Math-08 Homework #11 Solutions

Reading

- Text book section 2.4-2.7

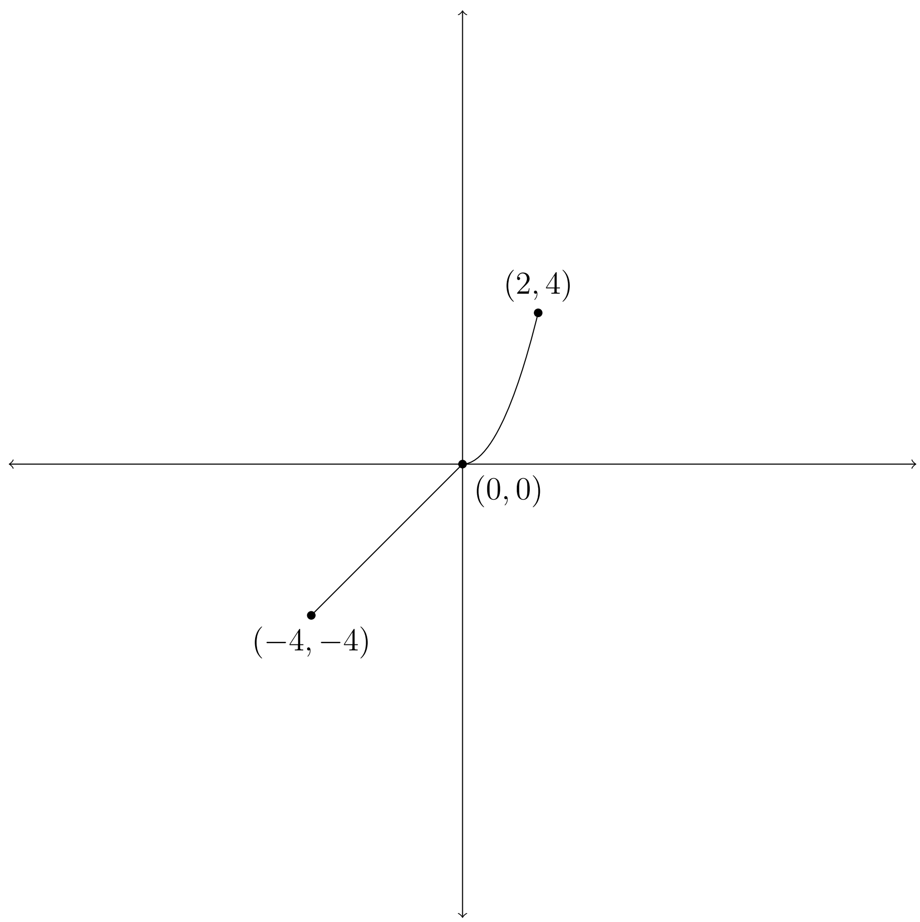
Problems

Note that all sketches of graphs must have all found intercepts and discontinuities labeled. All domains and ranges must be expressed in interval notation. Remember, sketches do not have to be to scale!

1). Consider the following piecewise function:

$$f(x) = \begin{cases} x, & (-4, 0) \\ x^2, & (0, 2) \end{cases}$$

a). Sketch the graph for $f(x)$.



b). List the transformations for $g(x) = -2f(x-1) + 3$ in the proper order.

