Math-08 Homework #12 Solutions

Reading

- Text book section 3.1, 3.2

Problems

Note that all sketches of graphs must have all found intercepts and discontinuities labeled. All domains and ranges must be expressed in interval notation. Remember, sketches do not have to be to scale!

1). Some kids are playing with a toy that launches balls straight up into the air. The balls leaving the launcher with a velocity of 96 ft/s. How high do the balls go?

We start with the motion under gravity formula:

$$h(t) = h_0 + v_0 t - 16t^2$$

This time, $h_0 = 0$ feet (on the ground) and $v_0 = 96$ ft/s:

$$h(t) = 96t - 16t^2$$

a). Find the answer by completing the square.

$$\begin{aligned} h(t) &= -16(t^2 - 6t) \\ &= -16(t^2 - 6t + 9) + 16(9) \\ &= -16(t-3)^2 + 144 \end{aligned}$$

This is an inverted parabola with vertex at $(3, 144)$. Thus, the ball reaches its maximum height of 144 feet after 3 seconds.

b). Find the answer using the $-\frac{b}{2a}$ shortcut.

$x = -\frac{96}{2(-16)} = \frac{96}{32} = 3$
$y = 96(3) - 16(3^2) = 288 - 16(9) = 288 - 144 = 144$

This tells us that the vertex occurs at $(3, 144)$, and thus we get the same answer as above.

2). Consider the polynomial:

$$f(x) = (x-1)^3(x+2)^2(x-3)$$

a). What is the end behavior?

The leading term is: $x^3x^2x = x^6$, so we have an even power with a positive coefficient. Therefore:

$$f(x) \to \infty \text{ as } x \to \pm\infty$$

b). What are the x-intercept(s)?

These are the zeros of the polynomial function:

$$(0,1), (0,-2), (0,3)$$

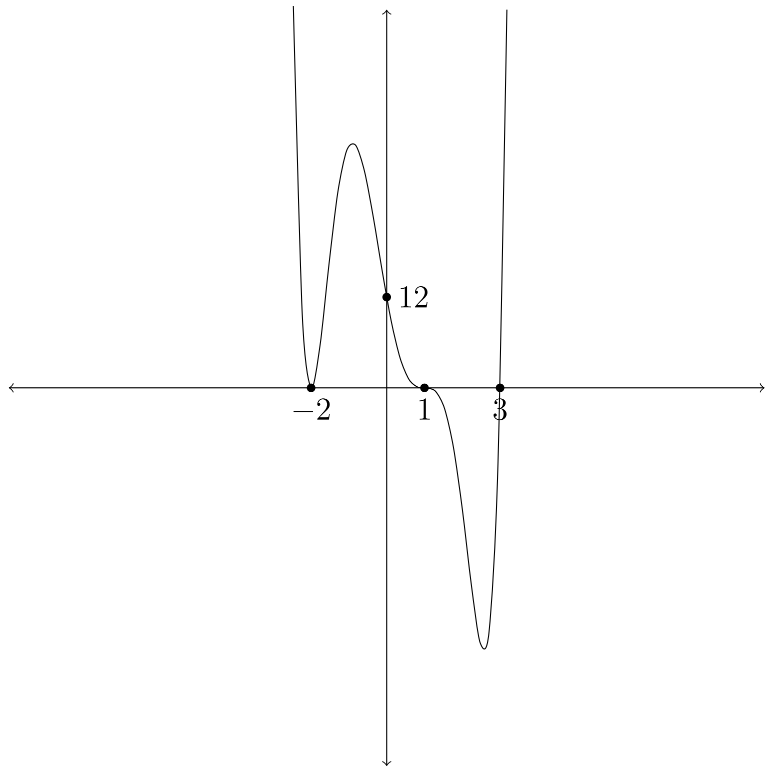
c). What are the y-intercept(s)?

$$\begin{aligned} f(0) &= (0-1)^3(0+2)^2(0-3) \\ &= (-1)^3(2^2)(-3) \\ &= (-1)(4)(-3) \\ &= 12 \end{aligned}$$

Thus, the y-intercept is $(0,12)$.

