Math-19 Homework #2

<u>Problems</u>

1). Let:

$$P \;:=\; 0 \text{ is a positive number}$$
$$Q \;:=\; 2 \geq 2$$
$$R \;:=\; \forall n, m \in \mathbb{N}, n + m \in \mathbb{N}$$

Determine whether the following (compound) statements are true or false:

Statement	T/F
P	
Q	
R	
not P	
not Q	
not R	
P and Q	
P and R	
Q and R	
P or Q	
P or R	
Q or R	

2). Convert $10.2\overline{45}$ to rational form.

3). Let:

$$A \;=\; \text{the set of all positive real numbers}$$
$$B \;=\; \text{the set of real numbers between -3 (exclusive) and 3 (inclusive)}$$

a). Graph each set on the real number line.

b). Represent each set using set-builder notation.

c). Represent each set using interval notation.

d). Graph $A \cup B$ and represent it in interval notation.

e). Graph $A \cap B$ and represent it in interval notation.

f). Graph $A - B$ and represent it in interval notation.

4). A careful solution of $4(x+2) = 11$ is given below. Give the rationale for each step from the ten real number rules (AC,AA,A0,AI,MC,MA,M1,MI,LD,RD) and the additional rules (SUB,WD).

$4(x+2) = 11$
$4x + 8 = 11$ _____
$(4x+8) - 8 = 11 - 8$ _____
$(4x+8) - 8 = 3$ _____
$4x + (8-8) = 3$ _____
$4x + 0 = 3$ _____
$4x = 3$ _____
$\frac{1}{4}(4x) = \frac{1}{4}(3)$ _____
$\frac{1}{4}(4x) = \frac{3}{4}$ _____
$(\frac{1}{4}4)x = \frac{3}{4}$ _____
$1x = \frac{3}{4}$ _____
$x = \frac{3}{4}$ _____

5). Consider the statement: $\forall\, a, b \in \mathbb{R}, |a - b| = |b - a|$

a). Give a careful proof of this statement. You will need to use one of the distributive rules (hint: factor out a -1), one of the properties in the box at the top of page 9 of your textbook, and the definition of absolute value.

b). What does this statement mean (what are the semantics)? (Hint: think distance)