Lab 6: Domains

The domain for a function can be stated either explicitly or must be determined implicitly. The latter can be a tricky proposition. In lecture, we discussed three rules/guidelines for determining domains. We start by assuming all real numbers and then throw out values that violate one of these three rules:

1). No zero denominators

2). No negative radicands in even radicals in numerator factors

3). No negative or zero radicands in even radicals in denominator factors

Consider the following example:

$$f(x) = \frac{\sqrt{(x-1)(x+3)}}{(x-2)\sqrt[3]{x+1}}$$

We need to examine this problem bit by bit to see where rule violations can occur. In the denominator, it is clear that $x \neq 2$ because it would violate rule 1. But do we need to worry about the $\sqrt[3]{x+1}$ factor? Since it is an odd root we don't have to worry about rule 3, but we still need $x \neq -1$ so as to not violate rule 1. As far as the numerator goes, we must solve the inequality $(x-1)(x+3) \geq 0$ so as to not violate rule 2. This yields $(-\inf, -3] \cup [1, \infty)$.

So we have three criteria that we must meet:

1). $x \neq 2$

2). $x \neq -1$

3). $x \in (-\inf, -3] \cup [1, \infty)$

All three of these criteria must hold, so we actually want the intersection of these three things. Note that the third criterion already excludes $x = -1$, so we just need to make a hole at $x = 2$:

$$x \in (-\inf, -3] \cup [1, 2) \cup (2, \infty)$$

Another important rule for determining domains is that domains must be determined based on how the function is initially presented to you. You must keep in mind that simplifications may hide domain issues! Consider the following example:

$$f(x) = \frac{\sqrt{(x-1)(x-2)}}{\sqrt{(x-1)(x-3)}}$$

It is very tempting to rewrite this as:

$$f(x) = \sqrt{\frac{(x-1)(x-2)}{(x-1)(x-3)}}$$

And then cancel the $x - 1$ to get:

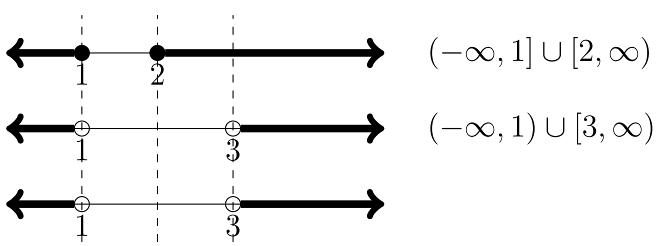
$$f(x) = \sqrt{\frac{x-2}{x-3}}$$

But this would be incorrect for domain purposes! Consider the value $x = 1$. This value looks perfectly fine for the reduced form, but would violate rule 3 in the original form, so we make sure that we exclude it from the domain. Likewise, $x = 2$ is fine for the reduced form, but violates rule 3 in the original form.

Instead, we must determine domain criteria for the numerator and denominator separately and then take the intersection. Doing this yields the following:

numerator	$(-\infty, 1] \cup [2, \infty)$
denominator	$(-\infty, 1) \cup [3, \infty)$

Note that in the numerator the endpoints are included, but in the denominator the endpoints are excluded (so as to not get a zero denominator). So we need the intersection of these two criteria. The best way to find this is to plot them on top of each other and use the endpoints to divide up the number line into regions and then only include regions (and endpoints) that occur in both criteria:



So the final answer is that the domain of the original function is

$$(-\infty, 1) \cup [3, \infty)$$

Note that this is the same as the denominator criterion, since the denominator criterion is a subset of the numerator criterion.

Make sure you can follow all of the steps in the above problem!

Now you try a problem. Consider the rather ugly function:

$$f(x) = \frac{(x^2-49)\sqrt[4]{x^2+3x+2}\sqrt{x^2-9}}{(x-7)\sqrt[5]{x^2-5x+6}\sqrt[6]{x^2-5x-6}}$$

Lot's of stuff going on here. The first step is to identify everything that may be a problem. We do this factor by factor. Here are the six factors in the problem. Write a number from 1 to 3 by each factor to indicate which of the three rules might apply. If no rules apply then write "none." Be sure to distinguish between how the rules apply to even roots versus odd roots.

1). $x^2 - 49$

2). $\sqrt[4]{x^2+3x+2}$

3). $\sqrt{x^2-9}$

4). $x - 7$

5). $\sqrt[5]{x^2-5x+6}$

6). $\sqrt[6]{x^2-5x-6}$

The next step is to factor each of the individual factors. Make sure that you get this correct or the whole problem goes haywire!

1).

2).

3).

4).

5).

6).

Next, determine the criteria for each factor. Note that for even roots you will need to solve a ≥ 0 inequality with the radicand. Your answers should be either: "none", $x \neq a$ for some $a \in \mathbb{R}$, or something in interval notation.

1).

2).

3).

4).

5).

6).

Three of the above answers should have been in interval notation. Graph them one on top of each other. Make sure that the endpoint values are positioned relative to one another. Then, use the endpoints to break up the number line into regions and take the intersection. The final intersection should be a union of two intervals. Be careful with inclusion/exclusion of endpoints.

Now review the other criteria to see if they introduce any holes. There is indeed one. Where is there a hole in the above intersection?

Graph the final answer for the domain of $f(x)$ and write the answer in interval notation:

$x \in$