Math-19 Homework #5 Solutions

Problems

1). Solve each of the following for x. Express your answers both graphically and in interval notation:

a). $x^2 + 9x - 36 = 0$

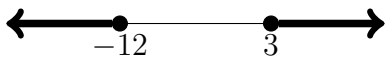
$(x + 12)(x - 3) = 0$

$x = -12, 3$

b). $x^2 + 9x - 36 < 0$

$x \in (-12, 3)$

c). $x^2 + 9x - 36 \geq 0$



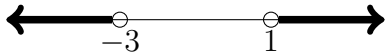
$x \in (-\infty, -12] \cup [3, \infty)$

2). Solve each of the following for x. Express your answers both graphically and in interval notation:

a). $\dfrac{x+3}{x-1} = 0$

$x = -3$

b). $\dfrac{x+3}{x-1} > 0$



$x \in (-\infty, -3) \cup (1, \infty)$

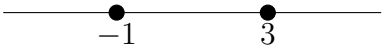
c). $\dfrac{x+3}{x-1} \leq 0$

$x \in [-3, 1)$

3). Solve each of the following for x. Express your answers both graphically and in interval notation:

a). $3\,|x-1|+1=7$

$3\,|x-1|=6$
$|x-1|=2$
$x-1=\pm 2$
$x=-1,3$

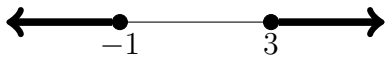


$x=-1,3$

b). $3\,|x-1|+1\leq 7$

$x \in [-1, 3]$

c). $3\,|x-1|+1\geq 7$



$x \in (-\infty, -1] \cup [3, \infty)$

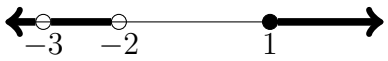
4). Find the domain of the following expressions. Express your answers both graphically and in interval notation:

a). $\sqrt{\dfrac{x^2+2x-3}{x^2+5x+6}}$

This is an even root, so the radicand must be ≥ 0.

$$\begin{aligned} \frac{x^2+2x-3}{x^2+5x+6} &\geq 0 \\ \frac{(x+3)(x-1)}{(x+2)(x+3)} &\geq 0 \\ \frac{x-1}{x+2} &\geq 0 \end{aligned}$$

Note: $x \neq -3$



$x \in (-\infty, -3) \cup (-3, -2) \cup [1, \infty)$

b). $\sqrt[3]{\dfrac{x^2+2x-3}{x^2+5x+6}}$

This is an odd root, so we are only concerned a zero denominator.

$(x+2)(x+3) \neq 0$

$x \neq -2, -3$

$x \in (-\infty, -3) \cup (-3, -2) \cup (-2, \infty)$

5). Muri is a shopkeeper that specializes in pickled vegetables. She has determined over the years that the best brine (salt solution) for pickling vegetables is 2 kg of salt per liter of water (2 kg/L). One day, she has her not-so-bright nephew helping her and he uses too much salt, resulting in a 5 kg/L solution. If her nephew made up 10 liters of the too-salty solution, how much pure water must he add to it to get the ideal 2 kg/L solution? For full credit, show the mixture equation and the appropriate values for each concentration and volume value in the equation.

$c_1 v_1 + c_2 v_2 = c_3(v_1 + v_2)$

c_1	5 kg/L
v_1	10 L
c_2	0 kg/L
v_2	x
c_3	2 kg/L
$v_1 + v_2$	$10 + x$

$5(10) + 0x = 2(10 + x)$
$50 = 2(10 + x)$
$x + 10 = 25$

$x = 15$

Thus, 15 L of pure water must be added to the 10 L of 5 kg/L solution in order to dilute it to 2 kg/L.