Common Graph Classes

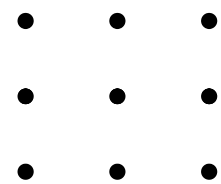
1. Empty (E_n)

$$\begin{aligned}
&V(E_n) = \{1, \ldots, n\} \\
&E(E_n) = \emptyset \\
&|V(E_n)| = n \\
&|E(E_n)| = 0 \\
&E_n \text{ is connected} \iff n = 1
\end{aligned}$$

Examples

E_1

E_4



E_9

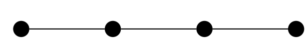
The null graph is represented by E_0.

2. Path (P_n)

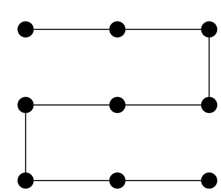
$$\begin{aligned}
&V(P_n) = \{1, \ldots, n\} \\
&E(P_n) = \{\{1, 2\}, \{2, 3\}, \ldots, \{n-1, n\}\} \\
&|V(P_n)| = n \\
&|E(P_n)| = n - 1 \\
&P_n \text{ is connected} \\
&\operatorname{diam}(P_n) = n - 1
\end{aligned}$$

Examples

P_1



P_4

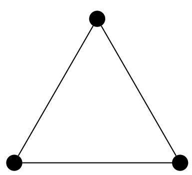


P_9

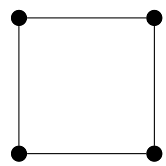
3. Cycle (C_n)

$$
\begin{aligned}
&V(C_n) = \{1, \dots, n\} \\
&E(C_n) = \{\{1,2\}, \{2,3\}, \dots, \{n-1, n\}, \{n,1\}\} \\
&|V(C_n)| = n \geq 3 \\
&|E(C_n)| = n \geq 3 \\
&C_n \text{ is connected} \\
&\operatorname{diam}(P_n) = \left\lfloor \frac{n}{2} \right\rfloor
\end{aligned}
$$

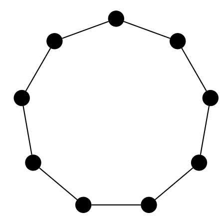
Examples



C_3



C_4



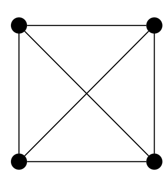
C_9

4. Complete (K_n)

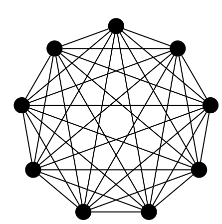
$$
\begin{aligned}
&V(K_n) = \{1, \dots, n\} \\
&E(K_n) = \mathcal{P}_2\left(V(K_n)\right) \\
&|V(K_n)| = n \\
&|E(K_n)| = \binom{n}{2} = \frac{n(n-1)}{2} \\
&K_n \text{ is connected} \\
&\operatorname{diam}(P_n) = 1 \iff G = K_n
\end{aligned}
$$

Examples

K_1



K_4

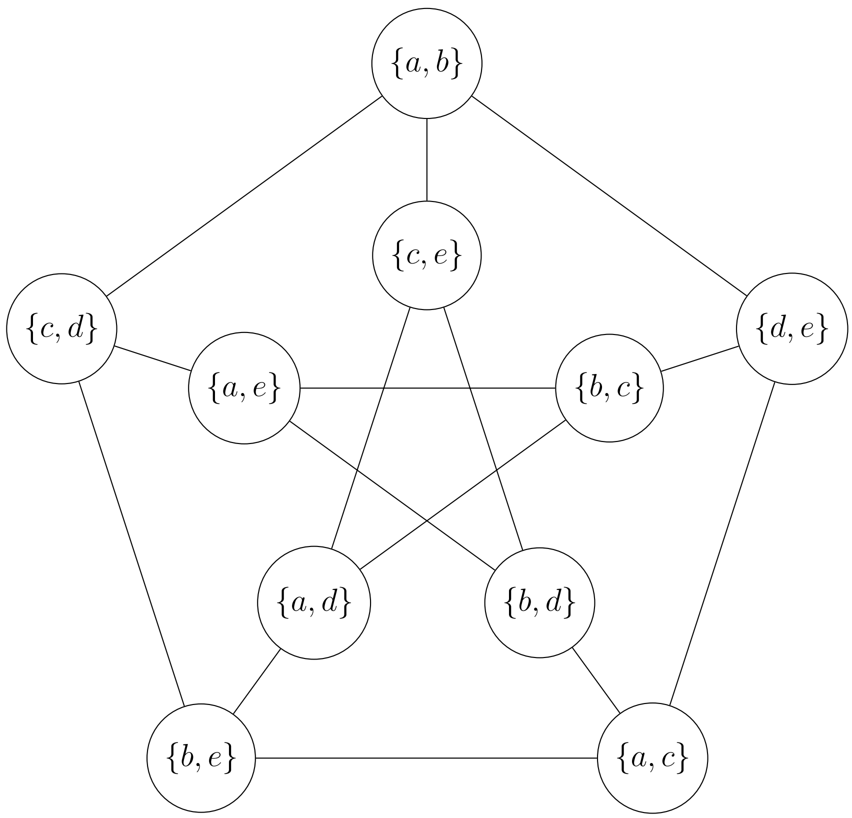


K_9

5. Petersen Graph (PG)

$$|V(PG)| = 10$$
$$|E(PG)| = 10$$
PG is connected

The vertices represents the 2-subsets of a 5 element set and the edges indicate disjoint sets.

