

Homework #9 Solutions

Problem

You work for a large conglomerate of 100 associated companies. In order to avoid antitrust issues with the DOJ, you need to divide the 100 companies into 4 different groups such that:

1. Companies within the same group cannot do business with each other.
2. Companies in different groups can do business with each other.

What organization of the 100 companies into the 4 groups maximizes the number of business opportunities?

- a) Start by labeling the groups X , Y , Z , and W . Let x = the number of companies assigned to group X and so on for the other groups. Construct an equation in x for the number of business opportunities for a company in group X — i.e., how many companies can that company do business with?

Each company in X can do business with the other $100 - x$ companies.

- b) Now build an equation in x for the total number of business opportunities for all companies in group X .

There are x companies, each of which can do business with $100 - x$ companies, for a total of $x(100 - x)$ business opportunities for all of the companies in X .

- c) Do likewise for the remaining groups and construct a function $f(x, y, z, w)$ that gives the total number of business opportunities across all the groups.

$$f(x, y, z, w) = x(100 - x) + y(100 - y) + z(100 - z) + w(100 - w)$$

- d) What is the constraint on x , y , z , and w ?

$$x + y + z + w = 100$$

- e) Introduce a Lagrange multiplier λ and determine $f_x = \lambda g_x$, where g is the function constructed from the above constraint.

$$100 - 2x = \lambda$$

- f) Do likewise for f_y , f_z , and f_w , and combine them with the constraint so that you have 5 equations in 5 unknowns.

$$100 - 2x = \lambda$$

$$100 - 2y = \lambda$$

$$100 - 2z = \lambda$$

$$100 - 2w = \lambda$$

$$x + y + z + w = 100$$

g) Use substitution to determine a value for λ .

$$2x = 100 - \lambda$$

$$x = 50 - \frac{\lambda}{2}$$

$$y = 50 - \frac{\lambda}{2}$$

$$z = 50 - \frac{\lambda}{2}$$

$$w = 50 - \frac{\lambda}{2}$$

$$4 \left(50 - \frac{\lambda}{2} \right) = 100$$

$$50 - \frac{\lambda}{2} = 25$$

$$\frac{\lambda}{2} = 25$$

$$\lambda = 50$$

h) Substitute the value for λ into the other equations to determine the optimal distribution of companies into the 4 groups.

$$x = 50 - \frac{50}{2} = 50 - 25 = 25$$

$$y = 25$$

$$z = 25$$

$$w = 25$$