Spring 2020 Math 30 Makeup Final Exam

Name: ____

1. Determine the limits:

   (a) $\displaystyle\lim_{x\to 0^+} \frac{|x|}{\sin x}$

   (b) $\displaystyle\lim_{x\to \infty} \frac{x \arctan x}{1-2x}$

   (c) $\displaystyle\lim_{x\to 0} \frac{x^3+2}{xe^{x^2}}$

2. Using the limit definition of the derivative, find $f'(x)$ where:

$$f(x) = \frac{x}{2x+3}$$

Show your work and *do NOT use differentiation rules.*

3. You are designing sealed plexiglass tanks to be used as part of a biology experiment. The tanks are rectangular in shape and have a square base. The tanks must have a volume of $1000\,\mathrm{cm}^3$. The material that will be used to make the base (bottom) costs $5\,\mathrm{cents/cm}^2$. The material that will be used to make the lid costs $4\,\mathrm{cents/cm}^2$. The material that will be used to make the walls costs $10\,\mathrm{cents/cm}^2$.

   (a) Determine the dimensions of the tank that minimize the material costs. Round you answers to the nearest tenth of a centimeter.

   (b) What is the minimum cost, rounded to the nearest cent?

   (c) Justify that your answer is indeed the minimum cost.

4. Let $f(x) = \ln(x \ln(1 + x^2))$. Find $f'(x))$.

5. Show all necessary steps:

   (a) Differentiate:

   $$g(x) = \cos(\arctan(\sqrt{x}))$$

   (b) Find the most general antiderivative of the function:

   $$f(x) = \frac{x^2 2^x + 6}{x^2}$$

6. A test missile passes over a monitoring station at an altitude of $5\,\mathrm{mi}$. The missile is traveling at $520\,\mathrm{mph}$. You are standing at a position that is $12\,\mathrm{mi}$ from the monitoring station. At what rate is the distance from the monitoring station to the missile changing when the missile passes over you?

7. Find the tangent line to the function:

$$f(x) = \ln(\cos x)$$

at $x = -\frac{\pi}{4}$.

8. Let $y = x\ln(1+x^2)$:

   (a) Find y'.

   (b) Find y''.

9. Let $y + 2x\cos(y^2 - x) = 1$. Find $\frac{dy}{dx}$ at the point $(1, -1)$.

10. The number of cases of influenza in New York City from the beginning of 1960 to the beginning of 1964 is modeled by the function:

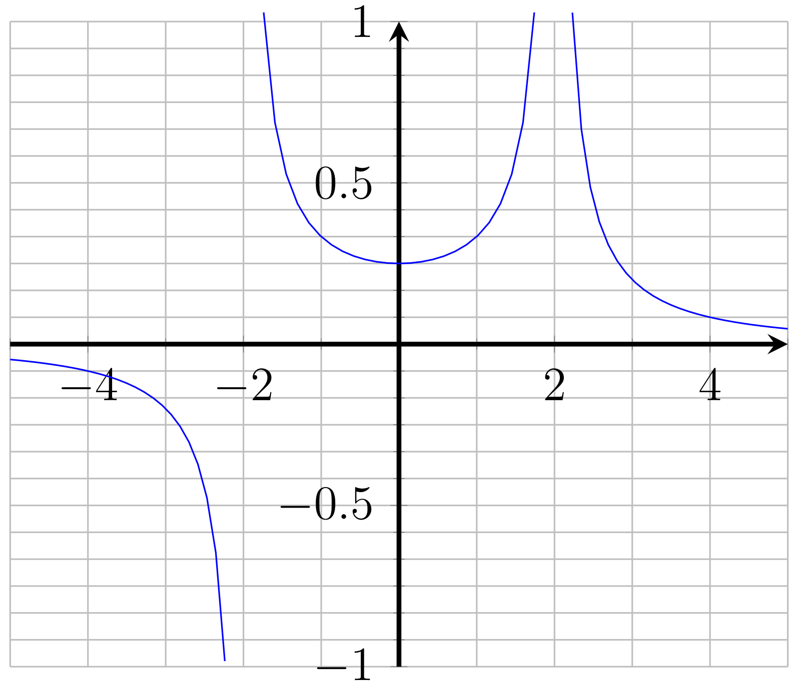
$$N(t) = 5.3e^{(0.093t^2 - 0.87t)} \qquad 0 \le t \le 4$$

where $N(t)$ gives the number of cases (in thousands) and t is measured in years, with $t = 0$ corresponding to the beginning of 1960.

(a) Find $N(0)$ and $N(4)$. Explain what these values indicate about the disease in New York City.

(b) Find $N'(0)$ and $N'(3)$. Explain what these values indicate about the disease in New York City.

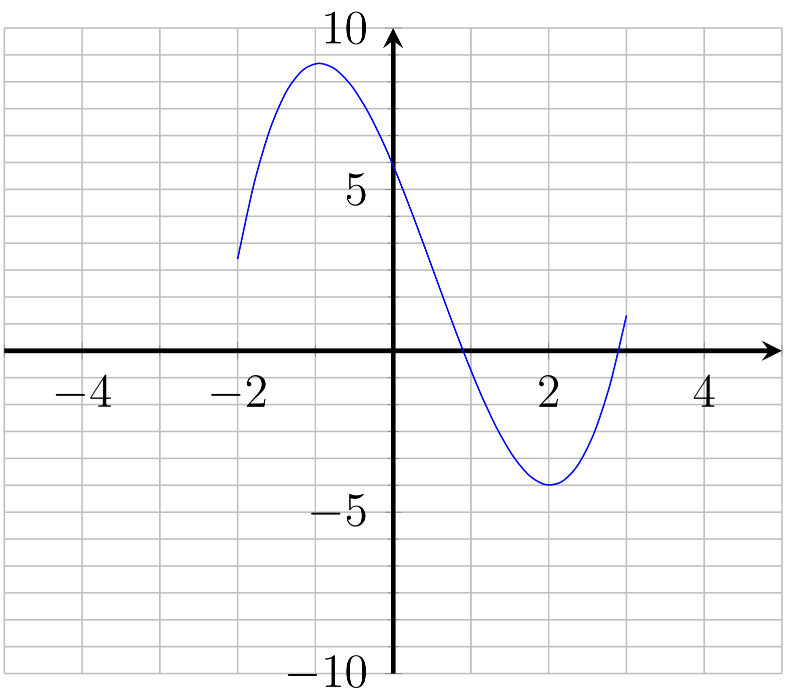
11. Suppose that $f(x)$ has the graph shown below. Sketch the graph of $f'(x)$. Make sure that you clearly show any critical point(s) or asymptote(s).



12. Is it possible to sketch a graph that satisfies ALL of the following conditions? If yes then sketch the graph of f below. Otherwise, explain (in one or two sentences) why not.

- $f(0) = 0$
- $\lim_{x \to -2^-} f(x) = 4$ and $\lim_{x \to -2^+} f(x) = -4$
- f is continuous from the left at $x = -2$
- $f(x) \to \infty$ as $x \to 4^-$ and $f(x) \to -\infty$ as $x \to 4^+$
- f is continuous at all values of x other than $x = -2$ and $x = 4$

13. Suppose that $f(x)$ is a function whose derivative $f'(x)$ is graphed below on the domain $-2 \leq x \leq 3$.



(a) Find the interval(s) where f is increasing and the interval(s) where f is decreasing. Justify your answer.

(b) Find the x value of the relative/local maximum(s) of f. Justify your answer.

(c) Find the interval(s) of concavity of f. Justify your answer.

(d) Find the x value of the point(s) of inflection of f. Justify your answer.