Math-71 Sections 9, 11, 12

Final Exam

Name: _____

This exam is closed book and notes. You may use a scientific calculator; however, no other electronics are allowed. You may also use the instructor-provided cheatsheet. Show all work; there is no credit for guessed answers. Simplify your answers unless told otherwise. In particular, all answers should contain no negative or rational exponents. All numerical answers should be in exact form unless you are specifically asked for a decimal value.

1. Consider the function:

$$f(x) = \frac{x - 2}{x^2 + 3x - 10}$$

   (a) Determine $\lim_{x \to 2} f(x)$ if it exists. Otherwise, state *DNE*.

   (b) Using interval notation, describe where $f(x)$ is continuous.

2. Determine $f'(x)$ for $f(x) = 3x^4$.

3. Determine $f'(x)$ for $f(x) = \dfrac{x}{\pi}$.

4. Determine $f'(x)$ for $f(x) = \dfrac{e^2}{x}$.

5. Determine $f'(x)$ for $f(x) = \dfrac{3e^{x^2}}{2}$.

6. Determine $f'(x)$ for $f(x) = \ln(x^2 + 2)$

7. Determine $f'(x)$ for $f(x) = 2x^3 - 3x^2 + 50x - 100$

8. Using the product and chain rules, determine $f'(x)$ for $f(x) = \ln(x)\sqrt{x^2 + 1}$

9. Using the quotient and chain rules, determine $f'(x)$ for $f(x) = \frac{e^x}{x^3 - 2}$

10. Determine $\int 3x^4 dx$.

11. Determine $\int \frac{x}{\pi} dx$.

12. Determine $\int \frac{e^2}{x} dx$.

13. Determine $\int_0^1 (2x^3 - 3x^2 + 50x - 100)dx$

14. Determine $\int 2xe^{x^2}\,dx$.

15. Determine $\int \dfrac{2x}{x^2+1}\,dx$.

16. You get your first paycheck from a new job. On March 1, you go to the bank and open a savings account that pays interest of 3% annually, compounded monthly, with an initial deposit of $1000. You set up an auto-deposit from your paycheck that will deposit $500 each month on the first of the month to the account, starting in April. In May, you need to withdraw $750 from your account to pay some bills. What is your account balance on June 2?

17. After watching a famous episode of the Nickelodean TV show iCarly, you decide to start your own penny tee t-shirt business. You decide that a good monthly demand function (with respect to selling price, in dollars) is given by:

$$n(p) = 2000 - 100p$$

You expect monthly fixed costs to be $1000 and variable costs to be $1 per shirt.

(a) Determine the monthly revenue function $R(p)$.

(b) Determine the monthly cost function $C(p)$.

(c) Determine the monthly profit function $P(p)$.

(d) At what per-shirt sales price is profit maximized?

(e) Prove (either by test points or concavity) that your answer is in fact a maximum.

18. You are working quality control at a precision ball-bearing factory. The radius of each ball-bearing as it comes off of the production line follows a normal distribution as follows:

$$p(x) = \frac{1}{0.01\sqrt{2\pi}} e^{\frac{-(x-2)^2}{0.0002}}$$

where the radius x is measured in millimeters.

(a) What is the mean radius of the ball-bearings?

(b) What is the standard deviation of the radius of the ball-bearings?

(c) At what x value does the corresponding bell curve have its absolute maximum?

(d) At what x values does the corresponding bell curve have its points of inflection?

(e) What is the probability that a ball-bearing radius is between 1.97 mm and 2.03 mm?

19. A new eBike company called Telfon, run by entrepreneur and international playboy Allen Tusk, is going to build its bikes at two factories: one in Singapore and one in South Korea. Let x be the number of bikes made in the Singapore plant and y be the number of bikes made in the South Korea plant. The cost function for making the bikes is given by:

$$C(x, y) = \frac{1}{2}x^2 + 4x + \frac{1}{4}y^2 + 8y$$

The initial combined order will be for 10,000 bikes. Using the Lagrange multiplier method, determine how these 10,000 bikes should be split between the two factories in order to minimize cost.

20. You work in the advertising department for a new fast-food restaurant that is launching in the bay area. You decide to run a major"branding" advertising campaign with a new radio spot (commercial). The goal is for people to both hear the commercial and remember the company name and business when asked - thus establishing your brand in their mind. This makes it more likely that people will try your restaurant when they are hungry. Your advertising agency estimates that the rate of change of people successfully branded is given by:

$$\frac{dx}{dt} = xe^{-x^2}$$

where x is millions of people branded and t is the number of months that the planned advertising campaign is running. How many potential customers are branded after 2 months?