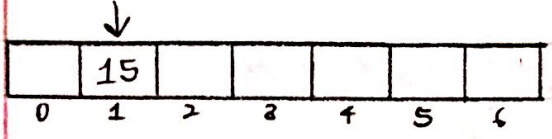
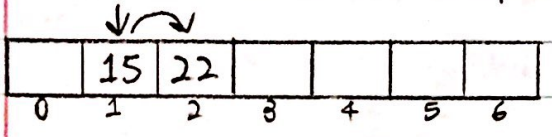
Problem 2 (Hash Table Analysis, 25%):

a) Insert 15 : $h(15) = 15\%7 = 1$ ※ Is position 1 empty? Yes.



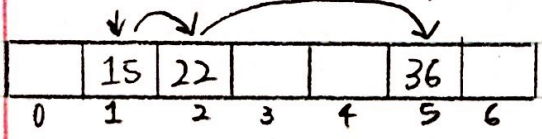
Load Factor $\alpha = \frac{\#\text{items in table}}{\text{table size}} = 1/7 < 1/2$

Insert 22 : $h(22) = 22\%7 = 1$. Is position 1 empty? No. Is position $(1+1^2)\%7 = 2$ empty? Yes.



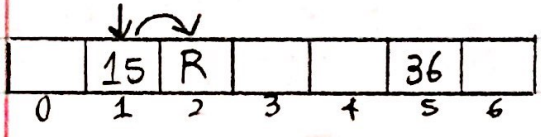
Load Factor $\alpha = 2/7 < 1/2$

Insert 36 : $h(36) = 36\%7 = 1$. Is position 1 empty? No. Is position 2 empty? No. Is position $(1+2^2)\%7 = 5$ empty? Yes.



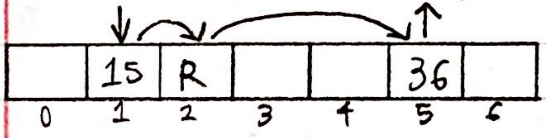
Load Factor $\alpha = 3/7 < 1/2$

Remove 22: $h(22) = 22\%7 = 1$. Is 22 at position 1? No. Is 22 at position 2? Yes.



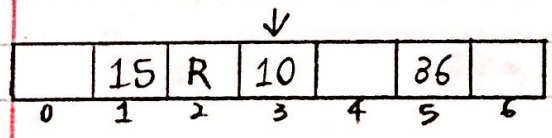
Load Factor $\alpha = 3/7 < 1/2$

Find 36: $h(36) = 36\%7 = 1$. Is 36 at position 1? No. Is 36 at position 2? No. Is 36 at position 5? Yes. Returned Found.



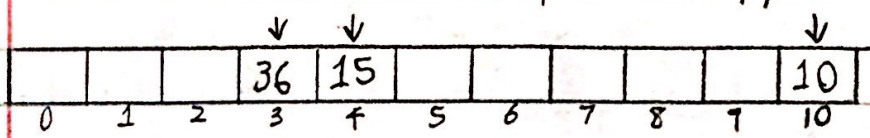
Load Factor $\alpha = 3/7 < 1/2$

Insert 10: $h(10) = 10\%7 = 3$. Is position 3 empty? Yes.



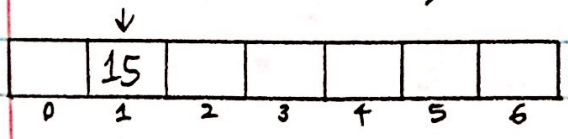
Load Factor $\alpha = 4/7 > 1/2$ ∴ Resize to table size 11!

Insert 15 : $h(15) = 15\%11 = 4$. Is position 4 empty? Yes.
Insert 10 : $h(10) = 10\%11 = 10$. Is position 10 empty? Yes.
Insert 36 : $h(36) = 36\%11 = 3$. Is position 3 empty? Yes.



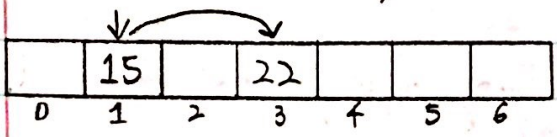
Load Factor $\alpha = 3/11 < 1/2$

b) Insert 15: $h(15) = 15\%7 = 1$, $h_2(15) = 3 - (15\%3) = 3$. Is position 1 empty? Yes.



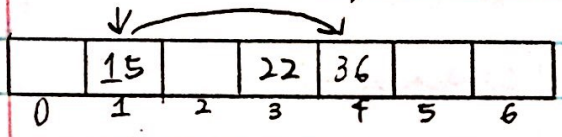
Load factor $\alpha = \frac{\#\text{ items in table}}{\text{table size}} = 1/7 < 1/2$

Insert 22: $h(22) = 22\%7 = 1$, $h_2(22) = 3 - (22\%3) = 2$. Is position 1 empty? No. position $[h(22) + 1 * h_2(22)]\%7$ $= 3$ empty? Yes.



