Math-42 Worksheet #22

Generalized Permutations and Combinations

1. How many 10-letter strings can be made from the 26-letter alphabet when:
   (a) Repetition is allowed.
   (b) Repetition is not allowed.
   (c) The word must be a sequence of zero or more Z's, followed by a sequence of zero or more A's, followed by a sequence of zero or more T's.

2. How many distinct strings can be constructed by arranging all the letters in MISSISSIPPI? Previously, you learned how to do this by placing letters via selection. This time, use the formula for arrangement of indistinguishable objects.

3. A hot dog cart offers chili-cheese dogs, mustard dogs, and kraut dogs. How many different ways are there to select a dozen hot dogs when:
   (a) The cart can make up to twenty of each type.
   (b) You want to buy at least four chili-cheese dogs.
   (c) You want to buy at least two of each type.
   (d) The cart can only make three more chili-cheese dogs. You could partition this into cases of 0, 1, 2, or 3 chili-cheese dogs selected; however, there is an easier way using a complement.

4. How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 = 23$ where the x_i are nonnegative integers and:
   (a) $x_1, x_2, x_3, x_4 \geq 0$
   (b) $x_1, x_2, x_3, x_4 > 0$
   (c) $x_1, x_3, x_5 \geq 2$ and $x_2, x_4 \geq 1$.

5. The so-called *ball-and-walls* method for selection with repetition is fairly easy when there are

no bounds or just lower bounds for each box. It is a bit harder when there are upper bounds. Let's work the following problem step by step:

You have three boxes in which you must place red balls in the first box, green balls in the second box, and blue balls in the third box. The total number of balls is limited to 19 and the number of balls allowed in each box is:

$$
\begin{aligned}
&1 \leq |b_1| \leq 4 \\
&2 \leq |b_2| \leq 6 \\
&5 \leq |b_3|
\end{aligned}
$$

How many ways are there to arrange the 19 balls in the three boxes?

(a) Start by allocating the lower bounds to each box and restate the problem with the lower bounds removed.

(b) Focusing on the new problem, how many ways total are there if the upper bounds are ignored?

(c) How many ways are there if there are 4 or more red balls in the first box, ignoring the upper bound for the second box?

(d) How many ways are there if there are 5 or more green balls in the second box, ignoring the upper bound for the first box?

(e) How many ways are there for 4 more red balls in the first box *and* 5 or more green balls in the second box?

(f) Can you put these results together to obtain the answer for the original problem? (Hint: PIE).