THE GEORGE WASHINGTON UNIVERSITY

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Segmentation and Predictive Scheduling Policy under Multiple Sources

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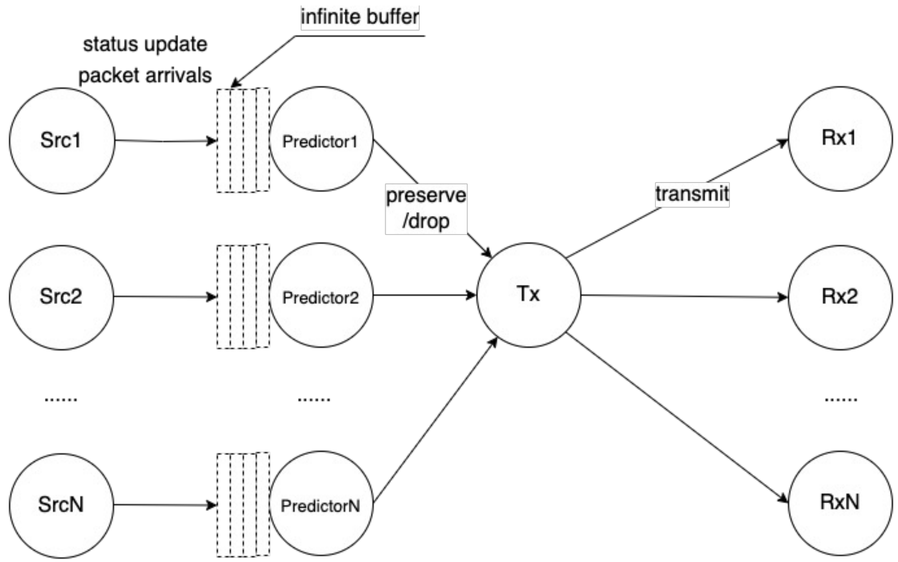
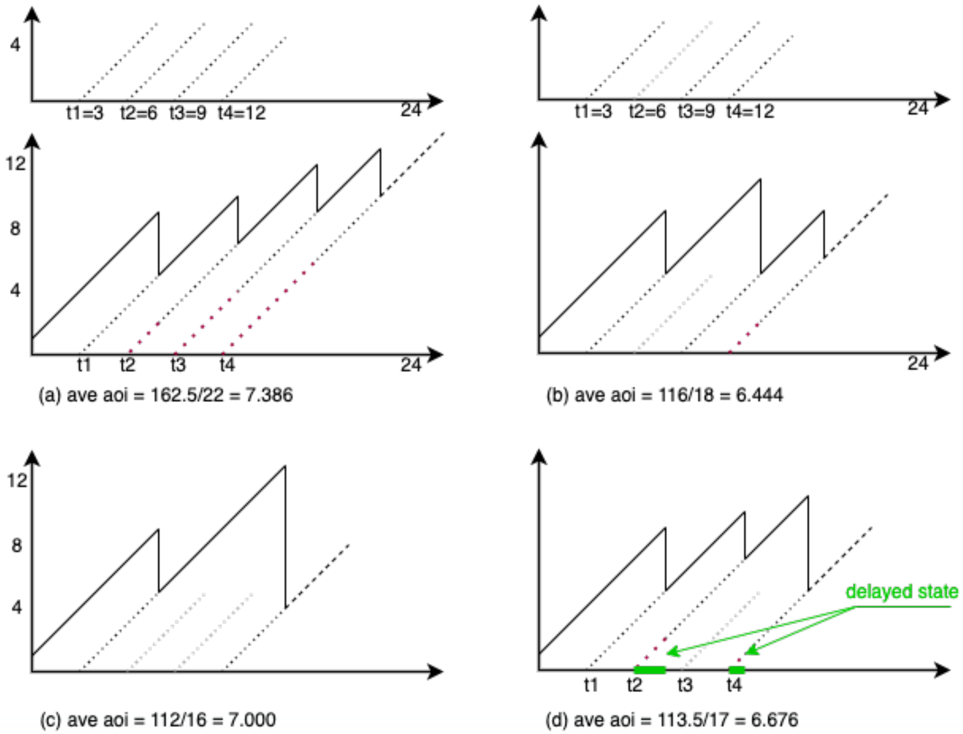
System Model



