Sample Final Exam
Fall 2015

1. $f(x) = x^2 - 7$
$g(x) = 2x + 5$

$f(g(-3)) = ?$

$g(-3) = 2(-3) + 5$
$= -6 + 5$
$= -1$
$f(g(-3)) = f(-1)$
$= (-1)^2 - 7$
$= 1 - 7$
$= -6.$

2. $x + x^2y^2 - 7y + 5 = 0$
Find x intercepts.

Set $y = 0$
$x + 5 = 0$
$x = -5$
$(-5, 0)$

3. $\frac{2}{x-3} = \frac{7}{x+5}$
$2(x+5) = 7(x-3)$
$2x + 10 = 7x - 21$
$5x = 31$
$x = \frac{31}{5}$

4. $\log_7(7^{15}7^{32}) = \log_7 7^{47} = 47$

5. $(f \circ g)(x) = \sqrt{1 + e^{3x}}$

$g(x) = 1 + e^{3x}$ (inner)
$f(x) = \sqrt{x}$ (outer).

6. $x + 5y + 3z = 7$
$y - 2z = 4$
$z = 1$

$y - 2(1) = 4$
$y - 2 = 4$
$y = 6$

$x + 5(6) + 3(1) = 7$
$x + 30 + 3 = 7$
$x = -26$

$(x, y, z) = (-26, 6, 1)$

7. Increasing on $[-2, 0]$ and $[2, 4]$

8. $f(x) = 2x^4 - x^3 + 12x + 35$

degree = 4 (even)
leading coefficient = 2 > 0
so, by the LCT, f(x) end behavior is like x^2, so (c).

9.

$$\begin{array}{r|l} & x - 2 \\ \hline x^2 - x + 2 & x^3 - 3x^2 + 7x + 5 \\ & \underline{x^3 - x^2 + 2x} \\ & -2x^2 + 5x + 5 \\ & \underline{-2x^2 + 2x - 4} \\ & 3x + 9 \end{array}$$

