# Math-19 Homework #1 Solutions

## Reading

Please read sections 1.1 through 1.5 and do all concept problems in the posted sections on webassign. This written homework is based on section 1.1 material only.

## Problems

1). Let:

$$\begin{aligned} P &:= 0 \text{ is a positive number} \\ Q &:= 2 \geq 2 \\ R &:= \forall n, m \in \mathbb{N}, n + m \in \mathbb{N} \end{aligned}$$

Determine whether the following compound statement is true or false:

P and Q and R or P and not Q and R or not P and Q and R

Start by rewriting the statement with parentheses to show operation order, then substitute the truth value for each individual statement, and then show the stepwise evaluation to the final result.

P is false. 0 is neither positive or negative.

Q is true. $2 \leq 2$ means $2 < 2$ *or* $2 = 2$. Since $2 = 2$ (reflexive property) and since only one of the two statements has to be true, Q is true.

R is true. It is the statement of closure of addition on the natural numbers.

(P and Q and R) or (P and (not Q) and R) or ((not P) and Q and R)

(F and T and T) or (F and (not T) and T) or ((not F) and T and T)

F or (F and F and T) or (T and T and T)

F or F or T

T

2). Convert $10.2\overline{45}$ to rational form.

$$
\begin{aligned}
x &= 10.2\overline{45} \\
10x &= 102.\overline{45} \\
1000x &= 10245.\overline{45} \\
\\
990x &= 10143 \\
x &= \frac{10143}{990} = \frac{1127}{110}
\end{aligned}
$$

3). Let:

$$
\begin{aligned}
A &= \text{the set of all positive real numbers} \\
B &= \text{the set of real numbers between -3 (exclusive) and 3 (inclusive)}
\end{aligned}
$$

a). Graph each set on the real number line.

b). Represent each set using set-builder notation.

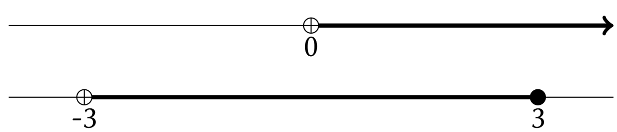
c). Represent each set using interval notation.

d). Graph $A \cup B$ and represent it in interval notation.

e). Graph $A \cap B$ and represent it in interval notation.

f). Graph $A - B$ and represent it in interval notation.

a). Graphs:




b). Setbuilder notation:

$$A = \{x \in \mathbb{R} \mid x > 0\}$$

$$B = \{x \in \mathbb{R} \mid -3 < x \leq 3\}$$

c). Interval notation:

$$A = (0, \infty)$$

$$B = (-3, 3]$$

d). $A \cup B = (-3, \infty)$

e). $A \cap B = (0, 3]$

f). $A - B = (3, \infty)$

4). A careful solution of $4(x+2) = 11$ is given below. Give the rationale for each step from the ten real number rules (AC,AA,A0,AI,MC,MA,M1,MI,LD,RD) and the additional rules (SUB,LCAN,RCAN).

| | |
|---|---|
| $4(x+2) = 11$ | |
| $4x + 8 = 11$ | LD |
| $(4x+8) - 8 = 11 - 8$ | CAN |
| $(4x+8) - 8 = 3$ | SUB |
| $4x + (8-8) = 3$ | AA |
| $4x + 0 = 3$ | AI |
| $4x = 3$ | A0 |
| $\frac{1}{4}(4x) = \frac{1}{4}(3)$ | CAN |
| $\frac{1}{4}(4x) = \frac{3}{4}$ | SUB |
| $(\frac{1}{4}4)x = \frac{3}{4}$ | MA |
| $1x = \frac{3}{4}$ | MI |
| $x = \frac{3}{4}$ | M1 |

5). Give a careful proof of $|a-b| = |b-a|$. You will need to use one of the distributive rules, the definition of absolute value, and some of the properties in box at the top of page 9. Be sure to justify each step.

| | |
|---|---|
| $\lvert a-b\rvert = \lvert -(b-a)\rvert$ | Prop of Neg #6 |
| $\lvert a-b\rvert = \lvert (-1)(b-a)\rvert$ | Prop of Neg #1 |
| $\lvert a-b\rvert = \lvert -1\rvert \, \lvert b-a\rvert$ | Prop of AV #3 |
| $\lvert a-b\rvert = 1 \cdot \lvert b-a\rvert$ | Def of AV |
| $\lvert a-b\rvert = \lvert b-a\rvert$ | M1 |