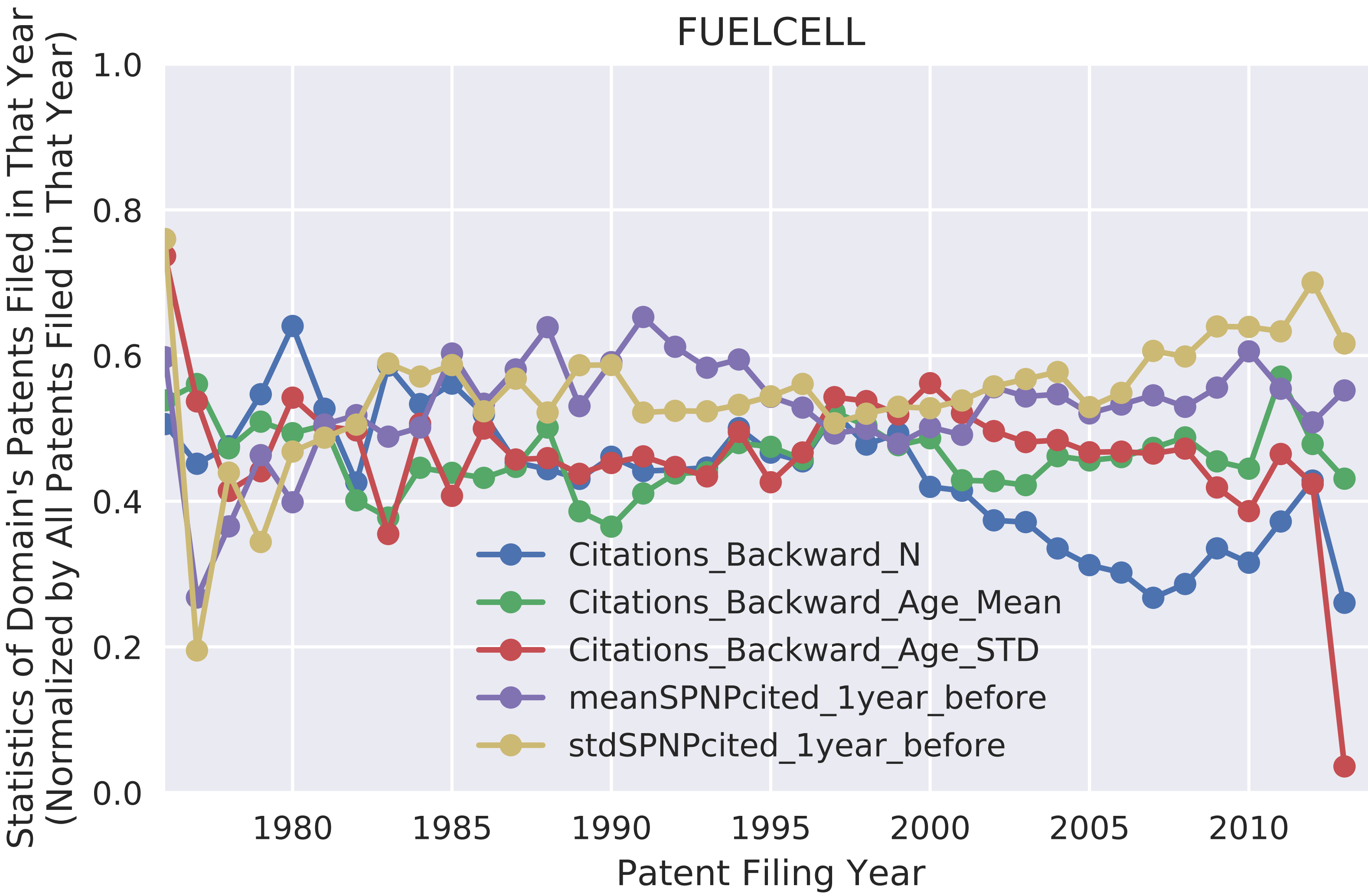
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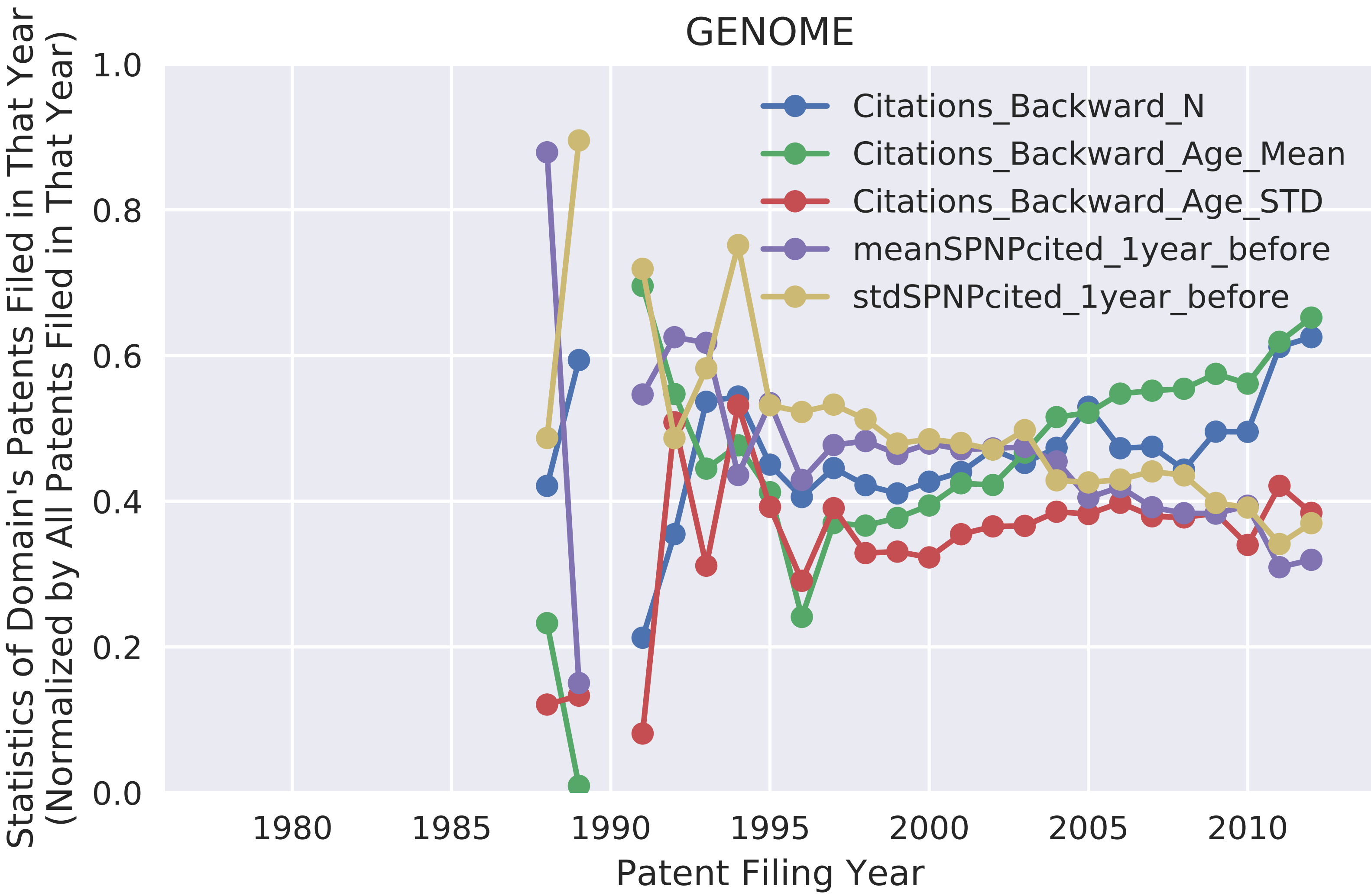

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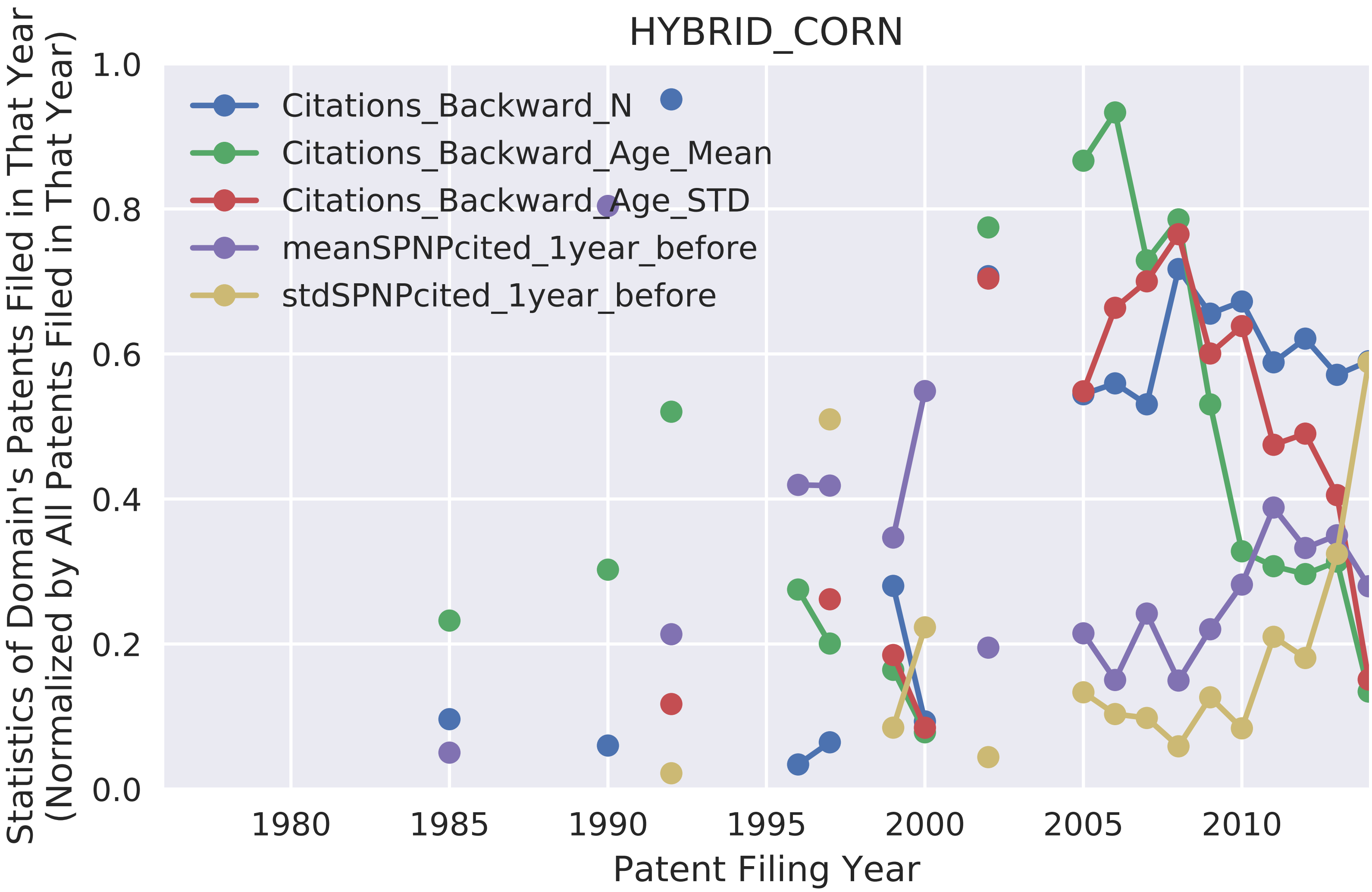

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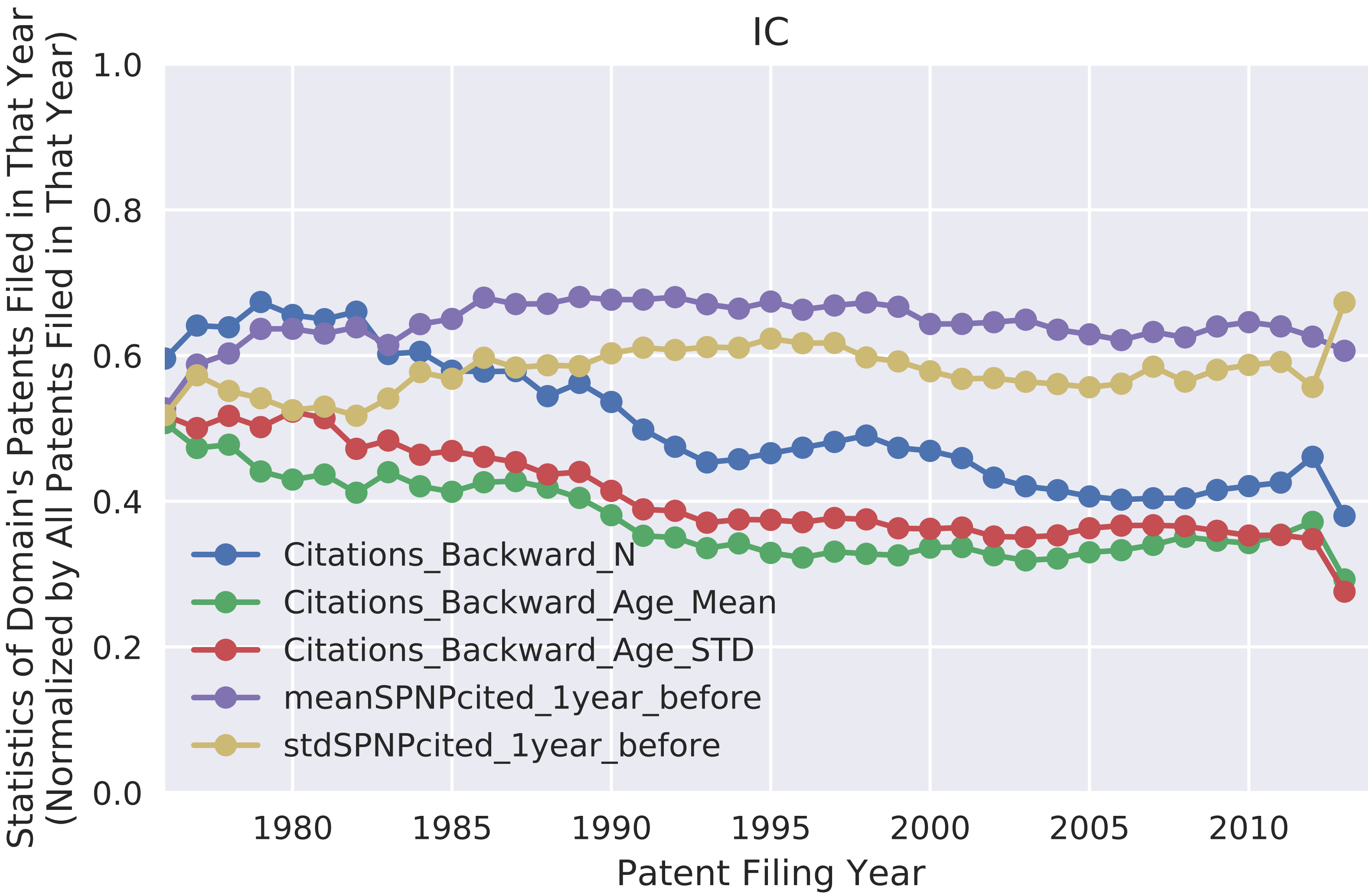

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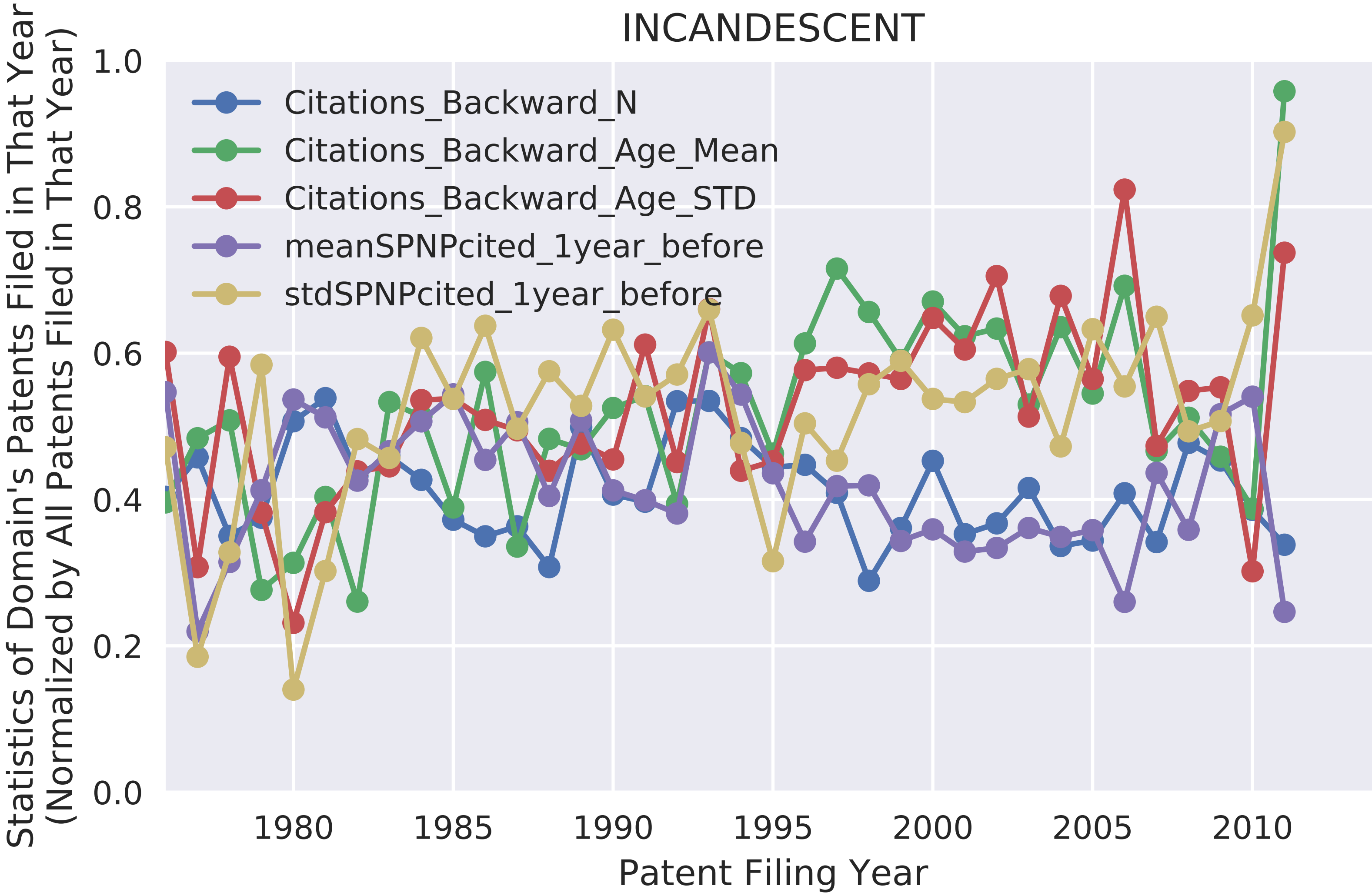
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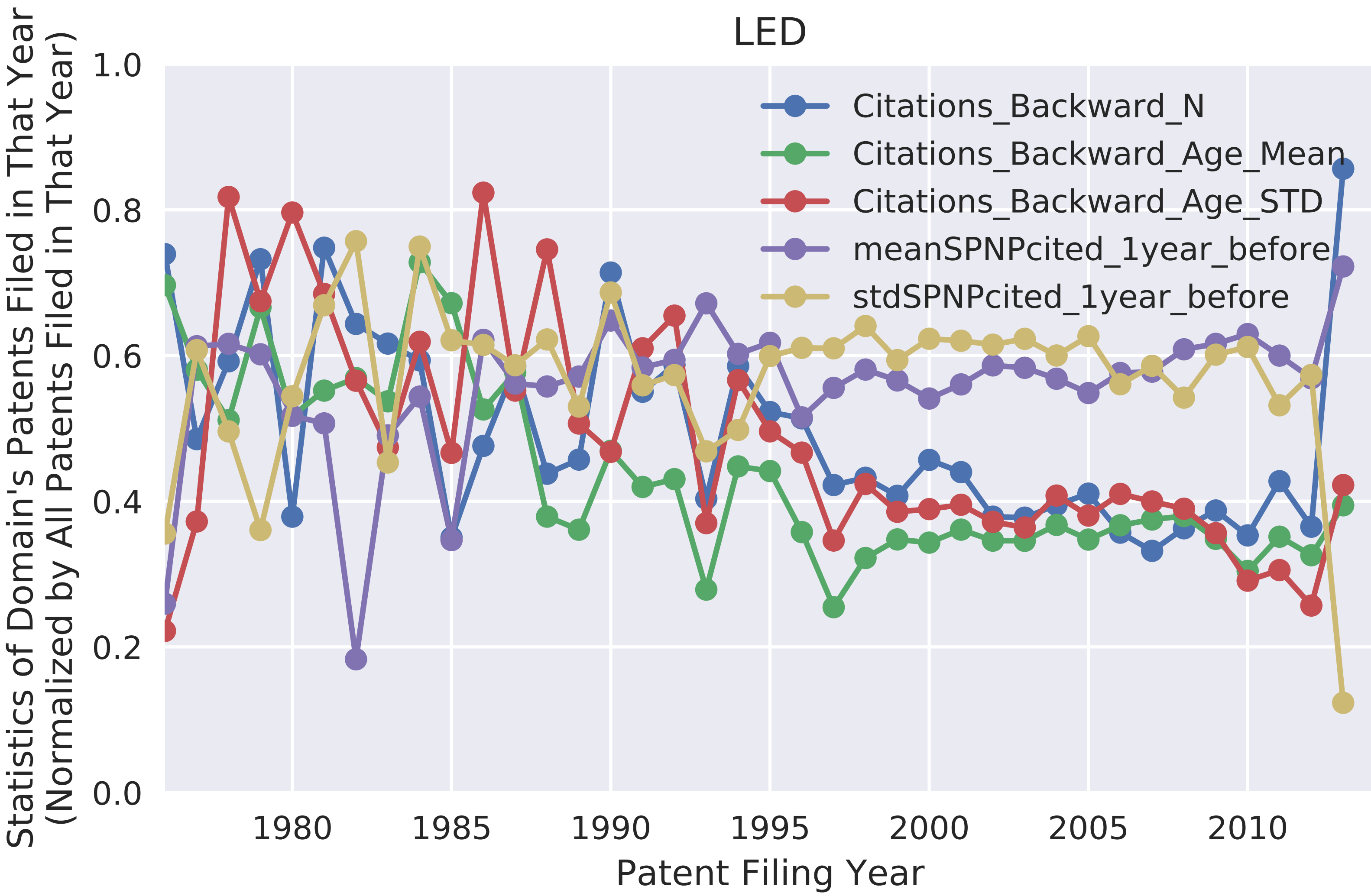
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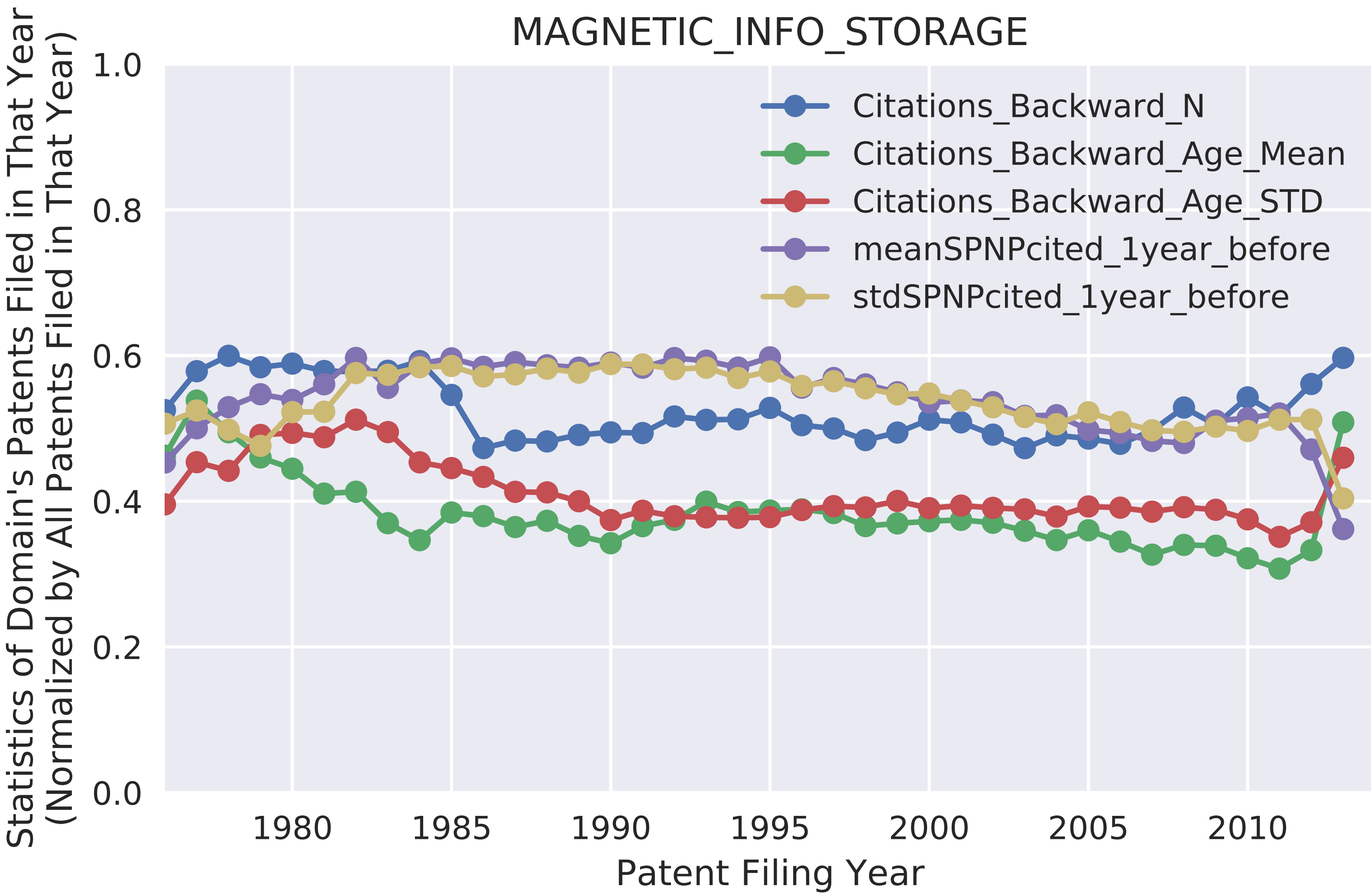

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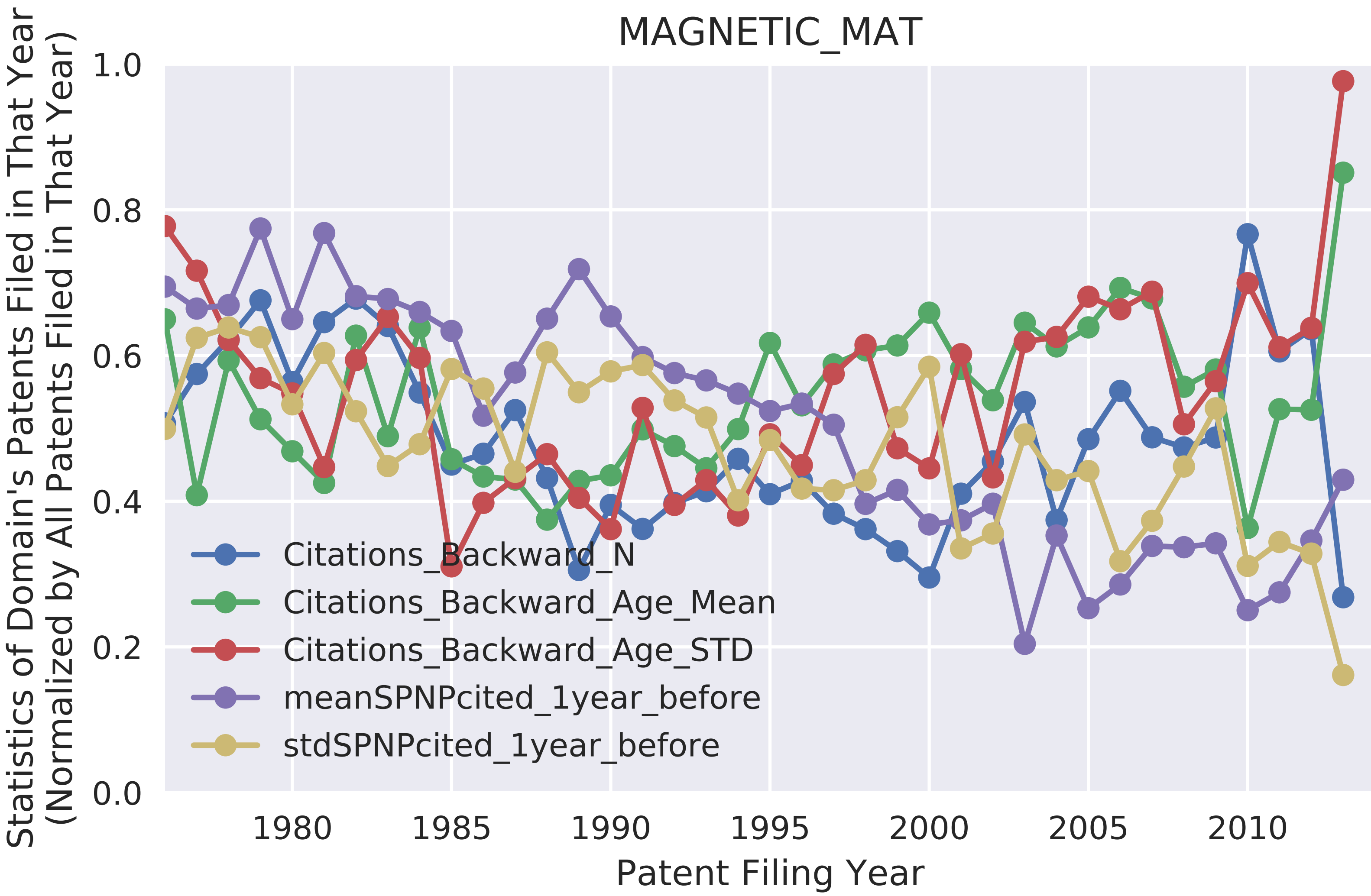
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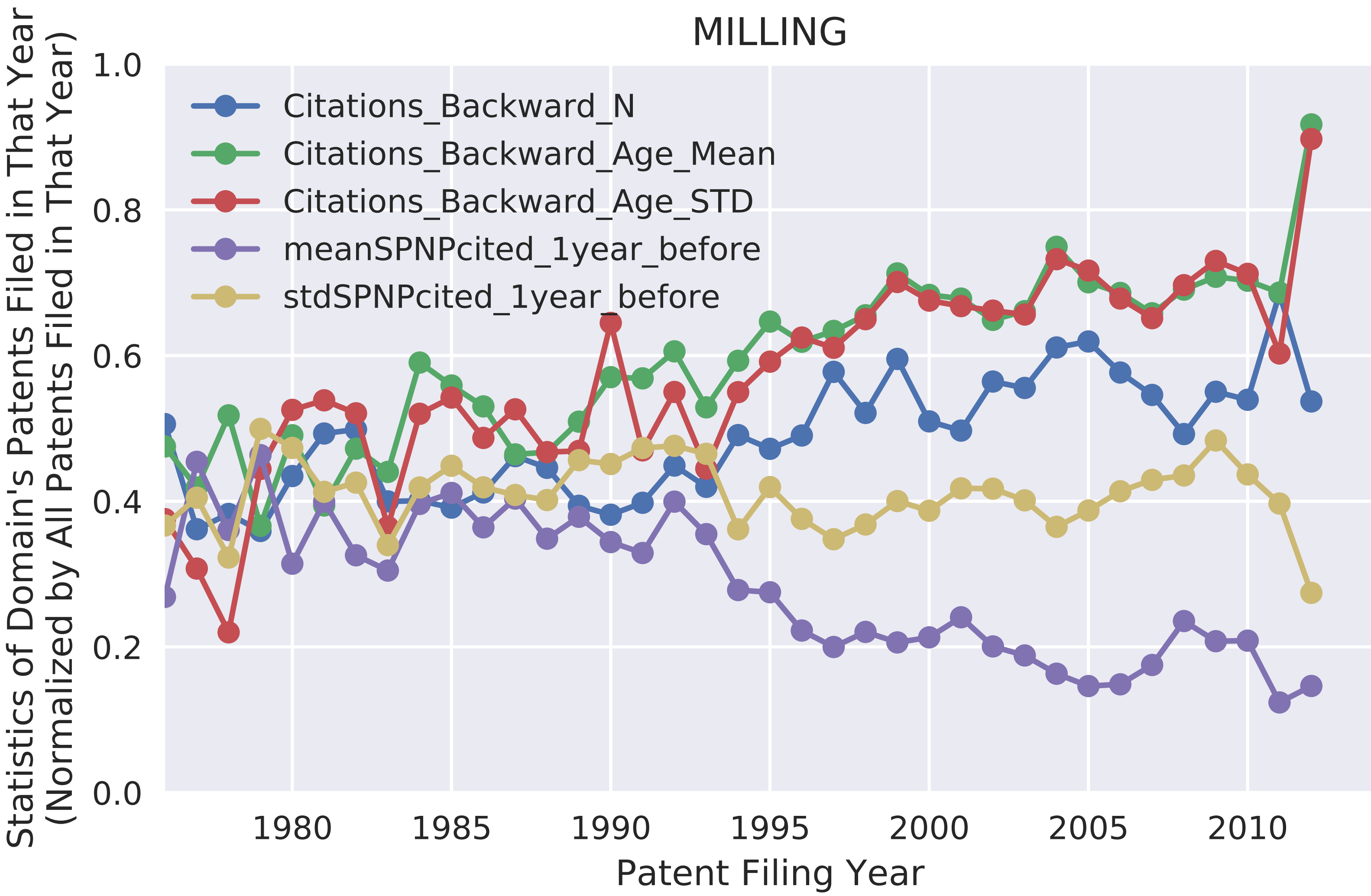
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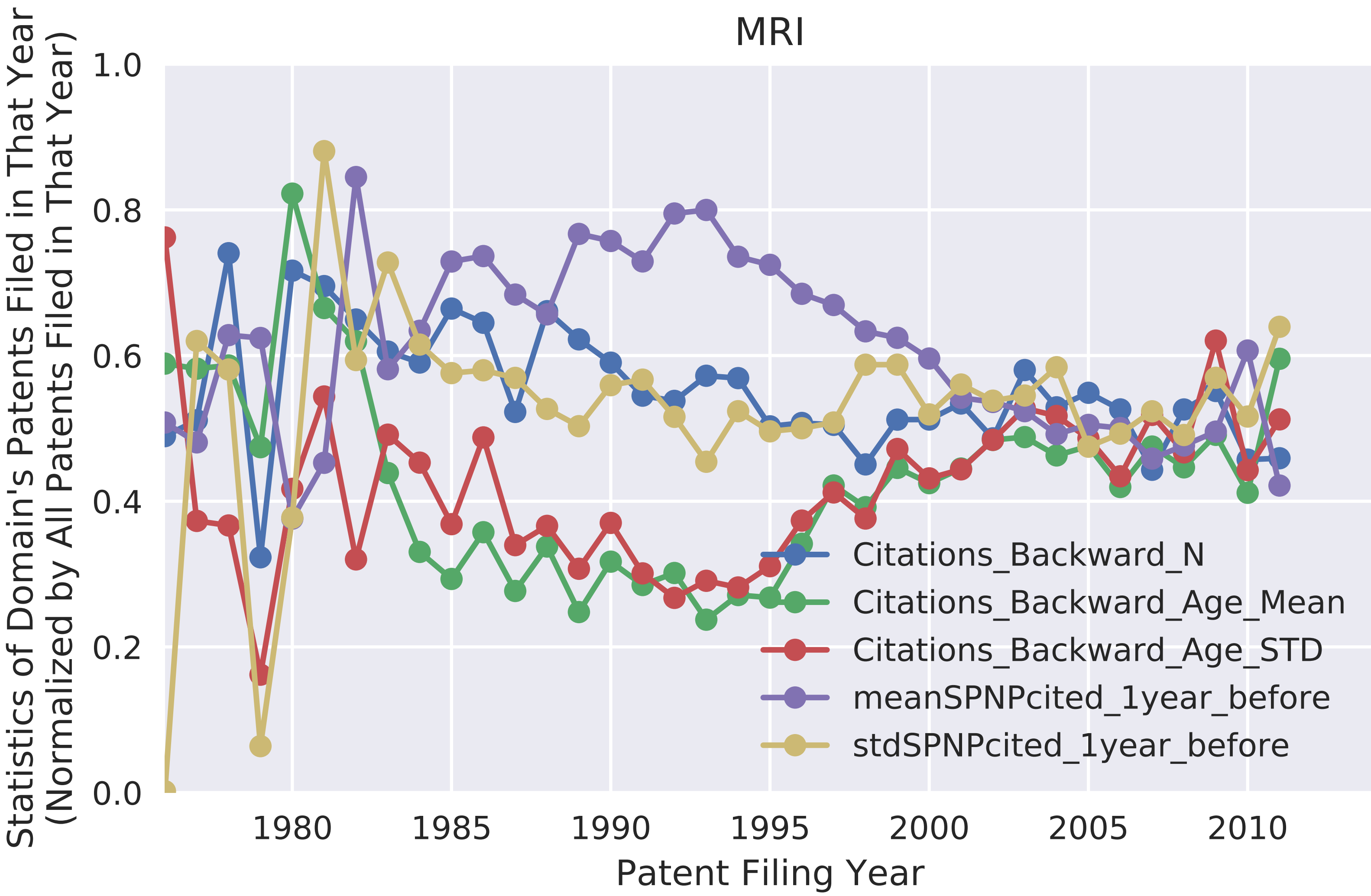
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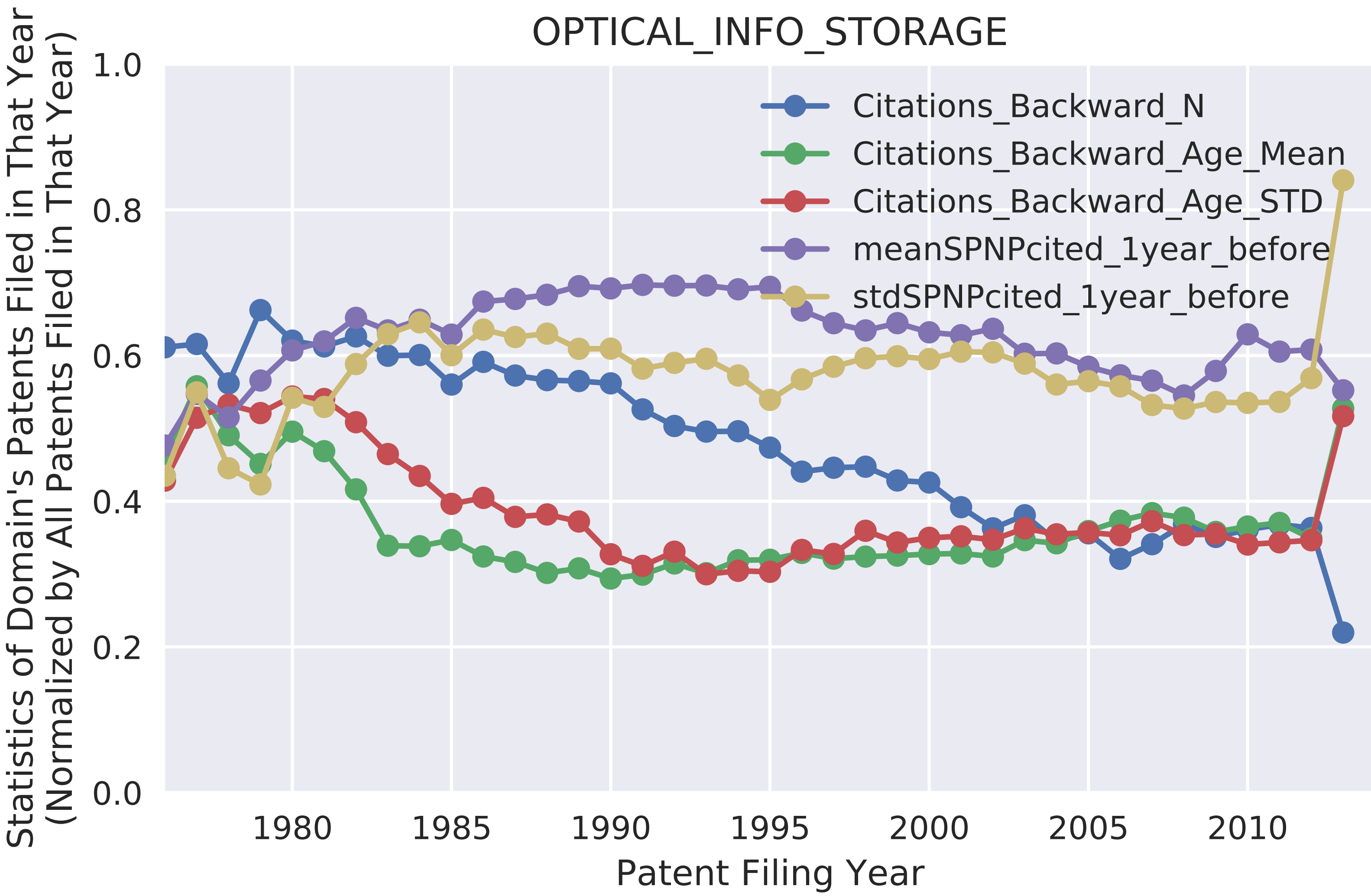

