Math-19 Homework #1

1. Let:

$$
\begin{aligned}
P &:= 0 \text{ is a positive number} \\
Q &:= 2 \geq 2 \\
R &:= \forall n, m \in \mathbb{N}, n + m \in \mathbb{N}
\end{aligned}
$$

   Determine whether the following compound statement is true or false:

   P and Q and R or P and not Q and R or not P and Q and R

   Start by rewriting the statement with parentheses to show operation order, then substitute the truth value for each individual statement, and then show the stepwise evaluation to the final result.

2. There is a theorem called DeMorgan's Theorem that helps us negate complex logical statements.

   (a) Consider the statement "not (A and B)". We know (A and B) is false whenever either A, B, or both are false, so not (A and B) = (not A) or (not B). Find a similar result for not (A or B).

   (b) Remember that $\forall x, P(x)$ can be viewed as a big compound AND statement. Since it will be false whenever there exists an x value for which $P(x)$ is false, we can conclude that: not $(\forall x, P(x)) = \exists x, (not\ P(x))$. Find a similar result for not $(\exists x, P(x))$.

3. Classify each of the listed numbers by putting an 'X' in the appropriate columns (Hint: some numbers will be in more than one set).

	$\mathbb{N}$	$\mathbb{W}$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{R} - \mathbb{Q}$	$\mathbb{R}$
0						
$\frac{4}{2}$						
-3						
1.036						
$10.14\overline{23}$						
$\sqrt{2}$						
$-\pi$						

4. Decimal to rational form conversion.

   (a) Convert $0.14\overline{23}$ to rational form.

   (b) Show that $0.\overline{1} = \frac{1}{9}$. If this is so, then $\frac{2}{9}$ should equal $0.\overline{2}$, $\frac{3}{9}$ should equal $0.\overline{3}$, and so on until $\frac{8}{9}$ should equal $0.\overline{8}$. So, what does $0.\overline{9}$ equal? Show that this is so by converting $0.\overline{9}$ to rational form.

5. Let:

$$\begin{aligned} A &= \text{the set of all positive numbers} \\ B &= \{x \in \mathbb{R} | -3 < x \leq 3\} \end{aligned}$$

Represent each set in interval notation and graph each set on separate real number lines. Determine $A \cup B$, $A \cap B$, and $A - B$, showing the results in both interval notation and graph form.

6. Solve by finding the LCM:

$$\frac{3}{8} + \frac{2}{9} - \frac{1}{12}$$

Show the prime factorization for each denominator and how you used the prime factorizations to determine the LCM.

7. Simplify completely:

$$\frac{\frac{5}{6} - \left(\frac{1}{2} + \frac{2}{3}\right)}{\frac{1}{10} + \frac{3}{15}}$$

8. Prove: $\forall a \in \mathbb{R} - \{0\}$, the multiplicative inverse a^{-1} is unique.

9. Prove: $\forall a, b \in \mathbb{R}, a(-b) = -(ab)$. You are only allowed to use the properties in the box on page 3 and properties 1 and 2 in the box on page 4. Make sure that your proof is syntactically complete, with a justification for each step.

10. Prove: $\forall a, b \in \mathbb{R}, |a - b| = |b - a|$ using the properties up to and including those in the box at the top of page 9. Make sure that your proof is syntactically complete, with a justification for each step. Do *not* just justify this based on the definition of distance.