Interest Problems

What is Interest?

Interest is money that is paid on the outstanding (unpaid) portion of a loan over time. For every loan there is a *borrower*, the person who borrows the money and is responsible for paying the money back with interest, and the *lender*, the person that loans the money and receives the payments and interest.

Important Values

Every loan has the following values:

Principal: The original loan amount, as well as any unpaid balance over the course of the loan.

Rate A percentage of the principal that is paid by the borrower to the lender for the privilege of using the loaned funds. Rates are usually stated as a particular percentage over time - e.g., 5% per year. Loans can have either a *fixed* interest rate, one that stays the same over the life of the loan, or a *variable* interest rate, one that can adjust over time. For all of our problems, we will deal with fixed interest rates only.

Time The amount of time that there is unpaid principal. Some loans have a set time period - all of the principal must be repaid within that period. Other loans are open-ended and allow the borrow to make payments as long as there is unpaid principal.

Loan Types

The first part of any problem is to determine who is the borrower and who is the lender. Here are a couple of common scenarios:

Savings Account Although a savings account may not seem like loan, it really is one: the saver is lending money to the bank and the bank in turn pays interest for the privilege of using the money for its own investments. This is a special type of loan, since the lender (saver) can demand full repayment (i.e., a total withdrawal) at any time.

Mortgage Since house prices are so high, people tend to need help purchasing a home. The type of loan used to buy a house is called a mortgage. In this case, the lender is the bank and the borrower is the home buyer. Mortgages tend to have a monthly repayment plan. Sometimes, this monthly payment plan doesn't cover the full payment of the loan, leaving a large last payment, typically called a *balloon* payment. One special feature of a mortgage is that they can be renegotiated so that a borrower can add more principle to the loan for things like home improvements (e.g., a bathroom or kitchen remodel).

Credit Card Everyone is probably familiar with credit cards; they have become such an integral part of our everyday lives. With a credit card, the bank is the lender and the credit card holder is the borrower. Every time the card holder purchases something with the card, new principal is added to the account. Every time the card hold makes a payment to the bank, the principal is reduced. Payments are usually monthly and interest is paid on any unpaid amount past the statement due date.

Home Equity Line Some borrowers take out a second mortgage (on top of the first) that acts like a credit card and is used primarily for home improvements. Such loans are called home equity lines, and are a simpler alternative to first mortgage renegotiations.

Corporate and Government Bonds Corporations and governments issue bonds to raise money for expansion or special projects. Government bonds can be issued by cities, counties, states, the federal government, and foreign governments. The corporation or government is the borrower and the bond buyer is the lender. There are all types of bonds, but a popular type is one that pays interest only over the life of the loan, followed by a balloon payment of the principal at the end of the loan.

Interest Models

There are two types of interest: simple and compounded.

Simple Interest

Simple interest is paid on the *principle only* and follows the equation:

$$A = P\left(1 + rt\right)$$

where r is the annual interest rate, t is the number of years, P is the outstanding principal, and A is the total amount (value) of the loan.

Compounded Interest

Compounded interest is paid on the *principal* plus all unpaid, previously accrued (earned) *interest* and follows the equation:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

The value n is the number of times interest is paid over the period of the interest rate. Thus, the $\frac{r}{n}$ represents the interest rate per compoundin period and the exponent nt represents the number of compounding periods. Since the period if most interest rates in one year, examples of compounding periods are:

daily	365
weekly	52
monthly	12
quarterly	4
semi-annually	2
yearly	1