EXECUTIVE SUMMARY

In a dynamic system, the equation of motion can give a glimpse of the movements of the overall system. However, as the system becomes more advanced and becomes dependent on two different motions, the equation of motion becomes more complicated. To study this, the vibrations and dynamics of a simplified washing machine is investigated.

In assuming a single degree of freedom (DOF) mechanical system, most complex dynamic studies can be simplified into a few manageable, and calculatable equations. The design and operation of a vibration absorber is studied through an ideal washing machine, elevated through a pedestal as seen in Figure 1. This system can be idealized into a single DOF mechanical system as seen in Figure 2, after assuming a constant equivalent mass and lateral stiffness from the pedestal. In order to counter act the deflection in the pedestal due to the offset, rotating mass, a vibration absorber is attached below the pedestal (as seen in Figure 3); the carbon steel pipe's length can be calculated to match its bending stiffness to counteract the rotating mass. This creates a 2-DOF mechanical system (as seen in Figure 4), that cannot be idealized into a 1-DOF mechanical system.

After performing said calculations, the addition of a vibration absorber as seen in Figure 7, does not perform a better job compared to an existing washer without one. Seen in Figure 5 and Figure 9, the 2-DOF system is only achieving a smaller amplitude of vibration during the natural frequency of the original 1-DOF system.

METHOD

A special note regarding the units of mass. In Imperial 'English' units, the unit for mass is defined to be $lb_f * s^2/in$. To convert a mass from lb_f, simply divide the number by $g = 32.2$ and then divide by 12 to convert from feet to inches.

The secondary mass of a carbon steel pipe with a $0.25\ in$ nominal diameter, Schedule 40, has a moment of inertia of $0.00331\ in^4$ and an elastic modulus of $32{,}634\ ksi$ [6][2]. The pipe has a diameter internal of $0.364\ in$ [6]. This pipe has a yield strength of $61.35\ ksi$ [2].

1-DOF Analysis:

The equation of motion (EOM) for a 1-DOF system can be written as follows:

$$M_e\ddot{y} + C_e\dot{y} + K_e y = F(t) \quad (1)$$

From basic steady state responses of systems due to periodic loads, the following equations can be assumed and stay true:

$$M_e = M + m_u \quad (2)$$

$$\omega_n = \sqrt{\frac{K_e}{M_e}} \quad (3)$$

$$\zeta = \frac{C_e \omega_n}{2K_e} \quad (4)$$

$$r = \frac{\omega}{\omega_n} \quad (5)$$

$$J(r) = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \quad (6)$$

From a response due to a mass imbalance in a single DOF, the system periodic response can be represented as the following:

$$Y(t) = e * \frac{m}{M} * J(r) * \sin(\omega t + \psi) \quad (7)$$

2-DOF Analysis:

With the addition of a 10 lbm carbon steel pipe (1/4” nominal diameter, Schedule 40), the system becomes 2-DOF. The bending stiffness of the pipe, K_A, will be found in order to match the natural frequency of the original system in Figure 2 to $\sqrt{\frac{K_A}{M_A}}$.

The equation of motion (EOM) for a 2-DOF system can be written as follows:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{x_1} \\ \ddot{x_2} \end{bmatrix} + \begin{bmatrix} c & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} f_1 t \\ 0 \end{bmatrix} \tag{8}$$

Additional eigenvalue analysis will be performed to find the natural frequencies of the two masses and find the amplitudes of vibration of the 2-DOF system.

