Math-1003b Homework #12 Solutions

Reading

- Sections 10.2 and 10.3

Problems

1). A student writes the following statements. Determine if each is either correct or incorrect (or misleading). Explain why incorrect statements are incorrect.

a). $\sqrt{9} = \pm 3$

This is incorrect. When we write $\sqrt{9}$ we want the principal (positive) root only, not the negative root. The correct form is:

$$\sqrt{9} = 3$$

b). $\left(x^{\frac{1}{2}}\right)^2 = |x|$

This is misleading. Because there is an even fractional root in the inside, the implied domain for x is $[0, \infty)$. Thus, the result is always positive and the absolute value is not needed.

c). $(x^2)^{\frac{1}{2}} = x$

This is incorrect. Notice that x went from an even power to an odd power. Consider what would happen if $x < 0$ - the LHS would be positive but the RHS would be negative. We need absolute value here:

$$\left(x^2\right)^{\frac{1}{2}} = |x|$$

d). $(x^3)^{\frac{1}{3}} = |x|$

This is incorrect. We don't use absolute value with odd powers/roots. Consider what would happen if $x < 0$ - the LHS would be negative, so we definitely don't want the absolute value on the RHS:

$$\left(x^3\right)^{\frac{1}{3}} = x$$

e). $\left(x^{\frac{1}{3}}\right)^3 = x$

This is correct. No absolute value is needed with odd powers/roots.

2). Simplify completely. Your answer should contain no negative exponents and should include absolute values where appropriate:

$$\left(\frac{x^4}{y^{-6}}\right)^{-\frac{1}{2}}\left(x^3y^{-2}\right)$$

$$\begin{aligned}\left(\frac{x^4}{y^{-6}}\right)^{-\frac{1}{2}}\left(x^3y^{-2}\right) &= \left[\frac{(x^4)^{-\frac{1}{2}}}{(y^{-6})^{-\frac{1}{2}}}\right]\left(x^3y^{-2}\right)\\ &= \left(\frac{x^{-2}}{y^3}\right)\left(x^3y^{-2}\right)\\ &= xy^{-5}\\ &= \frac{x}{y^5}\end{aligned}$$

Now we need to check for the need for absolute value. Note that x has an odd power in the original problem so no absolute value needed. But y goes from an even power in the original problem to an odd power in the result, so absolute value is needed:

$$\frac{x}{|y|^5}$$