Math-19 Homework #10 Solutions

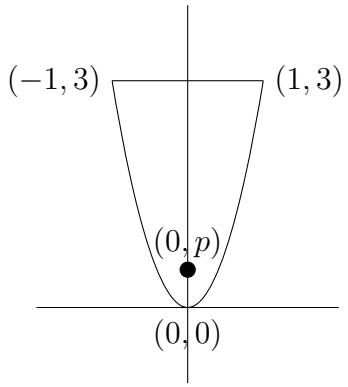
Reading

Please read sections 11.1-11.4, then do all concept problems in the posted sections on webassign.

Problems

1). You are designing a new spotlight with a parabolic reflector. The diameter of the reflector is 2 feet and the height is 3 feet. How high above bottom of the dish should the lightbulb be placed.

The lightbulb should be placed at the focus. The problem looks like this:



$$x^2 = 4py$$

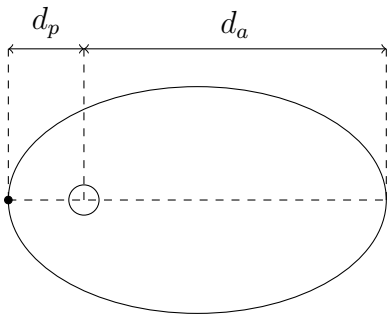
$$1^2 = 4p(3)$$

$$1 = 12p$$

$$p = \frac{1}{12}$$

So the focus should be located 1/12 ft above the bottom, or 1 inch.

2). The earth's orbit around the sun is elliptical with the sun at one of the foci. The closest the earth gets to the sun is about 91 million miles. The eccentricity of the earth's orbit is about 0.0167. What is the farthest distance between the earth and the sun?



The closest point is called the *perihelion*; call this distance d_p. The farthest point is called the *aphelion*; call this distance d_a.

We are given:

$$d_p = a - c = 91 \text{ million miles}$$

$$e = \frac{c}{a} = 0.0167$$

Note that:

$$d_a = 2a - (a - c) = a + c$$

But we can solve for a and c:

$$\begin{aligned}
c &= ae \\
d_p &= a - ae \\
d_p &= a(1 - e) \\
a &= \frac{d_p}{1 - e} \\
c &= d_p \left(\frac{e}{1 - e} \right) \\
d_a &= \frac{d_p}{1 - e} + d_p \left(\frac{e}{1 - e} \right) \\
&= \frac{d_p + d_p e}{1 - e} \\
&= d_p \left(\frac{1 + e}{1 - e} \right) \\
&= 91 \left(\frac{1 + 0.0167}{1 - 0.0167} \right) \\
&\approx 94
\end{aligned}$$

$d_a \approx 94$ million miles

3). Consider the following parabola:

$$y^2 - 6y - 12x + 33 = 0$$

a). What is the vertex?

$$\begin{aligned} y^2 - 6y &= 12x - 33 \\ y^2 - 6y + 9 &= 12x - 33 + 9 \\ (y-3)^2 &= 12x - 24 \\ (y-3)^2 &= 12(x-2) \end{aligned}$$

$V(2,3)$

b). What is the axis of symmetry?

$y = 3$

c). What is the focus?

$4p = 12$

$p = 3$

$F(2+p, 3) = F(5,3)$

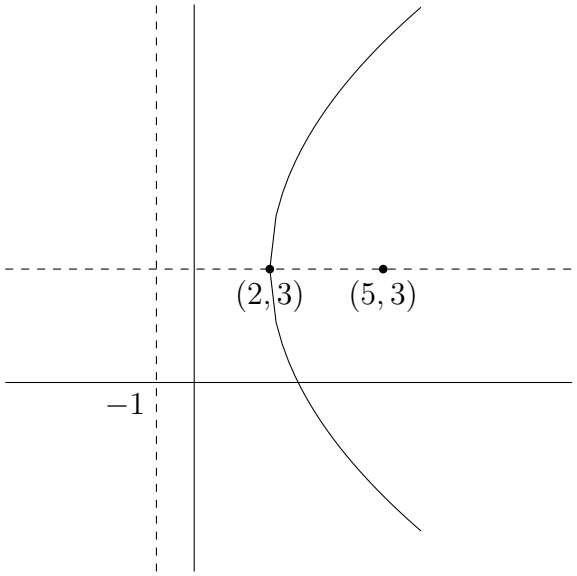
d). What is the directrix?

