Age of Information for Intelligent Reconfigurable Surface with Presence of Error

Jin Zhang[1], Ali Maatouk[2], Peng Zou[1] and Suresh Subramaniam[1]
[1]ECE Department, George Washington University, Washington DC, 20052, USA
[2]Paris Research Center, Huawei Technologies, Boulogne-Billancourt, France
[1]{*zhangjin, pzou94, suresh*}@*gwu.edu* [2]{*ali.maatouk@huawei.com*}

***Abstract*—As the metric of Age of Information (AoI) has gained its popularity in recent years, the application of AoI concept to intelligent reconfigurable surfaces (IRS), especially in scenarios with packet transmission errors, offers a nuanced approach to optimizing networking quality of timeliness. Intelligent reconfiguration surfaces, which actively manipulate electromagnetic waves to improve signal propagation and increase coverage, become even more effective when informed by up-to-date age of information. However, packet transmission errors, inherent in wireless networks with Rayleigh fading channel, pose a challenge by potentially increasing AoI, thereby degrading the timeliness and relevance of information used to reconfigure these surfaces.**

Beginning with network fundamentals, we analyzed the probability of successful packet transmission at each time slot to derive conditional mathematical AoI expression starting from a time slot. This expression calculates conditional AoI given the known transmission success probability. Using the Markov chain method, we solved Markov states′ probabilities for each initial condition at a time slot and combined them to obtain the general unconditional average AoI. Experiments and numerical results validated this unconditional average AoI expression. The experiments also explored the optimal timing for IRS phase reconfiguration to achieve the minimum average AoI.

I. Introduction

Age of Information (AoI) [1] is a novel performance metric designed to measure the freshness of information in time-sensitive communication systems. Unlike traditional metrics such as delay or throughput, AoI quantifies the time elapsed since the most recent update was generated and successfully delivered to its destination. This makes it particularly relevant in applications requiring real-time data, such as autonomous vehicles [2] [3], industrial IoT [4], healthcare monitoring systems [5] and UAVs [6]. By focusing on the timeliness of updates, AoI provides a more accurate assessment of system performance in dynamic environments, ensuring that the transmitted information remains relevant and up-to-date for effective decision-making [7]. The integration of AoI into applications is critical for optimizing data freshness and real-time communication in dynamic wireless networks, ensuring efficient resource allocation and improving communication reliability across various fields, such as IRS in 6G systems [7] [8] [9].

An Intelligent Reconfigurable Surfaces (IRS) is an advanced technology comprising numerous passive meta-elements [10] [11] capable of dynamically altering the phase, amplitude, and polarization of reflected electromagnetic waves. By adjusting these properties, IRS enhances wireless communication by facilitating beamforming, improving signal quality, and expanding coverage [12] [13] [14], thus enabling more efficient and adaptable wireless networks. The excitation time of phase shifts in an IRS plays a critical role in determining its real-time performance.

Incorporating AoI into the IRS system's design allows for a more dynamic and responsive adjustment of the reflecting surface configurations [15], prioritizing the transmission of packets that significantly impact the system's performance. This ensures that the system adapts to changing conditions in real time, enhancing the prioritization of critical data. By quantifying the freshness of information, scheduling policies can be devised to mitigate the effects of transmission errors [16], such as through error correction techniques or by shifting the reflection phase based on the current AoI. To further investigate these dynamics, recent studies explore specific AoI-related metrics under varying system conditions. In UAV communication systems, researchers focus on the interplay of robustness and timeliness. Papers [6] [17] examine robustness and timeliness in UAV networks for cooperative lossy communications by characterizing AoI for the given outage probability. Paper [10] proposes an IRS-assisted NOMA downlink transmission design. Previous studies [6] [10] [16] [17] have either kept the error rate constant when calculating AoI or used IRS without considering real-time AoI performance indicators.

As communication frequencies increase in 5G and 6G wireless networks, signal propagation faces greater challenges in bypassing obstacles. A practical approach to enhancing signal quality involves the use of IRS, composed of meta-units that create a Rayleigh fading channel. To mitigate the adverse effects of Rayleigh fading [18] [19], periodic pilot symbols are introduced to enable accurate channel estimation [20]. Consequently, IRS-enhanced communication channels exhibit cyclic behavior, with transmission success probabilities fluctuating based on pilot reception. In this paper, we establish a mathematical relationship between average AoI and time-varying transmission probabilities, also evaluating the impact of phase-shifting delays in IRS. We derive a closed-form expression for average AoI under packet transmission loss, which facilitates the optimization of IRS phase-shifting timing to minimize long-term average AoI. Unlike prior research that assumes static transmission probabilities, this work addresses time-varying probabilities across time slots and employs the general AoI metric rather than Peak AoI [16]. For periodic transmission probabilities, a geometric series approach is applied to solve the conditional closed-form equation, which is further extended using the Markov chain method to derive an unconditional closed-form equation. Our experimental results validate both the derived expressions and the optimal IRS phase-shift scheduling.

II. System Model

We consider a time-slotted communication system with a single source and a destination. Packets arrive to the source with a probability p at each time slot. The source then transmits these packets without a buffer following the first-come, first-served (FCFS) policy with the presence of transmission errors. If a packet is not successfully transmitted, it will be dropped instead of being stored for future retransmission. As illustrated in Fig. 1, the arriving packet will be transmitted in one time slot with success probability q_k for slot k.

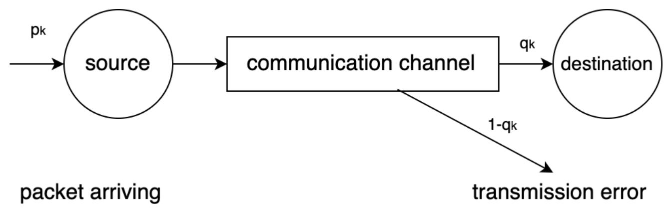

Fig. 1. Status update packets arrive at the source without buffering, and transmissions are subject to errors.

While systems with multiple buffers can accommodate more packets instead of dropping them, and their benefits have been explored in our previous work [21], we adopt the assumption of a zero-buffer system due to its proven optimality in minimizing the average AoI for systems with a single information source [22]. The periodic behavior of transmission success probabilities, denoted by $q_k = q_{k+w}$, is considered over slots with a period of w. This assumption is motivated by the Rayleigh Fading Channel with reconfiguration in IRS systems where the channel is fading over time and reconfigured every w slots. An example is analyzed in Appendix A [23] to demonstrate the feasibility of this assumption.

In our time-slotted model, we assume that the AoI increases by one at the beginning of each time slot and remains constant throughout the duration of that time slot. If an update packet arrives and is successfully delivered during time slot k, the AoI drops to one at the end of time slot k, which is equivalent to the beginning of time slot $k+1$. Based on this, and by letting $\Delta(k)$ denote the AoI at time slot k, we have

$$\Delta(k+1) = \begin{cases} 1, \text{ if } a(k) = 1 \text{ and } \ b(k) = 1, \\ \Delta(k) + 1, \quad \text{else}, \end{cases} \tag{1}$$

where $a(k)$ is an indicator of packet arrival and $b(k)$ is an indicator of transmission success at time slot k.

Fig. 2 illustrates an example of the AoI evolution. We index the packets that are delivered to the receiver by i, and we let X_i denote the number of time slots elapsed between the delivery of packets with indices i and $i+1$. In Fig. 2, the first packet is successfully transmitted at time slot t_1 and the second packet is successfully transmitted at time slot t_2; therefore $X_1 = t_2 - t_1$. Let us define Q_i as the cumulative AoI between t_i and t_{i+1}. For example, Q_1 is shown as the shaded area in Fig. 2. Therefore we have:

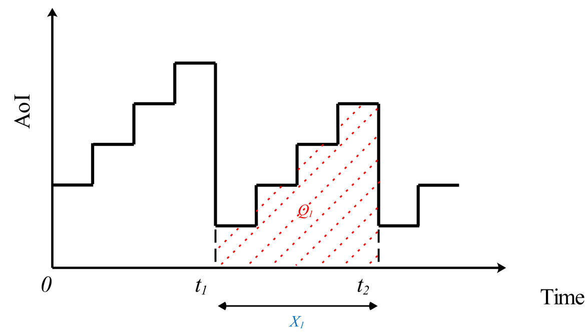

Fig. 2. Evolution of AoI over time slots.

$$Q_i = \frac{1}{2} X_i [X_i + 1] = \frac{1}{2} X_i^2 + \frac{1}{2} X_i. \tag{2}$$

The average AoI for source i can then be written as

$$\bar{\Delta} = \lim_{t\to\infty} \frac{\sum_{i=1}^{n^*(t)} Q_i}{\sum_{i=1}^{n^*(t)} X_i} = \lim_{t\to\infty} \frac{\sum_{i=1}^{n^*(t)} Q_i / n^*(t)}{\sum_{i=1}^{n^*(t)} X_i / n^*(t)},$$

where $n^*(t)$ is the index of the most recent delivered packet at time slot t. Therefore, we have

$$\bar{\Delta} = \frac{E[Q]}{E[X]} = \frac{E[X^2]}{2E[X]} + \frac{1}{2}. \tag{3}$$

Note that inter-delivery time X_i depends on the probability of success associated with the ith packet.