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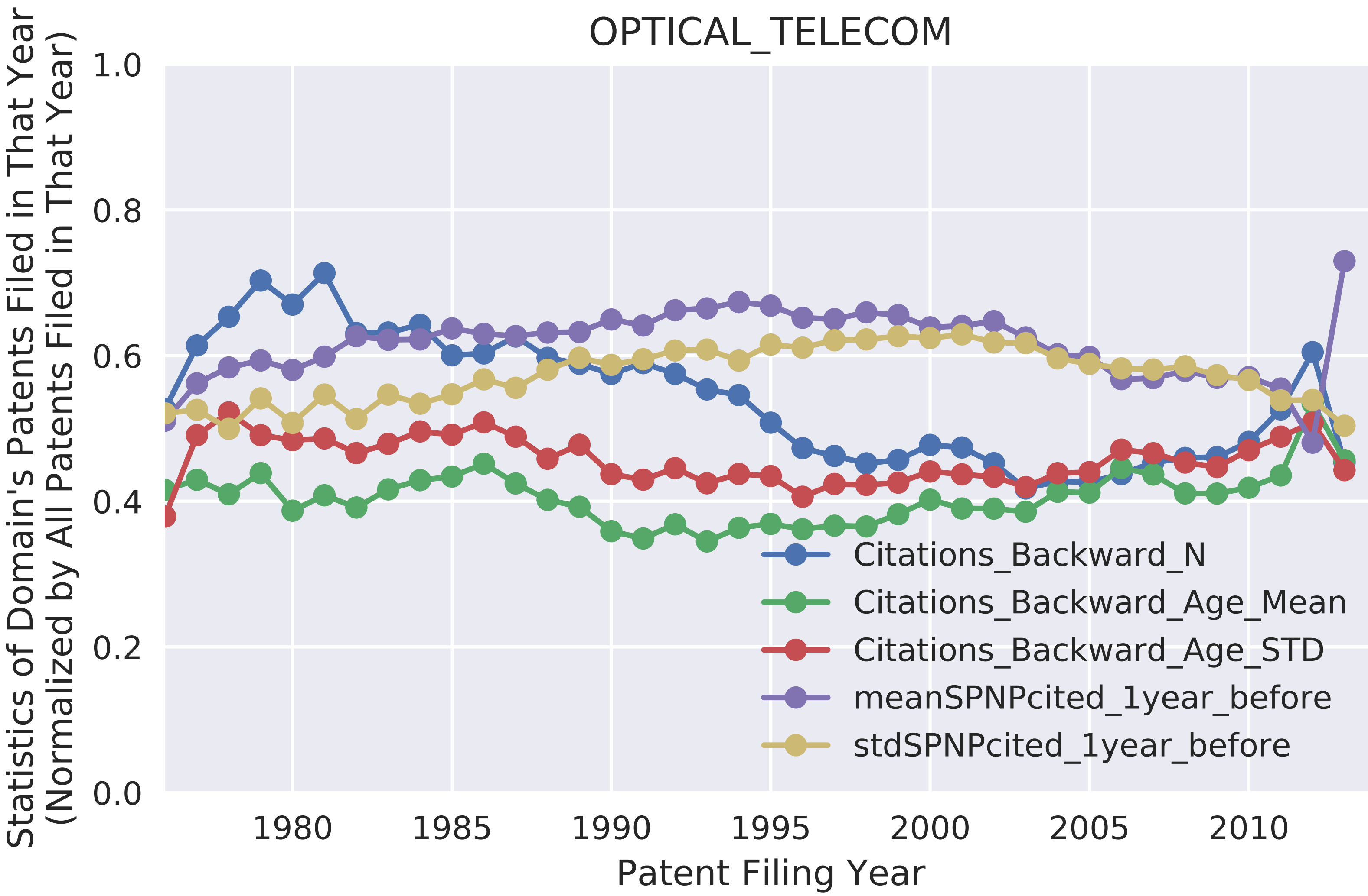
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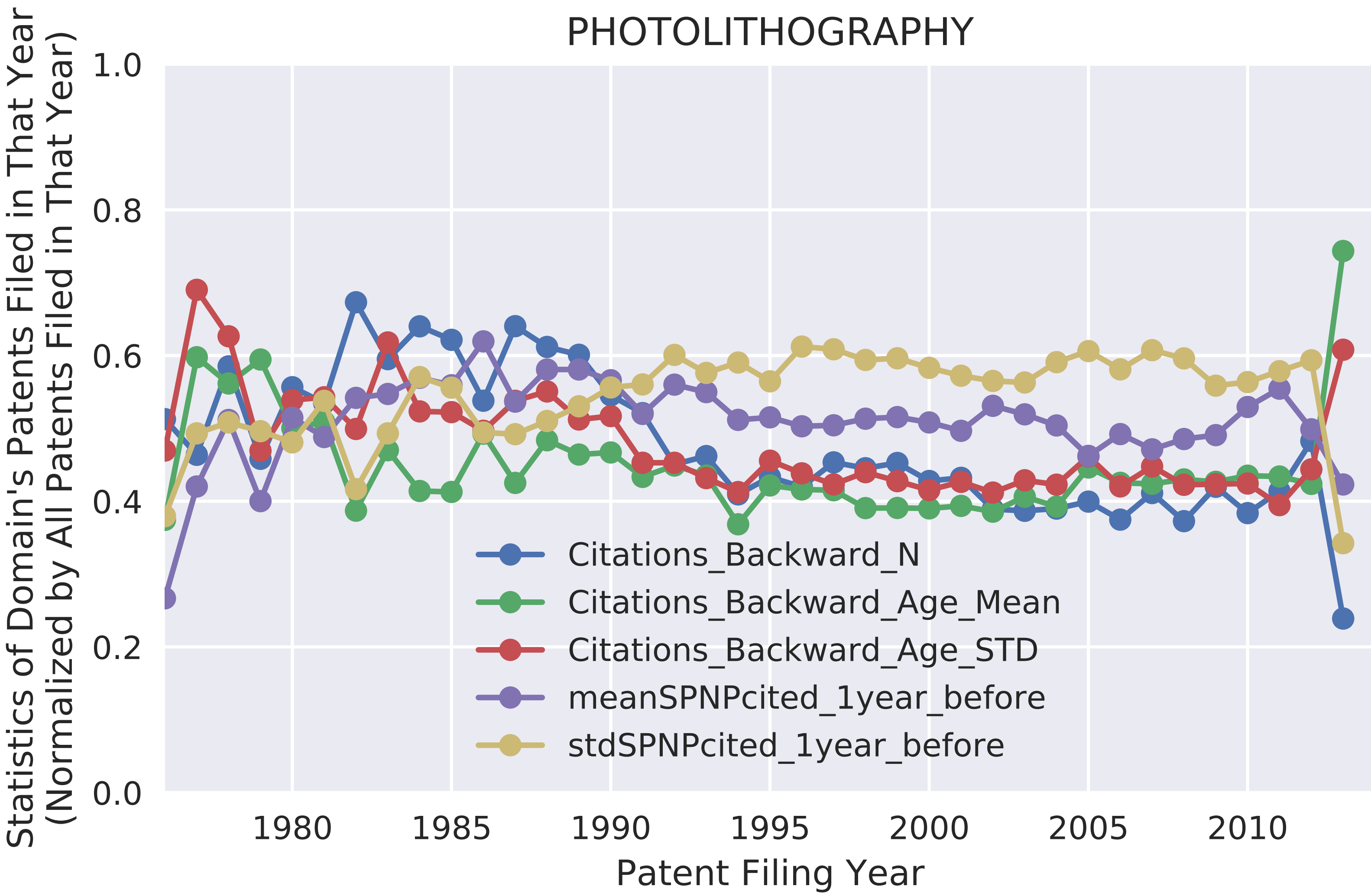

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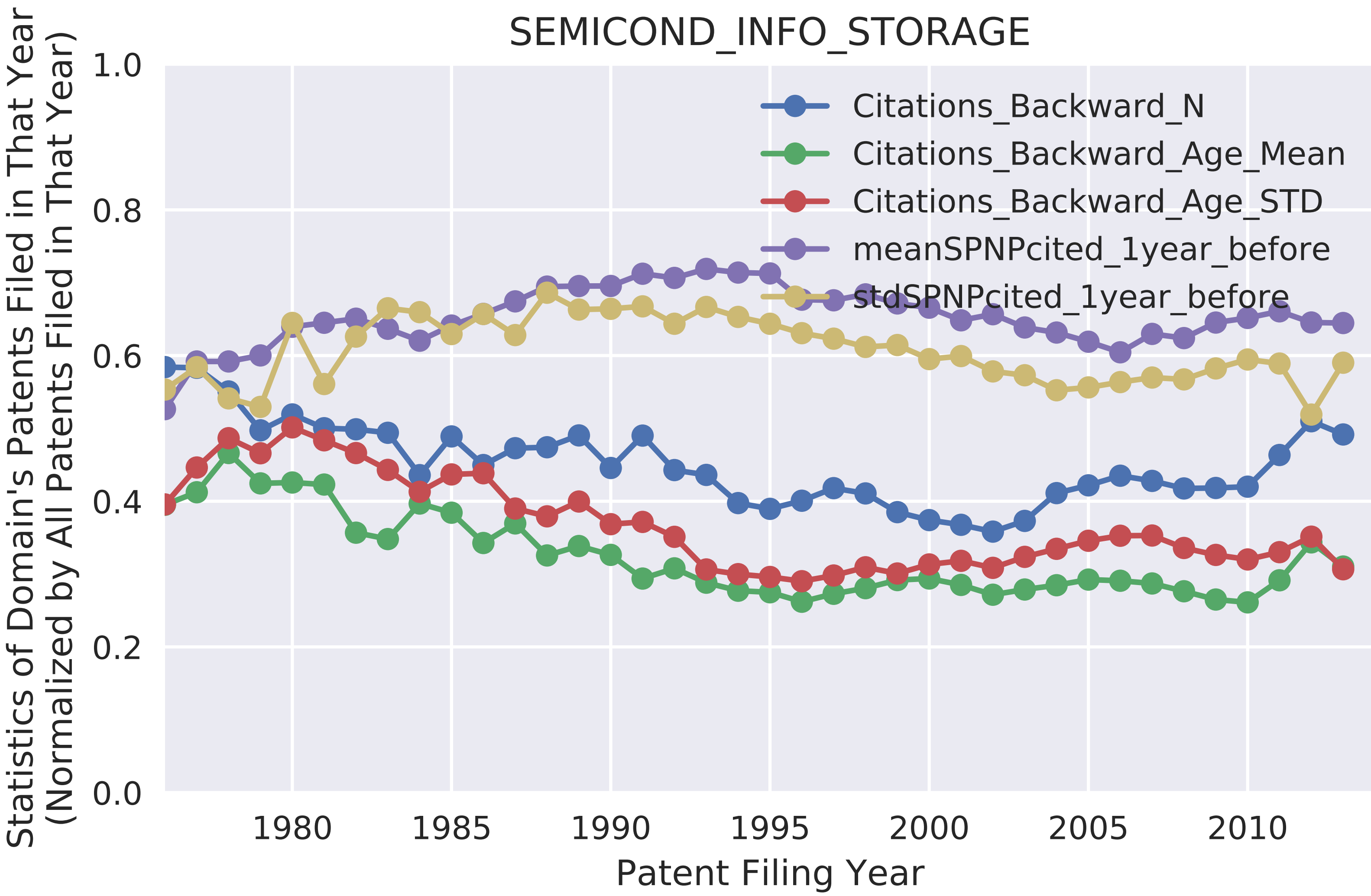
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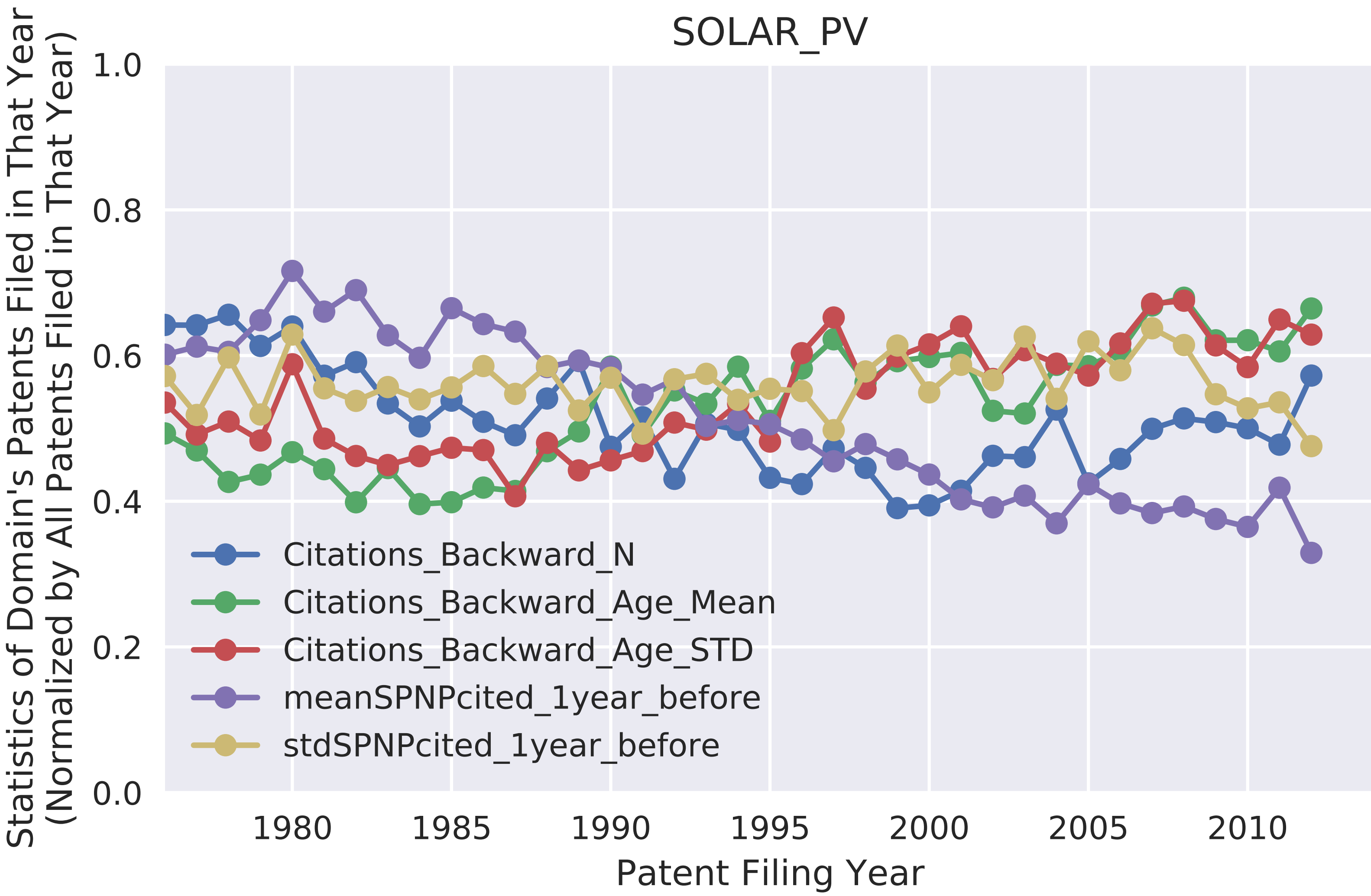
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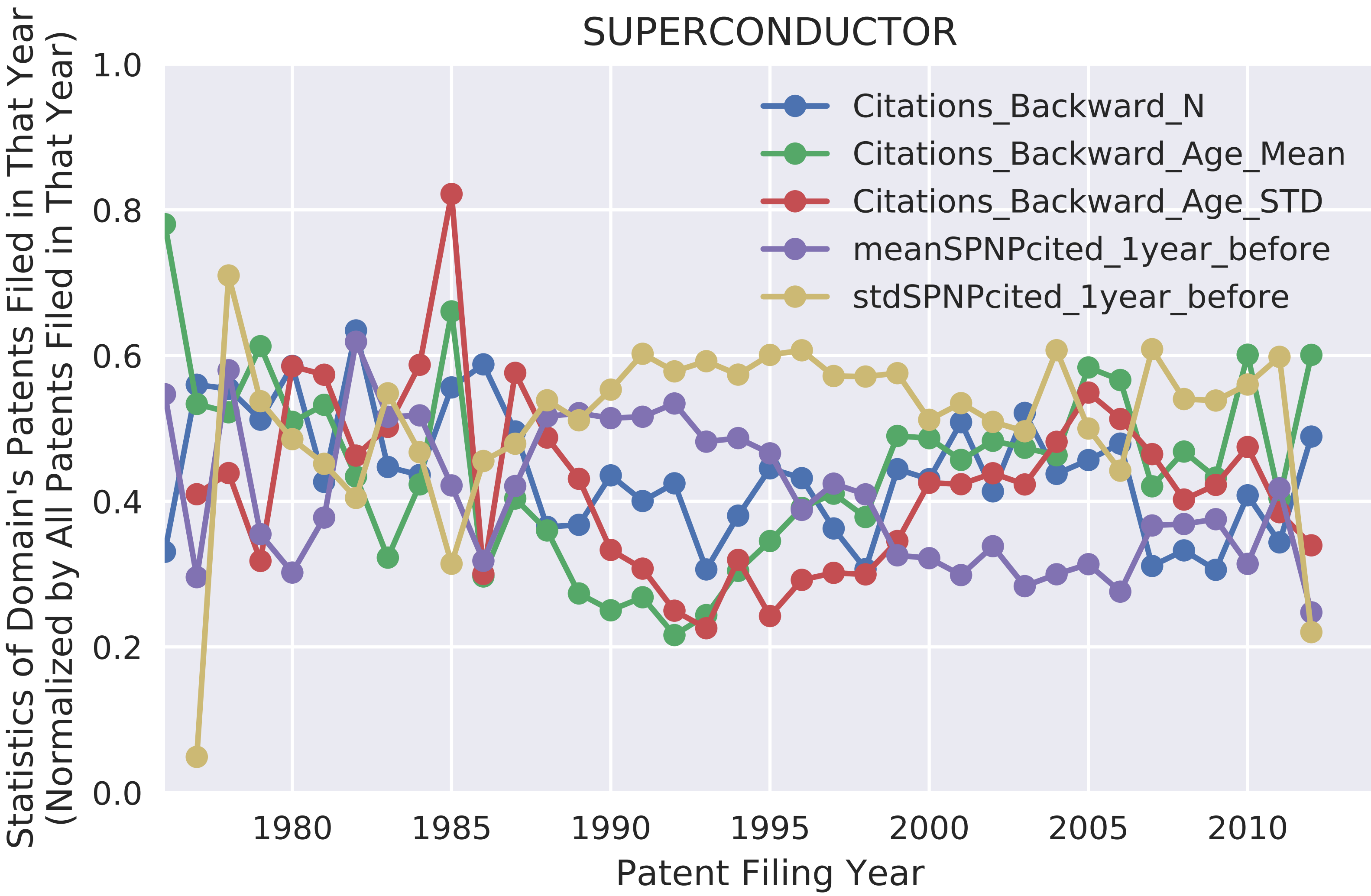
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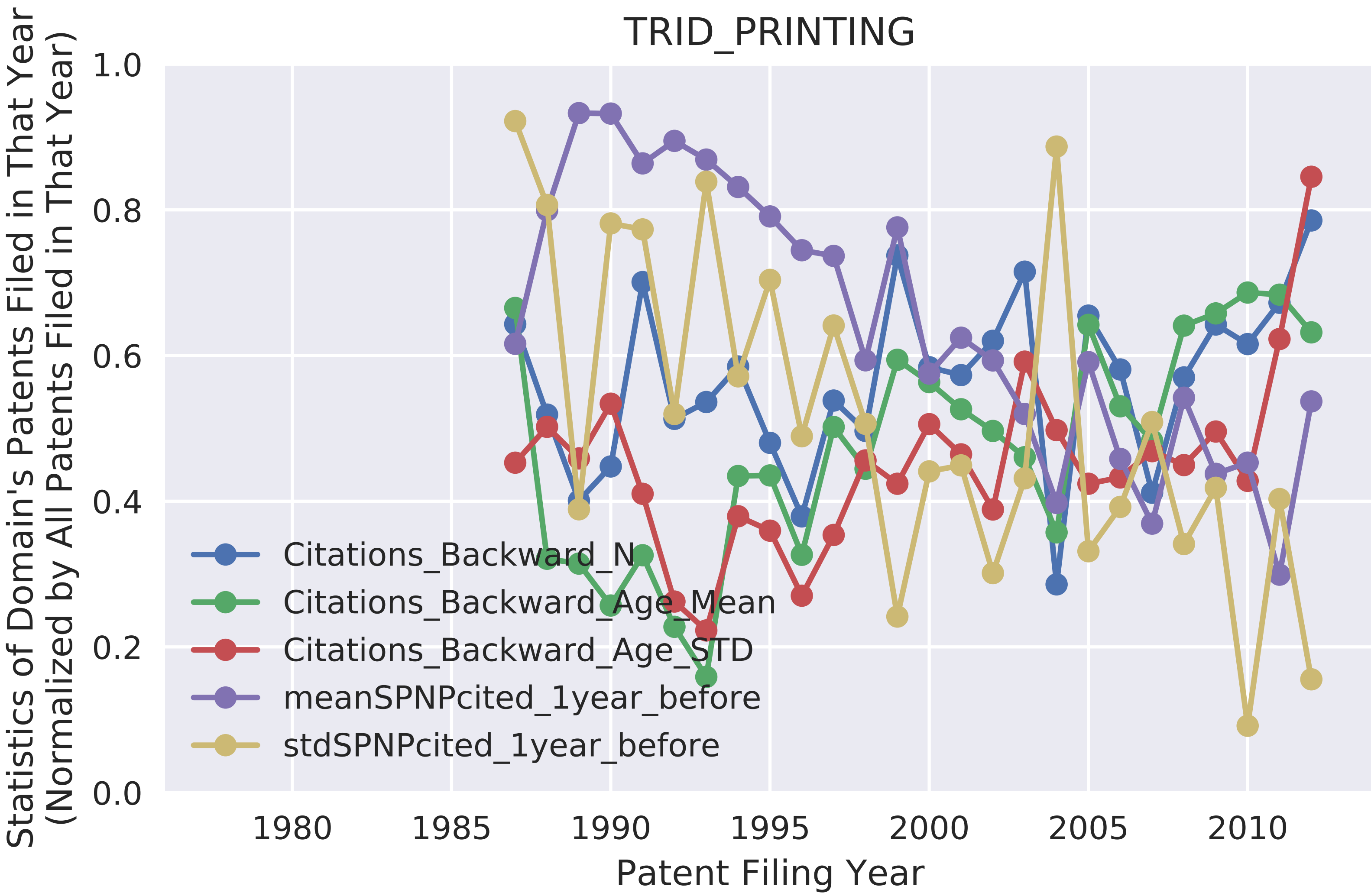

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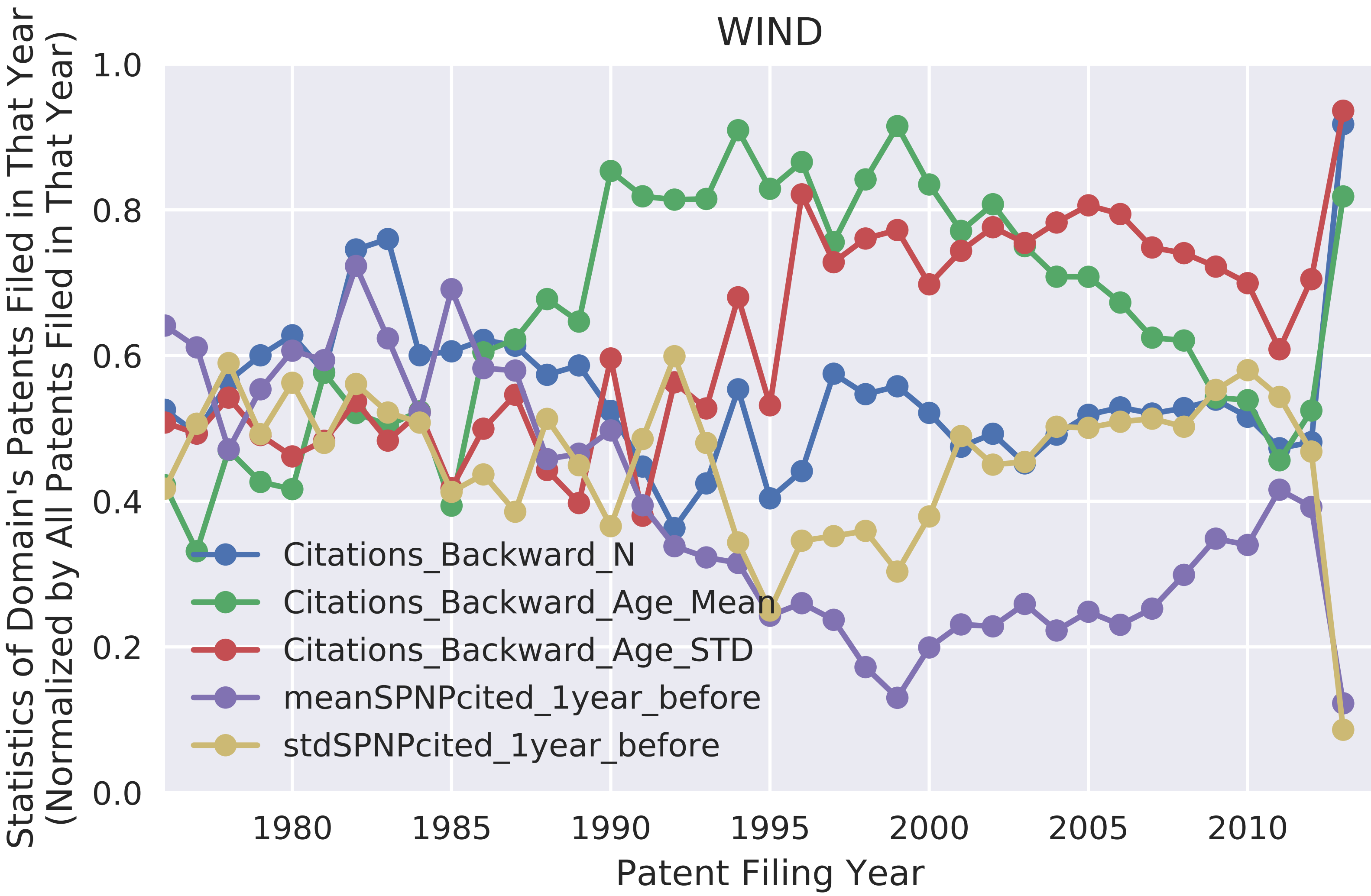
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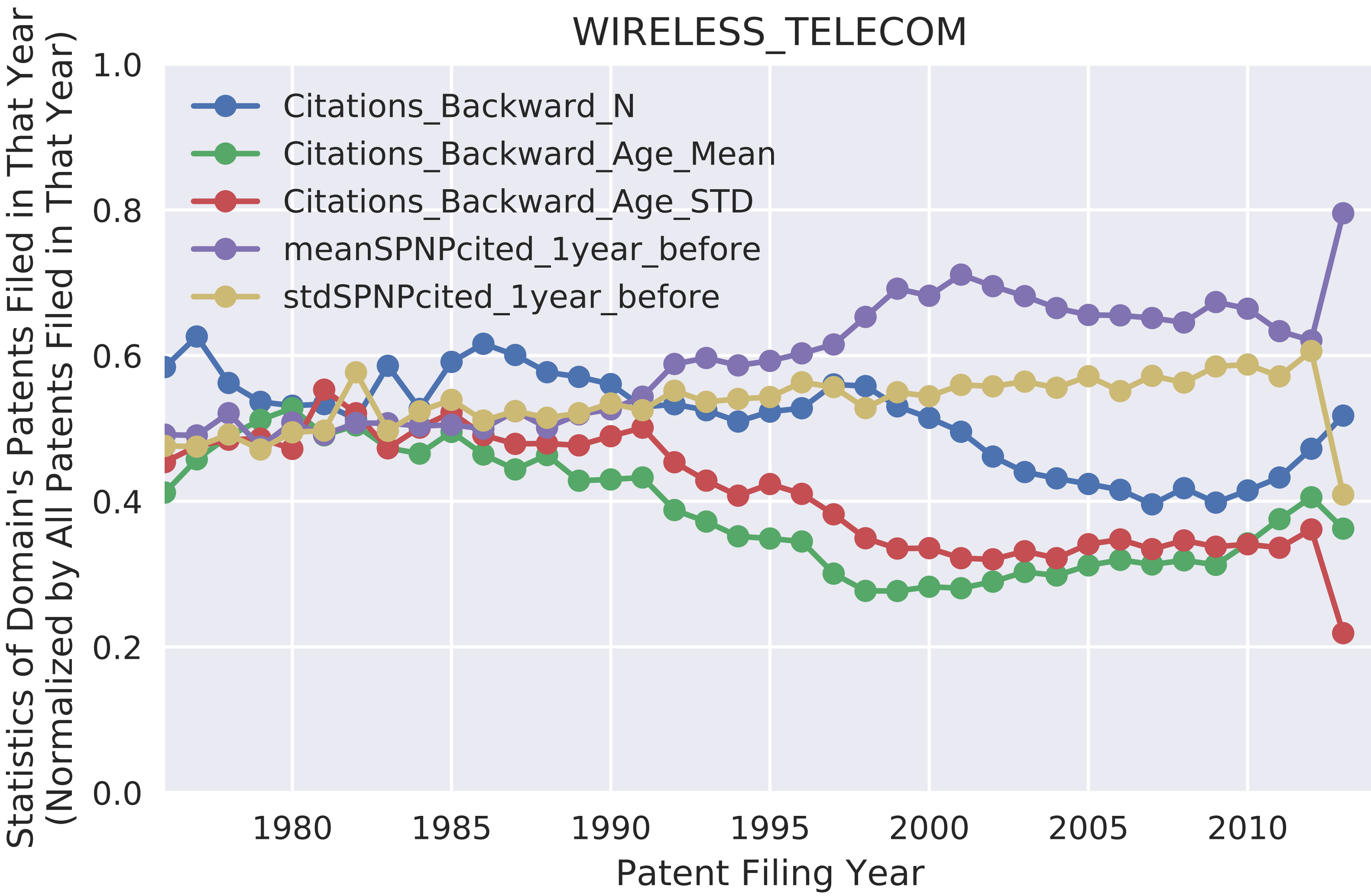
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A2 Technical Materials

A2.1 Mathematical Models

A2.1.1 ARIMA

The time series models were all modifications of ARIMA models (AutoRegressive Integrative Moving Average). The Integrative aspect says that instead of trying to model the level of a time series (Y) at year t (Y_t) we are modeling its change from the previous year ($Y_t - Y_{t-1}$). The simple model of a fixed improvement every year with noise is:

$Y_t - Y_{t-1} \sim \mu + \epsilon_t$

$\epsilon_t \sim normal(0, \sigma)$

and we have prior beliefs on these parameters of:

$\mu \sim normal(0, 4)$

$\sigma \sim cauchy(0, 4)$

The basic ARIMA model can be expanded by adding a Autoregressive element, which says the percent change in the previous year also contributes:

$Y_t - Y_{t-1} \sim \mu + \epsilon_t + \phi(Y_{t-1} - Y_{t-2})$

with a prior of

$\phi \sim normal(0, 4)$

This can be expanded to multiple steps backward in time, or to specific years. Here is what we term an ARIMA([1,5], 0) model[20]:

$Y_t - Y_{t-1} \sim \mu + \epsilon_t + \phi_1(Y_{t-1} - Y_{t-2}) + \phi_5(Y_{t-5} - Y_{t-6})$

The basic ARIMA model can also be expanded by adding a Moving Average element, which says the noise in the previous year also contributes:

$Y_t - Y_{t-1} \sim \mu + \epsilon_t + \theta\epsilon_{t-1}$

with a prior of

$\theta \sim normal(0, 4)$

This can be also expanded to multiple steps backward in time, or to specific years. Here is what we term an ARIMA(0, [1,5]) model:

$Y_t - Y_{t-1} \sim \mu + \epsilon_t + \theta_1\epsilon_{t-1} + \theta_5\epsilon_{t-5}$

[20] Typical nomenclature would denotes an ARIMA model by ARIMA(p, d, q), with p the number of AR elements, d the number of differencing used, and q the number of MA elements. Since all the models used in the present study are $d = 1$ we drop them from the notation, and we go in more detail for p and q as to which exact lags are being used for the AR and MA elements

The Autoregressive and Moving Average elements can be combined, such as in this ARIMA([1,5], [1,5]) model:

$Y_t - Y_{t-1} \sim \mu + \epsilon_t + \phi_1(Y_{t-1} - Y_{t-2}) + \phi_5(Y_{t-5} - Y_{t-6}) + \theta_1\epsilon_{t-1} + \theta_5\epsilon_{t-5}$

For performance data, a constraint was added that the change ($Y_t - Y_{t-1}$) could only be positive.

These were the models used in Figures A2.1 and A2.2.

A2.1.2 Partial Pooling

In partial pooling, the overall model structure is the same, but the parameter values for each individual trend Y_i are collectively drawn from a multinormal distribution:

$Y_{i,t} - Y_{i,t-1} \sim \mu_i + \epsilon_t$

$\epsilon_t \sim normal(0, \sigma_i)$

$[\mu_i, \sigma_i] \sim [\hat{\mu}, \hat{\sigma}] + multinormal(0, diag(\tau) * \Omega * diag(\tau))$

Priors:

$\hat{\mu} \sim normal(0, 4)$

$\hat{\sigma} \sim cauchy(0, 4)$

$\tau \sim cauchy(0, 1)$ (How much each parameter varies across the time series)

$\Omega \sim LKJ(1)$ (How the parameters correlate with each other across the time series)

This could be extended to the various kinds of ARIMA models by adding Autoregressive or Moving Average elements, and this was done for the models shown in Figures A2.3 and A2.4.

