Math-1005a Homework #5

Decimals and Percents

Problems

1). Normally you will be asked to supply your answers to math problems using exact, non-decimal values. This includes rational numbers, square roots, π, etc. However there are times when a decimal value is more appropriate. The main case is an answer to a word problem. For example, saying that something is $\frac{131}{16}$ feet away is not as meaningful as saying that it is about 8.2 feet away.

To convert from a rational number to a decimal number, simply divide the numerator by the denominator. Although you can use the long division that you learned in elementary school, it is much easier these days to just use a calculator when the numbers aren't nice.

Furthermore, don't go wild on decimal digits. For example, saying that something is 1.2456789128273 feet long is silly. Instead, round to a reasonable number of significant figures (usually 2 or 3). Most problems will state how many decimal digits are required.

Convert the following rational numbers to decimal numbers. Round your answers to at most four decimal places:

a). $\frac{1}{2}$

b). $-\frac{3}{8}$

c). $\frac{1}{9}$

d). $-\frac{135}{116}$

e). $\frac{38}{178}$

2). When converting a decimal number to rational form, determine how many places the decimal point must be moved to the right and then multiply and divide by that power of 10. For example, 1.234 requires the decimal point to be moved to the right by 3 places, so:

$$1.234 = 1.234 \cdot \frac{1000}{1000} = \frac{1234}{1000} = \frac{617}{500}$$

Remember that whole numbers are already rational and thus don't require any conversion.

Convert the following decimal numbers to rational numbers. Your answers should be reduced (improper) fractions or whole numbers - not mixed numbers:

a). 5.2

b). 4

c). 0.89

d). 38.7

e). 3.14

3). The percent sign (%) is a shorthand for "divide by 100". When the percent value is a whole number, it can be converted into a rational number by dividing by 100. For example:

$$12\% = \frac{12}{100} = \frac{3}{25}$$

and:

$$120\% = \frac{120}{100} = \frac{6}{5}$$

Evaluate the following expressions. Your answers should be reduced (improper) fractions or whole numbers - not mixed numbers:

a). 17%

b). -130%

c). 98%

d). -50%

e). 112%

4). Since "percent" means divide by 100, to convert from percent to decimal just move the decimal point over two places. For example:

$$12\% = 0.12$$

and:

$$123.4\% = 1.234$$

and:

$$0.01\% = 0.0001$$

Convert the following percents to decimal numbers:

a). 66%

b). 645%

c). -58.7%

d). 0%

e). -0.08%

5). When comparing values in different forms, convert to a common form and then compare.

For each of the following problems, determine whether the first value is less than, equal to, or greater than the second value:

a). $52\% \square 1$

b). $3.4 \square 34\%$

c). $90\% \square \frac{9}{10}$

d). $0.75\% \square \frac{7}{8}$

e). $11\% \square -11\%$

6). All percentage-related problems at some point come down to the following statement:

$a\%$ of b is c

Note that this is the same as the equation:

$$\frac{a}{100} \cdot b = c$$

You are usually given two of the values and your job is to find the third. When a and b are given then multiply to find c. For example, 50% of 25 is:

$$\frac{50}{100} \cdot 25 = 12.5$$

When either a or b is not known then divide c by the known value to find the unknown value. For example, to find out what percent 2 is of 16:

$$\frac{2}{16} = 0.125 = 12.5\%$$

Similarly, to determine that 2 is 12.5% of what number:

$$\frac{2}{12.5\%} = \frac{2}{0.125} = 16$$

Fill in the blanks with the appropriate numbers. Round all decimal numbers to four decimal points:

a). 13% of 200 is _____

b). 9% of _____ is 36

c). _____$\%$ of 150 is 38.125

d). 63.1% of 270 is _____

e). 98.3% of _____ is 322.5

f). _____$\%$ of 34 is 2.89

g). 110.7% of $\frac{110}{7}$ is _____

h). 80.9% of _____ is $\frac{400}{3}$

i). _____$\%$ of 363 is 275.88

7). Aja, Nella, and Cai are buying sandwiches at Lee's Sandwich Shop. Each sandwich normally sells for $\$9.00$; however, each girl has a discount coupon. Aja's coupon gives her $\$2.25$ off the normal price. Bella's coupon gives her a 25% discount. Cai's coupon lets her save $\frac{1}{3}$ of the normal price.

a). How much does each girl pay for her sandwich? Your answers should be in dollars and cents (i.e., rounded to 2 decimal places).

b). Who pays the most for their sandwich?

