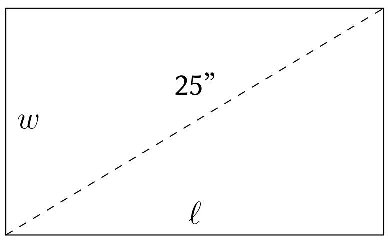
Math-08 Homework #8 Solutions

Reading

- Text book section 1.2,1.6,1.7

Problems

1). You are a product manager at an electronics firm in charge of a proposed new line of 25-inch monitors (i.e., the length of the diagonal across the screen is 25 inches):



You realize that the most appealing ratio for the dimensions of the screen would follow the golden ratio:

$$\frac{\ell}{w} = \frac{1+\sqrt{5}}{2} \approx 1.6 = \frac{8}{5}$$

a). Using the estimate of $8/5$, determine the dimensions ($\ell \times w$) for the new monitor. Round each dimension to two decimal places.

Note that we have a right triangle with a hypotenuse of 25 inches. Thus:

$$w^2 + \ell^2 = 25^2$$

Let's pick w to be our key unknown. So we need to define l in terms of w:

$$\frac{\ell}{w} = \frac{8}{5}$$

$$5\ell = 8w$$

$$\ell = \frac{8}{5}w$$

Now, plug in and solve:

$$w^2 + \left(\frac{8}{5}w\right)^2 = 25^2$$

$$w^2 + \frac{64}{25}w^2 = 625$$

$$\frac{89}{25}w^2 = 625$$

$$w^2 = \frac{625(25)}{89}$$

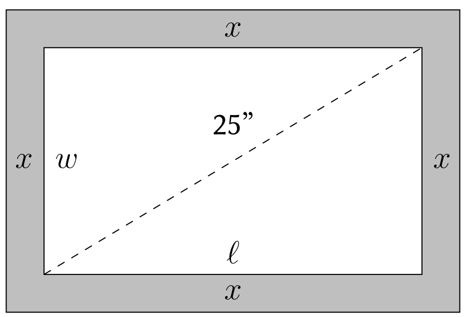
$$w = \sqrt{\frac{625(25)}{89}}$$

$$w = 13.25$$

$$\ell = \frac{8}{5}13.25 = 21.20$$

Dimensions$=21.20$ in $\times 13.25$ in

b). There needs to be an equal amount of casing around the edges of the screen and the packaging department would like the monitor to have a total area of 400 square inches.



Determine the width of the casing (x) around the screen. Round your answer to two decimal places.

$$(2x + 21.20)(2x + 13.25) = 400$$

$$4x^2 + 68.9x + 280.9 = 400$$

$$4x^2 + 68.9x - 119.1 = 0$$

$$x = \frac{-68.9 \pm \sqrt{68.9^2 - 4(4)(-119.1)}}{2(4)} = 1.58$$

The border should be 1.58 in.

2). A man stands atop a 256 foot cliff with a ball.

a). How long does it take for the ball to hit the ground if he simply releases the ball?

$$0 = 256 - 16t^2$$

$$16t^2 = 256$$

$$t^2 = 16$$

$$|t| = 4$$

$$t = \pm 4$$

The ball hits the ground in 4 seconds.

b). How long does it take for the ball to hit the ground if he throws the ball up with a velocity of 16 ft/s? (Hint: keep the negative solution around for later).

$$0 = 256 + 16t - 16t^2$$

$$0 = 16 + t - t^2$$

$$t^2 - t - 16 = 0$$

$$t = \frac{1 \pm \sqrt{(-1)^2 - 4(-1)(16)}}{2(1)} = 4.5, -3.5$$

The ball hits the ground in 4.5 seconds.

c). How long does it take for the ball to hit the ground if he throws the ball down with a velocity of 16 ft/s? (Hint: no additional calculations are needed).

The ball hits the ground in 3.5 seconds (the negative result from the previous part).

d). Assume that a lady is standing on the ground below the cliff and throws a ball up so that it passed the man on the cliff at a velocity of 16 ft/s. How long would it be before the ball hits the ground? (Hint: you already have all the information that you need).

The ball hits the ground in $4.5 + 3.5 = 8 seconds$, which is the sum of the two paths in the previous two parts.

