Math-13 Sections 01 and 02

Homework #1 Solutions

1. Indicate which subsets of the real numbers that the following values are members of:

VALUE	$\mathbb{N}$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{R}-\mathbb{Q}$	$\mathbb{R}$
0		✓	✓		✓
$\sqrt{2}$				✓	✓
$\sqrt{9}$	✓	✓	✓		✓
123.4			✓		✓
-123		✓	✓		✓
π				✓	✓
$12.34\overline{56}$			✓		✓

2. Convert the rational value $12.34\overline{56}$ to integer ratio form using the algorithm that we learned in class.

   (a) $x = 12.34\overline{56}$. The goal is to find the alternate syntax for x.

   (b) Capture all non-repeating digits: $100x = 1234.\overline{56}$.

   (c) Capture one set of repeated digits: $10000x = 123456.\overline{56}$.

   (d) Subtract the two equations (note that the repeating part cancels) and solve for x:

$$\begin{aligned}(10000-100)x &= 123456-1234\\ 9900x &= 122222\\ x &= \frac{122222}{9900}\end{aligned}$$

   Therefore, $12.34\overline{56} = \frac{122222}{9900}$.

3. Convert $24.57\overline{9}$ to integer ratio form *without* using the algorithm and justify your answer. (Hint: look for an alternate syntax for the value.)

   Note that $24.57\overline{9}$ becomes arbitrarily close to 24.58, and is thus equal to it. Now, use the algorithm to convert this finite decimal value to an integer ratio:

$$24.58\left(\frac{100}{100}\right) = \frac{2458}{100}$$

4. Is $\dfrac{123.4}{56.99}$ a rational number? If so, then explain why *without* using the algorithm.

   Note that:

$$\frac{123.4}{56.99} = \frac{\frac{1234}{10}}{\frac{5699}{100}} = \left(\frac{1234}{10}\right)\left(\frac{100}{5699}\right) = \frac{12340}{5699}$$

   This is an integer ratio where the denominator is not zero, and is therefore a rational number.