$$\frac{f(x)}{d(x)} = (x-2) + \frac{3x+9}{x^2-x+2}$$

10. $P_1(3,-7)$ $P_2(12,6)$

$m = \frac{6+7}{12-3} = \frac{13}{9}$

$y+7 = \frac{13}{9}(x-3)$ or $y-6 = \frac{13}{9}(x-12)$

11. $2x+5 \le 7x-8$

$5 \le 5x-8$

$13 \le 5x$

$\frac{13}{5} \le x$

$x \ge \frac{13}{5}$

$[\frac{13}{5}, \infty)$

12. $x = \sqrt{5x-6} = (5x-6)^{1/2}$

$x^2 = 5x-6$ $\quad (x^{\frac{1}{2}})^2 \rightarrow$ no need to worry about +/-

$x^2 - 5x + 6 = 0$

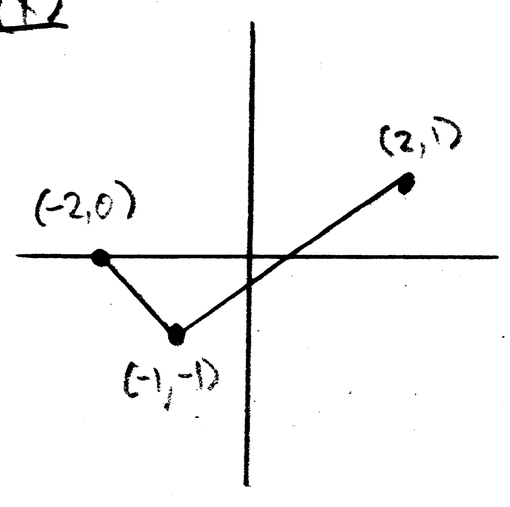
$(x-3)(x-2) = 0$

$x = 2, 3$

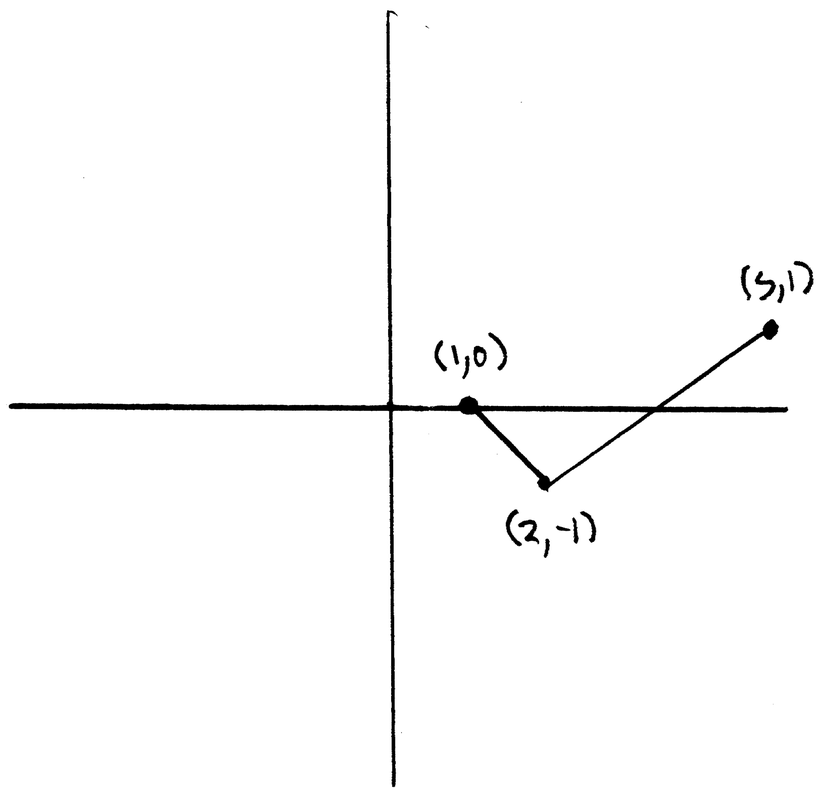
Checking for domain violation, 2 and 3 are both OK.

13. $y = 2 + f(x-3)$.