PROCEDURE

<u>1-DOF Analysis:</u>

From Figure 1 and Figure 2, the equivalent mass and lateral stiffness of the pedestal can be found as shown:

$$k_{pedestal} = \frac{F}{Max\ deflection} = \frac{100}{0.013} = 7692.307\ lbf/in \tag{9}$$

$$m_{pedestal} = \frac{k}{{\omega_n}^2} = \frac{7692.307}{(44.7 * 2 * \pi)^2} = 0.0975173\ snails = 37.68\ lb \tag{10}$$

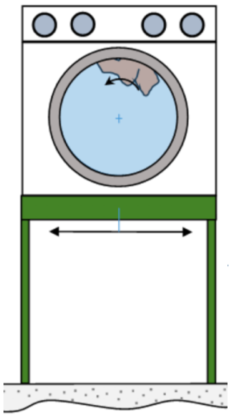

Figure 1: 1-DOF System

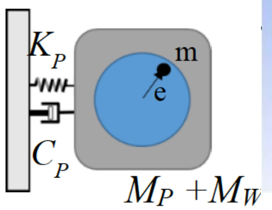

Figure 2: Equivalent System

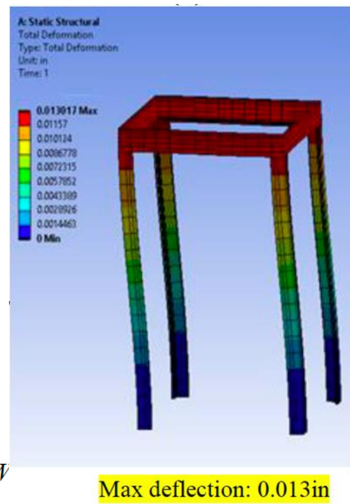

Figure 3: Max Deflection

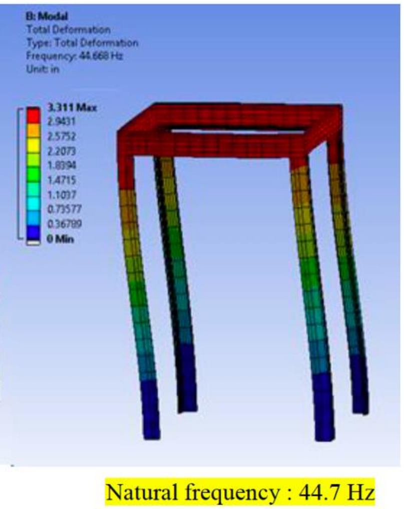

Figure 4: Natural Frequency

Delgado [1]

Assuming the small towel, with a mass of 0.125 lb, is located on the inner edge of the washing machine, making the imbalance displacement equal to half of the inner diameter. If the vibration amplitude of 0.25 in occurs at the maximum speed of the spinning cycle of 1400 rpm (146.61 $rad/$s), the system damping ratio can be back calculated.

$$M = m_{pedestal} + m_{towel} + m_{washer} = 137.806\ lb = 0.35664\ snails \tag{11}$$

$$\omega_n = \sqrt{\frac{K}{M}} = \sqrt{\frac{7692.307}{0.35664}} = 146.86\ rad/s \tag{12}$$

$$r = \frac{\omega}{\omega_n} = \frac{146.61}{146.86} = 0.998259 \tag{13}$$

$$0.25 = 7 * \frac{\frac{\frac{0.125}{32.2}}{12}}{0.35664} J(r = 0.998259) \tag{14}$$

$$J(r = 0.998259) = 39.373 \quad (15)$$

$$J(r = 0.998259) = 39.373 = \frac{0.998259^2}{\sqrt{(1 - 0.998259^2)^2 + (2 * \zeta * 0.998259)^2}} \quad (16)$$

$$\zeta = 0.01255 \quad (17)$$

The damping ratio, ζ, can be used with equation (4) to calculate C_e.

$$C_e = \frac{2\zeta K_e}{\omega_n} = \frac{2\zeta K_e}{\omega_n} = 1.315\ lbf\ s/in \quad (18)$$

Putting together, the EOM for the system in Figure 2 is as follows:

$$0.35664\ddot{y} + 1.315\dot{y} + 7692.307y = F(t) \quad (19)$$

For a generic drum speed, ω, the amplitude of displacement and phase lag can be calculated with equation (7). As seen in Figure 5, the amplitude of displacement is plotted against the drum speed, and as seen in Figure 6, the phase lag is plotted against the drum speed.

