

Multiplying Terms

Section 0.5 starts with a definition of a polynomial. Please read it; however, I won't cover it formally until chapter 3, which is all about polynomials.

But what is important is the notion of combine like factors, which is an application of the distributive rules:

$$ac + bc = (a + b)c$$

Example

$$2x + 3x = (2 + 3)x = 5x$$

$$5\sqrt{(x+1)} + \pi\sqrt{(x+1)} = (5 + \pi)\sqrt{(x+1)}$$

In some problems you need to match up terms with like factors:

Example

$$(x^2 + 5x - 3) - (2x^2 - 3x + 5) = -x^2 + 8x - 8$$

And when going the other way, remember to apply the common factor to each and every term in the parentheses:

Example

$$5x(x + 1) = 5x^2 + 5x$$

$$2xy(3xz + 2y + 1) = 6x^2yz + 4xy^2 + 2xy$$

$$x^{\frac{1}{2}}(x^2 + x^{\frac{1}{3}} + 1) = x^{\frac{5}{2}} + x^{\frac{5}{6}} + x^{\frac{1}{2}}$$

A common problem is to multiply two terms by two terms:

$$(a + b)(c + d) = (a + b)c + (a + b)d = ac + bc + ad + bd$$

This is the so-called FOIL method (first, outer, inner, last).

Example

$$(x + 2)(x - 3) = x^2 + 2x - 3x - 6 = x^2 - x - 6$$

More generally, each term in the first gets multiplied by each term in the second:

Example

$$(2xy + z)(3yz - 2x - 1) = 6xy^2z - 4x^2y - 2xy + 3xyz - 2xz - z$$

$$(x^2 - 3x + 1)(x^2 + 2x + 3) = \begin{array}{r} x^4 + 2x^3 + 3x^2 \\ -3x^3 - 6x^2 - 9x \\ x^2 + 2x + 3 \end{array} = x^4 - x^3 - 2x^2 - 7x + 3$$

Special Products:

1). Sum/Difference

$$(a + b)(a - b) = a^2 + ab - ab + b^2 = a^2 - b^2$$

Example

$$(x + 2)(x - 2) = x^2 - 4$$

$$(x + y)(x - y) = x^2 - y^2$$

$$(2x^2 + 3\sqrt{y})(2x^2 - 3\sqrt{y}) = 4x^4 - 9y$$

Note in the last example, the even root is on the inside, so y is assumed to be ≥ 0 , so the absolute value is not needed.

2). Two terms squared

$$(a + b)^2 = a^2 + 2ab + b^2$$

Example

$$(x + 3)^2 = x^2 + 6x + 9$$

$$(2x - 1)^2 = (2x + (-1))^2 = 4x^2 - 4x + 1$$

$$(\sqrt{x} + \sqrt{y})^2 = x + 2\sqrt{xy} + y$$

3). Two terms cubed

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

Example

$$(x + 1)^3 = x^3 + 3x^2 + 3x + 1$$

$$(2x - 3)^3 = (2x + (-3))^3 = 8x^3 - 36x^2 + 54x - 27$$

$$(x - y)^3 = (x + (-y))^3 = x^3 - x^2y + xy^2 - y^3$$