1) Start with $f(x)$

2) Translate right by 1

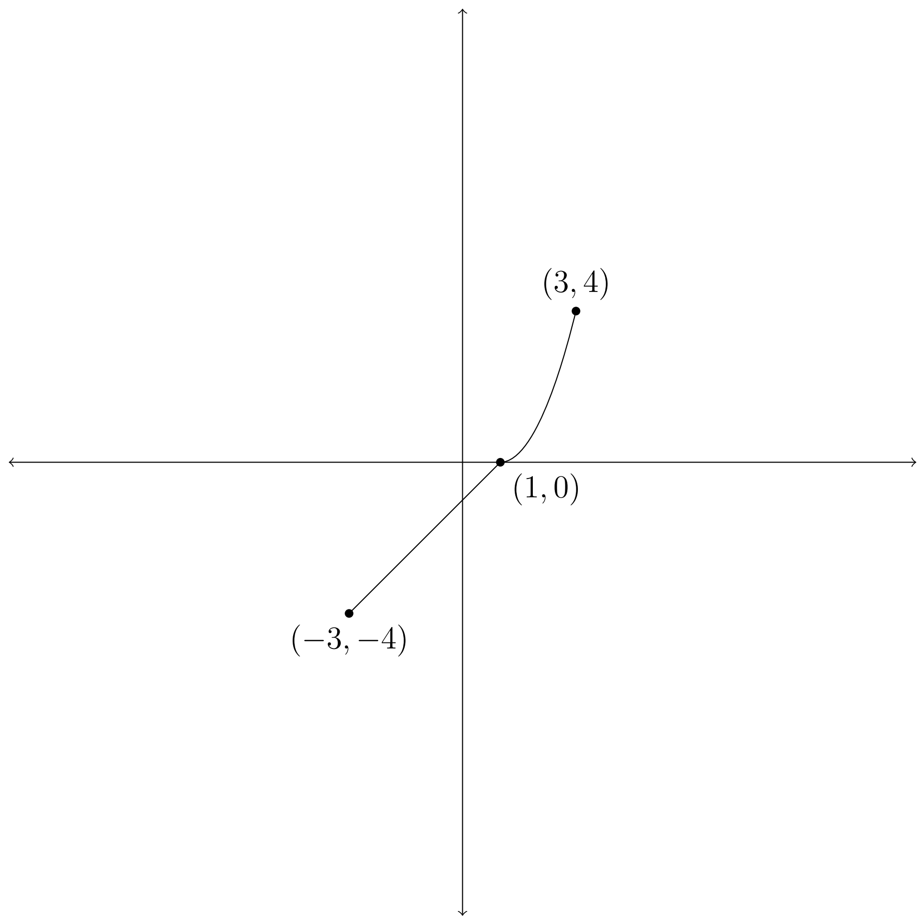
3) Scale by 2

4) Reflex across x-axis

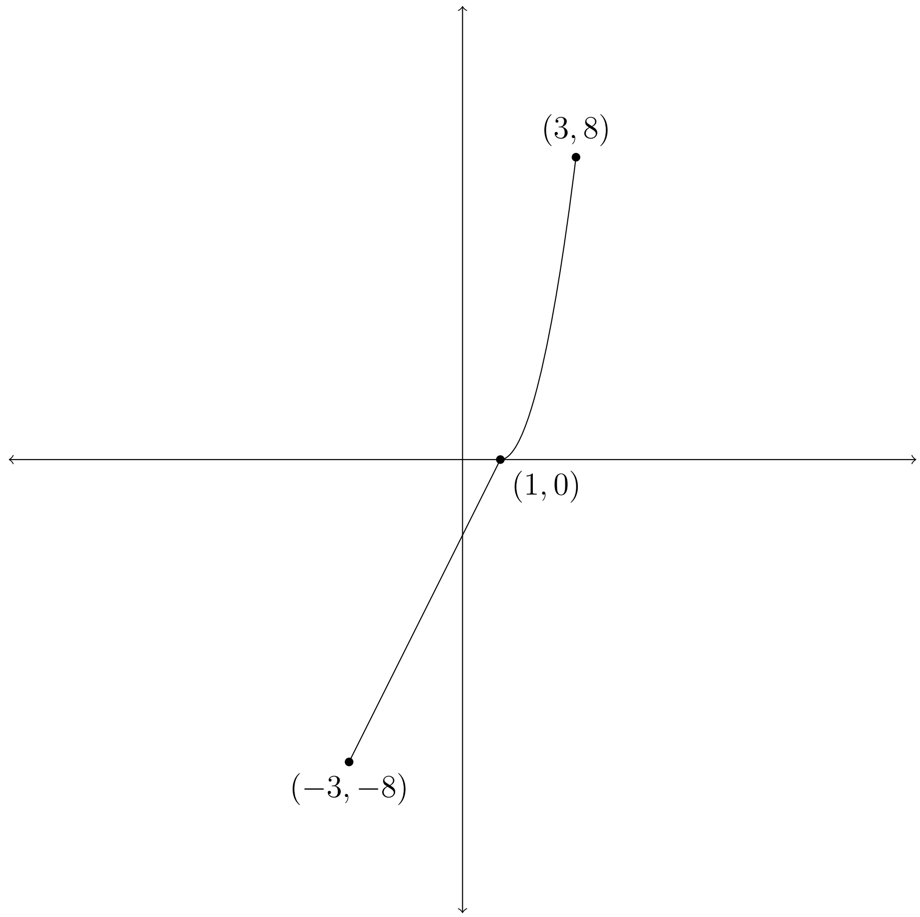
5) Translate up 3

c). Sketch the graph for $g(x)$.