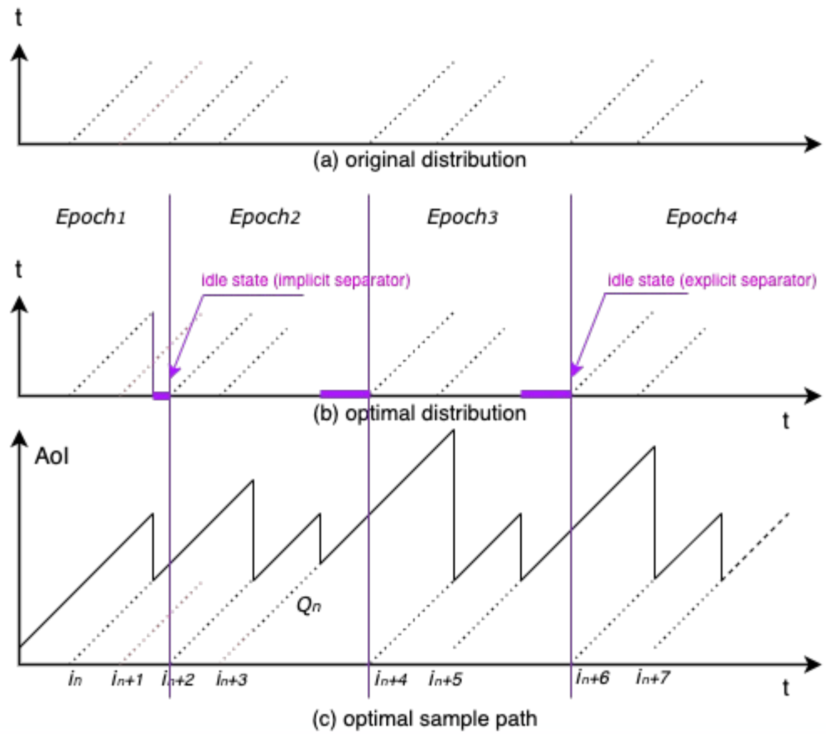
- predictor estimates inter-arrival time
- determine preserving or dropping
- known each inter-arrival and service time
- infinite buffer assumption
- multiple source-destination pairs
- packets arriving independent from others
- one channel shared by multiple sources

Problem Definition



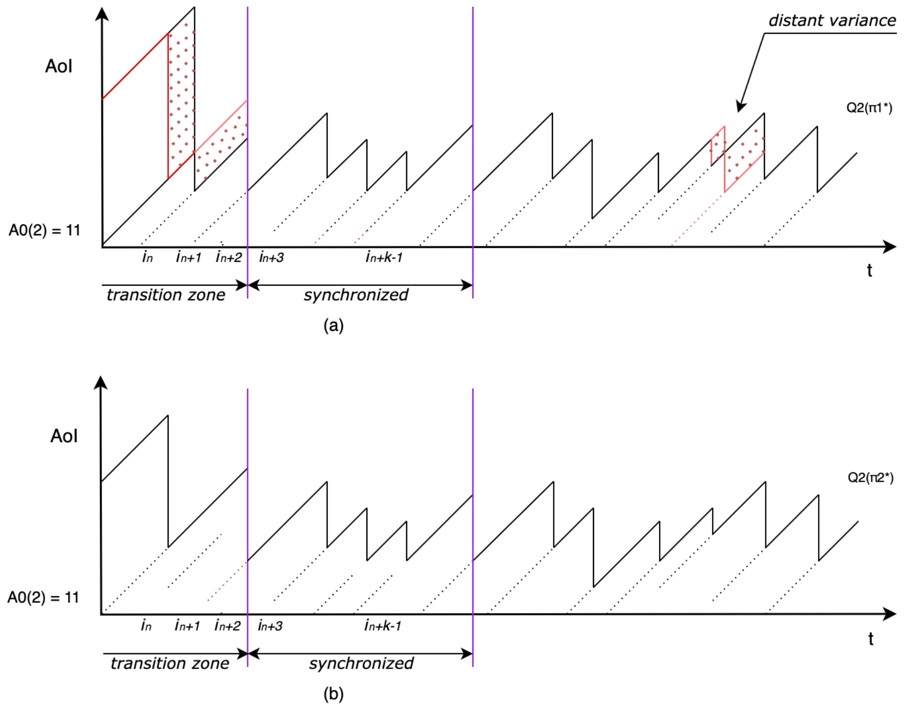
- large possible combinations of 2^k
- significant length of sequence k
- unaffordable time, energy and computing power