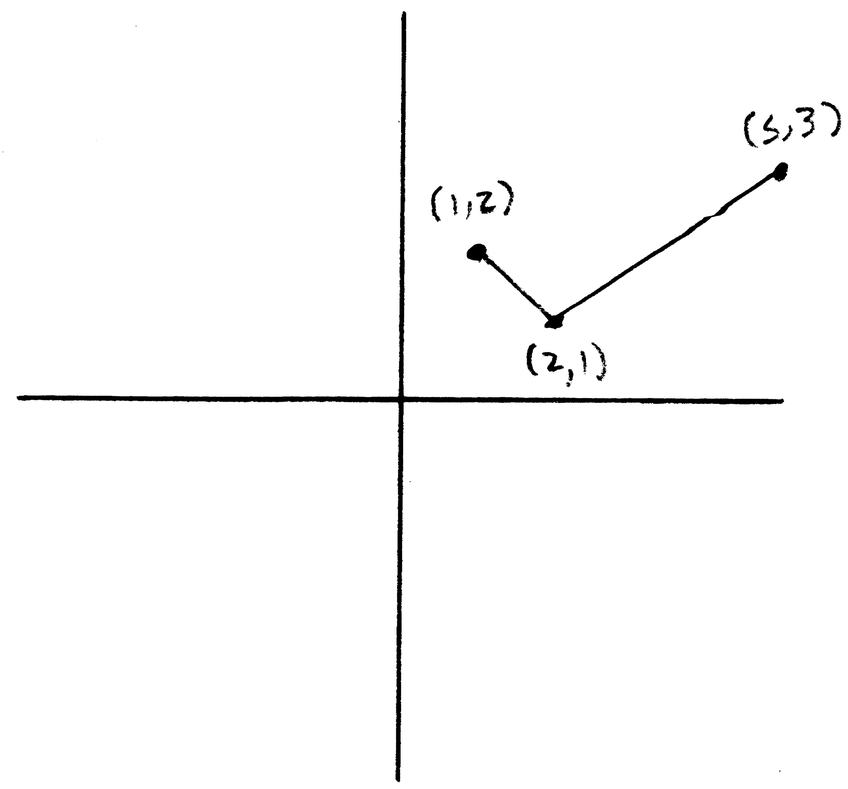
$f(x)$



$f(x-3)$ translate right 3



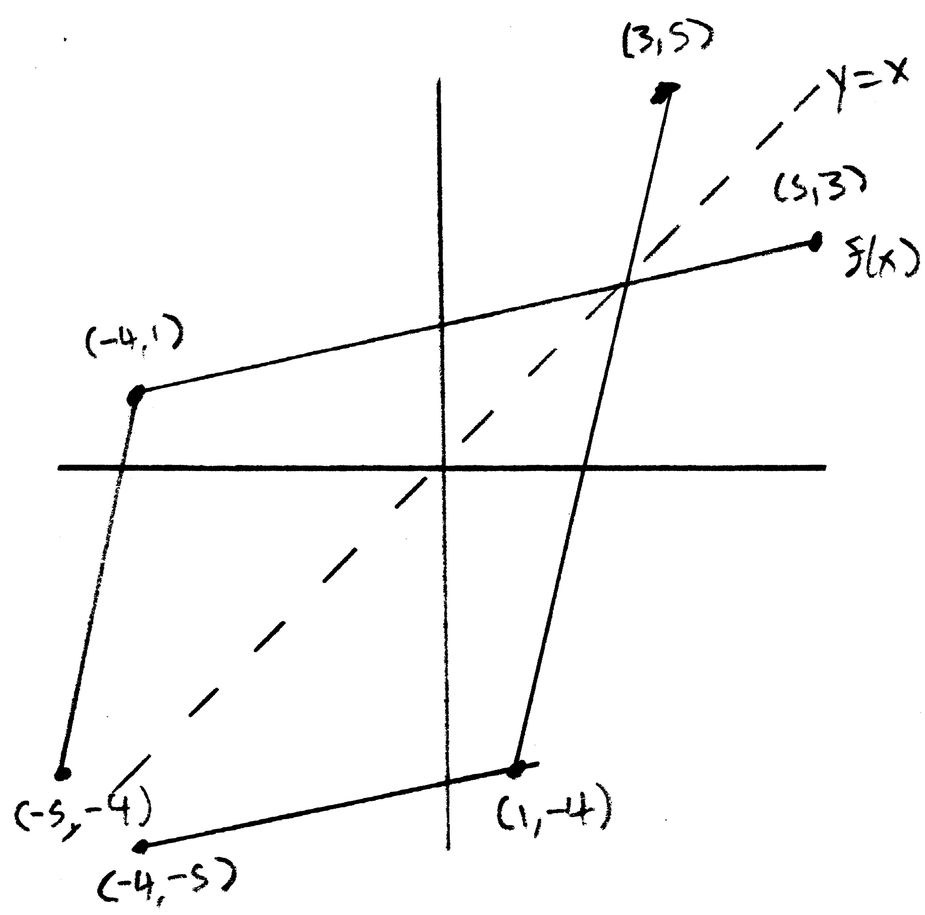
$2+f(x-3)$ translate up 2



14. $4\log(x+5)+2\log x-3\log(x-2) = \log(x+5)^4+\log x^2-\log(x-2)^3$
$= \log\frac{(x+5)^4x^2}{(x-2)^3}$

15. Since $(3,0)$ is on the graph $x=3$ is an x-intercept and thus $(x-3)$ divides $f(x)$. $\therefore$ remainder $=0$.

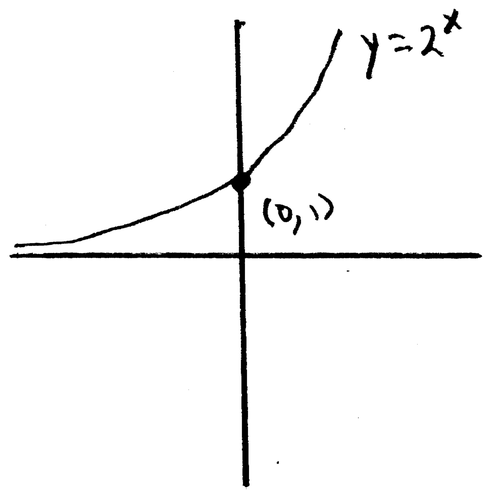
16.

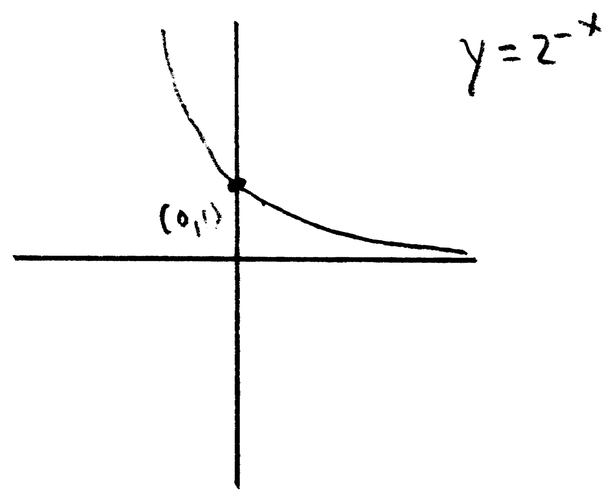


Swap x and y to reflect about $y=x$
i.e, $f(5)=3$ means $f^{-1}(3)=5$
also note that we know an inverse exists because $f(x)$ passes the horizontal line test.