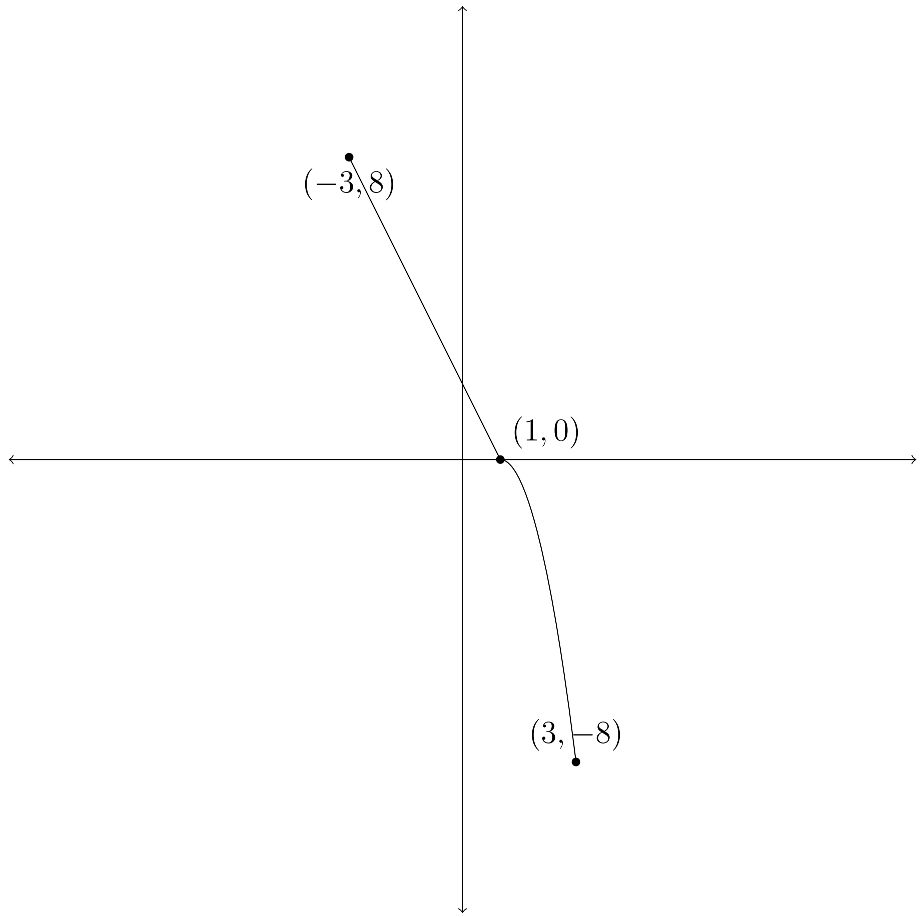
We start with $f(x)$ as above and then apply the first translation: right by 1:



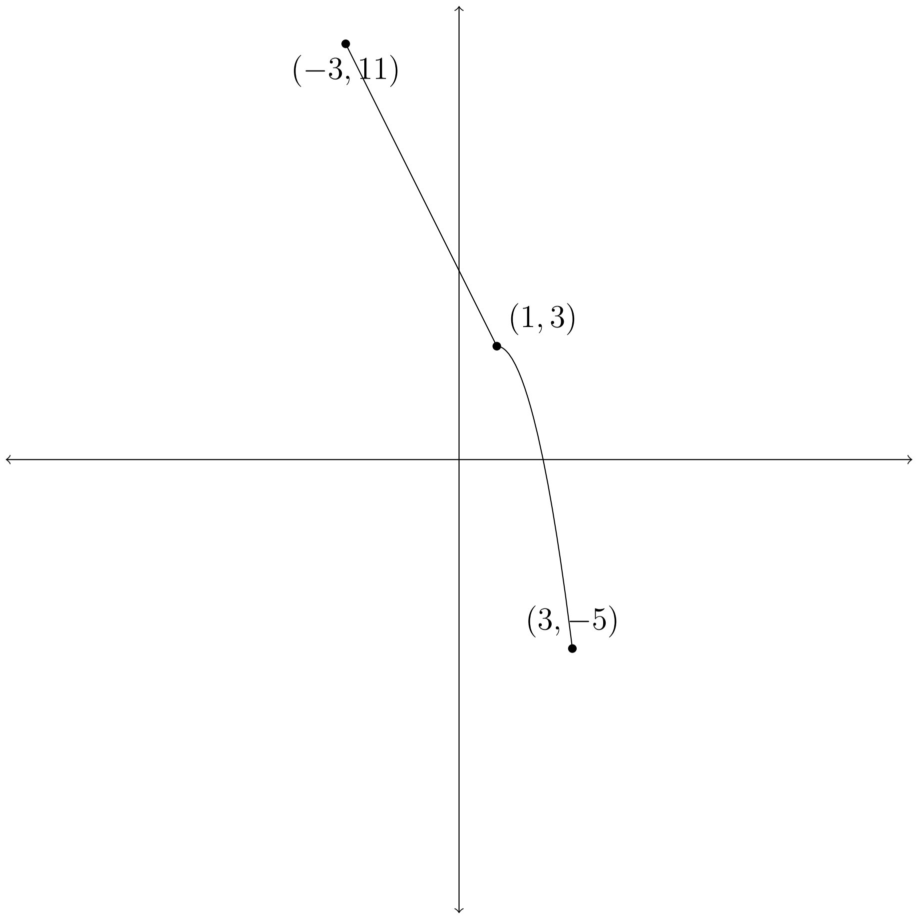
Next, we scale by 2. Note that this affects the y-coordinate only:



Now, reflect across the x-axis: all y values become $-y$:



Finally, translate up by 3 - this also only affects the y-coordinates:



d). What are the x and y intercepts for $g(x)$ (if any)?

When determining the intercepts, we need to know on which part of the piecewise graph they occur. This is evident from our sketch.

To find the y-intercept, which occurs on the linear part, set $x = 0$:

$g(0) = -2f(0-1) + 3 = -2f(-1) + 3 = -2(-1) + 3 = 2 + 3 = 5$

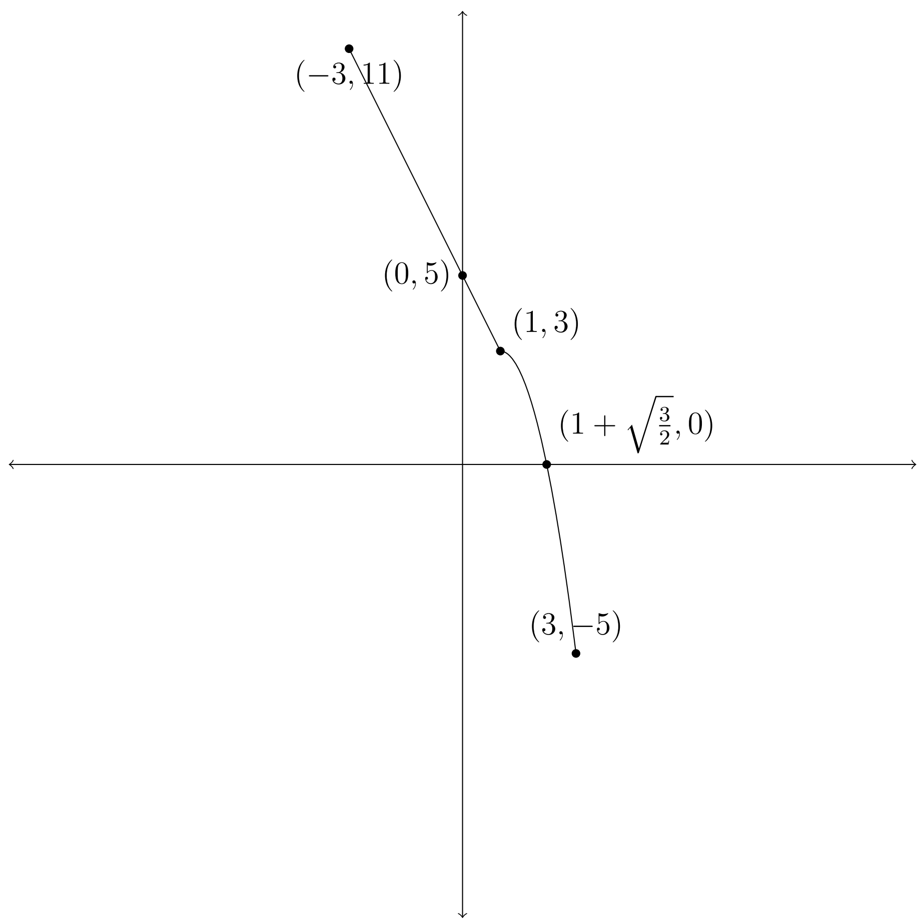
Thus, the y-intercept is (0,5).