Prediction and Segmentation



- minimum AoI in an epoch n
- synchronized initial age and final age
- separator under multiple sources
  - (idle waiting time, synchronization)

Comparison between Transition Zones



- compare between possible policies
- minimum epoch-AoI but different final ages
- an arbitrary AoI of n^{th} attempt epoch under any major scheduling policy π^M and any minor scheduling policy π^T:
  - $\phi_n(\pi^M, \pi^T) = \min_{\pi^M}\left[\sum_{i=1}^{k}\left(\varphi_i^{\pi^M}\right)\right] + \min_{\pi^T}\left[\sum_{j=1}^{k}\left(\varphi_j^{\pi^T}\right)\right]$
  - $\phi_n^*(\pi^{M*}, \pi^{T*}) \leq \phi_n(\pi_u^M, \pi_u^T), u \in \{1 \quad 2 \quad \cdots \quad 2^{k_n}\}$
- sample path synchronization under any two initial ages $A_0^{(1)}$ and $A_0^{(2)}$

Asymptotic Synchronization

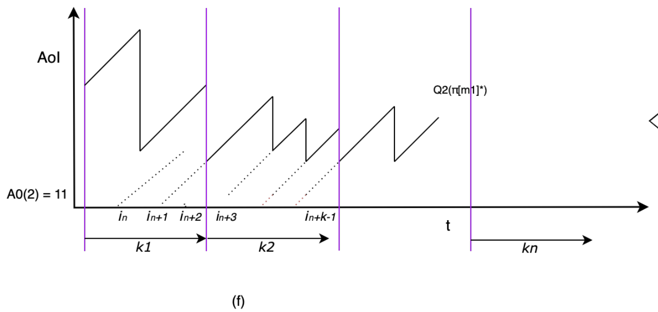
- Statement: For two conditions with different initial ages, but identical inter-arrival times and service times, the two optimized AoI processes under these two conditions shall be completely synchronized after a certain number of packets.

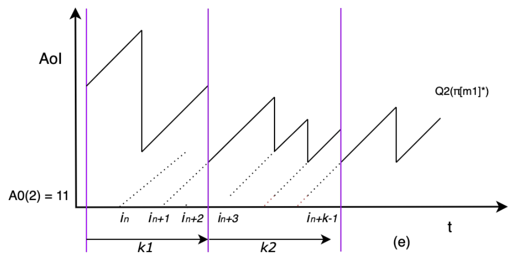
Sync Proof (1) – scheduling policy set

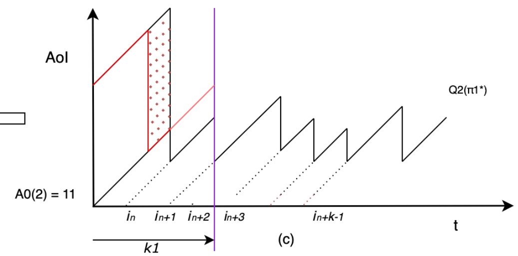
- any scheduling policies $\{\pi_i\}$ for a given attempt epoch n
  - $\forall\, i \in [1 \quad 2 \quad \cdots \quad 2^{l_n}],\, l_n = k_{n+1} - k_n$
- rearrange according to their corresponding $cAoI$ value
  - $\pi_i \in \{\pi_0^* \quad \pi_1 \quad \pi_2 \quad \cdots \quad \pi_{2^{l_n}}\}, i \in N$
  - $i \in \{0 \quad 1 \quad 2 \quad \cdots \quad i \quad i+1 \quad i+2 \quad \cdots \quad 2^{l_n}\}$
- sort scheduling policies in descending order of $cAoI$ value
  - $\{\phi(n)_0^* \quad \phi(n)_1 \quad \phi(n)_2 \quad \cdots \quad \phi(n)_i \quad \phi(n)_{i+1} \quad \phi(n)_{i+2} \quad \cdots \quad \phi(n)_{2^{l_n}}\}$
  - $\phi(n)_0^* \le \phi(n)_1 \le \phi(n)_2 \le \cdots \le \phi(n)_k \le \phi(n)_{k+1} \le \phi(n)_{k+2} \le \cdots \le \phi(n)_{2^n}$
- assume that the difference between any two adjacent AoI (steps) $\phi(n)_{\mathrm{i}}$ and $\phi(n)_{\mathrm{i+1}}$
  - $\Delta\phi(n)_{\mathrm{i}} = |\phi(n)_{\mathrm{i}} - \phi(n)_{\mathrm{i+1}}| \gg \varepsilon > 0$

Sync Proof (2) – AoI convergence

- **Lemma 1 – age of information convergence:**
  As the number of packets included into the iterative optimization increases, it gradually converges and approximates to optimal policy.





