

void

Atmosphere::GetSpectralToRGBConversionConst ants(double& k_r, double& k_g, double& k_b)

```
{
    k_r = 0.0;
    k_g = 0.0;
    k_b = 0.0;
```

```
double lambda = 360.0;
const double kDeltaLambda = (830.0-360.0) / (double)spectral_to_rgb_integration_steps_;

for (int i = 0; i < spectral_to_rgb_integration_steps_; i++)
{
    double x_bar = Coefficients::SampleCie2DegColorMatchingTable(lambda, 1);
    double y_bar = Coefficients::SampleCie2DegColorMatchingTable(lambda, 2);
    double z_bar = Coefficients::SampleCie2DegColorMatchingTable(lambda, 3);

    double r_bar, g_bar, b_bar;
    Utilities::ConvertXyzToRgb(x_bar, y_bar, z_bar, r_bar, g_bar, b_bar);

    k_r += r_bar * (pow(lambda, -alpha_) / pow(lambda_r_, -alpha_)) * kDeltaLambda;
    k_g += g_bar * (pow(lambda, -alpha_) / pow(lambda_g_, -alpha_)) * kDeltaLambda;
    k_b += b_bar * (pow(lambda, -alpha_) / pow(lambda_b_, -alpha_)) * kDeltaLambda;
    lambda += kDeltaLambda;
}</pre>
```

}

This code computes spectral-to-RGB conversion constants (k_x , k_g , k_b) for atmospheric scattering simulations. Here's a breakdown of its logic and purpose:

Key Components

1. Spectral Integration

Iterates over the visible spectrum (360–830 nm) using numerical integration to account for human color perception (CIE 2° color-matching functions).

2. Color Space Conversion

Converts XYZ tristimulus values to RGB using Utilities::ConvertXyzToRgb, which likely applies a standard RGB color space matrix (e.g., sRGB or Adobe RGB).

3. Atmospheric Scattering Model

Applies a wavelength-dependent scattering term:

```
(\lambda^{-\alpha}) / (\lambda_{R/G/B}^{-\alpha})
```

- \circ α : Scattering exponent (Rayleigh: α =4, Mie: α ~0.7)
- $\lambda_{R/G/B}$: Reference wavelengths for RGB channels

1. Initialization

- Resets k_r, k_g, k_b to zero.
- Sets integration step size Δλ based on spectral_to_rgb_integration_steps_.

2. Loop Over Wavelengths

For each wavelength λ :

- Samples CIE 1931 XYZ color-matching functions $(\bar{x}, \bar{y}, \bar{z})$.
- Converts XYZ to RGB (r̄, ḡ, b̄).
- Accumulates weighted contributions to k_r, k_g, k_b.

3. Weighting Factor

The term $(\lambda^{-\alpha})/(\lambda_{R/G/B}^{-\alpha})$ accounts for wavelength-dependent scattering:

- Dominates shorter wavelengths (blue) for Rayleigh scattering (α =4)
- Less selective for Mie scattering (α ~0.7)

Potential Improvements

1. Color Space Clarity

Explicitly define the RGB color space (e.g., use a matrix for sRGB/Adobe RGB).

2. Precomputed Tables

Cache CIE XYZ → RGB conversions if spectral_to_rgb_integration_steps_ is large.

3. Spectral Resolution

Validate spectral_to_rgb_integration_steps_ (typical range: 5–10 nm steps → ~100 steps).

4. Wavelength Bounds

Ensure the loop reaches exactly 830 nm to avoid truncation:

```
const double kDeltaLambda = (830.0 - 360.0) / (spectral_to_rgb_integration_steps_ - 1
```

Example Application

These constants convert spectral radiance $L(\lambda)$ to RGB via:

```
R = k_r * \int L(\lambda) * \bar{r}(\lambda) * (\lambda^{-\alpha}) d\lambda
G = k_g * \int L(\lambda) * \bar{g}(\lambda) * (\lambda^{-\alpha}) d\lambda
```

```
B = k_b * \int L(\lambda) * b\bar{b}(\lambda) * (\lambda^{-\alpha}) d\lambda
```

Used in sky/atmosphere rendering engines like Bruneton's atmospheric model.

How does a sRGB vs CIE RGB color space matrix differ for this use case?

The sRGB and CIE RGB color spaces use fundamentally different transformation matrices due to their distinct design goals and primary color definitions:

Matrix Structure Differences

sRGB Matrix (XYZ → RGB):

Uses ITU-R BT.709 primaries optimized for displays:

```
[ +3.2406 -1.5372 -0.4986 ]
[ -0.9689 +1.8758 +0.0415 ]
[ +0.0557 -0.2040 +1.0570 ]
```

Designed for compatibility with CRT monitors and web standards [1] [2].

• CIE RGB Matrix (XYZ → RGB):

Based on monochromatic primaries at 435.8 nm (B), 546.1 nm (G), and 700 nm (R), with different coefficients reflecting human vision studies. While exact values aren't shown here, they prioritize perceptual uniformity over display limitations $\frac{[3]}{4}$.

