Math-13 Sections 01 and 02

Homework #2 Solutions

1. You throw a ball straight up into the air using an airgun. The ball eventually slows down due to gravity, stops, and then falls back to earth. Let h be the height of the ball (in feet) at time t (in seconds) such that the height of the ball is given by $48t - 16t^2$.

   (a) Represent this situation using function notation.

   $$h(t) = 48t - 16t^2$$

   (b) Identify the independent and dependent variables.

   Independent: x

   Dependent: h

   (c) Where is the ball at $t = 2$ seconds?

   $$h(2) = 48(2) - 16(2)^2 = 96 - 64 = 32\,\text{ft}$$

   (d) How long does it take the ball to reach its maximum height of 36 ft?

   $$\begin{aligned}
   &48t - 16t^2 = 36\\
   &12t - 4t^2 = 9\\
   &4t^2 - 12t + 9 = 0\\
   &(2t-3)^2 = 0\\
   &2t - 3 = 0\\
   &2t = 3\\
   &t = \frac{3}{2} = 1.5\,\text{sec}
   \end{aligned}$$

2. The popular hamburger chain Bun-N-Burger has finally decided to go public. Their stock opens on the NASDAQ at $25 per share and increases in price according to the function $p(t) = 25 + 5t$, where t is the number of hours that the market has been open.

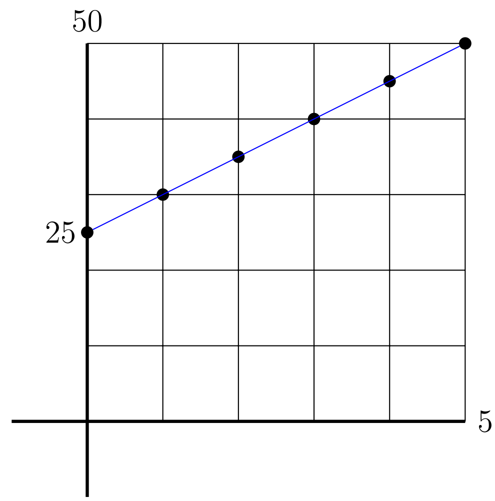
   (a) Represent the function using a table with values from $t = 0$ to $t = 5$.

t	$p(t)$
0	25
1	30
2	35
3	40
4	45
5	50

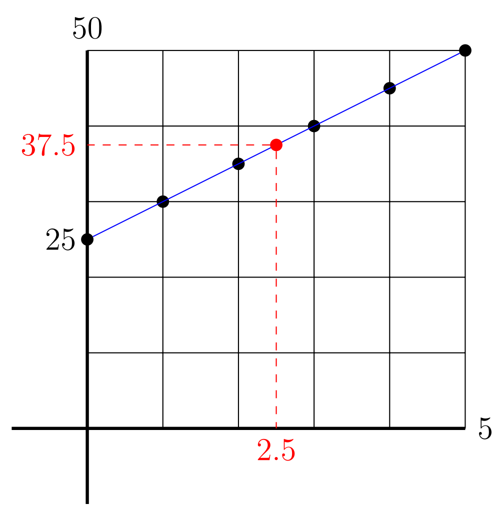
(b) Interpret the statement: $p(3) = 40$.

After 3 hours, the stock price is \$$40$.

(c) Sketch a graph of the function.



(d) From your graph, estimate the price of the stock at $t = 2.5$ hrs and show that point on the graph.



(e) What does the point $(4, 45)$ on the graph represent?

$f(4) = 45$

3. Let $f(x) = 2x + \sqrt{x+1}$. Solve for $f(x) = 8$.

$$\begin{aligned}
&2x + \sqrt{x+1} = 8 \\
&\sqrt{x+1} = 8 - 2x \\
&x + 1 = (8-2x)^2 \\
&x + 1 = 64 - 32x + 4x^2 \\
&4x^2 - 33x + 63 = 0 \\
&(4x-21)(x-3) = 0 \\
&x = 3, \frac{21}{4}
\end{aligned}$$

Check for extraneous solutions:

$$2(3) + \sqrt{3+1} = 6 + \sqrt{4} = 6 + 2 = 8$$

$$2\left(\frac{21}{4}\right) + \sqrt{\frac{21}{4}+1} = \frac{21}{2} + \sqrt{\frac{25}{4}} = \frac{21}{2} + \frac{5}{2} = \frac{26}{2} = 13 \neq 8$$

So $x = \frac{21}{4}$ is extraneous and the only solution is $x = 3$.

4. Determine the implicit domain for the function:

$$f(x) = \sqrt{\frac{x^2+x-2}{x^2-4}}$$

First, turn this into an inequality:

$$\frac{x^2+x-2}{x^2-4} \geq 0$$

Now, factor:

$$\frac{(x+2)(x-1)}{(x+2)(x-2)} \geq 0$$

Next, cancel the common factor, but remember that $x \neq -2$.

$$\frac{x-1}{x-2} \geq 0$$

Note that there is a zero at $x = 1$ and a pole at $x = 2$. Using test points, we get the following:

Finally, remember to leave the hole at $x = -2$.

So the final domain is:

$$(-\infty, -2) \cup (-2, 1] \cup (2, \infty)$$