Math-42 Sections 01, 02, 05

Homework #8 Solutions

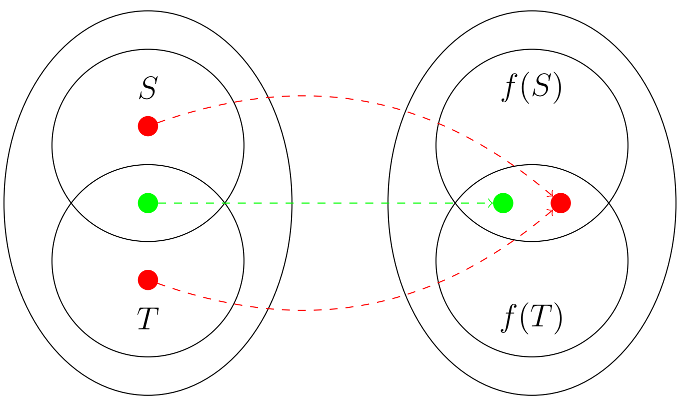
Problems

Consider $f : A \to B$ and let $S, T \subseteq A$.

1. Prove: $f(S \cap T) \subseteq f(S) \cap f(T)$

   Assume $y \in f(S \cap T)$
   $\exists x \in S \cap T, y = f(x)$
   $x \in S, y = f(x)$ and $x \in T, y = f(x)$
   $y \in f(S)$ and $y \in f(T)$
   $y \in f(S) \cap f(T)$
   $\therefore f(S \cap T) \subseteq f(S) \cap f(T)$

2. Draw a diagram that shows why this is a subset relationship and not set equality. In other words, show why there can be elements in $f(S) \cap f(T)$ that are not in $f(S \cap T)$.



3. How can f be limited so that equality occurs. In other words, how do you eliminate the problem in your drawing?

   Require that f be injective. Thus, the problem case in the diagram does not occur.

4. Which step in your proof is not reversible?

   The transition shown in red is the problem. Going in the forward direction, we are talking about the same x. Going in the reverse direction, there could be two separate x values.