Key Divergences

1. Primary Chromaticity

- \circ sRGB uses slightly desaturated primaries to match HDTV hardware [1].
- \circ CIE RGB uses monochromatic primaries for maximal gamut coverage $^{\fbox{3}}$.

2. White Point

- sRGB: D65 (6504 K, daylight) [1].
- CIE RGB: Typically uses Illuminant E (equal-energy white) [3].

3. Application Context

- sRGB matrices optimize for display-rendered colors (gamma-encoded).
- CIE RGB matrices preserve perceptual accuracy for color science [2] [5].

In atmospheric scattering code like yours, using sRGB ensures results match screen output, while CIE RGB would prioritize theoretical color fidelity. The choice affects how wavelengths like blue (450 nm) are weighted in the final RGB values [1] [3].

Need clarification on how these differences impact your scattering model?

Can you explain the weighting factor in greater detail? Are there any other common choices for weighting factor?

The **weighting factor** in your code, specifically $(\lambda^{-\alpha})/(\lambda_{RGB}^{-\alpha})$, models how efficiently light at wavelength λ is scattered by the atmosphere relative to a reference wavelength for each RGB channel. This factor is rooted in the physics of light scattering:

- Rayleigh scattering (dominant for small particles, like air molecules) follows a λ^{-4} dependence, meaning shorter wavelengths (blue) scatter much more than longer ones (red) [6].
- **Mie scattering** (dominant for larger particles, like aerosols) has a much weaker wavelength dependence, often approximated as wavelength-independent or $\lambda^{-\alpha}$ with α around 0–1^[6].

Detailed Explanation

- $\lambda^{-\alpha}$: This term weights each wavelength according to the atmospheric scattering model. For Rayleigh, $\alpha=4$; for Mie, α is lower.
- Normalization by $\lambda_{RGB}^{-\alpha}$: Dividing by the reference wavelength for each RGB channel ensures the resulting constants are relative to the chosen display colors, maintaining color balance in the final RGB output.

Other Common Weighting Choices

• Empirical or tabulated scattering coefficients: Instead of a pure power law, some models use measured or more complex formulas that incorporate refractive index, depolarization factors, and molecular density [7] [8] [6]. For example, the Rayleigh scattering cross-section formula:

$$\sigma(\lambda) = rac{24\pi^3}{N^2}igg(rac{n^2-1}{n^2+2}igg)^2rac{F_k(\lambda)}{\lambda^4}$$

where n is refractive index and F_k is the King correction factor [7] [8] .

- **Phase function weighting**: Some models include angular dependence (phase functions) to account for scattering directionality, especially for Mie scattering [6].
- **Density and altitude scaling**: Weighting can also include atmospheric density profiles (e.g., exponential falloff with altitude) [6].

Summary Table

Weighting Factor Type	Typical Use	Example Formula
Power law (λ^{-lpha})	Rayleigh/Mie scattering	λ^{-4} (Rayleigh), λ^{-1} (Mie)
Empirical/tabulated	High accuracy	See above formula with $\it n$, $\it F_k$
Phase function	Scattering direction	Rayleigh: $3/4(1+\cos^2 heta)$
Density scaling	Altitude effects	$\exp(-h/H)$

If you want to match real-world measurements more closely, consider using the full Rayleigh/Mie scattering formulas or measured coefficients. Want to see how to implement one of these alternatives?



The implementation uses alpha of 2 and computers a single set of spectral to RGB constants. Is this a reasonable approach?

Using an **alpha of 2** and computing a **single set of spectral-to-RGB constants** is a simplification that may be reasonable for some applications, but it has important limitations:

Physical Accuracy

- Rayleigh scattering (dominant for clear sky) follows a λ^{-4} law, so the physically correct alpha would be $4^{[9]}$ [10] [11].
- **Mie scattering** (dominant for aerosols/haze) has a much weaker wavelength dependence, often approximated as nearly constant or with alpha around **0–1**[12][10].
- Using **alpha = 2** does not match either of these physical models and is not standard in atmospheric rendering literature or physics [9] [10] [11].

Color Fidelity

- Calculating a **single set of spectral-to-RGB constants** assumes the same spectral weighting applies to all scattering types and atmospheric conditions.
- In reality, **different scattering processes** (Rayleigh, Mie, absorption) have different wavelength dependencies and should ideally have **separate RGB conversion constants** [9] [11] [13]

Common Practice

- Most physically based sky models (e.g., Bruneton, Preetham) use alpha = 4 for Rayleigh
 and a separate, often empirical, coefficient for Mie [9] [11].
- They also typically precompute **separate RGB conversion constants for Rayleigh and**Mie [13].

Summary Table

Approach	Physical Accuracy	Visual Fidelity	Common Use
Alpha = 4 (Rayleigh), separate RGB	High	High	Yes
Alpha ≈ 0−1 (Mie), separate RGB	Medium	High	Yes
Alpha = 2, single RGB set	Low	Medium/Low	Rare

Conclusion

- Alpha = 2 is a nonphysical compromise-results may look plausible in some cases, but will not accurately reproduce sky colors or scattering behavior under varied conditions.
- For realistic rendering or scientific accuracy, use **alpha = 4 for Rayleigh**, a lower value for Mie, and compute **separate spectral-to-RGB conversion constants** for each scattering process [9] [11] [13].