8). Sometimes we need to take a percentage/fraction of a percentage/fraction of something (two steps). Just multiply everything. For example, 10% of 50% of 75 is:

$$10\% \cdot 50\% \cdot 75 = 0.1 \cdot 0.5 \cdot 75 = 3.75$$

Compute 40% of $\frac{1}{9}$ of 4.

9). The city of Fremont has a lake in its Central Park called Lake Elizabeth. This summer, the city estimated the following breakdown of the bird population around the lake:

ducks	40%
geese	20%
cormorants	10%
herons	30%

What percentage of the birds that are not geese are herons? Round your answer to one decimal place.

10). A common problem is calculating how much of something after a percent increase. Suppose that you invest $\$1000$ (called the principal) at 5% simple interest per year. The interest after one year would be:

$$\$1000 \cdot 0.05 = \$50$$

So to determine how much you have after one year your would add the interest to the original principal:

$$1000 + 1000 \cdot 0.05 = 1000(1 + 0.05) = 1000(1.05) = 1050$$

for a total of $\$1050$.

Note the $\left(1 + \frac{p}{100}\right)$ factor; you can use this as a shortcut for percent growth problems.

You buy a one-year CD (certificate of deposit) for $\$13000$ that pays 3.5% interest. How much do you collect when you cash it in at the end of the year?

11). Percent decrease problems are similar to percent increase problems, except this time you subtract the percentage from the original and the key factor is $\left(1\frac{p}{100}\right)$.

The decrease in population in rural cities and towns has always been a problem as the children of residents seek better jobs in the city. Smallville, USA had a population of 1842 people at the start of 2016, but by the end of the year the population had dropped 6%. What was the population of Smallville at the start of 2017?

Remember, people don't come in fractions, so round to a whole number.

12). Sometimes we have a situation that changes from time to time. For example, suppose that you have an $\$1000$ investment that pays 10% in the first year and then 5% in the second year. After the first year the value of your investment would be:

$$\$1000(1 + 0.10) = \$1000(1.1) = \$1100$$

The important thing to note is that this value is the starting value for the next year, so after year two you would have:

$$\$1100(1 + 0.05) = \$1100(1.05) = \$1155$$

As a short cut, note that this is the same as:

$$\$1000(1 + 0.10)(1 + 0.05) = \$1000(1.1)(1.05) = \$1155$$

So the multiple growth and decrease factors can be multiplied together to obtain the total growth/decrease factor.

Suppose that you decide to play the stock market, so you invest $\$60000$ in Google, Inc. During the first month, Google stock increased 2% in value; however, during the second month the stock decreased by 2%. What is the value of your investment after the second month?

(Hint: the answer is not $\$60000$)

13). An important application of percentages in lab work and manufacturing is percent error. This is the amount that an actual measured value deviates from an estimated or required value and is calculated as follows:

$$\%\text{error} = \frac{\text{desired} - \text{actual}}{\text{desired}} \cdot 100$$

Note that the percent error can be positive or negative depending on whether the actual value is less than or greater than the desired value.

You are a quality control officer for a manufacturing firm that makes parts for the aircraft industry. Your plant is currently working on a bolt that is used in an aircraft wing. The design engineers specify that the bolt must be 4 cm long with a percent error of 0.05%. You receive 5 parts from the line and test them for compliance. Compute the percent error (to at most four decimal places) for each part and specify whether each part passes or fails inspection:

required (cm)	measured(cm)	% error	pass/fail
4	3.9984		
4	4.0024		
4	3.9976		
4	4.0004		
4	3.9992		