Midterm Exam

Name: _____

This exam is closed book and notes. You may use a TI-84 graphing calculator; however, no other electronics are allowed. You may also use a double-sided 8.5×11 cheatsheet of your making. Show all work; there is no credit for guessed answers. Simplify your answers unless told otherwise. In particular, all answers should contain no negative or rational exponents. All numerical answers should be in exact form unless you are specifically asked for a decimal value.

1. Consider the expression:

$$-\sqrt{\frac{7.\overline{9}}{2}}$$

   (a) Without using a calculator, evaluate the expression.

   (b) Circle the subsets of the real numbers to which the value belongs.

   $\mathbb{N}$ $\qquad\qquad$ $\mathbb{Z}$ $\qquad\qquad$ $\mathbb{Q}$ $\qquad\qquad$ $\mathbb{R} - \mathbb{Q}$ $\qquad\qquad$ $\mathbb{R}$

2. Complete the following:

   (a) $\left(x^{\frac{1}{2}}\right)^{2} =$

   (b) $\left(x^{2}\right)^{\frac{1}{2}} =$

   (c) $\left(x^{\frac{1}{3}}\right)^{3} =$

   (d) $\left(x^{3}\right)^{\frac{1}{3}} =$

3. You own a store that sells candy and nuts. You currently sell peanuts at $5 per pound and cashews at $7.50 per pound. The cashews aren't selling very well because their price is too high, so you decide to make a peanut/cashew mix. You want to have 10 pounds of this mix to sell at $6.50 per pound. How many pounds of peanuts and how may pounds of cashews should be in the mix?

4. Consider the following general form equation for a circle:

$$x^2 - 6x + y^2 + 2y + 1 = 0$$

(a) Convert the equation to standard form.

(b) What are the center and radius of the circle?

5. Let $f(x) = 2x^2 + 3x - 1$. Calculate the following:

$$\frac{f(x+h) - f(x)}{h}$$

6. You decide to start your own penny tee business. You estimate that the fixed costs per month are $1000 and the variable costs are $2.50 per shirt. Assuming a linear cost model, how much does it cost to produce 500 shirts each month?
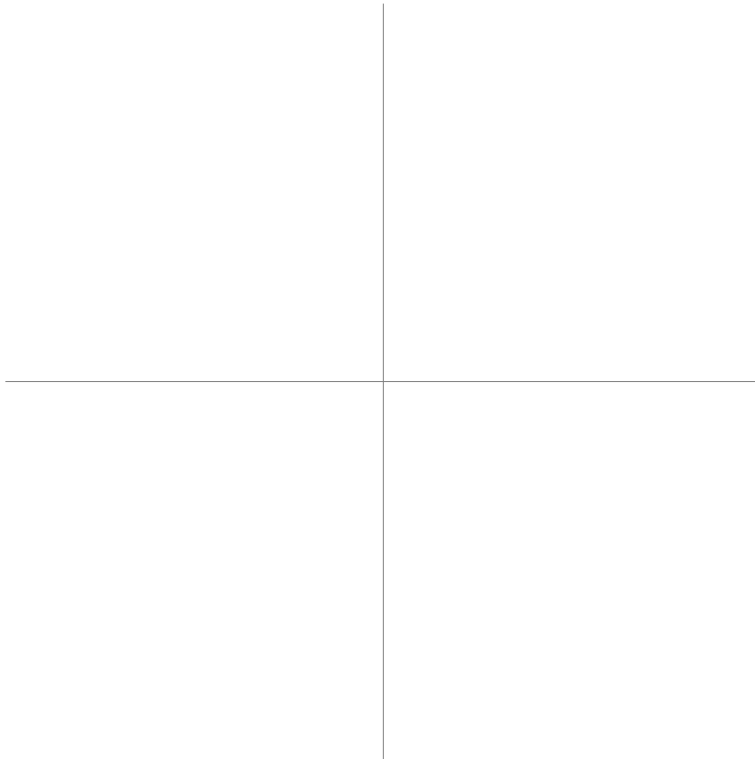
7. Consider the function:

$$y = -2\sqrt{x+1} + 3$$

(a) Starting with the basic function, list the transformations in the order that they should be applied.

