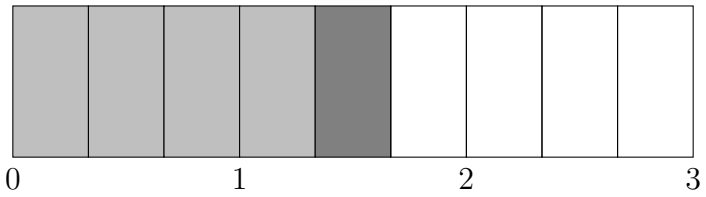
Math-1005a Homework #4

Problems

1). When we add two rational numbers that already have a common denominator then we are simply counting the number of fractional units:

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

What is the rational number equation that is suggested by the following diagram? Your answers should be in the form of (improper) fractions and not mixed numbers. You do not need to reduce your answers:



2). When adding and subtracting rational numbers (and fractions in general) that have different denominators we need to find a common denominator. One can always use the formula:

$$\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$$

but this can result in unnecessarily large values when adding and subtracting rational numbers. A better way is to use the least common multiple (LCM) of the denominators.

Evaluate the following expressions. Your answers should be reduced (improper) fractions or whole numbers - not mixed numbers:

a). $\frac{7}{13} + \frac{11}{2}$

b). $\frac{13}{14} - \frac{1}{21}$

c). $\frac{1}{2} + \frac{3}{4}$

d). $\frac{5}{121} - \frac{5}{22}$

e). $10 + \frac{2}{3}$

3). When adding and subtracting mixed numbers, we have two options. The first is to convert to improper fractions first. Alternatively, remember that mixed number notation is a shorthand for the results of the division algorithm:

$$n\frac{p}{q} = n + \frac{p}{q}$$

Thus, unlike multiplication, it is possible to add/subtract the two parts separately and then adjust so that the fractional part is proper. For example:

$$1\frac{1}{2} + 2\frac{3}{4} = 1 + 2 + \frac{1}{2} + \frac{3}{4} = 3\frac{5}{4} = 4\frac{1}{4}$$

Be careful when subtraction is involved - the negative sign applies to both the whole part and the fractional part:

$$-1\frac{1}{2} = -1 - \frac{1}{2}$$

For example:

$$1\frac{3}{4} - 2\frac{1}{2} = 1 - 2 + \frac{3}{4} - \frac{1}{2} = -1 + \frac{1}{4} = -\frac{3}{4}$$

Evaluate the following expressions. Your answers should be reduced (improper) fractions, whole numbers, or mixed numbers:

a). $4\frac{1}{2} + 5\frac{1}{3}$

b). $16\frac{7}{15} - 7\frac{2}{5}$

c). $3\frac{1}{5} + 1\frac{2}{3}$

d). $17\frac{3}{4} - 16\frac{5}{6}$

e). $7\frac{4}{9} + 3$

4). Sometimes it is easy to tell when one rational number is less than, greater than, or equal to another rational number. When the two numbers have the same denominator then you only need to compare the numerators. For example:

$$\frac{1}{4} < \frac{3}{4}$$

Sometimes the relative magnitude of the two numbers is obvious:

$$\frac{100}{9} > \frac{2}{3}$$

A fraction increases with its numerator and decreases with its denominator. Thus:

$$\frac{1}{3} < \frac{2}{3} \text{ and } \frac{1}{3} > \frac{1}{4}$$

Furthermore, negative numbers are always less that positive numbers:

$$-\frac{3}{4} < \frac{1}{4}$$

In all other cases, we need to find a common denominator and then compare the numerators.

For each of the following, indicate whether the first number is less than, equal to, or greater than the second number:

a). $\frac{8}{3} \square \frac{11}{3}$

b). $\frac{16}{5} \square \frac{48}{15}$

c). $\frac{4}{15} \square -\frac{4}{15}$

d). $\frac{2}{3} \square \frac{5}{6}$

e). $-\frac{2}{3} \square -\frac{5}{6}$

5). When adding and subtracting three or more rational numbers (and fractions in general), we need to find the least common denominator for all of the denominators. For example:

$$\frac{1}{2} + \frac{2}{3} - \frac{1}{4} = \frac{6}{12} + \frac{8}{12} - \frac{3}{12} = \frac{11}{12}$$

Evaluate the following expressions. Your answers should be reduced (improper) fractions or whole numbers - not mixed numbers:

a). $\frac{2}{3} + \frac{4}{3} + \frac{5}{3}$

b). $\frac{11}{5} + \frac{7}{5} - \frac{6}{5}$

c). $\frac{1}{8} - \frac{3}{4} - \frac{1}{3}$

d). $\frac{15}{7} - \frac{3}{4} + \frac{5}{2}$

e). $\frac{1}{3} + \frac{5}{2} + \frac{7}{5}$

6). Adrian and Stacey try to see who can run the farthest without stopping. Adrian is able to run $3\frac{1}{3}$ miles and Stacey is able to run $1\frac{5}{6}$ miles.

a). What is there combined mileage?

b). How much farther did Adrian run that Stacey?

Your answers can be reduced (improper) fractions, whole numbers, or mixed numbers.

7). After such a good workout, Adrian and Stacey go to the state fair. The fair is running a special: buy a souvenir bag for unlimited refills of popcorn. The friends decide to buy one and share it. At the end of the day, they realize that together they have eaten 8 bags of popcorn. Adrian estimates that she alone at $5\frac{1}{4}$ bags.

a). How much popcorn did Stacey eat?

b). How much more popcorn did Adrian eat than Stacey?

Your answers can be reduced (improper) fractions, whole numbers, or mixed numbers.

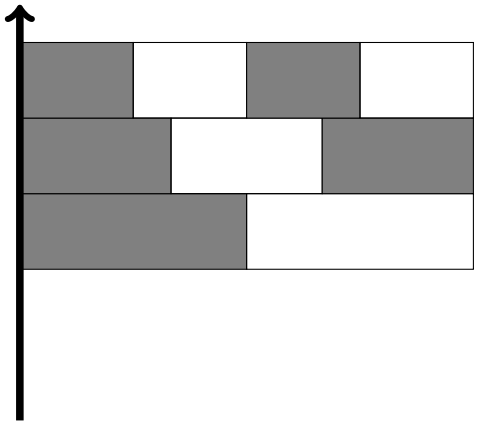
8). Jorge, Roberto, and Jose decide to eat lunch at Mountain Mikes all-you-can-eat pizza buffet. Jorge eats 6 slices, Roberto eats 8 slices, and Jose eats 9 slices. A large pizza normally has 12 slices. How many large pizzas did the three friends eat?

Your answer can be a reduced (improper) fraction, whole number, or mixed number.

9). Elizabeth owns a graphics design business with two employees: Jack and Joe. The company has just completed and been paid for a logo design project for a major Silicon Valley company. Elizabeth keeps half of the money for herself and the company. Of the remaining half, she gives $\frac{4}{5}$ to Jack. How much did Jack and Joe receive of the total?

Remember to use multiplication to find the fraction of a fraction. Your answers should be reduced fractions.

10). The flag of Westeros (shown below) is divided into three stripes:



Each stripe represents a region of the country, and is divided into an equal number of rectangles, where each rectangle represents a county in that region. The counties where manufacturing is predominant are colored gray. The remaining counties are rural/agricultural and are colored white.

How much of Westeros is devoted to manufacturing?

Remember to use multiplication to find the fraction of a fraction. Your answer should be a reduced fraction.