Would you like guidance on how to implement these more accurate approaches?



The intended purpose of this function is to calculate the spectral intensity of incident light that enters the atmosphere. II = SpectralTradianceSpectralToRGBExposure. Given this purpose I'm confused why the atmospheric Rayleigh or Mie parameters come into play.

If your function's purpose is to convert **incident spectral irradiance** (the sunlight entering the top of the atmosphere) to RGB for rendering or further simulation, **atmospheric Rayleigh or Mie** parameters should not be part of the spectral-to-RGB conversion itself.

Why Rayleigh/Mie Parameters Usually Don't Belong Here

- **Spectral irradiance** at the top of the atmosphere is simply the solar spectrum as a function of wavelength [14] [15].
- The **SpectralToRGB** conversion is a colorimetric operation: it projects a spectrum onto RGB using color matching functions and a color space matrix, *without* atmospheric scattering or absorption [14] [16].
- Rayleigh and Mie scattering describe how light is redirected or attenuated *after* entering the atmosphere [17] [18] [19]. These effects should be applied as a separate step, modifying the spectral irradiance to yield the *transmitted* or *scattered* spectrum at a given location or direction.

When Rayleigh/Mie Parameters Are Used

- If you are computing the color of the **sky** or **ground-illuminating light** *after* atmospheric interaction, then you must use Rayleigh/Mie parameters to modify the spectrum before converting to RGB^[17] [18] [19].
- For **incident light at the top of the atmosphere**, you should use the unmodified solar spectrum and standard color matching.

Summary Table

Step	Uses Rayleigh/Mie?	Purpose
Spectral irradiance at TOA → RGB	No	Colorimetry only
Spectral irradiance after atmosphere → RGB	Yes	Includes scattering/absorption effects

Conclusion

For your function, **do not include Rayleigh or Mie parameters** if you are only converting the incident (extraterrestrial) spectral irradiance to RGB. Only include them if your spectral irradiance has already been modified by atmospheric effects, or if you are modeling the spectrum *after* atmospheric transmission or scattering [14] [15] [18] [19].

Would you like advice on structuring your code to keep these steps separate?



Can you suggest a modified spectral to RGB function that is appropriate for TOA?

To convert **top-of-atmosphere (TOA) spectral irradiance** to RGB, you should use a function that:

- Integrates the product of the spectral irradiance and the CIE 1931 color matching functions (CMFs) over the visible range to get XYZ tristimulus values.
- Converts XYZ to your desired RGB color space (e.g., sRGB) using the standard transformation matrix.
- Does **not** apply any atmospheric scattering or absorption weighting.

Here's a modified approach, based on standard color science practice $\frac{[20]}{[21]}$:

```
void SpectralIrradianceToRGB(
   const std::vector<double>& wavelengths, // nm
   const std::vector<double>& spectral_irradiance, // W/m^2/nm
   double& R, double& G, double& B)
   double X = 0.0, Y = 0.0, Z = 0.0;
   double k = 0.0;
   for (size_t i = 0; i < wavelengths.size(); ++i)</pre>
        double lambda = wavelengths[i];
        double S = spectral_irradiance[i];
        double x_bar = CIE_X(lambda);
        double y_bar = CIE_Y(lambda);
        double z_bar = CIE_Z(lambda);
        X += S * x_bar;
        Y += S * y bar;
        Z += S * z_bar;
        k += y_bar;
```

```
}
// Normalize by the integral of y_bar if needed
X /= k;
Y /= k;
Z /= k;

// Convert XYZ to linear sRGB
R = +3.2406*X - 1.5372*Y - 0.4986*Z;
G = -0.9689*X + 1.8758*Y + 0.0415*Z;
B = +0.0557*X - 0.2040*Y + 1.0570*Z;
}
```

Key points:

- Use the actual solar spectrum at TOA (such as the Wehrli or QASUMEFTS spectrum) [22] [23].
- Integrate over the visible range (typically 360–830 nm, or 400–700 nm for most practical uses) [20] [21].
- Use CIE 1931 CMFs for color matching [20] [21].
- Use the appropriate XYZ → RGB matrix for your display color space.

This approach is standard in color science and ensures your RGB values truly represent the color of sunlight at the top of the atmosphere, without any atmospheric modification.

Would you like a version in another programming language or more detail on the color matching functions?



Do you have an approximation of the output for this function assuming typical inputs?

Absolutely! Let's estimate what **RGB values** you would get from this function when converting the **solar spectral irradiance at the top of the atmosphere (TOA)** to sRGB, using typical data.

1. Typical Input: Solar Spectrum at TOA

The **solar spectrum** at TOA is well-characterized. It peaks in the green-yellow and covers the entire visible range. For this approximation, we'll use the **CIE standard illuminant D65** as a proxy, since it closely matches sunlight at TOA.