A2.1.3 Vectorizing

For adding in patent data, a multidimensional or vectorized version of the models were used. Each trend is actually a vector with D dimensions, and those different dimensions could affect each other with some lag:

$\vec{Y_t} = [Y_{1,t}, Y_{2,t}, Y_{3,t}...Y_{D,t}]$

$\vec{Y_t} - \vec{Y}_{t-1} \sim \vec{\mu} + \vec{\epsilon_t} + \mathbf{P}(\vec{Y}_{t-1} - \vec{Y}_{t-2})$

$\vec{\epsilon_t} \sim normal(0, \vec{\sigma})$

where $\mathbf{P}$ is a DxD matrix

Priors (for each element in the vector or matrix):

$\vec{\mu} \sim normal(0, 4)$

$\vec{\sigma} \sim cauchy(0, 4)$

$\mathbf{P} \sim normal(0, 4)$

This could be extended by adding Moving Average elements, or even partially pooled.

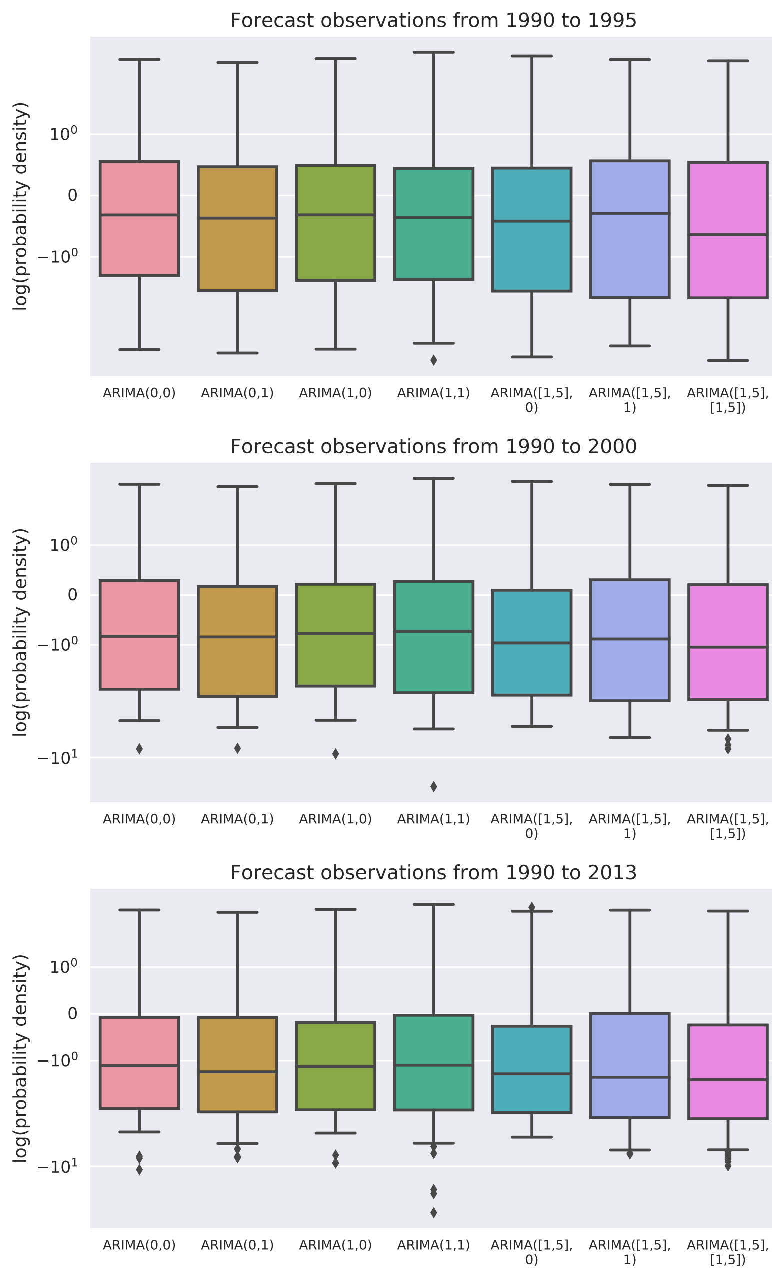


Figure A2.1: **Distribution of forecasts' accuracies for different models of technology price.**

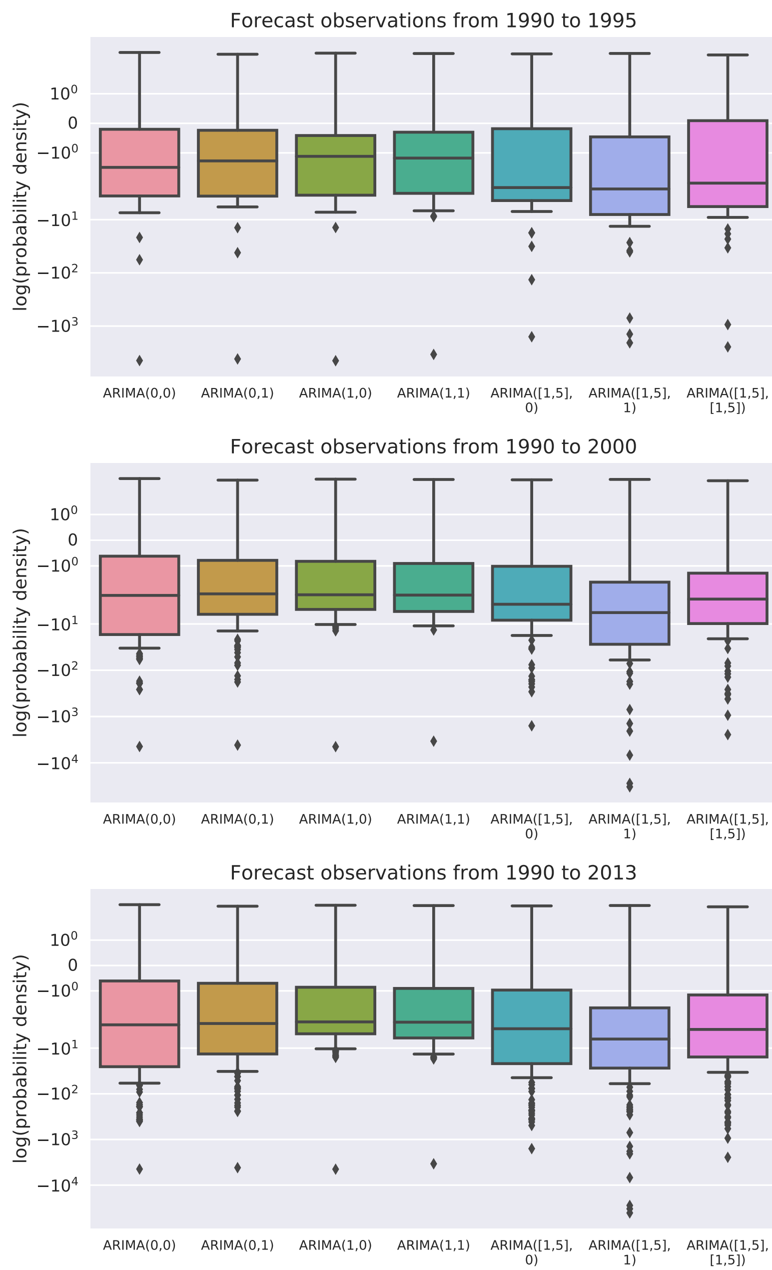


Figure A2.2: **Distribution of forecasts' accuracies for different models of technology performance.**

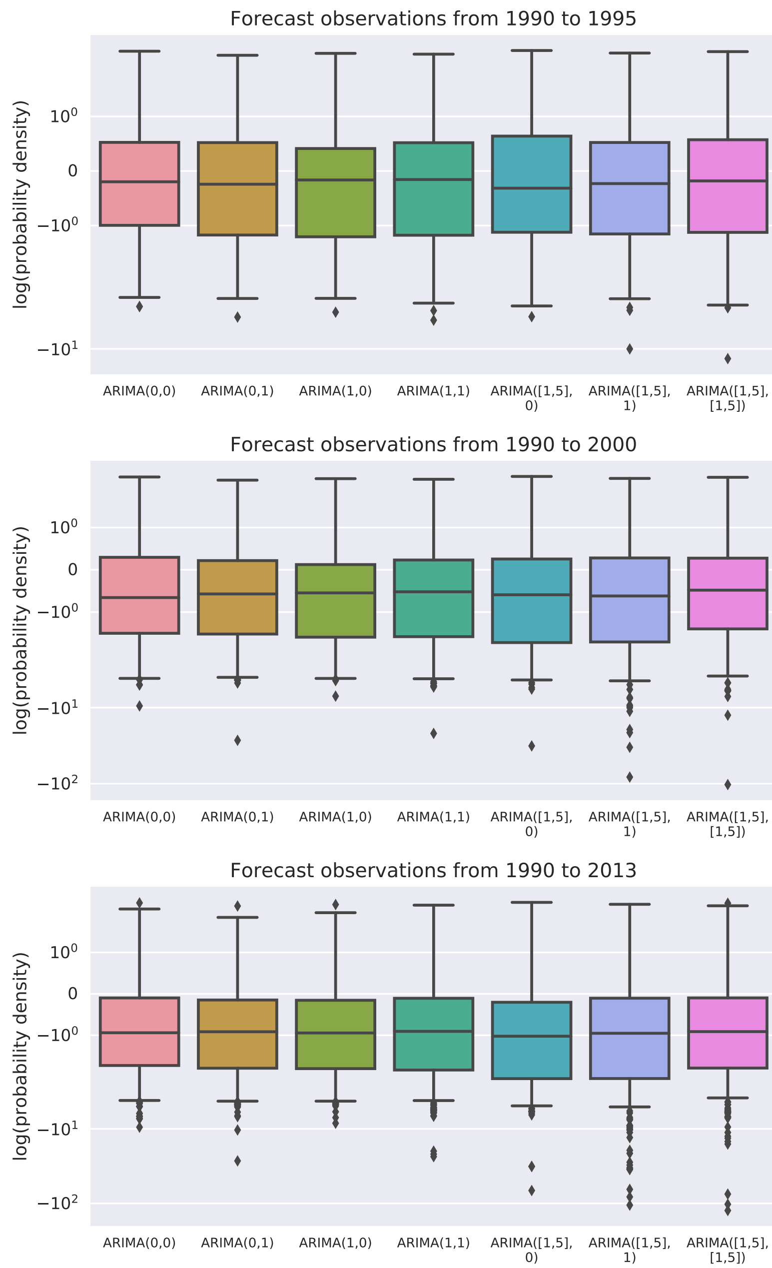


Figure A2.3: **Distribution of forecasts' accuracies for different models of technology price, using partial pooling.**

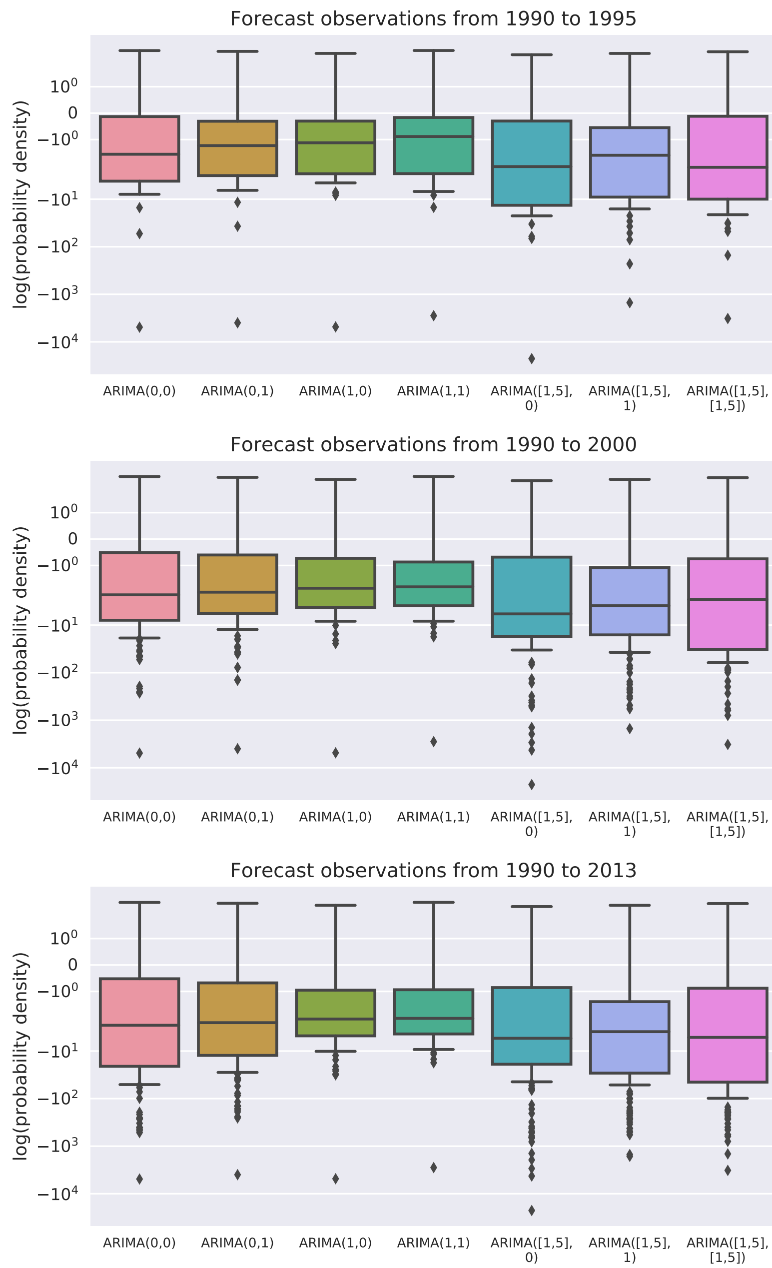


Figure A2.4: **Distribution of forecasts' accuracies for different models of technology performance, using partial pooling.**

In this study, a different model was created for each of the candidate predictors created from the patent data (e.g. number of citations). One dimension of $\vec{Y}$ was the patent predictor, and the other was either price or performance. It would also be possible to put all the predictors together with both price and performance into a single, large $\vec{Y}$, but this would take much longer to run, and the negative results from the simpler models didn't indicate there was any signal from this patent data.

This style of model is commonly called a VAR (Vector AutoRegression) model, though typically the errors of the different dimensions are correlated by being drawn collectively from a multinormal:

$\vec{\epsilon}_t \sim multinormal(0, diag(\tau) * \Omega * diag(\tau))$

This would be particularly appropriate if in the system being modeled noise/shocks from one dimension could affect another dimension at a timescale faster than that being sampled (i.e. faster than the time difference between t and t_1). That did not seem likely in for the technology trend data in the current study, and so we left the errors uncorrelated.

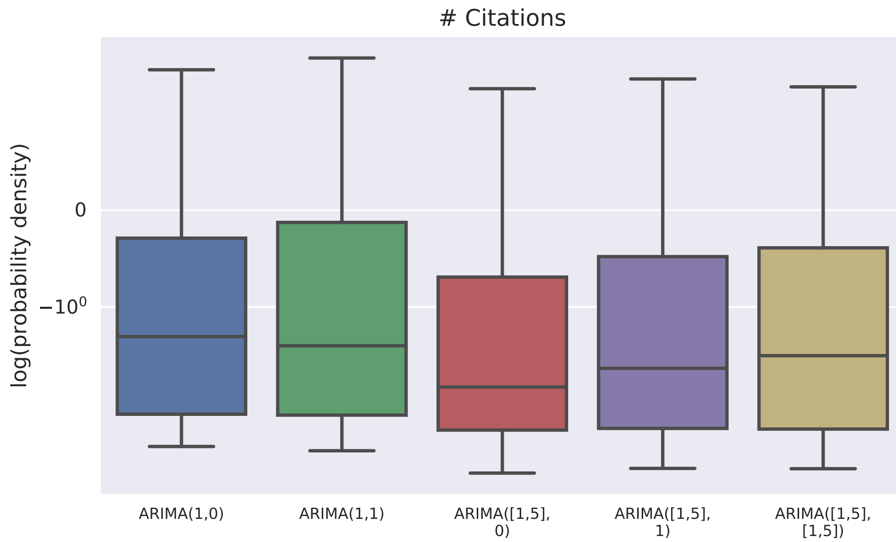


Figure A2.5: **Distribution of forecasts' accuracies for different models of technology price, using associated patents' number of citations.**

A2.2 Software

In order to perform the analyses in this study, we wrote a statistical software package in Python named `pystan_time_series`. The code for this package is freely available online at `https://github.com/jeffalstott/pystan_time_series`. The package uses Bayesian inference to fit time series models to data, allowing for a wide variety of options to add on to a basic ARIMA model. The package uses `stan`[21] for the inference, and uses the `pystan`[22] interface with `stan` from Python.

Additionally, all code to perform this study is available at `github.com/jeffalstott/technologytimeseries_forecasting`. As the patent data is too large to host on Github, that data is available upon request.

[21] `http://mc-stan.org/`
[22] `https://pystan.readthedocs.io/`

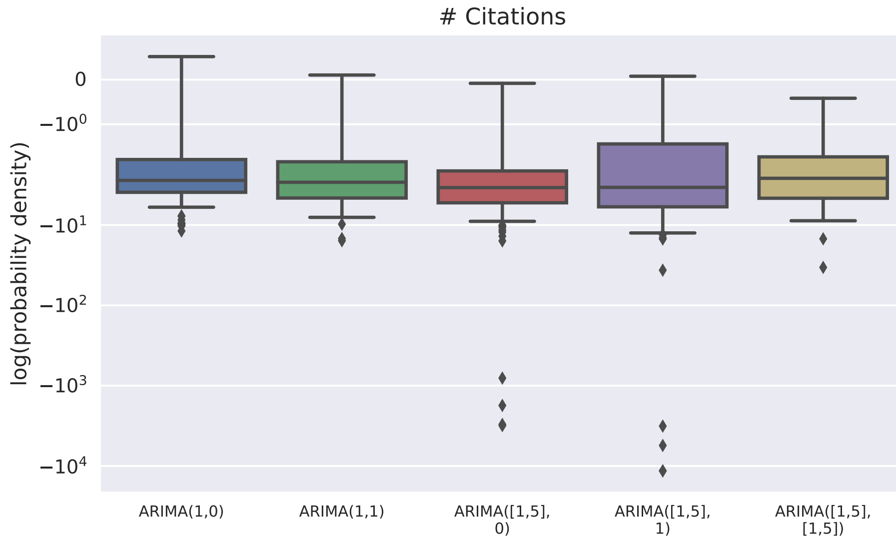


Figure A2.6: **Distribution of forecasts' accuracies for different models of technology performance, using associated patents' number of citations.**

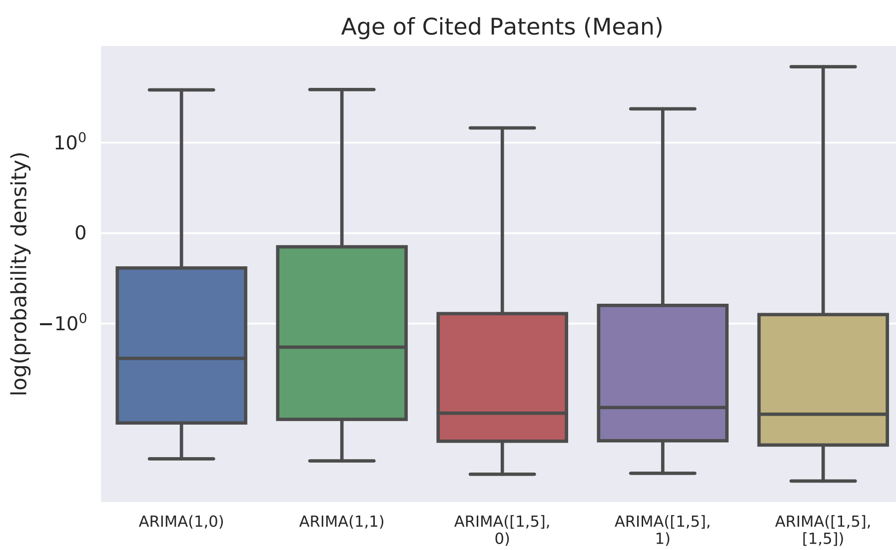


Figure A2.7: **Distribution of forecasts' accuracies for different models of technology price, using associated patents' mean age of cited patents.**

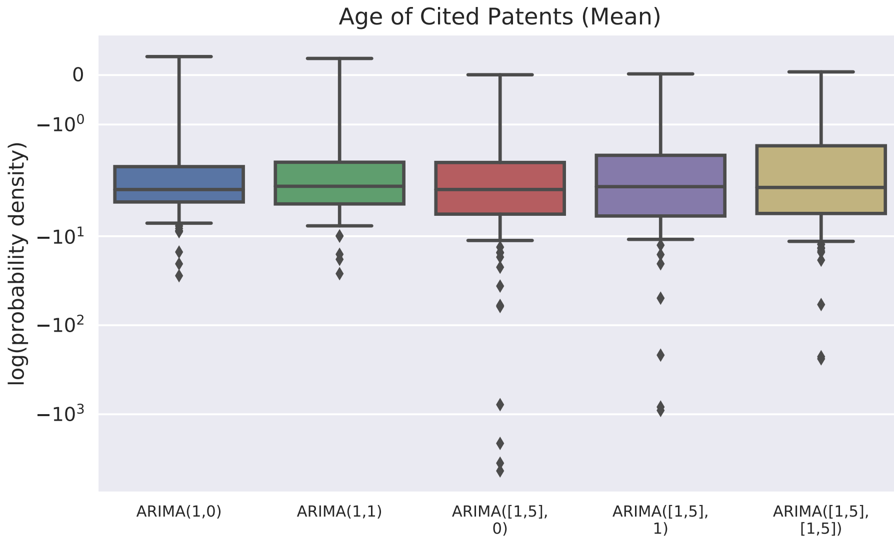


Figure A2.8: **Distribution of forecasts' accuracies for different models of technology performance, using associated patents' mean age of cited patents.**

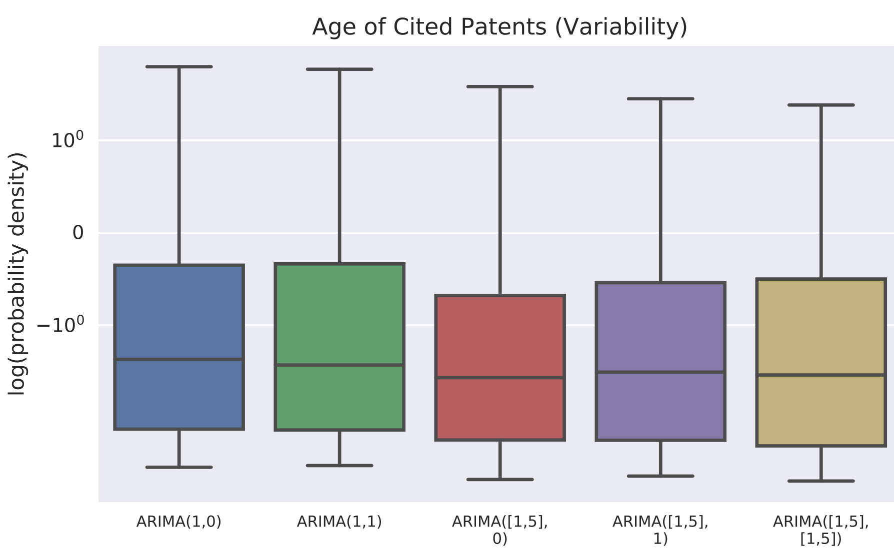


Figure A2.9: **Distribution of forecasts' accuracies for different models of technology price, using associated patents' variability of age of cited patents.**

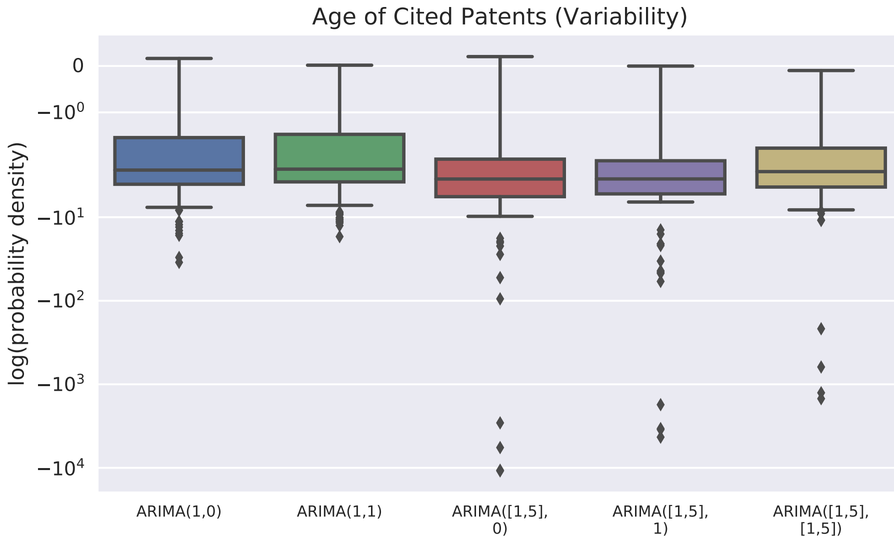


Figure A2.10: **Distribution of forecasts' accuracies for different models of technology performance, using associated patents' variability of age of cited patents.**

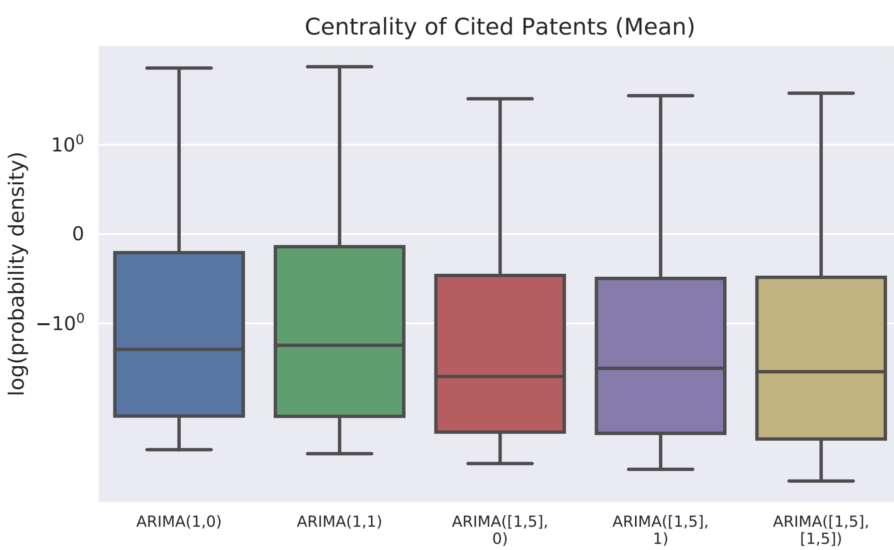


Figure A2.11: **Distribution of forecasts' accuracies for different models of technology price, using associated patents' mean centrality of cited patents.**

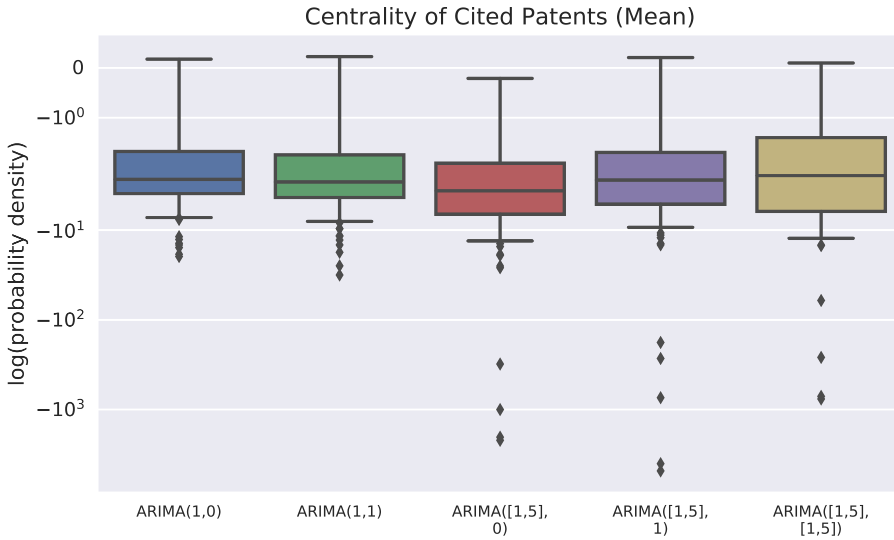


Figure A2.12: **Distribution of forecasts' accuracies for different models of technology performance, using associated patents' mean centrality of cited patents.**

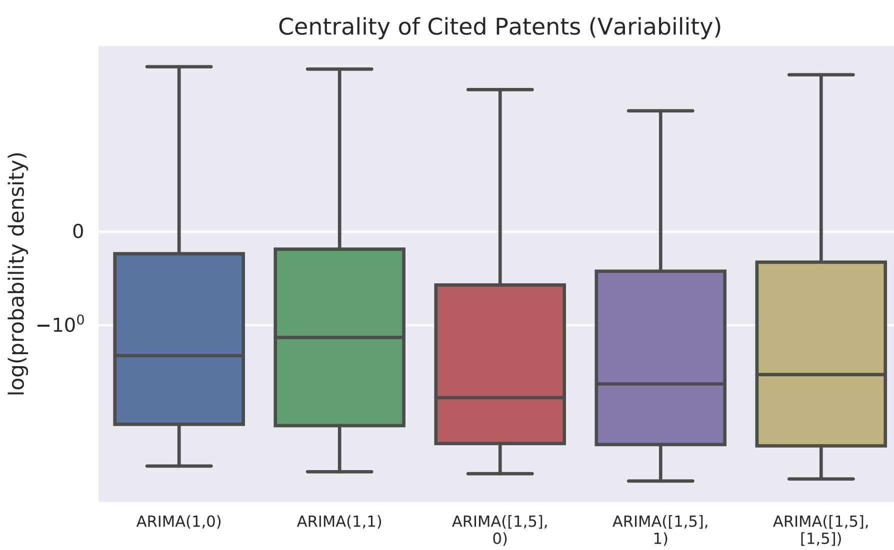


Figure A2.13: **Distribution of forecasts' accuracies for different models of technology price, using associated patents' variability of centrality of cited patents.**

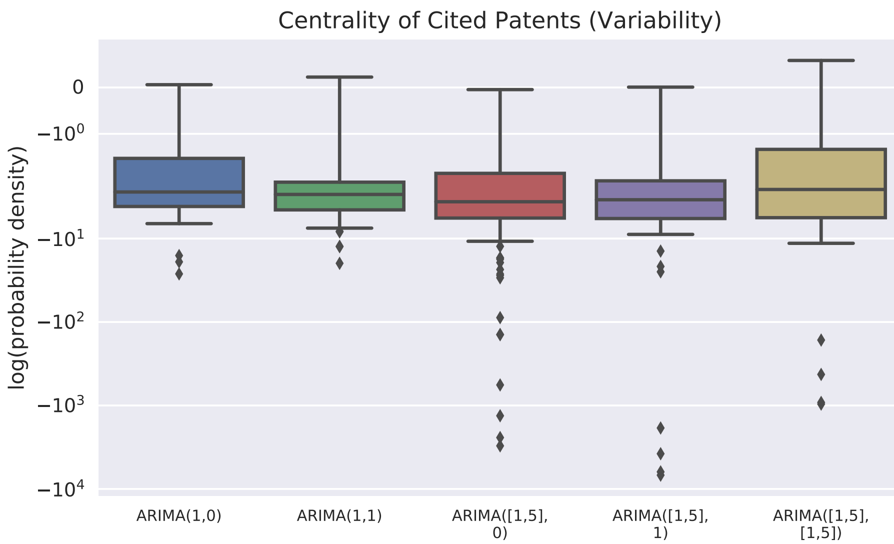


Figure A2.14: **Distribution of forecasts' accuracies for different models of technology performance, using associated patents' variability of centrality of cited patents.**

A3 Bonus Analysis: Forecasts of AI performance on Atari games

During the course of this study the Electronic Frontier Foundation began a project to compile and publish historical data on Artificial Intelligence performance on various tasks.[23] We used the trend extrapolation process described in this study to make forecasts on a portion of this data.

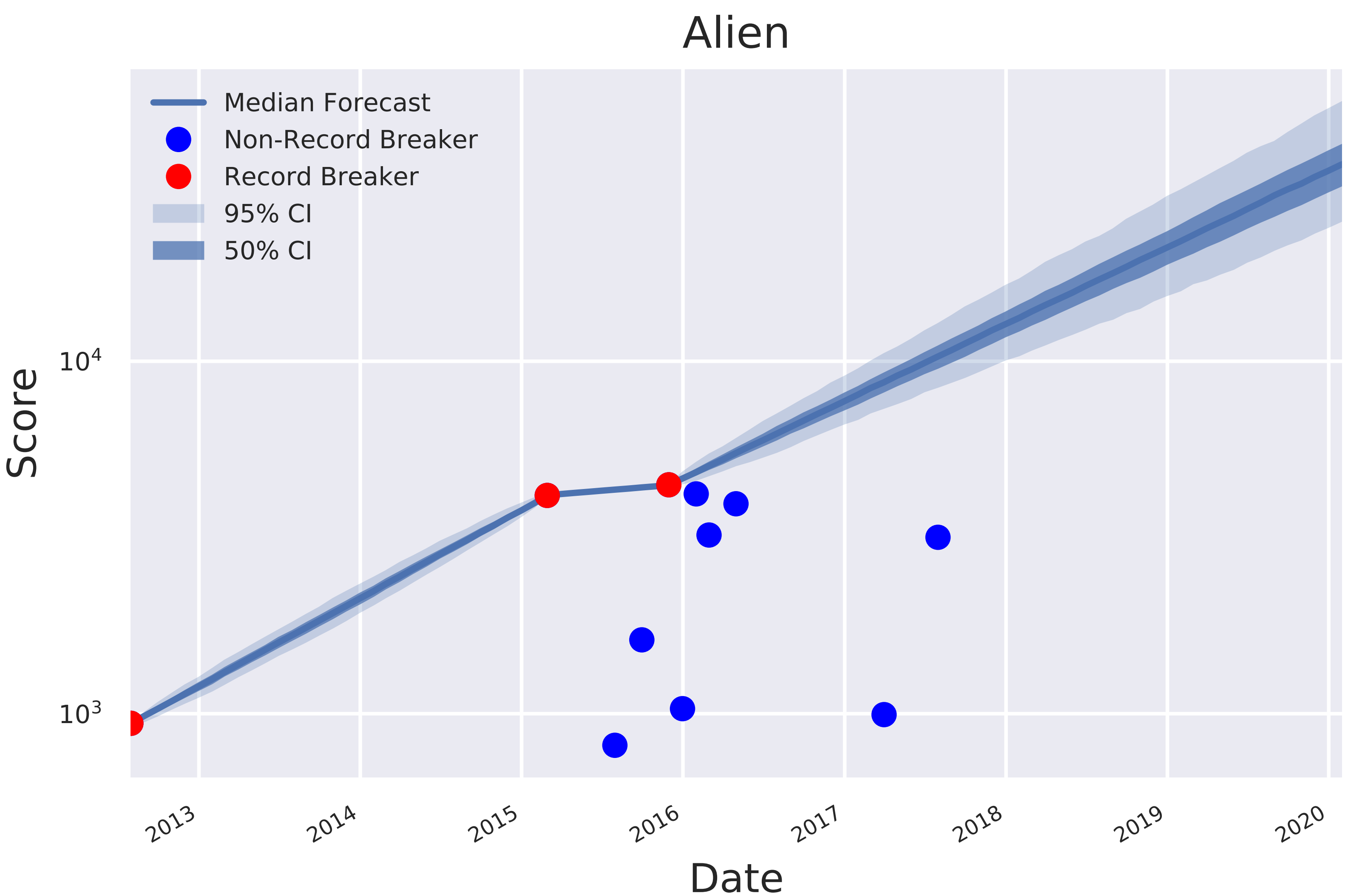
We forecasted AI performance on Atari games. Atari games have been a test bed for developing new machine learning techniques, and so there is a growing data set of AI performance over time for dozens of games. The data all came from research articles describing new AI techniques, which came with both scores on a suite of Atari games and a timestamp of when the paper was submitted.

We used the data from 49 games (253 total data points) to do trend extrapolation of the record-breaking AI scores expected until 2020. These forecasts were made with partial pooling, which allowed for making forecasts for individual games even if there is little data available for that game. For example, the game "Montezuma's Revenge" has data for only one top score (the first-recorded score has not been beaten), and so taken by itself this lack of trend data would prohibit making a forecast. However, because we have data from all the other games, this allowed us to infer what an average improvement rate is across all games, and then use that for making forecasting for "Montezuma's Revenge."

