Math-08 Homework #3 Solutions

Reading

- Text book section 0.3 and 0.4.

Problems

1). This problem investigates the meaning of a^b when both a and b are irrational.

a). Type $\pi^{\sqrt{2}}$ into your calculator and write down the answer to five decimal places.

$$\pi^{\sqrt{2}} = 5.04750$$

b). Build a table like we have done in class to show how finer and finer approximations of π and $\sqrt{2}$ result in an answer that is arbitrarily close to $\pi^{\sqrt{2}}$. The first column should be approximations of π. The second column should be approximations of $\sqrt{2}$. The third column should be a calculation based on your current approximated values. Do this for up to five decimal places.

π	$\sqrt{2}$	$\pi^{\sqrt{2}}$
3	1	3.00000
3.1	1.4	4.87423
3.14	1.41	5.01962
3.141	1.414	5.04492
3.1415	1.4142	5.04721
3.14159	1.41421	5.04747

2). Simplify:

$$\sqrt{75} - \sqrt{27}$$

$$\begin{aligned}\sqrt{75} - \sqrt{27} &= \sqrt{25 \cdot 3} - \sqrt{9 \cdot 3} \\ &= \sqrt{25}\sqrt{3} - \sqrt{9}\sqrt{3} \\ &= 5\sqrt{3} - 3\sqrt{3} \\ &= (5-3)\sqrt{3} \\ &= 2\sqrt{3}\end{aligned}$$

3). Simplify:

$$\frac{\sqrt{\sqrt[3]{x+1}xy^2}}{(x+1)x^{-\frac{3}{2}}y^3}$$

Your answer should have no negative exponents each factor should appear only once. Do not rationalize the denominator. Beware of even roots of even powers!

The first step is to replace the radicals with fractional exponents. Since we have nested radicals here, let's do the inner one first:

$$\frac{\sqrt{(x+1)^{\frac{1}{3}}xy^2}}{(x+1)x^{-\frac{3}{2}}y^3}$$

Now, let's do the outer one:

$$\frac{[(x+1)^{\frac{1}{3}}xy^2]^{\frac{1}{2}}}{(x+1)x^{-\frac{3}{2}}y^3}$$

We now need to apply the $\frac{1}{2}$ exponent to all factors in the square brackets:

$$\frac{[(x+1)^{\frac{1}{3}}]^{\frac{1}{2}}x^{\frac{1}{2}}(y^2)^{\frac{1}{2}}}{(x+1)x^{-\frac{3}{2}}y^3}$$

Now reduce by *multiplying* the exponents. Note that alarm bells should should go off for the y term!:

$$\frac{(x+1)^{\frac{1}{3}\cdot\frac{1}{2}}x^{\frac{1}{2}}y^{2\cdot\frac{1}{2}}}{(x+1)x^{-\frac{3}{2}}y^3} = \frac{(x+1)^{\frac{1}{6}}x^{\frac{1}{2}}\,|y|}{(x+1)x^{-\frac{3}{2}}y^3}$$

Now, combine terms with like bases by subtracting exponenets. Note that we cannot combine the $|y|$ in the numerator with the y^3 in the denominator because the bases are different!:

$$\frac{(x+1)^{\frac{1}{6}-1}x^{\frac{1}{2}+\frac{3}{2}}\,|y|}{y^3} = \frac{(x+1)^{-\frac{5}{6}}x^2\,|y|}{y^3}$$

Finally, we get rid of the negative exponent in the numerator by moving the factor to the denominator:

$$\frac{x^2\,|y|}{(x+1)^{\frac{5}{6}}y^3}$$

4). On your calculator, store the value 1 into the variable x and the value -1 into the variable y. Then type the original expression (not your simplied one) from problem (3) into your calculator. Note that you will need to type $(x+1)^{\frac{1}{3}}$ instead of $\sqrt[3]{x+1}$. Make sure that you do this all in only 3 steps: 2 store operations and then the expression. Turn in a screenshot showing all 3 steps. (Hint: the answer should be $-0.56123\ldots$)

(See file on canvas)