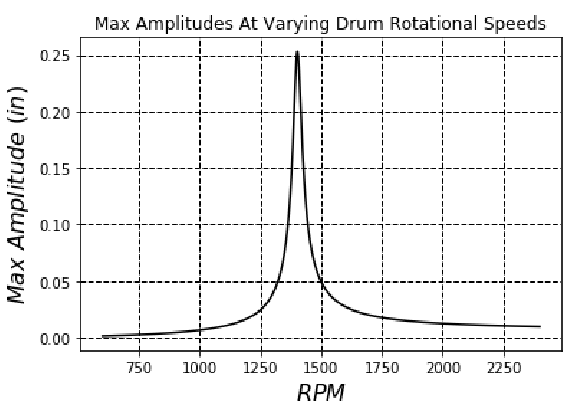

Figure 5: Max Amplitude

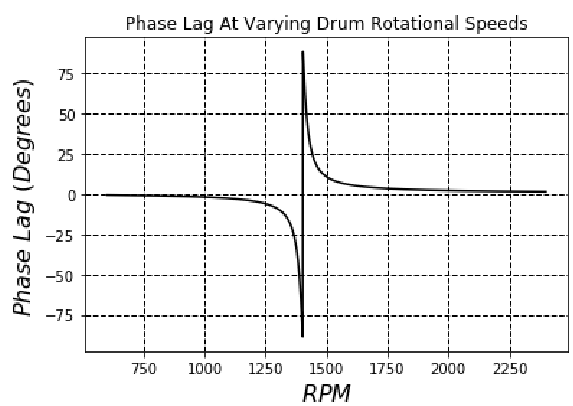

Figure 6: Phase Lag

<u>2-DOF Analysis:</u>

With the addition of the steel pipe, as seen in Figure 7, the system is now 2-DOF. The system can be reduced to Figure 8.

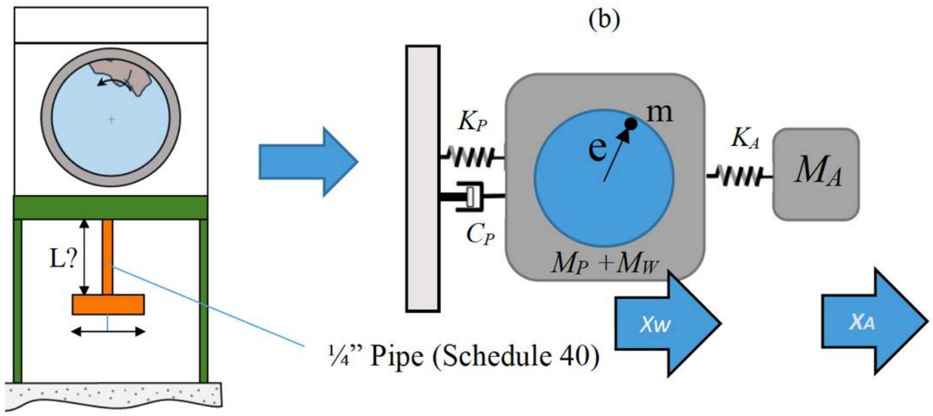

Figure 7: 2-DOF System

Figure 8: Equivalent System

Delgado [1]

Finding the bending stiffness of the pipe is as follows:

$$\sqrt{\frac{K_A}{M_A}} = \omega_n \tag{20}$$

$$K_A = 558.17\ lbf/in \tag{21}$$

Using this, the length of the pipe can be calculated as follows:

$$\sigma_{max} = \sigma_{yield} = \frac{FL}{I} r = \frac{KY_{op}L}{I} r \tag{22}$$

$$L = 7.9\ in \tag{23}$$

From this length, the max deflection can be found as follows:

$$\delta = \frac{kxL^3}{3EI} = 0.21485 \text{ in} \tag{24}$$

In order the derive the EOM for this 2-DOF, a matrix format is used as shown.