3). For each of the following inequalities, graph the solution set and state the solution set in interval notation.

a). $2\,|5 - 3x| + 7 < 21$

We want to get the inequality into standard form first:

$$2\,|5 - 3x| < 14$$

$$|5 - 3x| < 7$$

Now that it is in standard form, and is a "less than" problem, we can make the corresponding compound inequality and solve simultaneously for x:

$$-7 < 5 - 3x < 7$$

$$-12 < -3x < 2$$

$$4 > x > -\frac{2}{3}$$

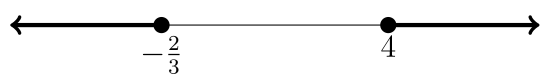
$$-\frac{2}{3} < x < 4$$

Note that in the last steps we multiplied by a negative number, so we needed to flip the inequality signs. The resulting graph is as follows:

And the corresponding interval notation is: $\left(-\frac{2}{3}, 4\right)$

b). $2\,|5 - 3x| + 7 \geq 21$

Instead of the inside, we want the outside. Also, since we are allowing equality, we include the endpoints. The resulting graph is as follows:



And the resulting interval notation is: $\left(-\infty, -\frac{2}{3}\right] \cup [4, \infty)$

4). Solve for x, stating the solution in interval notation.

$$\frac{x+1}{x-2} < \frac{x-3}{x+4}$$

Remember, we cannot cross multiply here because we don't yet know if we might be multiplying by a negative number. Instead, we bring everything to one side and combine using the fraction rules from 0.2:

$$\frac{x+1}{x-2} - \frac{x-3}{x+4} < 0$$

$$\frac{(x+1)(x+4) - (x-2)(x-3)}{(x-2)(x+4)} < 0$$

$$\frac{(x^2+5x+4) - (x^2-5x+6)}{(x-2)(x+4)} < 0$$

$$\frac{(10x-2)}{(x-2)(x+4)} < 0$$

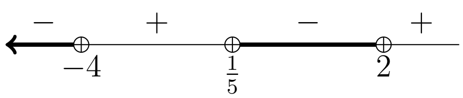
As an optional step, let's factor out the 10 from the factor in the numerator. Note that it is a positive constant, so it will not affect the sign. In fact, if we multipl both sides by $\frac{1}{10}$, it just goes away:

$$\frac{10(x-\frac{1}{5})}{(x-2)(x+4)} < 0$$

$$\frac{(x - \frac{1}{5})}{(x-2)(x+4)} < 0$$

Remember, if it had been a -10 then we would need to flip the inequality sign. We are now ready to build a sign table. Remember to include both the zeros and the poles, since we can change sign across either:

test	$(x-\frac{1}{5})$	$(x-2)$	$(x+4)$	sign
-5	$-$	$-$	$-$	$-$
0	$-$	$-$	$+$	$+$
1	$+$	$-$	$+$	$-$
3	$+$	$+$	$+$	$+$



Since the inequality is "less than" we want all of the negative intervals. Note that since equality is not allowed, all endpoints are not included:

$$(-\infty, -4) \cup \left(\frac{1}{5}, 2\right)$$

5). Determine the domain for each of the following expressions, stating each in interval notation.

a).

$$\sqrt{\frac{x^2 - 3x - 10}{x^2 - 9x + 20}}$$

This is a square (even) root, so negative radicands are not allowed. We turn this into an inequality:

$$\frac{x^2 - 3x - 10}{x^2 - 9x + 20} \geq 0$$

We need the numerator and denominator in factored form so that we can build a sign table:

$$\frac{(x-5)(x+2)}{(x-5)(x-4)} \geq 0$$

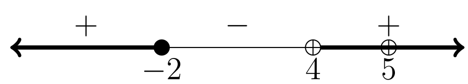
Now we note that the $(x-5)$ factor cancels; however, we need to remember that 5 is not in the domain - we will need a hole there:

$$\frac{x+2}{x-4} \geq 0$$

We are now ready to build the sign table:

test	$(x+2)$	$(x-4)$	sign
-3	$-$	$-$	$+$
0	$+$	$-$	$-$
5	$+$	$+$	$+$

And don't forget the hole at 5:



Note that equality is allowed here, so zeros (-2) are included, but poles (4)are still excluded. So the domain is:

$$(-\infty, -2] \cup (4, 5) \cup (5, \infty)$$

b).

$$\sqrt[3]{\frac{x^2 - 3x - 10}{x^2 - 9x + 20}}$$

This is an *odd* root, so there is no need to worry about a negative radicand. We still need to be cautious of a zero denominator, through, so based on the answer to the previous part, we just need to leave holes at $x = 4, 5$:

$$(-\infty, 4) \cup (4, 5) \cup (5, \infty)$$