

Math-71 Sections 9, 11, 12

Homework #3 Solutions

**Due: 2/19/2019 5:45pm**

**Problem**

A thin board is to be rested up against a large rock such that it touches the rock at exactly one point. Let the ground be the  $x$  direction and assume that the origin is at the start of the rock. The surface of the rock follows the function:

$$s(x) = \sqrt{x}$$

where  $s(x)$  is the height of the surface of the rock (in feet) at position  $x$  (also in feet).

1. Use the definition of the derivative (i.e., the difference quotient from Section 7.3) to determine  $s'(t)$ .

$$\begin{aligned} s'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \\ &= \lim_{h \rightarrow 0} \frac{(x+h) - x}{h(\sqrt{x+h} + \sqrt{x})} \\ &= \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})} \\ &= \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} \\ &= \frac{1}{\sqrt{x} + \sqrt{x}} \\ &= \frac{1}{2\sqrt{x}} \end{aligned}$$

2. Assuming that the board intersects with the rock surface at  $x = 4$ , determine where one end of the board touches the ground.

We need to determine an equation of a line for the board. Note that  $s'(x)$  is the slope of the tangent line to the rock surface at position  $x$ , and so:

$$s'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{2 \cdot 2} = \frac{1}{4}$$

Now, since we know that the board touches the rock at the point  $(4, \sqrt{4}) = (4, 2)$ , use the point-slope form of the line and convert to the slope-intercept form:

$$(y - 2) = \frac{1}{4}(x - 4)$$

$$y - 2 = \frac{1}{4}x - 1$$

$$y = \frac{1}{4}x + 1$$

Finally, we need the  $x$  intercept of the line, so set  $y = 0$ :

$$0 = \frac{1}{4}x + 1$$

$$\frac{1}{4}x = -1$$

$$x = -4$$

Thus, the board touches the ground at  $x = -4$  ft.