Math-19 Homework #15 Solutions

Problems

1). Consider the function:

$$f(x) = 2\tan(4\pi x - \pi) + 1 = 2\tan\left[4\pi\left(x - \frac{1}{4}\right)\right] + 1$$

a). What is the period P?

$$P = \frac{\pi}{4\pi} = \frac{1}{4}$$

b). What is the horizontal translation b?

$\frac{1}{4}$ to the right

c). What is the phase angle ϕ?

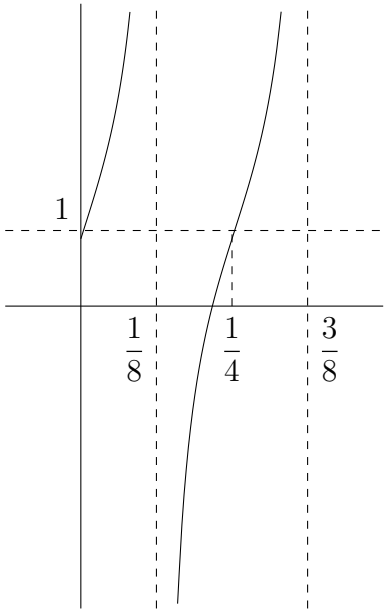
$$\pi = -\pi$$

d). What is the y-intercept?

$$f(0) = 2\tan(-\pi) + 1 = 0 + 1 = 1$$

$(0, 1)$

e). Sketch one cycle of the graph in the interval $\left(b + \frac{P}{2}, b - \frac{P}{2}\right)$ and then extend the sketch back to the y-intercept.



2). Two 1 kg masses are each suspended on a spring with $k = \pi^2$ and are stretched downward by 2 units. The first spring is released at $t = 0$. The second spring is released at $t = 3$.

a). Find $f_1(t)$ for the first mass.

$$f_1(t) = 2\cos\left(\sqrt{\frac{\pi^2}{1}}t\right) = 2\cos \pi t$$

b). Find $f_2(t)$ for the second mass.

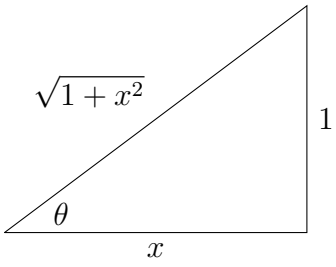
$$f_2(t) = 2\cos \pi(t-3) = 2\cos(\pi t - 3\pi)$$

c). What is the phase difference between the two masses? The phase angle in the previous part is 3π; however, since the period is only 2π, we can state the phase angle as $3\pi - 2\pi = \pi$ or $180°$.

3). Evaluate:

$$\cot\left(\cos^{-1}\frac{x}{\sqrt{1+x^2}}\right)$$

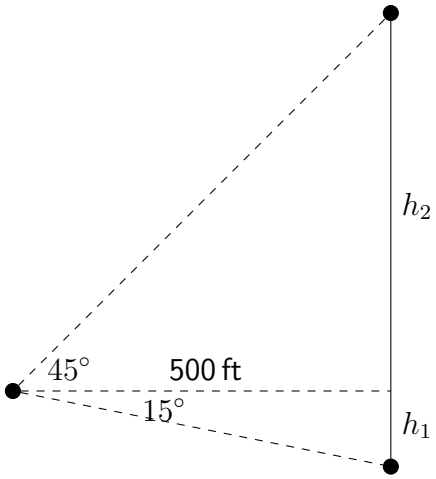
First, we repeat to ourselves, "the angle whose cosine is ...." Next, we draw a right triangle corresponding to our angle:



Note that by using the Pythagorean theorem we determine that the length of the opposite side is 1. We now take the cotangent of our angle:

$$\cot\left(\cos^{-1}\frac{x}{\sqrt{1+x^2}}\right) = \frac{x}{1} = x$$

4). A water tower is located 500 ft from a building. An observer looks at the tower from a window in the building. The angle of depression to the bottom of the tower is $45°$. The angle of elevation to the top of the tower is $15°$. How tall is the tower (to the nearest foot)?



$\tan(15°) = \dfrac{h_1}{500}$ and so $h_1 = 500\tan(15°)$.

$\tan(45°) = \dfrac{h_2}{500}$ and so $h_2 = 500\tan(45°)$.

$h = h_1 + h_2 = 500[\tan(15°) + \tan(45°)] \approx 634$ ft