# Hypothesis Testing

## Definition: Statistical Hypothesis

A *statistical hypothesis* is a claim concerning the parameter(s) or form of a probability distribution.

## Example

Consider a farm that produces brown eggs. The farm claims that the weight of each egg is normally distributed with $\mu = 65$ g and $\sigma = 2$ g.

Suppose a carton of eggs is purchased from the farm in the example and the average weight of the sample is only 61.5 g. Is the difference due to randomness or is this significant evidence against the farm's claim of 65 g?

## Definition: Hypothesis Test

A *hypothesis test* compares a favored statistical hypothesis, called the *null hypothesis* and denoted $H_0$, and a contradictory statistical hypothesis, called the *alternate hypothesis* and denote $H_a$. A sample is used to obtain statistical information related to $H_0$. If the evidence against $H_0$ is strong enough then $H_0$ is rejected in favor of $H_a$. Otherwise, the test fails to reject $H_0$. Thus, the two outcomes of a hypothesis test are:

1. Reject $H_0$
2. Fail to reject $H_0$

## Example

In the above example, the two hypotheses are:

$H_0$**:** $\mu = 65$

$H_a$**:** $\mu \neq 65$

## Definition: Null Value

The *null value* of a hypothesis test, denoted $\theta_0$, is the claimed parameter value in the null hypothesis.

Thus, for a null hypothesis: $H_0 : \theta = \theta_0$ there is a two-sided and two one-sided alternate hypotheses:

1. $H_a : \theta \neq \theta_0$
2. $H_a : \theta < \theta_0$
3. $H_a : \theta > \theta_0$

For the two one-sided alternates, the null hypothesis is assumed to be the fully contradictory $\theta \geq \theta_0$ or $\theta \leq \theta_0$; however, during calculations the equality form is simplier and deemed sufficient.

**Example**

In the above example, the FDA wishes to enforce that $\mu = 65$ by making sure that the true value just is not less than the claimed value - i.e., $H_a : \mu < 65$. A true value greater than 65 simply benefits the customer.

## Hypothesis Test Procedure

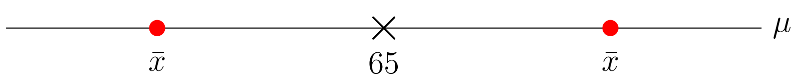
1. Construct $H_0$ and $H_a$ in terms of some null value $\theta_0$.
2. Select an unbiased point estimator $\hat{\theta}$ for $\theta_0$.
3. Collect sample data.
4. Compare the observation to the claim to see if there is sufficient reason to reject $H_0$.

**Example**

In the above example, for $H_0 : \mu = 65$, select $\bar{X}$ as a suitable (unbiased) point estimator for $\mu$. A sample is then collected and $\bar{x}$ is calculated. The three possible alternatives would then be judged as follows:

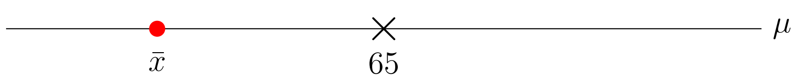
1. $H_a : \mu \neq 65$

   Reject $H_0$ if $\bar{x}$ is either too small or too large:

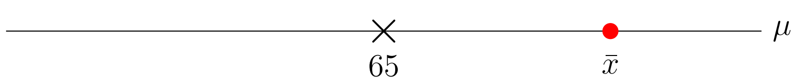
2. $H_a : \mu < 65$

   Reject $H_0$ if $\bar{x}$ is too small.

3. $H_a : \mu > 65$

   Reject $H_0$ if $\bar{x}$ is too large.

Notions of "too small" and "too large" will be developed by balancing test error and using $p$-values.