(b) What are the final transformed coordinates of the key point?

(c) What are the x-intercepts, if any?

(d) What are the y-intercepts, if any?

(e) Sketch the graph. The transformed key point and all intercepts must be clearly shown and labeled for full credit.

(f) What is the domain (in interval notation)?

(g) What is the range (in interval notation)?

8. Consider the general form parabolic function:
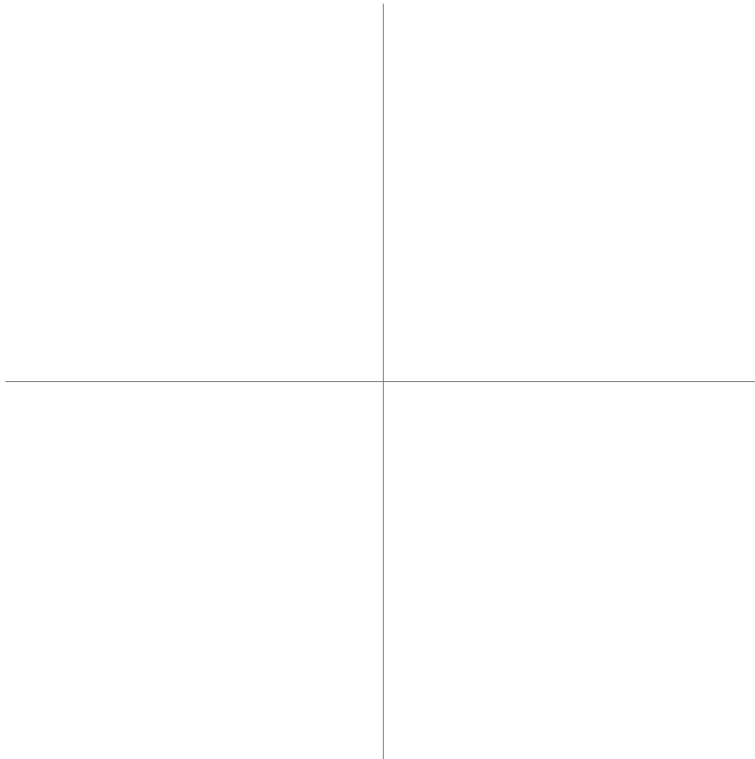
$$y = -2x^2 + 4x + 3$$

(a) Convert the equation to standard form.

(b) What are the coordinates of the vertex?

(c) What are the x-intercepts, if any?

(d) What are the y-intercepts, if any?

(e) Sketch the graph. The vertex and all intercepts must be clearly shown and labeled for full credit.

(f) What is the domain (in interval notation)?

(g) What is the range (in interval notation)?

9. Consider the polynomial:

$$y = 2x^4 + 3x^3 - x$$

(a) Completely factor by determining candidate zeros and using long or synthetic division. There is no credit for factoring by grouping.