We can express X_i as $X_i(q_k)$ where q_k represents the probability of successful delivery of the ith packet. With the periodic behavior of q_k, there exist w cases of $X_i(q_k)$ based on different values of q_k. Therefore, conditioned on these w cases of different success probability q_k of the previous packet in the inter-delivery time, we can obtain the following:

$$E[X] = \sum_{k=1}^{w} \pi_k E[X(q_k)] \tag{4}$$

$$E[X^2] = \sum_{k=1}^{w} \pi_k E[X(q_k)^2] \tag{5}$$

where π_k is the stationary probability that a packet is successfully delivered with probability q_k.

In the next section, we show the analysis for the conditioned expectation for any success probability q_k and finally derive a closed-form expression for long-term average AoI.

III. Closed-Form Expressions Analysis

In this section, we start with the conditioned expectation of inter-deliver time $X_i(q_k)$ and show a discrete Markov chain analysis for stationary distribution that a packet is delivered with probability q_k. For simplicity, in the rest of the paper, we omit the index i for $X_i(q_k)$ and present it as $X(q_k)$. With those results, we can finally derive the closed-form expression for average AoI in this system.

A. conditioned $E[X(q_k)]$ and $E[X(q_k)^2]$

Conditioned on the event $X(q_k)$, which indicates that the previous packet was successfully delivered with probability q_k, we can derive:

$$P\{X(q_k) = 1\} = pq_{k+1},$$

which reflects the fact that in the next time slot, a packet is generated with probability p and successfully delivered with probability q_{k+1}. Therefore, when $X(q_k) = 2$, consider the event that in the first time slot, either no packet is generated or a packet is generated but its transmission fails, we have:

$$\begin{aligned} P\{X(q_k) = 2\} &= [(1-p) + p(1-q_{k+1})](pq_{k+2}) \\ &= (1-pq_{k+1})(pq_{k+2}). \end{aligned}$$

Following this approach, we present the probability expression for general case when $X(q_k) = m$ in Lemma 1.

Lemma 1 *The probability that inter-deliver time equals m slots conditioned on the first packet with success probability q_k can be expressed as follows,*

$$P\{X(q_k) = m\} = (pq_{k+m})\prod_{h=1}^{m-1}(1-pq_{h+k}) \tag{6}$$

Proof: In eq.1, the term pq_{k+m} is the probability that after m slots, a packet is generated and successfully delivered. Meantime, the product $\prod_{h=1}^{m-1}(1-pq_{h+k})$ is the probability that in each of those slots between, either no packet is generated or a packet is generated but its transmission fails. ■

Relying on Lemma 1, the closed-form expressions for the expected inter-delivery time and the expectation of its square will be presented by Lemma 2 and Lemma 3, respectively.

Lemma 2 *The expectation of inter-delivery time $E[X(q_k)]$ is given as,*

$$\begin{aligned} E[X(q_k)] = \Big(\frac{1}{1-r}\Big)\sum_{h=1}^{w}\Bigg\{\Bigg[pq_{h+k} \\ \times \prod_{j=1}^{h-1}\Big(1-pq_{j+k}\Big)\Bigg]\Bigg(h+\frac{wr}{1-r}\Bigg)\Bigg\} \end{aligned} \tag{7}$$

where $r = \prod_{j=1}^{w}(1-pq_j)$, and edge product $\prod_{j=1}^{0}(\cdot) = 1$.

Proof: The details can be found in Appendix B [23]. ■

Similarly, the expectation $E[X(q_k)^2]$ of second order moment of inter-delivery time is provided as follows.

Lemma 3 *The expectation $E[X(q_k)^2]$ of squared inter-delivery time $X(q_k)^2$ under the condition that the first packet with success probability q_k is given by,*

$$\begin{aligned} E[X^2] = \Big(\frac{1}{1-r}\Big)\sum_{h=1}^{w}\Bigg\{\Big[pq_{h+k}\times \\ \prod_{j=1}^{h-1}(1-pq_{j+k})\Big]\Bigg[h^2+\frac{2wr}{1-r}h+\frac{r(1+r)w^2}{(1-r)^2}\Bigg]\Bigg\} \end{aligned} \tag{8}$$

Proof: The details can be found in Appendix C [23]. ■

The next subsection presents a Markov Chain analysis to derive the stationary distribution π_k that a packet is delivered with probability q_k.

B. Stationary Distribution Analysis for π_k

To derive the stationary distribution of which a packet is delivered with probability q_k, we construct a discrete Markov chain with w states as shown in Fig. 3. We define the state space $\boldsymbol{S} = \{S_1, S_2, ..., S_w\}$ where a state S_k denotes that a packet is successfully delivered with probability q_k.

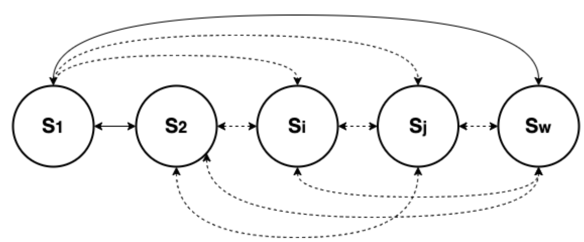

Fig. 3. Each state can either transit to the next or transition to another with Markov stationary probability.

The transition probability between any two states in this Markov chain is determined by the transmission success of packets, with each packet's transmission success being independent of the others. Therefore, state transitions within the state space $\boldsymbol{S}$ are independent of past evolution processes. We define β_{ij} as the state transition probability from S_i to S_j and its expression is presented in Lemma 4:

Lemma 4 *Any state transition probability β_{ij} from state i to state j is given by,*

$$\beta_{ij} = \begin{cases} \frac{q_j\prod_{h=i+1}^{j-1}(1-q_h)}{1-\prod_{h=1}^{w}(1-q_h)}, & i<j, \\ \frac{q_j\prod_{h=i+1}^{w}(1-q_h)\prod_{h=1}^{j-1}(1-q_h)}{1-\prod_{h=1}^{w}(1-q_h)}, & i \geq j. \end{cases} \tag{9}$$

where edge product $\prod_{h=j}^{j-1}(\cdot) = 1$.

Proof: The details can be found in Appendix D [23]. ■

As the state transitions are derived, we can obtain the stationary distribution by solving the global balance equations. The global balance equations are formulated with stationary distribution set $\boldsymbol{\pi}^T = \{\pi_1, \pi_2, \cdots, \pi_w\}$ and state transition matrix $\boldsymbol{T}$ as follow:

$$\begin{bmatrix} \pi_1 \\ \pi_2 \\ \vdots \\ \pi_w \end{bmatrix} = \begin{bmatrix} \beta_{11} & \beta_{21} & \cdots & \beta_{w1} \\ \beta_{12} & \beta_{22} & \cdots & \beta_{w2} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{1w} & \beta_{2w} & \cdots & \beta_{ww} \end{bmatrix} \begin{bmatrix} \pi_1 \\ \pi_2 \\ \vdots \\ \pi_w \end{bmatrix}, \tag{10}$$

where the matrix equation is constrained with $\sum_{k=1}^{w} \pi_k = 1$.

The solution of this equation (10) is presented in Lemma 5.