17. $g(x) = 2^{-x}$

a.





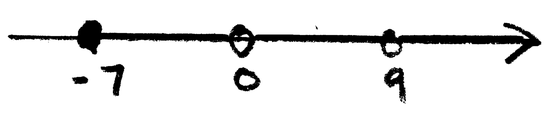
(reflect across y axis)

$g(x) \to \infty$ as $x \to -\infty$
$g(x) \to 0$ as $x \to \infty$

b. Domain = $\mathbb{R}$
Range = $(0, \infty)$

18. $g(x) = \frac{\sqrt{x+7}}{x^2-9x} = \frac{\sqrt{x+7}}{x(x-9)}$

$x \neq 0, 9$
$x + 7 \geq 0$
$x \geq -7$



Domain = $[-7, 0) \cup (0, 9) \cup (9, \infty)$

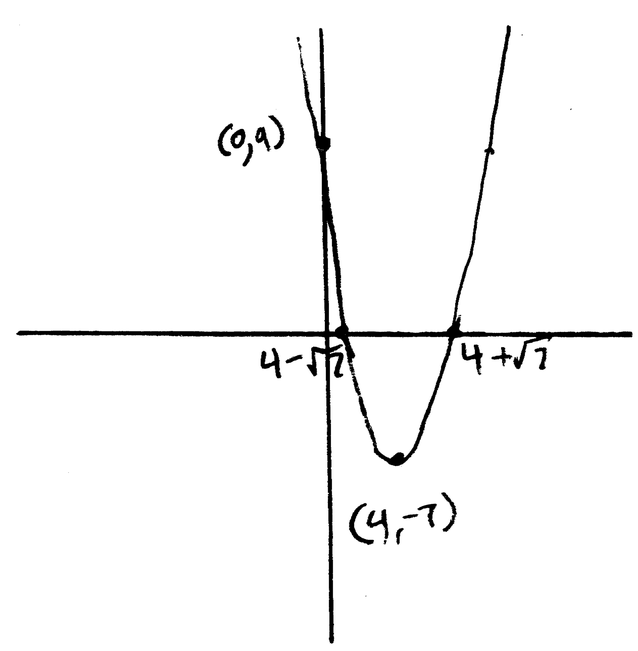
19. $f(x) = 3x^2$

$$\frac{f(5+h) - f(5)}{h} = \frac{3(5+h)^2 - 3(5)^2}{h}$$
$$= \frac{3(25 + 10h + h^2) - 75}{h}$$
$$= \frac{\cancel{75} + 30h + 3h^2 - \cancel{75}}{h}$$
$$= 30 + 3h.$$

20. $f(x) = x^2 - 8x + 9$
$= (x^2 - 8x + 16) + 9 - 16$
$f(x) = (x-4)^2 - 7$
vertex = $(4, -7)$

$f(0) = 0^2 - 8(0) + 9 = 9$

$x^2 - 8x + 9 = 0$
$$x = \frac{8 \pm \sqrt{8^2 - 4(1)(9)}}{2(1)}$$
$$= \frac{8 \pm \sqrt{64 - 36}}{2}$$
$$= \frac{8 \pm \sqrt{28}}{2}$$
$$= \frac{8 \pm 2\sqrt{7}}{2}$$
$x = 4 \pm \sqrt{7}$