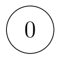


6. Cube Graph (Q_n)

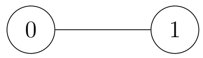
$$|V(PG)| = 2^n$$
$$|E(PG)| = n2^n$$
Q_n is connected

The vertices represents the bit strings of length n and the edges indicate a one bit difference.

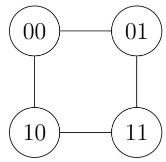
Examples


0

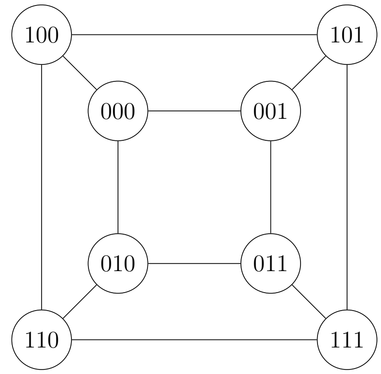
Q_0


0
1

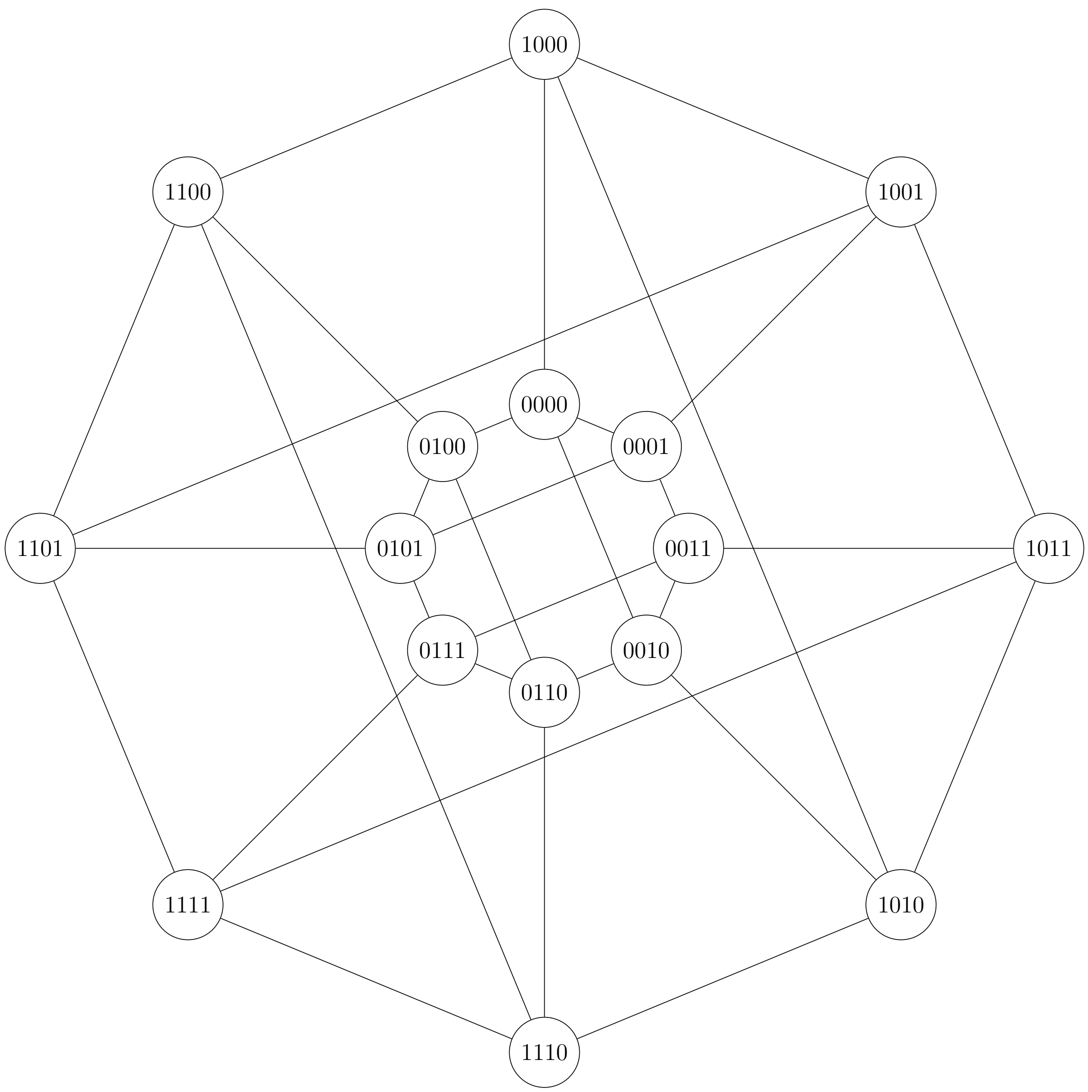
Q_1


00
01
10
11

Q_2


100
101
000
001
010
011
110
111

Q_3


1000
1100
1001
0000
0100
0001
1101
0101
0011
1011
0111
0010
0110
1111
1010
1110

Q_4