Subsets and Equality

Definition

To say that a set A is a *subset* of a set B, denoted $A \subseteq B$, means:

$$\forall a \in A, a \in B$$

or more conveniently for proofs:

$$x \in A \implies x \in B$$

Definition

To say that a set A equals a set B, denoted $A = B$, means:

$$A \subseteq B \text{ and } B \subseteq A$$

or more conveniently for proofs:

$$x \in A \iff x \in B$$

Definition

To say that A is a *proper* subset of B, denoted $A \subset B$, means that $A \subseteq B$ but $A \neq B$. If $A = B$ then A is called an *improper* subset of B.

Theorem

For all sets A:

1). $\emptyset \subseteq A$

2). $A \subseteq A$

3). $A \subseteq \mathcal{U}$

The proofs follow trivially from the definitions.