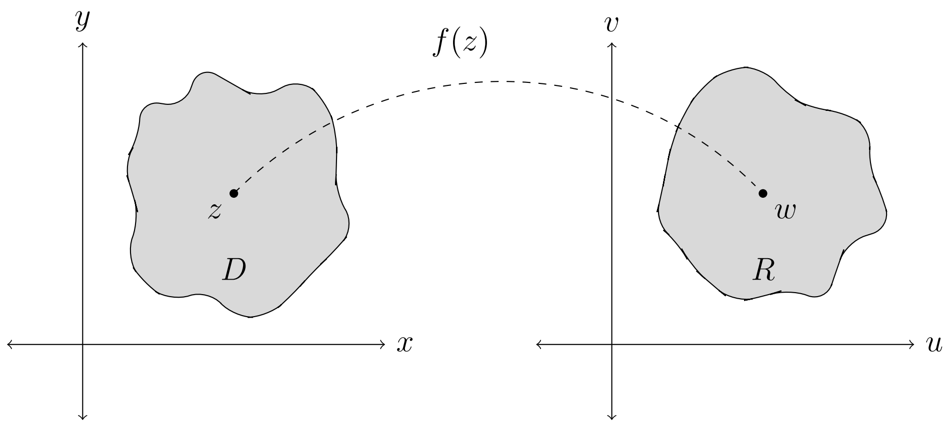
Mappings

- A function f is a *mapping* from the z-plane to the w-plane, such that the coordinates (x, y) are mapped to coordinates (u, v).

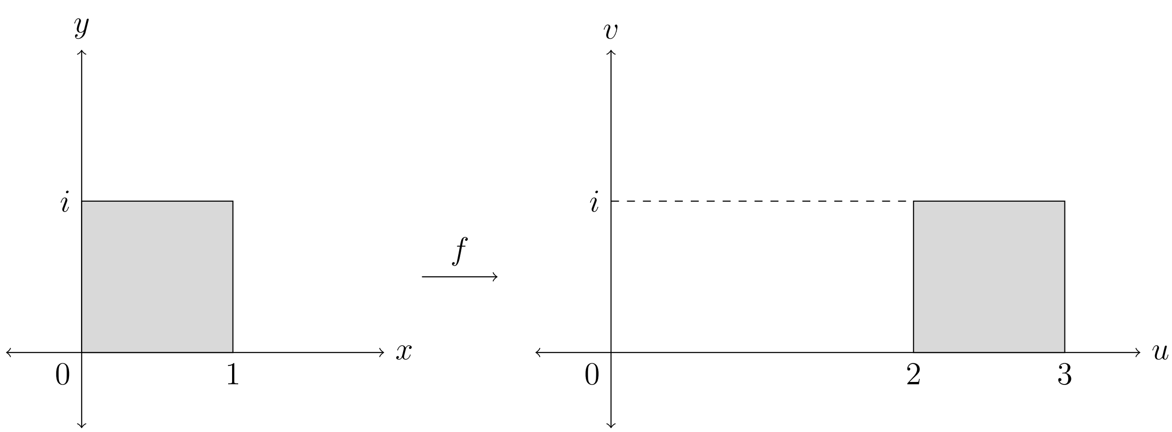


- The region D is called the *domain* and the region R is called the range.
- The value w is called the *image* of z under f, and the value z is called the *pre-image* of w.
- Functions can be *multi-valued*; there can be more than one pre-image for every image.

