Math-19 Section 1

Homework #6 Solutions

Problems

Consider the transformed function:

$$y = 2f\left(-\frac{1}{3}(x-1)\right) + 1$$

For each of the following choices of $f(x)$, determine the final coordinates of the key point, position of all asymptotes, and any x and y intercepts and then sketch the final graph.

Here are the transformations in the order that they are applied and how the affect the key point:

TRANSFORMATION	e^x	$\ln(x)$
Start with basic function	$(0,1)$	$(1,0)$
Right 1	$(1,1)$	$(2,0)$
H scale $\frac{1}{3}$	$(1,1)$	$(4,0)$
H reflect	$(1,1)$	$(-2,0)$
V scale 2	$(1,2)$	$(-2,0)$
Up 1	$(1,3)$	$(-2,1)$

1. $f(x) = e^x$

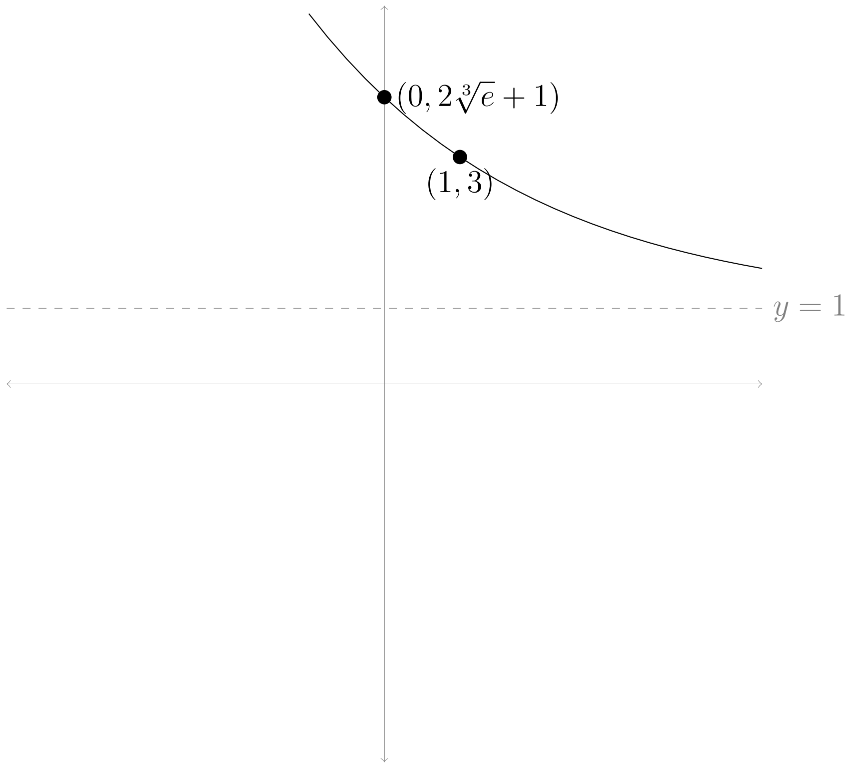
   The vertical translation moves the HA up 1. The x-intercept is found as follows:

   $$0 = 2e^{-\frac{1}{3}(x-1)} + 1$$
   $$e^{-\frac{1}{3}(x-1)} = -\frac{1}{2}$$

   But the exponential is always greater than 0, as so there is no x-intercept. For the y-intercept:

   $$y = 2e^{-\frac{1}{3}(0-1)} + 1 = 2e^{\frac{1}{3}} + 1 = 3.791$$

   So the y-intercept is at $(0, 3.791)$.



2. $f(x) = \ln(x)$

   The horizontal translation move the VA right 1. This causes the key point, which is at $(2, 0)$ after the horizontal translation, to be scale 3 times away from the VA. Thus, the distance of 1 is scaled to 3 and the key point moves to $(4, 0)$. The horizontal reflection moves it 3 to the left of the VA to -2. We can test that we have the correct x-coordinate of the final key point by plugging it in to the horizontal transformations and making sure that we get back to the original value of 1:

$$-\frac{1}{3}(-2-1) = -\frac{1}{3}(-3) = 1$$

   The x-intercept is found as follows:

$$\begin{aligned}
&0 = 2\ln\left[-\frac{1}{3}(x-1)\right] + 1 \\
&\ln\left[-\frac{1}{3}(x-1)\right] = -\frac{1}{2} \\
&-\frac{1}{3}(x-1) = e^{-\frac{1}{2}} \\
&x - 1 = -3e^{-\frac{1}{2}} \\
&x = 1 - \frac{3}{\sqrt{e}} \\
&x = -0.820
\end{aligned}$$

And so the x-intercept is at $(-0.820, 0)$. For the y-intercept:

$$y = 2\ln\left[-\frac{1}{3}(0-1)\right] + 1 = 2\ln\left(\frac{1}{3}\right) + 1 = 1 - 2\ln(3) = -1.197$$

So the y-intercept is at $(0, -1.197)$.