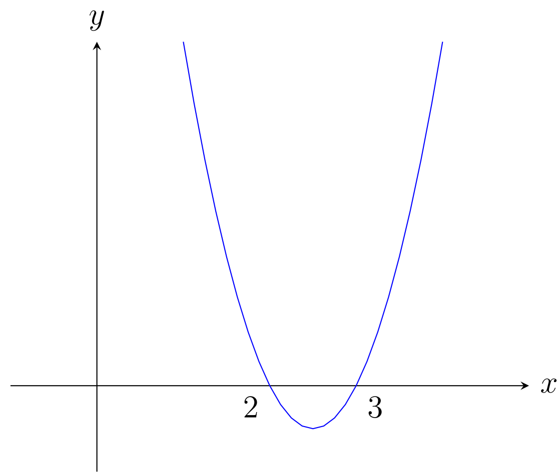
Limits

Example

Consider the quadratic function $f(x) = x^2 - 5x + 6$:



What happens to $f(x)$ as $x \to 2$, but $x \neq 2$?

x	$f(x)$
2.1	-0.09
2.01	-0.0099
2.001	-0.000999
2	
1.999	0.001001
1.99	0.0101
1.9	0.11

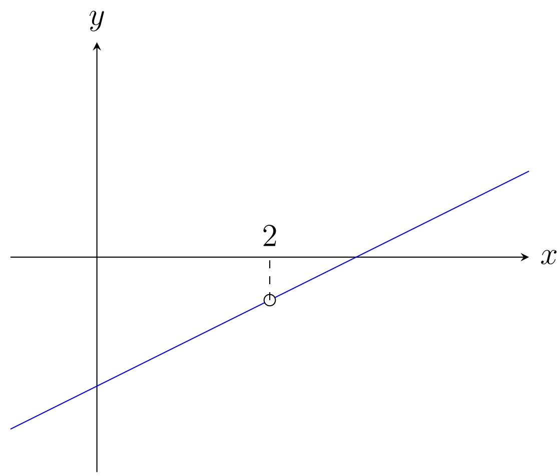
It appears that $f(x) \to 0$ as $x \to 2$ (from either direction).

In the previous example, it turns out that $f(x)$ is actually defined at $x = 2$ and furthermore, $f(2) = 0$. This special case will be used later as a formal definition of *continuity*. However, as previously stated, we don't actually care about the function value at $x = 2$. In fact, the function might not even be defined at the x value in question.

Example

Consider the rational function:

$$f(x) = \frac{x^2 - 5x + 6}{x - 2}$$

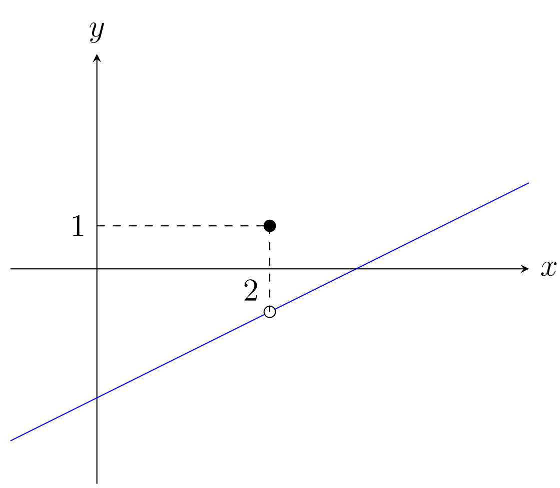


Now, as $x \to 2$:

x	$f(x)$
2.1	-0.9
2.01	-0.99
2.001	-0.999
2	
1.999	-1.001
1.99	-1.01
1.9	-1.1

It appears that $f(x) \to -1$ as $x \to 2$ (from either direction), even though $f(2)$ is not defined. To reiterate, we do not care what actually happens at $x = 2$. In fact, let's define $f(2) = 1$:

$$f(x) = \begin{cases} \frac{x^2-5x+6}{x-2}, & x \neq 2 \\ 1, & x = 2 \end{cases}$$

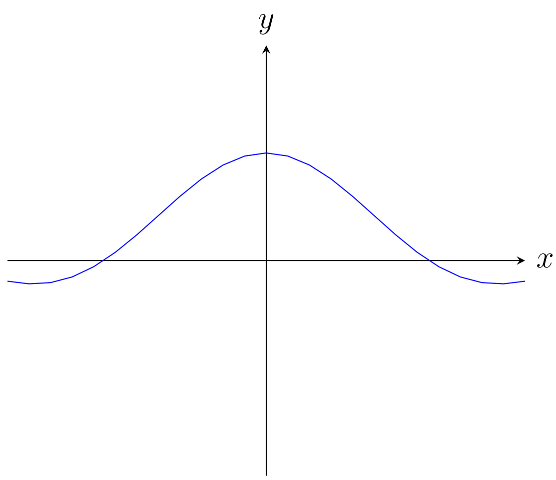


Still, $f(x) \to -1$ as $x \to 2$, regardless of the fact that $f(2) = 1$. Once again, we do not care about the function at $x = 2$; we only care what happens near $x = 2$.

Example

Consider the function:

$$f(x) = \frac{\sin x}{x}$$



As $x \to 0$:

x	$f(x)$
1	0.841471
0.1	0.998334
0.01	0.999983
0	
-0.01	0.999983
-0.1	0.998334
-1	0.841471

It appears that $f(x) \to 1$ as $x \to 0$. Note that at $x = 0$, $f(x) = \frac{0}{0}$, which is a so-called *indeterminate form*; we cannot tell if the function is actually defined at $x = 0$ or not. In this case it is and $f(0) = 1$.