Math-19 Homework #13 Solutions

Problems

1). Given:

$$\log_b 2 = 0.6931$$

$$\log_b 3 = 1.0986$$

$$\log_b 5 = 1.6094$$

find $\log_b\left(\frac{75}{4}\right)$. You must use each one of the given values, you are not allowed to determine the value of b, and you must show exactly how you obtained the answer.

$$\begin{aligned}
\log_b\left(\frac{75}{4}\right) &= \log_b 75 - \log_b 4 \\
&= \log_b(3\cdot 25) - \log_b 2^2 \\
&= \log_b 3 + \log_b 25 - 2\log_b 2 \\
&= \log_b 3 + \log_b 5^2 - 2\log_b 2 \\
&= \log_b 3 + 2\log_b 5 - 2\log_b 2 \\
&= 1.0986 + 2(1.6094) - 2(0.6931) \\
&= 2.9312
\end{aligned}$$

2). Consider the equation: $y = \log_a x$

a). Derive the change of base formula for some arbitrary base b.

$$\begin{aligned}
y &= \log_a x \\
a^y &= x \\
\log_b a^y &= \log_b x \\
y \log_b a &= \log_b x \\
y &= \frac{\log_b x}{\log_b a}
\end{aligned}$$

b). Use your formula with $b = e$ and your calculator to compute $\log_7 100$.

$$\log_7 100 = \frac{\ln 100}{\ln 7} = 2.3666$$

c). Assume that you made a mistake and used the common log key instead of the natural log key in the above calculation. Would you get a different answer? Why or why not?

No, because the equation is good for any base.

3). Researchers tend to prefer exponential (base e) equations. For example, the normal equation for the radioactive decay of Carbon-14, which has a half-life of 5730 years, would be:

$$A = A_0 \cdot 2^{-\frac{t}{5730}}$$

Find a value for x such that:

$$A = A_o e^{xt}$$

is an equivalent equation.

$$\begin{aligned}
2^{-\frac{t}{5730}} &= e^{xt} \\
\ln 2^{-\frac{t}{5730}} &= \ln e^{xt} \\
-\frac{t}{5730} \ln 2 &= xt \\
x &= -\frac{\ln 2}{5730} \\
&\approx -1.21 \times 10^{-4}
\end{aligned}$$

4). The San Francisco earthquake of 1906 was estimated at 7.8 on the Richter Scale. Current building codes have resulted in skyscrapers that can withstand an 8.0 earthquake. How much strong is this than the 1906 quake?

The equation for a reading on the Richter scale is:

$$r = \log \frac{I}{I_0}$$

where I_0 is the reference intensity.

Let I_1 be the intensity of an 8.0 earthquake and I_2 be the intensity of a 7.8 earthquake:

$$\begin{aligned}
8.0 - 7.8 &= \log \frac{I_1}{I_0} - \log \frac{I_2}{I_0} \\
0.2 &= \log \frac{\frac{I_1}{I_0}}{\frac{I_2}{I_0}} \\
0.2 &= \log \left(\frac{I_1}{I_0} \cdot \frac{I_0}{I_2} \right) \\
0.2 &= \log \frac{I_1}{I_2} \\
10^{0.2} &= 10^{\log \frac{I_1}{I_2}} \\
\frac{I_1}{I_2} &= 10^{0.2}
\end{aligned}$$

So: $\dfrac{I_1}{I_2} \approx 1.6$

Therefore, today's skyscrapes can withstand an earthquake that is 1.6 times as strong as the 1906 earthquake.