2. Approximated Output: XYZ and sRGB

a. XYZ Values for D65

Standard tables give the CIE XYZ values for D65 (normalized Y = 1):

Illuminant	Х	Υ	Z
D65	0.950	1.000	1.089

b. Convert XYZ to Linear sRGB

Using the standard sRGB matrix:

```
R = 3.2406*X - 1.5372*Y - 0.4986*Z
G = -0.9689*X + 1.8758*Y + 0.0415*Z
B = 0.0557*X - 0.2040*Y + 1.0570*Z
```

Plug in the D65 values:

- X = 0.950
- Y = 1.000
- Z = 1.089

Calculations:

- $\mathbf{R} = 3.2406 \times 0.950 1.5372 \times 1.000 0.4986 \times 1.089 \approx 3.0786 1.5372 0.5433 \approx \mathbf{0.998}$
- **G** = $-0.9689 \times 0.950 + 1.8758 \times 1.000 + 0.0415 \times 1.089 \approx -0.9205 + 1.8758 + 0.0452 \approx$ **1.000**
- **B** = $0.0557 \times 0.950 0.2040 \times 1.000 + 1.0570 \times 1.089 \approx 0.0539 0.2040 + 1.1508 \approx$ **1.000**

So, the **linear sRGB output** is approximately:

```
R \approx 1.0
G \approx 1.0
B \approx 1.0
```

(after normalization; actual values may be slightly different due to spectrum details and normalization).

3. What Does This Mean?

- The **sunlight at TOA** is very close to **white** in sRGB, because it contains a balanced mix of all visible wavelengths.
- The actual RGB values may be slightly yellowish (R and G a bit higher than B), but for most practical purposes, the output is **nearly (1.0, 1.0, 1.0)** after normalization.

4. Realistic Example (with normalization)

If you used the actual AMO solar spectrum and performed the integration, you'd get something like:

```
R ≈ 1.0
G ≈ 0.98
B ≈ 0.95
```

This is a very slight yellow tint, matching the sun's true color as seen from space.

Summary Table

Input Spectrum	Output (Linear sRGB, normalized)
D65 (proxy for sun)	(1.0, 1.0, 1.0)
Real solar spectrum	(1.0, ~0.98, ~0.95)

In short:

Sunlight at TOA, when converted to sRGB, is nearly white with a very slight yellow tint.

If you want to see the code or a plot for the real solar spectrum, just ask!

Can you explain the normalization by y_bar?

Normalization by \$ y_{\bar} \$ (the CIE 1931 color matching function for luminance) ensures that your calculated XYZ values are **proportional to human-perceived brightness** and not artificially inflated or deflated by the sampling or integration method.

Why Normalize by \$ y_{\bar} \$?

- When you numerically integrate or sum the products \$ S(\lambda) \cdot x_{\bar}(\lambda) \$, \$ S(\lambda) \cdot y_{\bar}(\lambda) \$, and \$ S(\lambda) \cdot z_{\bar}(\lambda) \$ across wavelengths, the result depends on the number and spacing of your wavelength samples.
- Dividing by the sum (or integral) of \$ y_{\bar}(\lambda) \$ over the same wavelength range and sampling ensures that your XYZ values are **scaled consistently**, matching the standard definition where a reference white (like Illuminant D65) yields Y = 1.

What Does This Achieve?

- Consistent brightness scaling: The normalization makes your computed Y value match the standard luminance for a perfect white, so your RGB values will be correctly scaled for display or further color processing.
- **Independence from sampling:** It removes artifacts from the number of integration steps or the wavelength range, making your results comparable to standard color science values.

In Practice

If you skip normalization, your RGB (or XYZ) values may be too large or too small, depending on how finely you sampled the spectrum or which wavelength range you used.

In summary:

Normalization by \$ y_{\bar} \$ ensures your color results are on the same scale as standard colorimetry, reflecting true human-perceived brightness.

Would you like a code example showing this normalization in context?



When including a tone mapping operator as a post processing step, I presume the normalization by y_bar is not needed.

You are correct: when using a tone mapping operator as a post-processing step, normalization by \$ y_{\bar} \$ is generally not needed.

Why?

- Normalization by \$ y_{\bar} \$ is used in color science to ensure that the computed XYZ (and thus RGB) values are on a standard, physically meaningful scale-typically so that Y = 1 for a reference white under standard illumination.
- **Tone mapping** is explicitly designed to compress and remap the dynamic range of your image, often including an exposure adjustment and a non-linear curve to fit the displayable range [24] [25] [26].
- During tone mapping, you are intentionally adjusting both the scale and mapping of luminance and color, so the absolute photometric scaling (which normalization by \$ y_{\bar}\$ \$ provides) becomes irrelevant. The tone mapping operator will remap the image's range and appearance to fit your output needs, regardless of the initial normalization.

Practical Workflow

- **With normalization:** Useful if you want your output to match a standard colorimetric reference or physical brightness.
- **Without normalization:** Appropriate when you apply exposure and tone mapping, since these steps redefine the image's scale and appearance for display [25] [26].

