Introduction

There are some problems that algebra alone cannot solve. A new principle is needed in order to solve these harder problems.

Arbitrarily Large

Infinity (∞) is not an actual number, but instead is indicative of a process:

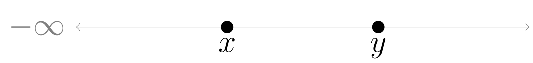
1. Select a positive number.
2. Now select a next number that is larger than the previous number.
3. Go to 2.

This is possible because the real numbers are unbounded: for every $y \in \mathbb{R}$ there exists some $x \in \mathbb{R}$ such that $x > y$.

Definition: Arbitrarily Large

To say that a value $x \in \mathbb{R}$ is *arbitrarily large*, denoted by $x \to \infty$, means that for every $y \in \mathbb{R}$, $x > y$.

This also works in the negative direction. For $x \to -\infty$, select a negative number and then continually select numbers that are less than the previous number. In other words, for every $y \in \mathbb{R}, x < y$.



Arbitrarily Small

A number can also be said to be arbitrarily small. Like infinity, this is not an actual number, but is indicative of a process:

1. Select a positive number.
2. Now select a next positive number that is smaller than the previous number.
3. Go to 2.

This is possible because between any two real numbers there are an infinite number of real numbers. Thus, for any value $y > 0$ there exists some x such that $0 < x < y$.

Definition: Arbitrarily Small

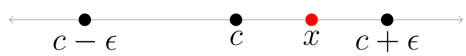
To say that a value $x \in \mathbb{R}^+$ is *arbitrarily small*, denoted by $x \to 0^+$, means that for every $y \in \mathbb{R}^+, 0 < x < y$.

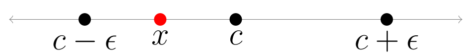
The Greek letters epsilon (ϵ) and delta (δ) are typically used to represent arbitrarily small values.

Arbitrarily Close

Definition: Arbitrarily Close

To say that a value $x \in \mathbb{R}$ is *arbitrarily close* to another value $c \in \mathbb{R}$, denoted by $x \to c$, means that for all $\epsilon > 0, |x - c| < \epsilon$. In other words: $c - \epsilon < x < c + \epsilon$.





Problems

Slope

Area

Sequences and Series