Lemma 5 *The stationary distribution π_k is given by,*

$$\pi_k = \frac{pq_k}{\sum_{h=1}^{w} pq_h} = \frac{q_k}{\sum_{h=1}^{w} q_h} \tag{11}$$

Proof: The details can be found in Appendix E [23]. ■

Since π_k is derived, we leverage them to obtain the closed-form expression for average AoI in the following Theorem:

Theorem 1 *The closed-form expression for average AoI in this error channel with periodic success probability is:*

$$\bar{\Delta} = \frac{\sum_{k=1}^{w} \left\{ \frac{q_k}{\sum_{i=1}^{w} q_i} \sum_{h=1}^{w} \Big(G(h,k) H(h,r,w) \Big) \right\}}{2 \sum_{k=1}^{w} \left\{ \frac{q_k}{\sum_{i=1}^{w} q_i} \sum_{h=1}^{w} \Big(G(h,k) B(h,r,w) \Big) \right\}} + \frac{1}{2} \tag{12}$$

where $G(h,k) = pq_{h+k} \prod_{i=1}^{h-1} (1 - pq_{i+k})$, $H(h,r,w) = h^2 + \frac{2wr}{1-r} h + \frac{r(1+r)w^2}{(1-r)^2}$, $B(h,r,w) = h + \frac{wr}{1-r}$ and $r = \prod_{i=1}^{w} (1 - pq_i)$

Proof: By substituting Eq. (7), Eq. (8), and Eq. (11) into Eq. (4) and Eq.(5), we have:

$$E[X] = \sum_{k=1}^{w} \left\{ \frac{q_k}{\sum_{i=1}^{w} q_i} \left(\frac{1}{1-r} \right) \times \sum_{h=1}^{w} \Big(G(h,k) B(h,r,w) \Big) \right\} \tag{13}$$

$$E[X^2] = \sum_{k=1}^{w} \left\{ \frac{q_k}{\sum_{i=1}^{w} q_i} \left(\frac{1}{1-r} \right) \times \sum_{h=1}^{w} \Big(G(h,k) H(h,r,w) \Big) \right\} \tag{14}$$

Next by substituting Eq.(13) and Eq.(14) into Eq.(3), we have:

$$\bar{\Delta} = \frac{\sum_{k=1}^{w} \left\{ \frac{q_k}{\sum_{i=1}^{w} q_i} \sum_{h=1}^{w} \Big(G(h,k) H(h,r,w) \Big) \right\}}{2 \sum_{k=1}^{w} \left\{ \frac{q_k}{\sum_{i=1}^{w} q_i} \sum_{h=1}^{w} \Big(G(h,k) B(h,r,w) \Big) \right\}} + \frac{1}{2}$$

■

In the next subsection, we consider the reconfiguration in the system which is assumed to be few time slots with zero successful probability at each end of the period.

C. AoI Analysi for IRS with Reconfiguration model

Assume that the reconfiguration of IRS is not negligible and requires u time slots per period. During these reconfiguration intervals, transmission is halted, and the transmission success probability becomes zero. Consequently, Theorem 2 provides a closed-form expression for average AoI.

Theorem 2 *Given a period w of transmission success probability and additional reconfiguration time u during which the transmission probability is zero, the average AoI is given by,*

$$\bar{\Delta} = \frac{\sum_{k=1}^{w} \left\{ \frac{q_k}{\sum_{i=1}^{w} q_i} \sum_{h=1}^{w} \Big(G(h,k) H'(h,r,w,u) \Big) \right\}}{2 \sum_{k=1}^{w} \left\{ \frac{q_k}{\sum_{i=1}^{w} q_i} \sum_{h=1}^{w} \Big(G(h,k) B'(h,r,w,u) \Big) \right\}} + \frac{1}{2} \tag{15}$$

where $G(h,k) = pq_{h+k} \prod_{i=1}^{h-1} (1 - pq_{i+k})$, $H'(h,r,w,u) = h^2 + \frac{2(w+u)r}{1-r} h + \frac{r(1+r)(w+u)^2}{(1-r)^2}$, $B'(h,r,w,u) = h + \frac{(w+u)r}{1-r}$ and $r = \prod_{i=1}^{w} (1 - pq_i)$

Proof: The details can be found in Appendix F [23]. ■

IV. NUMERICAL RESULTS

Experiments validate that calculated results from the closed-form expression of conditional average AoI for initial condition q_{k^*} match the corresponding simulation results.

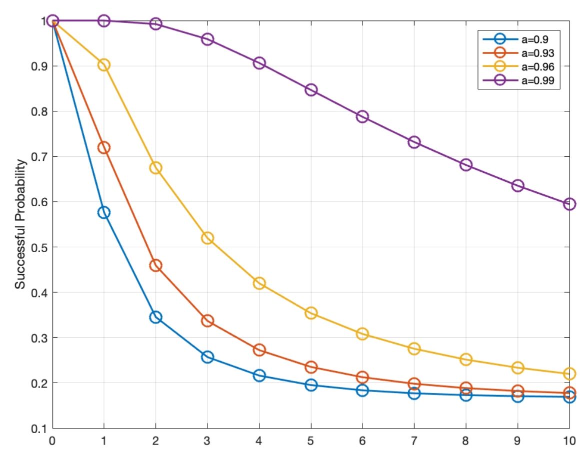

Fig. 4. *The simulated periodic transmission success probability with cycle w.*

Initially, adhering to the theoretical model of Rayleigh fading wireless networks outlined in Appendix A, a simulation yielded the transmission success probability for a Rayleigh fading channel a pilot symbol period of $1200\mu s$ and $w = 10$

slots, as shown in Fig. 4, where each slot is $120\mu s$. To compute the theoretical results, the channel transmission success probabilities were substituted into the closed-form expression for conditional average AoI. For simulation results, using the same periodic transmission success probabilities, a dataset was generated by simulating the success or failure of one million packet transmissions over the communication channel. From this dataset, all of cumulative AoIs $\{Q_l^{(s)}\}$ for the initial condition q_s was extracted, summed, and averaged to obtain the simulation results.

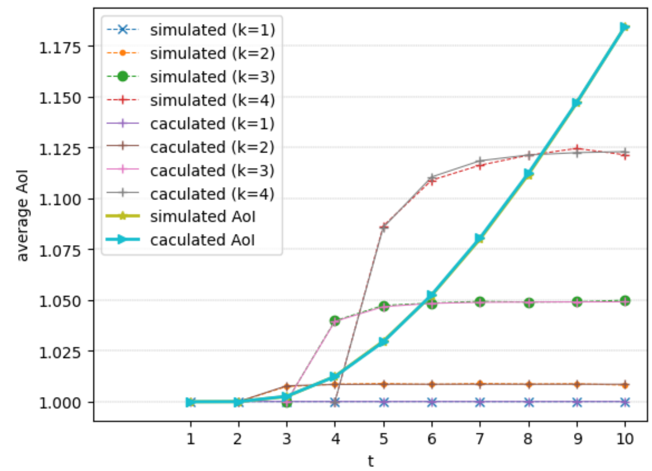

Fig. 5. *Simulated and calculated conditional or unconditional average AoIs.*

Fig. 5 presents simulated and calculated conditional average AoIs for distinct initial conditions at slot $s = 1, 2, 3, 4$ within a period of $w = 10$. The simulated and calculated unconditional average AoIs are composed of the conditional average AoIs with Markov chains. The dashed lines with markers represent the simulation results for distinct initial conditions aligning with closed-form expression calculations in solid lines, while the starred thick solid lines depict the unconditional simulation results matching the theoretical calculations.

With the transmission success probability cycle expanded by the delay of IRS phase shifting, resulting in a new cycle length of $(w + u)$, we set the phase adjustment delay of a 1600-element IRS to $1600\mu s$ and $u = 0, 1, 5, 8, 13$ slots. To obtain the results from the closed-form expression, the same channel transmission probabilities used for validating both conditional and unconditional average AoI expressions, along with zero transmission probability during the delay period, were substituted into the closed-form expression for average AoI with IRS phase shifting delay. Using a method analogous to that for simulating the unconditional average AoI, as demonstrated in Fig. 6, the simulation incorporated extended probability cycle with added delay, yielding simulation results with delay. The results from the theoretical expression align with the simulation outcomes.

V. Conclusion

This study investigated a time-slotted status update system where the probability of transmission success is considered.

