Spring 2020 Math 30 Makeup Final Exam

Name: ____

1. Determine the limits:

(a) $\displaystyle\lim_{x\to -1^+} \frac{x^5 - 1}{x^5 + 1}$

(b) $\displaystyle\lim_{x\to +\infty} \arctan \frac{x^{10} - 3x^5 - 2}{x^{10} + 4x^3 - 1}$

(c) $\displaystyle\lim_{x\to e^+} \frac{1}{x \ln(x-1)}$

(d) $\displaystyle\lim_{x\to +\infty} \frac{2x}{(x-1)\ln x}$

2. Using the limit definition of the derivative, find $f'(x)$ where:

   $$f(x) = 6x^3 + 2x$$

   Show your work and *do NOT use differentiation rules.*

3. You need to find the dimensions of a cylindrical can (much like a soda or soup can) so that its volume is 81 cubic inches. The metal used for the side of the can costs 50 cents per square inch, the metaul used for the bottom of the can costs 65 cents per square inch, and the metal used for the top of the can costs 85 cents per square inch. Find the dimensions of the can that minimize the cost. What is the minimum cost? How do you know that your answer gives a minimum?

4. Find an equation for the tangent line to the curve:

$$f(x) = 3\sqrt{9x} - 6e^{(2-x)}$$

at $x = 2$.

5. Show all necessary steps:

   (a) Differentiate:

   $$g(x) = \arctan\sqrt{\sin x}$$

   (b) Find the most general antiderivative of the function:

   $$f(x) = \frac{x^2 + x + 1}{x}$$

6. Two trains, the red train and the blue train, start at Wooster Station and travel to nearby towns. The red train travels East at 40 mph while the blue train North at 60 mph. The red train leaves at noon. 30 minutes later, the blue train leaves. How fast is the distance between the two trains changing at 1 pm?

7. Find the following derivatives:

   (a) Find the first derivative:

   $$y = \sqrt[3]{x} \cdot \sec \sqrt{x}$$

   (b) Find the second derivative:

   $$f(x) = \ln(3 + x^4)$$

8. Find y':

$$y = x^{x^2}$$

9. Find $\frac{dy}{dx}$ at the point $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$ on the curve:

$$2\cos x \sin y = 1$$

10. The position of a particle moving along a coordinate axis is given by:

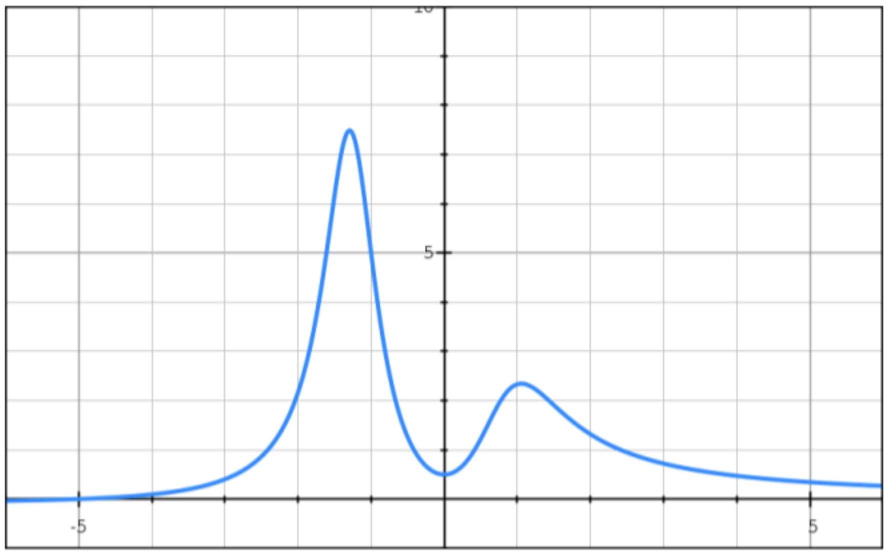
$$s(t) = t^3 - 9t^2 + 24t + 4 \qquad t \geq 0$$

(a) Find the velocity $v(t)$.

(b) Find the acceleration $a(t)$.

(c) Over what time intervals is the particle moving from left to right? From right to left?

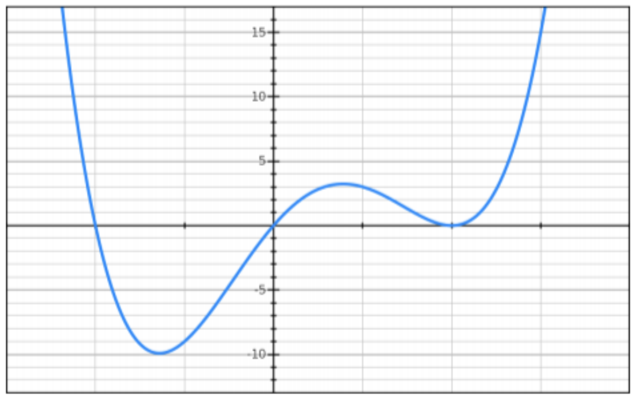
11. Suppose that $f(x)$ has the graph shown below. Sketch the graph of $f'(x)$. Make sure that you clearly show any critical point(s) or asymptote(s).



12. Is it possible to sketch a graph that satisfies ALL of the following conditions? If yes then sketch the graph of f below. Otherwise, explain (in one or two sentences) why not.

   - $f(0) = 0$
   - $\lim_{x \to -4^-} f(x) = 2$ and $\lim_{x \to -4^+} f(x) = -2$
   - f is continuous from the left at $x = -4$
   - $f(x) \to \infty$ as $x \to 2^-$ and $f(x) \to -\infty$ as $x \to 2^+$
   - f is continuous at all values of x other than $x = -4$ and $x = 2$

13. Suppose that $f(x)$ is a function whose derivative $f'(x)$ is graphed below on the domain $-3 \leq x \leq 4$.



(a) Find the interval(s) where f is increasing and the interval(s) where f is decreasing. Justify your answer.

(b) Find the x value of the relative/local maximum(s) of f. Justify your answer.

(c) Find the interval(s) of concavity of f. Justify your answer.

(d) Find the x value of the point(s) of inflection of f. Justify your answer.