

Math-13 Sections 01 and 02

Homework #13 Solutions

Using substitution, evaluate the following definite integral:

$$\int_1^2 x(2x^2 + 1)^3 dx$$

Let $t = 2x^2 + 1$, and so $dt = 4x dx$. This changes the limits as follows:

$$t(1) = 2(1)^2 + 1 = 3$$

$$t(2) = 2(2)^2 + 1 = 9$$

Now, adjust with the needed constant, substitute, and solve:

$$\begin{aligned}\int_1^2 x(2x^2 + 1)^3 dx &= \frac{1}{4} \int_1^2 (2x^2 + 1)^3 (4x dx) \\ &= \frac{1}{4} \int_3^9 t^3 dt \\ &= \frac{1}{4} \left[\frac{1}{4} t^4 \right]_3^9 \\ &= \frac{1}{16} t^4 \Big|_3^9 \\ &= \frac{1}{16} (9^4 - 3^4) \\ &= \frac{1}{16} (6561 - 81) \\ &= \frac{1}{16} (6480) \\ &= 405\end{aligned}$$