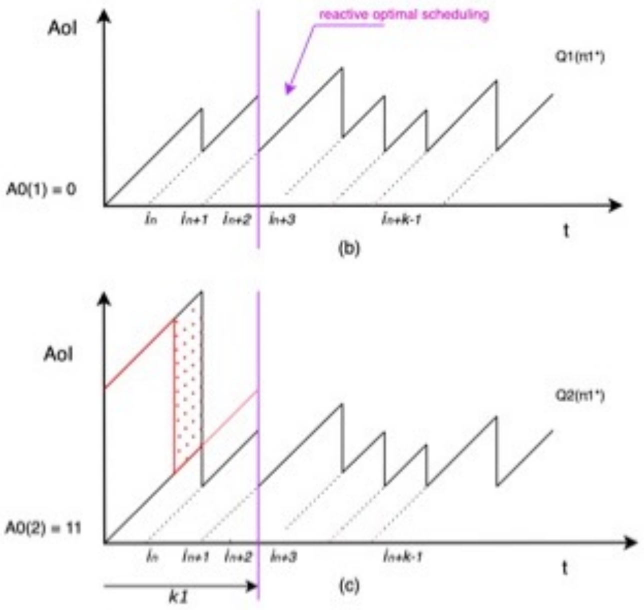


optimal

near-optimal at (k-1) step

near-optimal at k-step

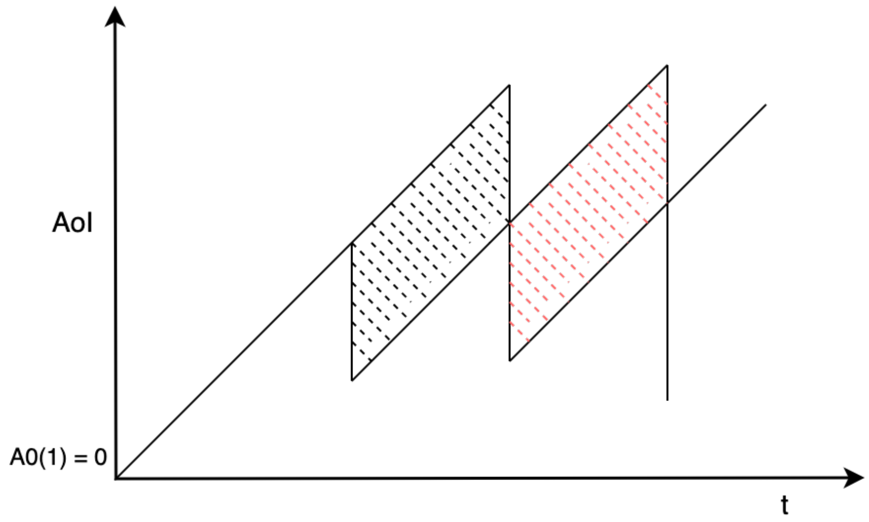
Sync Proof (3) – one near-optimal case with k-step optimum



- optimal scheduling policy π^* and optimal sample path $\mathcal{P}^* = s(\pi^*, 0)$ for a given sequence with the initial age 0.
- optimal scheduling policies $\pi^*_{A_0}$ and optimal sample path $\mathcal{P}^*_{A_0} = s\left(\pi^*_{A_0}, A_0\right)$for a given sequence with the initial age A_0.
- obtain intermediate sample path $\mathcal{P}^*_{I-A_0} = s(\pi^*, A_0)$, by substituting optimal scheduling zero-policy π^* into the optimization with initial age A_0.
- distance between near-optimal $\phi(n)^*$ and $\phi(n)_{\mathrm{i}}$, equivalent to difference between $\mathcal{P}^*_{I-A_0}$ and $\mathcal{P}^*$.

