Math-19 Homework #9 Solutions

<u>Problems</u>

1). Consider the quadratic function: $y = 2 - 5x - 3x^2$.

a). By completing the square, convert the general form to standard form.

$$
\begin{aligned}
y &= 2 - 5x - 3x^2 \\
&= -3x^2 - 5x + 2 \\
&= -3\left(x^2 + \frac{5}{3}x\right) + 2 \\
&= -3\left(x^2 + \frac{5}{3}x + \frac{25}{36}\right) + 2 + 3\left(\frac{25}{36}\right) \\
&= -3\left(x + \frac{5}{6}\right)^2 + 2 + \frac{25}{12} \\
y &= -3\left(x + \frac{5}{6}\right)^2 + \frac{49}{12}
\end{aligned}
$$

b). What are the coordinates of the vertex?

$$\left(-\frac{5}{6}, \frac{49}{12}\right)$$

c). Is the parabola open up or open down? How do you know?

Open down because $a = -3 < 0$.

d). What are the x-intercepts (if any)?

$$
\begin{aligned}
-3\left(x + \frac{5}{6}\right)^2 + \frac{49}{12} &= 0 \\
3\left(x + \frac{5}{6}\right)^2 &= \frac{49}{12} \\
\left(x + \frac{5}{6}\right)^2 &= \frac{49}{36} \\
x + \frac{5}{6} &= \pm\frac{7}{6} \\
x &= -\frac{5}{6} \pm \frac{7}{6} \\
x &= -2, \frac{1}{3}
\end{aligned}
$$

$(-2, 0)$ and $\left(\frac{1}{3}, 0\right)$

e). What are the y-intercepts (if any)?

$(0, 2)$

f). What is the maximum value (if any)?

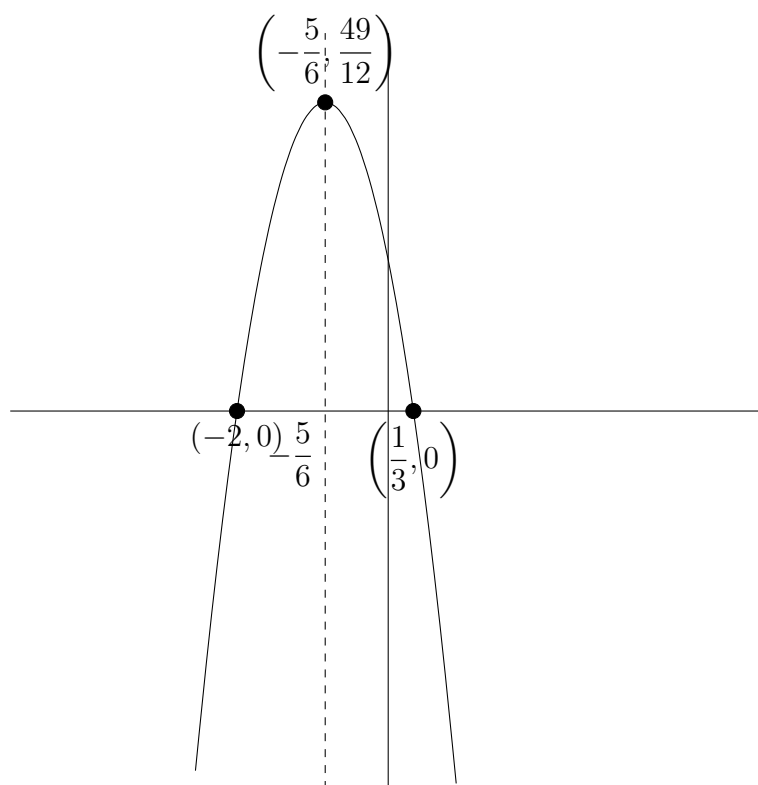This is the y-coordinate of the vertex: $\dfrac{49}{12}$

g). What is the minimum value (if any)?

none.

h). Where is the axis of symmetry?

$x = -\dfrac{5}{6}$

i). Sketch the parabola. All key points must be labeled.

2). Consider the following polynomial function in factored form:

$$y = x^2(1 - x)^2(2 + x)(2 - x)^3$$

a). What is the degree?

First, let's rewrite this with the factors in the proper order:

$$y = -x^2(x - 1)^2(x + 2)(x - 2)^3$$

So the leading term is $(x^2)(x^2)(x)(x^3) = x^8$.

degree$= 8$

b). What is the leading term?

$-x^8$ (see above)

c). What is the end behavior? How do you know?

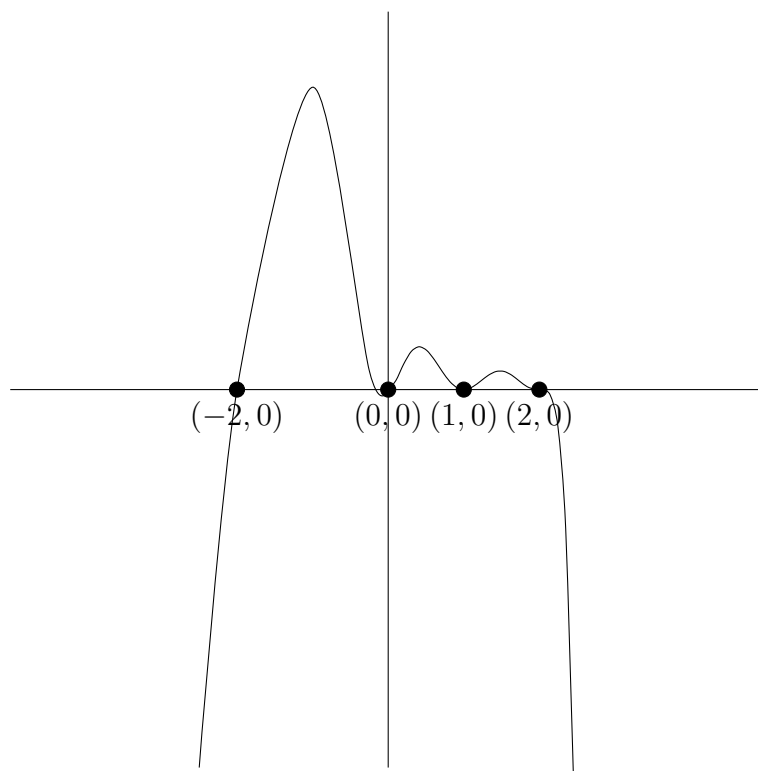Like $-x^2$ because the power of the leading term (8) is even and the sign of the leading coefficient (-1) is < 0.

d). What are the x-intercepts (if any)?

$(0,0), (1,0), (\pm 2, 0)$

e). What are the y-intercepts (if any)?

$(0,0)$

f). Sketch the polynomial.

Note the shape at each x-intercept.

g). Attach a screenshot showing the determination of at least one relative minimum or maximum.

Here is the big local maxima (but you can do any of them).