To find the x-intercept, which occurs on the parabola, set $y = 0$:

$$\begin{aligned}
-2f(x-1)+3 &= 0\\
-2(x-1)^2+3 &= 0\\
2(x-1)^2 &= 3\\
(x-1)^2 &= \frac{3}{2}\\
x-1 &= \pm\sqrt{\frac{3}{2}}\\
x &= 1\pm\sqrt{\frac{3}{2}}
\end{aligned}$$

Note that we only need the positive one: $\left(1+\sqrt{\frac{3}{2}},0\right)$

So the final sketch looks like this:



e). What are the domain and range of $g(x)$?

Domain: $[-3,3]$
Range: $[-5,11]$

2). Consider the function:

$$h(x) = \sqrt{x+1} - 3$$

a). Write $h(x)$ as a composition of two functions $f \circ g$, neither of which is just x.

The two most straightforward answers would be:

$f(x) = \sqrt{x} - 3$ and $g(x) = x + 1$ or

$f(x) = x - 3$ and $g(x) = \sqrt{x+1}$

b). Determine the x and y intercepts for $h(x)$ (if any).

For the y-intercept, set $x = 0$:

$$y = \sqrt{0+1} - 3 = \sqrt{1} - 3 = 1 - 3 = -2$$

So the y-intercept is $(0, -2)$

For the x-intercept, set $y = 0$:

$$0 = \sqrt{x+1} - 3$$

$$\sqrt{x+1} = 3$$

$$x + 1 = 9$$

$$x = 8$$

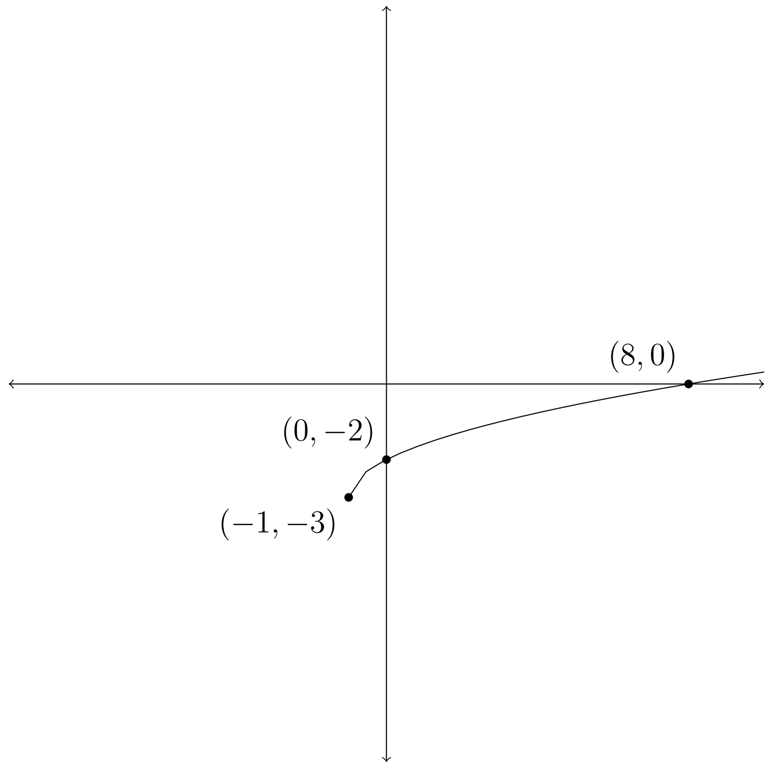
So the x-intercept is $(8, 0)$

c). Sketch the graph for $h(x)$.

The transformations for this graph are:

1) Start with $y = \sqrt{x}$
2) Translate left by 1
3) Translate down by 3

This graph is fairly simple, so let's just transform the key point at $(0, 0)$ and rely on our found intercepts:



d). Determine the domain and range for $h(x)$.

Domain: $[-1, \infty)$
Range: $[-3, \infty)$

3). Consider the function:

$$f(x) = \frac{1}{x-2} + 1$$

a). List the transformations, starting with one of the standard functions.

