Sample Final Exam
Spring 2015

1. $7x^2 - 3x + 5 = 0$

$b^2 - 4ac = (-3)^2 - 4(7)(5) = 9 - 140 = -131 < 0$
no real solutions.

2. $(f \circ g)(x) = \frac{1}{\sqrt{7-2x^2}}$

$g(x) = 7 - 2x^2$
$f(x) = \frac{1}{\sqrt{x}}$

3. $\log_2 4 = y \rightarrow y = 2 \quad (4, 2)$
$\log_2 x = 0 \rightarrow x = 1 \quad (1, 0)$

4. $f(x) = -7x^6 - 3x^2 + 2x - 15$

degree = 6 (even)
leading coefficient = $-7 < 0$
$\therefore$ by LCT, like $-x^2$, so d

5. $f(x) = 3x^4 - 7x^3 + 12x^2 - 8$

$p = \pm 1, \pm 2, \pm 4, \pm 8$
$q = \pm 1, \pm 3$

$\frac{p}{q} = \pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{8}{3}$.

6. $x - y + 3z = 1$
$y + 2z = 7$
$z = 3$

$y + 2(3) = 7$
$y + 6 = 7$
$y = 1$

$x - 1 + 3(3) = 1$
$x - 1 + 9 = 1$
$x = -7$

$(x, y, z) = (-7, 1, 3)$

7. $y^2 + x^3 y - x^2 + 3 = 0$

Find x-intercepts.

Set $y = 0$

$-x^2 + 3 = 0$

$x^2 = 3$

$x = \pm\sqrt{3}$

$(\pm\sqrt{3}, 0)$

8. $\frac{3}{x+2} - 6 = 5$

$\frac{3}{x+2} = 11$

$11(x+2) = 3$

$11x + 22 = 3$

$11x = -19$

$x = -\frac{19}{11}$.

9. $f(x) = -5x^2 - 2x$

$$\begin{aligned} f(7+h) - f(7) &= [-5(7+h)^2 - 2(7+h)] - [-5(7)^2 - 2(7)] \\ &= [-5(49 + 14h + h^2) - 14 - 2h] - (-245 - 14) \\ &= \cancel{-245} - 70h - 5h^2 \cancel{-14} - 2h \cancel{+245} \cancel{+14} \\ &= -72h - 5h^2 \end{aligned}$$

10. $f(x) = \frac{\sqrt{25 - x^2}}{x - 3}$

$x \neq 3$ (denominator).

$25 - x^2 \geq 0$

$x^2 - 25 \leq 0$ ← note we want −, not +

$(x+5)(x-5) \leq 0$

	$(x-5)$	$(x+5)$	
−6	−	−	+
0	−	+	−
6	+	+	+

Domain $= [-5, 3) \cup (3, 5]$

11. $f(x) = x^2 - 7$

$g(x) = \frac{5}{x+3}$

$$
\begin{aligned}
(f \circ g)(x) &= f(g(x)) \\
&= f\left(\frac{5}{x+3}\right) \\
&= \left(\frac{5}{x+3}\right)^2 - 7 \\
&= \frac{25}{(x+3)^2} - 7 \\
&= \frac{25 - 7(x+3)^2}{(x+3)^2} \\
&= \frac{25 - 7(x^2 + 6x + 9)}{(x+3)^2} \\
&= \frac{25 - 7x^2 - 42x - 63}{(x+3)^2} \\
&= \frac{-7x^2 - 42x - 38}{(x+3)^2} \\
&= -\frac{7x^2 + 42x + 38}{(x+3)^2}
\end{aligned}
$$

12. $h(x) = -2$

From the graph, $x = -1.6$ (roughly).

So $h^{-1}(-2) = -1.6$

13. $P_1(-5, 7)$ $P_2(4, 3)$

$m = \frac{3-7}{4+5} = -\frac{4}{9}$

$y - 7 = -\frac{4}{9}(x+5)$ OR $y - 3 = -\frac{4}{9}(x-4)$

14. $\frac{2}{x+1} + \frac{3}{x+3} = 1$

In this case, instead of x-mult, multiply all by $(x+1)(x+3)$.

