Relations

- Physical phenomena are characterized by certain quantities and the how those quantities are *related* to each other.
- Each quantity is associated with a set that contains the possible values for that quantity. In precalculus and calculus these sets are almost always subsets of $\mathbb{R}$.
- Each quantity is assigned a variable that can assume elements of the associated set.
- To show that $a \in A$ *is related to* $b \in B$, use the ordered pair (a, b). This answers the question, "What is the value of the B quantity given the value of the A quantity." In fact, a is acting like an *input* value and b is acting like an *output* value.
- If each possible $a \in A$ is associated with exactly one $b \in B$ then the relation is *well-defined*. Otherwise, the relation is *not well-defined*.

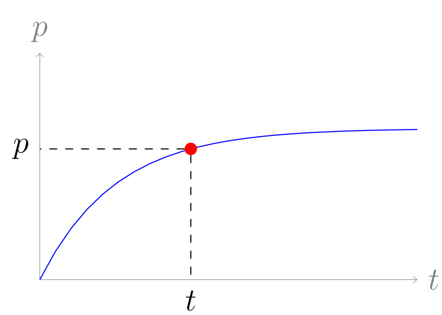
Example: A Chemical Reaction

Quantities:

- Mass of a reactant: $r \in [0, \infty]$
- Mass of a product: $p \in [0, \infty]$
- Heat energy absorbed (endothermic) or emitted (exothermic): $h \in \mathbb{R}$
- Time: $t \in [0, \infty)$

Relations:

- How much product has been produced by a given time?: (t, p), well-defined.
- How much time has passed when a certain amount of product has been produced?: (p, t), well-defined.
- How much energy has been released when a certain amount of a reactant has been consumed?: (r, h), well-defined.



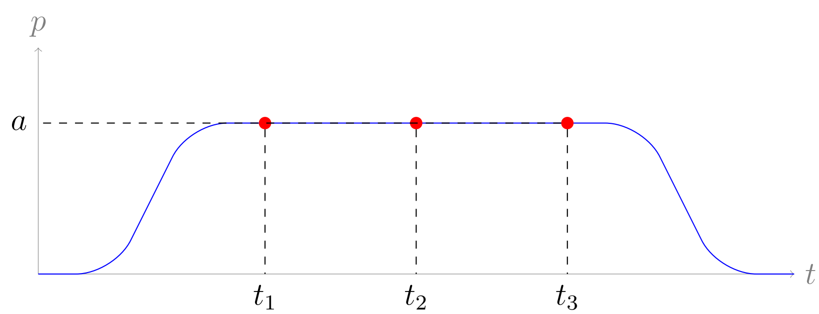
Example: The Flight of an Aircraft

Quantities:

- Distance traveled: $d \in [0, \infty)$
- Altitude: $a \in [0, \infty)$
- Airspeed: $s \in [0, \infty)$
- Time: $t \in [0, \infty)$

Relations:

- What is the aircraft's altitude at a given time?: (t,a), well-defined.
- At what times is the aircraft at a particular altitude?: (a,t), not well-defined.



Definition: Relation

A relation $\mathcal{R}$ between a set A and a set B is set that is a subset of $A \times B$:

$$\mathcal{R} \subseteq A \times B = \{(a, b) \,|\, a \in A \text{ and } b \in B\}$$

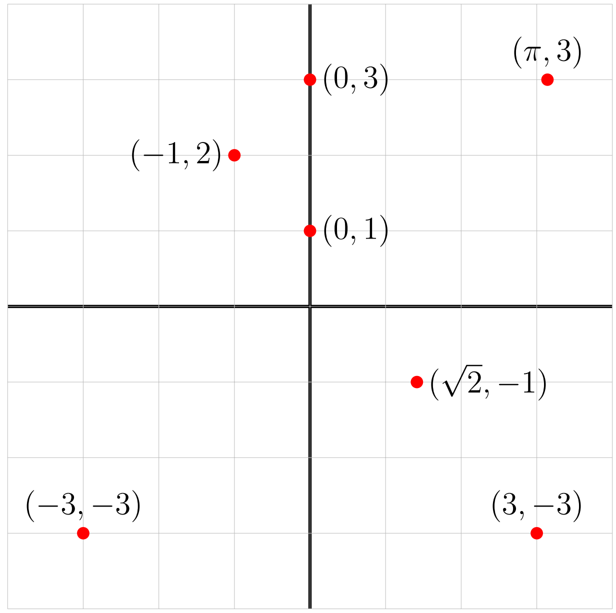
In precalculus and calculus, relations are almost always subsets of $\mathbb{R} \times \mathbb{R} = \mathbb{R}^2$, which is also known as the Cartesian plane.

Example

There is no limitation on which ordered pairs are included in a relation. In fact, elements of the relation can be selected arbitrarily. Let $\mathcal{R} \subset \mathbb{R}^2$ where:

$$\mathcal{R} = \{(0, 1), (0, 3), (-1, 2), (\pi, 3), (\sqrt{2}, -1), (-3, -3), (3, -3)\}$$

Note that $\mathcal{R}$ is a set of discrete elements and that the values from the two sets can be reused without limitations.



Example

Relations are normally constrained by physical phenomena and can result from measurements taken during an experiment or from well-known formulas that *model* the phenomena. Consider an object thrown into the air with a speed of 64 ft/s. Gravity slows and eventually stops the object at a maximum height of 64 ft and then the object falls back to earth:

$$\mathbb{R} = \left\{(t,h) \,\middle|\, h = 64t - 16t^2\right\}$$

Note that this is a continuous phenomenon and so the relation is an infinite set and thus cannot be specified by roster.