Dear Professor Bejarano,

I am Jefferson Chandra, a student in your CS 25100 class, and when I was doing homework 4 question 6 I had an epiphany that I think is really cool to me. *Disclaimer: maybe someone found this already and I am just stating the obvious*. I will try to develop this theorem more and maybe incorporate it in the Dijkstra's Algorithm so that it might be able to find the shortest path of a weighted graph with negative weight edges (even if it is not supposed to, and even though we already have the Bellman-Ford's Algorithm). I know that proving question 6 with a counterexample is enough, but now that I have taken Discrete Mathematics and Real Analysis, I am not very satisfied and think that we need an actual reason and correlation between the two graphs, the original and the modified, rather than just proving by counterexample, even though we are taught in CS 182 that it is enough; so here is what I found. Please feel free to email me at jchandr@purdue.edu or reply to the message if you have any questions about my method, I would be very happy to discuss this with you. Thank you in advance and enjoy, hopefully :)

Sincerely,
Jefferson Chandra

Theorem:
$X - Y = X' - Y' - Z(n - m)$

- X = initial shortest path (distance)
- Y = initial longer path (distance)
- X' = X after graph is modified
- Y' = Y after graph is modified
- n = number of edges in path X
- m = number of edges in path Y
- $Z = \forall k \in \mathbb{Z}$ such that k is the number added to every edge in the graph, creating the modified graph, to "eliminate" negative weight edges

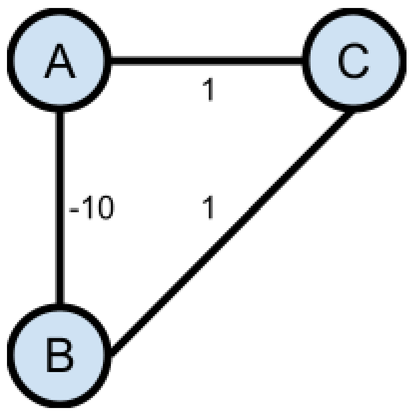
Suppose X and Y are different paths with same source and destination in a weighted graph G. Graph G has both positive and negative weight edges. Let the smallest weight in G be some negative integer $z < 0$.

Question 6 from Homework 4 proposes that by adding some integer $Z = ((-z) + 1)$ to each of the edges in graph G, creating graph G', the shortest path would be the same; and proven by counterexample, this proposition is false, as Y' can be shorter than X'.

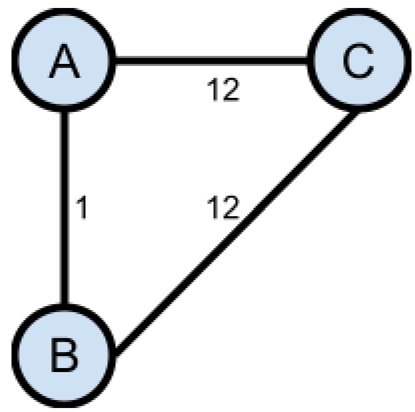
However, the theorem above has an additional component that might help determine the actual shortest path: $Z(n - m)$.

Example:

Original (G)



Add $Z = 11$ to every edge (G')



Suppose we are looking for the shortest path from vertex A to vertex C
The shortest path of G is $A \rightarrow B \rightarrow C$ with distance -9
The shortest path of G' is $A \rightarrow C$ with distance 12

Using the equation: $X - Y = X' - Y' - Z(n - m)$
$X = -9$ $(A \rightarrow B \rightarrow C)\ in\ G$
$Y = 1$ $(A \rightarrow C)\ in\ G$
$X' = 13$ $(A \rightarrow B \rightarrow C)\ in\ G'$
$Y' = 12$ $(A \rightarrow C)\ in\ G'$
$n = 2$ (count the arrows of the path X)
$m = 1$ (count the arrows of the path Y)
$Z = 11$

Therefore,
$X - Y = X' - Y' - Z(n - m)$
$-9 - 1 = 13 - 12 - 11(2 - 1)$
$-10 = 13 - 12 - 11(1)$
$-10 = 1 - 11$
$-10 = -10$
Q.E.D.

The reason is because the number that is added to every edge in the graph to “eliminate” negative weight edges, in this case 11, is only added once to the overall shortest path of the modified graph, but is added twice to the original graph, or that 22 is added to the overall path, which creates a longer path, because 11 < 22. By subtracting the shortest path of the modified path with z(n-m), with z being any integer that is added to every edge in the graph to “eliminate” negative weight edges, in this case 11, n being the number of edges in the original shortest path, and m being the number of edges in the original longer path, we can find the actual shortest path after running Dijkstra Algorithm.