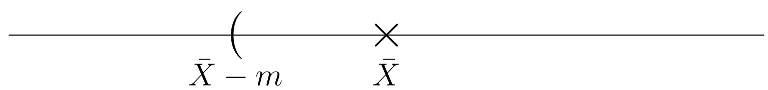
One-sided Confidence Intervals

Sometimes only a one-sided confidence interval is needed:

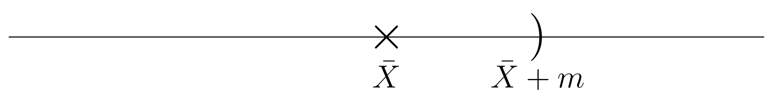
- Lower confidence bound

$$1 - \alpha = P(\bar{X} < \mu) = P\left(\mu \in (\bar{X} - m, \infty)\right)$$



- Upper confidence bound

$$1 - \alpha = P(\mu < \bar{X}) = P\left(\mu \in (-\infty, \bar{X} + m)\right)$$



<u>**Theorem**</u>

Let $X_i \overset{\text{iid}}{\sim} \mathrm{N}(\mu, \sigma^2)$ such that μ is unknown. If σ is known then the one-sided $1 - \alpha$ confidence intervals are given by:

- Upper

$$\mu < \bar{x} + z_\alpha \frac{\sigma}{\sqrt{n}}$$

- Lower

$$\mu > \bar{x} - z_\alpha \frac{\sigma}{\sqrt{n}}$$

If σ is unknown then the one-sided $1 - a$ confidence intervals are given by:

- Upper

$$\mu < \bar{x} + t_{\alpha, n-1} \frac{s}{\sqrt{n}}$$

- Lower

$$\mu > \bar{x} - t_{\alpha, n-1} \frac{s}{\sqrt{n}}$$

<u>**Example**</u>

A sample carton of brown eggs from a farm has $\bar{x} = 65.5$. Assuming a normal population with $\sigma^2 = 4$, obtain 95% upper and lower confidence intervals for μ.

$$z_{0.05} = \Phi(0.95) = 1.645$$

$$z_{0.05}\frac{\sigma}{\sqrt{n}} = 1.645 \cdot \frac{2}{\sqrt{12}} = 0.95$$
$$65.5 + 0.95 = 66.45$$
$$65.5 - 0.95 = 64.55$$