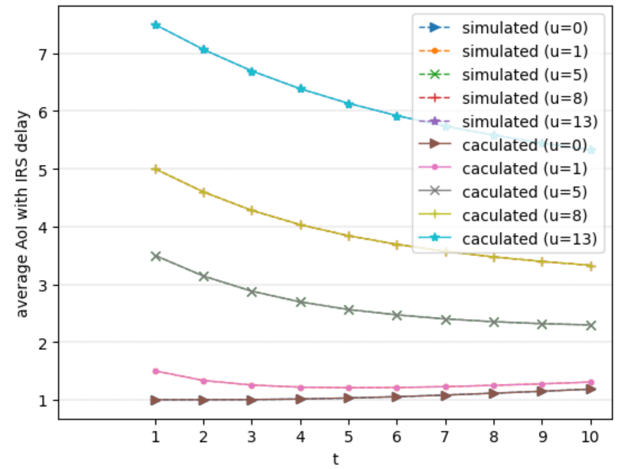

Fig. 6. *Simulated and calculated average AoIs with delay of shifting IRS phase.*

The impact of packet losses on information freshness was assessed through the analysis of both conditional and unconditional AoI. The periodic characteristics of transmission success probability within an IRS-assisted wireless network were also examined. Future research directions include the exploration of additional queuing models with finite buffers. Moreover, the optimization of AoI performance through different numbers of re-transmissions in both single-source and multi-source scenarios will be further investigated.

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Appendix A
Periodic probability of transmission success

The time-varying nature of transmission success probabilities is analyzed across time slots, a behavior commonly observed in Rayleigh fading channels with channel re-estimation using pilot symbols. The periodicity of transmission success has been derived for IRS-enhanced networks. In this context, the received signal in a Rayleigh fading channel can be mathematically represented as follows,

$$y(t) = h(t)x(t) + n(t) \tag{16}$$

where $t = 1, 2, 3, \cdots$, $h(t)$ and $n(t)$ are independent zero mean circular complex Gaussian random variables with variances σ_h^2 and σ_n^2, respectively. For $h(0)$ given, the fading coefficient $h(t)$ in Rayleigh fading channel is modeled as,

$$\begin{aligned} h(t) &= \alpha h(t-1) + z(t) \\ &= \alpha^t h(0) + \sum_{k=1}^{t} \alpha^{t-k} z(t) \end{aligned} \tag{17}$$

where $z(t)$'s are i.i.d. circular complex Gaussian variables with zero mean and variance equal to $Var[z(t)] = (1-\alpha^2)\sigma_h^2$. Assume that the original signal is $x_0(t)$, then we have the transmitted signal as $x(t) = \frac{h(0)^*}{\|h(0)\|^2} x_0(t)$. Therefore, the SNR(signal-to-noise ratio) is given as,

$$\begin{aligned} \gamma &= \frac{\|h(t)x(t)\|^2}{n(t)^2} = \frac{\|h(t)x(t)\|^2}{\|n(t)\|^2} \\ &= \frac{\left\| \left(\alpha^t h(0) + \sum_{k=1}^{t} \alpha^{t-k} z(t) \right) \frac{h(0)^*}{\|h(0)\|^2} x_0(t) \right\|^2}{\|n(t)\|^2} \\ &= \left\| \frac{h(0)^*}{\|h(0)\|^2} x_0(t) \right\|^2 \frac{\left\| \left(\alpha^t h(0) + \sum_{k=1}^{t} \alpha^{t-k} z(t) \right) \right\|^2}{\|n(t)\|^2} \\ &= \frac{\|x_0(t)\|^2}{\|h(0)\|^2 \|n(t)\|^2} \left\| \left(\alpha^t h(0) + \sum_{k=1}^{t} \alpha^{t-k} z(t) \right) \right\|^2 \\ &= \frac{b}{\|n(t)\|^2} \left\| \alpha^t h(0) + \sum_{k=1}^{t} \alpha^{t-k} z(t) \right\|^2 \end{aligned} \tag{18}$$

where $b = \frac{\|x_0(t)\|^2}{\|h(0)\|^2}$.

Note that $z(t) \sim \mathcal{CN}(\mu, \sigma_z^2)$, therefore, we have $\alpha^{t-k} z(t) \sim \mathcal{CN}(0, \alpha^{2(t-k)} \sigma_z^2)$ and $\sum_{k=1}^{t} \alpha^{t-k} z(t) \sim \mathcal{CN}(0, \sum_{k=1}^{t} \alpha^{2(t-k)} \sigma_z^2)$. Now, let $Z = \alpha^t h(0) + \sum_{k=1}^{t} \alpha^{t-k} z(t)$, we can have,

$$Z \sim \mathcal{CN}\left(\alpha^t h(0), \frac{1-\alpha^{2t}}{1-\alpha^2} \sigma_z^2 \right) \tag{19}$$

$\|Z\|^2 \sim$ Non-centered Chi-Squared $(2, \lambda)$ with $\lambda = \frac{\|\alpha^t h(0)\|^2}{\sigma_Z^2}$, where $\sigma_Z^2 = \frac{1-\alpha^{2t}}{1-\alpha^2} \sigma_z^2$.

Now let $\hat{Z} = \|Z\|^2$, $\hat{N} = \|n(t)\|^2$, we have $\hat{N} \sim exp(\frac{1}{\sigma_n^2})$. Consider a threshold for SNR as γ_{th} that when γ is less than γ_{th}, the transmission failed. Therefore, the probability of failure is the probability if SNR is less than the threshold. So we have,

$$\begin{aligned} P(\gamma \le \gamma_{th} | h(0)) =& P\left(\frac{b\hat{Z}}{\hat{N}} \le \gamma_{th} \middle| h(0) \right) \\ =& \int_{-\infty}^{+\infty} \int_{\frac{b\hat{Z}}{\gamma_{th}}}^{+\infty} f_{\hat{Z}}(\hat{z}) f_{\hat{N}}(\hat{n}) d\hat{n} d\hat{z} \\ =& \int_{0}^{+\infty} \int_{\frac{b\hat{Z}}{\gamma_{th}}}^{+\infty} f_{\hat{Z}}(\hat{z}) \frac{1}{\sigma_n^2} e^{\frac{1}{\sigma_n^2}} d\hat{n} d\hat{z} \\ =& \int_{0}^{+\infty} f_{\hat{Z}}(\hat{z}) e^{-\frac{b\hat{Z}}{\gamma_{th}} \frac{1}{\sigma_n^2}} d\hat{z} \\ =& M_{\hat{Z}}(-\frac{b}{\gamma_{th}\sigma_n^2}) = M_{\hat{Z}}(B) \end{aligned} \tag{20}$$

where $B = -\frac{b}{\gamma_{th}\sigma_n^2}$ and $M_{\hat{Z}}(B)$ is the moment generating function of $\hat{Z}$ at B and we have,

$$P(\gamma \le \gamma_{th} | h(0)) = M_{\hat{Z}}(B) = \frac{1}{1-\sigma_Z^2} e^{\frac{\lambda B}{1-\sigma_Z^2}} \tag{21}$$

where λ and σ_Z^2 are given before. As shown in the equation (21). The probability q_k of transmission success over time slots follows exponential distribution with $\frac{\lambda B}{1-\sigma_Z^2}$ in a Rayleigh fading channel in wireless networks.