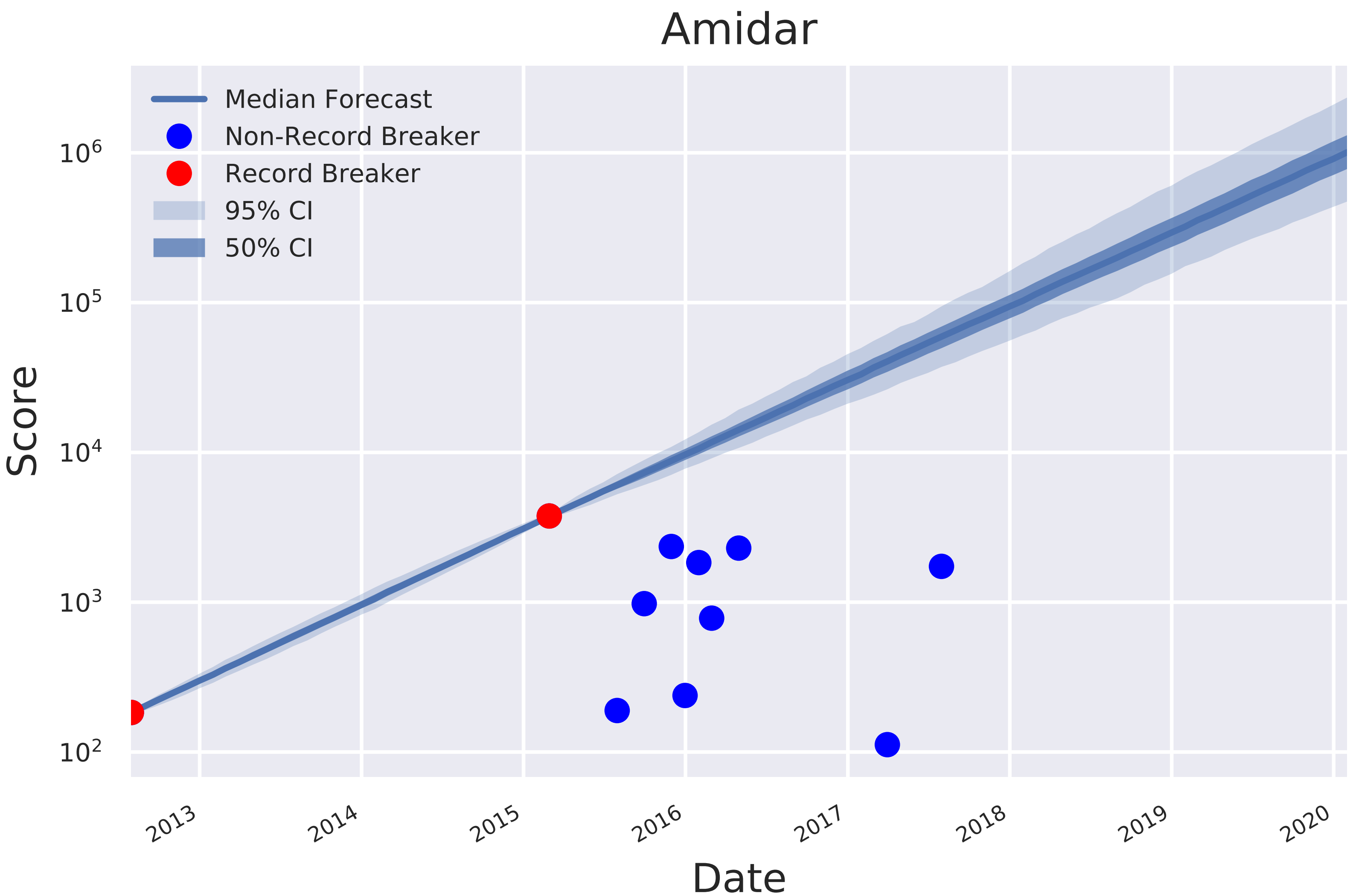
Modeling this Atari data likely has similar problems to modeling the performance data earlier in this study: the model assumes the performance improves relatively smoothly, while the reality has jumps and steps. Again, the model doesn't try to capture the probability of there being no improvement on a given year. At best, it models what the record breaker's value will be, conditional on it showing up.

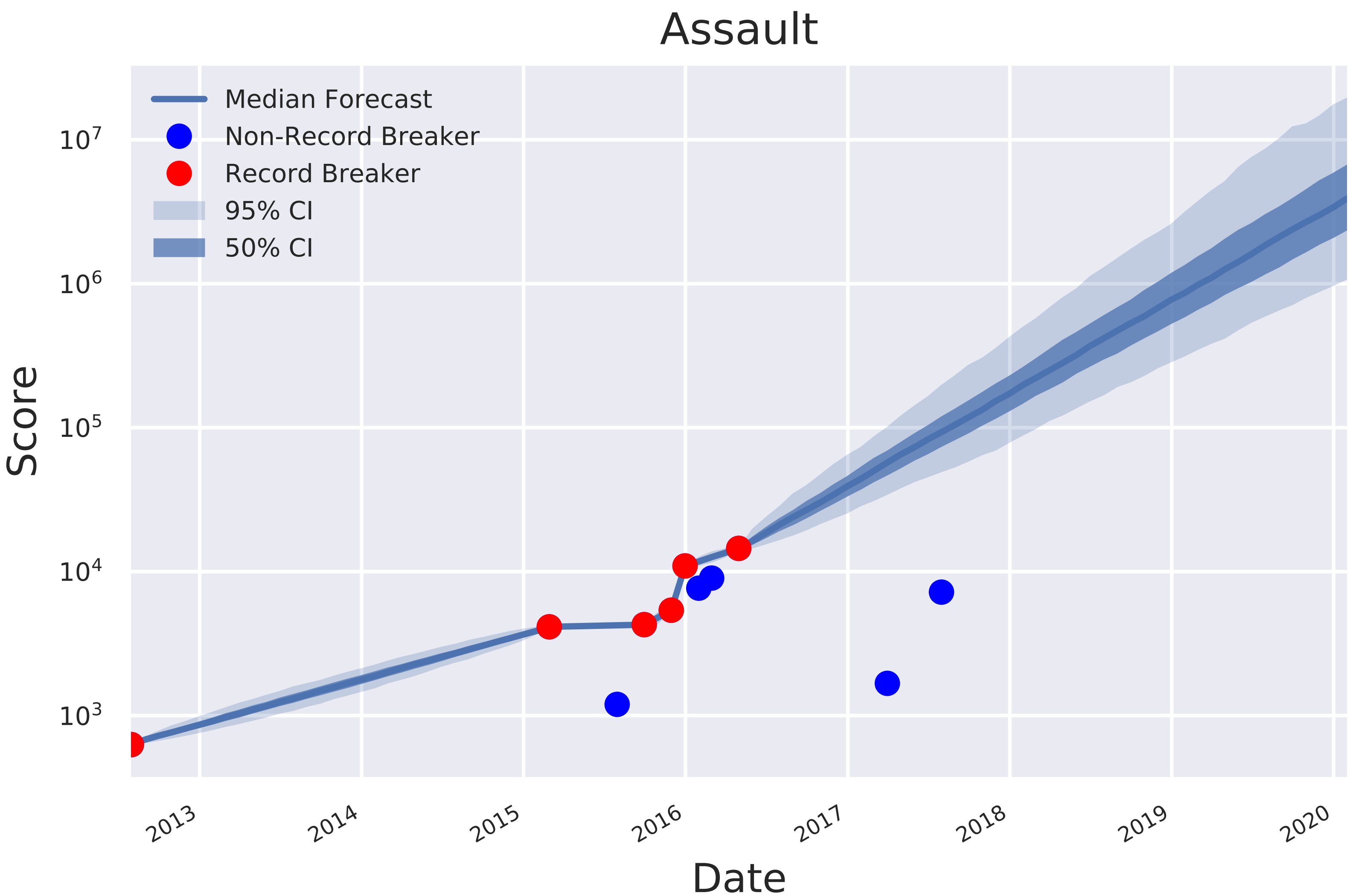
But this Atari data is even more complex than the earlier performance data, in that it is clear that the AI researchers aren't even trying to set records, per se. Instead, the researchers are developing many new AI techniques and are showing off what they can do on a pre-existing suite of games. Sometimes a technique will set a new record on a game, but that wasn't the goal. The discrepancy in goals is made particularly clear by the fact that often the new AI techniques being advertised could be combined, but they are not. Instead, the articles seek to report the performance of the new technique in isolation. An ecosystem of researchers that were truly trying to push the scores on the Atari games as high as they could go would likely yield different results. Accurately forecasting record breakers does not necessarily require considering all this internal complexity, but there is ample opportunity to experiment with slightly more complex models[24] in the pursuit of better prediction of technology performance.

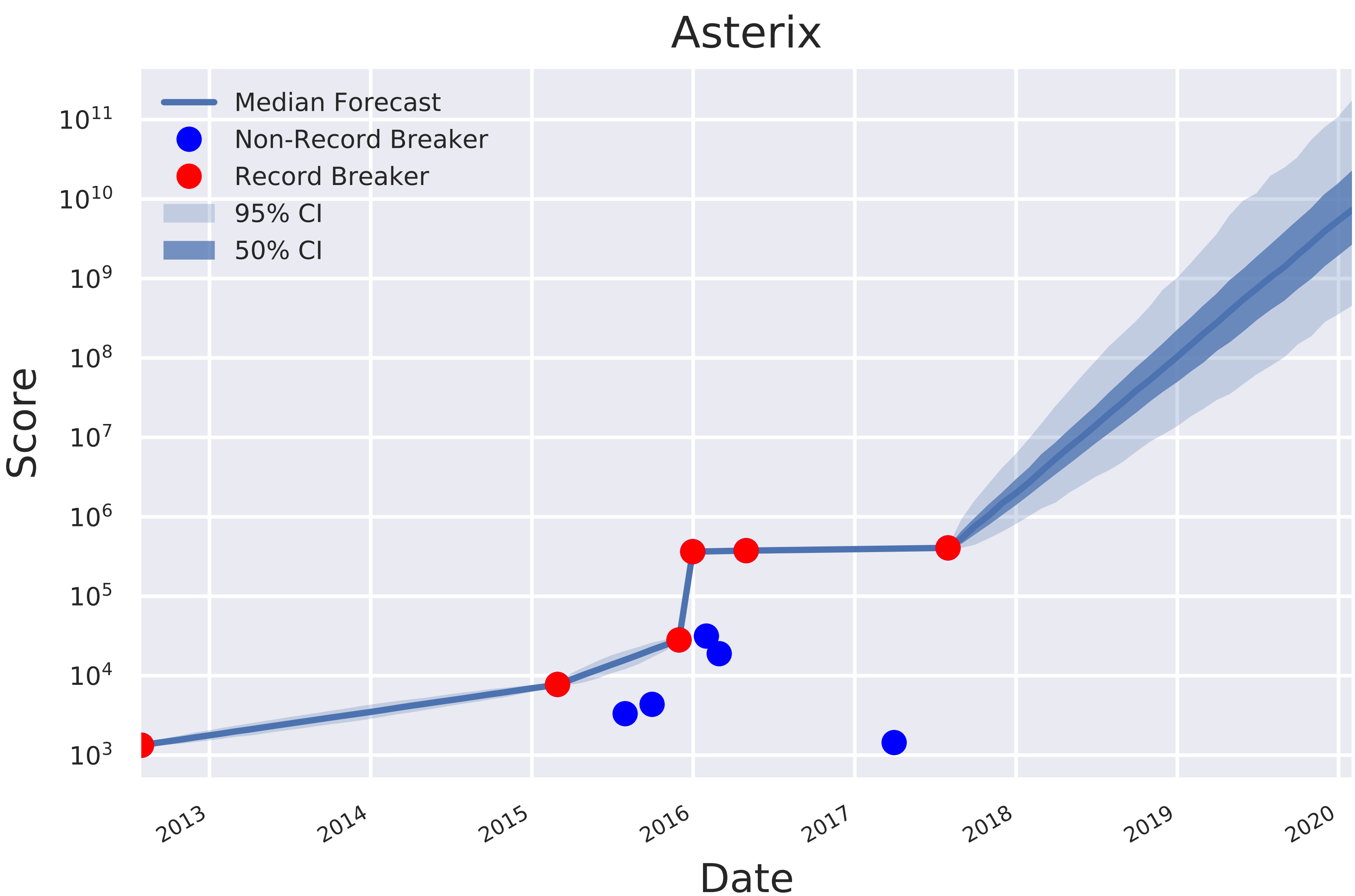
[23] `https://www.eff.org/files/AI-progress-metrics.html`
[24] Berdahl et al., see n. 16



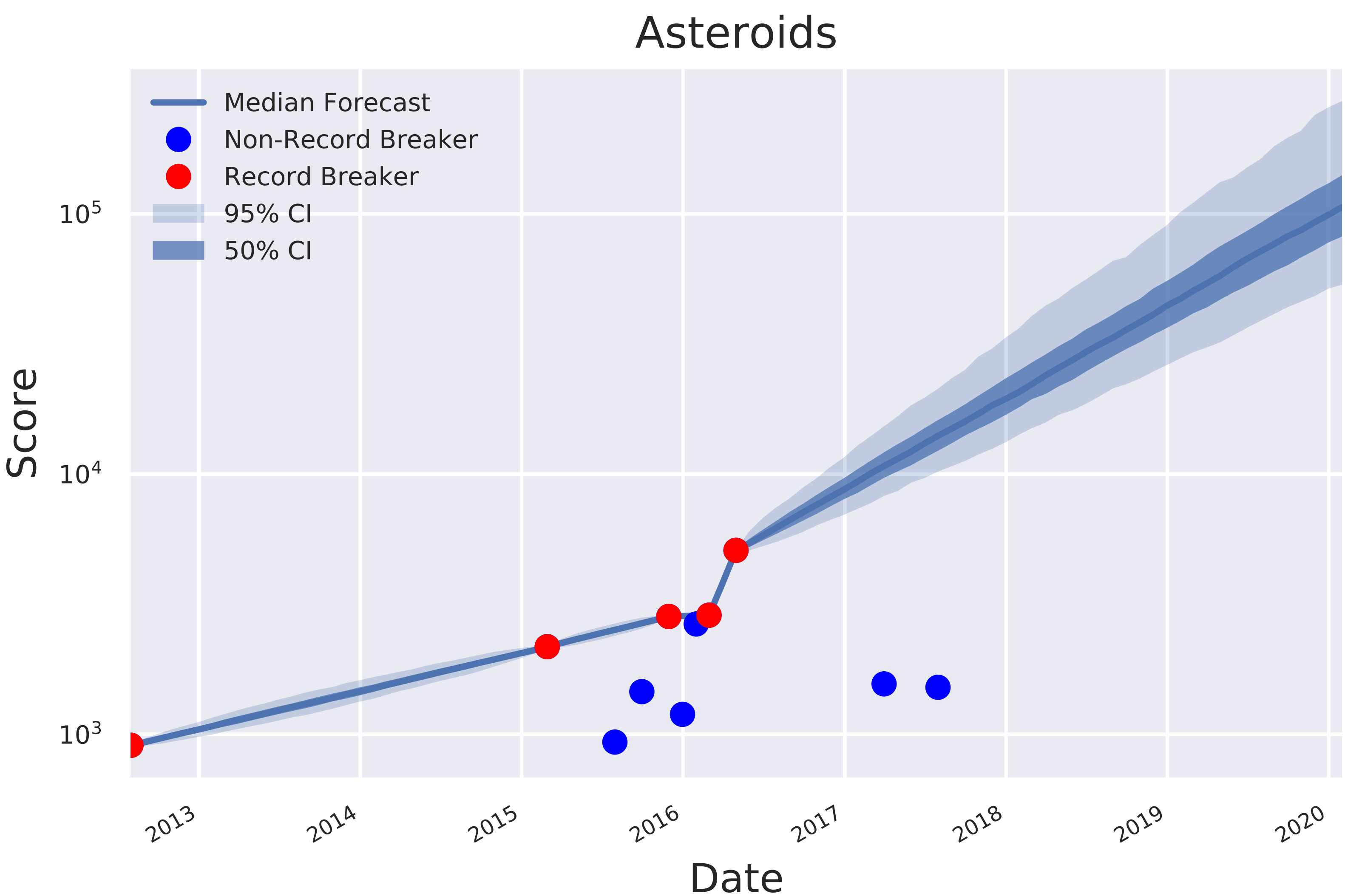
Alien
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
Score
10^4
10^3
2013
2014
2015
2016
2017
2018
2019
2020
Date


Amidar
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
10^6
10^5
10^4
10^3
10^2
Score
2013
2014
2015
2016
2017
2018
2019
2020
Date

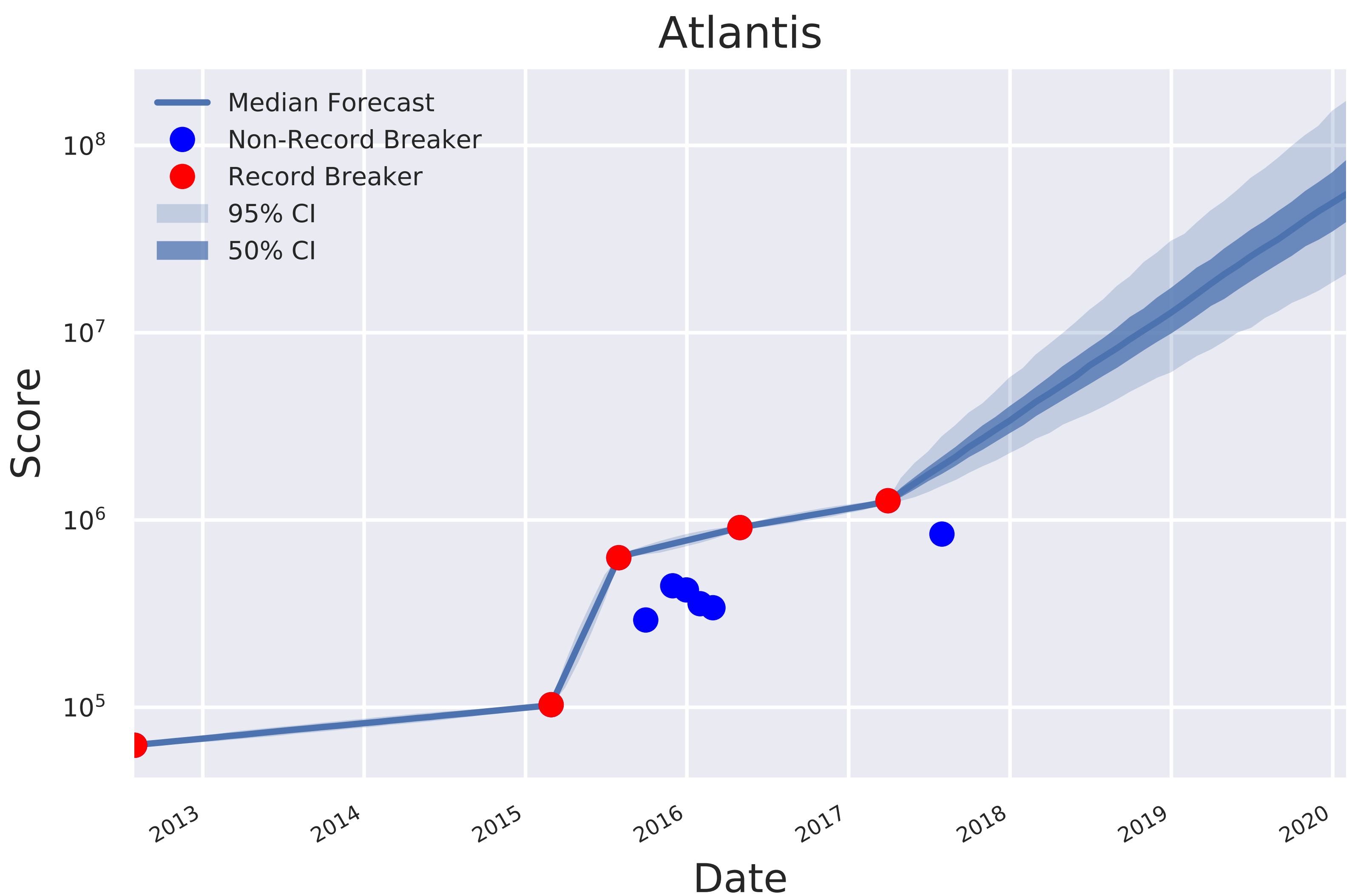

Assault
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
10^7
10^6
10^5
10^4
10^3
Score
2013
2014
2015
2016
2017
2018
2019
2020
Date



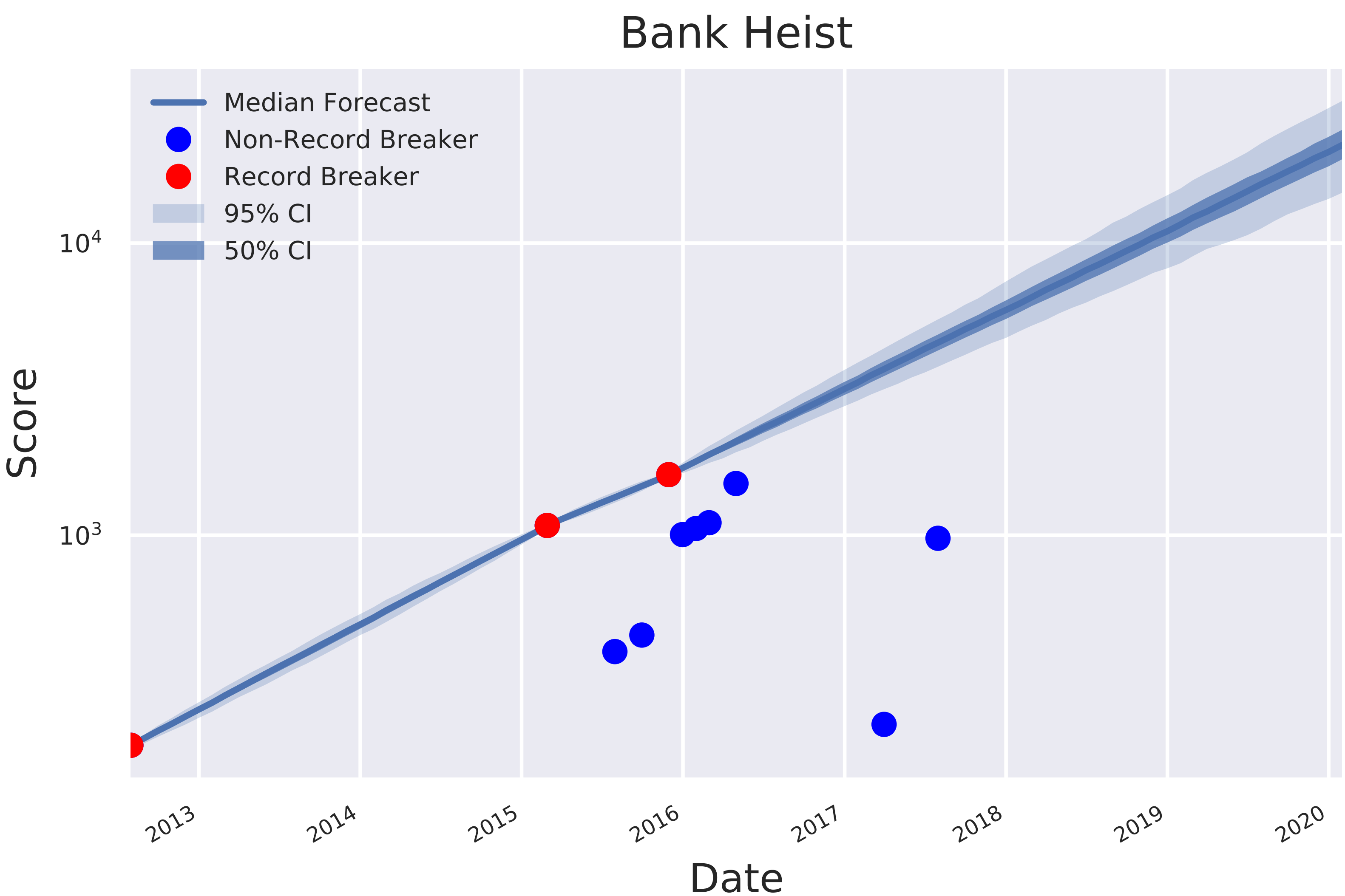
Asterix
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
Score
10^{11}
10^{10}
10^{9}
10^{8}
10^{7}
10^{6}
10^{5}
10^{4}
10^{3}
2013
2014
2015
2016
2017
2018
2019
2020
Date

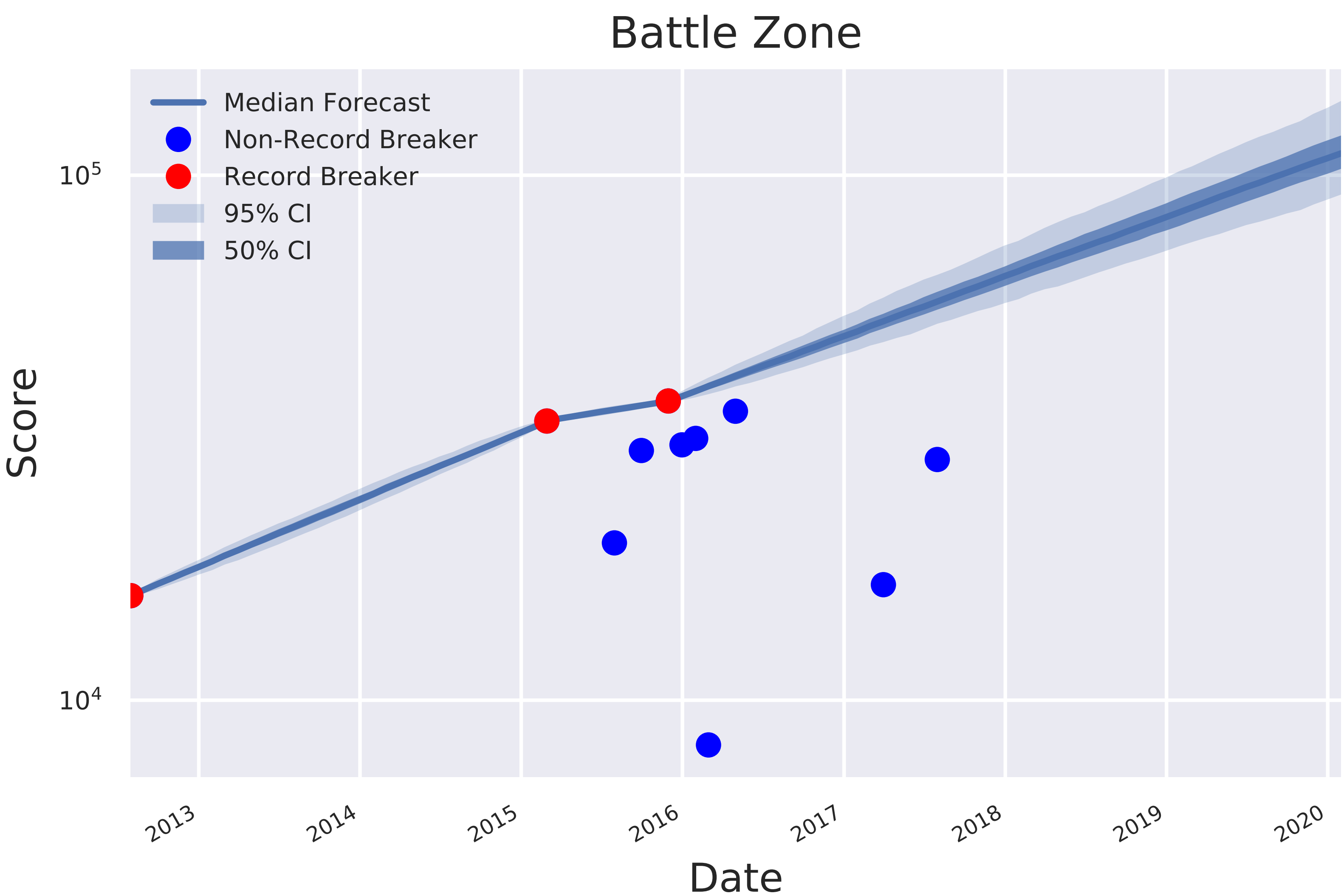


Asteroids
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
10^5
10^4
10^3
Score
2013
2014
2015
2016
2017
2018
2019
2020
Date

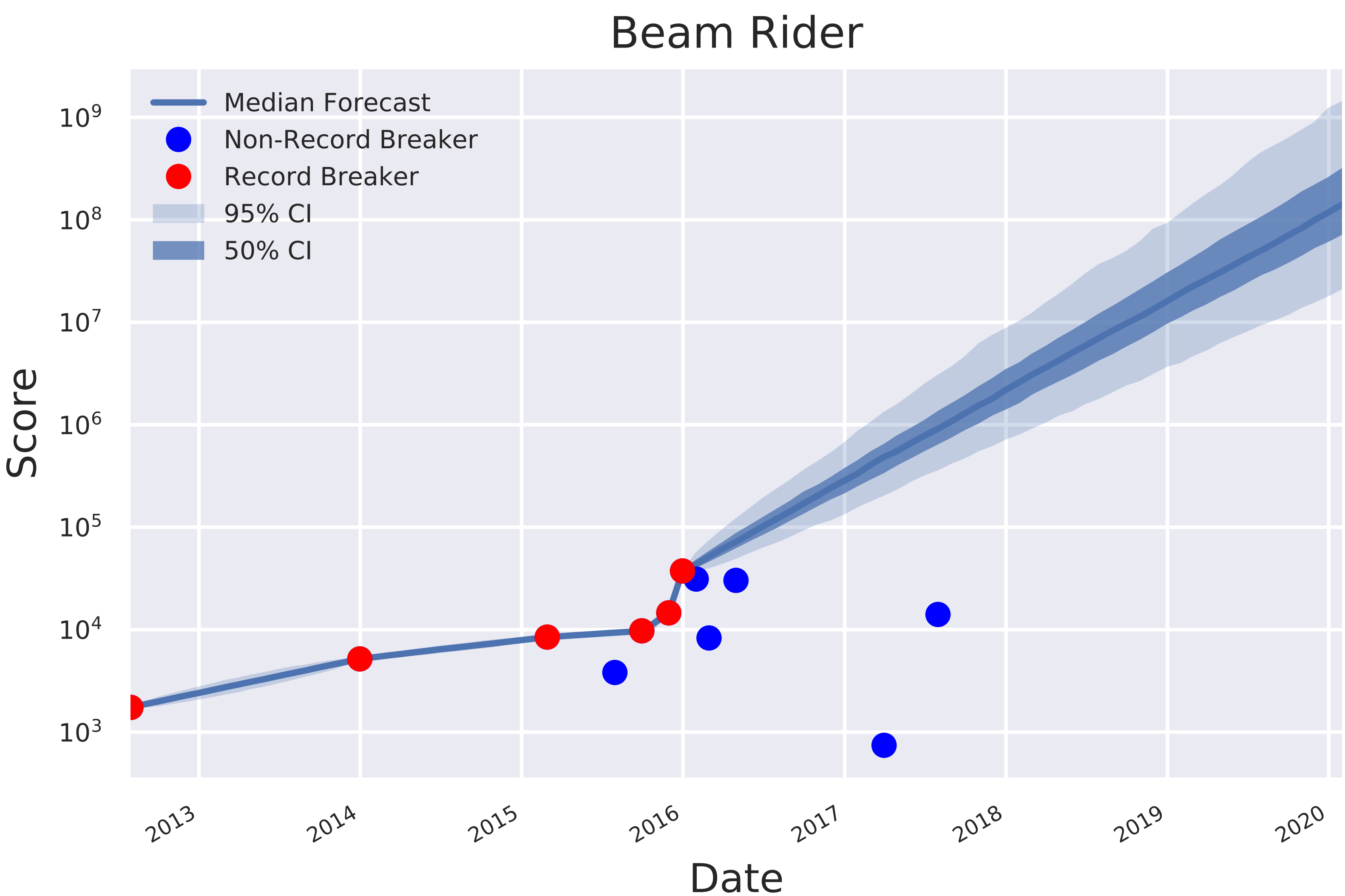


Atlantis
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
10^8
10^7
10^6
10^5
Score
2013
2014
2015
2016
2017
2018
2019
2020
Date

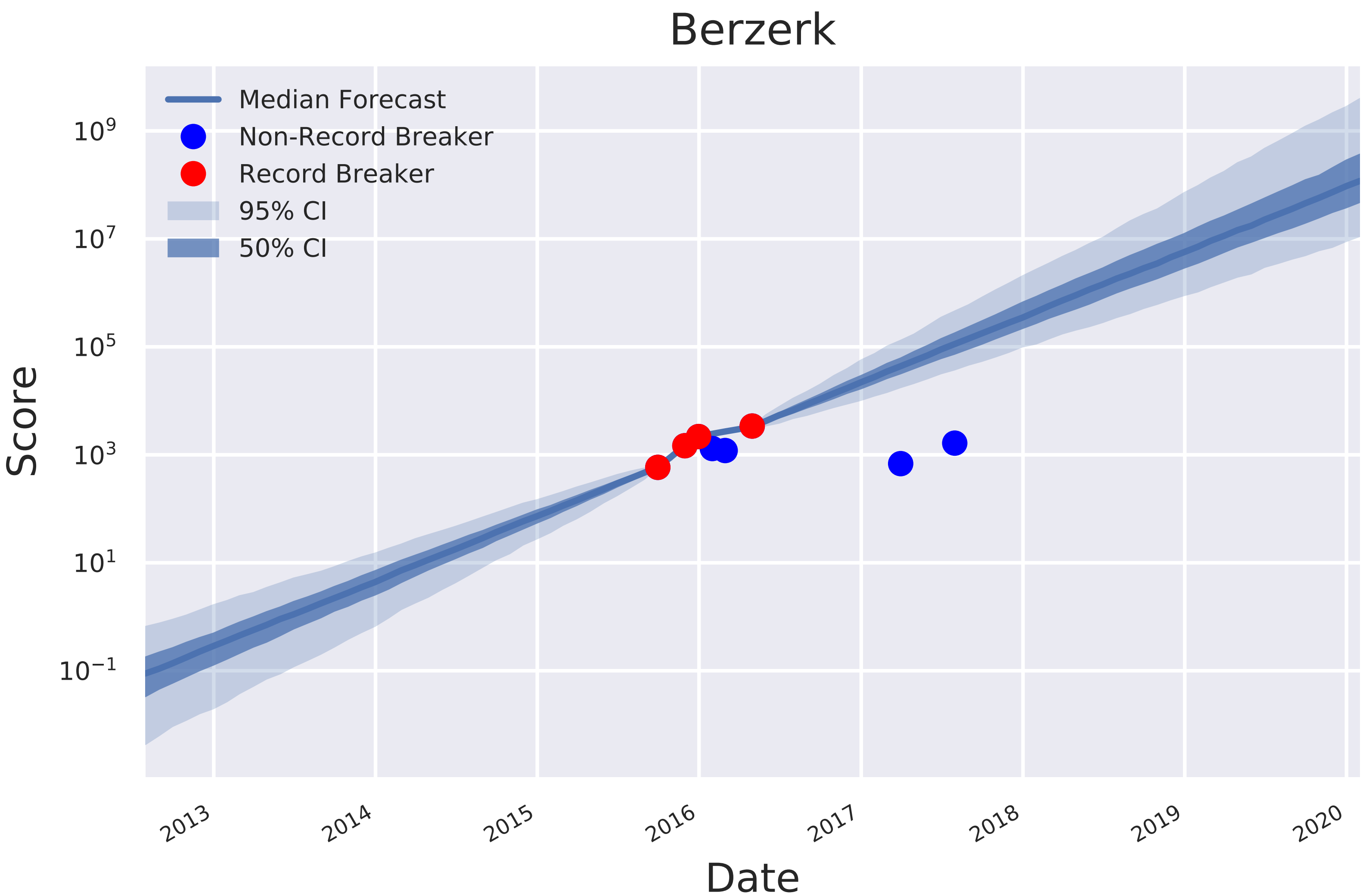

Bank Heist
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
10^4
10^3
Score
2013
2014
2015
2016
2017
2018
2019
2020
Date



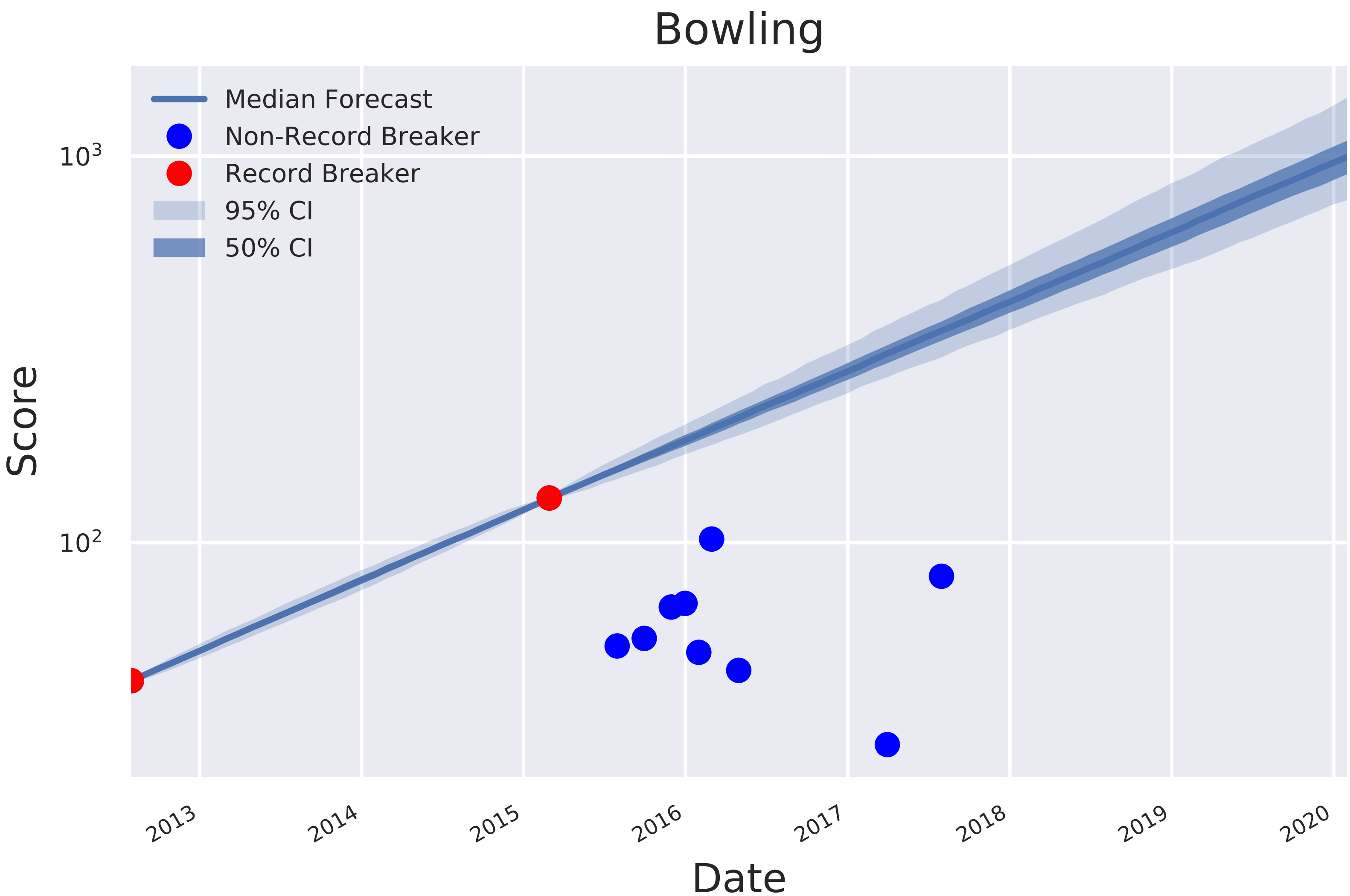
Battle Zone
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
10^5
10^4
Score
2013
2014
2015
2016
2017
2018
2019
2020
Date