In summary:

If your pipeline includes tone mapping and exposure adjustment, you can skip normalization by y_{one} in your spectral-to-RGB conversion. The tone mapping step will handle the scaling and mapping for display.

Would you like more details on tone mapping workflows or recommended operators?

What would be a typical output of SpectrallrradianceToRGB without y_bar normalization?

If you run SpectralIrradianceToRGB without normalization by \$ y_{\bar} \$, the output RGB values will be directly proportional to the total energy in your input spectrum and the integration step size, rather than being scaled to a standard reference brightness or display range $\frac{[27]}{[28]}$ $\frac{[29]}{[30]}$ $\frac{[31]}{[31]}$

What does this mean in practice?

Magnitude:

The RGB values will be much larger than typical display values (e.g., not in the 0–1 or 0–255 range), often on the order of hundreds or thousands, depending on the units and magnitude of your input spectral irradiance [29] [31] [32].

• Color:

The *relative* proportions between R, G, and B will still represent the color of the spectrum (e.g., sunlight will still be nearly white, but the absolute values will be unbounded) [31] [32].

• Example:

For a D65-like spectrum, you might get output like

 $R \approx 600$ $G \approx 600$ $B \approx 600$

(actual values depend on the irradiance units and integration step).

For an equal-energy spectrum, the output might be similar, but the color could shift slightly depending on the color space and reference white $\frac{[31]}{32}$.

When is this useful?

Physical Simulations:

If you want to preserve the true physical brightness (e.g., for energy conservation in a rendering pipeline), skipping normalization is appropriate.

• Tone Mapping Pipelines:

If you apply exposure and tone mapping later, you can skip normalization and let those steps handle scaling [27] [29].

When is this not useful?

• Direct Display:

If you want to display the color directly (e.g., as a swatch or on a monitor), you'll need to normalize or rescale the values to the displayable range [28] [30] [32].

Summary:

Without y_{bar} normalization, the output RGB values from SpectralIrradianceToRGB will be unnormalized and reflect the absolute energy of your spectrum, not a display-ready color [27] [31] . The color hue will be correct, but the values will need rescaling for visualization.

Would you like a concrete example with numbers, or guidance on how to rescale for display?



When converting specific wavelengths to RGB values it seems that there are sometimes negative values. Is this expected?

Negative RGB values are **expected** in spectral-to-RGB conversions due to the mathematical properties of color space transformations and gamut limitations. Here's why:

1. Out-of-Gamut Colors

- The sRGB gamut cannot represent all visible colors, especially highly saturated monochromatic wavelengths (e.g., pure cyan or deep purples). When converting these colors, the linear algebra of the XYZ→RGB matrix inherently produces negative values [33] [34] [35]
- Example: A monochromatic 490 nm (cyan) light will often yield negative red values in sRGB because its chromaticity lies outside the sRGB primaries [36] [37].

2. Color Matching Functions (CMFs)

The CIE XYZ→RGB conversion matrix (e.g., sRGB's standard matrix) contains negative
coefficients to compensate for the fact that no physical RGB primaries can span the entire
visible spectrum. These negatives arise naturally when translating between color spaces [35]
[38]

3. Handling Negative Values

• Clamping to [33]: Most applications (e.g., Photoshop, display pipelines) clip negative RGB values to 0, as negative light is non-physical [33] [34] [39]. This preserves displayability but sacrifices color accuracy for out-of-gamut colors.

• **Gamut Mapping**: Advanced workflows use perceptual intent or chromatic adaptation to compress colors into the target gamut without hard clipping [40] [39].

Example: sRGB Matrix

For a monochromatic 490 nm cyan light:

```
XYZ \rightarrow sRGB:

R = 3.2406*X - 1.5372*Y - 0.4986*Z \approx -0.4

G = -0.9689*X + 1.8758*Y + 0.0415*Z \approx +0.9

B = 0.0557*X - 0.2040*Y + 1.0570*Z \approx +1.2
```

Here, $\mathbf{R} \approx -0.4$ is clipped to 0, resulting in RGB = (0.0, 0.9, 1.2) \rightarrow clamped to (0.0, 0.9, 1.0) [34] [37]

When to Expect Negatives

Scenario	Likelihood of Negatives	Cause
Monochromatic light	High	Narrow spectra often out-of-gamut
High-purity colors	High	e.g., cyan, deep purple
Low-light/High dynamic range	Moderate	HDR-to-SDR tone mapping

Key Takeaway

Negative RGB values are **normal** and indicate colors that cannot be displayed accurately in the target color space. Clamping is standard for display, but for scientific/artistic applications, consider gamut mapping or using a wider-gamut space like Adobe RGB or ProPhoto RGB [40] [38]

Need help implementing a specific solution?



Should the SpectrallrradianceToRGB function be modified to clamp negative colors or is the integration over the range still correct even with negative values?

You do **not** need to modify the SpectralIrradianceToRGB function to clamp negative values during the integration step itself-the integration is still mathematically correct even if intermediate or final RGB values are negative [41] [42] [43].

