Math-13 Sections 01 and 02

Homework #10 Solutions

Consider the function:

$$f(x) = x^{\frac{2}{3}} - x$$

on the closed interval $[0, 8]$.

1. Determine $f'(x)$.

$$f'(x) = \frac{2}{3}x^{-\frac{1}{3}} - 1 = \frac{2}{3\sqrt[3]{x}} - 1 = \frac{2 - 3\sqrt[3]{x}}{3\sqrt[3]{x}}$$

2. Determine the critical points on the interval.

   To find the zeros:

$$\begin{aligned}
&2 - 3\sqrt[3]{x} = 0 \\
&3\sqrt[3]{x} = 2 \\
&\sqrt[3]{x} = \frac{2}{3} \\
&x = \left(\frac{2}{3}\right)^3 = \frac{8}{27}
\end{aligned}$$

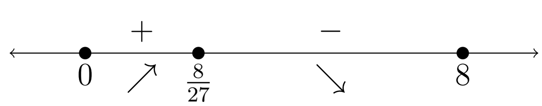
   Therefore there is a zero at $x = \frac{8}{27}$ and a poll at $x = 0$.

3. Calculate $f(x)$ at each endpoint and critical point.

$$\begin{aligned}
f(0) &= 0^{\frac{2}{3}} - 0 = 0 \\
f\left(\frac{8}{27}\right) &= \left(\frac{8}{27}\right)^{\frac{2}{3}} - \frac{8}{27} = \frac{4}{9} - \frac{8}{27} = \frac{4}{27} \\
f(8) &= 8^{\frac{2}{3}} - 8 = 4 - 8 = -4
\end{aligned}$$

   Therefore, the endpoints are $(0, 0)$ and $(8, -4)$ and there is one critical point at $\left(\frac{8}{27}, \frac{4}{27}\right)$.

4. Determine where $f(x)$ is increasing and decreasing over the interval. You must prove your result by evaluating the derivative at proper test points. Summarize this information with a real number graph.



$$f'\left(\frac{1}{27}\right) = \frac{2}{3}\left(\frac{1}{27}\right)^{-\frac{1}{3}} - 1 = \frac{2}{3}(3) - 1 = 2 - 1 = 1 > 0$$
$$f'(1) = \frac{2}{3}(1)^{-\frac{1}{3}} - 1 = \frac{2}{3} - 1 = -\frac{1}{3} < 0$$

5. Classify each endpoint and derivative critical point as either a relative or absolute minimum or maximum or point of inflection.

   Since $f(x)$ is increasing on $(0, \frac{8}{27})$ and decreasing on $(\frac{8}{27}, 8)$, the critical point at $(\frac{8}{27}, \frac{4}{27})$ is a relative maximum. Based on the function values, we have the following:

point	*rmin*	*rmax*	*amin*	*amax*	*poi*
$(0,0)$	✓				
$(\frac{8}{27}, \frac{4}{27})$		✓		✓	
$(8,-4)$	✓		✓		

6. Sketch the graph on the interval. Be very specific near $x = 0$.