Math-42 Sections 01, 02, 05

Exam #2

Name: _____

This is a take-home exam. It is open textbook and open notes; however, no collaboration is allowed. Show all work; there is no credit for guessed answers. Upload the completed exam to canvas by the deadline.

1. Let $A = \{\pi, e, \sqrt{2}\}$ and $B = \{e, 42\}$. Evaluate the following:

   (a) $A \cup B$

   (b) $B \cap A$

   (c) $A - B$

   (d) $B \times A$

   (e) $\mathcal{P}(A)$

2. Prove: $A - B = A \cap \bar{B}$

3. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^3 + 1$:

   (a) Prove that f is injective.

   (b) Prove that f is surjective.

   (c) Prove that f is bijective.

   (d) Prove that f is invertible.

4. Let A and B be sets and let $T \subseteq B$. Also let $f : A \to B$.

   (a) Use a diagram to show that $f(f^{-1}(T)) \subseteq T$.

   (b) Under what condition does equality hold?

5. Find a closed form for the sequence: $a_n = a_{n-1} + 5$ where $a_0 = 3$.

6. Find a closed form for the sequence: $a_n = 5a_{n-1}$ where $a_0 = 3$.

7. Evaluate:

$$\sum_{k=50}^{100} (\sqrt{k} - \sqrt{k-1})$$

8. Evaluate:

$$\sum_{k=1}^{100}(2k+1)$$

9. Evaluate:

$$\sum_{k=2}^{\infty}\left(-\frac{1}{4}\right)^{k}$$

10. Prove: A and B countable $\implies$ $A \times B$ countable.