Example

Translation of the Unit Square

$f(z) = z + 2$

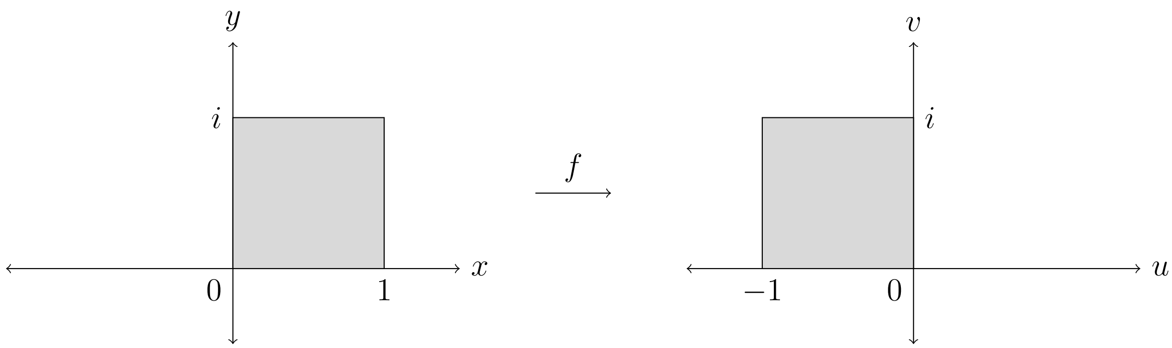


Example

Rotation by $90°$

$f(z) = iz$

$f(z) = ire^{i\theta} = e^{i\frac{\pi}{2}}re^{i\theta} = re^{i(\theta+\frac{\pi}{2})}$



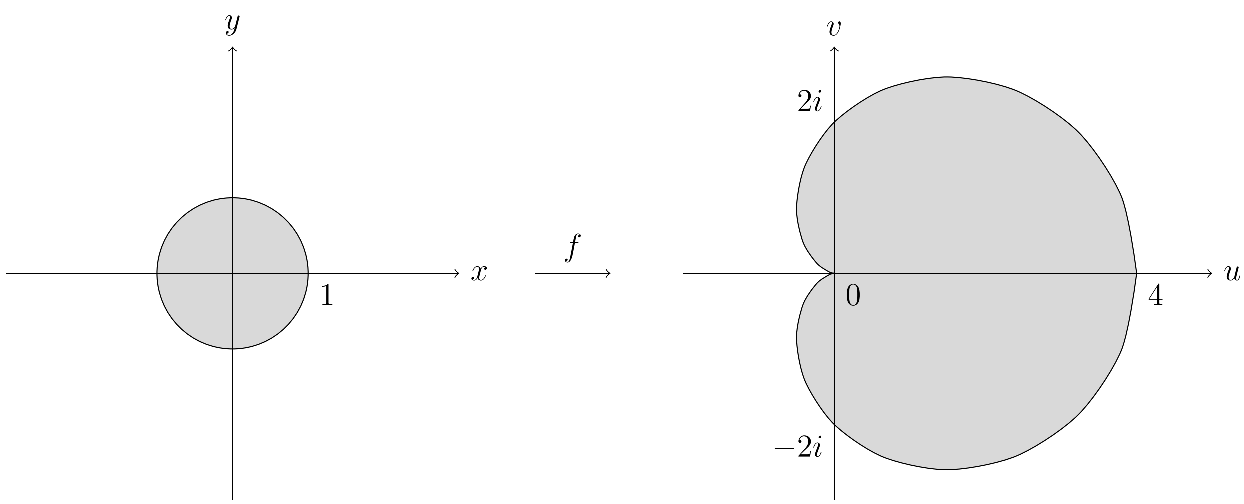
<u>**Example**</u>

$f(z) = (1+z)^2$ on the unit disk

Consider the boundary:

$$
\begin{aligned}
z &= e^{i\theta} \\
w &= (1+e^{i\theta})^2 \\
&= (1+\cos\theta + i\sin\theta)^2 \\
&= \left(2\cos^2\frac{\theta}{2} + i2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)^2 \\
&= \left(2\cos\frac{\theta}{2}\right)^2\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)^2 \\
&= \left(4\cos^2\frac{\theta}{2}\right)\left(e^{i\frac{\theta}{2}}\right)^2 \\
&= 2(1+\cos\theta)e^{i\theta}
\end{aligned}
$$

