Factoring

Factoring is the application of the distributive property to remove a set of common factors from a collection of terms:

$$t_1 + t_2 + t_3 + \ldots = (f_1 + f_2)(f_3 + f_4)\cdots$$

Sometimes the factors are obvious, sometimes not.

But why do we want to do this? Usually, because we want to compare the factors to zero:

Example

$$x^2 + 3x - 10 = (x+5)(x-2) = 0$$

Due to our property of zero, we know that at least one of the factors must be zero.

Techniques:

1). Obvious common factors

Example

$$3x + 4x - 5x = (3 + 4 - 5)x = 2x$$

$$2xy^2z + 4xy - 10xz + 2x = 2x(y^2z + 2y - 5z + 1)$$

2). Difference of Squares

$$a^2 - b^2 = (a+b)(a-b)$$

Example

$$x^2 - 4 = (x+2)(x-2)$$

$$x^4 - y^2 = (x^2 + y)(x^2 - y)$$

$$x - y = (\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y})$$

3). Sum/Difference of Cubes

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Example

$$x^3 + 1 = (x+1)(x^2 - x + 1)$$

$$x^3 - 8 = (x-2)(x^2 + 2x + 4)$$

$$x^3 + y^3z^3 = (x + yz)(x^2 - xyz + y^2z^2)$$

4). Perfect Square

$$a^2 + 2ab + b^2 = (a+b)^2$$

$$a^2 = 2ab + b^2 = (a-b)^2$$

Example

$x^2 + 2x + 1 = (x+1)^2$

$4x^2 - 4x + 1 = (2x-1)^2$

$x^4 + 2x^2y^2 + y^4 = (x^2+y^2)^2$

5). By inspection (backwards FOIL) $ax^2 + bx + c = (nx+m)(rx+s)$

a). If the leading term is negative, factor out a (-1).

b). Determine possible FIRST

c). Determine possible LAST

d). Try to match up possible LAST with OUTER+INNER

Example

$x^2 + 3x + 2 = (x+2)(x+1)$

$-4x^2 - 16x + 16 = -(2x+4)^2$

Case 1: $c > 0, b > 0$

Must be $+, +$

Example

$x^2 + 3x + 2 = (x+2)(x+1)$

Case 2: $c > 0, b < 0$

Must be $-, -$

$x^2 - 3x - 2 = (x-2)(x-1)$

Case 3: $c < 0, b > 0$

Must be $+, -$ (larger,smaller)

$x^2 + 3x - 10 = (x+5)(x-2)$

Case 4: $c < 0, b < 0$

Must be $-, +$ (larger,smaller)

$$x^2 - 3x - 10 = (x-5)(x+2)$$

6). Grouping

Read on your own. Comes up rarely.

An important note: due to the way that the real numbers work, you can factor anything from anything: multiply above and below by the thing that you want to factor out.

Example

$2 = 3 \cdot \frac{2}{3}$

$2x^2 + 2x + \frac{1}{2} = 2(x^2 + x + \frac{1}{4}) = 2(x + \frac{1}{2})^2$

$x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + x^{-\frac{1}{2}} = x^{-\frac{1}{2}}(x^2 + 2x + 1) = \frac{(x+1)^2}{\sqrt{x}}$

$x^{\frac{1}{2}} + x^{\frac{1}{3}} = x^{\frac{1}{3}}(x^{\frac{1}{6}} + 1)$

When factoring out rational exponents, always pick the smallest (negative) exponent.