Math-42 Sections 01, 02, 05

Exam #1

Name: ____

This exam is closed book and notes. You may use a calculator; however, no other electronics are allowed. A cheatsheet with a table of logical equivalences is provided on the last page. Show all work; there is no credit for guessed answers.

1. Prove the following logical equivalence using a truth table. Be sure to show all intermediate steps and the final tautology.

$$p \leftrightarrow q \equiv p \wedge q \vee \neg p \wedge \neg q$$

2. You are in the land of knights and knaves. Remember that knights always tell the truth (i.e., make a true statement) and knaves always lie (i.e., make a false statement). You meet two men: A and B. A says, "If I am a knight then he is a knave." B says, "He is a knave." Determine whether each person is either a knight or a knave. Be sure to explain why.

3. Prove using logical equivalences in a step by step fashion (i.e., do not skip steps) and justify each step:

$$(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$$

You are only allowed to use the equivalence of the implication and the rules stated in the cheatsheet on the last page.

4. Consider the eight types of rules of inference that we studied:

   (a) Modus Ponens

   (b) Modus Tollens

   (c) Hypothetical Syllogism

   (d) Disjunctive Syllogism

   (e) Addition

   (f) Simplification

   (g) Conjunction

   (h) Resolution

   Identify the rule of inference used in each of the following arguments:

   ____ $n \in \mathbb{N}$ or $n < 0$. $n \notin \mathbb{N}$ or $n = 1$. Therefore $n < 0$ or $n = 1$.

   ____ If n is odd then n^2 is odd. n is odd. Therefore n^2 is odd.

   ____ $n \in \mathbb{N}$ and $n \in \mathbb{Z}$. Therefore $n \in \mathbb{Z}$.

   ____ $a < x$ and $x < b$. Therefore $a < x < b$.

   ____ If $n < n^2$ then $n \neq 1$. If $n \neq 1$ then $n + 5 \neq 6$. Therefore if $n < n^2$ then $n + 5 \neq 6$.

   ____ If n is even then n^2 is even. n^2 is odd. Therefore n is odd.

   ____ $a \leq b$. $a \neq b$. Therefore $a < b$.

   ____ $a < b$. Therefore $a \leq b$.

5. To say that a function $f(x)$ is continuous at a point $x = a$ means:

$$\forall \epsilon > 0, \exists \delta > 0, \forall x \in \mathbb{R}, |x - a| < \delta \to |f(x) - f(a)| < \epsilon$$

   State the definition that says a function $f(x)$ is discontinuous (i.e., not continuous) at a point $x = a$.

6. State the following definitions:

   (a) n is an even integer.

   (b) n is an odd integer.

7. State the definition for $x \in \mathbb{Q}$.

8. Prove by direct proof:

$$\forall\, n, m \in \mathbb{Z}, (n, m \text{ odd} \to nm \text{ odd})$$

9. Prove:

$$\forall\, n, m \in \mathbb{Z}, (nm \text{ even} \to n \text{ even or } m \text{ even})$$

10. Prove:

    The set of rational numbers is closed under multiplication.

Logical Equivalences:

EQUIVALENCE	NAME
$p \wedge T \equiv p$ $p \vee F \equiv p$	Identity
$p \vee T \equiv T$ $p \wedge F \equiv F$	Domination
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent
$\neg(\neg p) \equiv p$	Double Negation
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	DeMorgan
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption
$p \vee \neg p = T$ $p \wedge \neg p = F$	Negation