$$\begin{bmatrix} 0.3567 & 0 \\ 0 & 0.026 \end{bmatrix} \begin{bmatrix} \ddot{x_w} \\ \ddot{x_A} \end{bmatrix} + \begin{bmatrix} 1.315 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x_w} \\ \dot{x_A} \end{bmatrix} + \begin{bmatrix} 8{,}250.48 & -558.2 \\ -558.2 & 558.2 \end{bmatrix} \begin{bmatrix} x_w \\ x_A \end{bmatrix} = \begin{bmatrix} F(t) \\ 0 \end{bmatrix} \tag{25}$$

If the two masses are elastically couples and the system is treated as undamped, the natural frequencies for the system can be found. This reduces to the following when assuming a periodic, and harmonic motion of $x = \text{acos}\,(\omega t)$:

$$-\omega^2 \begin{bmatrix} 0.35664 & 0 \\ 0 & 0.0259 \end{bmatrix} \begin{bmatrix} x_w \\ x_A \end{bmatrix} + \begin{bmatrix} 8{,}250.477 & -558.17 \\ -558.17 & 558.17 \end{bmatrix} \begin{bmatrix} x_w \\ x_A \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{26}$$

Hence, the determinate of the eigenvalue problem must be set to 0, giving the following:

$$\begin{vmatrix} -m_1\omega^2 + k_1 + k_2 & -k_2 \\ -k_2 & -m_2\omega^2 + k_2 \end{vmatrix} = 0 \tag{27}$$

After algebra and quadratic factorization, the values for ω_{n1} and ω_{n2} can be found:

$$\omega_{n1} = 167.9398 \frac{rad}{s} \tag{28}$$

$$\omega_{22} = 128.3786 \frac{rad}{s} \tag{29}$$

The modal vectors are also found to be the following:

$$X1 = \begin{bmatrix} -1.101 \\ -4.681 \end{bmatrix}, X2 = \begin{bmatrix} -1.262 \\ 4.086 \end{bmatrix} \tag{30}$$

The 2-DOF system will have amplitudes of motion seen in the figure below:

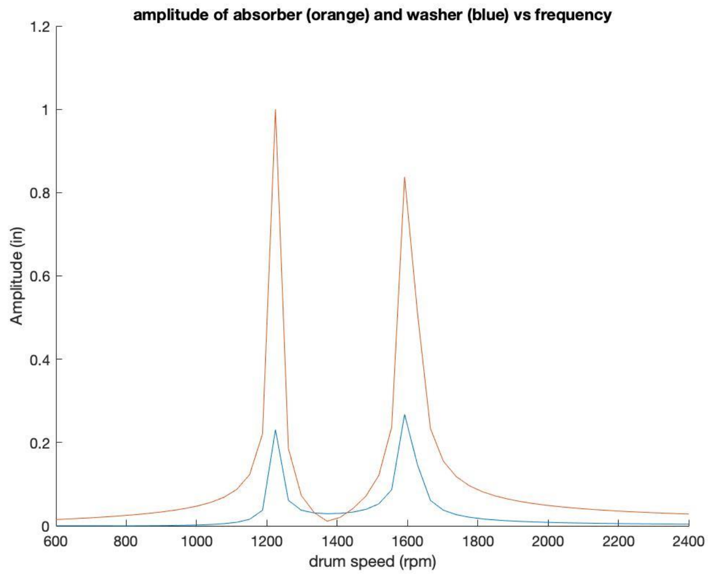

Figure 9: 2-DOF Amplitude Response

RESULTS AND DISCUSSION

In a system where an amplitude of vibration is caused by a mass imbalance rotating, the two key findings are the systems amplitude and phase of motion. The amplitude is the displacement the system will achieve due to the imbalance rotation, and the phase of motion is related to the differences between the location of the washing machine with respect to the pedestal. The natural frequency relates to the motions produced when operating a washing machine with no load and no external forces. However, when the washing machine is operated, the users have a choice of increasing speeds above or below this natural frequency.

The max amplitude of motion in the 1-DOF system is approximately $0.253\ in$ as seen in Figure 5, which occurs at roughly $1375\ rpm$.

