

Math-42 Sections 01, 02, 05

## Homework #5

**Due: Week of 3/2**

### Reading

Section 1.6-1.8

### Problem

Prove the following theorem:

#### Theorem

$\forall n \in \mathbb{Z}, n$  is either even or odd (but not both).

Your proof must address the following points:

1.  $n$  is even or odd (and nothing else).
2.  $n$  is odd  $\implies n$  is not even (hint: contradiction).
3.  $n$  is even  $\implies n$  is not odd (hint: contrapositive).

The first point is a bit more difficult. Start by making a statement about 0. Then assuming that  $n$  is even, what can you say about  $n - 1$  and  $n + 1$ ? Likewise, assuming that  $n$  is odd, what can you say about  $n - 1$  and  $n + 1$ . Can you organize these facts into an argument that shows that you have accounted for all possible  $n \in \mathbb{Z}$ ?