San José State University
Fall 2015
Math-8: College Algebra
Section 03: MW noon–1:15pm
Section 05: MW 4:30–5:45pm

Quiz #12 (Solutions)

You may use your book, notes, and homework, but please do not work together or ask for help from others.

1. A system of linear equations can have **zero**, **one**, or **infinite** solutions.

2. Find all points of intersection:

$$\begin{aligned} x^2 + y^2 - 6x - 2y - 6 &= 0 \\ x - y &= 0 \end{aligned}$$

This is the intersection between a circle and a line. This can happen in zero places, one place (tangent), or two places (chord). Substitution is the best way to solve this. Let $y = x$ and plug into the circle equation:

$$\begin{aligned} x^2 + x^2 - 6x - 2x - 6 &= 0 \\ 2x^2 - 8x - 6 &= 0 \\ x^2 - 4x - 3 &= 0 \\ x &= \frac{4 \pm \sqrt{4^2 - 4(1)(-3)}}{2(1)} \\ &= \frac{4 \pm \sqrt{16+12}}{2} \\ &= \frac{4 \pm \sqrt{28}}{2} \\ &= \frac{4 \pm 2\sqrt{7}}{2} \\ x &= 2 \pm \sqrt{7} \end{aligned}$$

Thus, the points of intersection are $(2-\sqrt{7}, 2-\sqrt{7})$ and $(2+\sqrt{7}, 2+\sqrt{7})$ (since y=x).

3. Why doesn't the answer in problem 2 contradict the statement in problem 1?

Because the system in (2) is not a linear system. Thus, we can have something other than 0, 1, or ∞ solutions — in this case two solutions.

4. Solve using substitution, elimination, row operations, or matrices. You must show all steps for full credit:

$$\begin{aligned} x+y+z+w &= 6 \\ 2x+3y-w &= 0 \\ -3x+4y+z+2w &= 4 \\ x+2y-z+w &= 0 \end{aligned}$$

Start by transferring the coefficients into an augmented matrix:

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 6 \\ 2 & 3 & 0 & -1 & 0 \\ -3 & 4 & 1 & 2 & 4 \\ 1 & 2 & -1 & 1 & 0 \end{array}\right)$$

Now, use the 1 in the (1,1) pivot position to get rid of everything in the column below it. The row operations are as follows:

$$-2 * R1 + R2 \to R2$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 6 \\ 0 & 1 & -2 & -3 & -12 \\ -3 & 4 & 1 & 2 & 4 \\ 1 & 2 & -1 & 1 & 0 \end{array}\right)$$

$$3 * R1 + R3 \to R3$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 6 \\ 0 & 1 & -2 & -3 & -12 \\ 0 & 7 & 4 & 5 & 22 \\ 1 & 2 & -1 & 1 & 0 \end{array}\right)$$

$$-1 * R1 + R4 \to R4$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 6 \\ 0 & 1 & -2 & -3 & -12 \\ 0 & 7 & 4 & 5 & 22 \\ 0 & 1 & -2 & 0 & -6 \end{array}\right)$$

Now, use the 1 in the (2,2) pivot position to get rid of everything in the column below it:

$$-7 * R2 + R3 \rightarrow R3$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 6 \\ 0 & 1 & -2 & -3 & -12 \\ 0 & 0 & 18 & 26 & 106 \\ 0 & 1 & -2 & 0 & -6 \end{array}\right)$$

$$-1 * R2 + R4 \rightarrow R4$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 6 \\ 0 & 1 & -2 & -3 & -12 \\ 0 & 0 & 18 & 26 & 106 \\ 0 & 0 & 0 & 3 & 6 \end{array}\right)$$

Now, do some clean-up:

$$-frac12 * R3 \rightarrow R3$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 6 \\ 0 & 1 & -2 & -3 & -12 \\ 0 & 0 & 9 & 13 & 53 \\ 0 & 0 & 0 & 3 & 6 \end{array}\right)$$

$$-frac13 * R4 \rightarrow R4$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 6 \\ 0 & 1 & -2 & -3 & -12 \\ 0 & 0 & 9 & 13 & 53 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right)$$

The matrix is now in row eschelon form! The new system of equations is:

$$\begin{aligned} x + y + z + w &= 6 \\ y - 2z - 3w &= -12 \\ 9z + 13w &= 53 \\ w &= 2 \end{aligned}$$

Now, using back substitution:

$$\begin{aligned}
9z + 13(2) &= 53 \\
9z + 26 &= 53 \\
9z &= 27 \\
z &= 3
\end{aligned}$$

$$\begin{aligned}
y - 2(3) - 3(2) &= -12 \\
y - 6 - 6 &= -12 \\
y - 12 &= -12 \\
y &= 0
\end{aligned}$$

$$\begin{aligned}
x + 0 + 3 + 2 &= 6 \\
x + 5 &= 6 \\
x &= 1
\end{aligned}$$

So, the final answer is $(x, y, z, w) = (1, 0, 3, 2)$.