$2(x+3) + 3(x+1) = (x+1)(x+3)$

$2x + 6 + 3x + 3 = x^2 + 4x + 3$

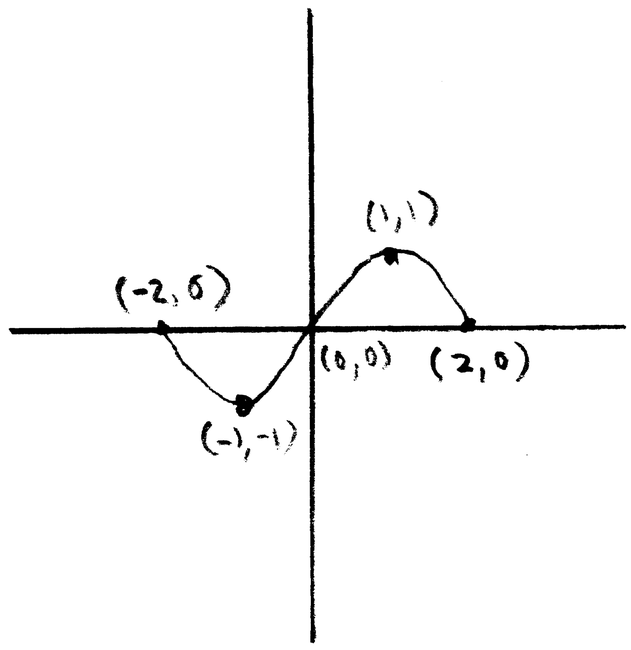
$5x + 9 = x^2 + 4x + 3$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

