Cavallaro, Jeffery
Math 231a
Homework #0

1. $A = \{1, 2, 3\} \qquad B = \{1, 3, 4\}$

$$A \cap B = \{1, 3\}$$
$$A \cup B = \{1, 2, 3, 4\}$$

2. How many different ways?

   (a) Arrange 5 people in a row:

$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$$

   (b) Select 4 people from a group of 10:

$$\begin{aligned}\binom{10}{4} &= \frac{10!}{4!(10-4)!} \\ &= \frac{10!}{4!6!} \\ &= \frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2 \cdot 1} \\ &= 5 \cdot 3 \cdot 2 \cdot 7 \\ &= 210\end{aligned}$$

3. $f(x) = \dfrac{1}{1 + \sqrt{x}}$

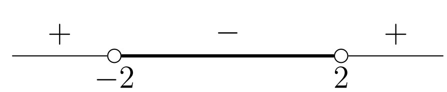
Domain: $x \in [0, \infty)$
Range: $x \in (0, 1]$

4. Solve:

$$-1 < \frac{3-x}{2} < 2$$
$$-2 < 3 - x < 4$$
$$-4 < x - 3 < 2$$
$$-1 < x < 5$$

$$x \in (-1, 5)$$

$$x^2 < 4$$
$$x^2 - 4 < 0$$
$$(x+2)(x-2) < 0$$



$$x \in (-2, 2)$$

5. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges for $p > 1$

6. Determine:

$$\sum_{i=0}^{n} \binom{n}{i} a^i b^{n-i} = (a+b)^n$$

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r} \qquad |r| < 1$$

$$\sum_{n=0}^{\infty} \frac{A^n}{n!} = e^A \qquad A \in \mathbb{R}$$

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{1+n} \right) = 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \cdots = 1$$

7. Evaluate:

   (a)

$$\begin{aligned}
\int_1^{\infty} \frac{2}{x^3} dx &= \lim_{b \to \infty} \int_1^b 2x^{-3} dx \\
&= \lim_{b \to \infty} 2 \left(\frac{x^{-2}}{-2} \right) \Big|_1^b \\
&= \lim_{b \to \infty} \left(-x^{-2} \right) \Big|_1^b \\
&= \lim_{b \to \infty} \left(-\frac{1}{b^2} + 1 \right) \\
&= 0 + 1 \\
&= 1
\end{aligned}$$

(b)

$$\begin{aligned}
\int_0^1 x(1-x)^3 dx &= \int_0^2 x(1-3x+3x^2-x^3)dx \\
&= \int_0^1 (x-3x^2+3x^3-x^4)dx \\
&= \left(\frac{x^2}{2} - x^3 + \frac{3x^4}{4} - \frac{x^5}{5}\right)\Bigg|_0^1 \\
&= \frac{1}{2} - 1 + \frac{3}{4} - \frac{1}{5} \\
&= -\frac{1}{2} + \frac{3}{4} - \frac{1}{5} \\
&= -\frac{10}{20} + \frac{15}{20} - \frac{4}{20} \\
&= \frac{1}{20}
\end{aligned}$$

(c)

$$\int_0^\infty xe^{-2x}dx \qquad \text{(by parts)}$$

$$\begin{array}{ll}
u = x & dv = e^{-2x}dx \\
du = dx & v = -\frac{1}{2}e^{-2x}
\end{array}$$

$$\begin{aligned}
\int_0^\infty xe^{-2x}dx &= -\frac{1}{2}xe^{-2x}\Big|_0^\infty - \int_0^\infty \left(-\frac{1}{2}e^{-2x}\right) dx \\
&= 0 + \frac{1}{2}\int_0^\infty e^{-2x}dx \\
&= \frac{1}{2}\left(-\frac{1}{2}e^{-2x}\right)\Bigg|_0^\infty \\
&= -\frac{1}{4}\ e^{-2x}\big|_0^\infty \\
&= -\frac{1}{4}(0-1) \\
&= \frac{1}{4}
\end{aligned}$$

(d)

$$\begin{aligned}
\int_0^\infty xe^{-x^2}dx &= -\frac{1}{2}\int_0^\infty \left(-2xe^{-x^2}\right)dx \\
&= -\frac{1}{2}\, e^{-x^2}\Big|_0^\infty \\
&= -\frac{1}{2}(0-1) \\
&= \frac{1}{2}
\end{aligned}$$