Real Analysis

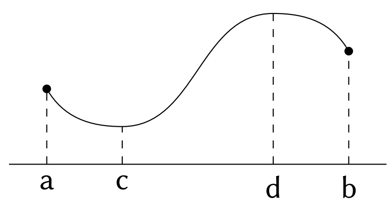
The prerequisites for the definitions and theorems of analysis are important. For example, consider the *Extreme Value Theorem*:

Theorem

If $f : [a, b] \to \mathbb{R}$ is continuous then:

1). $\exists\, c \in [a, b]$ such that $f(c)$ is an absolute minimum for f on $[a, b]$.

2). $\exists\, d \in [a, b]$ such that $f(d)$ is an absolute maximum for f on $[a, b]$.

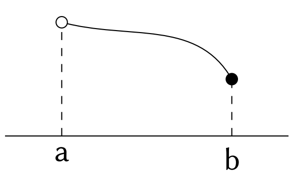


This is "deep" because:

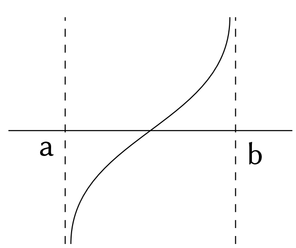
1). It is non-constructive.

2). It is almost false.

Consider what happens when the interval is not closed:

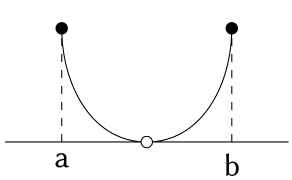


$(a, b]$

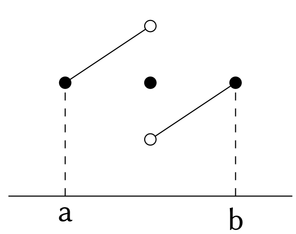


(a, b)

Consider what happens when the function is not continuous on the interval:



no min



no min or max