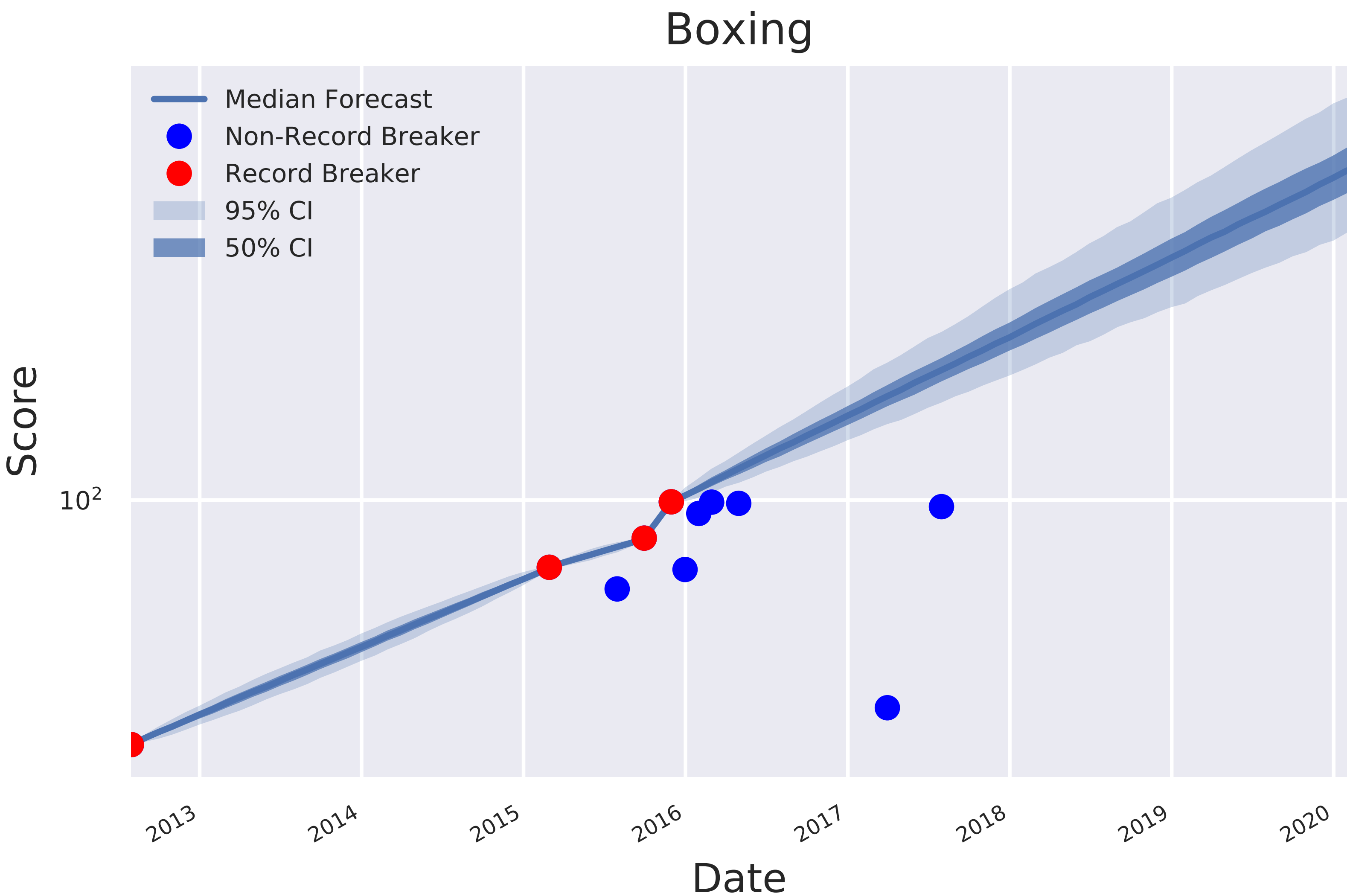
Beam Rider
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
Score
10^9
10^8
10^7
10^6
10^5
10^4
10^3
2013
2014
2015
2016
2017
2018
2019
2020
Date



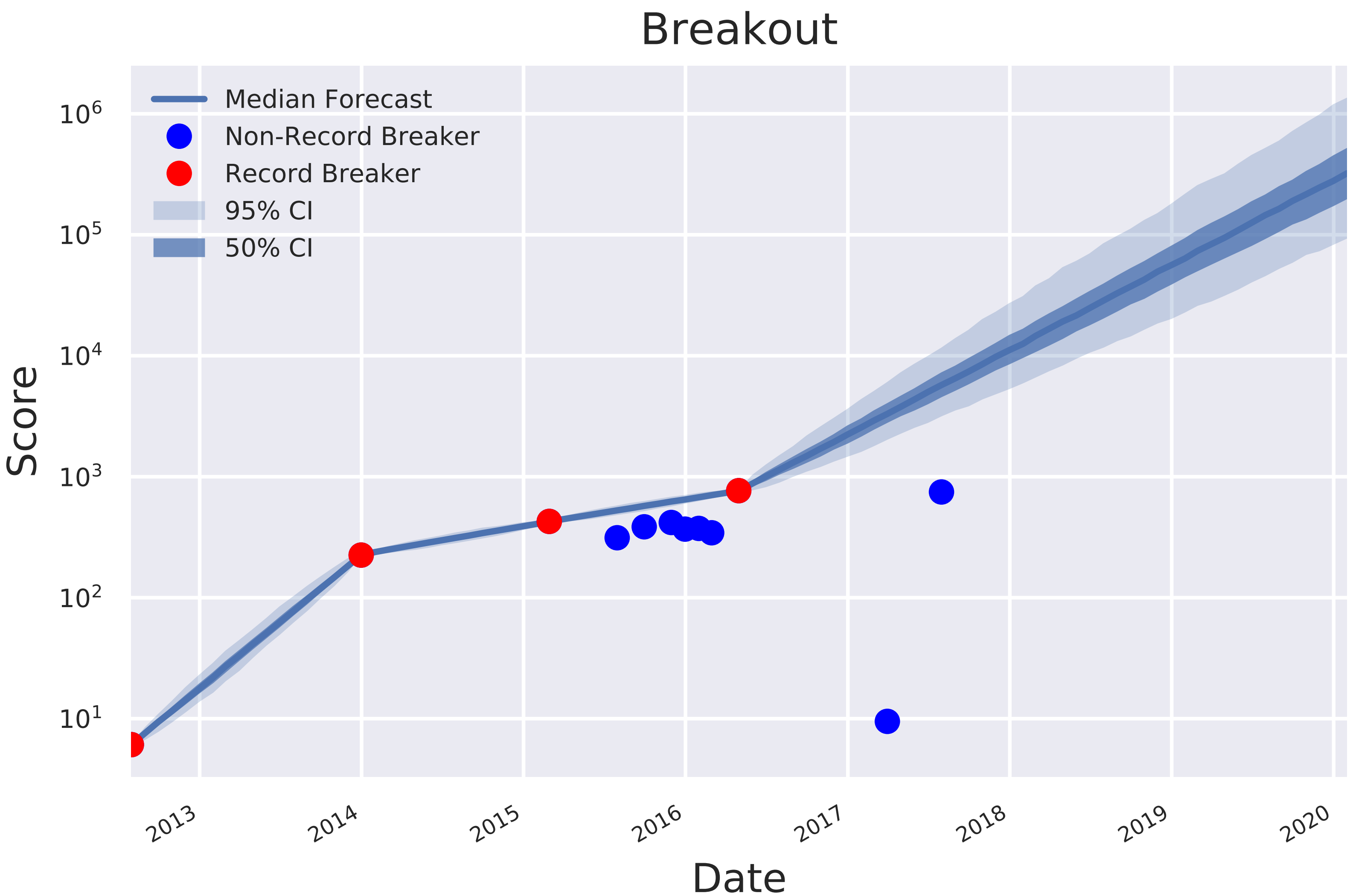
Berzerk
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
Score
Date
10^9
10^7
10^5
10^3
10^1
10^{-1}
2013
2014
2015
2016
2017
2018
2019
2020



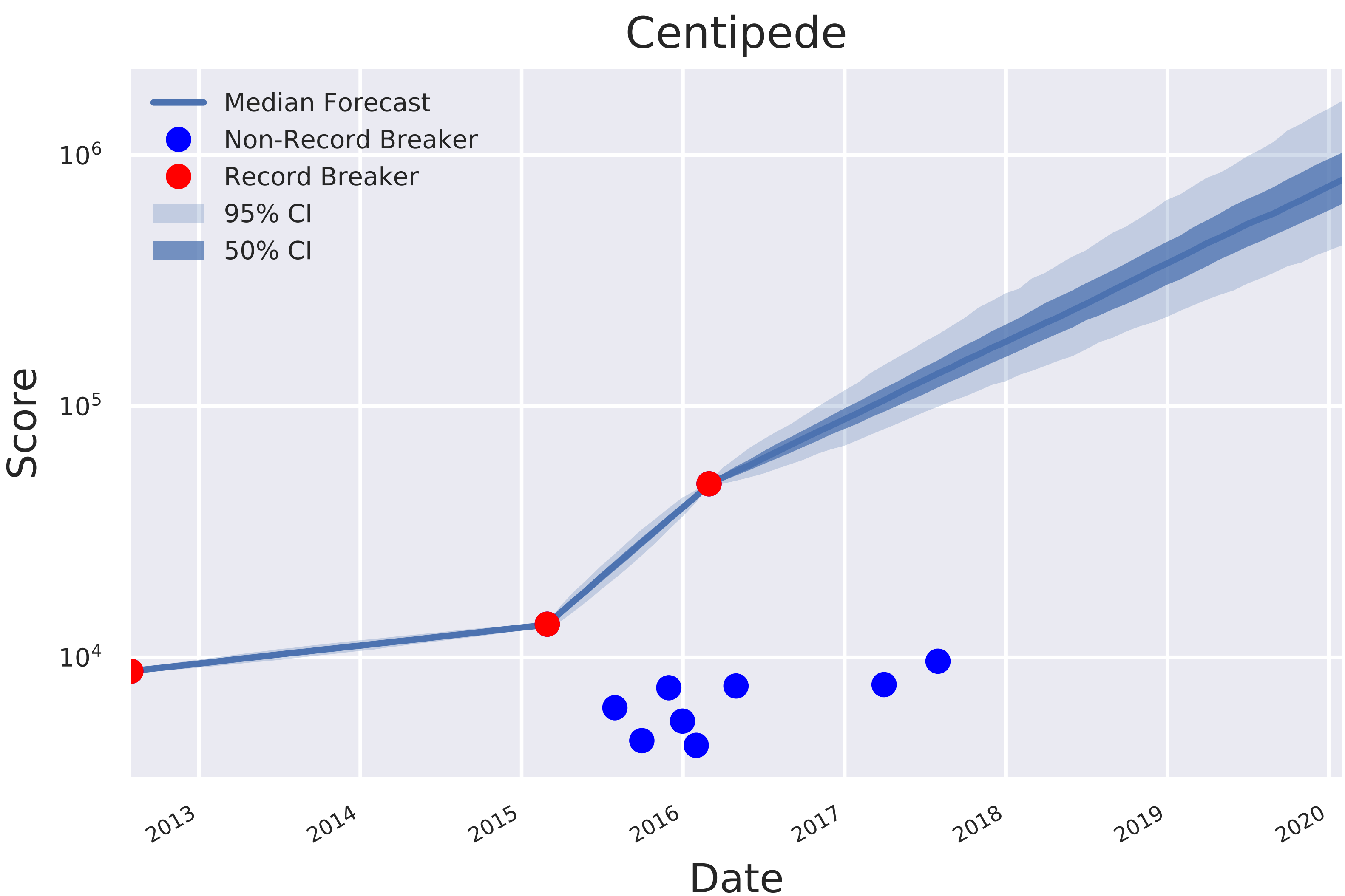
Bowling
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
10^3
10^2
Score
2013
2014
2015
2016
2017
2018
2019
2020
Date



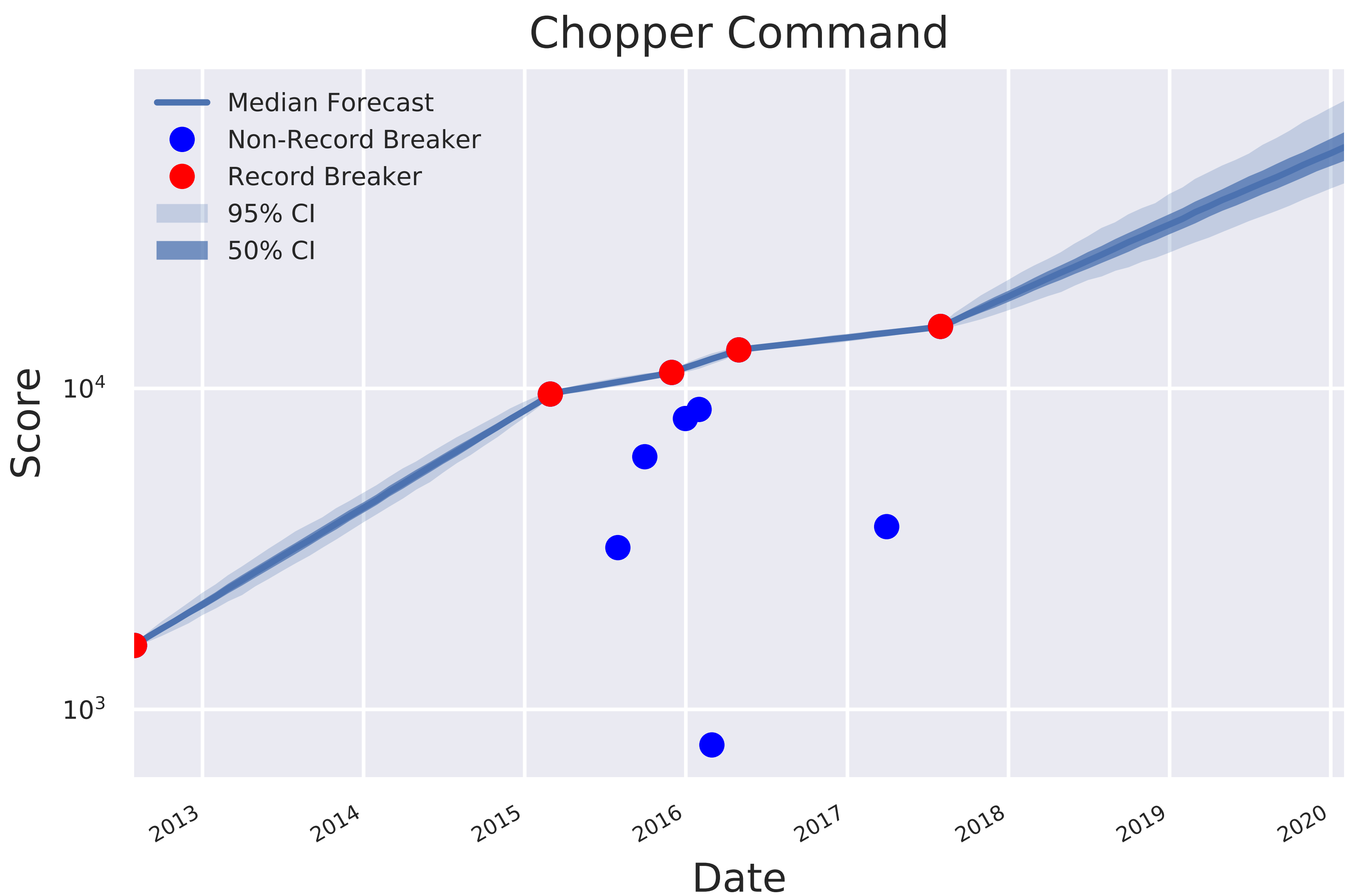
Boxing
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
Score
10^2
2013
2014
2015
2016
2017
2018
2019
2020
Date



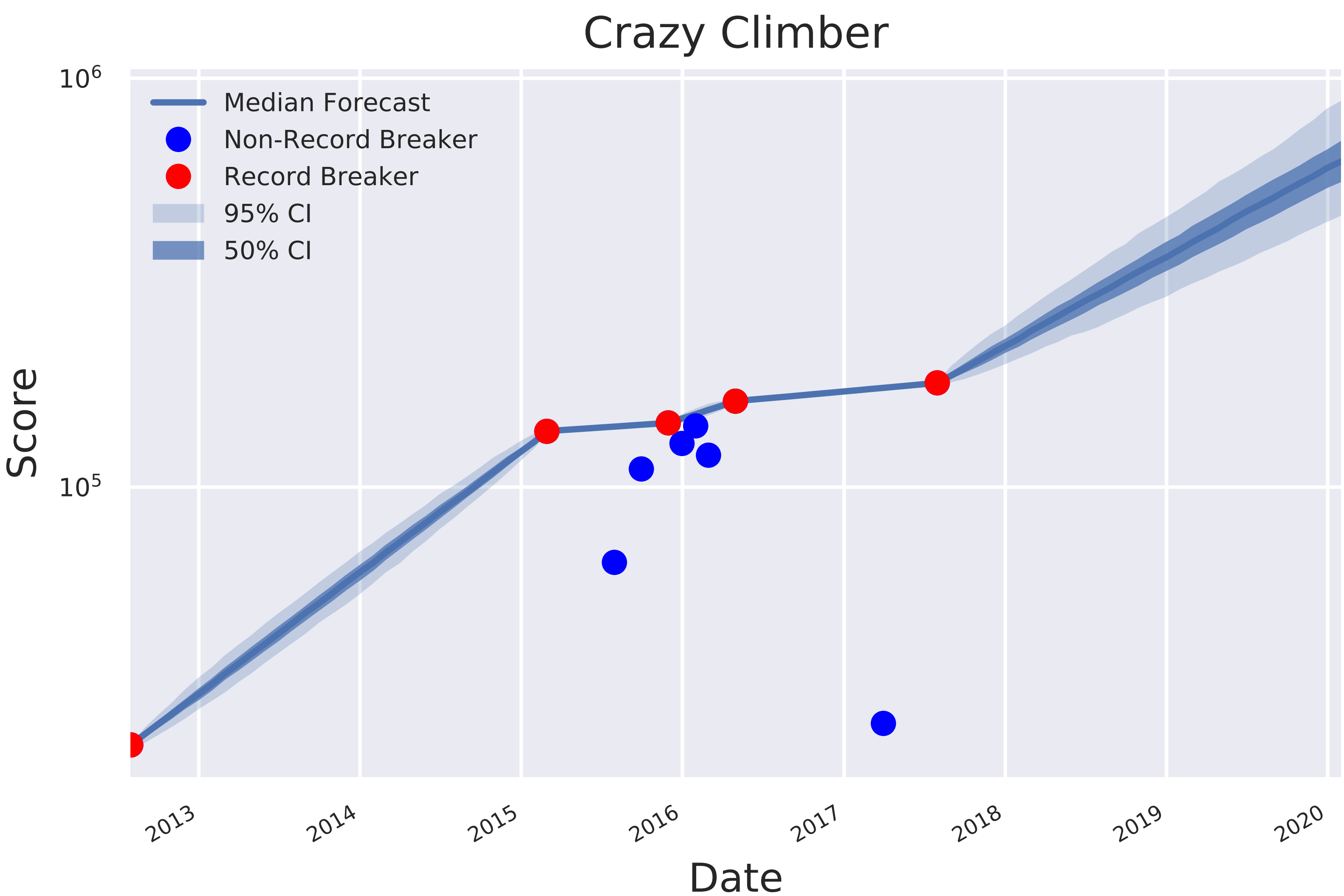
Breakout
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
Score
10^6
10^5
10^4
10^3
10^2
10^1
2013
2014
2015
2016
2017
2018
2019
2020
Date

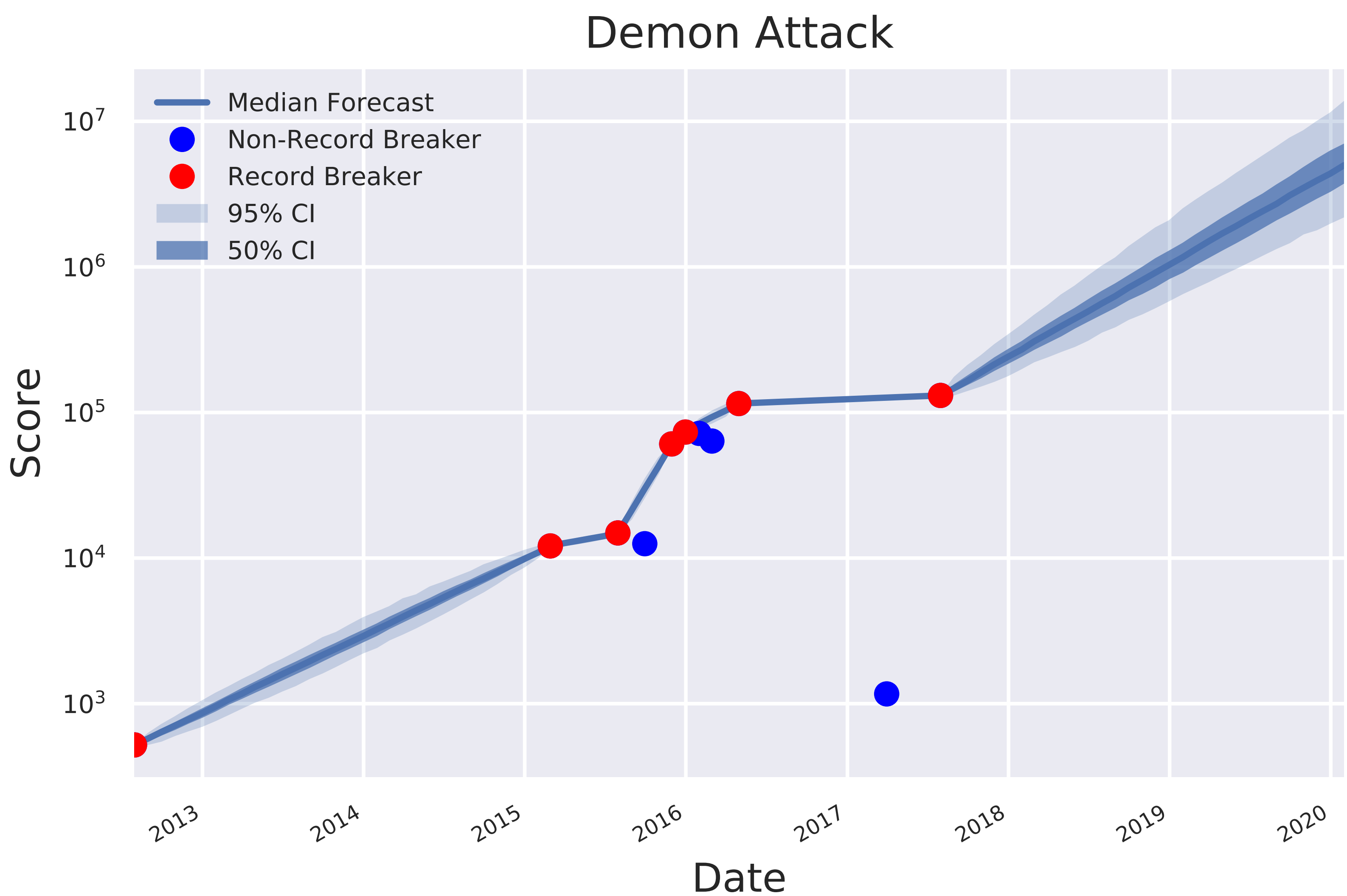


Centipede
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
10^6
10^5
10^4
Score
2013
2014
2015
2016
2017
2018
2019
2020
Date

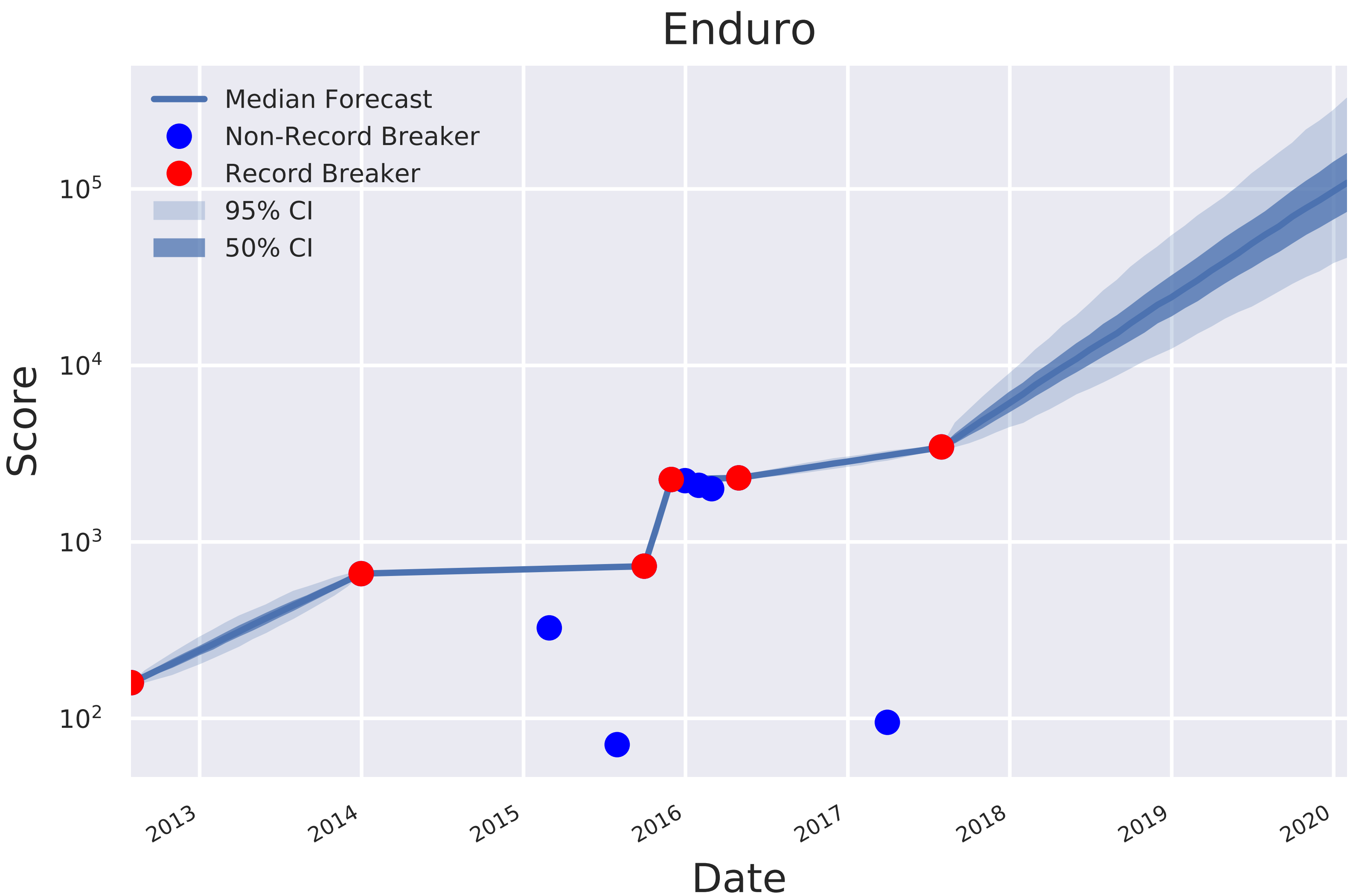


Chopper Command
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
Score
10^4
10^3
2013
2014
2015
2016
2017
2018
2019
2020
Date

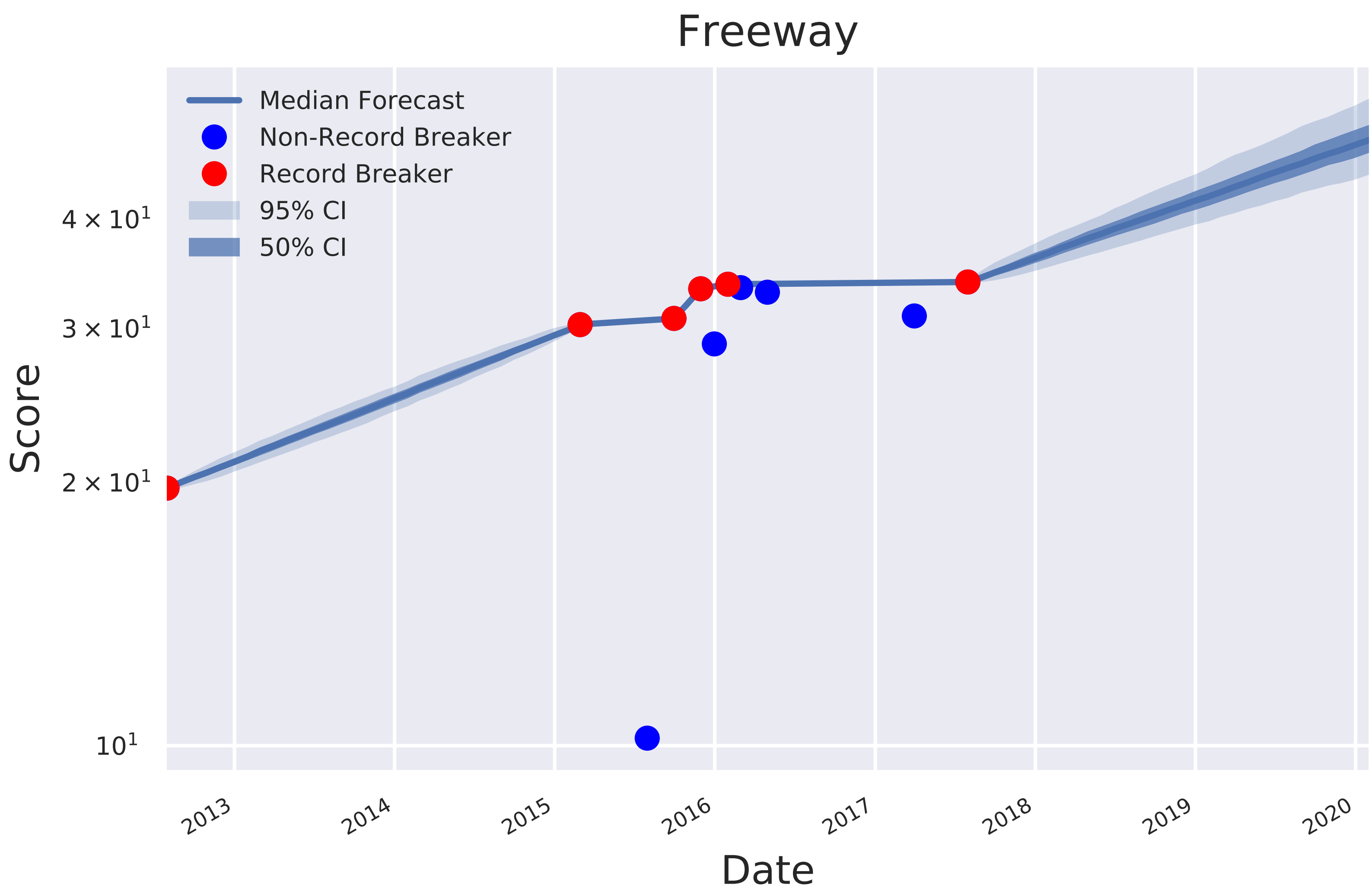

Crazy Climber
Score
Date
10^6
10^5
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
2013
2014
2015
2016
2017
2018
2019
2020

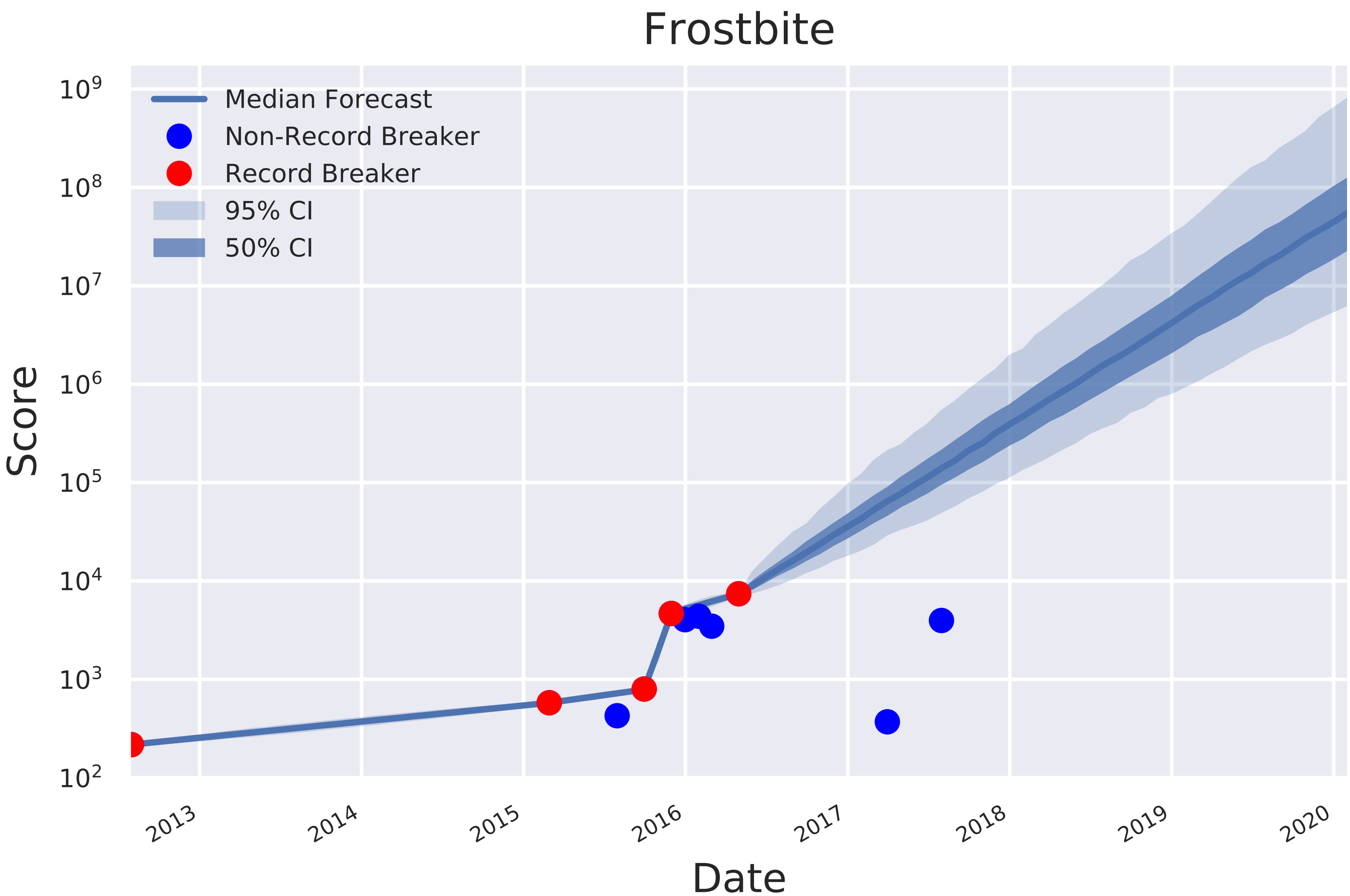


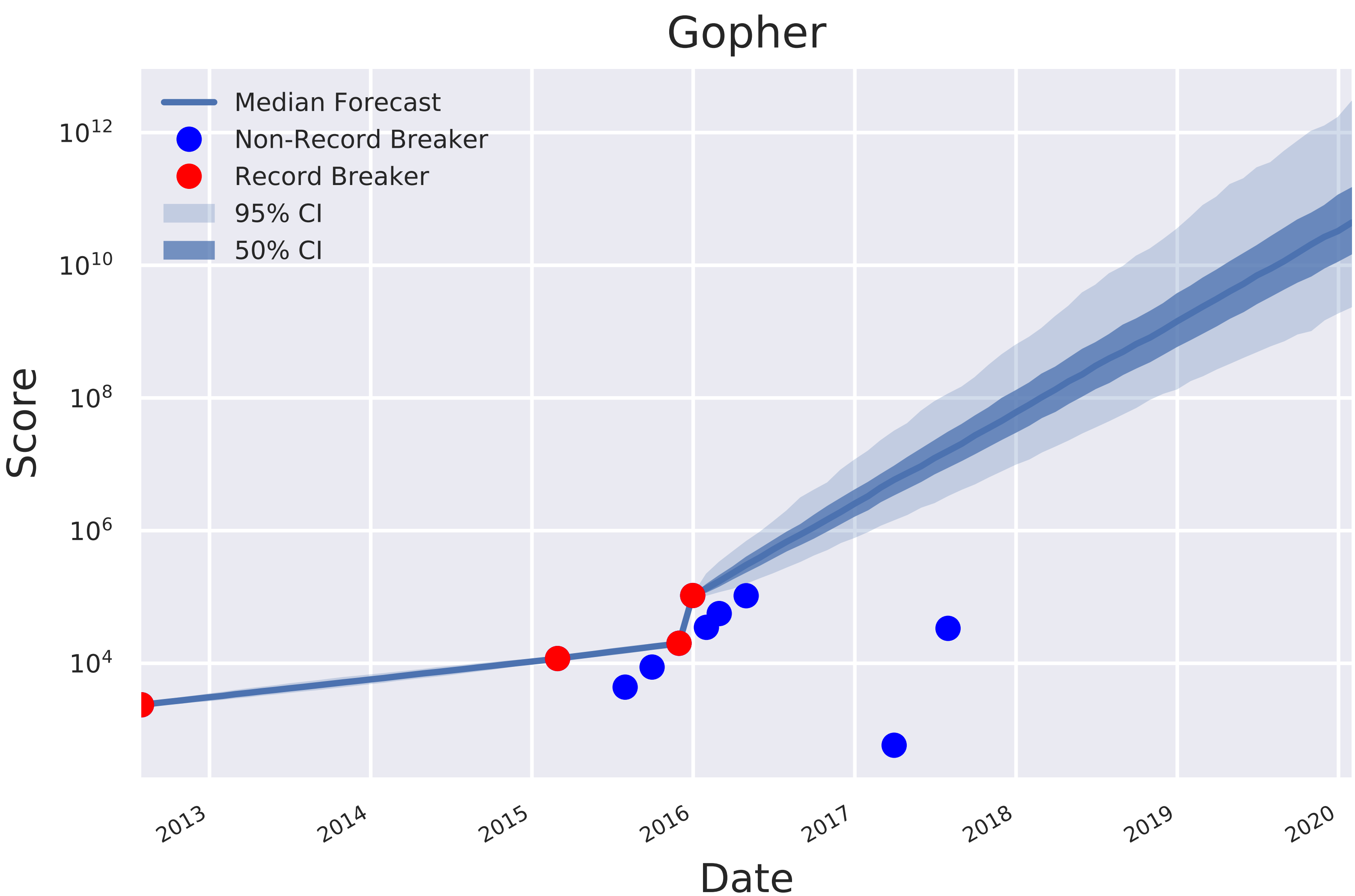
Demon Attack
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
Score
10^7
10^6
10^5
10^4
10^3
2013
2014
2015
2016
2017
2018
2019
2020
Date



