Math-08 Homework #15 Solutions

Reading

- Text book section 5.1-5.3

Problems

1). Consider the following system of equations:

$$x^2 + y^2 - 4x - 6y - 12 = 0$$

$$x - y + 6 = 0$$

a). Determine the points of intersection (if any).

We use the substitution method. First, solve the second equation for y:

$$y = x + 6$$

Now plug into the first equation and solve for x:

$$\begin{aligned} x^2 + (x+6)^2 - 4x - 6(x+6) - 12 &= 0 \\ x^2 + x^2 + 12x + 36 - 4x - 6x - 36 - 12 &= 0 \\ 2x^2 + 2x - 12 &= 0 \\ x^2 + x - 6 &= 0 \\ (x+3)(x-2) &= 0 \\ x &= -3, 2 \end{aligned}$$

Now plugging the found x values into the second equation for solve for y:

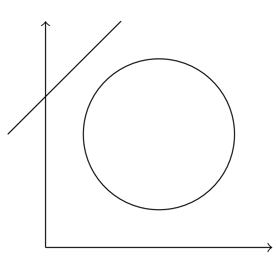
$$y = -3 + 6 = 3$$

$$y = 2 + 6 = 8$$

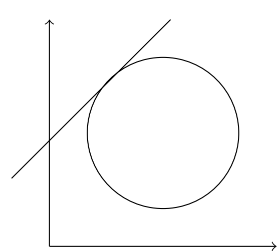
So, the points of intersection are: $(-3, 3)$ and $(2, 8)$

b). Give a geometric description of the problem - what are the geometric figures involved, what are the possibilities of intersection (there are three), and which of those possibilities is represented by the problem.

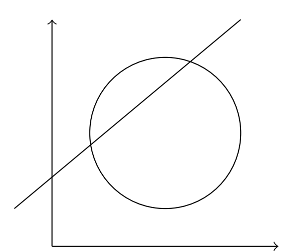
This system represents a line intersecting a circle. There are three possible ways that this can happen:



No intersection



1 point



2 points

Note that since the circle is not linear, we are not bound by the $0, 1, \infty$ solutions rule. Our particular case is the third one - intersection at two points.

2). Solve the following system of linear equations:

$$3x + 3y + 5z = 1$$

$$3x + 5y + 9z = 0$$

$$5x + 9y + 17z = 0$$

You may use row operations directly on the equations, or use a matrix. For each step, clearly indicate the row operation used and state of the equations/matrix after the row operation.

$$\left[\begin{array}{ccc|c} 3 & 3 & 5 & 1 \\ 3 & 5 & 9 & 0 \\ 5 & 9 & 17 & 0 \end{array}\right]$$

$-R_1 + R_2 \to R_2$

$$\left[\begin{array}{ccc|c} 3 & 3 & 5 & 1 \\ 0 & 2 & 4 & -1 \\ 5 & 9 & 17 & 0 \end{array}\right]$$

$\frac{1}{3}R_1 \to R_1$
$\frac{1}{2}R_2 \to R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & \frac{5}{3} & \frac{1}{3} \\ 0 & 1 & 2 & -\frac{1}{2} \\ 5 & 9 & 17 & 0 \end{array}\right]$$

$-5R_1 + R_3 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & \frac{5}{3} & \frac{1}{3} \\ 0 & 1 & 2 & -\frac{1}{2} \\ 0 & 4 & \frac{26}{3} & -\frac{5}{3} \end{array}\right]$$

$-4R_2 + R_3 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & \frac{5}{3} & \frac{1}{3} \\ 0 & 1 & 2 & -\frac{1}{2} \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} \end{array}\right]$$

$\frac{3}{2}R_3 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & \frac{5}{3} & \frac{1}{3} \\ 0 & 1 & 2 & -\frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \end{array}\right]$$

We can now back-solve:

$$\begin{aligned} x + y + \frac{5}{3}z &= \frac{1}{3} \\ y + 2z &= -\frac{1}{2} \\ z &= \frac{1}{2} \end{aligned}$$

$$z = \frac{1}{2}$$

$$\begin{aligned} y + 2(\frac{1}{2}) &= -\frac{1}{2} \\ y + 1 &= -\frac{1}{2} \\ y &= -\frac{3}{2} \end{aligned}$$

$$\begin{aligned} x + (-\frac{3}{2}) + \frac{5}{3}(\frac{1}{2}) &= \frac{1}{3} \\ x - \frac{3}{2} + \frac{5}{6} &= \frac{1}{3} \\ x - \frac{2}{3} &= \frac{1}{3} \\ x &= 1 \end{aligned}$$

Solution: $\left(1, -\frac{3}{2}, \frac{1}{2}\right)$