(b) Sketch the polynomial. All intercepts must be clearly marked and labeled and the end behavior must be clearly visible.
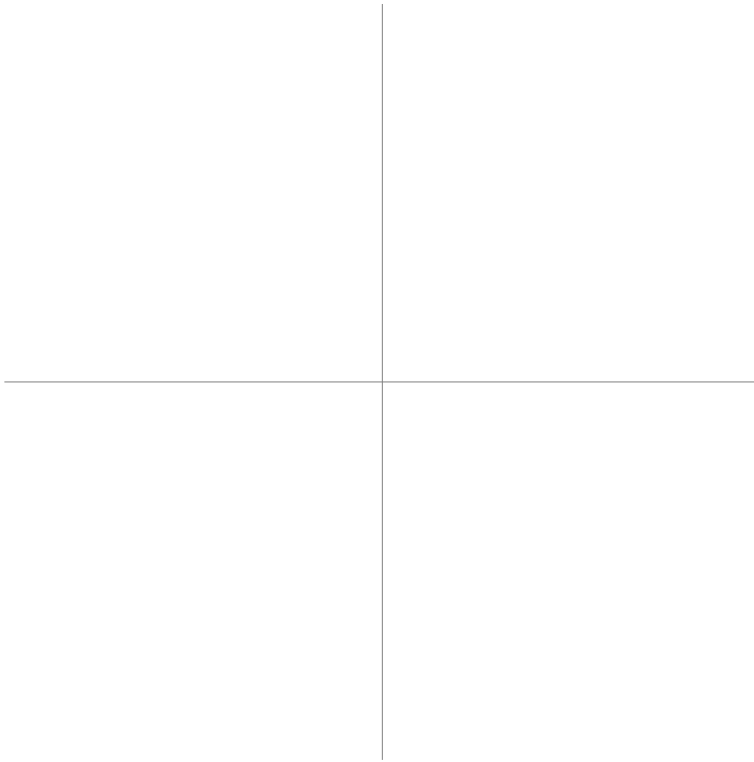
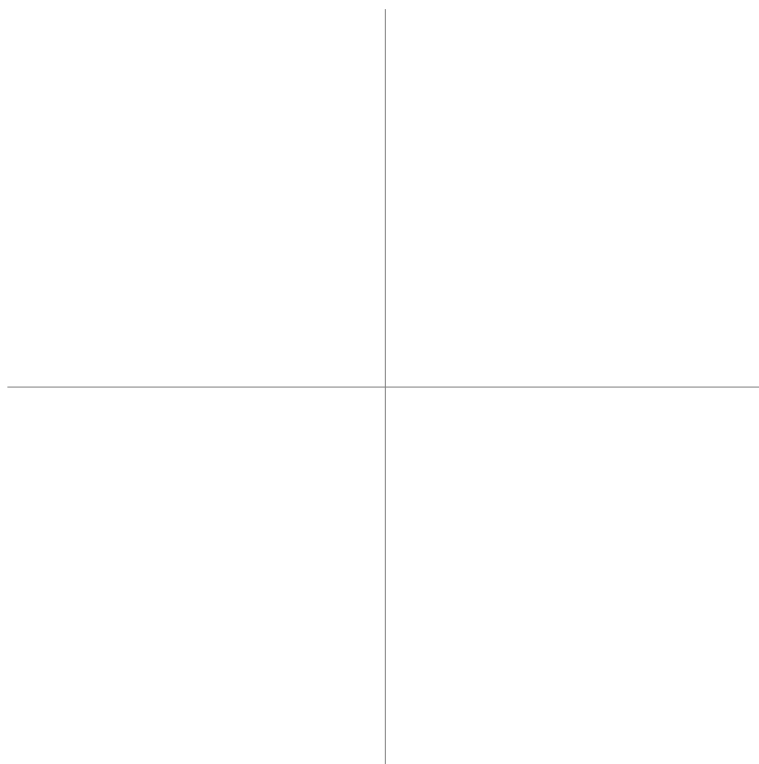(c) Using your calculator, determine any extrema and mark them on your sketch (if any).

10. Consider the rational function:

$$y = \frac{x^2 - 4x + 3}{x^2 - 5x + 6}$$

(a) What are the zeros?

(b) What are the poles?

(c) Where are the x-intercepts (if any)?

(d) Where are the y-intercepts (if any)?

(e) Where are the vertical asymptotes (if any)?

(f) Where are the horizontal asymptotes (if any)?

(g) What is the end behavior as $x \to \infty$ and $x \to -\infty$? Be sure to specify direction - i.e., from above $(+)$ or below $(-)$.

(h) Sketch the graph. All intercepts, asymptotes, and holes must be clearly marked and labeled for full credit.

(i) Using your calculator, determine any extrema and mark them on your sketch (if any).