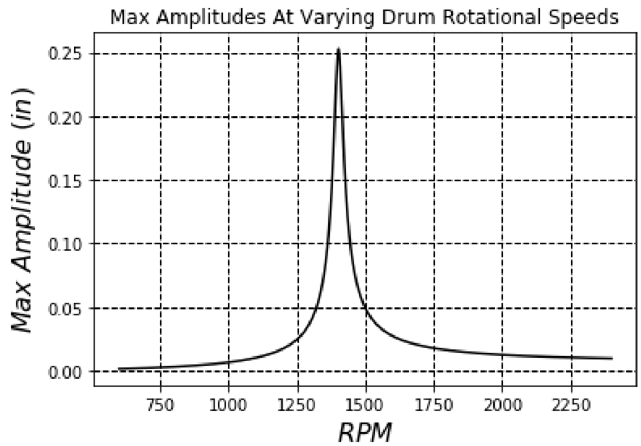

Figure 5: Max Amplitude

With the addition of a steel pipe, seen in Figure 7, the length of the pipe is to be $7.9\ in$ to act as a vibration absorber.

As seen in Figure 9, the addition of the secondary mass, the absorber system, does not actually perform a good job. During the natural frequency of the original, 1-DOF system, the addition of the absorber system does decrease the amplitude of frequency response. These frequencies range from approximately $22\ Hz$ to $25\ Hz$.

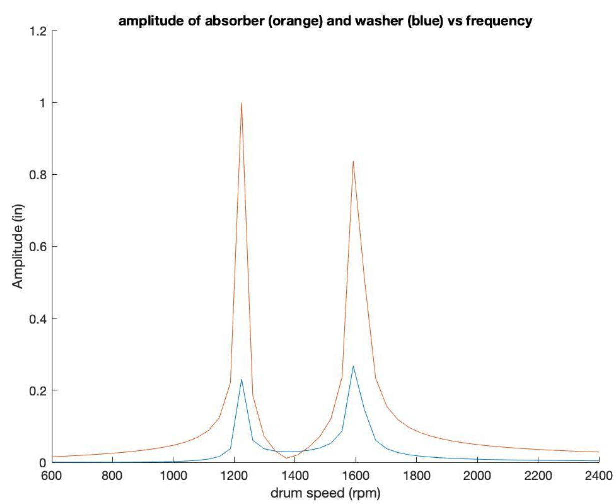

Figure 9: 2-DOF Amplitude Response

Although the max amplitude of vibration of the whole system decreases at the washer's natural frequency, the max amplitude becomes much larger at other frequencies. At the natural frequency of the 2-DOF system, $\omega_{n1} = 167.9398 \frac{rad}{s}$ and $\omega_{n2} = 128.3786 \frac{rad}{s}$, the maximum amplitude is nearly four times as large when no absorber system is attached. However, at the operation speed of $1400\ rpm$, the vibration magnitude is less than $0.05\ in$. This could be improved by changing the placement of the vibration absorber.

Most standard washing machines have built in vibration absorbers. This is usually located on the bottom of the machine, also acting as a padding between the surface and the machine. This can be observed in Figure 10 through Knivel's washing machine foot pads.

Figure 10: Washing Machine Foot Pads, Knivel [3]

From a lack of pedestal, the system is more efficient in reducing vibration as there is no medium to create vibrations. The pedestal itself acts as a stiffness spring, and with the removal of this, the maximum deflection is reduced.

If a pedestal is attached to the washing machine, it usually includes a base to absorb as much vibration before reaching the stable, ground. As seen in Figure 11, the commercially sold Samsung FlexWash pedestal is flat in all faces.

Figure 11: Samsung FlexWash pedestal, Samsung [7]

From these findings, the system used in Figure 7 does not accurately represent a typical washer and pedestal system. It is found that most washing machines commercially sold include a shock absorber that transmits the vibrations from the drum to the frame. These absorbers are attached on the base of the frame and connected to the outer tub [5]. A picture of one is seen in Figure 12.