Proof (4) – basic blocks & discrete deduction

- **Lemma 2 – discrete deduction each step:** Regardless of any scheduling policy, each packet is selected for scheduling and successfully delivered to the destination. The reduction in AoI value caused by this action must be discrete and must be the same between optimal policies with or without identical initial ages. If the packets, determined by the packet sequence combination (decision vector) derived from this discontinuous age deduction, are successfully transmitted, the cumulative AoI value caused by deliveries must also be a discrete jump.



Proof (5) – limited number of basic blocks

- any $\phi(n)$ constructed by a sample path $\mathcal{P}$ under any scheduling policy π
- possible combination of selections from the same set of basic blocks, except for the only initial basic block
- limited number of common basic blocks for constructing any sample path $\mathcal{P}$
- difference $\Delta\phi(n)'_u$ between k-step optimizing sample path $\mathcal{P}^*_{I-A_0}$ with $\phi(n)^*$ and optimal sample path $\mathcal{P}^*$ with $\phi(n)_k$
  - $\Delta\phi'_u = \phi'_u - (\phi')^* = s(A'_0, \quad \pi'_u) - s(A'_0, \quad (\pi')^*)$
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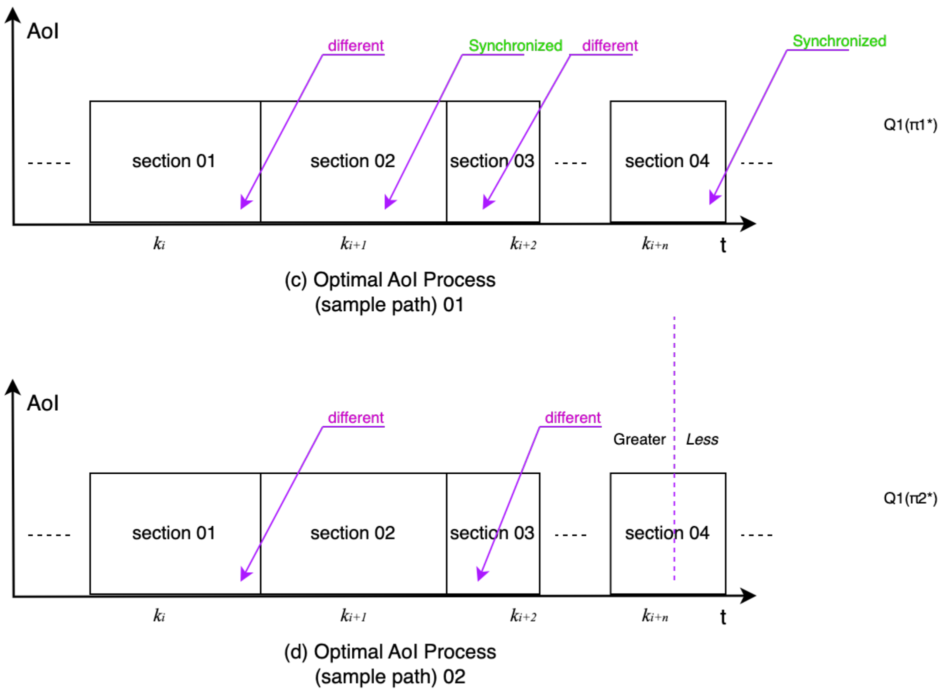
Proof (6) – different scope with limitation

- difference $\Delta\phi(n)'_u$ between k-step optimizing sample path $\mathcal{P}^*_{I-A_0}$ with $\phi(n)^*$ and optimal sample path $\mathcal{P}^*$ with $\phi(n)_k$
  - $\Delta\phi'_u = \phi'_u - (\phi')^* = s(A'_0, \quad \pi'_u) - s(A'_0, \quad (\pi')^*)$
  - $\phi^* = s(0, \quad \pi^*) \leq (\phi')^*$
  - $\Delta\phi'_u = \phi'_u - (\phi')^* \leq \phi'_u - \phi^* = s(A'_0, \quad \pi'_u) - s(0, \quad \pi^*)$
  - $\Delta\phi'_u \leq s(A'_0, \quad \pi'_u) - s(0, \quad \pi^*) = A'_0 \times (I_{l1} + X_{l1})$
  - $\Delta\phi'_u = \phi'_u - (\phi')^* \leq A'_0 \times (I_{l1} + X_{l1})$
  - $0 \leq \Delta\phi'_u \leq A'_0 \times (I_{l1} + X_{l1})$
- limited number of basic blocks between near-optimal and optimal
  - $k = \Delta\phi'_u / \phi_b$
  - $0 \leq k \leq [A'_0 \times (I_{l1} + X_{l1})] / \phi_b \ll \infty$