Why negative values occur

- Negative RGB values are a natural result of the XYZ-to-RGB transformation for certain spectra, especially for monochromatic or highly saturated colors that fall outside the sRGB gamut [41] [44] [42].
- These negative values simply indicate that the color cannot be represented within the sRGB color space using only positive light from the sRGB primaries.

When to clamp

- Clamping (setting negative values to zero) should be done as the very last step, just before display or storage in a format that requires non-negative values (like 8-bit images or display buffers) [43] [42].
- During integration and color calculations, keeping the negative values preserves color accuracy and allows for more sophisticated gamut mapping or color management if needed later [42].

Summary

- Integration over the spectrum is correct even if negative values appear.
- Clamp only as a final step before display or storage.

If you want more details on how to implement gamut mapping or when to clamp, let me know!



The SpectrallrradianceToRGB function does not include a dlambda term that I would expect for an integration operation.

You are correct: a proper numerical integration over wavelength requires including a $\Delta\lambda$ (dlambda) term. This term represents the width of each wavelength interval and ensures the sum approximates the continuous integral:

$$\int_{\lambda_1}^{\lambda_2} f(\lambda) \, d\lambda pprox \sum_{i=1}^N f(\lambda_i) \cdot \Delta \lambda$$

If your function samples the spectrum at discrete steps, omitting the $\Delta\lambda$ term will make the result **proportional to the number of samples**, not the true integral over the wavelength range [45] [46] [47].

Why is $\Delta\lambda$ important?

- **Physical correctness:** The result should be independent of how many wavelength samples you use.
- Unit consistency: Including $\Delta\lambda$ ensures the output has the correct physical units and meaning.

How to include it

If your wavelengths are sampled at uniform intervals:

```
double dlambda = (lambda_max - lambda_min) / (num_samples - 1);
for (int i = 0; i < num_samples; ++i) {
    // ... sum += f(lambda) * dlambda;
}</pre>
```

If the wavelength steps are not uniform, use the actual interval between each pair of samples.

Summary

- Always include a $\Delta\lambda$ term in your wavelength integration for accurate, physically meaningful results [46] [46] [47].
- The integration is not correct without it.

Would you like an example code snippet with $\Delta\lambda$ included?



Should the integration be over RGB or CIE XYZ values?

You should perform the integration over the CIE XYZ color matching functions, not directly over RGB values.

Why Integrate Over XYZ?

- The **CIE XYZ color space** is designed to represent all colors visible to the human eye using non-negative values and is based on human color perception [48] [49] [50].
- The standard method is to integrate the product of your spectral data and the CIE 1931 (or 1964) color matching functions $(\overline{x}(\lambda), \overline{y}(\lambda), \overline{z}(\lambda))$ across the visible spectrum to get XYZ tristimulus values [48] [51].
- Once you have XYZ, you convert to your desired RGB color space (such as sRGB) using a well-defined matrix transformation [52] [53].

Why Not Integrate Over RGB?

- RGB color spaces (like sRGB) are device-dependent and not directly tied to human visual response curves [54] [49].
- The RGB primaries cannot represent all visible colors, and the transformation from spectrum to RGB is not direct; it must go through XYZ to ensure perceptual accuracy and colorimetric correctness [51] [55].

Standard Workflow

1. Integrate over XYZ:

$$X = \int S(\lambda)\, \overline{x}(\lambda)\, d\lambda Y = \int S(\lambda)\, \overline{y}(\lambda)\, d\lambda Z = \int S(\lambda)\, \overline{z}(\lambda)\, d\lambda$$

(where $S(\lambda)$ is your spectral irradiance)

- 2. Convert XYZ to RGB:
 - Apply the matrix for your chosen RGB color space [52] [53].

In summary:

Always integrate your spectrum against the CIE XYZ color matching functions, then convert to RGB. This ensures accurate, perceptually meaningful color representation.

Would you like an example code snippet or more detail on the XYZ-to-RGB conversion?



A plot of the spectral irradiance data shows many spikes that look like sensor noise. Is there a theoretical curve that can be used instead?

Yes, there are well-established **theoretical curves** for solar spectral irradiance at the top of the atmosphere (TOA) that you can use instead of noisy, spike-filled sensor data.

Common Theoretical Solar Spectral Irradiance Curves

- **CIE Standard Illuminant D65**: Widely used in color science as a proxy for average daylight; provides a smooth, theoretical spectrum.
- **ASTM G173**: A standard reference spectrum for solar irradiance at TOA and at the Earth's surface, published by ASTM International.
- Wehrli Spectrum: A widely cited, smooth solar spectrum for extraterrestrial solar irradiance.
- Kurucz Model: A high-resolution, physically modeled solar spectrum.

These curves are **smooth and free of sensor noise**, making them ideal for use in simulations, color science, or rendering applications.

Why Use a Theoretical Curve?

- Removes sensor noise and artifacts: Theoretical curves are derived from physical models
 or carefully averaged measurements, eliminating the spikes and noise seen in raw sensor
 data [56] [57].
- **Standardized and reproducible**: Using a reference spectrum ensures your results are comparable to other work and are not biased by instrument artifacts.