Appendix B
Expected Inter-Delivery Time in Closed-Form Expression

By the definition of expectation of a probabilistic variable, the expected inter-delivery time $E[X_k] = \Sigma_{m=1}^{\infty} m P\{X_k = m\}$ can be expanded as,

$$\begin{aligned} E[X_k] =& \sum_{m=1}^{N} \left[m(p_{m+k}q_{m+k}) \prod_{i=1}^{m-1} (1 - p_{i+k}q_{i+k}) \right] \\ =& (1) \times p_{k+1}q_{k+1} + (2) \times p_{k+2}q_{k+2}(1 - p_{k+1}q_{k+1}) \\ &+ (3) \times p_{k+3}q_{k+3}(1 - p_{k+1}q_{k+1})(1 - p_{k+2}q_{k+2}) + \dots \\ &+ (w)p_{k+w}q_{k+w}(1 - p_{k+1}q_{k+1})(1 - p_{k+2}q_{k+2}) \dots \\ &\quad (1 - p_{k+w-1}q_{k+w-1}) \\ &+ (w+1)p_{k+w+1}q_{k+w+1}(1 - p_{k+1}q_{k+1})(1 - p_{k+2}q_{k+2}) \dots \\ &\quad (1 - p_{k+w}q_{k+w}) \\ &+ \cdots\cdots \\ &+ (bw)p_{k+bw}q_{k+bw}(1 - p_{k+1}q_{k+1})(1 - p_{k+2}q_{k+2}) \\ &\quad \cdots (1 - p_{k+bw-1}q_{k+bw-1}) \\ &+ (bw+1)p_{k+bw+1}q_{k+bw+1}(1 - p_{k+1}q_{k+1})(1 - p_{k+2}q_{k+2}) \\ &\quad \cdots (1 - p_{k+bw}q_{k+bw}) \end{aligned}$$

$$+\cdots\cdots$$
$$+(bw+c)p_{k+bw+c}q_{k+bw+c}(1-p_{k+1}q_{k+1})(1-p_{k+2}q_{k+2})\cdots(1-p_{k+bw+c-1}q_{k+bw+c-1}).$$

Based on the above expansion, the expectation $E[X_k]$ can be rewritten as,

$$\begin{aligned}E[X_k]=&\sum_{m=1}^{N}\left[m(p_{k+m}q_{k+m})\prod\nolimits_{i=1}^{m-1}(1-p_{i+k}q_{i+k})\right]\\=&\sum_{b=0}^{N/w}\left[p_{k+1}q_{k+1}(bw+1)r^b\right]\\&+\cdots\cdots\\&+\sum_{b=0}^{N/w}\Big[(1-p_{k+1}q_{k+1})(1-p_{k+2}q_{k+2})\\&\cdots(1-p_{k+w-1}q_{k+w-1})p_{k+w}q_{k+w}(bw+w)r^b\Big]+\mathbb{R}_2.\end{aligned}$$

The residue $\mathbb{R}_2$ of $E[X_k]$ is expressed as,

$$\begin{aligned}\mathbb{R}_2=&(bw+1)p_{k+bw+1}q_{k+bw+1}(1-p_{k+1}q_{k+1})\\&(1-p_{k+2}q_{k+2})\dots(1-p_{k+bw}q_{k+bw})\\&+(bw+2)p_{k+bw+2}q_{k+bw+2}(1-p_{k+1}q_{k+1})\\&(1-p_{k+2}q_{k+2})\dots(1-p_{k+bw+1}q_{k+bw+1})\\&+\cdots\cdots\\&+(bw+c)p_{k+bw+c}q_{k+bw+c}(1-p_{k+1}q_{k+1})\\&(1-p_{k+2}q_{k+2})\dots(1-p_{k+bw+c-1}q_{k+bw+c-1})\\=&(bw+1)p_{k+1}q_{k+1}r^b\\&+\cdots\cdots\\&+(bw+c)p_{k+c}q_{k+c}(1-p_{k+1}q_{k+1})\\&(1-p_{k+2}q_{k+2})\dots(1-p_{k+c-1}q_{k+c-1})r^b\end{aligned}$$

where number of periods will be of $b=\lfloor\frac{m}{n}\rfloor$, and the residue c of mod m with period of w, $m=bw+c$ and $r=(1-p_1q_1)(1-p_2q_2)\dots(1-p_wq_w)$.

The expression $E[X_k]$ can be further simplified as,

$$\begin{aligned}E[X_k]=&\sum_{m=1}^{N}\left[m(p_{m+k}q_{m+k})\prod\nolimits_{i=1}^{m-1}(1-p_{i+k}q_{i+k})\right]\\=&p_{k+1}q_{k+1}\sum_{b=0}^{N/w}\left[(bw+1)r^b\right]\\&+(1-p_{k+1}q_{k+1})p_{k+2}q_{k+2}\sum_{b=0}^{N/w}\left[(bw+2)r^b\right]+\cdots\cdots\\&+(1-p_{k+1}q_{k+1})(1-p_{k+2}q_{k+2})\dots(1-p_{k+w-1}q_{k+w-1})\\=&\sum_{h=1}^{w}\left\{\left[p_{h+k}q_{h+k}\prod\nolimits_{i=1}^{h-1}(1-p_{i+k}q_{i+k})\right]\sum_{b=0}^{N/w}\left[(bw+h)r^b\right]\right\}\\&+\mathbb{R}_2\end{aligned}\tag{22}$$

The summation of the other mutated series with each term nr^n is given by,

$$S_M=\sum_{n=0}^{M}nr^n=\frac{r(1-r^{M+1})-r^{M+1}(M+1)(1-r)}{(1-r)^2}.\tag{23}$$

To verify summation equation (23), the general term $a_{n+1}=(n+1)r^{n+1}$ is also derived by the difference between the sum of the first $n+1$ terms and the first n terms, so as to validate the expression for the sum of first n terms.

$$\begin{aligned}a_{n+1}=&S_{n+1}-S_n\\=&\frac{r(1-r^{(n+1)+1})-r^{(n+1)+1}((n+1)+1)(1-r)}{(1-r)^2}\\&-\frac{r(1-r^{n+1})-r^{n+1}(n+1)(1-r)}{(1-r)^2}\\=&(n+1)r^{n+1}\end{aligned}$$

After the equation (23) is applied, let $M=\lfloor N/w\rfloor$, the summation Z_N of the first N terms of a geometric series is simplified as,

$$\begin{aligned}Z_N=&\sum_{b=0}^{\lfloor N/w\rfloor}(bw+h)r^b=\sum_{b=0}^{\lfloor N/w\rfloor}\left(bwr^b+hr^b\right)\\=&w\sum_{b=0}^{M}br^b+h\sum_{b=0}^{M}r^b\\=&w\left[\frac{r^{M+1}(M+1)(1-r)+r(1-r^{M+1})}{(1-r)^2}\right]\\&+h\left(\frac{1-r^{M+1}}{1-r}\right)\end{aligned}$$

The limit of $\mathbb{R}_2$ is given by $\lim_{N\to\infty}\mathbb{R}_2=0$. When the number $M=\lfloor N/w\rfloor$ of cycles goes to infinity, the limit $\lim_{N\to\infty}Z_N$ is given by,

$$\lim_{N\to\infty}Z_N=\frac{wr}{(1-r)^2}+\left(\frac{1}{1-r}\right)h\tag{24}$$

After the limit of summation Z_N of the mutated geometric series is substituted into equation (22), a closed-form expression of expected inter-delivery time is presented by,

$$\begin{aligned}E[X_k]=&\sum_{h=1}^{w}\left\{\left[p_{h+k}q_{h+k}\prod\nolimits_{i=1}^{h-1}(1-p_{i+k}q_{i+k})\right]\right.\\&\left.\left[\frac{wr}{(1-r)^2}+\left(\frac{1}{1-r}\right)h\right]\right\}\\=&\left(\frac{1}{1-r}\right)\sum_{h=1}^{w}\left\{\left[p_{h+k}q_{h+k}\prod\nolimits_{i=1}^{h-1}(1-p_{i+k}q_{i+k})\right]\right.\\&\left.\times\left[h+\frac{wr}{(1-r)}\right]\right\}\end{aligned}$$

where $r=(1-p_1q_1)(1-p_2q_2)\cdots(1-p_wq_w)$, and the boundary product is defined as $\prod_{i=1}^{0}(\cdot)=1$.

The proof of Lemma 2 is completed.

