$z = e^{i\theta} \implies z + \frac{1}{z} = 2\cos\theta$
$z^n - 1 = \prod_{k=0}^{n-1}\left(z - e^{i\frac{2\pi k}{n}}\right)$
$\cos(\theta) = \cosh i\theta$
$i\sin(\theta) = \sinh i\theta$
$\cosh(\theta) = \cos i\theta$
$i\sinh(\theta) = \sin i\theta$
Cauchy-Schwarz:
$|\sum a_k b_k|^2 \le \left(\sum |a_k|^2\right)\left(\sum |b_k|^2\right)$
$\sum \left|a_k - c\overline{b_k}\right|^2, c = \frac{\sum a_k b_k}{\sum |b_k|^2}$
Cauchy-Riemann:
$f'(z) = u_x + iv_x = v_y - iu_y$
$u_x = v_y$ and $v_x = -u_y$
$f'(z) = f_x = -if_y$
$f'(z) \implies f_{\bar{z}} = 0$
partials exist, cont, CR $\implies f'(z)$
Polar Cauchy-Riemann:
$f'(z) = e^{-i\theta}(u_r + iv_r) = \frac{1}{r}e^{-i\theta}(v_\theta - iu_\theta)$
$ru_r = v_\theta$ and $u_\theta = -rv_r$
$f'(z) = e^{-i\theta}f_r = -i\frac{1}{r}e^{-i\theta}f_\theta$
Milne-Thompson:
$f(z) = 2u\left(\frac{z+\bar{z}}{2}, \frac{z-\bar{z}}{2i}\right) - u(x_0, y_0) + iC$
$f(z) = 2iv\left(\frac{z+\bar{z}}{2}, \frac{z-\bar{z}}{2i}\right) - iv(x_0, y_0) + C$
$f(z) = u(z, 0) + iv(z, 0)$
Harmonic:
$u_{xx} + u_{yy} = 0$
$r^2 u_{rr} + ru_r + u_{\theta\theta} = 0$
Torsion:
$Re[f(z)] - (\alpha x^2 + \beta y^2 + \gamma) = 0$
$\psi(x, y) = ARe[F(z)] + B(x^2 - y^2) + C$
$\psi(x, y) = \frac{1}{2}(x^2 - y^2)$
$\Psi(x, y) = \psi(x, y) - \frac{1}{2}(x^2 - y^2)$
$\psi(x, y) = \frac{1}{\alpha+\beta}[Re(f(z)) + \frac{1}{2}(\beta-\alpha)(x^2-y^2) - \gamma]$
$\Psi(x, y) = \frac{1}{\alpha+\beta}[Re[f(z)] - (\alpha x^2 + \beta y^2 + \gamma)]$
Green: $\int_C Mdx + Ndy = \iint_D (N_x - M_y)dxdy$
CG: $\int_\gamma f(z)dz = 0$
CGMCD: $\int_C f(z)dz = \sum_{k=1}^n \int_{\gamma_k} f(z)dz$
CIF: $f(a) = \frac{1}{2\pi i}\int_\Gamma \frac{f(z)}{z-a}dz$
CIFMCD: $f(a) = \frac{1}{2\pi i}\int_C \frac{f(z)}{z-a}dz - \sum_{k=1}^n \int_{\gamma_k} \frac{f(z)}{z-a}dz$
CIFD: $f^{(n)}(a) = \frac{n!}{2\pi i}\int_\Gamma \frac{f(z)}{(z-a)^{n+1}}$
CIFHP: $f(z) = \frac{y}{\pi}\int_{-\infty}^{\infty} \frac{f(\xi)}{(\xi-x)^2+y^2}$
Poisson: $f(a) = f(re^{i\theta}) = \frac{1}{2\pi}\int_0^{2\pi} \frac{(R^2-r^2)f(Re^{i\phi})}{R^2-2rR\cos(\theta-\phi)+r^2}d\phi = \frac{1}{2\pi}\int_0^{2\pi} \frac{(R^2-r^2)f(Re^{i\phi})}{\left|Re^{i\phi}-re^{i\theta}\right|^2}d\phi$
MVT Harm: $u(0) = \frac{1}{2\pi}\int_0^{2\pi} u(R, \phi)d\phi$
Harnack: $\frac{R-r}{R+r}u(0) \le u(r, \theta) \le \frac{R+r}{R-r}u(0)$
Parseval: $\frac{1}{2\pi}\int_0^{2\pi} \left|f(re^{i\theta})\right|^2 d\theta = \sum_{n=0}^\infty |a_n|^2 r^{2n}$
Schwarz: $f(z) = \frac{1}{2\pi i}\int_{|z|=R} \left(\frac{\zeta+z}{\zeta-z}\right)\frac{u(\zeta)}{\zeta}d\zeta$
Laurent:
$f(z) = \sum_{n=0}^\infty a_n(z-z_0)^n + \sum_{n=1}^\infty \frac{b_n}{(z-z_o)^n}$
$a_n = \frac{1}{2\pi i}\int_{C_2} \frac{f(\zeta)}{(\zeta-z_0)^{n+1}}d\zeta$
$b_n = \frac{1}{2\pi i}\int_{C_1} \frac{f(\zeta)}{(\zeta-z_0)^{-n+1}}d\zeta$
$a = \frac{z-z_0}{\zeta-z_0}$
Residue: $Res[f, z_0] = \frac{1}{(n-1)!}\lim_{z\to z_0} D^{(n-1)}[(z-z_0)^n f(z)]$
Fibonacci:
$\sum_{n=0}^\infty F_n z^n = \frac{z}{1-z-z^2}$
Bernoulli:
$\frac{z}{e^z-1} = \sum_{n=0}^\infty \frac{B_n}{n!}z^n$
$B_0 = 1, B_1 = -\frac{1}{2}, B_n = -\frac{1}{n+1}\sum_{k=0}^{n-1}\binom{n+1}{k}B_k$
$B_{2k+1} = 0, k > 0$
Euler:
$\operatorname{sech} z = \frac{2e^z}{e^{2z}+1} = \sum_{n=0}^\infty \frac{E_n}{n!}z^n$
$E_0 = 1, E_n = 1 - \frac{1}{2}\sum_{k=1}^n \binom{n}{k}E_{n-k}$
Bessel:
$e^{\frac{u}{2}(z-\frac{1}{z})} = \sum_{n=-\infty}^\infty J_n(u)z^n$