Math-19 Homework #8 Solutions

Problems

1). You are a cost accountant at a firm that manufactures pencils. You prepare next month's cost study for your boss as follows:

item	cost ($)
wood (per pencil)	0.05
graphite (per pencil)	0.02
paint (per pencil)	0.01
rubber (per pencil)	0.04
metal (per pencil)	0.03
lease on plant	10,000
labor	20,000
depreciation	10,000
utilities	5,000
other plant costs	7,500

Your marketing department's research indicates that you can sell your pencils to retailers at 25 cents per pencil.

a). Construct a linear model for this situation.

Variable Costs$= 0.05 + 0.02 + 0.01 + 0.04 + 0.03 = 0.15$

Fixed Costs$= 10000 + 20000 + 10000 + 5000 + 7500 = 52500$

$P(n) = R(n) - C(n) = 0.25n - (0.15n + 52500) = 0.10n - 52500$

b). How many pencils do you need to sell to break even?

$P(n) = 0 = 0.10n - 52500$
$0.10n = 52500$
$n = 525000$

2). Consider the function: $y = -|x-2| + 3$

a). List the starting standard function and the three transformation steps in the order that they should be applied.

i. Start with $y = |x|$

ii. Translate right 2

iii. Reflect x-axis

iv. Translate up 3

b). What are the x-intercepts (if any)?

$0 = -|x-2| + 3$
$|x-2| = 3$

$x - 2 = \pm 3$
$x = -1, 5$

$(-1, 0)$ and $(5, 0)$

c). What are the y-intercepts (if any)?

$y = -|0-2| + 3 = -|-2| + 3 = -2 + 3 = 1$

$(0, 1)$

d). What are the local maxima (if any)?

There is one local maximum at the translated vertex: $(2, 3)$

e). What are the local minima (if any)?

None.

f). What is the domain?

$\mathbb{R}$

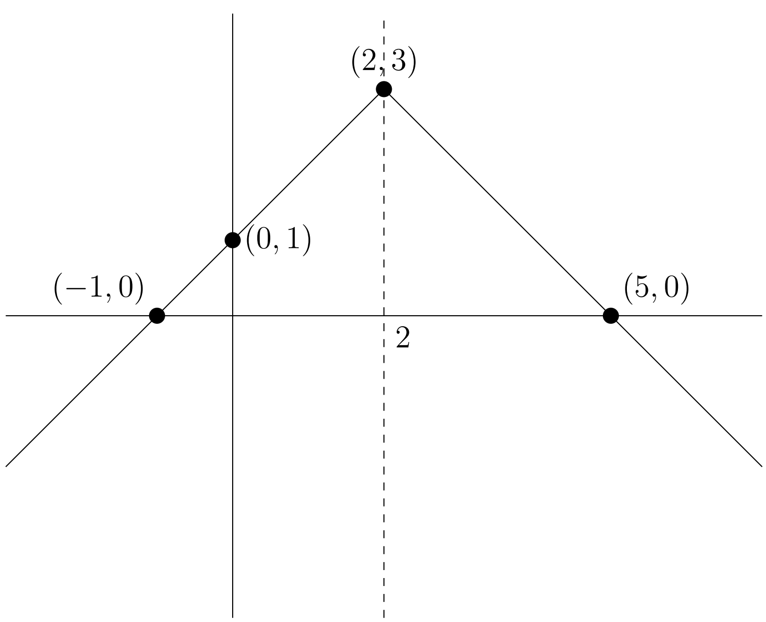
g). What is the range?

$(-\infty, 3]$

h). What is the axis of symmetry?

$x = 2$

i). Sketch the graph of the function. Be sure to label all important points.



3). Consider the function: $y = -\dfrac{1}{x+2} + 1$

a). List the starting standard function and the three transformation steps in the order that they should be applied.

   i. Start with $y = \dfrac{1}{x}$

   ii. Translate right 2

   iii. Reflect x-axis

   iv. Translate up 1

b). What is the equation of the horizontal asymptote?

$y = 1$

c). What is the equation of the vertical asymptote?

$x = -2$

d). What is the end behavior as $x \to \infty$ - be very specific.

$f(x) \to 1^-$

e). What is the end behavior as $x \to -\infty$ - be very specific.

$f(x) \to 1^+$

f). What are the x-intercepts (if any)?

$$0 = -\frac{1}{x+2} + 1$$
$$\frac{1}{x+2} = 1$$
$$x + 2 = 1$$
$$x = -1$$

$(-1, 0)$

g). What are the y-intercepts (if any)?

$$y = -\frac{1}{0+2} + 1 = -\frac{1}{2} + 1 = -\frac{1}{2}$$

$(0, -\frac{1}{2})$

h). What are the local maxima (if any)?

none.

i). What are the local minima (if any)?

none.

