

Math-42 Worksheet #18

The Basics of Counting

1. Using the letters in the word: STATISTICS
 - (a) How many ways are there to construct a sequence of two different letters?
 - (b) How many ways are there to construct a sequence of two different letters if the first letter must be a vowel and the second letter must be a consonant?
2. You invest in and open a sandwich shop. You offer 3 types of bread, 6 types of meat, 5 types of cheese, and 8 toppings (such as mustard, mayo, etc). How many possible sandwiches are there if each sandwich selects one type of bread, one type of meat, one type of cheese, and one topping?
3. How many different sequences of head and tails are possible if a coin is flipped 10 times?
4. How many ways are there to select a man and a woman who are not married to each other from a group of n married couples?
5. There are 4 roads between town A and town B , 3 roads between town B and town C , and 2 roads between town A and town C . How many possible routes are there from town A to town C , assuming that each road can be traversed at most one time? (Hint: partition the routes into those that go through town B and those that do not.)
6. You are creating a 6-letter (case-insensitive) password whose characters are limited to letters (26 possibilities) and digits (10 possibilities). At least one character must be a digit. How many possible valid passwords are there? (Hint: consider where the first digit appears.)
7. Five committee members need to be seated at a table facing an audience. The committee members' names are: Alice, Bob, Cerida, David, and Earl. For each of the following cases,

determine the number of ways to seat the committee at the table (note: the cases are all separate):

- (a) The members can be seated in any order.
- (b) Earl has to leave early, so he has requested an end chair.
- (c) Alice and Cerida are best friend and want to sit next to each other.
- (d) Bob and David had a big argument before the meeting and insist on sitting at opposite ends of the table.

8. You roll two standard six-sided dice: one red and one green. Assuming that the dice are distinct:

- (a) How many possible rolls are there?
- (b) How many ways are there to roll a seven?
- (c) How many ways are there to roll doubles?
- (d) How many ways are there to roll a seven or a double?
- (e) How many ways are there to roll an even number?
- (f) How many ways are there to roll an even number or a double?
- (g) How many ways are there to roll an eight?
- (h) How many ways are there to roll an eight or a double?

9. Let A be the set of positive integers less than 100 that are divisible by 3 and let B be the set of positive integers less than 100 that are divisible by 4. Determine the following:

- (a) $|A|$
- (b) $|B|$
- (c) $|A \cap B|$
- (d) $|A \cup B|$

10. You have 5 balls: 3 red, 1 blue, and 1 green. How many distinct color arrangements are there if the balls are to be arranged in a row? (Hint: how many times does the same arrangement occur because of the red balls?)