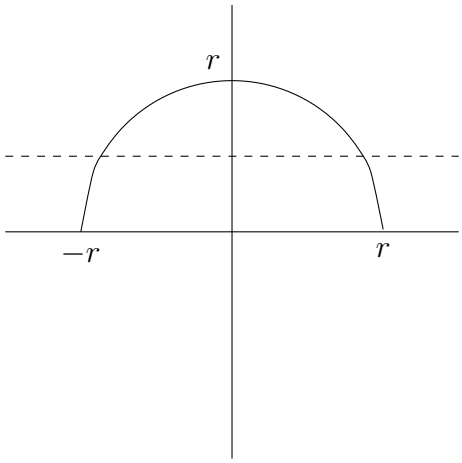
Math-19 Homework #12 Solutions

Problems

1). Consider the circle $x^2 + y^2 = r^2$ and remember that we needed to restrict the range in order to obtain the function $y = \sqrt{r^2 - x^2}$.

a). Sketch the half-circle function and demonstrate why it is not one-to-one?

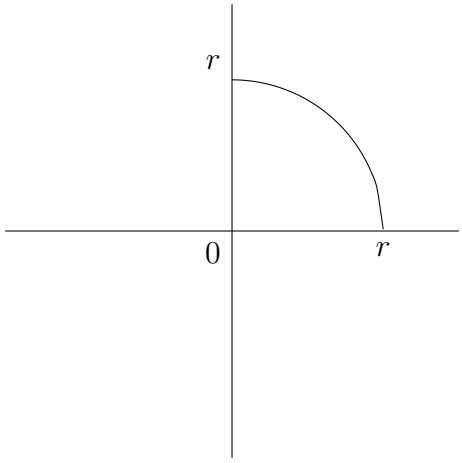


It fails the horizontal line test.

b). Suggest a how to limit the domain so that it is a one-to-one function.

Limit the domain so that we get only one of the quarter circles.

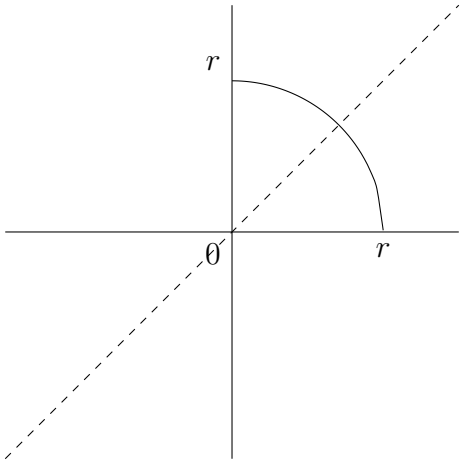
c). Sketch the new graph for the one-to-one function and state its domain and range.



Domain: $[0, r]$

Range: $[0, r]$

d). By observing the graph (and the line $y = x$), predict something about the inverse function.



Notice that if we reflect the graph around $y = x$, we get the same thing!

e). Derive the inverse to prove your prediction.

Swap x and y and then solve for y:

$$\begin{aligned} x &= \sqrt{r^2 - y^2} \\ x^2 &= r^2 - y^2 \\ y^2 &= r^2 - x^2 \\ y &= \sqrt{r^2 - x^2} \end{aligned}$$

2). You use \$1000 to open a savings account at your local bank on the first of February. The savings account has an interest rate of 1.5% per year and compounds monthly on the last day of the month. You set up an auto-deposit of \$100 from your paycheck to occur on the first of each month, starting with the second month (March). During April, you withdraw \$250 to purchase a new gameboy (gotta catch em all!).

a). Who is the lender and who is the borrower?

You are the lender and the bank is the borrower.

b). Calculate $x = 1 + \dfrac{r}{n}$

$$x = 1 + \frac{0.015}{12} = 1.00125$$

c). Construct a polynomial in x to determine the account value on July 2.

month	net
Feb	1000
Mar	100
Apr	-150
May	100
Jun	100
Jul	100

$A = 1000x^5 + 100x^4 - 150x^3 + 100x^2 + 100x + 100$

Note that we are asked for the balance on July 2, the day after the last autodeposit.

d). What is the account value on July 2?

$A = \$1256.58$

3). Consider the exponential function $y = -2e^{-(x+1)} - 3$

a). List the transformations in the order that they should be applied.

i. Start with $y = e^x$.

ii. Translate left 1.

iii. Horizontal reflection.

iv. Vertical scale by 2.

v. Vertical reflection.

vi. Translate down 3.

