Sets

Definition

A set is a *well-defined*, *distinct*, and *unordered* collection of members called elements, where the elements are selected from an all-inclusive set called a universe.

This definition is poor because it is circular, since it uses the word “set” in the definition, and incomplete because it appeals to the reader’s intuition of what it means to be a collection and a member of a collection. This is due to limitations in the recursive nature of the definition of things via language. Thus, this approach to sets is often referred to as naïve set theory.

Properties

1). To say that a set A is *well-defined* means that every element a from the universe $\mathcal{U}$ is unambiguously either in set A ($a \in A$) or not in set A ($a \notin A$):

$$\forall a \in \mathcal{U}, a \in A \text{ or } a \notin A$$

2). The elements of a set are distinct; if two elements are equal then they are considered to be the *same* element—not two separate elements.

3). The elements of a set are unordered; when listing the elements of a set, any convenient ordering is acceptable and all orderings represent the same set.

Notation

Sets can be specified in any of the following ways:

1). **Description**. The use of words to describe the elements of a set:

$$A = \text{the set of natural numbers from 2 to 5, inclusive}$$

2). **Roster**. A comma-separated list of the elements enclosed in curly-braces. Ellipses $(\ldots)$ can be used to represent omitted elements when the pattern of the elements is obvious, or in the case of infinite sets:

$$A = \{1, 2, 3\}$$

$$B = \{1, 2, 3, \ldots, 10\}$$

$$\mathbb{N} = \{1, 2, 3, \ldots\}$$

$$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

3). **Setbuilder Notation**.

a). Universe/Condition

$$E = \{e \in \mathbb{Z} \mid e = 2n, n \in \mathbb{Z}\}$$

Although sometimes the universe is omitted when obvious or understood:

$$E = \{e \mid e = 2n, n \in \mathbb{Z}\}$$

b). Pattern/Qualifier

$$E = \{2n \mid n \in \mathbb{Z}\}$$

Definition

The set with no elements is called the *empty set* and is denoted by $\emptyset$:

$$\emptyset = \{\}$$

Sometimes we will say something like, "let S be a set of...," where any possible such set is acceptable; however, any such set selected will always be well-defined.