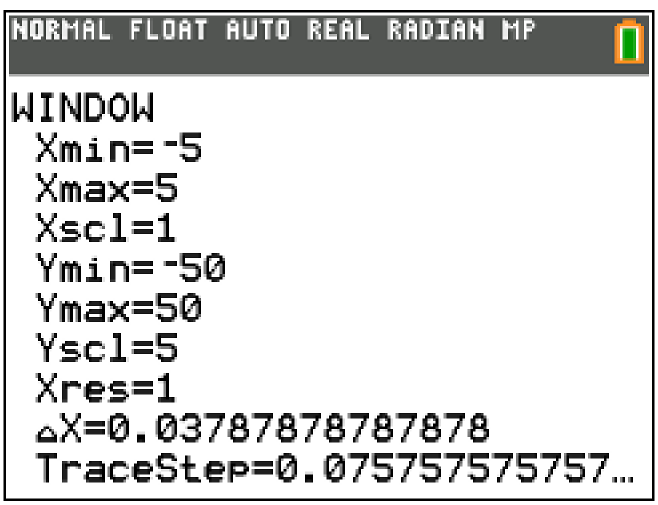
d). Sketch the graph. Be sure to mark all intercepts and show the proper shape at each zero. A sign table or multiplicity info must be included for credit.

- The multiplicity at $x = 1$ is 3 (odd), and so the graph changes sign with a point-of-inflection shape.
- The multiplicity at $x = -2$ is 2 (even), and so the graph does not change sign.
- The multiplicity at $x = 3$ is 1 (odd), and so the graph changes sign with a linear shape.

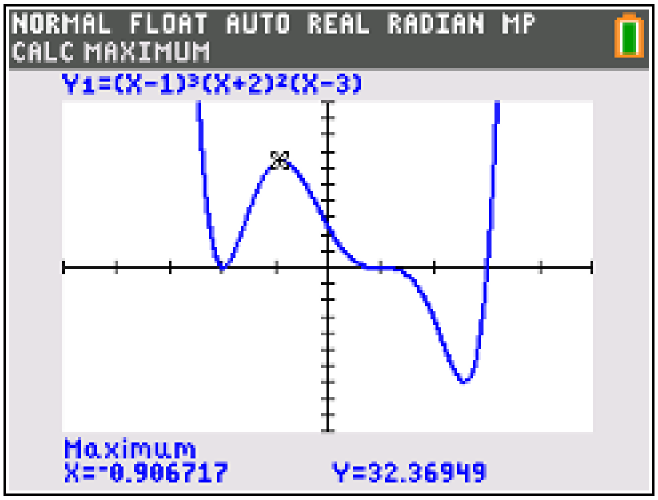


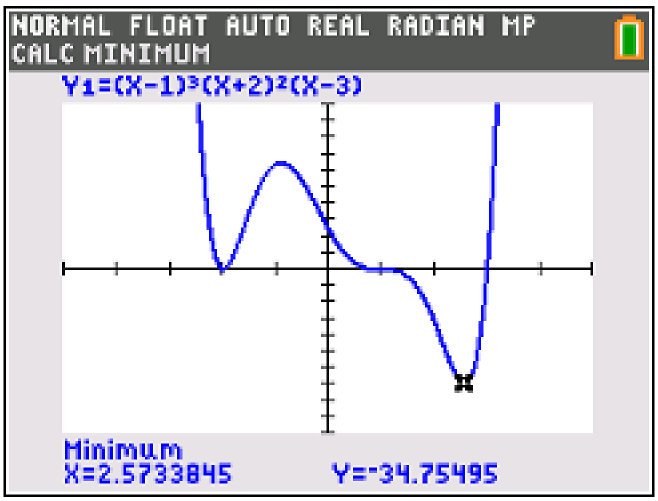
e). Use a calculator to determine any extrema on the graph using the minimum and maximum functions. Attach a screenshot showing the determination of each value.

In order to get a proper view of the graph, I had to use the following window settings:



We already know that we have a minimum at $(-2, 0)$ due to the even multiplicity. The other extrema are found as follows:


NORMAL FLOAT AUTO REAL RADIAN MP
CALC MAXIMUM
Y1=(X-1)³(X+2)²(X-3)
Maximum
X=⁻0.906717
Y=32.36949


NORMAL FLOAT AUTO REAL RADIAN MP
CALC MINIMUM
Y1=(X-1)³(X+2)²(X-3)
Minimum
X=2.5733845
Y=⁻34.75495