Math-13 Sections 01 and 02

Homework #6 Solutions

A ball is thrown upward off of a 100 ft cliff at a velocity of 64 ft/s. The equation of motion for the ball is:

$$H(t) = 100 + 64t - 16t^2$$

1. Using the *definition of the derivative* (i.e., not the derivative formulas), determine $H'(t)$.

$$\begin{aligned}
H'(t) &= \lim_{h\to 0} \frac{H(t+h) - H(t)}{h} \\
&= \lim_{h\to 0} \frac{[100 + 64(t+h) - 16(t+h)^2] - [100 + 64t - 16t^2]}{h} \\
&= \lim_{h\to 0} \frac{100 + 64t + 64h - 16(t^2 + 2th + h^2) - 100 - 64t + 16t^2}{h} \\
&= \lim_{h\to 0} \frac{\cancel{100} + \cancel{64t} + 64h - \cancel{16t^2} - 32th - 16h^2 - \cancel{100} - \cancel{64t} + \cancel{16t^2}}{h} \\
&= \lim_{h\to 0} \frac{64h - 32th - 16h^2}{h} \\
&= \lim_{h\to 0} (64 - 32t - 16h) \\
&= 64 - 32t
\end{aligned}$$

2. How fast is the ball traveling and in which direction (up or down) after 5 sec?

   Since $v(t) = H'(t)$:

$$v(5) = 64 - 32(5) = 64 - 160 = -96$$

   So the ball is traveling at 96 ft/s is the down direction.

3. What is the ball's height after 5 sec?

$$H(t) = 100 + 64(5) - 16(5)^2 = 100 + 320 - 16(25) = 420 - 400 = 20$$

   So the ball has fallen to a height of 20 ft.

4. What is the equation of the tangent line to the curve at $t = 5$?

   Using $m = -96$ and the point $(5, 20)$:

$$\begin{aligned}
H - 20 &= -96(t - 5) \\
H &= -96t + 480 + 20 \\
H &= -96t + 500
\end{aligned}$$

5. What is the equation of the normal line to the curve at $t = 5$?

   Using $m = \frac{1}{96}$ and the point $(5, 20)$:

$$H - 20 = \frac{1}{96}(t - 5)$$
$$H = \frac{1}{96}t - \frac{5}{96} + 20$$
$$H = \frac{1}{96}t + \frac{1915}{96}$$