CS 325 (Homework 2)

Hao Deng

Problem 1:

By using Muster Method Version 2:

$T(n) = 3T(n-1) + 1$ | a = 3, b = 1, f(n) = 1 | for a > 0, b > 0, d >= 0

$1 = O(n^d) \rightarrow O(n^0)$

If $f(n)$ is $\Theta(n^d)$ then

$$T(n) = \begin{cases} \Theta(n^d), & if\ a < 1, \\ \Theta(n^{d+1}), & if\ a = 1 \\ \Theta(n^d a^{n/b}), & if\ a > 1. \end{cases}$$

Since a > 1, we plug in the values into $\Theta(n^d a^{\frac{n}{b}})$ for solving the problem.

$\Theta(n^d a^{\frac{n}{b}}) \rightarrow \Theta(n^0 3^{\frac{n}{1}})$

Answer: $T(n) = \Theta(3^n)$, as 3^n is the asymptotic bounds for T(n).

Problem 2:

a). Pseudocode for Ternary Search Tree:

```
//Linked list will be used for the TST
ternarySearch(root, value){

        if not root      //If the root is NULL, return 0
                return false;

        if value equals to root->data //If the value is the same as the root value
                return true;
//If the value is less than the node, then we should search the left of TST recursively
        else if value less than root->data
                then return ternarySearch(root->left, value);

        else if value greater root->data)//If the value is greater, then go the right
                then return ternarySearch(root->right, value);

        else
                then return ternarySearch(root->middle, value);  //Else go to the middle
}
```

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b). Recurrence:

$T(n) = T(n/3) + 2$ or $T(n) = T(n/3) + c$

c). Solving Recurrence:

By using the Master Method,

a=1, b=3, f(n) = Θ. f(n) is a constant, so it's also Θ(1).

$\log_3 1 = 0 \rightarrow n^0 \rightarrow \Theta(1)$. In this comparison with f(n), we can use case 2.

$T(n) = \Theta(\lg n)$

Comparison to the run time of binary search algorithm, they both tend to have the time complexity of Θ(lgn).

Problem 3:

a). Pseudocode:

```
maxMin(array){

    if array size = 1
        return array[0];

    //Takes the array and divide it by 2 for two separate arrays
    left_array = first half of the array;
    right_array = second half of the array;

    //Finds the max and min from the left array
    (left_min, left_max) = maxMin(left_array);
    //Finds the max and min from the right array
    (right_min, right_max) = maxMin(right_array);

    //Compare the max and min from the left array with the right array to determine the actual max and min values
    If left_min <= right_min, then actual_min = left_min;
        Else actual_min = right_min;
    If left_max <= right_max, then actual_max = right_max;
        Else actual_max = left_max;

    Return (actual_min, actual_max);
}
```

b). Recurrence:

$T(n) = 2T(n/2) + 2$ or $T(n) = 2T(n/2) + c$

c). Solving the recurrence:
By using the Master Method,

a=2, b=2, f(n) = $\Theta(1)$.
$Log_2 2 = 1$, $n^{log_2 2} = n$, case 1 since $n > \Theta(1)$
$T(n) = \Theta(n)$.

By using recursion, the time is linear. With iterative algorithm, it will still going to $\Theta(n)$.

Problem 4:

a). Pseudocode for 4-way merge sort

```
mergeSort(array, left, right){

        if left value is less than right value{

                middle = the middle of the whole array
                left_middle = the middle of the first half array
                right_middle = the middle of the second half array

        //Since it's a 4 way merge sort, calling the function 4 times with different range pass in
        mergeSort(array, first left array)
        mergeSort(array, second left array)
        mergeSort(array, first right array)
        mergeSort(array, second right array)

        //calling the merging function to merge the arrays
        merging(array, all the middle points)
        }
}
```

b). Recurrence and Solution

Recurrence: $T(n) = 4*T(n/4)+n$

Solution: By using the Master Method,

a=4,b=4,f(n)=n;
$\log_4 4 = 1$, $n^{\log_4 4} = n$, case 2
$T(n) = \Theta(n \lg n)$.

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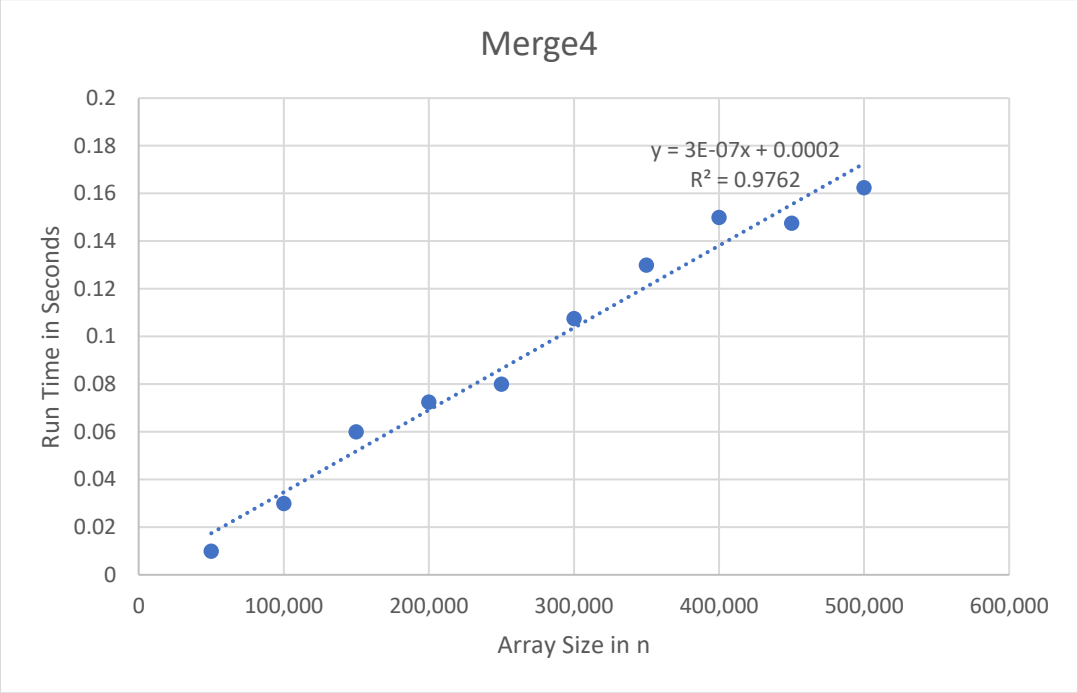
Problem 5:

c). 4 Way Merge Sort

Array Size	Run Time 1	Run Time 2	Run Time 3	Run Time 4
50000	0.01	0.01	0.01	0.01
100000	0.03	0.03	0.03	0.03
150000	0.06	0.06	0.06	0.06
200000	0.08	0.07	0.07	0.07
250000	0.08	0.08	0.08	0.08
300000	0.11	0.1	0.11	0.11
350000	0.13	0.13	0.13	0.13
400000	0.15	0.15	0.15	0.15
450000	0.12	0.17	0.18	0.12
500000	0.15	0.15	0.2	0.15

d).



The type of curve best fits to this data set would be a supposed linear curve despite the complexity is $O(n \log_4 n)$. Since this is presented in a large data scale, the n will conquer the $\log_4 n$. The line of best fit is labelled in the graph.

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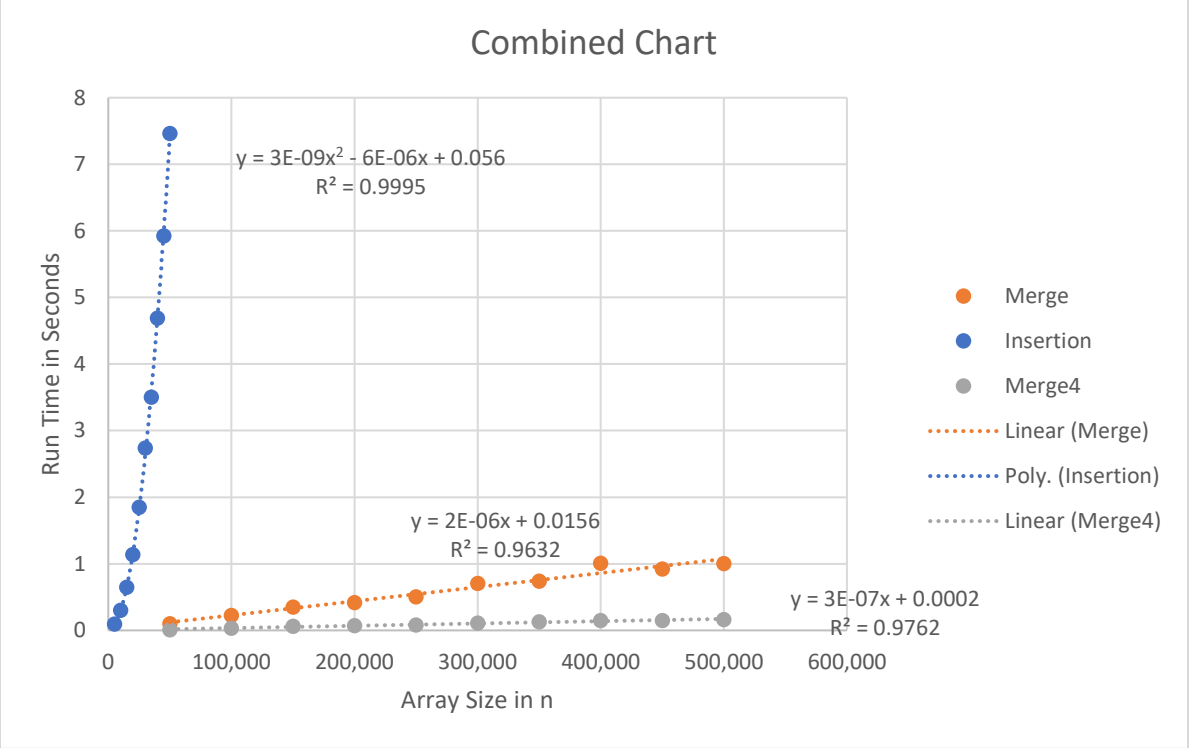
e).



4 way merge sorting is faster than both insertion and merge sort because 4 way merge breaks the array into four parts and sort them instead of just breaking down into two like the regular merge sort.

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Modified Timing Code

```
//Reference to my previous merge sort program
//Merge sort reference:
//https://www.geeksforgeeks.org/merge-sort/
int main(){

    srand (time(NULL));
     clock_t t;
    int array_size;
    int temp_var;

    int n =0;
    int random;

    for(int i=0;i<10;i++){
        n = n + 50000;
        int array[1000000];
        for(int j=0;j<n;j++){
            random = rand()%100001;
            array[j] = random;
        }


        t = clock();

            // Calling the merge function
        mergeSortFourWays(array, n-1);

        t = clock() - t;

        cout<<"Array Size: "<<n<<endl;
        cout<<"Time: "<<((float)t)/CLOCKS_PER_SEC << " seconds"<<endl;
        cout<<endl;

    }


    return 0;
}
```