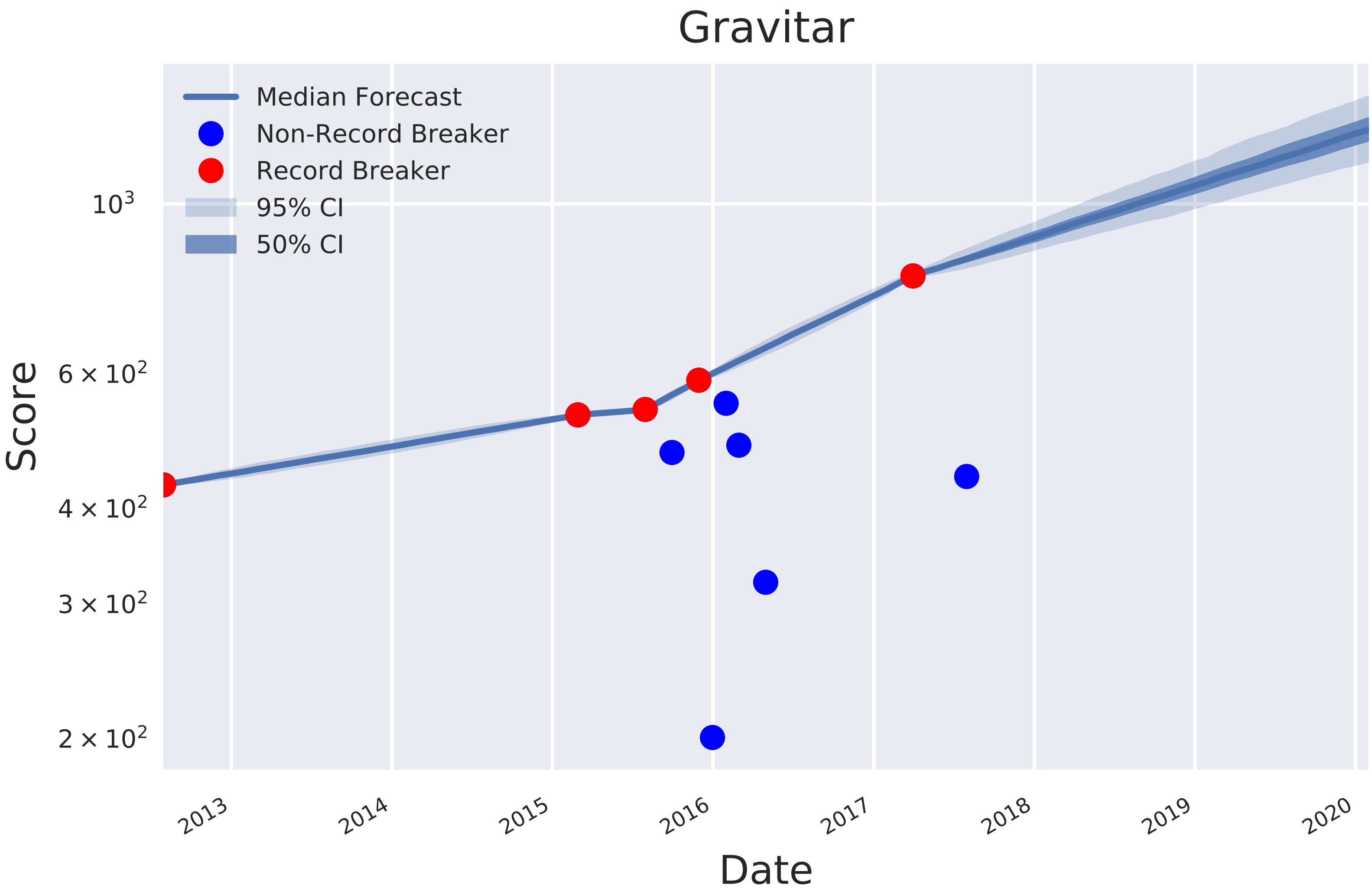
Enduro
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
Score
10^5
10^4
10^3
10^2
2013
2014
2015
2016
2017
2018
2019
2020
Date

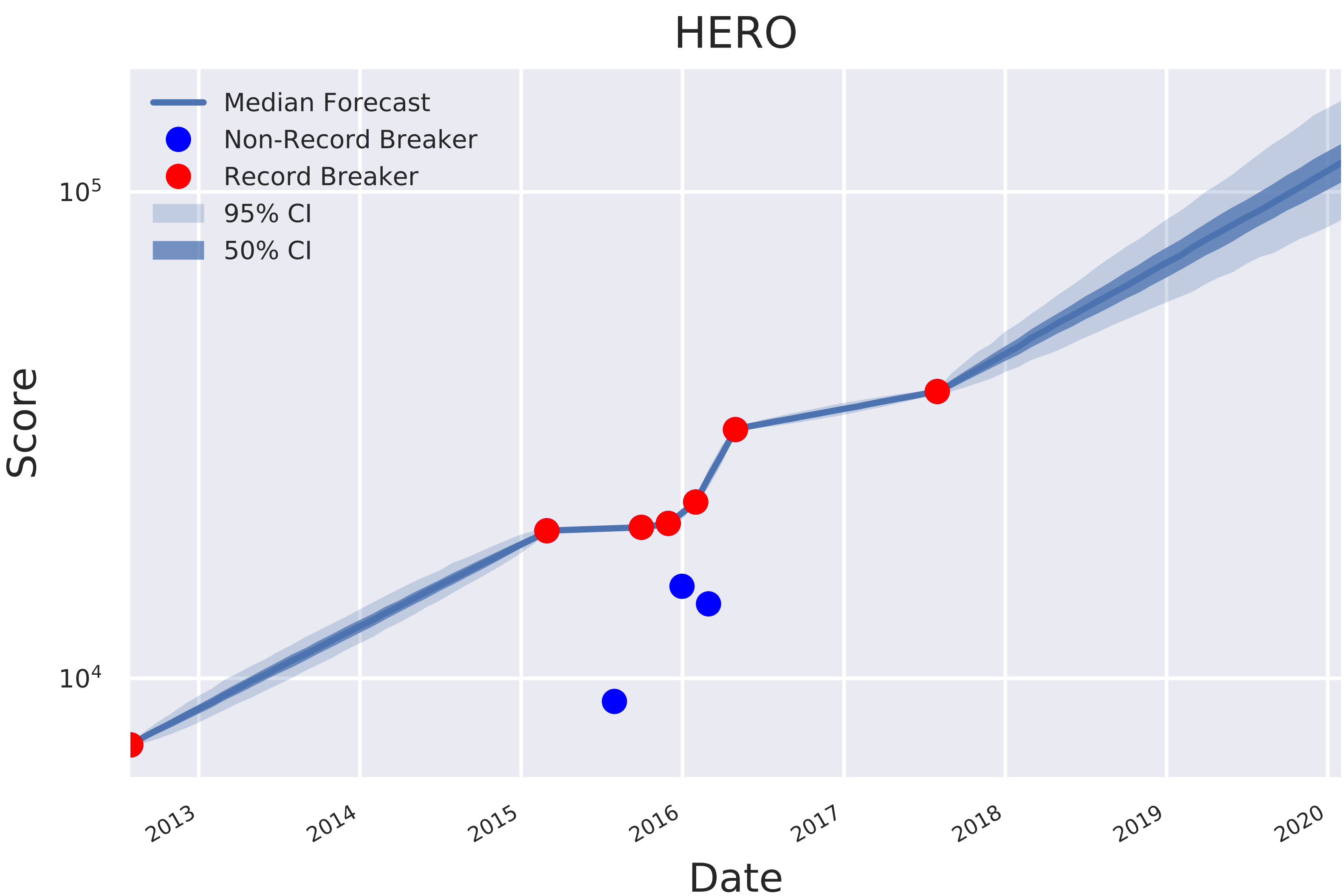

Freeway
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
Score
4×10^1
3×10^1
2×10^1
10^1
2013
2014
2015
2016
2017
2018
2019
2020
Date

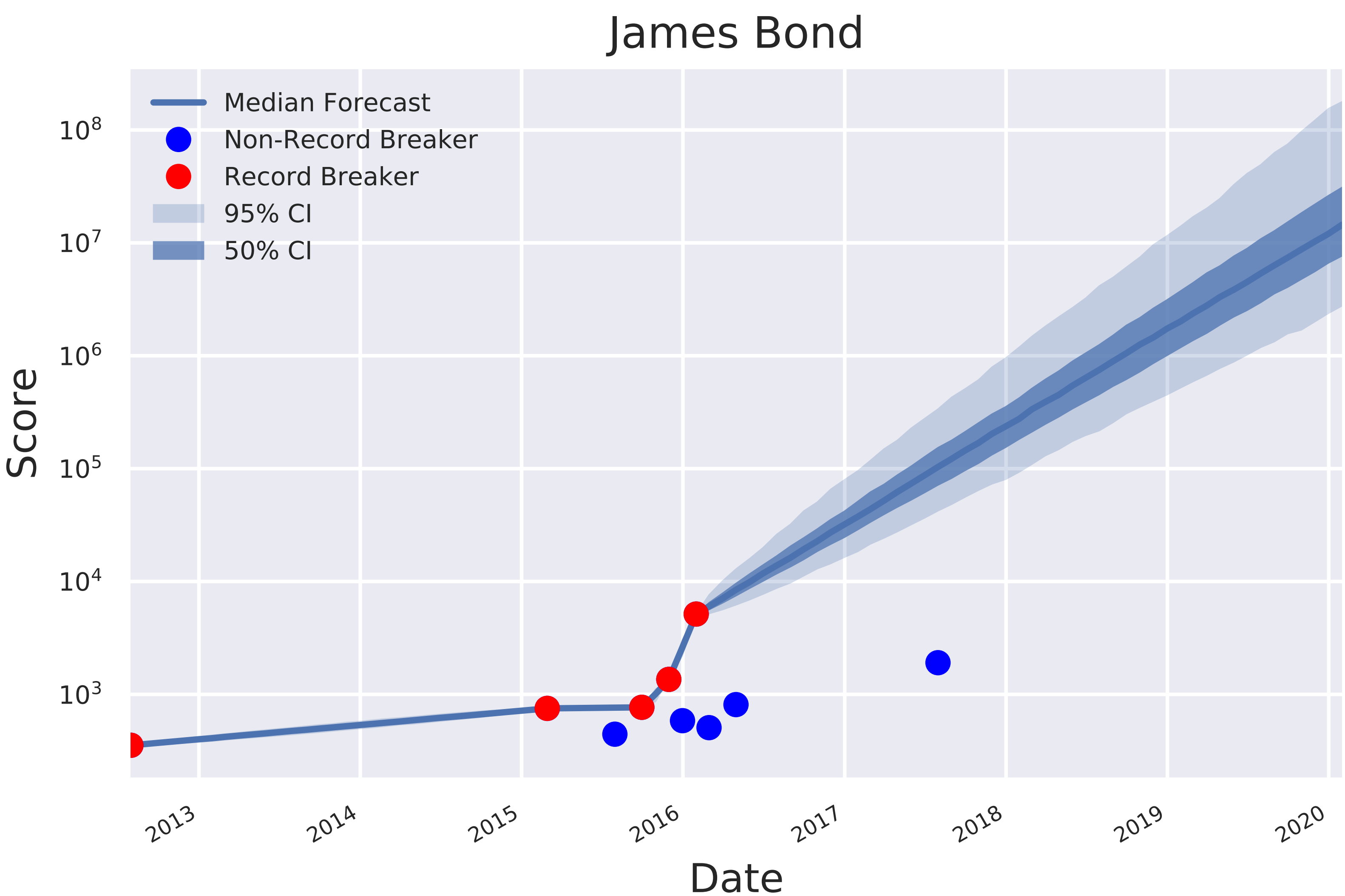

Frostbite
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
Score
10^2
10^3
10^4
10^5
10^6
10^7
10^8
10^9
2013
2014
2015
2016
2017
2018
2019
2020
Date

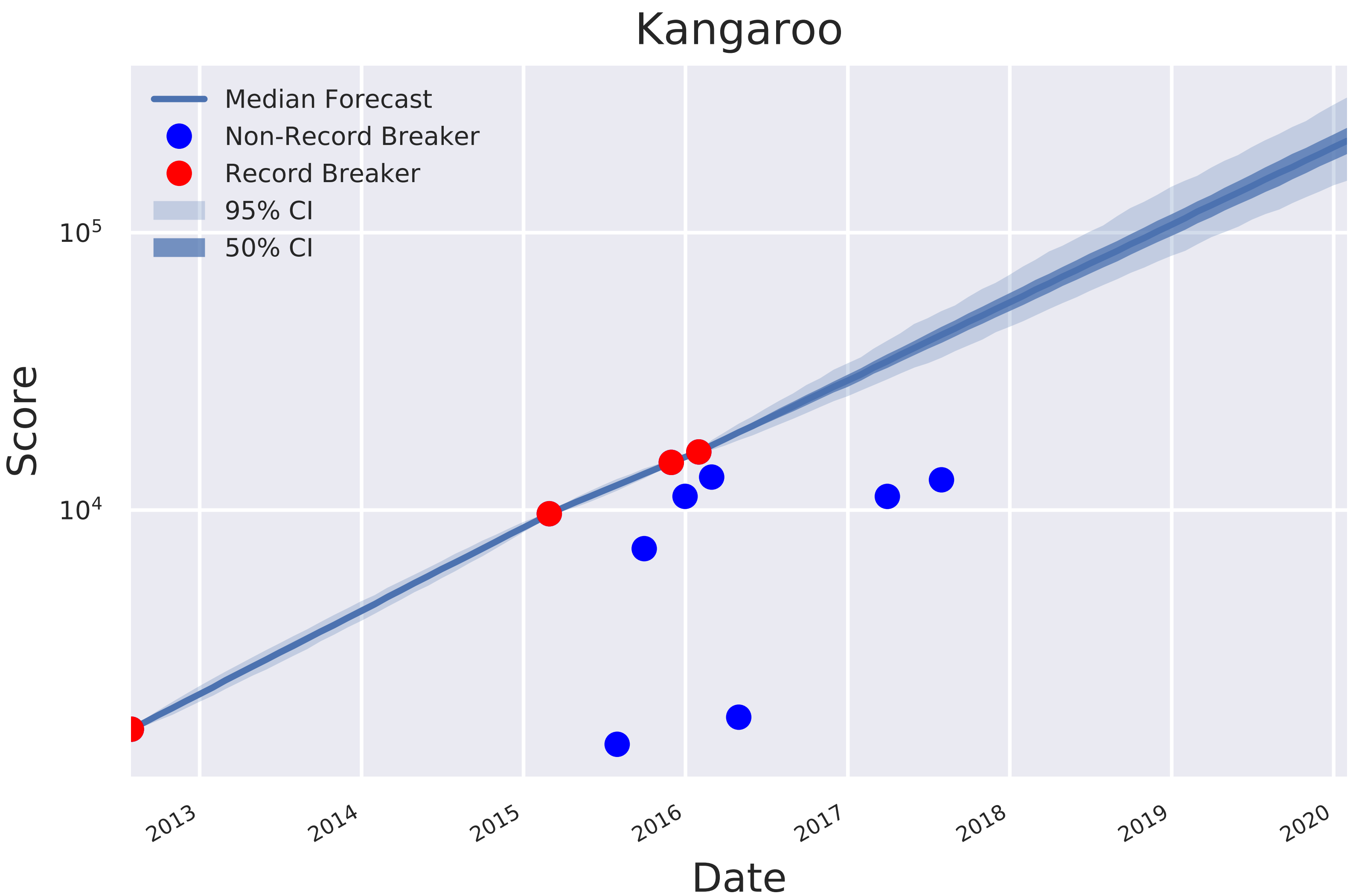


Gopher
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
Score
10^12
10^10
10^8
10^6
10^4
2013
2014
2015
2016
2017
2018
2019
2020
Date

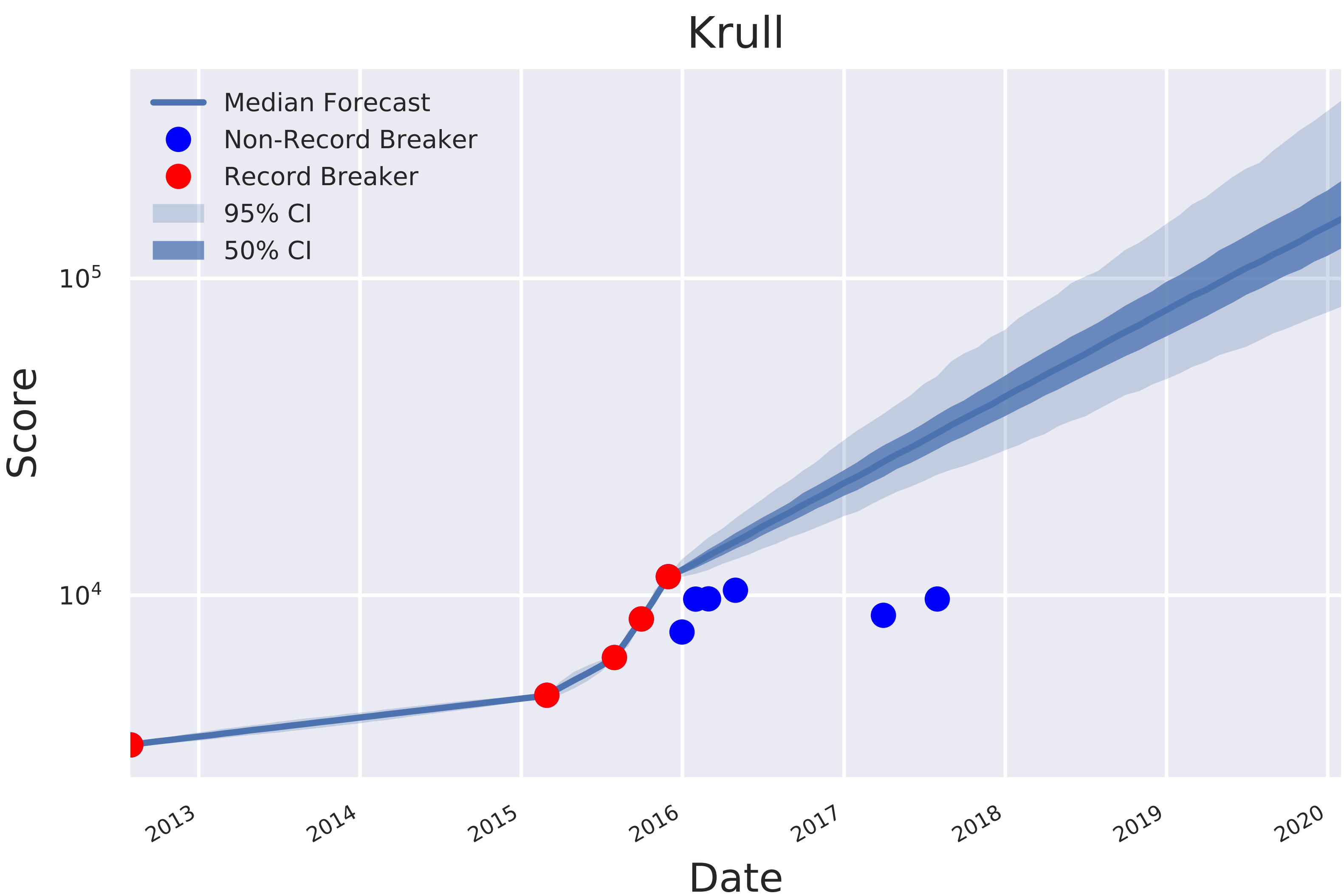

Gravitar
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
Score
10^3
6×10^2
4×10^2
3×10^2
2×10^2
2013
2014
2015
2016
2017
2018
2019
2020
Date

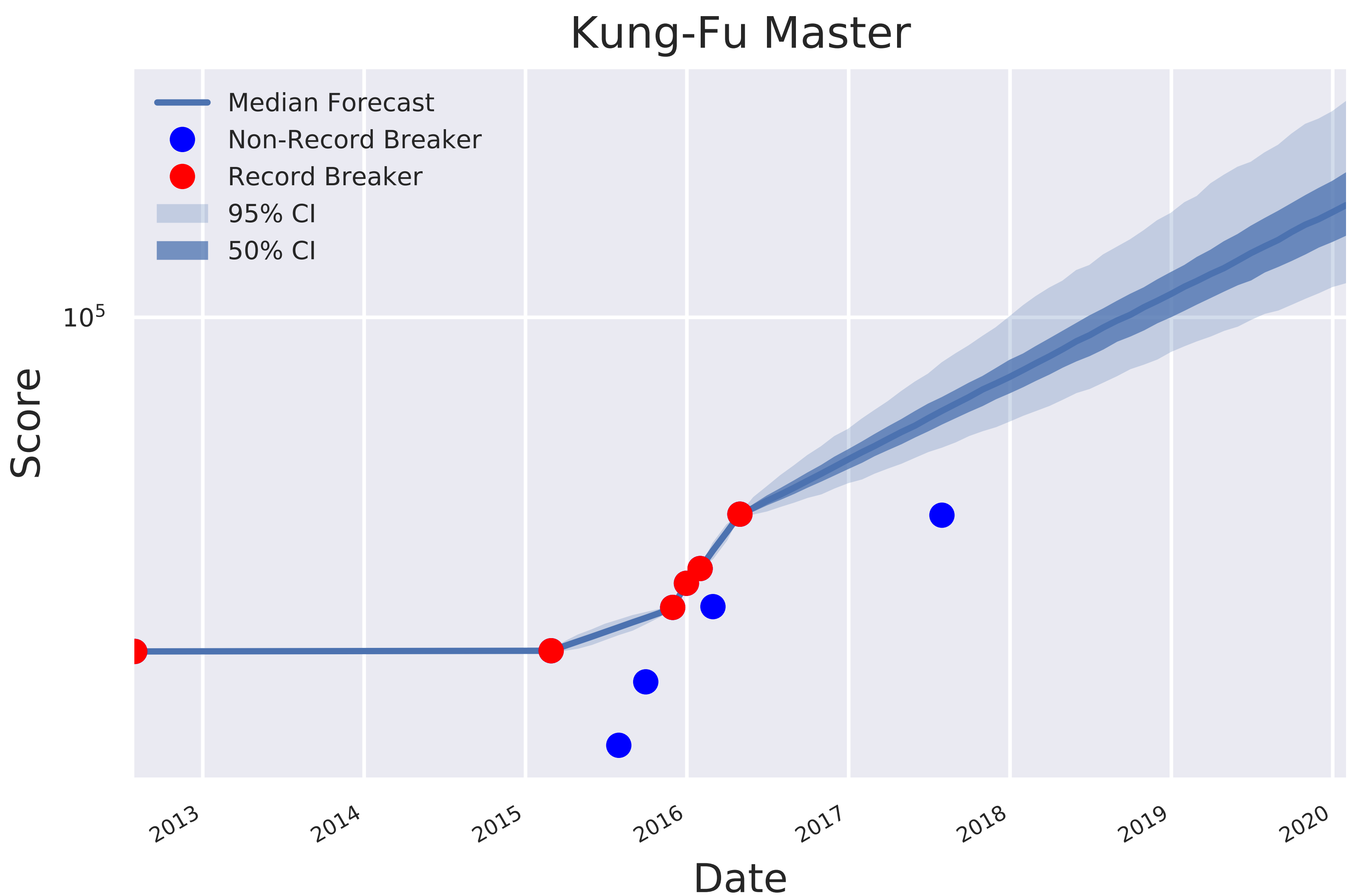

HERO
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
10^5
10^4
Score
2013
2014
2015
2016
2017
2018
2019
2020
Date

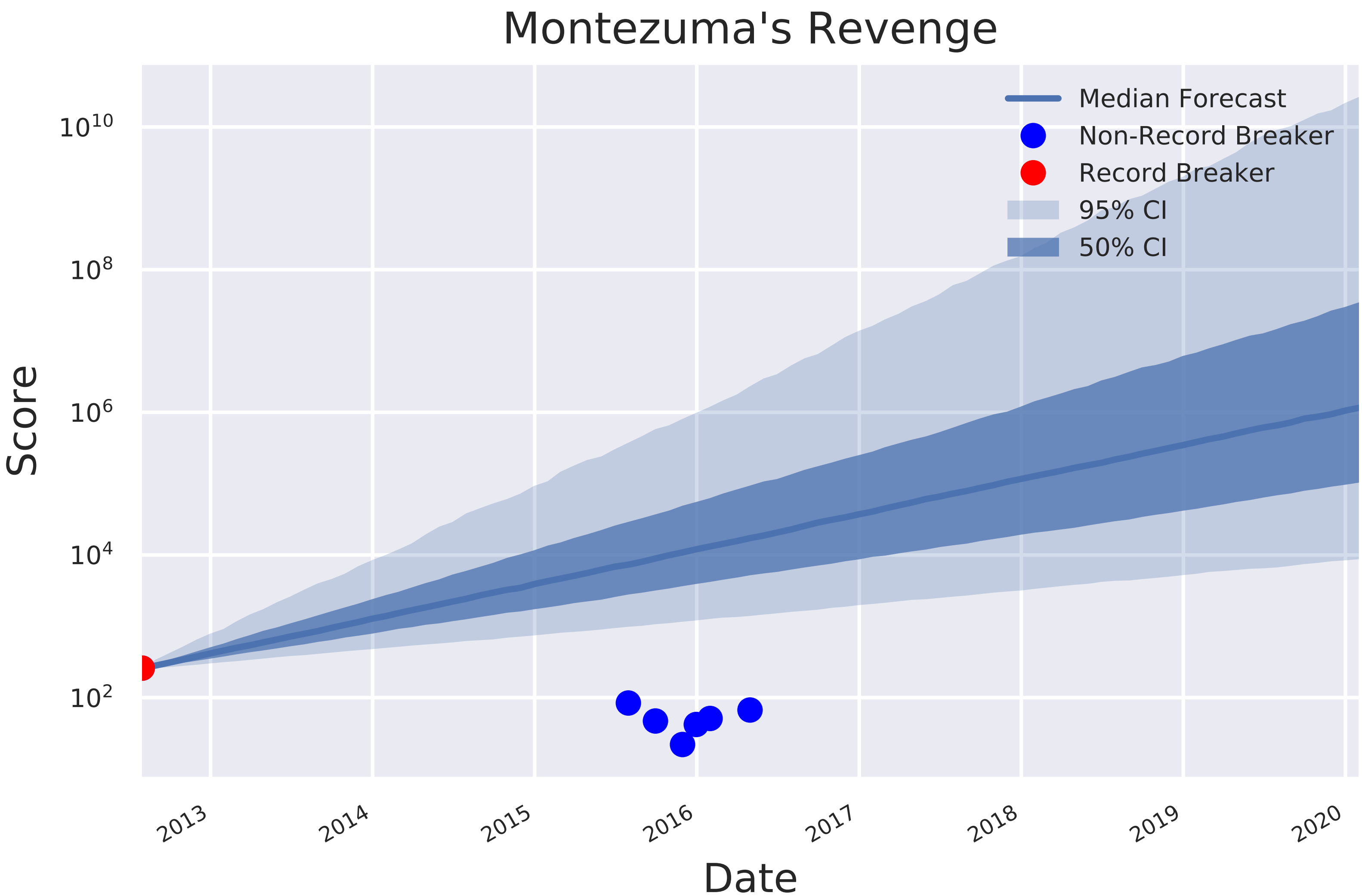

James Bond
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
10^8
10^7
10^6
10^5
10^4
10^3
Score
2013
2014
2015
2016
2017
2018
2019
2020
Date



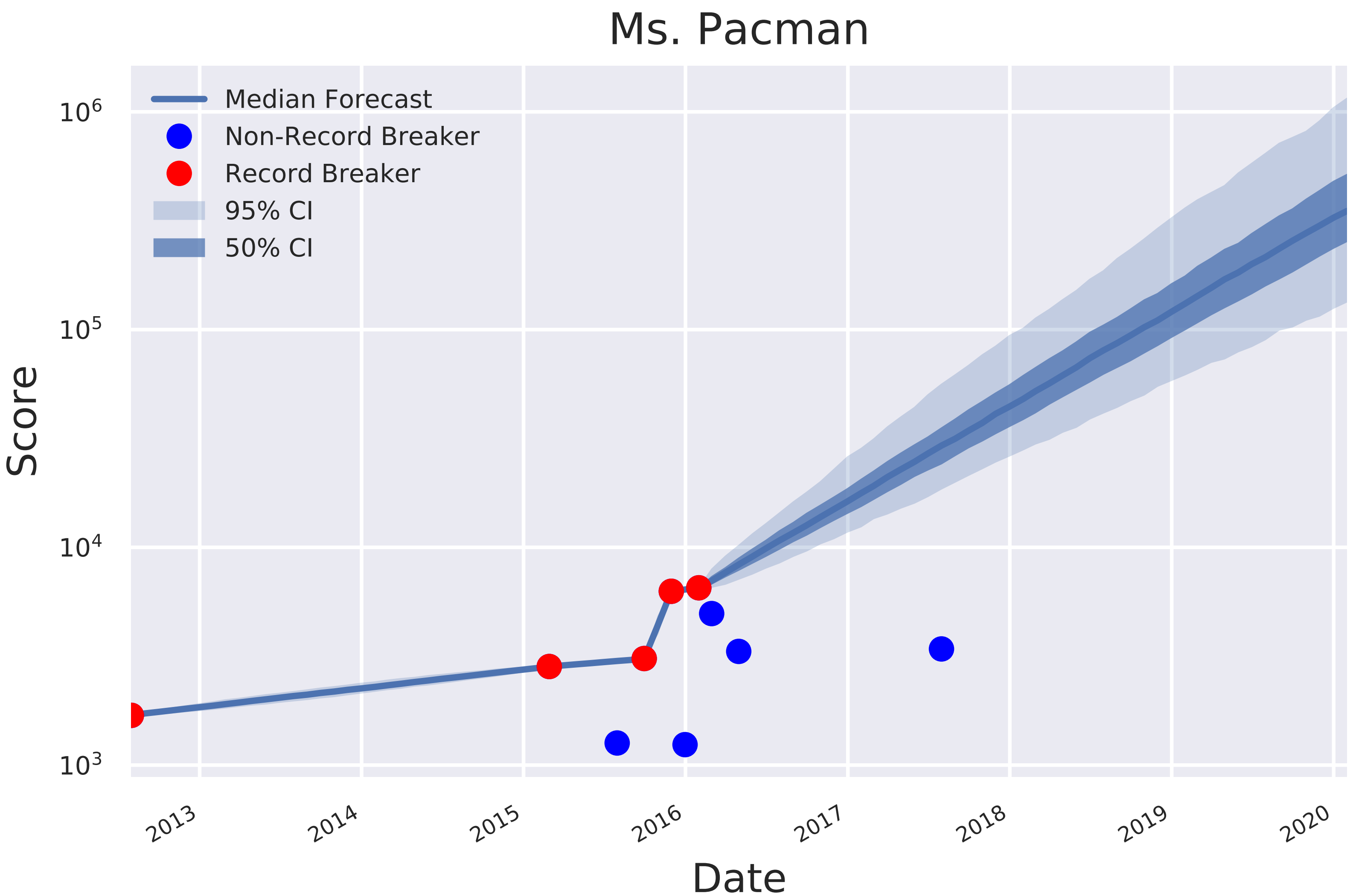
Kangaroo
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
10^5
10^4
Score
2013
2014
2015
2016
2017
2018
2019
2020
Date

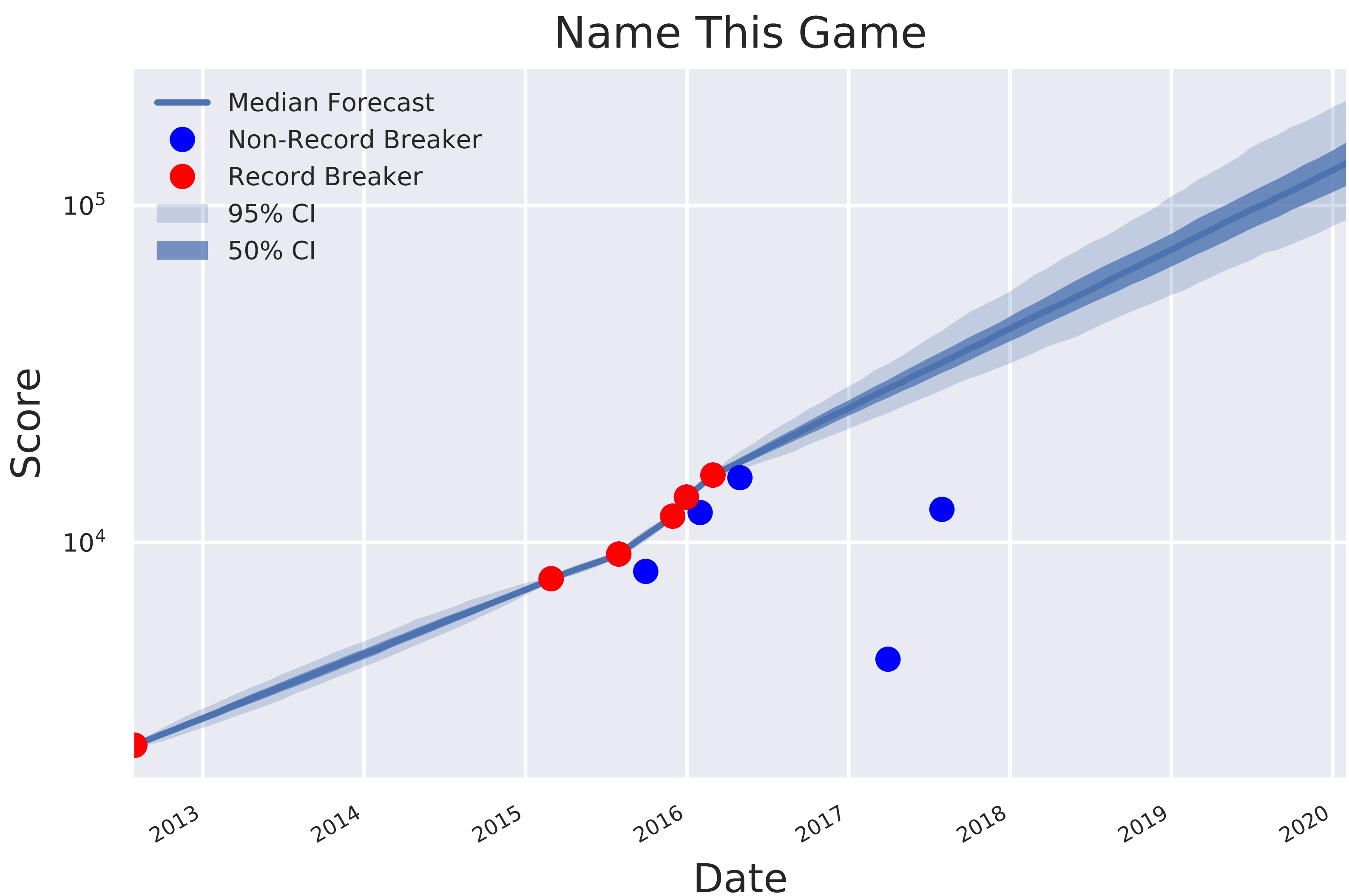

Krull
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
10^5
10^4
Score
2013
2014
2015
2016
2017
2018
2019
2020
Date