APPENDIX C
EXPECTED SQUARED INTER-DELIVERY TIME IN CLOSED-FORM EXPRESSION

In order to find a closed-form expression for the nominator of $\sum_{m=1}^{\infty}\left\{m^2(p_{m+k}q_{m+k})\prod_{i=1}^{m-1}(1-p_{i+k}q_{i+k})\right\}$, we simplify it as follows. The symbol w is defined as the period of probability of transmission success q_k.

$$E[X_k^2]=\sum_{m=1}^{N} m^2(p_{k+m}q_{k+m})\prod\nolimits_{i=1}^{m-1}(1-p_{i+k}q_{i+k})$$

Let $r=(1-p_1q_1)(1-p_2q_2)\dots(1-p_wq_w)=(1-p_{k+1}q_{k+1})(1-p_{k+2}q_{k+2})\dots(1-p_{k+w}q_{k+w})$, where $k\in N$, because of the periodicity property of probability of transmission success. Let $m=bw+c$, $b=\lfloor m/w\rfloor$, and $b\in\{0,1,2,3,\dots\}$, $c=m-bw$, $c\in N$. We can expand summation of the first N terms of simplified numerator of $\overline{\Delta}_k$ as following,

$$\begin{aligned}
E[X_k^2]&=\sum_{m=1}^{N}\left[m^2(p_{m+k}q_{m+k})\prod\nolimits_{i=1}^{m-1}(1-p_{i+k}q_{i+k})\right]\\
&=(1)^2p_{k+1}q_{k+1}+(2)^2p_{k+2}q_{k+2}(1-p_{k+1}q_{k+1})\\
&\quad\dots\dots\\
&\quad+(w)^2p_{k+w}q_{k+w}(1-p_{k+1}q_{k+1})(1-p_{k+2}q_{k+2})\\
&\quad\cdots(1-p_{k+w-1}q_{k+w-1})\\
&\quad+\cdots\cdots\\
&\quad+(w+w)^2p_{k+w+w}q_{k+w+w}(1-p_{k+1}q_{k+1})(1-p_{k+2}q_{k+2})\cdots\\
&\quad(1-p_{k+w}q_{k+w})\cdots(1-p_{k+2w-1}q_{k+2w-1})\\
&\quad+\dots\dots\\
&\quad+[(b-1)w]^2p_{k+(b-1)w}q_{k+(b-1)w}(1-p_{k+1}q_{k+1})\\
&\quad(1-p_{k+2}q_{k+2})\cdots(1-p_{k+(b-1)w-1}q_{k+(b-1)w-1})\\
&\quad+\dots\dots\\
&\quad+(bw)^2p_{k+bw}q_{k+bw}(1-p_{k+1}q_{k+1})\\
&\quad(1-p_{k+2}q_{k+2})\dots(1-p_{k+bw-1}q_{k+bw-1})\\
&\quad+(bw+1)^2p_{k+bw+1}q_{k+bw+1}(1-p_{k+1}q_{k+1})\\
&\quad(1-p_{k+2}q_{k+2})\dots(1-p_{k+bw}q_{k+bw})\\
&\quad+\dots\dots\\
&\quad+(bw+c)^2p_{k+bw+c}q_{k+bw+c}(1-p_{k+1}q_{k+1})\\
&\quad(1-p_{k+2}q_{k+2})\dots(1-p_{k+bw+c-1}q_{k+bw+c-1}).
\end{aligned}$$

The expected expression $E[X_k^2]$ of squared inter-delivery time can be rewritten as,

$$\begin{aligned}
E[X_k^2]&=\sum_{m=1}^{N}\left[m^2(p_{k+m}q_{k+m})\prod\nolimits_{i=1}^{m-1}(1-p_{i+k}q_{i+k})\right]\\
&=\sum_{b=0}^{N/w}\left[p_{k+1}q_{k+1}(bw+1)^2r^b\right]\\
&\quad+\sum_{b=0}^{N/w}\left[(1-p_{k+1}q_{k+1})p_{k+2}q_{k+2}(bw+2)^2r^b\right]\\
&\quad+\dots\dots\\
&\quad+\sum_{b=0}^{N/w}\Big[(1-p_{k+1}q_{k+1})(1-p_{k+2}q_{k+2})\\
&\quad\dots(1-p_{k+w-1}q_{k+w-1})p_{k+w}q_{k+w}(bw+w)^2r^b\Big]+\mathbb{R}_1.
\end{aligned}$$

The residue $\mathbb{R}_1$ of $E[X_k^2]$ is expressed as,

$$\begin{aligned}
\mathbb{R}_1&=(bw+1)^2p_{k+bw+1}q_{k+bw+1}(1-p_{k+1}q_{k+1})\\
&\quad(1-p_{k+2}q_{k+2})\dots(1-p_{k+bw}q_{k+bw})\\
&\quad+(bw+2)^2p_{k+bw+2}q_{k+bw+2}(1-p_{k+1}q_{k+1})\\
&\quad(1-p_{k+2}q_{k+2})\dots(1-p_{k+bw+1}q_{k+bw+1})\\
&\quad+\dots\dots\\
&\quad+(bw+c)^2p_{k+bw+c}q_{k+bw+c}(1-p_{k+1}q_{k+1})\\
&\quad(1-p_{k+2}q_{k+2})\dots(1-p_{k+bw+c-1}q_{k+bw+c-1})\\
&=(bw+1)^2p_{k+bw+1}q_{k+bw+1}r^b\\
&\quad+(bw+2)^2p_{k+bw+2}q_{k+bw+2}(1-p_{k+bw+1}q_{k+bw+1})r^b\\
&\quad+\dots\\
&\quad+(bw+c)^2p_{k+bw+c}q_{k+bw+c}(1-p_{k+bw+1}q_{k+bw+1})\\
&\quad(1-p_{k+bw+2}q_{k+bw+2})\dots(1-p_{k+bw+c-1}q_{k+bw+c-1})r^b\\
&=(bw+1)^2p_{k+1}q_{k+1}r^b\\
&\quad+(bw+2)^2p_{k+2}q_{k+2}(1-p_{k+1}q_{k+1})r^b\\
&\quad+\dots\dots\\
&\quad+(bw+c)^2p_{k+c}q_{k+c}(1-p_{k+1}q_{k+1})\\
&\quad(1-p_{k+2}q_{k+2})\dots(1-p_{k+c-1}q_{k+c-1})r^b
\end{aligned}$$

where the number of complete cycles, denoted as b, is determined by the floor of the division of m by w , thus $b=\lfloor\frac{m}{w}\rfloor$. Additionally, the remainder c is derived from the modulus of m with the period w , leading to the relationship $m=bw+c$. Here, r is calculated as the product of the probabilities of unsuccessful transmissions across one cycle, where $r=(1-p_1q_1)(1-p_2q_2)\dots(1-p_wq_w)$.

The expected expression $E[X_k^2]$ of squared inter-delivery time can be further simplified as,

$$\begin{aligned}
E[X_k^2]&=\sum_{m=1}^{N}\left[m^2(p_{m+k}q_{m+k})\prod\nolimits_{i=1}^{m-1}(1-p_{i+k}q_{i+k})\right]\\
&=p_{k+1}q_{k+1}\sum_{b=0}^{N/w}\left[(bw+1)^2r^b\right]\\
&\quad+(1-p_{k+1}q_{k+1})p_{k+2}q_{k+2}\sum_{b=0}^{N/w}\left[(bw+2)^2r^b\right]\\
&\quad+\dots\dots
\end{aligned}$$

$$
\begin{aligned}
&+ (1-p_{k+1}q_{k+1})(1-p_{k+2}q_{k+2}) \\
&\quad \cdots p_{k+w}q_{k+w} \sum_{b=0}^{N/w} \left[(bw+w)^2 r^b\right] + \mathbb{R}_1 \\
&= \sum_{h=1}^{w} \Bigg\{ \left[p_{h+k}q_{h+k} \prod\nolimits_{i=1}^{h-1} (1-p_{i+k}q_{i+k})\right] \\
&\quad \times \sum_{b=0}^{N/w} \left[(bw+h)^2 r^b\right] \Bigg\} + \mathbb{R}_1
\end{aligned}
$$

The summation of the first M terms of a mutated geometric series is obtained by summing each term with a product of n^2 and r^n, where n is natural number.

$$
\begin{aligned}
S_M = \sum_{n=0}^{M} n^2 r^n = &\frac{r + r^2 - r^{M+2} - r^{M+3}}{(1-r)^3} \\
&- \frac{(1-r^2)r^{M+1}(M+1) + r^{M+1}(1-r)^2 M(M+1)}{(1-r)^3}
\end{aligned} \tag{25}
$$

To validate the formula (25), the expression for a general term a_{n+1} is derived by differentiating the sum of the first $n+1$ terms from the first n terms, thereby confirming the expression for the summation of first n terms.

$$
\begin{aligned}
a_{n+1} =& S_{n+1} - S_n \\
=& \frac{r + r^2 - r^{(n+1)+2} - r^{(n+1)+3}}{(1-r)^3} \\
&- \frac{(1-r^2) r^{(n+1)+1}((n+1)+1)}{(1-r)^3} \\
&- \frac{r^{(n+1)+1}(1-r)^2 (n+1)((n+1)+1)}{(1-r)^3} \\
&- \frac{r + r^2 - r^{n+2} - r^{n+3} - (1-r^2) r^{n+1}(n+1)}{(1-r)^3} \\
&+ \frac{r^{n+1}(1-r)^2 n(n+1)}{(1-r)^3} \\
=& \frac{(1-r) r^{n+1} (1-r)^2 (n+1)^2}{(1-r)^3} = (n+1)^2 r^{n+1}
\end{aligned}
$$

Thus, the general term $a_{n+1} = (n+1)^2 r^{n+1}$ of the mutated series by differentiating two summations of n and $n+1$ terms is verified.