$x = 2 - p = 2 - 3 = -1$

e). What is the focal diameter?

$d = |4p| = 4 \cdot 3 = 12$

f). Sketch the parabola, labeling all of the above items.



4). Consider the following ellipse:

$$4x^2 + 25y^2 - 50y - 75 = 0$$

a). What is the center?

$$\begin{aligned} 4x^2 + 25(y^2 - 2y) &= 75 \\ 4x^2 + 25(y^2 - 2y + 1) &= 75 + 25 \\ 4x^2 = 25(y-1)^2 &= 100 \\ \frac{x^2}{25} + \frac{(y-1)^2}{4} &= 1 \\ \frac{x^2}{5^2} + \frac{(y-1)^2}{2^2} &= 1 \end{aligned}$$

$C(0, 1)$

b). What is the length of the major axis?

$a = 5$

$2a = 10$

c). What is the length of the minor axis?

$b = 2$

$2b = 4$

d). What are the four vertices?

$V_1(5, 1)$
$V_2(-5, 1)$
$V_3(0, 3)$
$V_4(0, -1)$

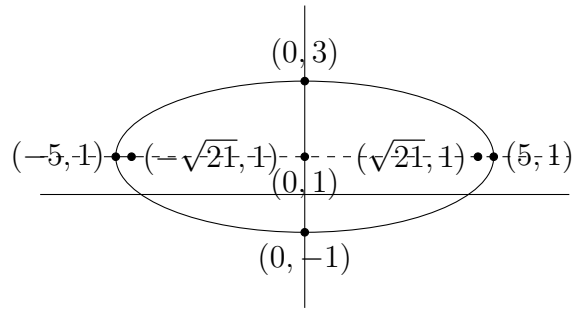
e). What are the two foci?

$c^2 = a^2 - b^2 = 25 - 4 = 21$

$c = \sqrt{21}$

$F(\pm\sqrt{21}, 1)$

f). Sketch the ellipse, labeling all of the above items.



5). Consider the following hyperbola:

$$36x^2 + 72x - 4y^2 + 32y + 116 = 0$$

a). What is the center?

$$\begin{aligned} 36(x^2 + 2x) - 4(y^2 - 8y) &= -116 \\ 36(x^2 + 2x + 1) - 4(y^2 - 8y + 16) &= -116 + 36 - 64 \\ 36(x+1)^2 - 4(y-4)^2 &= -144 \\ \frac{(y-4)^2}{36} - \frac{(x+1)^2}{4} &= 1 \\ \frac{(y-4)^2}{6^2} - \frac{(x+1)^2}{2^2} &= 1 \end{aligned}$$

$C(-1, 4)$

b). What is the length of the horizontal axis?

$b = 2$

$2b = 4$

c). What is the length of the vertical axis?

$a = 6$

$2a = 12$

d). What are the two vertices?

$V_1(-1, 10)$
$V_2(-1, -2)$

e). What are the two foci?

$c^2 = a^2 + b^2 = 36 + 4 = 40$

$c = \sqrt{40} = 2\sqrt{10}$

$F(-1, 4 \pm 2\sqrt{10})$

f). What are the two asymptotes?

$$(y-4) = \frac{6}{2}(x+1) \qquad (y-4) = -\frac{6}{2}(x+1)$$

$$y = 3x+7 \qquad y = -3x+1$$

g). Sketch the hyperbola, labeling all of the above items.