San José State University
Fall 2015
Math-8: College Algebra
Section 03: MW noon–1:15pm
Section 05: MW 4:30–5:45pm

Quiz #15 (Solutions)

1. Assume that we know the following:

$$\begin{aligned} \log_b 2 &= 0.43068 \\ \log_b 7 &= 1.20906 \end{aligned}$$

a. Without determining b, what is $\log_b 28$?

$$\begin{aligned} \log_b 28 &= \log_b(2^2 \cdot 7) \\ &= \log_b(2^2) + \log_b 7 \\ &= 2\log_b 2 + \log_b 7 \\ &= 2(0.43068) + 1.20906 \\ &= 2.07042 \end{aligned}$$

b. What is b?

$$\begin{aligned} \log_b 2 &= 0.43068 \\ \frac{\ln 2}{\ln b} &= 0.43068 \text{ (change of base formula)} \\ \frac{\ln b}{\ln 2} &= \frac{1}{0.43068} \\ \ln b &= \frac{\ln 2}{0.43068} \\ e^{\ln b} &= e^{\frac{\ln 2}{0.43068}} \\ b &= 5 \end{aligned}$$

2. Solve for x:

$$\log_2(2x) + \log_2(x-3) = 3$$

$$\begin{aligned} log_2[2x(x-3)] &= 3 \\ 2^{log_2[2x(x-3)]} &= 2^3 \\ 2x(x-3) &= 8 \\ 2x^2-6x &= 8 \\ 2x^2-6x-8 &= 0 \\ (x+1)(x-4) &= 0 \\ x &= -1, 4 \end{aligned}$$

Before we can conclude that both solutions are correct, we must make sure that we have not violated any domain restrictions. Indeed, plugging in $x = -1$ results in a log or a negative value, and thus must be discarded. The other solution, $x = 4$, is OK. Therefore, the final answer is $x = 4$.

3. Derive the change-of-base formula:

$$\log_b x = \frac{log_a x}{log_a b}$$

$$\begin{aligned} y &= \log_b x \\ b^y &= x \\ \log_a b^y &= \log_a x \\ y \log_a b &= \log_a x \\ y &= \frac{\log_a x}{\log_a b} \end{aligned}$$

4. In class we talked about carbon dating and noted that the equation typically used:

$$y = \frac{1}{10^{12}} e^{-\frac{t}{8223}}$$

is slightly different than the normal half-life-based equation:

$$y = \frac{1}{10^{12}} \left(\frac{1}{2}\right)^{\frac{t}{5700}}$$

where 5700 years is the half-life of C_{14}. Show that these two equations are equivalent.

Let's equate the two expressions, replace 8223 with x, and then solve for x:

$$
\begin{aligned}
\frac{1}{10^{12}}\left(\frac{1}{2}\right)^{\frac{t}{5700}} &= \frac{1}{10^{12}}e^{-\frac{t}{x}} \\
\left(\frac{1}{2}\right)^{\frac{t}{5700}} &= e^{-\frac{t}{x}} \\
\ln\left(\frac{1}{2}\right)^{\frac{t}{5700}} &= \ln\left(e^{-\frac{t}{x}}\right) \\
\frac{t}{5700}\ln\left(\frac{1}{2}\right) &= -\frac{t}{x} \\
\frac{1}{5700}\ln\left(\frac{1}{2}\right) &= -\frac{1}{x} \\
\frac{5700}{\ln\left(\frac{1}{2}\right)} &= -x \\
x &= -\frac{5700}{\ln\left(\frac{1}{2}\right)} \\
x &= 8223
\end{aligned}
$$