1) Start with $y = \frac{1}{x}$

2) Translate right 2

3) Translate up 1

b). Determine the x and y intercepts for $f(x)$ (if any).

$$0 = \frac{1}{x-2} + 1$$

$$\frac{1}{x-2} = -1$$

$$-(x-2) = 1$$

$$-x + 2 = 1$$

$$x = 1$$

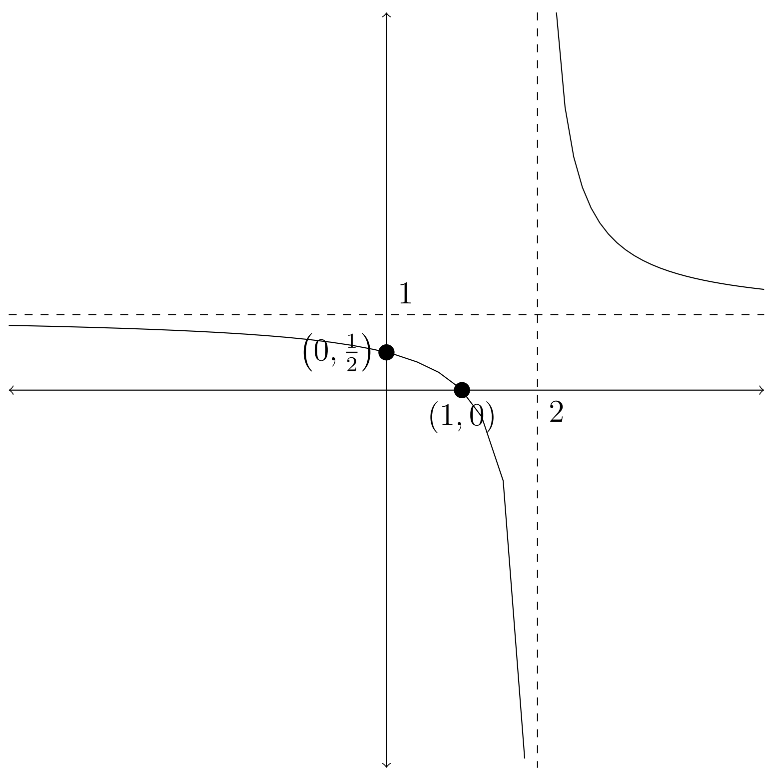
So the x-intercept is at $(1, 0)$

$$y = \frac{1}{0-2} + 1 = -\frac{1}{2} + 1 = \frac{1}{2}$$

So the y-intercept is at $\left(0, \frac{1}{2}\right)$

c). Sketch the graph for $f(x)$.

The trick here is to determine how the asymptotes move. The vertical asymptote moves with the horizontal translation from $x = 0$ to $x = 2$. The horizontal asymptote at moves with the vertical translation from $y = 0$ to $y = 1$:



d). Determine the domain and range for $f(x)$.

Note that the domain has a hole at $x = 2$ and the range has a hole at $y = 1$:

Domain: $(-\infty, 2) \cup (2, \infty)$
Range: $(-\infty, 1) \cup (1, \infty)$

4). Consider the following two functions:

$$f(x) = \sqrt{x} + 1$$

$$g(x) = x^2$$

Determine the following and state the domain for each:

a). $f + g$

$(f+g)(x) = \sqrt{x} + 1 + x^2$

The domain cannot include negative values because of the square root:

Domain: $[0, \infty)$

b). fg

$(fg)(x) = (\sqrt{x} + 1)x^2$

Once again, the domain cannot include negative values because of the square root:

Domain: $[0, \infty)$

c). $\frac{f}{g}$

$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x}+1}{x^2}$

This time, the value 0 is excluded as well because of the denominator:

Domain: $(0, \infty)$

d). $\frac{f}{f}$

$\left(\frac{f}{f}\right)(x) = \frac{\sqrt{x}+1}{\sqrt{x}+1} = 1$

Although the simplified function is constant and thus can accept all real values, we still need to honor the domains of the original functions:

Domain: $[0, \infty)$

e). $f \circ g$

$(f \circ g)(x) = f(g(x)) = f(x^2) = \sqrt{x^2} + 1 = |x| + 1$

Even though the simplified function can take all real numbers, there may be a problem with the original functions; however, in this case, the limitation (no negatives) is in the outer function - the inner function scrubs all negative values out of its range, which becomes the domain for the second function. Thus, the second function will never see a negative number:

Domain: $\mathbb{R}$