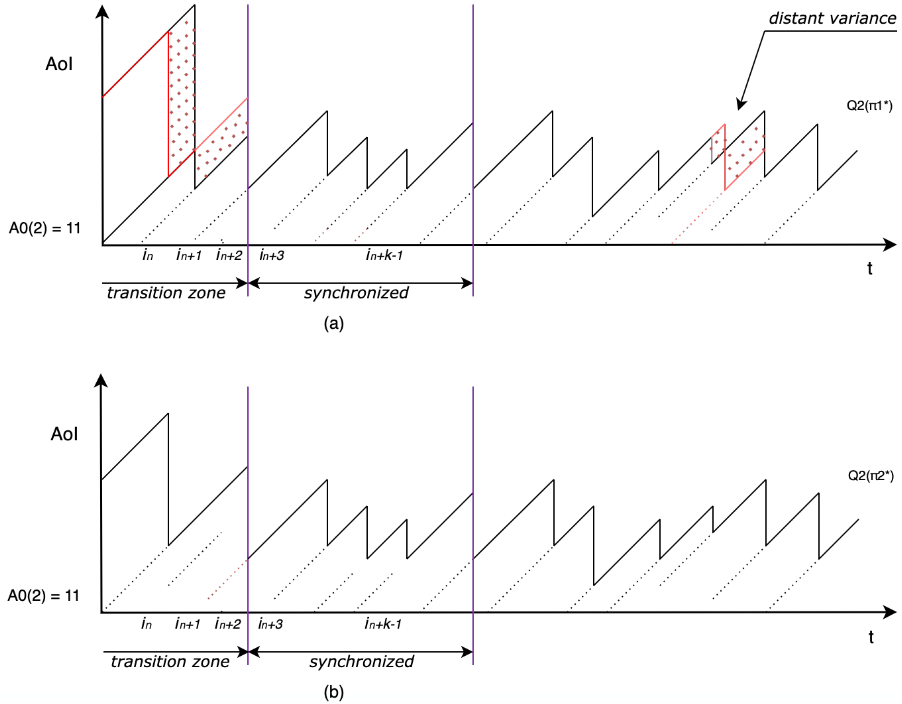
Proof (7) – syn property within sequence



- **Lemma 3 – synchronous and asynchronous:** With respect to cumulative age of information and under optimal policies, when comparing two optimal policies with identical initial ages, inter-arrival times and service times, any unbalanced parts of cumulative age of information of sample path within any section have to be counteracted within same section before completely synchronized sample paths under its optimal policy.

- **Definition**: Complete synchronization is defined as every packet delivered in same time instants and identical AoI sample paths.

Proof (8) – vicinity mutation



- **Lemma 4 – vicinity mutation but distantness invariant:** For two packet sequences with distinct initial ages but identical inter-arrival times and service times, in terms of the two corresponding two optimal AoIs, the difference in process between the two should only occur during the period from initiation up to, but not including, complete synchronization, and should not be present after synchronization is achieved.

Proof (9) – idle state tolerance

- **Lemma 5 – idle state tolerance:** when approximating optimal policies, a near-optimal policy can be obtained by mutating the optimal policy. Because of an idle state in the optimal policy has some extent tolerance in terms of policy mutation, the new near-optimal policy won’t have infinite impact on the optimal policy after policy mutation.