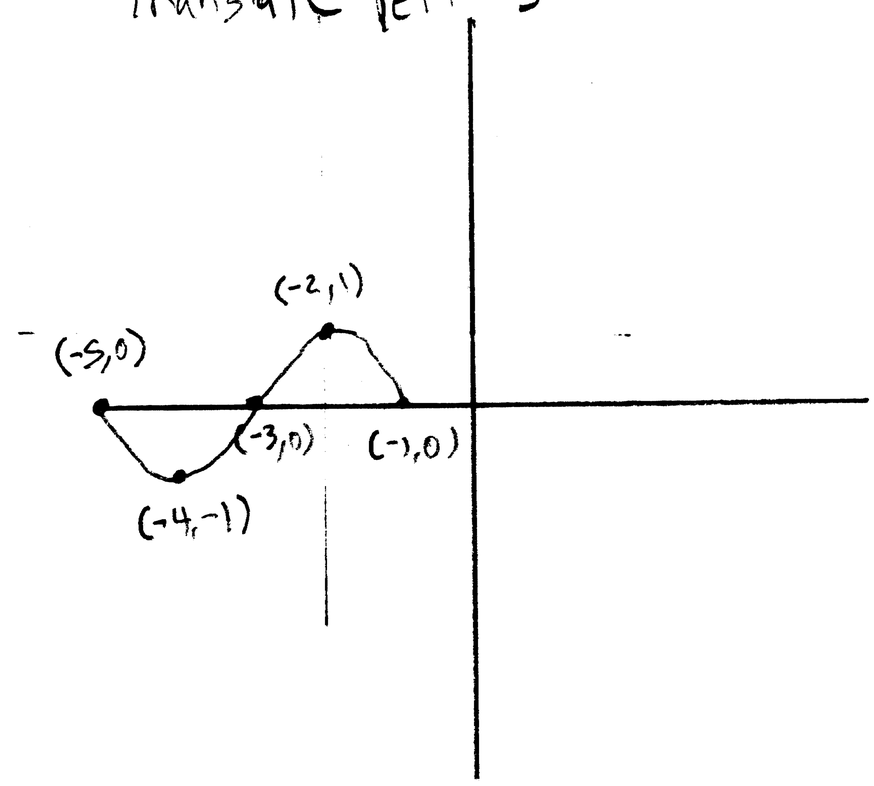
$x = -2, 3$

15. $y = 2f(x+3)$

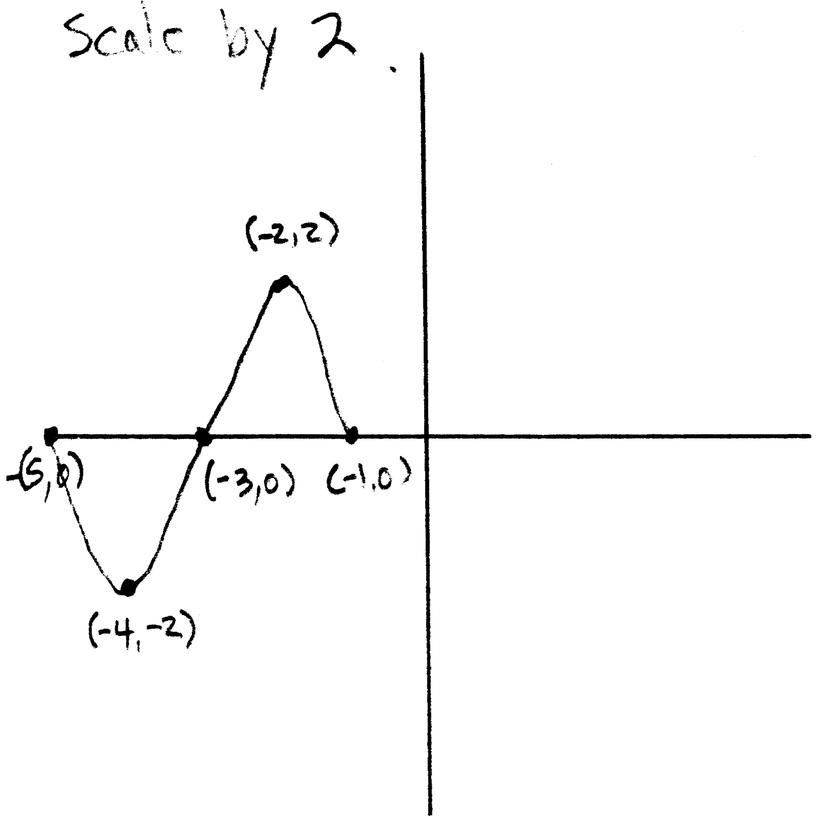


$f(x)$

translate left 3



Scale by 2.



16. $\log\left(\frac{\sqrt[3]{x+3}}{y^5 z}\right) = \log\left[(x+3)^{\frac{1}{3}}\right] - \log(y^5 z)$

$= \frac{1}{3}\log(x+3) - [\log y^5 + \log z]$

$= \frac{1}{3}\log(x+3) - 5\log y - \log z$

17. $2x - 5y = 7$

$3x + 4y = 2$

$6x - 15y = 21$

$-6x - 8y = -4$

$-23y = 17$

$y = -\frac{17}{23}$

$2x - 5\left(-\frac{17}{23}\right) = 7$

$2x + \frac{85}{23} = 7$

$2x = 7 - \frac{85}{23} = \frac{161}{23} - \frac{85}{23} = \frac{76}{23}$

$x = \frac{76}{46} = \frac{38}{23}$

$(x, y) = \left(\frac{38}{23}, -\frac{17}{23}\right)$ (yes, really)

18. $P(x) = -0.0002x^2 + 40x - 130000$

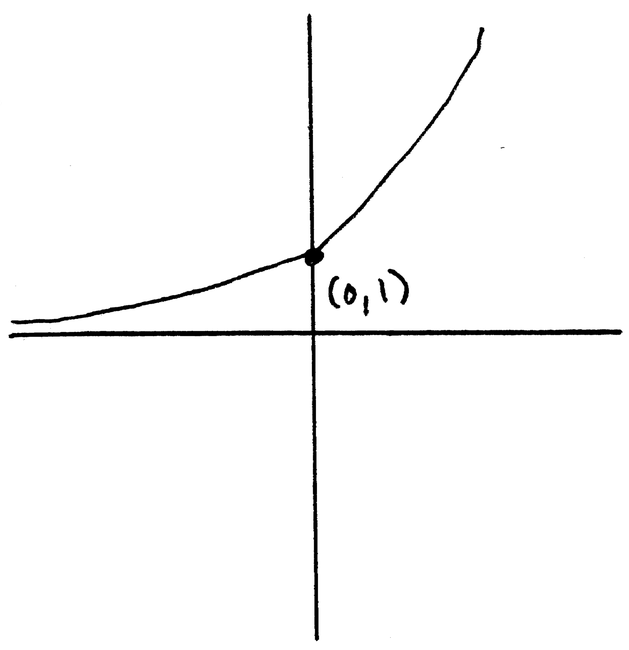
We want to maximize this – i.e., find the vertex of this upside down parabola. Although the $-\frac{b}{2a}$ method may be easier here, I will complete the square:

