Homework #8 Solutions

Consider the function:

$$f(x) = (2x^2 + 1)\sqrt{3x - 5}$$

Determine $f'(x)$ and simplify.

Note that $f(x) = g(x)h(x)$ where:

$$g(x) = 2x^2 + 1$$
$$h(x) = \sqrt{3x - 5} = (3x - 5)^{\frac{1}{2}}$$

So we are going to use the product rule. We will need the derivatives of the two parts:

$$g'(x) = 4x$$
$$h'(x) = \frac{1}{2}(3x - 5)^{-\frac{1}{2}}(3) = \frac{3}{2}(3x - 5)^{-\frac{1}{2}}$$

Note that we used the chain rule for $h'(x)$. Now, putting the parts together and simplifying:

$$
\begin{aligned}
f'(x) &= g'(x)h(x) + g(x)h'(x) \\
&= 4x(3x - 5)^{\frac{1}{2}} + (2x^2 + 1)\frac{3}{2}(3x - 5)^{-\frac{1}{2}} \\
&= (3x - 5)^{-\frac{1}{2}}\left[4x(3x - 5) + \frac{3}{2}(2x^2 + 1)\right] \\
&= (3x - 5)^{-\frac{1}{2}}\left[12x^2 - 20x + 3x^2 + \frac{3}{2}\right] \\
&= (3x - 5)^{-\frac{1}{2}}\left[15x^2 - 20x + \frac{3}{2}\right] \\
&= \frac{1}{2}(3x - 5)^{-\frac{1}{2}}(30x^2 - 40x + 3) \\
&= \frac{30x^2 - 40x + 3}{2\sqrt{3x - 5}}
\end{aligned}
$$