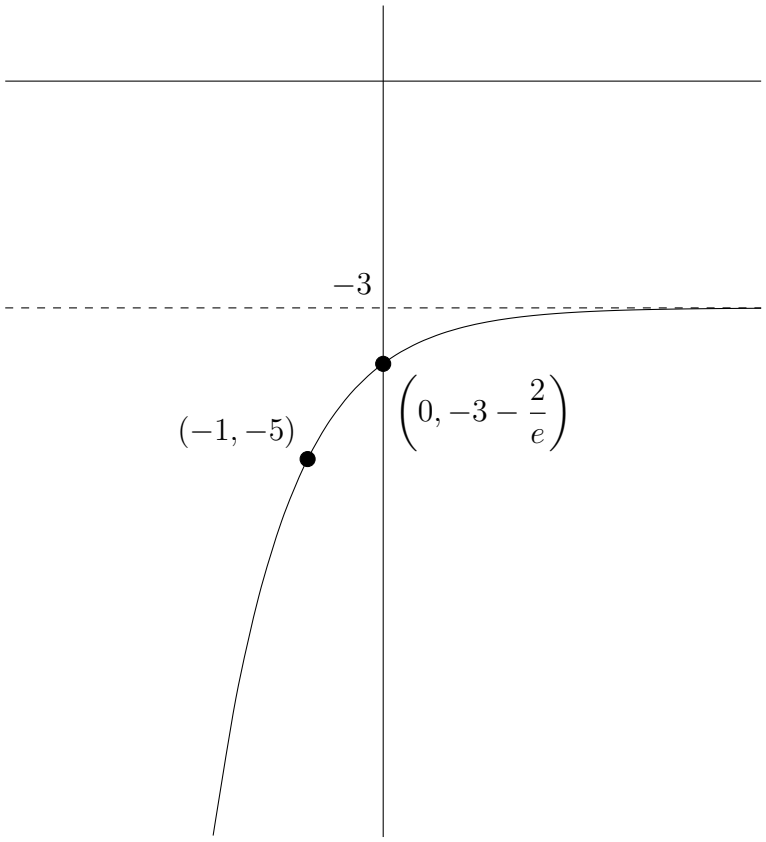
b). What is the y-intercept (if any)?

$y(0) = -2e^{-(0+1)} - 3 = -2e^{-1} - 3 = -\left(3 + \frac{2}{e}\right) \approx -3.74$

$\left(0, -\left(3 + \frac{2}{e}\right)\right)$

c). What is the domain (in interval notation)?

In order to see the domain and range, we need to sketch the graph:



Thus, the domain is $\mathbb{R}$.

d). What is the range (in interval notation)?

$(-\infty,-3)$

4). Consider the logarithmic function $y=\log(-2(x+1))+3$

a). List the transformations in the order that they should be applied.

i. Start with $y=\log(x)$.

ii. Translate left 1.

iii. Horizontal compression by a factor of 2.

iv. Horizontal reflection.

v. Translate up 3.

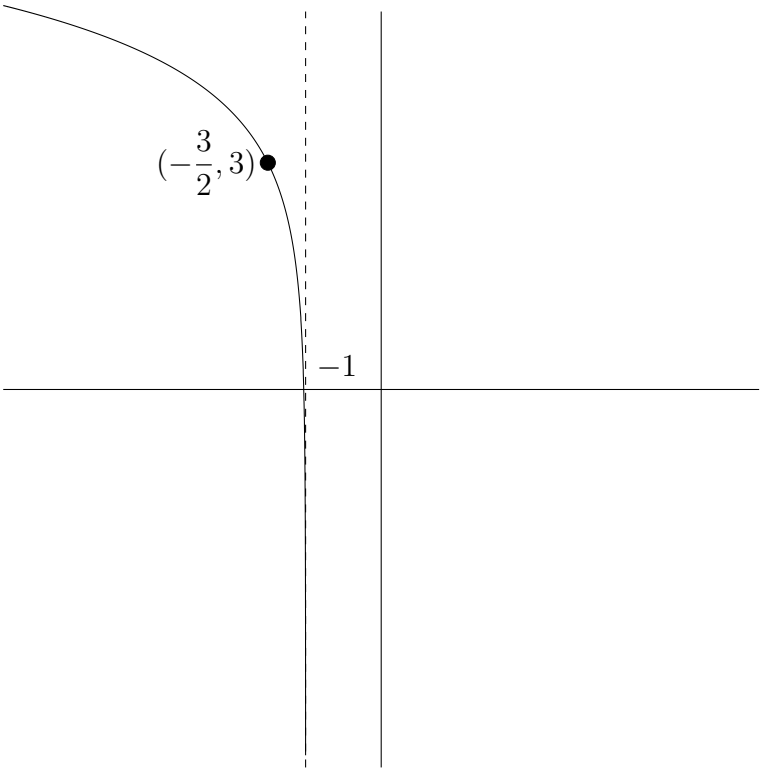
b). What is the y-intercept (if any)?

$y(0)=\log(-2(0+1))+3=\log(-2)+3$

none (cannot take the $\log$ of a negative number.

c). What is the domain (in interval notation)?

In order to see the domain and range, we need to sketch the graph:



Thus, the domain is $(-\infty, -1)$.

d). What is the range (in interval notation)?

$\mathbb{R}$