Math-19 Homework #2 Solutions

Problems

1). Let:

$$\begin{aligned} P &:= 0 \text{ is a positive number} \\ Q &:= 2 \geq 2 \\ R &:= \forall n, m \in \mathbb{N}, n + m \in \mathbb{N} \end{aligned}$$

Determine whether the following (compound) statements are true or false:

Statement	T/F	comment
P	F	Zero is neither positive nor negative.
Q	T	$\geq$ means "greater than OR equal to".
R	T	This is closure of the natural numbers under addition.
not P	T	not F = T
not Q	F	not T = F
not R	F	not T = F
P and Q	F	F and T = F
P and R	F	F and T = F
Q and R	T	T and T = T
P or Q	T	F and T = T
P or R	T	F and T = T
Q or R	T	T and T = T

2). Convert $10.2\overline{45}$ to rational form.

Let $x = 10.2\overline{45}$. We want $x = \frac{p}{q}$ for $p, q \in \mathbb{Z}$ and $q \neq 0$.

Capture all of the fixed digits:

$$10x = 102.\overline{45}$$

Now, capture all of the fixed digits, and one set of repeating digits.

$$1000x = 10245.\overline{45}$$

Solve for x by subtracting the first equation from the second:

$$\begin{aligned} 990x &= 10143 \\ x &= \frac{10143}{990} \end{aligned}$$

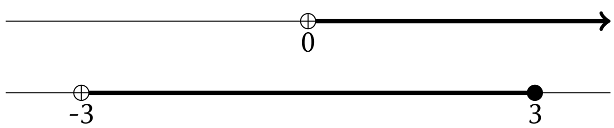
You can reduce, but not required here.

$$x = \frac{1127}{110}$$

3). Let:

$$\begin{aligned} A &= \text{the set of all positive real numbers} \\ B &= \text{the set of real numbers between -3 (exclusive) and 3 (inclusive)} \end{aligned}$$

a). Graph each set on the real number line.



b). Represent each set using set-builder notation.

$$A = \{x \in \mathbb{R} \mid x > 0\}$$

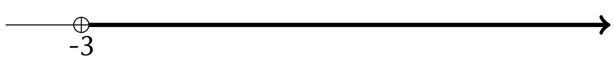
$$B = \{x \in \mathbb{R} \mid -3 < x \leq 3\}$$

c). Represent each set using interval notation.

$$A = (0, \infty)$$

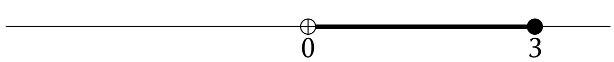
$$B = (-3, 3]$$

d). Graph $A \cup B$ and represent it in interval notation.



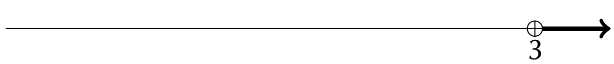
$$A \cup B = (-3, \infty)$$

e). Graph $A \cap B$ and represent it in interval notation.



$$A \cap B = (0, 3]$$

f). Graph $A - B$ and represent it in interval notation.



$$A - B = (3, \infty)$$

4). A careful solution of $4(x+2) = 11$ is given below. Give the rationale for each step from the ten real number rules (AC,AA,A0,AI,MC,MA,M1,MI,LD,RD) and the additional rules (SUB,WD).

$4(x+2) = 11$	
$4x + 8 = 11$	LD
$(4x+8) - 8 = 11 - 8$	CAN
$(4x+8) - 8 = 3$	SUB
$4x + (8-8) = 3$	AA
$4x + 0 = 3$	AI
$4x = 3$	A0
$\frac{1}{4}(4x) = \frac{1}{4}(3)$	CAN
$\frac{1}{4}(4x) = \frac{3}{4}$	SUB
$(\frac{1}{4}4)x = \frac{3}{4}$	MA
$1x = \frac{3}{4}$	MI
$x = \frac{3}{4}$	M1

5). Consider the statement: $\forall\, a, b \in \mathbb{R}, |a-b| = |b-a|$

a). Give a careful proof of this statement. You will need to use one of the distributive rules (hint: factor out a -1), one of the properties in the box at the top of page 9 of your textbook, and the definition of absolute value.

$\|a-b\| = \|-(b-a)\|$	Prop of Neg #6
$\|a-b\| = \|(-1)(b-a)\|$	Prop of Neg #1
$\|a-b\| = \|-1\|\,\|b-a\|$	Prop of AV #3
$\|a-b\| = 1 \cdot \|b-a\|$	Def of AV
$\|a-b\| = \|b-a\|$	M1

or:

$\|a-b\| = \|-(-a)-b\|$	Prop of Neg #2
$\|a-b\| = \|-(-a+b)\|$	Prop of Neg #5
$\|a-b\| = \|-(b+(-a))\|$	AC
$\|a-b\| = \|-(b-a)\|$	Definition of Subtraction
$\|a-b\| = \|b-a\|$	Prop of AV #2

b). What does this statement mean (what are the semantics)? (Hint: think distance)

The distance between a and b is the same as the distance between b and a.