Math-19 Lab #3

1). In class we did a proof to show that $\forall a \in \mathbb{R}$, the inverse $-a$ is unique:

<u>Proof</u>

Assume $a \in \mathbb{R}$
Assume that a has two inverses, call them a' and a''

$a + a' = 0$	A4
$a + a'' = 0$	A4
$a + a' = a + a''$	SUB
$\therefore a' = a''$	LCAN

Use this proof as an example and produce a similar proof that shows that the additive identity (i.e., 0) is also unique. Be sure to justify each step.

2). We know that addition and multiplication are closed operations on the real numbers: if you add or multiply two real numbers the result is a real number. We know that addition and multiplication are also closed for integers, but how about for rational numbers?

a). Prove that if you add two rational numbers then the result is a rational number. Start by assuming that you have two rational numbers in fractional form and add them. Make sure that you apply all parts of the definition of a rational number to show that the result is indeed a rational number.

b). Similarly, prove that if you multiple two rational numbers you also get a rational number.

c). Find some easy counterexamples to show that the set of irrational numbers is not closed under addition or multiplication. Come up with some irrational numbers that when you add them you get something that is not irrational. Repeat for multiplication.

d). Use part (a) to show that if you add a rational number and an irrational number then the result is irrational. Start by assuming that you have a rational and an irrational and you add then. Assume (incorrectly) that the result is in fact rational and use part (a) to arrive at a contradiction. Thus, the assumption that the result is rational is incorrect and the only alternative is that it is irrational.

3). Section 1.1 Problems 67-76 odd

4). Section 1.1 Problems 79-84 odd

5). Section 1.1 Problem 92