Norm and Trace

Definition: Conjugate

Let $\alpha = r + s\sqrt{d} \in \mathbb{Q}(\sqrt{d})$. The *conjugate* of α is given by:

$$\alpha' = r - s\sqrt{d} \in \mathbb{Q}(\sqrt{d})$$

Note that when $d < 0$ then $\alpha' = \overline{\alpha}$.

Definition

Let $\alpha = r + s\sqrt{d} \in \mathbb{Q}(\sqrt{d})$. The *norm* of α is given by:

$$N(\alpha) = \alpha\alpha'$$

The *trace* of α is given by:

$$T(\alpha) = \alpha + \alpha'$$

Note that $N(\alpha) = r^2 - ds^2$ and $T(\alpha) = 2r$, and so $N(\alpha), T(\alpha) \in \mathbb{Q}$.

Properties

Let $\alpha = r + s\sqrt{d}, \beta = u + v\sqrt{d} \in \mathbb{Q}(\sqrt{d})$:

1). $T(\alpha + \beta) = T(\alpha) + T(\beta)$

2). $N(\alpha\beta) = N(\alpha)N(\beta)$

3). $N(\alpha) = 0 \iff \alpha = 0$

4). $\alpha \neq 0 \implies \alpha^{-1} = \frac{\alpha'}{N(\alpha)}$

5). $d < 0 \implies N(\alpha) \geq 0$