$\rho = 2(1+\cos\theta) \qquad \phi = \theta$

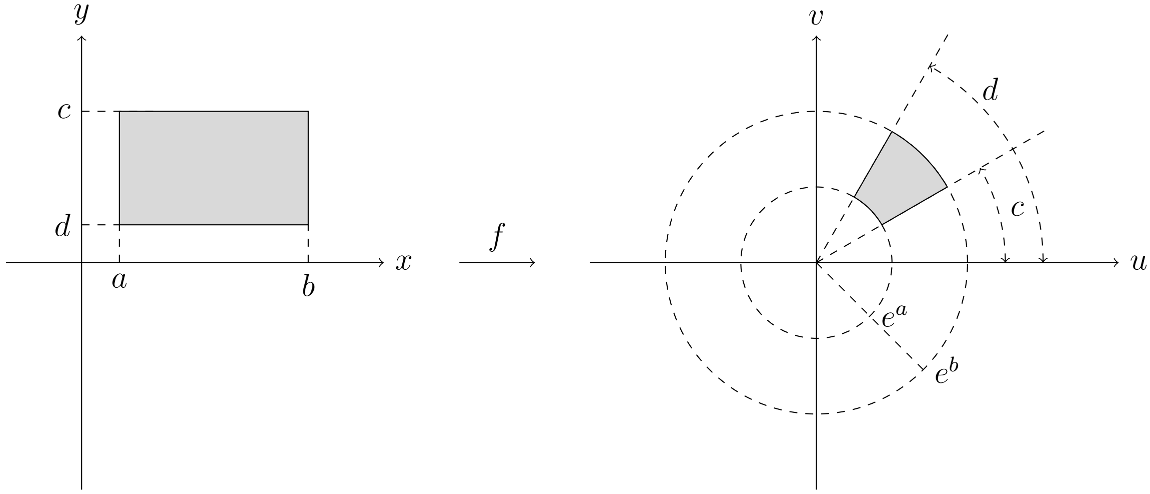


Example

$f(z) = e^z$ on a rectangular region

$w = e^z = e^{x+iy} = e^x e^{iy}$

$\rho = e^x \qquad \phi = y$



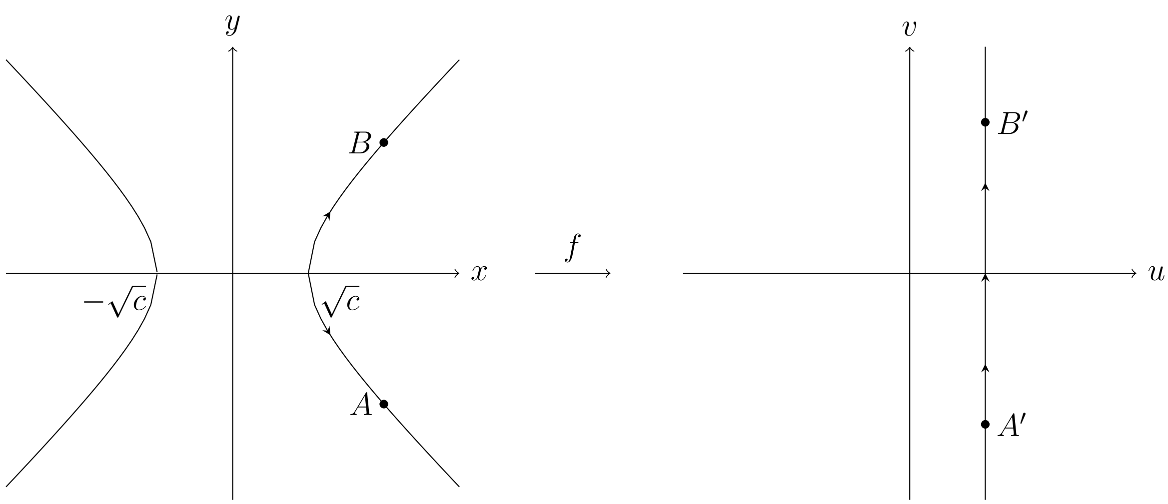
Example

$f(z) = z^2$

$f(z) = f(x+iy) = (x+iy)^2 = (x^2 - y^2) + i2xy$

$u = x^2 - y^2 \qquad v = 2xy$

Assume $u = x^2 - y^2 = c$ from $z = A$ to $z = B$ on the right arc
$v = 2y\sqrt{y^2 + c}$



Now, assume $v = 2xy = c$ from $z = A$ to $z = B$ on the QI arc
$u = x^2 - \frac{c^2}{4x^2}$