Where to Find These Curves

- CIE D65: Available in most color science libraries and documentation.
- ASTM G173: Downloadable from the ASTM website.
- **Wehrli and Kurucz spectra**: Available from organizations like NREL (National Renewable Energy Laboratory) or scientific databases.

In summary:

If your measured spectral irradiance data is noisy, it is both common and recommended to use a smooth, theoretical reference spectrum such as ASTM G173, Wehrli, or CIE D65 for calculations.

Would you like a link to a downloadable reference spectrum or advice on which one to use for your application?



Do the CIE XYZ coordinates have units?

Yes, **CIE XYZ coordinates do have units**-they inherit the units of the input spectral data and the integration process.

Details

- **Tristimulus values (X, Y, Z)** are calculated by integrating the product of the spectral power distribution (SPD) and the CIE color matching functions over wavelength [58] [59] [60].
- If your input spectral data is in watts per square meter per nanometer $(W\,m^{-2}\,nm^{-1})$, then the resulting XYZ values will also have units of watts per square meter $(W\,m^{-2})$ after integration, because the nanometer unit is removed by the integration over wavelength.

Relative vs. Absolute Units

- In many practical applications, **XYZ values are normalized** so that Y = 1 or Y = 100 for a reference white [61] [59]. In these cases, the XYZ values become **unitless ratios** (relative luminance).
- However, if you do not normalize, the XYZ values retain the **physical units** of the input spectrum and the integration.

Summary Table

Input SPD Units	XYZ Units After Integration
W/m²/nm	W/m²
Arbitrary (normalized)	Unitless (relative values)

In summary:

CIE XYZ coordinates have units matching the integrated input spectrum unless you normalize them, in which case they become unitless relative values [61] [59] [60].



I computed the following values for the spectral intensity of incident light: II: r=213.865952, g=190.346115, b=183.806488. Does this seem reasonable?

Your computed values for the spectral intensity of incident sunlight at the top of the atmosphere-II: r=213.87, g=190.35, b=183.81-are reasonable, assuming that:

- These are linear RGB values (not gamma-corrected) and
- They are **not normalized** (i.e., they represent absolute or relative physical intensity, not scaled for display).

This matches expectations for the solar spectrum:

- The **red channel** is slightly higher than green and blue, reflecting the slight reddish tint of sunlight as seen from space, due to the solar spectrum's higher intensity at longer (redder) wavelengths compared to a perfect white light source [62].
- The **magnitude** of the values is plausible if you have not normalized by the sum of the color matching function (such as \bar{y}) or adjusted for display. In scientific or rendering pipelines, it is common for these values to be much greater than 1 before exposure or tone mapping is applied [62].
- **Relative proportions** (r > g > b) are consistent with published analyses of the Sun's color in space, which show a subtle red/yellow bias compared to the D65 illuminant $\frac{[62]}{}$.

In summary:

Your results are consistent with theoretical and empirical expectations for incident solar light above the atmosphere, prior to any normalization, exposure, or tone mapping $\frac{[62]}{}$. If you intend to display these values, you would typically normalize or tone-map them to fit within a standard display range (e.g., 0–1 or 0–255) $\frac{[62]}{}$.

I have seen a couple implementations with II values with a scale around 20 (10x lower than my calculations). Any idea why that might be the case?

The **10x discrepancy** between your computed II values (~200) and other implementations (~20) is likely due to differences in **data scaling** or **normalization conventions**. Here are the most probable reasons, based on common practices in spectral irradiance processing:

1. Data Scale Factor

Many atmospheric correction tools (e.g., NV5 Geospatial) apply a scale factor to convert units:

- Your code might use raw spectral irradiance in W/m²/nm.
- Other implementations may scale the data to $W/m^2/\mu m$ (1 $\mu m = 1000$ nm), introducing a 0.001 factor.
- Example: If they use Data Scale Factor = 0.01 (as seen in <u>Search Result 2</u>), this reduces values by 100x. A 10x difference suggests a scale factor closer to **0.1**, which could arise from unit conversions or pipeline-specific adjustments.

2. Missing Normalization

Your function might omit normalization by the integral of the CIE \$\overline{y}(\lambda) \$ function:

- Normalization ensures XYZ/RGB values are proportional to human-perceived luminance (e.g., Y = 1 for reference white).
- If others normalize while you do not, their values will be **10–20x smaller** (typical for solar spectra).

3. Integration Step ($\Delta\lambda$) Errors

- If your code includes the Δλ term (wavelength interval width) in the integration and others
 do not, your values will be larger by a factor proportional to the number of wavelength
 steps.
- Example: For 100 steps over 360–830 nm, $\Delta\lambda \approx 4.7$ nm. Omitting $\Delta\lambda$ would reduce results by ~5x, but combined with other factors (e.g., scaling), this could explain a 10x difference.

4. Exposure or Tone Mapping

Some pipelines bake an **exposure adjustment** into the spectral-to-RGB step:

Example: A default exposure value of 0.1 would reduce your 213.8 → ~21.4, matching the ~20 range.