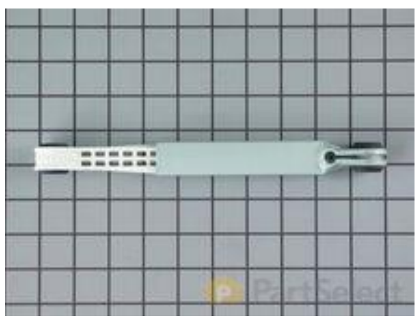

Figure 12: Shock Absorber, PartSelect [5]

These shock absorbers or vibration absorbers are useful in other situations besides mitigating movements of washing machines. These parts can be found in cars, trucks, and essentially all vehicles today. These damping forces reduce the movements when driving over potholes, bumps, and uneven surfaces. An example of a shock absorber on a car is seen in Figure 13.

Figure 13: A Car and Shock Absorber, Olive Press [4]

CONCLUSION

In a washing machine, vibrations will occur due to the mass imbalance of the load inside. The equation of motion for this example from Figure 1 is shown below.

$$0.35664\ddot{y} + 1.315\dot{y} + 7692.307y = F(t) \tag{31}$$

While observing this, a 1-DOF system can be used to simplify calculations. As seen in Figure 5, the maximum amplitude of vibration occurs at roughly $1375\ rpm$, with approximately a magnitude of 1/4th of an inch.

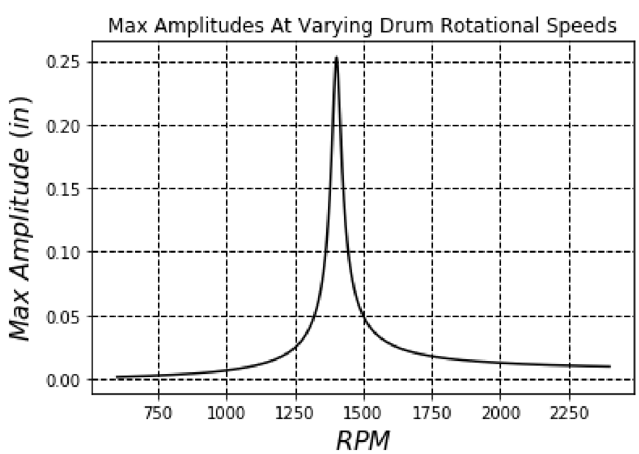

Figure 5: Max Amplitude

The phase angle, or lag, can also be calculated when observing the movement of the washing machine with respect to the pedestal. This can be seen in Figure 6.

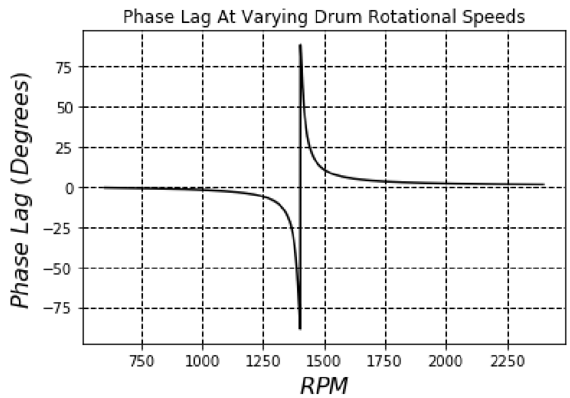

Figure 6: Phase Lag

To mitigate this vibration, it was believed that adding a pipe, found to be roughly $8in$ in length, will improve results. This changes the equation of motion to be as below.

$$\begin{bmatrix} 0.3567 & 0 \\ 0 & 0.026 \end{bmatrix} \begin{bmatrix} \ddot{x_w} \\ \ddot{x_A} \end{bmatrix} + \begin{bmatrix} 1.315 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x_w} \\ \dot{x_A} \end{bmatrix} + \begin{bmatrix} 8{,}250.48 & -558.2 \\ -558.2 & 558.2 \end{bmatrix} \begin{bmatrix} x_w \\ x_A \end{bmatrix} = \begin{bmatrix} F(t) \\ 0 \end{bmatrix} \quad (32)$$

