Math-42 Worksheet #8

Proof Methods and Strategy

1. Prove that 63 is not a perfect square.

2. Prove (by exhaustion) that for all positive integers $n \leq 5, n^2 < 25$.

3. Prove or disprove: $a, b \in \mathbb{Q} \implies a^b \in \mathbb{Q}$.

4. Prove or disprove: $a, b \in \mathbb{R} - \mathbb{Q} \implies a + b \in \mathbb{R} - \mathbb{Q}$

5. Prove by cases: if $\dfrac{x^2 - 4}{x + 1} \geq 0$ then $x \in [-2, -1) \cup [2, \infty)$. Be sure that any values not in the domain are separate cases.

6. Use the rational roots theorem and proof by exhaustion to prove that $\sqrt{3}$ is irrational.

7. Prove by contradiction that $\sqrt{3}$ is irrational. (Hint: start the same way that you would to prove that $\sqrt{2}$ is irrational.)

8. Prove that for all $x, y \in \mathbb{R}$:

$$\min\{x, y\} = \frac{x + y - |x - y|}{2}$$
$$\max\{x, y\} = \frac{x + y + |x - y|}{2}$$

   Is it necessary to uses cases? (Hint: AWLOG)

9. The real number axiom for the additive identity says:

   $$\exists\, 0 \in \mathbb{R}, \forall\, x \in \mathbb{R}, x + 0 = x$$

   Note that the axiom establishes existence, but says nothing about uniqueness. Prove that there is no other real number that is an additive identity. (Hint: start by assuming that there are at least two: 0 and $0'$, and then show that $0 = 0'$.

10. Likewise, the real number axiom for the additive inverse says:

    $$\forall\, x \in \mathbb{R}, \exists\, y \in \mathbb{R}, x + y = 0$$

    Prove that for any real number x, its additive inverse is unique.