Zeros

$f^{(n)}(a) = 0 \implies f(z) = 0$
$\left|f^{(n)}(a)\right| \leq \frac{Mn!}{r^n}$
$f(z)$ entire/bounded $\implies$ $f(z)$ constant (Liouville)
$f(z) = g(z)$ in E and AP z_0 then $f(z) = g(z)$ in D
$f(z)$ compact then finite zeros

Newton

$\sum_{k=0}^{n} a_k = -\frac{a_{n-1}}{a_n}$

Cauchy

$|z| = 1 + \max\{|a_k| \mid 0 \leq k \leq n\}$

Argument Principle

$N = \frac{1}{2\pi i} \int_\gamma \frac{f'(z)}{f(z)} dz$

$\frac{1}{2\pi i} \int_\gamma z \frac{f'(z)}{f(z)} dz = -\frac{a_{n-1}}{a_n}$

Rouche's Theorem

$f(x), g(x)$ analytic
$|g| < |f|$ on γ
Zeros of $f + g$ = zeros of f in γ

$|f - g| < |f|$ on γ
Zeros of f = zeros of g in γ

Enestrome Theorem

$0 < a_{k-1} < a_k < a_n$
Zeros of $p(x)$ inside $|z| = 1$

Extended Plane

$\int_C f(z) dz = 2\pi i \sum_{k=1}^{n} \operatorname{Res}[f, z_k] = -2\pi i \operatorname{Res}[f, \infty]$
$\operatorname{Res}[f, \infty]$ =-coeff of w in $f(\frac{1}{w})$

Schwarz's Lemma

$|f(z)| \leq |z| \frac{M}{R}$
$|f'(0)| \leq \frac{M}{R}$
Equality at $f(z) = cz\frac{M}{R}, |c| = 1$

Conformal Mappings

Regular=CD and $f'(z) \neq 0$
$J = |f'(z)|^2$
$f(z)$ analytic iff conformal

Cross Ratio

$(z_1, z_2, z_3, z_4) = \frac{(z_1 - z_3)(z_2 - z_4)}{(z_1 - z_4)(z_2 - z_3)}$
$\Delta = ad - bc$
$s(z_1) - s(z_2) = \frac{\Delta(z_1 - z_2)}{(cz_1 + d))(cz_2 + d)}$
$(z_1, z_2, z_3, z_4) = (w_1, w_2, w_3, w_4)$
$z \neq -\frac{d}{c} \implies s(z)$ conformal
$(z_1, z_2, z_3, z_4) \in \mathbb{R} \iff z_1, z_2, z_3, z_4$ on circle
LFT maps circle (or line) to circle (or line)
$\bar{(}z_1, z_2, z_3, z_4) = (\bar{(}z_1), \bar{(}z_2), \bar{(}z_3), \bar{(}z_4))$

Symmetry

$z^* = \frac{R^2}{(z-a)} + a$
$(z^* - a)(\bar{(}z) - \bar{(}a)) = R^2$
$\bar{(}z, z_1, z_2, z_3) = (z^*, z_1, z_2, z_3)$
$\bar{(}s(z), s(z_1), s(z_2), s(z_3)) = (s(z^*), s(z_1), s(z_2), s(z_3))$