Math-13 Sections 01 and 02

Homework #12 Solutions

A particle is moving with a velocity measured in meters per second given by the function:

$$v(t) = \frac{t^2 - 3t + 2}{\sqrt{t}}$$

Where is the particle's position at $t = 4$ s if the particle's position at $t = 1$ s is 5 m?

First, modify $v(t)$ by dividing by $t^{\frac{1}{2}}$ so that it is easier to integrate:

$$v(t) = t^{\frac{3}{2}} - 3t^{\frac{1}{2}} + 2t^{-\frac{1}{2}}$$

Now, find the antiderivative, which is the position:

$$s(t) = \frac{2}{5}t^{\frac{5}{2}} - 3\left(\frac{2}{3}\right)t^{\frac{3}{2}} + 2(2)t^{\frac{1}{2}} + C = \frac{2}{5}t^{\frac{5}{2}} - 2t^{\frac{3}{2}} + 4t^{\frac{1}{2}} + C$$

Next, use the initial condition to resolve C:

$$s(1) = \frac{2}{5}(1)^{\frac{5}{2}} - 2(1)^{\frac{3}{2}} + 4(1)^{\frac{1}{2}} + C = 5$$
$$\frac{2}{5} - 2 + 4 + C = 5$$
$$\frac{12}{5} + C = 5$$
$$C = 5 - \frac{12}{5} = \frac{13}{5}$$

And so the final function for the position is:

$$s(t) = \frac{2}{5}t^{\frac{5}{2}} - 2t^{\frac{3}{2}} + 4t^{\frac{1}{2}} + \frac{13}{5}$$

Finally, plug in 4 to determine the position after 4 s:

$$\begin{aligned} s(t) &= \frac{2}{5}(4)^{\frac{5}{2}} - 2(4)^{\frac{3}{2}} + 4(4)^{\frac{1}{2}} + \frac{13}{5} \\ &= \frac{2}{5}(32) - 2(8) + 4(2) + \frac{13}{5} \\ &= \frac{64}{5} - 16 + 8 + \frac{13}{5} \\ &= \frac{77}{5} - 8 \\ &= \frac{37}{5} \\ &= 7.4 \end{aligned}$$

Therefore, after 4 s, the particle is at position 7.4 m.