Math-42 Worksheet #10

Set Operations

1. Let $\mathcal{U} = \{n \in \mathbb{N} \,|\, 1 \leq n \leq 10\}$ be the universe and defined the following sets in $\mathcal{U}$:

$$A = \{1, 2, 4, 7, 9\}$$
$$B = \{2, 3, 4, 10\}$$
$$C = \{5, 8\}$$

Construct the following sets using roster notation:

(a) $A \cup B$

(b) $B \cup A$

(c) $A \cap B$

(d) $B \cap A$

(e) $A - B$

(f) $B - A$

(g) $A \cup C$

(h) $A \cap C$

(i) $A - C$

(j) $C - A$

(k) $A \cup \emptyset$

(l) $A \cap \emptyset$

(m) $A - \emptyset$

(n) $\emptyset - A$

(o) $\bar{A}$

(p) $\bar{B}$

(q) $\bar{C}$

(r) $\bar{\emptyset}$

2. Consider the following sets:

$$A = \{x \in \mathbb{R} \,|\, x < -1\}$$
$$B = \{x \in \mathbb{R} \,|\, x \geq 2\}$$

Graph each of the following sets and express them using interval notation:

(a) A

(b) $\bar{A}$

(c) B

(d) $\bar{B}$

(e) $A \cup B$

(f) $A \cap B$

(g) $A - B$

(h) $\bar{A} \cup \bar{B}$

(i) $\bar{A} \cap \bar{B}$

(j) $\bar{A} - \bar{B}$

(k) $\overline{A \cup B}$

(l) $\overline{A \cap B}$

(m) $\overline{A - B}$

(n) $\overline{\bar{A} \cup \bar{B}}$

(o) $\overline{\bar{A} \cap \bar{B}}$

(p) $\overline{\bar{A} - \bar{B}}$

3. Prove the identity: $A - B = A \cap \bar{B}$ using:

(a) The definitions of the set operators and logic. Remember, this is a set equality proof, so it is bidirectional. Instead of proving both directions, each of your proof steps should be an iff.

(b) A Venn diagram.

4. Consider the set expression: $\bar{A} \cup (\bar{B} \cap C)$

(a) Show the selection region(s) on a Venn diagram.

(b) Complement and simplify.

(c) Show the selection region(s) of the complement on a Venn diagram.

(d) Using your Venn diagrams, confirm that the complement is correct.

5. Prove the following using the set operation definitions and logic:

   (a) $A \subseteq B \implies A \cup B = B$

   (b) $A \subseteq B \implies A \cap B = A$

6. Let $\{A_k : k \in \mathbb{N}\}$ be the family of sets where $A_k = \left[-\frac{1}{k}, \frac{1}{k}\right]$. Determine each of the following sets:

   (a) $\displaystyle\bigcup_{k \in \mathbb{N}} A_k$

   (b) $\displaystyle\bigcap_{k \in \mathbb{N}} A_k$

7. DeMorgan works for finite and infinite generalized unions and intersections as well. Let $\{A_k : k \in I\}$ be some general family of sets and prove the following using careful logical proofs:

   (a) $\displaystyle\overline{\bigcup_{k \in I} A_k} = \bigcap_{k \in I} \overline{A_k}$

   (b) $\displaystyle\overline{\bigcap_{k \in I} A_k} = \bigcup_{k \in I} \overline{A_k}$