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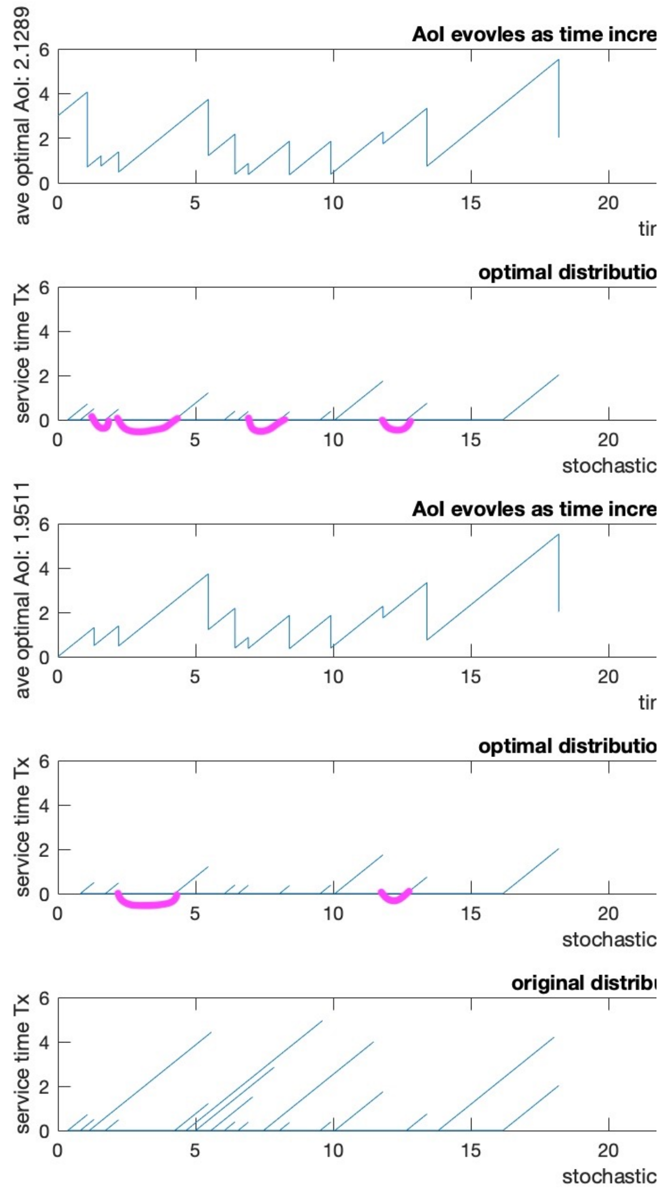
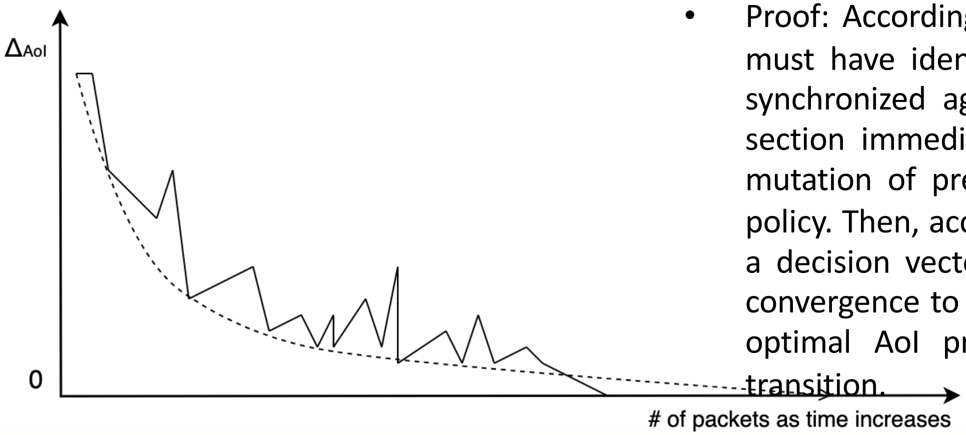


Proof (10) – synchronization occurrence

- **Theorem1 – asymptotic synchronization:** *For two conditions with different initial ages, but identical inter-arrival times and service times, the two optimized AoI processes under these two conditions must be completely synchronized after a certain packet.*

- Proof: According to Lemma3, because two sequences with different initial ages must have identical or different policies after the initial ages and will not be synchronized again in the third segment. According to Lemma 2, in the first section immediately following the initial age, near-optimal policy obtained by mutation of previous scheduling policy will gradually converge to the optimal policy. Then, according to Lemma 1, the AoI deduction caused by the decisions or a decision vector in the scheduling policy must be discrete. In the process of convergence to Q2(π2*), because of the discrete nature of convergence, the two optimal AoI processes will be completely consistent after period time of transition.



Proof (11) – syn with muti-sources

Long Sequence Segmentation

Theorem 2 – segmentation property: *For a given sequence with multiple source-receiver pairs by sharing one transmission channel, the global optimal sample path can be segmented into multiple local optimal epochs without jeopardizing future sample path evolution.*