Kung-Fu Master
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
10^5
Score
2013
2014
2015
2016
2017
2018
2019
2020
Date

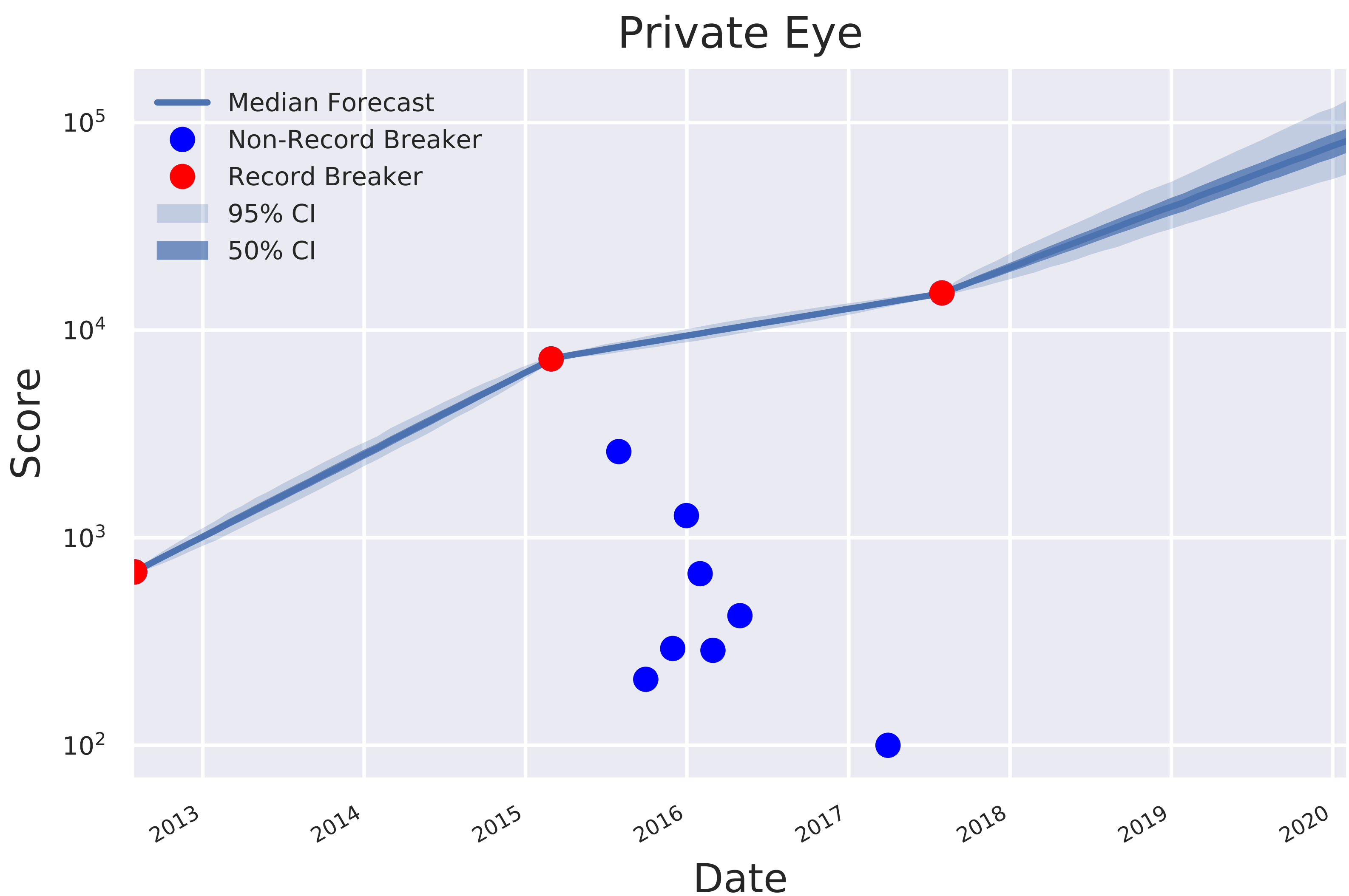


Montezuma's Revenge
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
10^{10}
10^{8}
10^{6}
10^{4}
10^{2}
Score
2013
2014
2015
2016
2017
2018
2019
2020
Date

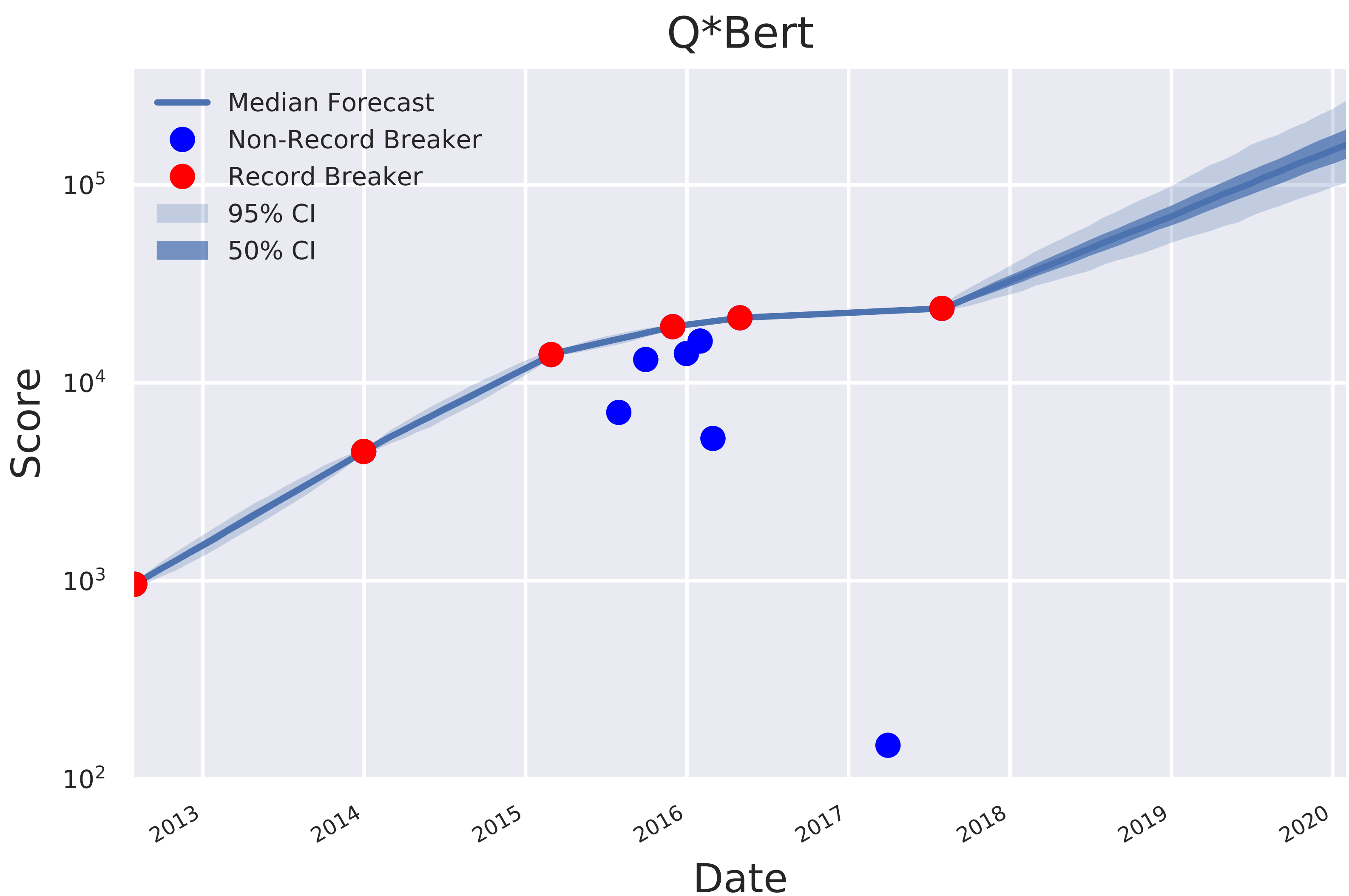

Ms. Pacman
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
10^6
10^5
10^4
10^3
Score
2013
2014
2015
2016
2017
2018
2019
2020
Date

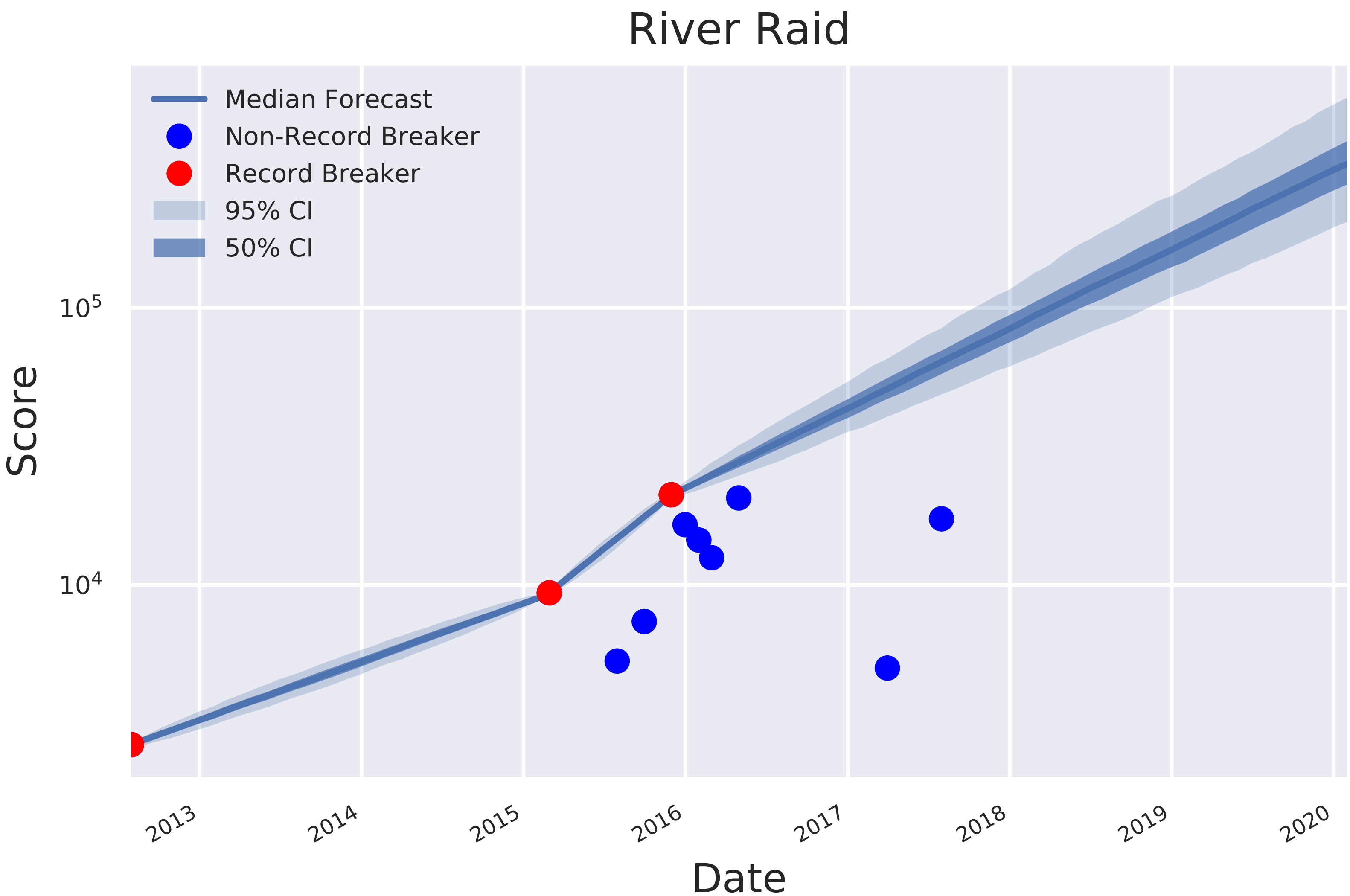


Name This Game
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
10^5
10^4
Score
2013
2014
2015
2016
2017
2018
2019
2020
Date

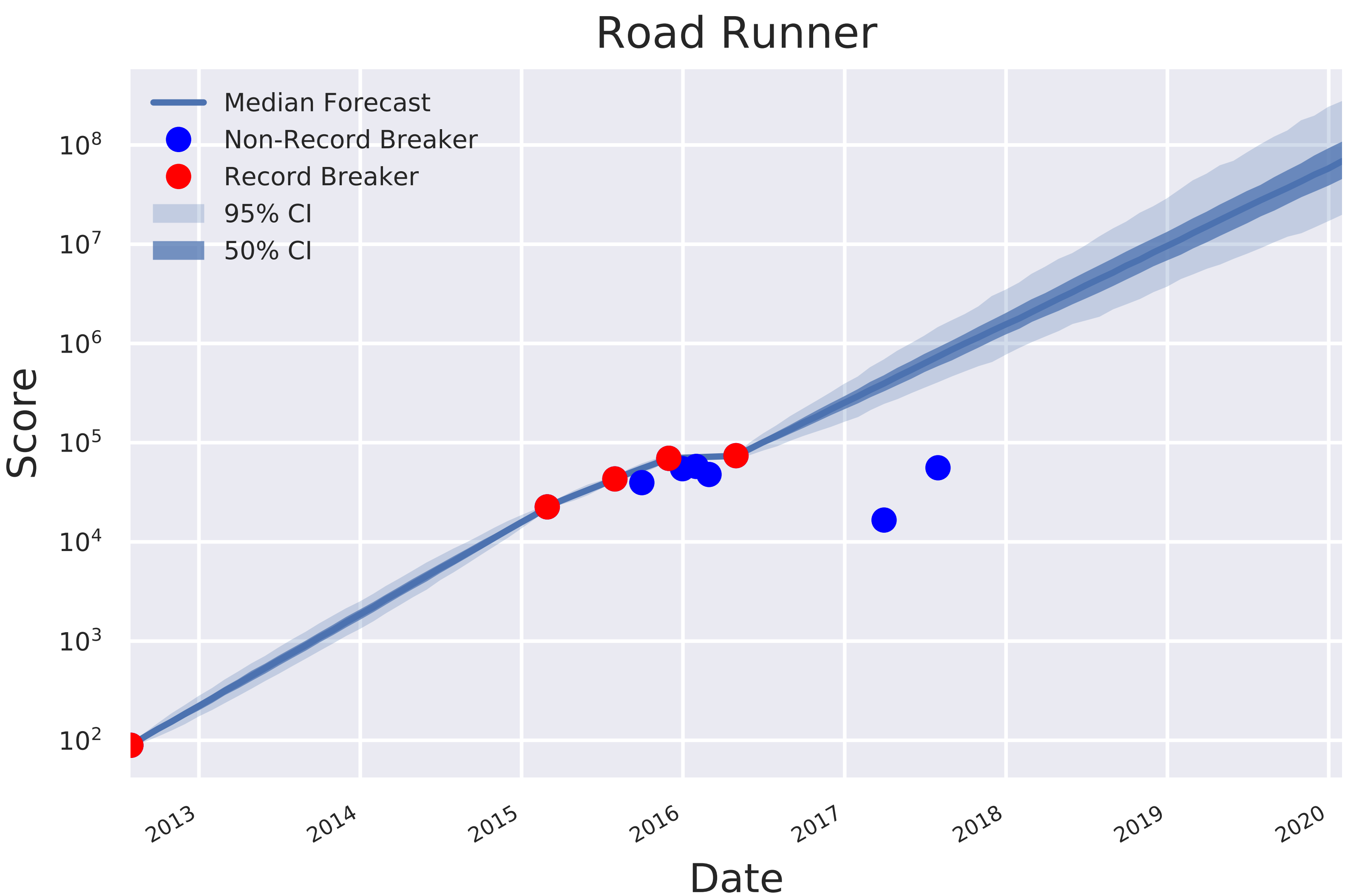


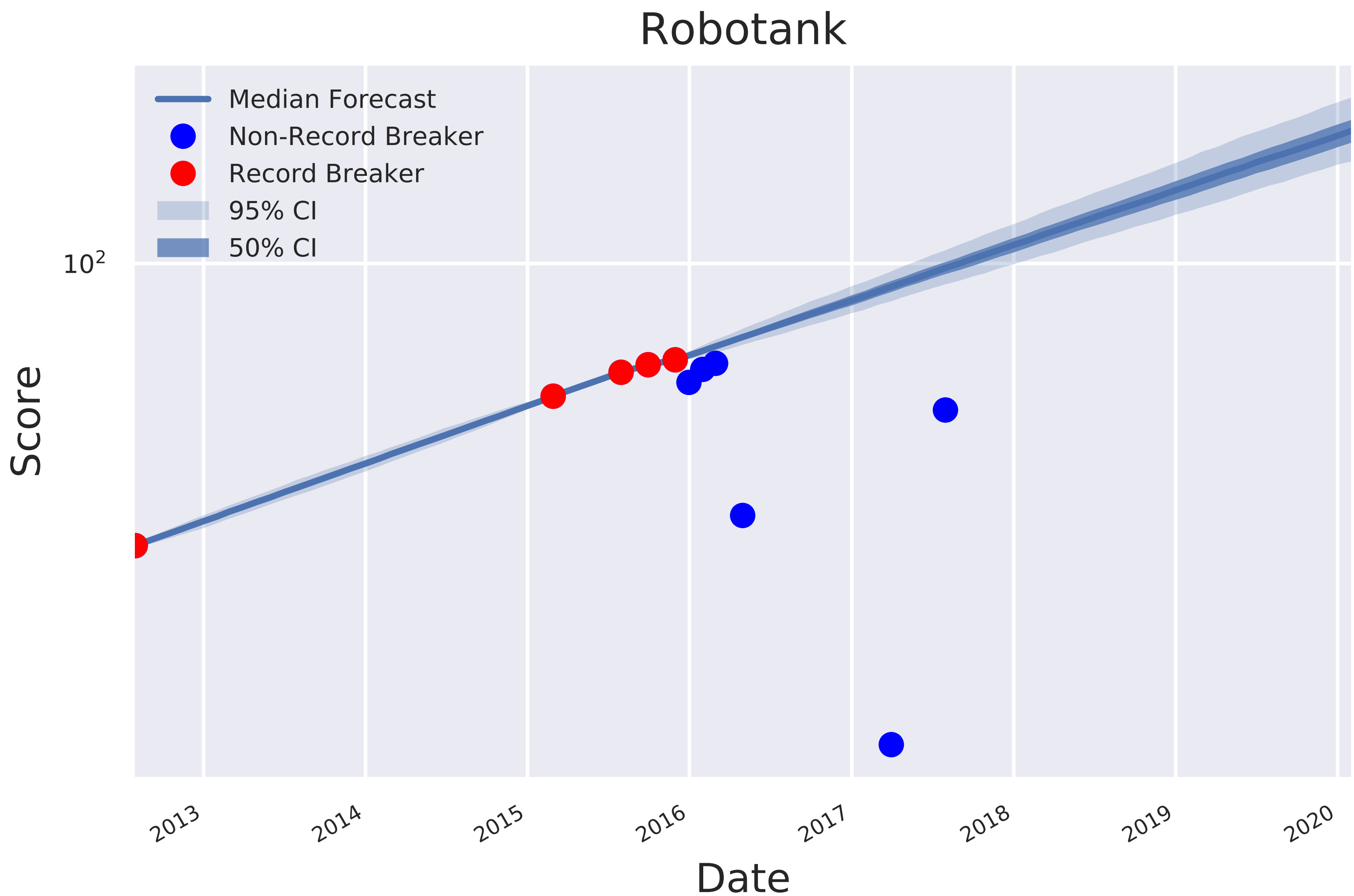
Private Eye
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
10^5
10^4
10^3
10^2
Score
2013
2014
2015
2016
2017
2018
2019
2020
Date

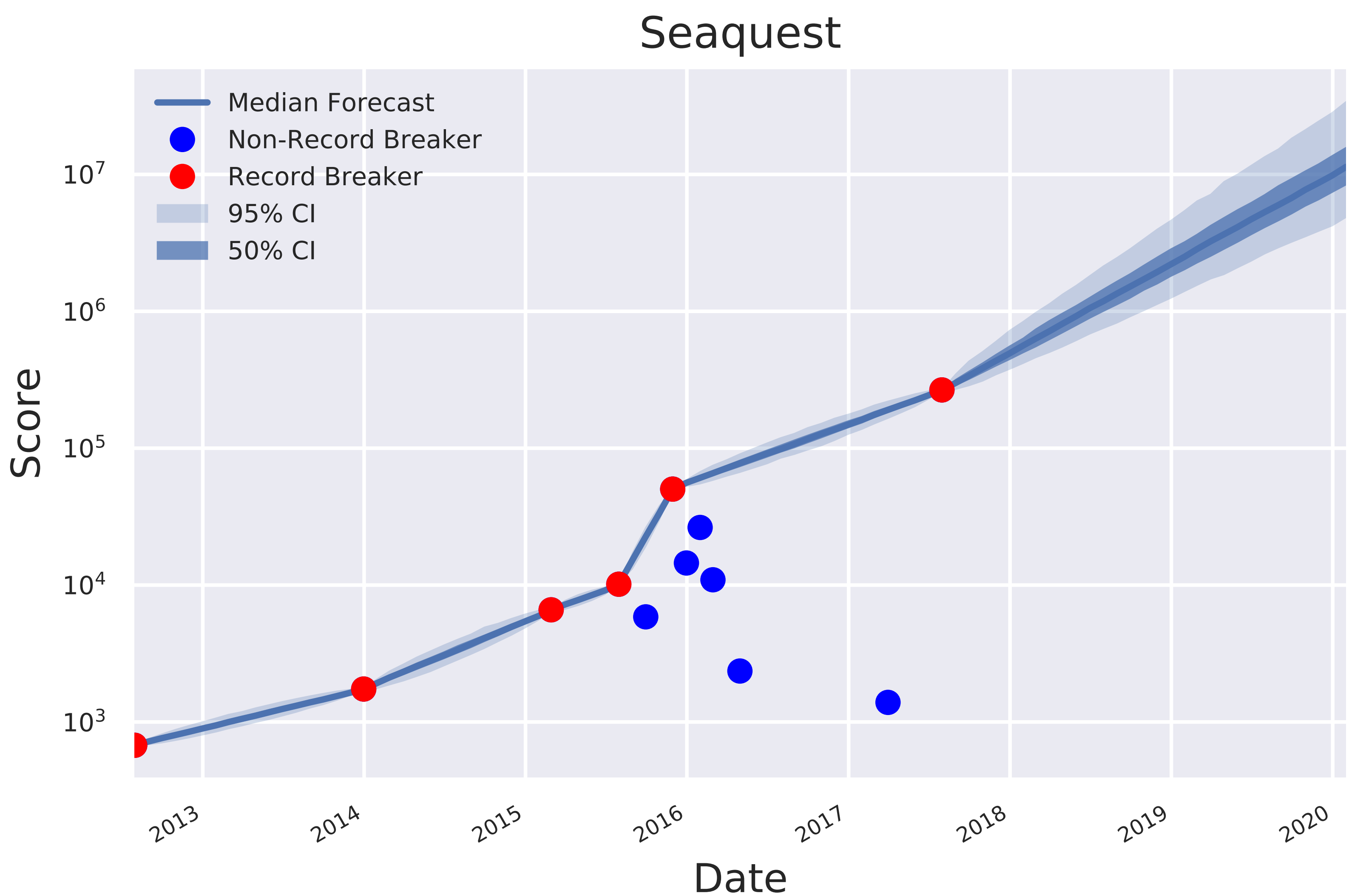

Q*Bert
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
Score
10^5
10^4
10^3
10^2
2013
2014
2015
2016
2017
2018
2019
2020
Date



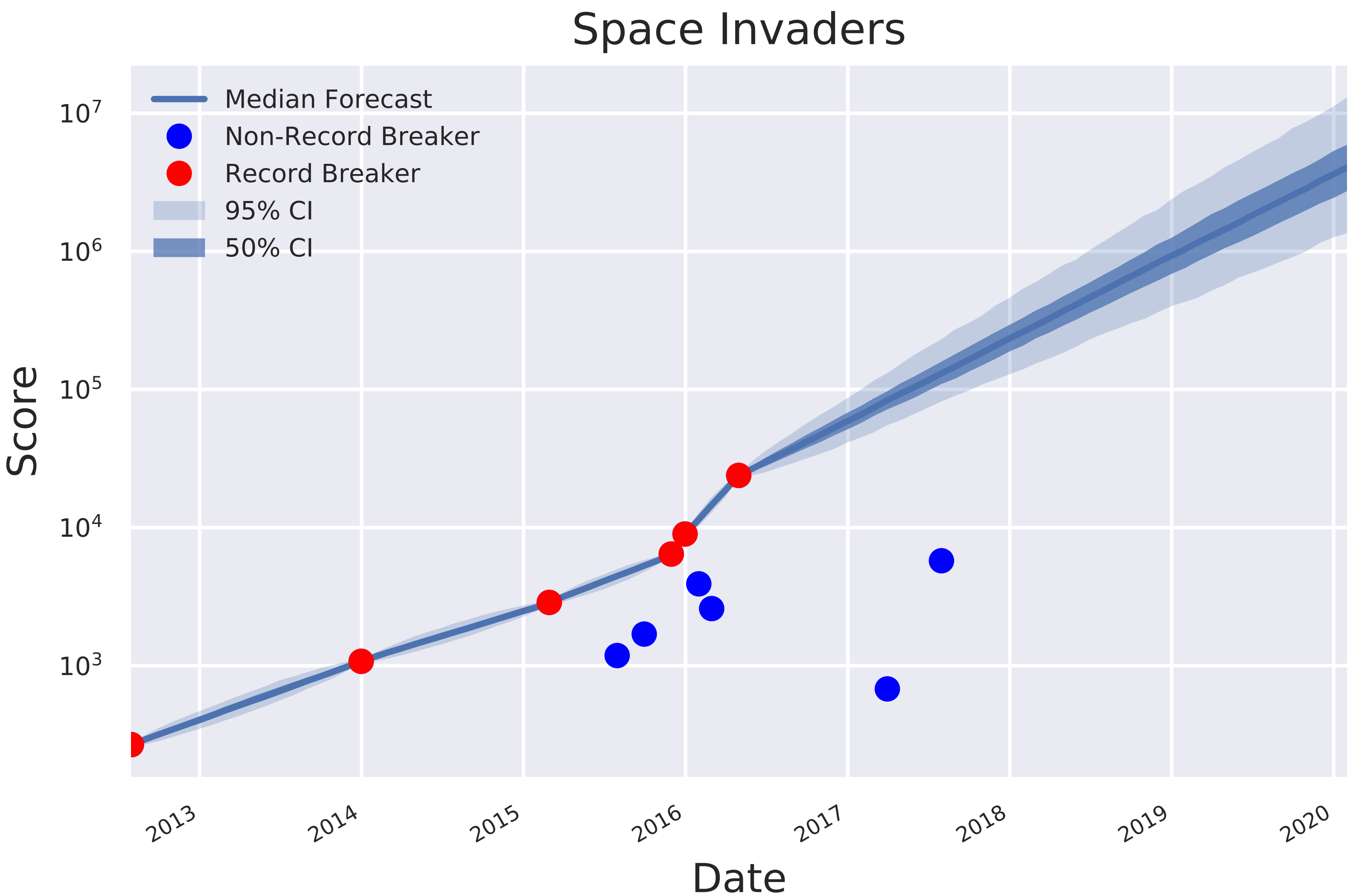
River Raid
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
10^5
10^4
Score
2013
2014
2015
2016
2017
2018
2019
2020
Date


Road Runner
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
Score
Date
10^8
10^7
10^6
10^5
10^4
10^3
10^2
2013
2014
2015
2016
2017
2018
2019
2020

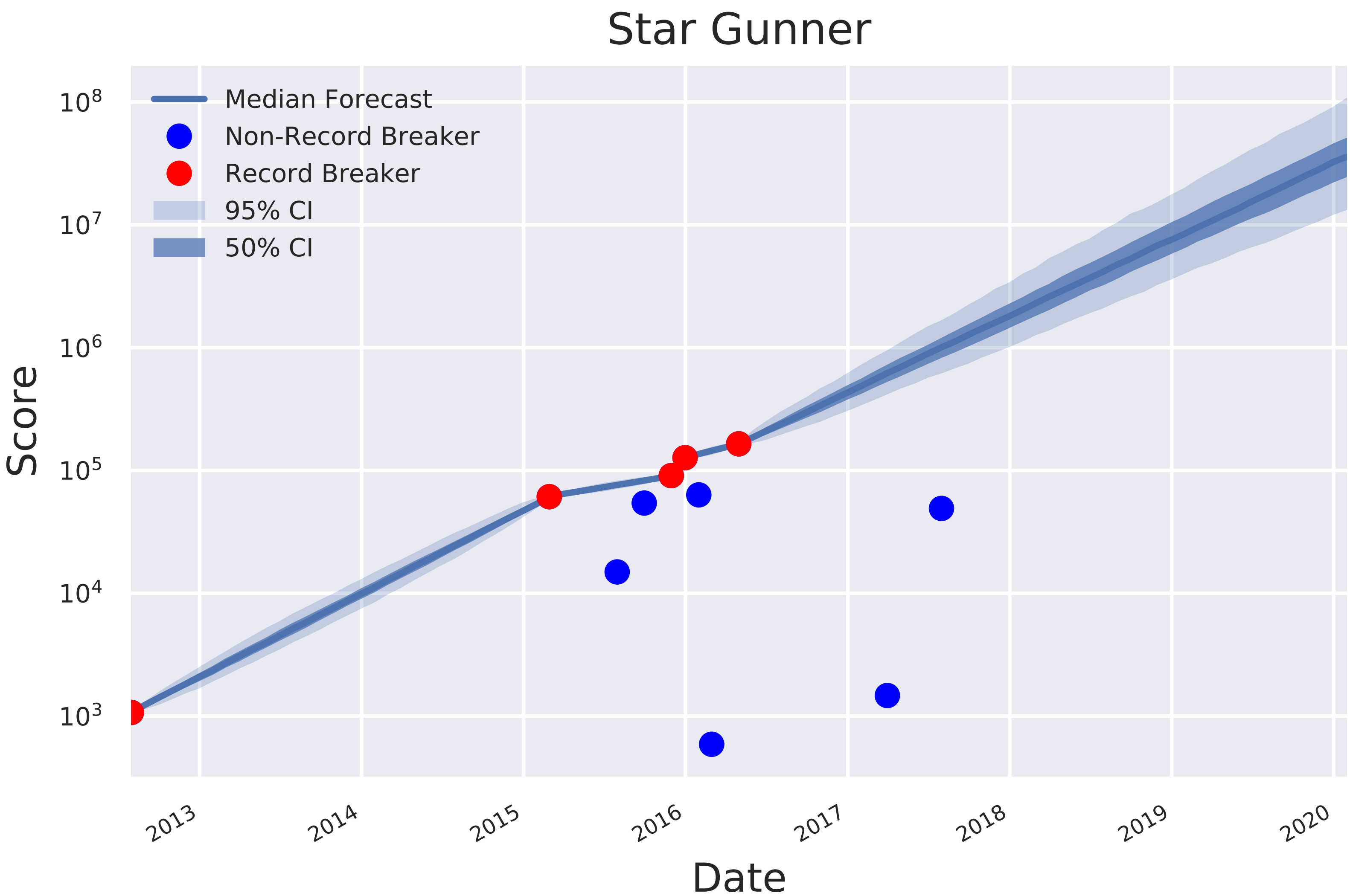

Robotank
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
10^2
Score
2013
2014
2015
2016
2017
2018
2019
2020
Date

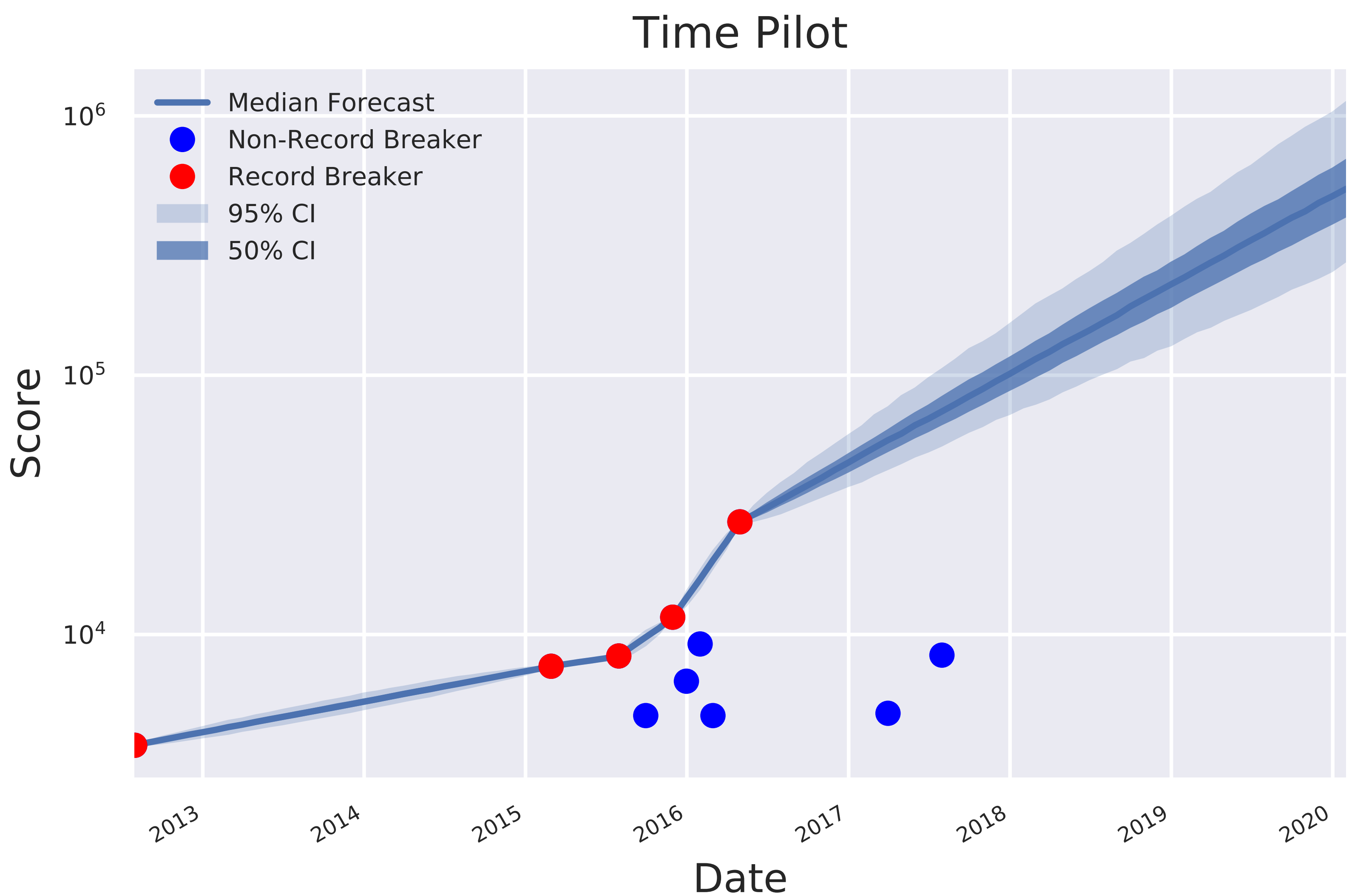


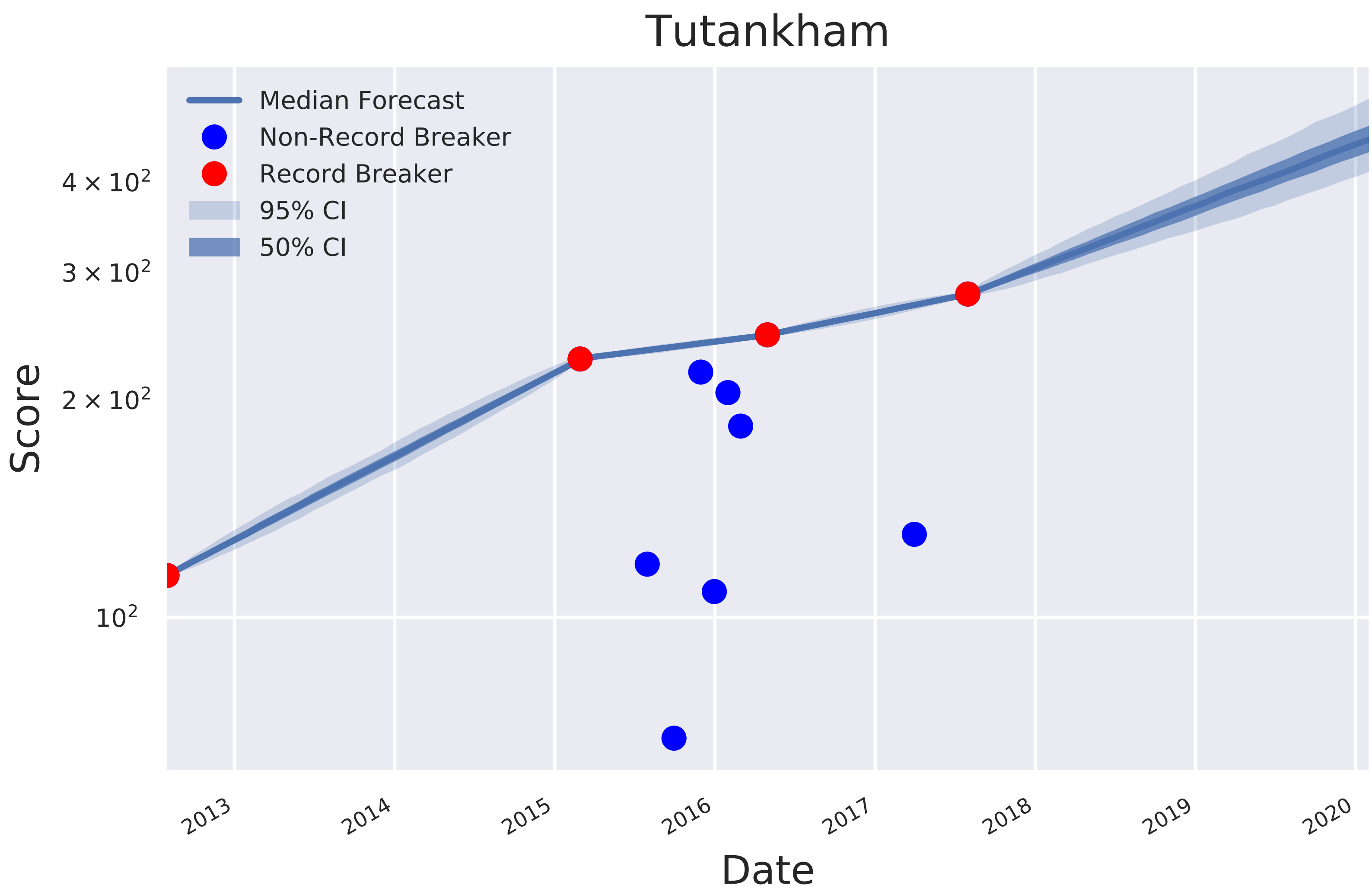
Seaquest
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
Score
10^3
10^4
10^5
10^6
10^7
2013
2014
2015
2016
2017
2018
2019
2020
Date



