# Math-08 Homework #6 Solutions

Reading

- Text book section 1.1, 1.3, 1.4

Problems

1). Determine if and explain why each of the following statements is either correct or incorrect (or misleading).

a). $\sqrt{9} = 3$

This is correct. This form asks for the *principle* root, which is positive.

b). $\sqrt{9} = \pm 3$

This is incorrect, since we only want the principle root, not the negative one. If we want the negative one, we must explicitly write $-\sqrt{9}$.

c). $\left(x^{\frac{1}{2}}\right)^2 = |x|$

This is misleading. By writing $x^{\frac{1}{2}}$, we are implicitly stating that $x \geq 0$ - x is never negative, since we cannot take the square root of a negative number. So, since $x$ is always positive, the absolute value is not needed here.

d). $\left(x^{\frac{1}{2}}\right)^2 = x$

This is correct. As stated above, $x \geq 0$ so no absolute value is needed.

e). $(x^2)^{\frac{1}{2}} = x$

Incorrect! This is the case where we need absolute value, since $x$ can be negative. To see this, plug in a negative number:

$$[(-2)^2]^{\frac{1}{2}} = 4^{\frac{1}{2}} = 2$$

Note that we started with a negative value but got a positive value, so we need the absolute value here: $|-2| = 2$.

f). $(x^2)^{\frac{1}{2}} = |x|$

Correct. Since $x$ can be negative, which is lost after squaring, we need the absolute value.

g). $(x^3)^{\frac{1}{3}} = |x|$

Incorrect! Remember, odd powers/roots preserve negative values, so the absolute value is wrong:

$$[(-2)^3]^{\frac{1}{3}} = (-8)^{\frac{1}{3}} = -2$$

h). $(x^3)^{\frac{1}{3}} = x$

Correct. Odd powers/roots preserve negative values.

2). Consider the quadratic equation: $2x^2 + x - 6 = 0$

a). Solve using factor by inspection.

$$(2x-3)(x+2) = 0$$
$$x = -2, \frac{3}{2}$$

b). Solve by manually completing the square.

$$\begin{aligned} 2x^2 + x &= 6 \\ x^2 + \frac{1}{2}x &= 3 \\ x^2 + \frac{1}{2}x + \frac{1}{16} &= 3 + \frac{1}{16} \\ \left(x + \frac{1}{4}\right)^2 &= \frac{49}{16} \\ \left|x + \frac{1}{4}\right| &= \frac{7}{4} \\ x + \frac{1}{4} &= \pm\frac{7}{4} \\ x &= \frac{-1 \pm 7}{4} \\ x &= -2, \frac{3}{2} \end{aligned}$$

c). Solve using the quadratic formula.

$$a = 2, b = 1, c = -6$$

$$\begin{aligned} x &= \frac{-1 \pm \sqrt{1^2 - 4(2)(-6)}}{2(2)} \\ &= \frac{-1 \pm \sqrt{1 + 48}}{4} \\ &= \frac{-1 \pm \sqrt{49}}{4} \\ &= \frac{-1 \pm 7}{4} \\ &= -2, \frac{3}{2} \end{aligned}$$

Note that the discriminant was a perfect square (not 0), so we get two rational roots.

d). Show how the answers from the quadratic formula can be used to come up with a factoring of the LHS of the original equation.

$$\left(x - \frac{3}{2}\right)(x - (-2)) = 0$$

$$\left(x - \frac{3}{2}\right)(x + 2) = 0$$

Note that this doesn't look quite the same since there is a missing factor of 2, so multiply both sides:

$$\begin{aligned} 2\left(x - \frac{3}{2}\right)(x + 2) &= 2 \cdot 0 \\ \left(2x - 2 \cdot \frac{3}{2}\right)(x + 2) &= 0 \\ (2x - 3)(x + 2) &= 0 \end{aligned}$$