$$P(x) = -2\times10^{-4}x^2 + 40x - 1.3\times10^5$$
$$= -2\times10^{-4}(x^2 - 2\times10^5 x) - 1.3\times10^5$$
$$= -2\times10^{-4}(x^2 - 2\times10^5 x + 10^{10}) - 1.3\times10^5 + (2\times10^{-4})(10^{10})$$
$$P(x) = -2\times10^{-4}(x - 10^5)^2 + 1.87\times10^6$$

Kawii should produce 100,000 games for a profit of \$1,870,000.

19. $g(x) = 5^x$

a.



$g(x) \to \infty$ as $x \to \infty$
$g(x) \to 0$ as $x \to -\infty$

b. Domain = $\mathbb{R}$
Range = $(0, \infty)$

20. a. Decreasing: $[-1, 0]$ and $[3, 5]$
b. Range: $[-2, 6]$

21. Let x = annual salary in 2012
$(x + 11000)(1.1) = 102300$
$x + 11000 = 93000$
$x = 82000$

22. $A(t) = Pe^{rt}$

$75000 = 50000e^{0.04t}$

$1.5 = e^{0.04t}$

$\ln 1.5 = \ln e^{0.04t}$

$0.04t = \ln 1.5$

$t = \frac{\ln 1.5}{0.04}$

$t = 10.14$ years.

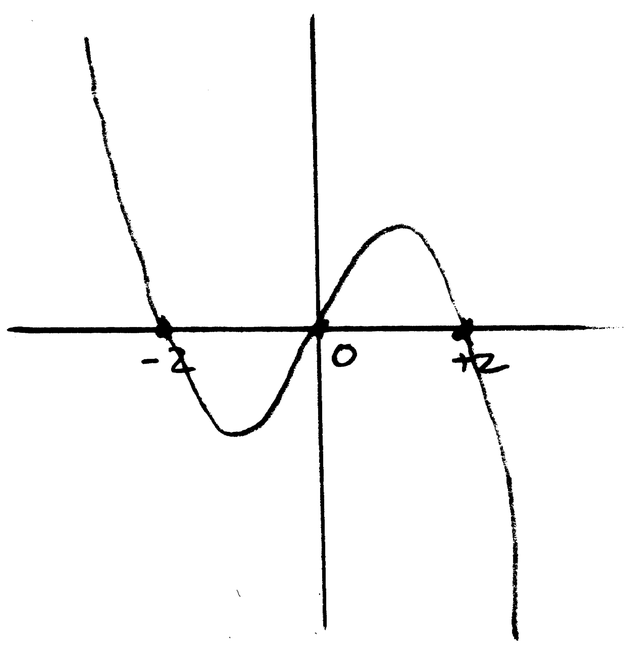
23. $f(x) = -2x^3 + 8x$

a. $f(x) = -2x(x^2 - 4)$

$= -2x(x+2)(x-2)$

b. $0, \pm 2$

c.



$f(0) = 0$

deg $= 3$ (odd)

coeff $= -2 < 0$

like $-x^3$ (LCT)

	-2 ← affects sign	x	$x-2$	$x+2$	
-1	$-$	$-$	$-$	$+$	$-$
1	$-$	$+$	$-$	$+$	$+$

24 $f(x) = x^3 - x^2 - 8x + 12$

$p = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

$q = \pm 1$

$\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

$f(1) = 1 - 1 - 8 + 12 \neq 0$

$f(-1) = -1 - 1 + 8 + 12 \neq 0$

$f(2) = 8 - 4 - 16 + 12 = 0$ ✓

$$
\begin{array}{r}
x^2 + x - 6 \\
x - 2 \overline{\big) x^3 - x^2 - 8x + 12} \\
\underline{x^3 - 2x^2} \\
x^2 - 8x \\
\underline{x^2 - 2x} \\
-6x + 12 \\
\underline{-6x + 12} \\
0
\end{array}
$$

$f(x) = (x-2)(x^2 + x - 6) = (x-2)(x-2)(x+3) = (x-2)^2(x+3)$

repeated root!