Systems of Linear Equations

Definition

A *linear equation* in n variables $\{x_1, x_2, \ldots, x_n\}$ is an equation of the form:

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = \sum_{k=1}^{n} a_kx_k = b$$

where the a_k and $b \in \mathbb{R}$ or $\mathbb{C}$ and are called *coefficients*.

A *system* of linear equations (SOLE) is a set of m equations in the same n variables.

Definition

A *solution* of a SOLE in n variables is a tuple $(s_1, s_2, \ldots, s_n)$ that makes each equation in the system a true statement when x_k is replaced by s_k.

The *solution set* of a SOLE is the set of all possible solutions of the SOLE.

To say that two SOLEs are *equivalent* means that they have the same solution set.

Example

$2x + y = 4$
$-x + 2y = 3$
$\overline{}$
$x = 2y - 3$
$2(2y - 3) + y = 4$
$5y = 10$
$y = 2$
$x = 2(2) - 3 = 1$

$(1, 2)$ is a unique solution

Two lines that intersect
at a single point

$x + 2y = 4$
$-x - 2y = -4$
$\overline{}$
$y = 2 - \frac{1}{2}x$

$(x, 2 - \frac{1}{2}x)$
Infinite solutions

Two coinciding lines

$x + 2y = 4$
$x + 2y = 10$

No solutions
Two parallel lines

Theorem

A SOLE has exactly one of the following possible solution sets:

1). No solution

2). Exactly one solution

3). Infinitely many solutions

Definition

To say that a SOLE is *consistent* means that it has at least one solution. Otherwise, the system is *inconsistent*.