After equations (23) and (25) are applied, let $M = \lfloor N/w \rfloor$, the summation T_N of the first N terms of a geometric series is simplified as,

$$
\begin{aligned}
T_N =& \sum_{b=0}^{\lfloor N/w \rfloor} (bw+h)^2 r^b \\
=& \sum_{b=0}^{\lfloor N/w \rfloor} \left[b^2 w^2 r^b + 2bwhr^b + h^2 r^b\right] \\
=& w^2 \sum_{b=0}^{M} b^2 r^b + 2wh \sum_{b=0}^{M} b r^b + h^2 \sum_{b=0}^{M} r^b \\
=& w^2 \left[\frac{r^{M+3} - r^2 + r^{M+2} - r - (r^2-1) r^{M+1}(M+1)}{(r-1)^3}\right] \\
&+ 2wh \left[\frac{r(1-r^{M+1}) - r^{M+1}(M+1)(1-r)}{(1-r)^2}\right] \\
&+ \frac{(1-r^M)}{1-r} h^2
\end{aligned} \tag{26}
$$

The limit of $\mathbb{R}_1$ is given by $\lim_{N\to\infty} \mathbb{R}_1 = 0$. When the number $M = \lfloor N/w \rfloor$ of cycles goes to infinity, the limit $\lim_{N\to\infty} T_N$ is given by,

$$
\lim_{N\to\infty} T_N = \left(\frac{1}{1-r}\right) h^2 + \frac{2wr}{(1-r)^2} h + \frac{w^2(r+r^2)}{(1-r)^3}
$$

After the limit of summation T_N of the mutated geometric series is substituted into equation (26), a closed-form expression of expected squared inter-delivery time is presented by,

$$
\begin{aligned}
E[X_k^2] =& \sum_{h=1}^{w} \Bigg\{ \left[p_{h+k}q_{h+k} \prod\nolimits_{i=1}^{h-1} (1-p_{i+k}q_{i+k})\right] \\
&\left[\left(\frac{1}{1-r}\right) h^2 + \frac{2wr}{(1-r)^2} h + \frac{w^2(r+r^2)}{(1-r)^3}\right] \Bigg\} \\
=& \left(\frac{1}{1-r}\right) \sum_{h=1}^{w} \Bigg\{ \left[p_{h+k}q_{h+k} \prod\nolimits_{i=1}^{h-1} (1-p_{i+k}q_{i+k})\right] \\
&\left[h^2 + \frac{2wr}{1-r} h + \frac{r(1+r)w^2}{(1-r)^2}\right] \Bigg\}
\end{aligned}
$$

The Lemma 3 is proved.

Appendix D
Proof of Lemma 4 Generic Formula for Any State Transition Probability

Given the state space $\{S_1, S_2, \cdots, S_w\}$ and the summation term $T_w = \sum_{n=0}^{\infty} \left[(1-q_1)(1-q_2)\cdots(1-q_w)\right]^n$, the state transition probability β_{w1} from state S_w to S_1 is expressed as,

$$
\beta_{w1} = P(S_w \to S_1) = q_1 \times T_w = \frac{q_1}{1 - \prod_{h=1}^{w}(1-q_h)}
$$

The state transition probability $\beta_{(w-1)1}$ from state S_{w-1} to state S_1 is given by,

$$
\begin{aligned}
\beta_{(w-1)1} =& P(S_{w-1} \to S_1) = q_1(1-q_w) \times T_w \\
=& \frac{q_1(1-q_w)}{1 - \prod_{h=1}^{w}(1-q_h)}
\end{aligned}
$$

Accordingly, any transition probability β_{i1} from any state S_i to state S_1 is given by,

$$
\begin{aligned}
\beta_{i1} =& P(S_i \to S_1) = q_1(1-q_{i+1})\cdots(1-q_w) \times T_w \\
=& \frac{q_1 \prod_{h=i+1}^{w}(1-q_h)}{1 - \prod_{h=1}^{w}(1-q_h)}
\end{aligned}
$$

An arbitrary transition probability from state i to state j when $i < j$,

$$\beta_{ij} = \frac{q_j \prod_{h=i+1}^{j-1}(1-q_h)}{1-\prod_{h=1}^{w}(1-q_h)}, \qquad i < j \tag{27}$$

where the edge case $\prod_{h=j}^{j-1}(\cdot) = 1$

Any transition probability from state i to state j when $i \geq j$,

$$\beta_{ij} = \frac{q_j \prod_{h=i+1}^{w}(1-q_h)\prod_{h=1}^{j-1}(1-q_h)}{1-\prod_{h=1}^{w}(1-q_h)}, \; i \geq j. \tag{28}$$

where the edge cases $\prod_{h=w+1}^{w}(\cdot) = 1$ and $\prod_{h=1}^{0}(\cdot) = 1$.

In general, any transition probability from state i to j is summarized as,

$$\beta_{ij} = \begin{cases} \frac{q_j \prod_{h=i+1}^{j-1}(1-q_h)}{1-\prod_{h=1}^{w}(1-q_h)}, & i < j, \\ \frac{q_j \prod_{h=i+1}^{w}(1-q_h)\prod_{h=1}^{j-1}(1-q_h)}{1-\prod_{h=1}^{w}(1-q_h)}, & i \geq j. \end{cases} \tag{29}$$

where $\prod_{h=j}^{j-1}(\cdot) = 1$, $\prod_{h=w+1}^{w}(\cdot) = 1$ and $\prod_{h=1}^{0}(\cdot) = 1$.

Appendix E
Proof of Lemma 5 of Markov Stationary State Probabilities

The vector $\boldsymbol{\pi}^T = \{\pi_1, \pi_2, \cdots, \pi_w\}$ denotes the Markov stationary state probability set and the state transition matrix $\boldsymbol{T}$ is defined as,

$$\boldsymbol{T} = \begin{bmatrix} \beta_{11} & \beta_{21} & \cdots & \beta_{w1} \\ \beta_{12} & \beta_{22} & \cdots & \beta_{w2} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{1w} & \beta_{2w} & \cdots & \beta_{ww} \end{bmatrix}$$

where β_{ij} denotes state transition probability from state i to j in Lemma 4 and w is transmission success probability cycle.

By the definition of Markov chain, the matrix state equation $\boldsymbol{\pi} = \boldsymbol{T}\boldsymbol{\pi}$ with constraint is expressed as,

$$\begin{bmatrix} \pi_1 \\ \pi_2 \\ \vdots \\ \pi_w \end{bmatrix} = \begin{bmatrix} \beta_{11} & \beta_{21} & \cdots & \beta_{w1} \\ \beta_{12} & \beta_{22} & \cdots & \beta_{w2} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{1w} & \beta_{2w} & \cdots & \beta_{ww} \end{bmatrix} \begin{bmatrix} \pi_1 \\ \pi_2 \\ \vdots \\ \pi_w \end{bmatrix}$$

where the matrix equation is constrained with $\sum_{s=1}^{w} \pi_s = 1$ and its generic sub-equation can be expanded as,

$$\pi_s = \sum_{h=1}^{w} \beta_{hs}\pi_h, \quad s \in w. \tag{30}$$

Lemma 6 *The probability that at least one packet is successfully delivered within a period w shall be mathematically expressed as either summation of possible cases or an alternative expression by excluding one case of no packet delivery,*

$$1 - \prod_{h=1}^{w}(1-q_h) = \left\{ \sum_{i=1}^{s-1} \left[q_i \prod_{h=i+1}^{s-1}(1-q_h) \right] + \sum_{i=s}^{w} \left[q_i \prod_{h=i+1}^{w}(1-q_h) \prod_{h=1}^{s-1}(1-q_h) \right] \right\}. \tag{31}$$

where any $s \in w$.

