Math-71 Sections 9, 11, 12

Homework #8 Solutions

Problem

Consider the logarithm function:

$$y = -3\ln\left[-\frac{1}{2}(x+2)\right] - 1$$

a) What is the final x-coordinate of the key point?

The horizontal transformation must bring the key point x-coordinate back to its original value of 1:

$$\begin{aligned} -\frac{1}{2}(x+2) &= 1 \\ x+2 &= -2 \\ x &= -4 \end{aligned}$$

b) What is the final y-coordinate of the key point?

The original key point y-coordinate is 0. So, the transformed value is:

$$-3(0) - 1 = -1$$

Therefore, the transformed key point is $(-4, -1)$.

c) What are the x-intercepts (if any)?

$$\begin{aligned} 0 &= -3\ln\left(-\frac{1}{2}(x+2)\right) - 1 \\ -3\ln\left(-\frac{1}{2}(x+2)\right) &= 1 \\ \ln\left(-\frac{1}{2}(x+2)\right) &= -\frac{1}{3} \\ -\frac{1}{2}(x+2) &= e^{-\frac{1}{3}} \\ x+2 &= -2e^{-\frac{1}{3}} \\ x &= -2 - 2e^{-\frac{1}{3}} \\ x &= -2\left(1 + e^{-\frac{1}{3}}\right) \\ x &\approx -3.4 \end{aligned}$$

d) What are the y-intercepts (if any)?

$$y = -3\ln\left[-\frac{1}{2}(0+2)\right] - 1$$
$$y = -3\ln(-1) - 1$$

But we cannot take the natural log of a negative number. Thus, DNE.

e) Where is the vertical asymptote?

The vertical asymptote moves with the horizontal translations. Thus:

$$x = -2$$

f) Sketching the final graph. Do not use tick marks — relative positioning only. The key point, all intercepts, and the vertical asymptote must be clearly shown and labeled.