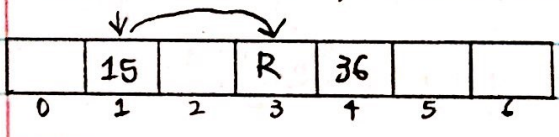
Load factor $\alpha = 2/7 < 1/2$

Insert 36: $h(36) = 36\%7 = 1$, $h_2(36) = 3 - (36\%3) = 3$. Is position 1 empty? No. position $[h(36) + 1 * h_2(36)]\%7$ $= 4$ empty? Yes.



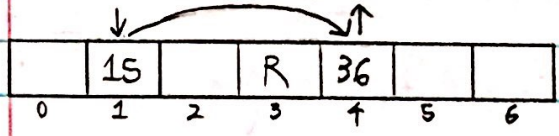
Load factor $\alpha = 3/7 < 1/2$

Remove 22: $h(22) = 22\%7 = 1$, $h_2(22) = 3 - (22\%3) = 2$. Is 22 at position 1? No. Is 22 at position $[h(22) + 1 * h_2(22)]\%7$ $= 3$? Yes.



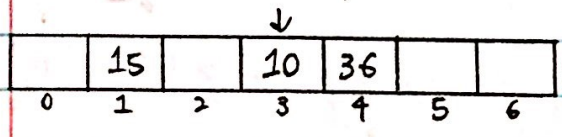
Load factor $\alpha = 3/7 < 1/2$

Find 36: $h(36) = 36\%7 = 1$, $h_2(36) = 3 - (36\%3) = 3$. Is 36 at position 1? No. Is 36 at position $[h(36) + 1 * h_2(36)]\%7$ $= 4$? Yes. Returned Found.



Load factor $\alpha = 3/7 < 1/2$

Insert 10: $h(10) = 10\%7 = 3$, $h_2(10) = 3 - (10\%3) = 2$. Is position 3 empty? Yes.



Load factor $\alpha = 3/7 < 1/2$

c) $\Pr[\text{All } j=3 \text{ positions we are checking for a key } k \text{ are set to true even though it's not in the set}] = (\frac{2}{3})^3$

For an element to be a false positive, all 3 hash functions must hit true in the Bloom filter, which takes up $2/3$ of the Bloom filter.

Let X be the total # of false positives out of 27. $x_i = \begin{cases} 1 \text{ if false positive} \\ 0 \text{ if not} \end{cases}$

$$\mathbb{E}[X] = \sum_{i=1}^{27} \mathbb{E}[x_i] = \sum_{i=1}^{27} \binom{27}{i} \left(\left(\frac{2}{3}\right)^3\right)^i \left(1 - \left(\frac{2}{3}\right)^3\right)^{27-i}$$

↳ Binomial Distribution

By Linearity of Expectations, $= 27 \times \left(\frac{2}{3}\right)^3$

$= 8$

∴ We expect 8 false positives on average.

Problem 3 (LRU Cache Implementation, 35%) Analysis:

1. Very Large Test – https://norvig.com/big.txt ≈ 1,088,424 words
   Large Test – Hamlet text ≈ 31,955 words
   Moderate Test 1 – 5,007 uniformly random words
   Moderate Test 2 – 5,326 short story (www.theshortstay.co.uk/wp-content/uploads/2016/02/short-stories-by-J-Goethe.pdf)

	4. Test(size)	2. Cache Capacity	# Left Rotations	# Right Rotations	3. Total Rotations	5. Avg # of Rotations	6. # Cache Misses	
Very Large Test	1mil	100	3,731,439	3,566,933	7,298,372	6	969,027	
	1mil	1,000	3,863,081	3,707,557	7,570,638	6	916,663	
	1mil	10,000	4,913,872	4,763,738	9,677,610	8	646,147	
Hamlet Text	31k	100	115,641	111,096	226,737	7	28,873	
	31k	1,000	153,831	149,706	303,537	9	22274	
	31k	10,000	219,124	217,745	436,869	13	0	
Mod. Test 1	5k	100	20,334	19,335	39,669	7	4,851	
	5k	1,000	41,978	41,431	83,409	16	3,643	
	5k	10,000	58,208	57,892	116,100	23	0	
Mod. Test 2	5k	100	20,934	20,197	41,131	7	4,219	
	5k	1,000	33,459	33,200	66,659	12	1,233	
	5k	10,000	32,902	32,775	65,677	12	0	

7. While the cache capacity increased by factors of 10 for both moderate-sized tests, the total # of rotations increased for the uniformly random data while it increased then decreased for the English text. There was a noticeable difference between the two tests.
   Average # of rotations: Moderate Test 1 had increasingly higher average # of rotations as cache capacity increased compared to Moderate Test 2 which stayed constant even as cache capacity increased by a factor of 10.