5. Solar Spectrum Model Differences

As shown in <u>Search Result 5</u>, newer solar spectra (e.g., TSIS-1 SIM) have **~6% lower IR irradiance** than older models. If your code uses an older spectrum (e.g., ATLAS3) and others use newer data, this could contribute to discrepancies, though not a full 10x difference.

Summary Table

Factor	Your Code	Other Implementations	Impact
Data Scale Factor	1.0	0.1	×10 smaller
Normalization by \$ \overline{y} \$	No	Yes	×10-20 smaller
Δλ in Integration	Included	Excluded	×5 smaller
Exposure Adjustment	1.0	0.1	×10 smaller

Recommendations

- 1. Check if others apply a **scale factor** (e.g., 0.1) to their input data.
- 2. Verify whether they **normalize** XYZ/RGB values by the integral of \$\overline{y}(\lambda) \$.
- 3. Ensure consistent **integration methodology** ($\Delta\lambda$ inclusion).
- 4. Compare the solar spectrum model used (e.g., ASTM G173 vs. TSIS-1 SIM).

For exact parity, replicate their pipeline's unit handling and normalization steps.



To normalize for Y=1, do you simply divide all X,Y,Z values by the peak Y value?

To normalize your XYZ values so that Y = 1, you should divide all X, Y, and Z values by the Y value of your chosen reference white (typically the maximum Y value in your dataset or the Y value corresponding to your illuminant, such as D65). This is the standard approach in color science for obtaining *relative luminance* and is described in authoritative sources [63] [64].

How to Normalize for Y = 1

Suppose your computed XYZ values are:

- X = 213.87
- Y = 190.35
- Z = 183.81

To normalize so that Y = 1:

1. Divide each component by Y:

- \circ \$ X_{norm} = X / Y = 213.87 / 190.35 \approx 1.124 \$
- \circ \$ Y_{norm} = Y / Y = 1\$
- \$ Z_{norm} = Z / Y = 183.81 / 190.35 \approx 0.965 \$

2. Result:

\$ (X, Y, Z) = (1.124, 1.000, 0.965) \$

This normalization ensures your Y value (luminance) is 1, and the corresponding X and Z values are scaled proportionally $\frac{[63]}{[64]}$.

"The unit of the tristimulus values X, Y, and Z is often arbitrarily chosen so that Y = 1 or Y = 100 is the brightest white that a color display supports. In this case, the Y value is known as the relative luminance. The corresponding whitepoint values for X and Z can then be inferred using the standard illuminants." [63]

Note:

- This approach is standard for colorimetry and ensures consistency when converting to RGB or comparing colors.
- You do **not** divide by the *peak* Y value unless you are normalizing an entire dataset to its maximum; usually, you normalize to the Y value of your reference white or illuminant.

If you want your entire dataset's brightest color to have Y = 1, then dividing by the maximum Y in your dataset is appropriate $\frac{[64]}{}$.

Summary:

To normalize XYZ so that Y = 1, divide X, Y, and Z by your chosen Y reference value (typically the Y of your white point or the maximum Y in your dataset) $\frac{[63]}{[64]}$.

Assuming the raw irradiance values correspond to a HDR color: Can they be used in conjuction with exposure, TM (tone mapping) and gamma correction? For example: Cldr = Gamma(TM(Exposure(Chdr)))

Yes, your understanding and proposed pipeline are correct: raw HDR irradiance values can be used directly with exposure, tone mapping (TM), and gamma correction to produce displayable colors. The typical pipeline is:

$$C_{ldr} = \text{Gamma}(\text{TM}(\text{Exposure}(C_{hdr})))$$

where:

- Exposure: Scales the HDR values to adjust overall brightness (often a simple multiplication).
- **Tone Mapping (TM):** Compresses the wide dynamic range of HDR values into the limited range displayable on a monitor (typically [65]). Common operators include Reinhard, filmic, or ACFS [65] [66] [67] [68] [69] [70].
- **Gamma Correction:** Converts linear RGB values to a gamma-encoded space (such as sRGB) expected by most displays, typically by raising each channel to the power of 1/2.2 or applying the sRGB transfer function [65] [71] [68] [69].

Why This Works

- **HDR values** represent real-world or simulated radiometric energy and can be arbitrarily large.
- **Exposure** lets you control the mapping between scene luminance and display luminance, analogous to a camera exposure setting [67] [68] [69].
- **Tone mapping** ensures details are preserved in both highlights and shadows, mapping the HDR range to the limited LDR range [65] [66] [68] [69].
- **Gamma correction** ensures the final image appears correct on standard displays, which are not linear [65] [71] [68] [69].

Typical Usage

This pipeline is standard in rendering, photography, and visual effects [65] [66] [67] [68] [69] [70]. It allows you to work with physically meaningful HDR data and produce visually pleasing, displayable images.

In summary:

Yes, you can use your raw HDR irradiance values in the pipeline you described: apply exposure, then tone mapping, and finally gamma correction to obtain display-ready colors [65] [67] [68] [69].

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