Math-42 Worksheet #19

# The Pigeonhole Principle

1. Show that in a class of 30 students, at least 2 will have a last name that starts with the same letter.

2. Show that in any set of $25000$ people with four-digit PINs needed to access their bank accounts from an ATM, at least three people will have the same PIN.

3. You have a messy sock drawer where $10$ pairs of socks are all mixed up. You need to get up early one morning when it is still dark, but the light in your room is not working. If you pick socks one-by-one blindly from the drawer, what is the minimum number of socks that you need to select before your are guaranteed to have a matched pair among the selected socks when:

   (a) You have five pairs of black socks and five pairs of blue socks.

   (b) You have six pairs of black socks and four pairs of blue socks.

   (c) You have two pairs each of black, blue, red, green, and white.

   (d) All the socks are white.

4. What is the minimum number of times a coin must be flipped in order to ensure at least $5$ heads or $5$ tails?

5. What is the minimum number of times two dice must be rolled to guarantee at least $2$ even rolls or $2$ odd rolls?

6. Let the set $A$ consist of any twenty positive integers. Show that there are at least two elements $a, b \in A$ such that $(a \bmod 17) = (b \bmod 17)$.

7. A conference has 20 different time periods in which to schedule 250 sessions. How many rooms will be needed?

8. Suppose that a soccer team scores at least one goal in 20 consecutive games. If the team scores a total of 30 goals in those 20 games, show that in some sequence of consecutive games the team scores exactly 9 goals.

9. Show that in any group of $10$ facebook users, at least two have the same number of friends within the group. (Hint: consider what happens if there is a user that is friends with no one in the group or is friends with all nine other members in the group.)

10. Given a square with sides of length $2$ and five points in the interior of the square, show that at least two of the points are within $\sqrt{2}$ of each other. (Hint: partition the square in some fashion.)