Math-13 Sections 01 and 02

# Homework #7 Solutions

A ball is thrown upward off of a 100 ft cliff at a velocity of 64 ft/s. The equation of motion for the ball is:

$$h(t) = 100 + 64t - 16t^2$$

1. Using the derivative formulas, determine $h'(t)$.

   $$h'(t) = 0 + 64(1) - 2(16)t^{2-1} = 64 - 32t$$

2. Using your answer to part (1) (and not the vertex formula $-\frac{b}{2a}$), at what time does the ball reach its maximum height (hint: how fast is the ball going at that point)?

   When the ball reaches its maximum height, gravity has stopped it so the velocity at the maximum height is 0:

   $$64 - 32t = 0$$
   $$32t = 64$$
   $$t = 2$$

   Therefore, the ball reaches its maximum height after 2 s.

3. What is the balls maximum height?

   $$h(2) = 100 + 64(2) - 16(2)^2 = 100 + 128 - 64 = 164$$

   Therefore, the ball reaches a maximum height of 164 ft

4. How long before the ball hits the ground?

   When the ball hits the ground, $h(t) = 0$:

   $$100 + 64t - 16t^2 = 0$$
   $$4t^2 - 16t - 25 = 0$$
   $$t = \frac{16 \pm \sqrt{(-16)^2 - 4(4)(-25)}}{2(4)}$$
   $$t = \frac{16 \pm \sqrt{256 + 400}}{8}$$
   $$t = \frac{16 \pm \sqrt{656}}{8}$$
   $$t = \frac{16 \pm 4\sqrt{41}}{8}$$
   $$t = \frac{4 \pm \sqrt{41}}{2}$$
   $$t \approx -1.2, 5.2$$

   Therefore, the ball hits the ground after 5.2 s.

5. Note that you probably got two solutions (one positive and one negative) for the part (4), and you should have selected the positive one. What is the meaning of the negative solution (hint: what is the meaning of negative time)?

   The negative solution corresponds to the ball being thrown up from the ground 1.2 s earlier with sufficient velocity such that it is going 64 ft/s as it passes the top of the cliff. Note that the ball will follow the same path once it passes that top of the cliff. The required initial velocity will be the same as the velocity when the ball hits the ground:

   $$v(5.2) = 64 - 32(5.2) = -102.4$$

   So the ball must be thrown upward from the ground at a velocity of 102.4 ft/s so that it follows the same path as a ball thrown upward from the cliff at 64 ft/s.