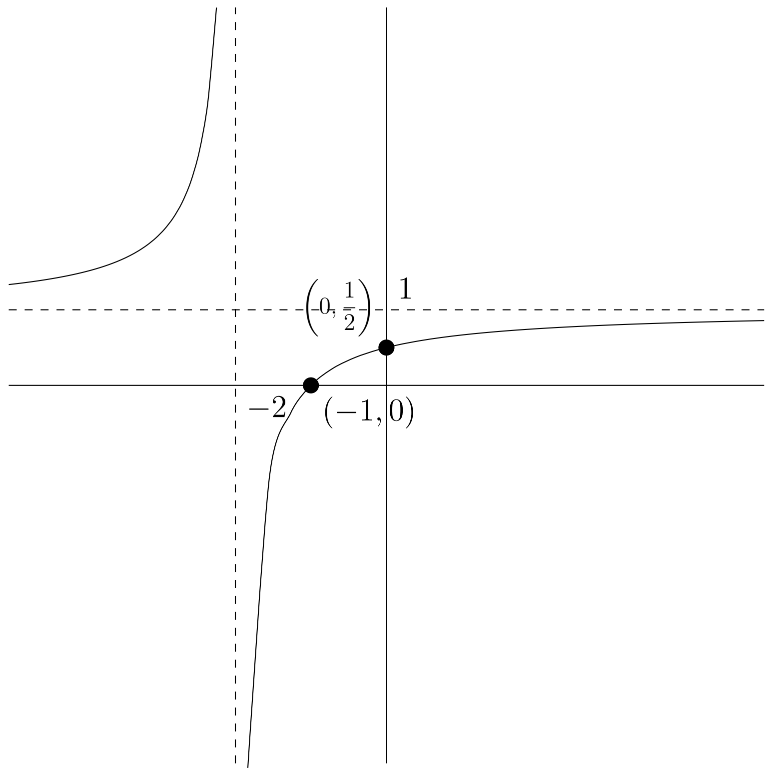
j). What is the domain?

$(-\infty, -2) \cup (-2, \infty)$

k). What is the range?

$(-\infty, 1) \cup (1, \infty)$

l). Sketch the graph of the function. Be sure to label all important points.



4). Evaluate the difference quotient for the function: $f(x) = x^2 + 3x - 1$.

$$
\begin{aligned}
\frac{f(x+h) - f(x)}{h} &= \frac{[(x+h)^2 + 3(x+h) - 1] - [x^2 + 3x - 1]}{h} \\
&= \frac{[x^2 + 2xh + h^2 + 3x + 3h - 1] - [x^2 + 3x - 1]}{h} \\
&= \frac{2xh + h^2 + 3h}{h} \\
&= 2x + 3 + h
\end{aligned}
$$

5). Consider the following functions:

$f(x) = \sqrt{x} + 1$

$g(x) = x^2$

$h(x) = 2x - 1$

Compute AND find the domain for the following functions:

a). $(g - h)(x)$

$(g - h)(x) = g(x) - h(x) = x^2 - (2x - 1) = x^2 - 2x + 1 = (x - 1)^2$

Domain: $\mathbb{R}$

b). $(fg)(x)$

$(fg)(x) = f(x)g(x) = (\sqrt{x}+1)x^2$

Domain: $[0, \infty)$

c). $\left(\frac{g}{h}\right)(x)$

$$\left(\frac{g}{h}\right)(x) = \frac{g(x)}{h(x)} = \frac{x^2}{2x-1}$$

Domain: $\left\{x \in \mathbb{R} \mid x \neq \frac{1}{2}\right\} = \left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$

d). $(f \circ g)(x)$

$(f \circ g)(x) = f(g(x)) = f(x^2) = \sqrt{x^2}+1 = |x|+1$

Note that $g(x)$ guarantees that only nonnegative values are fed to $f(x)$.

Domain: $\mathbb{R}$

e). $\left(\frac{f}{f}\right)(x)$

$$\left(\frac{f}{f}\right)(x) = \frac{f(x)}{f(x)} = \frac{\sqrt{x}+1}{\sqrt{x}+1} = 1$$

Note that even though the final result looks like it can take $\mathbb{R}$, we must honor the domain of $f(x)$.

Domain: $[0, \infty)$