21. $f(x) = x^3 - 5x^2 + 2x + 8$

$p: \pm 1, \pm 2, \pm 4, \pm 8$

$q: \pm 1$

$\frac{p}{q} = \pm 1, \pm 2, \pm 4, \pm 8$

$f(1) = 1 - 5 + 2 + 8 \neq 0$

$f(-1) = -1 - 5 - 2 + 8 = 0$ ✓

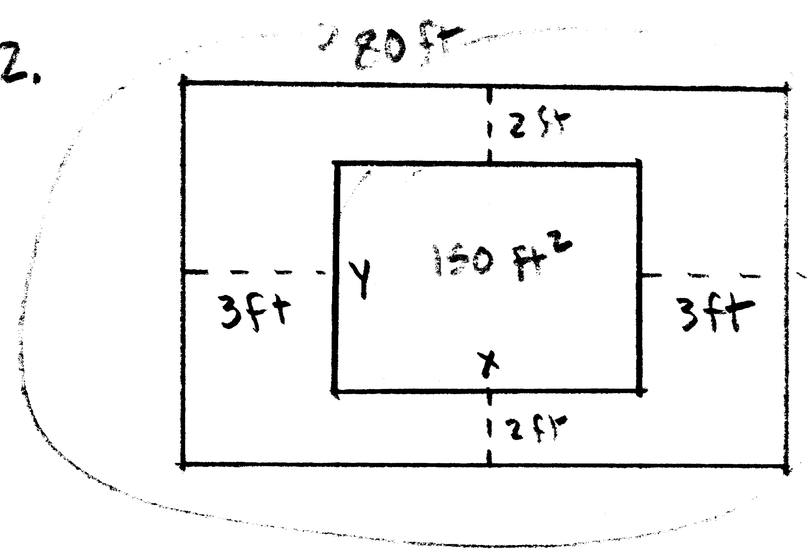
$$
\begin{array}{r|l}
 & x^2 - 6x + 8 \\
\hline
x+1 & x^3 - 5x^2 + 2x + 8 \\
 & \underline{x^3 + x^2} \\
 & -6x^2 + 2x \\
 & \underline{-6x^2 - 6x} \\
 & 8x + 8 \\
 & \underline{8x + 8} \\
 & 0
\end{array}
$$

$f(x) = (x+1)(x^2 - 6x + 8)$

$f(x) = (x+1)(x-2)(x-4)$

zeros $= -1, 2, 4$.

22.



$L(x) = 3 + x + 3 = 6 + x$

$W(y) = 2 + y + 2 = 4 + y$

$A(x,y) = xy = 150$

$P(x,y) = 2L(x) + 2W(y) = 80$

$2(6+x) + 2(4+y) = 80$

$12 + 2x + 8 + 2y = 80$

$2x + 2y = 60$

$x + y = 30$

$y = 30 - x$

$x(30 - x) = 150$

$30x - x^2 = 150$

$x^2 - 30x + 150 = 0$

$x = \frac{30 \pm \sqrt{30^2 - 4(1)(150)}}{2(1)}$

$= \frac{30 \pm \sqrt{900 - 600}}{2}$

$= \frac{30 \pm \sqrt{300}}{2}$

$= \frac{30 \pm 10\sqrt{3}}{2}$

$x = 15 \pm 5\sqrt{3}$

$x = 23.66, 6.34$

W L

$L = 23.66$ ft

$W = 6.34$ ft

23. $f(x) = (x^2-1)(x^2-9)$

a. $f(x) = (x+1)(x-1)(x+3)(x-3)$

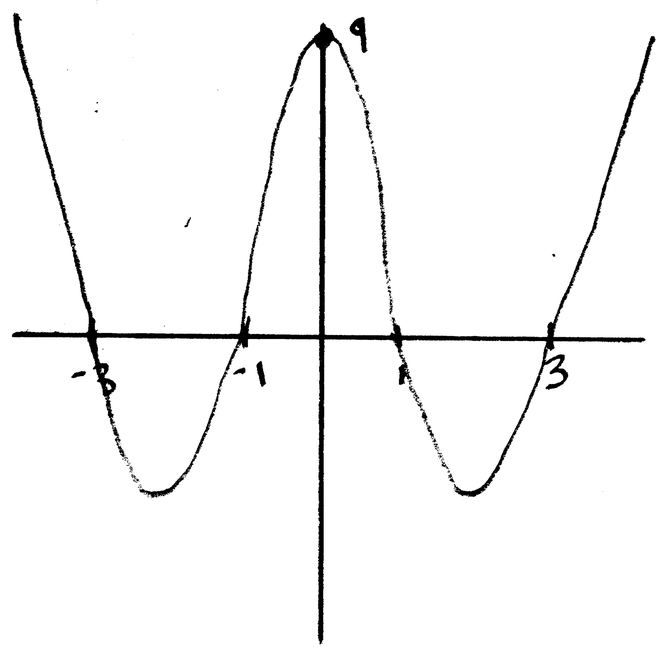
b. zeros: $\pm 1, \pm 3$

c.

	$(x+1)$	$(x-1)$	$(x+3)$	$(x-3)$	
-2	$-$	$-$	$+$	$-$	$-$
0	$+$	$-$	$+$	$-$	$+$
2	$+$	$+$	$+$	$-$	$-$

Leading term is x^4
so, by LCT, like x^2

$f(0) = (-1)(-9) = 9$



24. $W(t) = 5000e^{0.01t}$

$10^7 = 5 \times 10^3 e^{0.01t}$

$0.2 \times 10^4 = e^{0.01t}$

$2000 = e^{0.01t}$

$\ln 2000 = 0.01t$

$t = \frac{\ln 2000}{0.01}$

$t = 760.09$ days