Homework #9 Solutions

Problem

Find the two first partial and four second partial derivatives for the following function of two variables. Be sure to perform all reasonable simplifications prior to taking derivatives.

$$f(x, y) = \ln\left[x^2 e^{(x^3+1)y^2}\right]$$

First, let's simplify according to the log rules:

$$f(x, y) = \ln\left[x^2 e^{(x^3+1)y^2}\right]$$
$$= \ln(x^2) + \ln(e^{(x^3+1)y^2})$$
$$f(x, y) = 2\ln x + (x^3 + 1)y^2$$

Now, find the two first partials:

$$f_x = \frac{2}{x} + 3x^2 y^2$$

$$f_y = 2(x^3 + 1)y$$

Now, find the two unmixed second partials:

$$f_{xx} = -\frac{2}{x^2} + 6xy^2$$

$$f_{yy} = 2(x^3 + 1)$$

Finally, find the two mixed second partials:

$$f_{xy} = 6x^2 y$$

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Note that the two mixed second partials are the same, which is expected since all the partials are continuous.