From the addition of the hanging pipe, it was found that the pipe did not improve results. The vibrations increased up to almost 4 times larger at certain frequencies of operation. Only near the frequencies of 1320 to $1500\ rpm$, did the absorber reduce vibration. The new peaks of vibration correspond to the natural frequencies of the 2-DOF system, being $\omega_{n1} = 167.9398 \frac{rad}{s}$ and $\omega_{n2} = 128.3786 \frac{rad}{s}$. With an operation speed of $1400\ rpm$, the vibration magnitude is less than $0.05\ in$. This can be visualized in Figure 9.

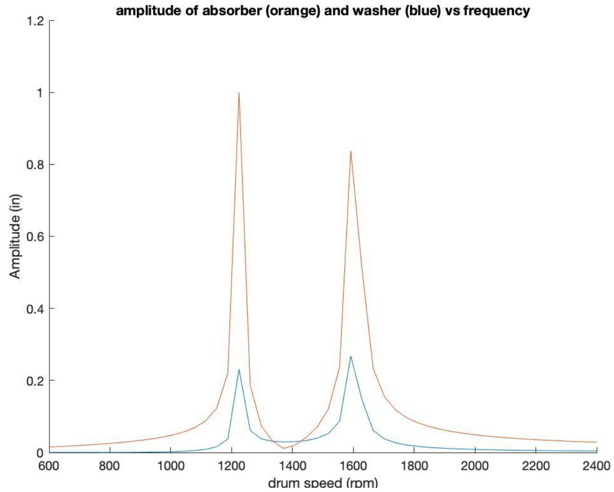

Figure 9: 2-DOF Amplitude Response

This of course is not an accurate representation of commercially sold washing machines. Most washing machines have internal mechanism such as shock absorbers, place in particulate locations, to reduce vibrations. In addition, pedestals added to washing machines are designed to reduce movements and create a solid foundation. The pedestals or washing machines may include padding to help reduce movement as well.

REFERENCES

[1] A Delgado. (2020). "MEEN 363 FALL 2020 – ASSIGNMENT # 2: Design and operation of a vibration absorber" Texas A&M University, 2020.

[2] Croxon, J. (n.d.). Schedule 40 Steel Pipe Technical Specifications. Retrieved from https://www.hunker.com/13401145/schedule-40-steel-pipe-technical-specifications

[3] Knivel. (n.d.). Washing Machine Foot Pads for Anti-vibration and Anti-Walk | Washer and Dryer Anti Slip Mat with Rubber Technology | Ideal for Vibration Reduction and Shock Absorber | Rubber Isolation Feet Set of 4 [Digital image]. Retrieved from https://www.amazon.com/Anti-vibration-Anti-Walk-Technology-Vibration-Reduction/dp/B07C6Z8B36/ref=asc_df_B07C6Z8B36/?tag=hyprod-20&linkCode=df0&hvadid=309760836873&hvpos=&hvnetw=g&hvrand=11663859519054323986&hvpone=&hvptwo=&hvqmt=&hvdev=c&hvdvcmdl=&hvlocint=&hvlocphy=9027902&hvtargid=pla-569410277499&psc=1

[4] Olive Press. (2019, December 26). What is a Shock Absorber and when do you need to change it? Retrieved from https://www.theolivepress.es/spain-news/2019/12/27/what-is-a-shock-absorber-and-when-do-you-need-to-change-it/

[5] PartSelect. (n.d.). How to Fix A Washer That Shakes and Moves. Retrieved from https://www.partselect.com/Repair/Washer/Shakes-Moves/

[6] Penn Machine. (n.d.). Steel Pipe Specifications Schedule 40. Retrieved from https://www.pennusa.com/products/schedule_40.php

[7] Samsung. (n.d.). 6 Inch Height FlexWash™/FlexDry™ Pedestal: Black Stainless Steel [Digital image]. Retrieved from https://www.ajmadison.com/cgi-bin/ajmadison/WE272NV.html