Math-42 Sections 01, 02, 05

Homework #10 Solutions

Problems

1. Prove: $\forall\, a, b, c \in \mathbb{Z}$:

   $$a|b \text{ and } b|c \implies a|c$$

   Assume that $a, b, c \in \mathbb{Z}$ such that $a|b$ and $b|c$.
   $\exists\, k \in \mathbb{Z}, b = ka$ and $\exists\, \ell \in \mathbb{Z}, c = \ell b$
   $c = \ell b = \ell(ka) = (k\ell)a$
   $\therefore a|c$

2. Give an example with $a = 7$ and $b, c \neq 7$.

   Let $a = 7$, $b = 21$, and $c = 84$.

   $7|21$ because $3 \cdot 7 = 21$.
   $21|84$ because $4 \cdot 21 = 84$.
   $7|84$ because $12 \cdot 7 = 84$.