Proof: The probability without any packet delivery within period w is expressed as $\prod_{h=1}^{w}(1-q_h)$ and thus at least one packet delivery will be $1 - \prod_{h=1}^{w}(1-q_h)$. The probability expression on the right side $\sum_{i=1}^{s-1}\left[q_i \prod_{h=i+1}^{s-1}(1-q_h)\right] + \sum_{i=s}^{w}\left[q_i \prod_{h=i+1}^{w}(1-q_h)\prod_{h=1}^{s-1}(1-q_h)\right]$ in equation (31) is simplified and both sides are the same. For example,

$$\begin{aligned} &\left\{ \sum_{i=1}^{s-1} \left[q_i \prod_{h=i+1}^{s-1}(1-q_h) \right] + \sum_{i=s}^{w} \left[q_i \prod_{h=i+1}^{w}(1-q_h) \prod_{h=1}^{s-1}(1-q_h) \right] \right\} \\ =&q_1(1-q_2) + q_2 + q_3(1-q_4)(1-q_5)(1-q_1)(1-q_2) \\ &+ q_4(1-q_5)(1-q_1)(1-q_2) + q_5(1-q_1)(1-q_2) \\ =&1 - (1-q_1)(1-q_2) + q_3(1-q_4)(1-q_5)(1-q_1)(1-q_2) \\ &+ q_4(1-q_5)(1-q_1)(1-q_2) + q_5(1-q_1)(1-q_2) \\ =&1 - (1-q_1)(1-q_2)(1-q_5) + q_3(1-q_4)(1-q_5) \\ &(1-q_1)(1-q_2) + q_4(1-q_5)(1-q_1)(1-q_2) \\ =&1 - (1-q_1)(1-q_2)(1-q_4)(1-q_5) \\ &+ q_3(1-q_4)(1-q_5)(1-q_1)(1-q_2) \\ =&1 - (1-q_1)(1-q_2)(1-q_3)(1-q_4)(1-q_5) \\ =&1 - \prod_{h=1}^{w}(1-q_h) \end{aligned}$$

where $s = 3$, $w = 5$.

The state transition probabilities β_{ij} and the Markov stationary state probability vector $\boldsymbol{\pi}$ are substituted into generic sub-equation (30). When Lemma 6 is applied, the expression π_s of an arbitrary stationary state probability is simplified as,

$$\begin{aligned} \pi_s &= \sum_{i=1}^{w} \{\beta_{is} \times \pi_i\} \\ &= \sum_{i=1}^{s-1} \left\{ \frac{q_s \prod_{h=i+1}^{s-1}(1-q_h)}{1-\prod_{h=1}^{w}(1-q_h)} \times \frac{q_i}{\sum_{h=1}^{w} q_h} \right\} \\ &\quad + \sum_{i=s}^{w} \left\{ \frac{q_s \prod_{h=i+1}^{w}(1-q_h)\prod_{h=1}^{s-1}(1-q_h)}{1-\prod_{h=1}^{w}(1-q_h)} \times \frac{q_i}{\sum_{h=1}^{w} q_h} \right\} \\ &= \frac{q_s}{\sum_{h=1}^{w} q_h} \end{aligned} \tag{32}$$

The general solution to Markov matrix equation is obtained in (32) and Lemma 5 is proved.

APPENDIX F
A CLOSED-FORM EXPRESSION OF AVERAGE AOI WITH IRS RECONFIGURATION CONVERSION TIME DELAY

The period of transmission success probability is extended by the delay time u of IRS reconfiguration conversion. After the extended period $w+u$ is substituted into equation (36) in Appendix **??**, the closed-form expression of average AoI with IRS conversion delay is given by,

$$\overline{\Delta}(u)=\frac{\sum_{h=1}^{w+u}\Big\{G(h,k)H(h,r,w,u)\Big\}}{2\sum_{h=1}^{w+u}\Big\{G(h,k)B(h,r,w,u)\Big\}}+\frac{1}{2} \tag{33}$$

where function $G(h,k)=p_{h+k}q_{h+k}\prod_{i=1}^{h-1}\big(1-p_{i+k}q_{i+k}\big)$, $H(h,r,w,u)=\big(\frac{1}{1-r}\big)h^2+\frac{2(w+u)r}{1-r}h+\frac{r(1+r)(w+u)^2}{(1-r)^3}$, $B(h,r,w,u)=(\frac{1}{1-r})h+\frac{(w+u)r}{(1-r)^2}$ and $r=\prod_{i=1}^{w}(1-p_iq_i)$.

The proof of Theorem 2 is completed.

Given period w of packet transmission success probability, let $n=bw+c$ represent the total number of successfully transmitted packets. The symbol n_s denotes the number of delivered packets with initial condition of q_s within the period w and the relationship between n and n_k is given by $n=\sum_{s=1}^{w}n_s$. The Markov stationary state probability at any state s is defined as $\pi_s=\frac{n_s}{n}$ with n and n_s.

By the definition of expectation, the expression $E[X_s]$ of expected conditional inter-delivery time and the expectation $E[X_s^2]$ of squared conditional inter-delivery time is given by,

$$E[X_s]=\frac{\sum_{b=0}^{\lfloor\frac{n}{w}\rfloor}X_{bw+s}}{n_s} \tag{34}$$

$$E[X_s^2]=\frac{\sum_{b=0}^{\lfloor\frac{n}{w}\rfloor}X_{bw+s}^2}{n_s} \tag{35}$$

After equation (34) is applied, the expectation $E[X]$ of unconditional inter-delivery time X is formulated as,

$$\begin{aligned}E[X]=&\frac{1}{n}\sum_{i=1}^{n}X_i=\frac{1}{n}\sum_{i=1}^{w\times\lfloor\frac{n}{w}\rfloor}X_i-\frac{1}{n}\sum_{i=w\times\lfloor\frac{n}{w}\rfloor+1}^{n}X_i\\=&\frac{1}{n}\sum_{s=1}^{w}\left[\sum_{b=0}^{\lfloor\frac{n}{w}\rfloor}X_{bw+s}\right]=\sum_{s=1}^{w}\left[\frac{\sum_{b=0}^{\lfloor\frac{n}{w}\rfloor}X_{bw+s}}{n}\right]\\=&\sum_{s=1}^{w}\left[\frac{n_s}{n}\times\frac{\sum_{b=0}^{\lfloor\frac{n}{w}\rfloor}X_{bw+s}}{n_s}\right]\\=&\sum_{s=1}^{w}\big\{\pi_s\times E[X_s]\big\}\end{aligned}$$

Accordingly, equation (35) is applied, the expectation $E[X^2]$ of the squared unconditional inter-delivery time X^2 is also derived as,

$$\begin{aligned}E[X^2]=&\frac{1}{n}\sum_{i=1}^{n}X_i^2=\frac{1}{n}\sum_{i=1}^{w\times\lfloor\frac{n}{w}\rfloor}X_i^2-\frac{1}{n}\sum_{i=w\times\lfloor\frac{n}{w}\rfloor+1}^{n}X_i^2\\=&\frac{1}{n}\sum_{s=1}^{w}\left[\sum_{b=0}^{\lfloor\frac{n}{w}\rfloor}X_{bw+s}^2\right]=\sum_{s=1}^{w}\left[\frac{\sum_{b=0}^{\lfloor\frac{n}{w}\rfloor}X_{bw+s}^2}{n}\right]\\=&\sum_{s=1}^{w}\left[\frac{n_s}{n}\times\frac{\sum_{b=0}^{\lfloor\frac{n}{w}\rfloor}X_{bw+s}^2}{n_s}\right]\\=&\sum_{s=1}^{w}\big\{\pi_s\times E[X_s^2]\big\}\end{aligned}$$

Therefore, the unconditional average AoI $\overline{\Delta}$ can be reconstructed with conditional AoIs under Markov stationary state probability π_s and expressed as,

$$\overline{\Delta}=\frac{E[X^2]}{2E[X]}+\frac{1}{2}=\frac{\sum_{s=1}^{w}\big\{\pi_sE[X_s]\big\}}{2\sum_{s=1}^{w}\big\{\pi_sE[X_s^2]\big\}}+\frac{1}{2} \tag{36}$$