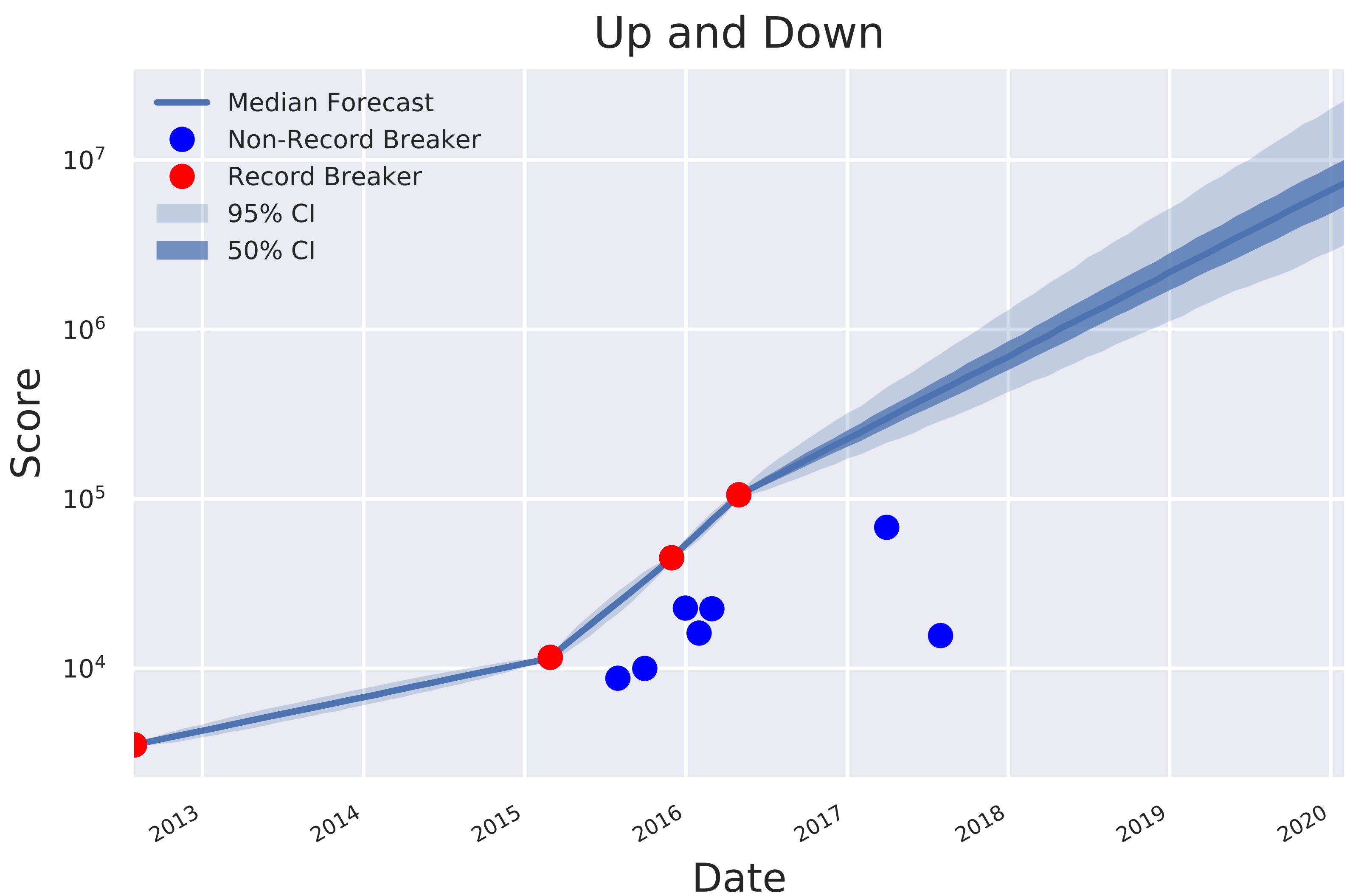
Space Invaders
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
10^7
10^6
10^5
10^4
10^3
Score
2013
2014
2015
2016
2017
2018
2019
2020
Date


Star Gunner
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
Score
10^8
10^7
10^6
10^5
10^4
10^3
2013
2014
2015
2016
2017
2018
2019
2020
Date

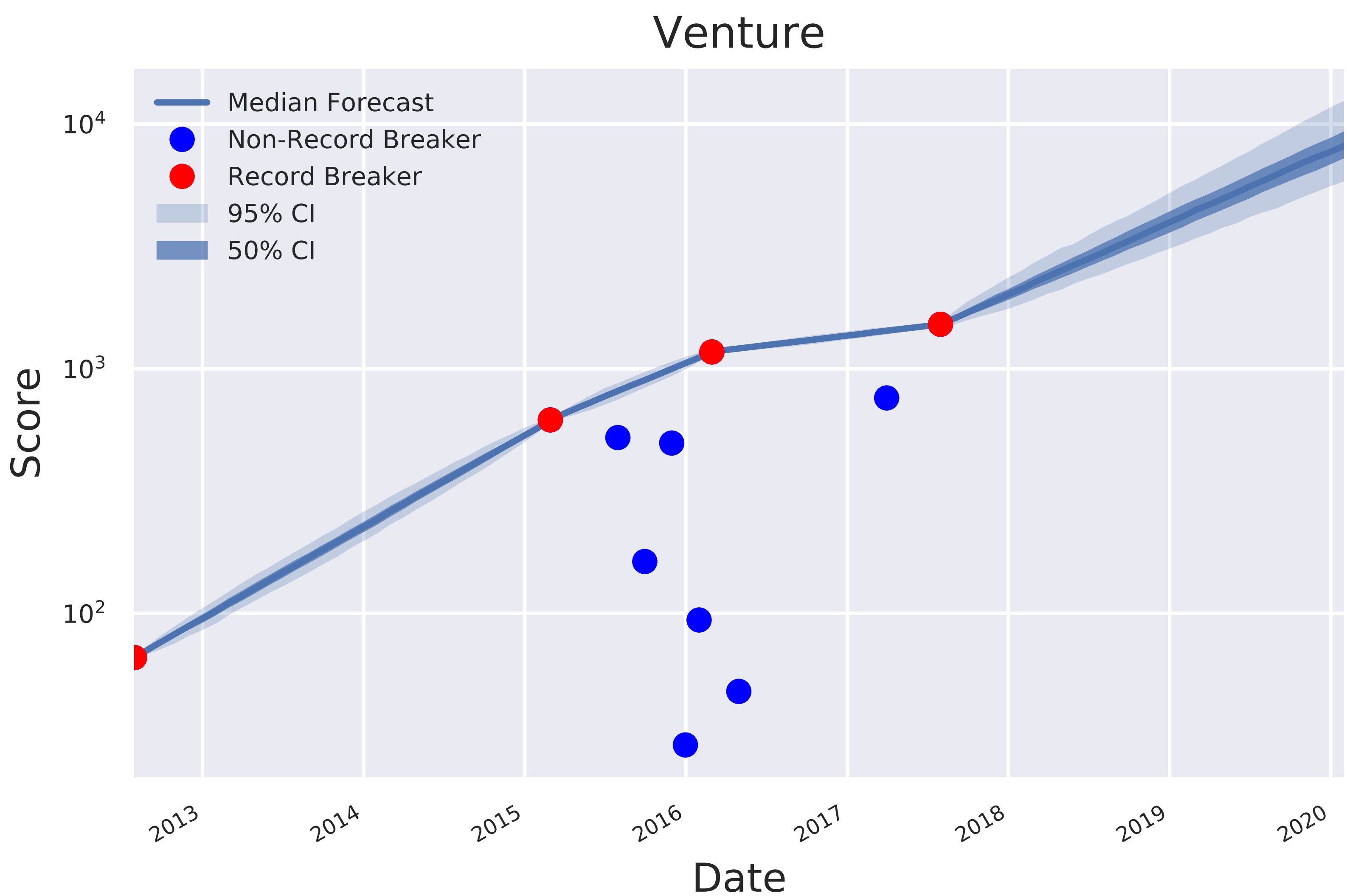

Time Pilot
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
10^6
10^5
10^4
Score
2013
2014
2015
2016
2017
2018
2019
2020
Date

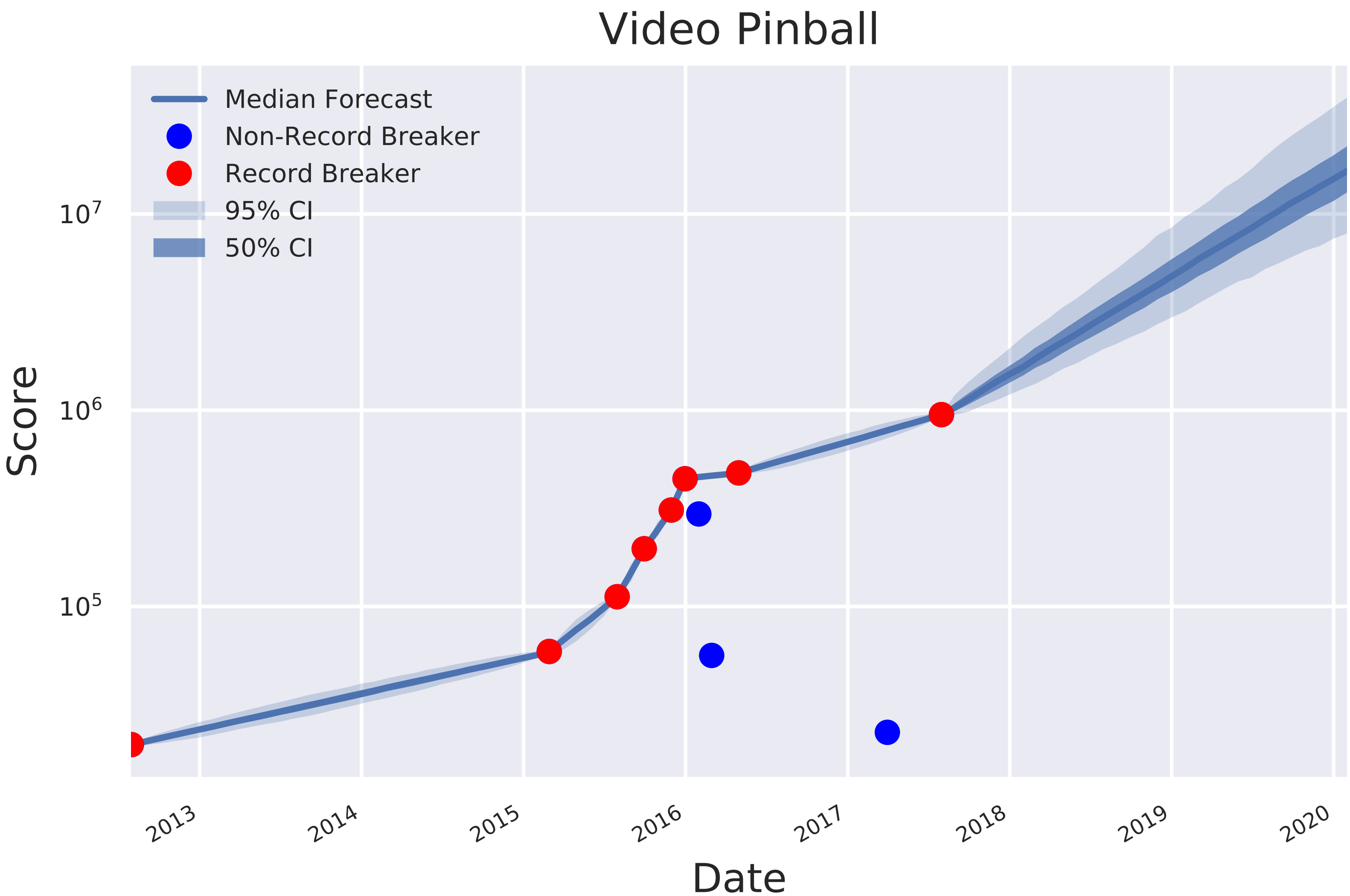


Tutankham
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
4×10^2
3×10^2
2×10^2
10^2
Score
2013
2014
2015
2016
2017
2018
2019
2020
Date

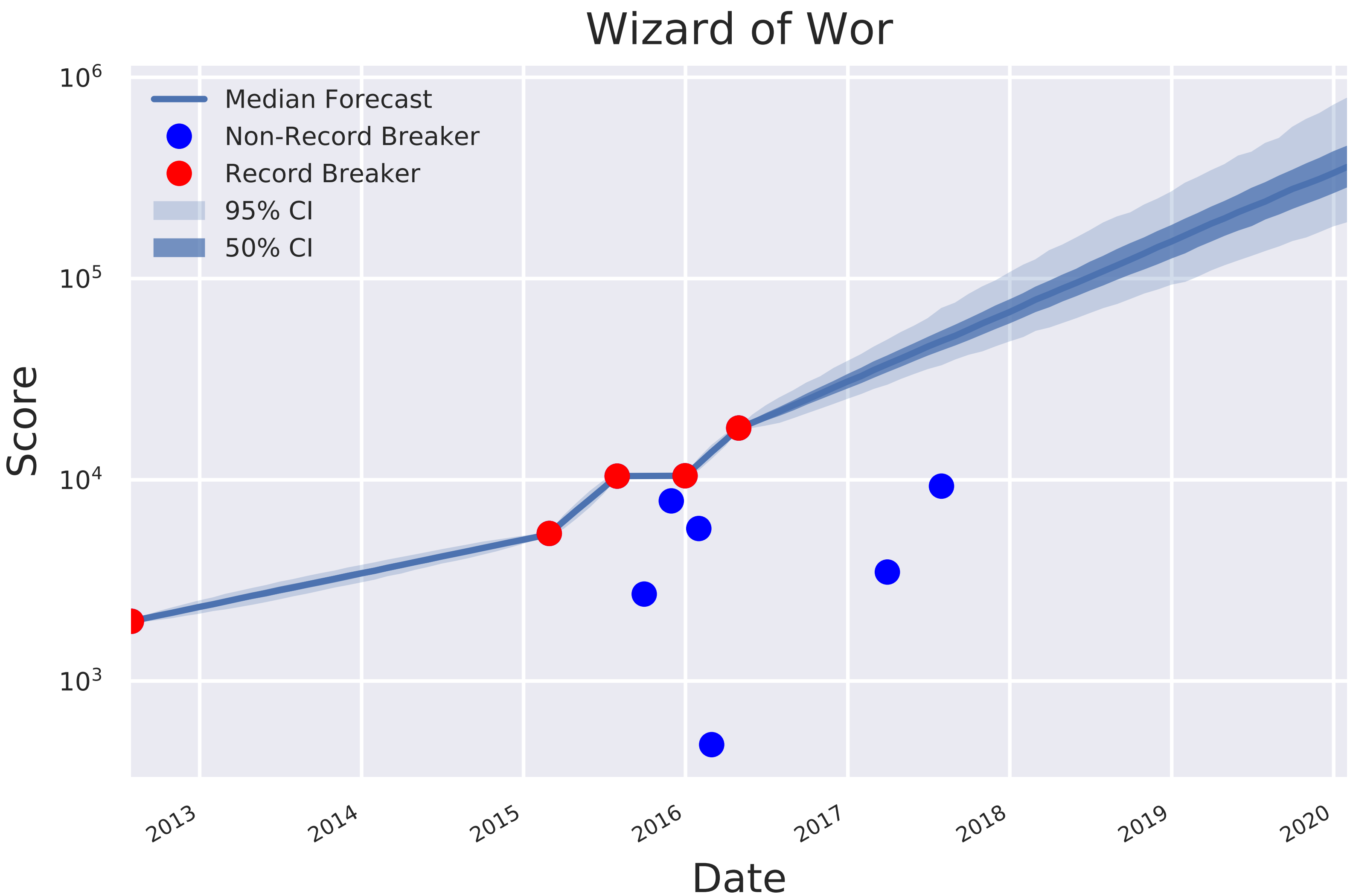


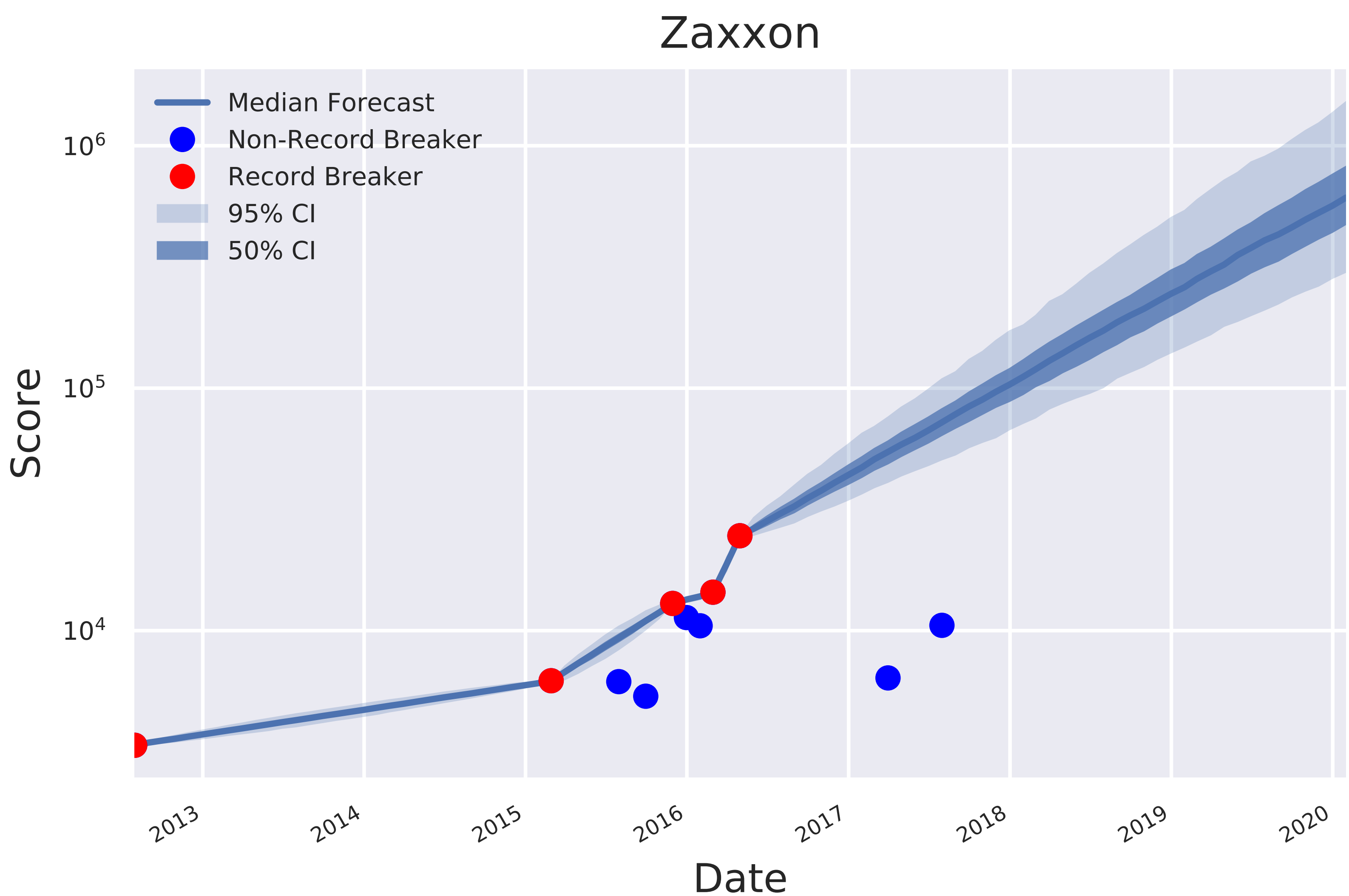
Up and Down
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
10^7
10^6
10^5
10^4
Score
2013
2014
2015
2016
2017
2018
2019
2020
Date


Venture
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
10^4
10^3
10^2
Score
2013
2014
2015
2016
2017
2018
2019
2020
Date

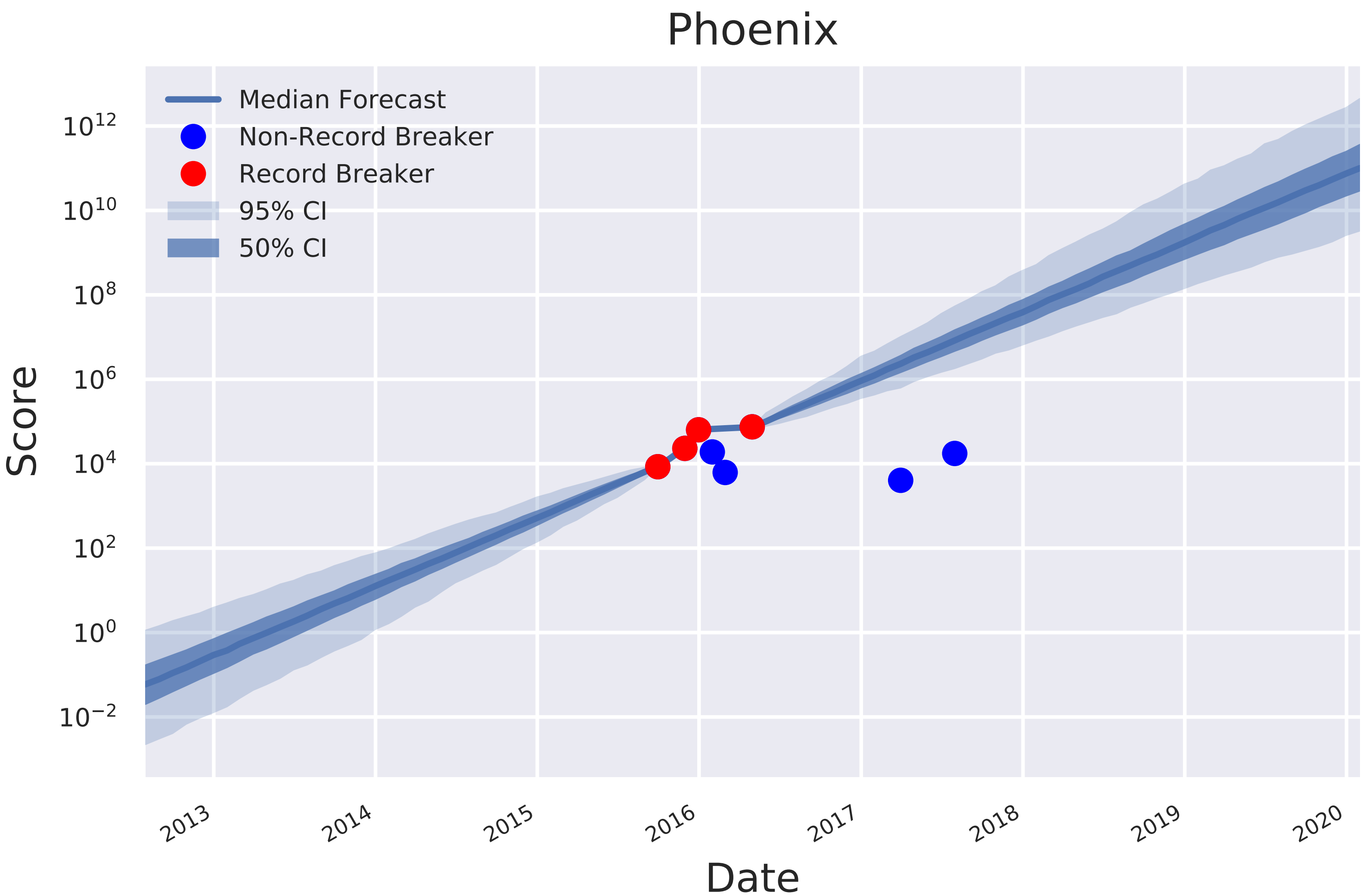


Video Pinball
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
10^7
10^6
10^5
Score
2013
2014
2015
2016
2017
2018
2019
2020
Date

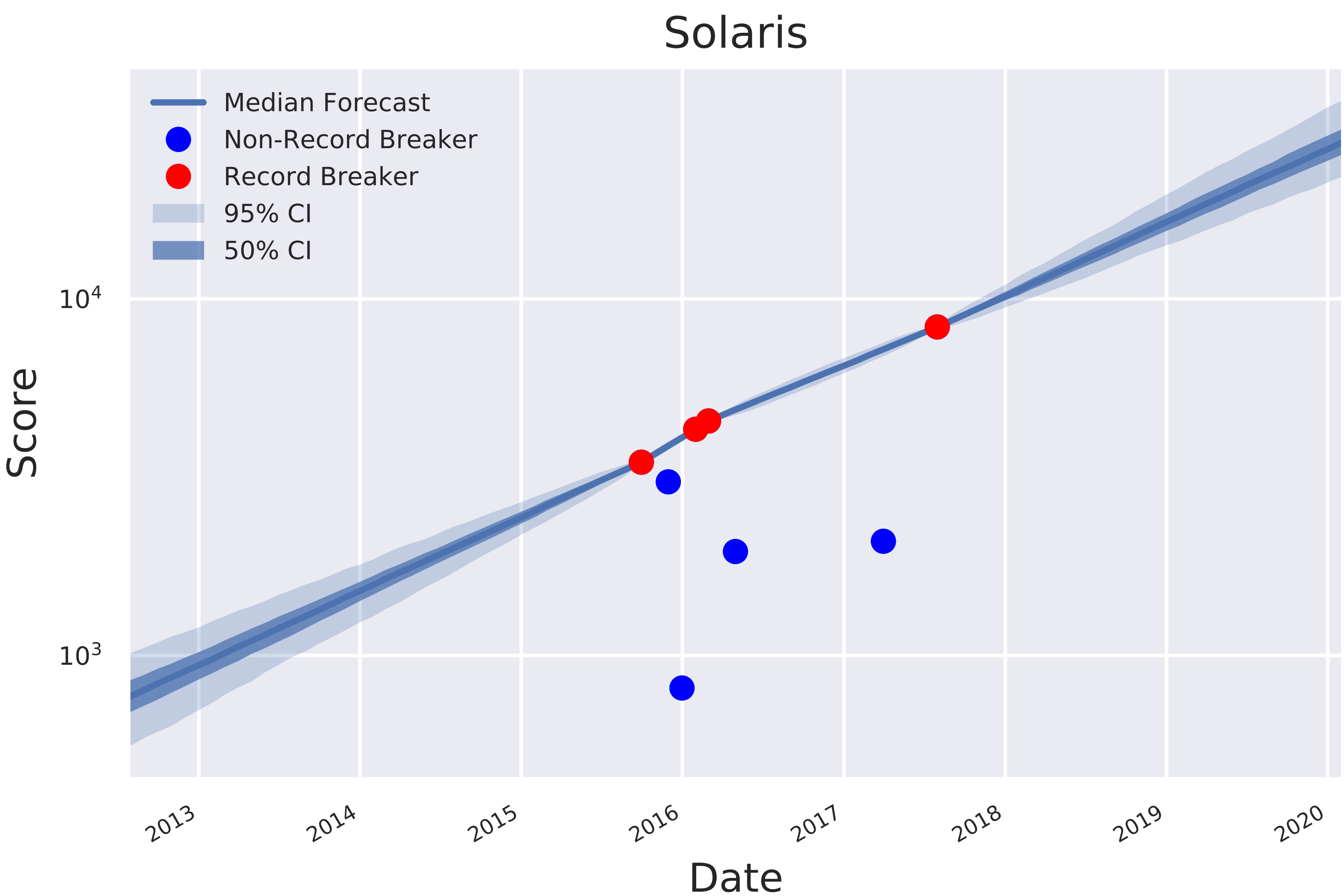

Wizard of Wor
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
10^6
10^5
10^4
10^3
Score
2013
2014
2015
2016
2017
2018
2019
2020
Date



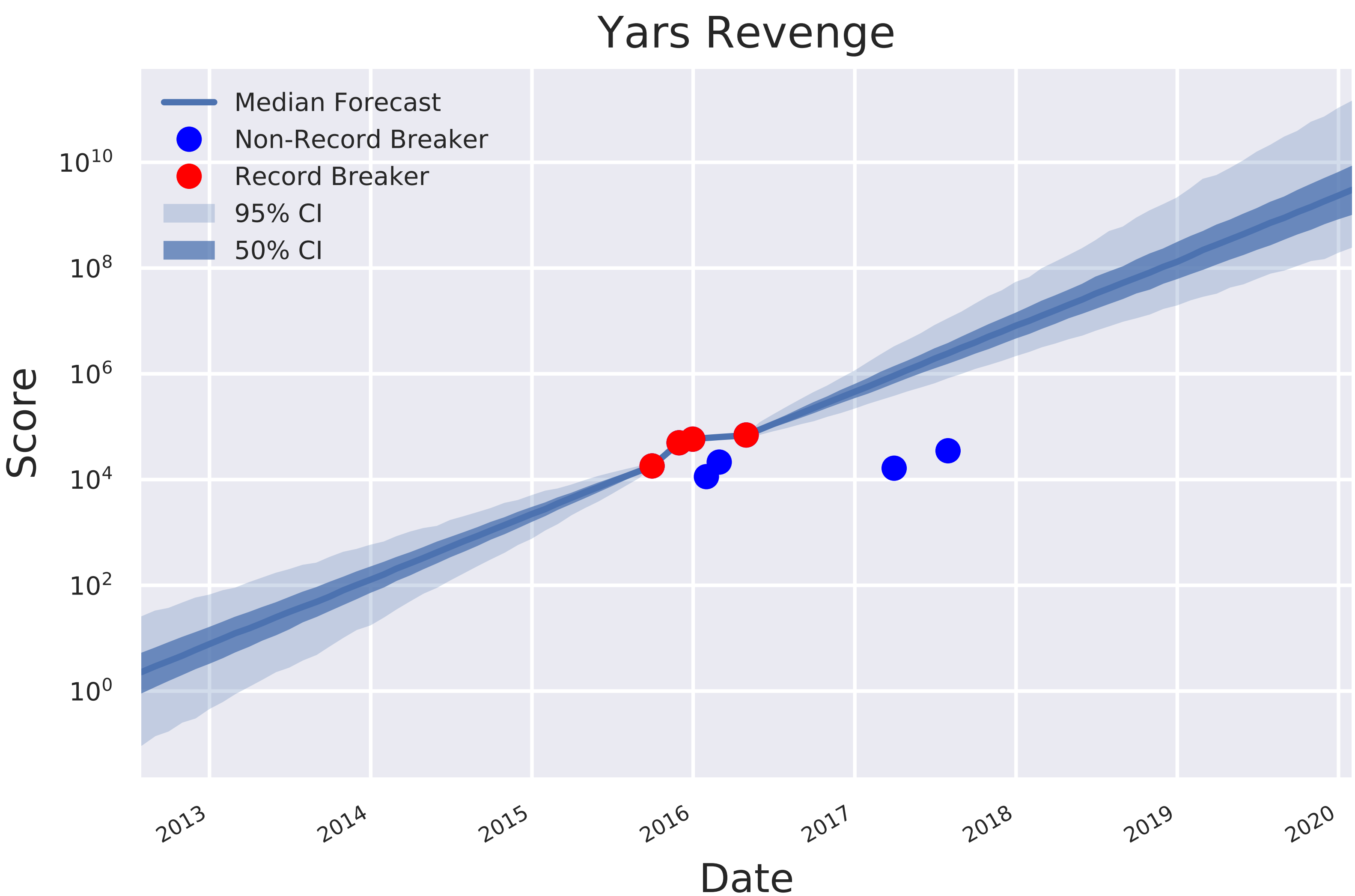
Zaxxon
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
10^6
10^5
10^4
Score
2013
2014
2015
2016
2017
2018
2019
2020
Date

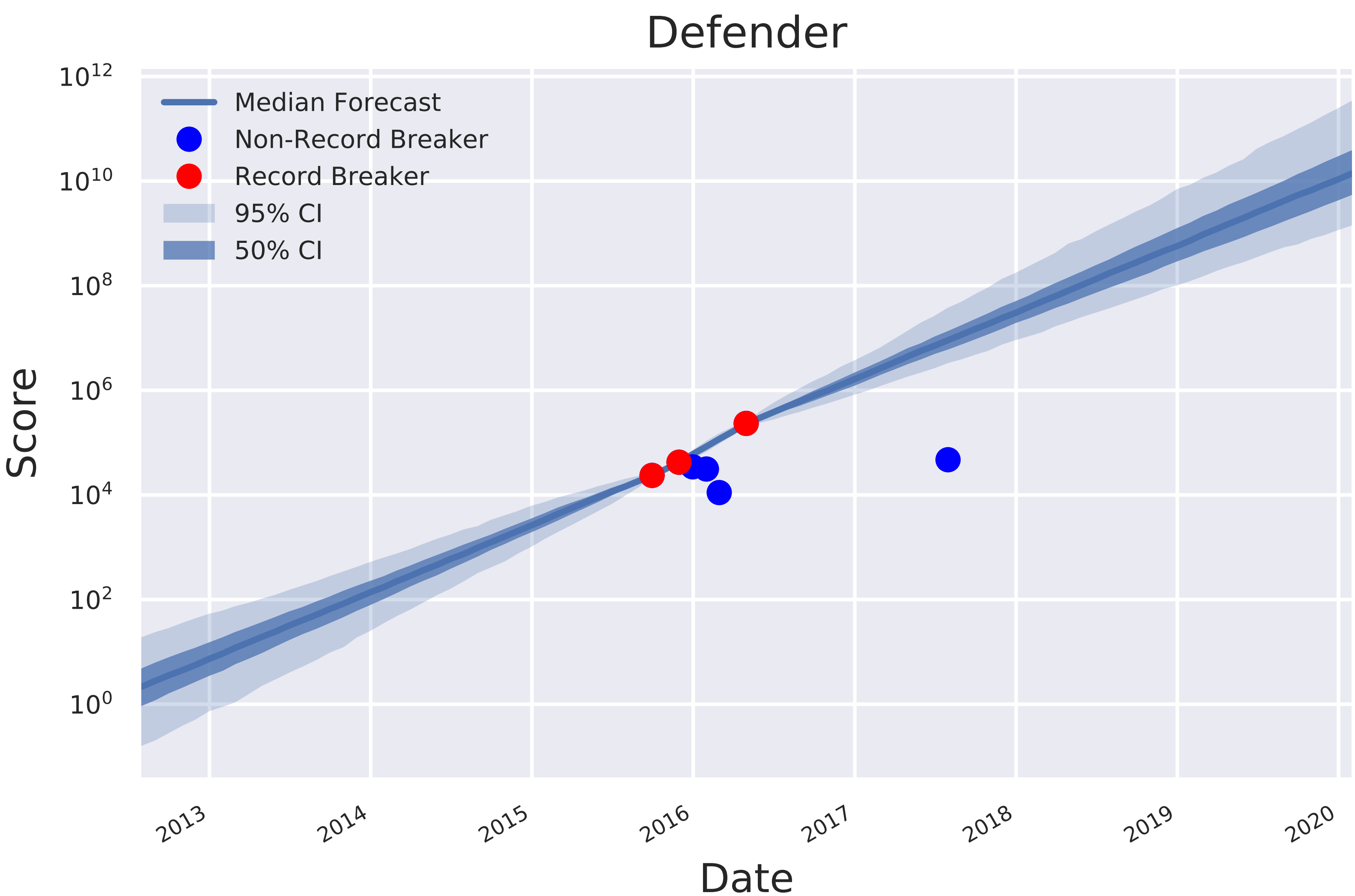


Phoenix
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
Score
10^{12}
10^{10}
10^{8}
10^{6}
10^{4}
10^{2}
10^{0}
10^{-2}
2013
2014
2015
2016
2017
2018
2019
2020
Date



Solaris
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
10^4
10^3
Score
2013
2014
2015
2016
2017
2018
2019
2020
Date


Yars Revenge
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
10^{10}
10^{8}
10^{6}
10^{4}
10^{2}
10^{0}
Score
2013
2014
2015
2016
2017
2018
2019
2020
Date


Defender
Median Forecast
Non-Record Breaker
Record Breaker
95% CI
50% CI
Score
10^{12}
10^{10}
10^{8}
10^{6}
10^{4}
10^{2}
10^{0}
2013
2014
2015
2016
2017
2018
2019
2020
Date