Math-19 Homework #2 Solutions

Reading

Please read sections 1.1 through 1.5 and 1.7 and then do all concept problems in the posted sections on webassign.

Problems

1). Simplify completely. Your answer should have no negative exponents and please rationalize the denominator. Don't worry if the exponents get messy.

$$\frac{\sqrt[4]{\sqrt{75}+\sqrt{27}}}{\sqrt{4\sqrt{20}\sqrt[3]{54}}}$$

This is really messy, so you need to take it step by step. Let's work with the numerator first:

$$\begin{aligned}
\sqrt[4]{\sqrt{75}+\sqrt{27}} &= (\sqrt{75}+\sqrt{27})^{1/4} \\
&= (\sqrt{25\cdot 3}+\sqrt{9\cdot 3})^{1/4} \\
&= (5\sqrt{3}+3\sqrt{3})^{1/4} \\
&= (8\sqrt{3})^{1/4} \\
&= (2^3 3^{1/2})^{1/4} \\
&= 2^{3/4}3^{1/8}
\end{aligned}$$

and now the denominator:

$$\begin{aligned}
\sqrt{4\sqrt{20}\sqrt[3]{54}} &= (4\sqrt{20}\sqrt[3]{54})^{1/2} \\
&= (2^2\sqrt{4\cdot 5}\sqrt[3]{27\cdot 2})^{1/2} \\
&= (2^2\cdot 2\sqrt{5}\cdot 3\sqrt[3]{2})^{1/2} \\
&= (2^3\sqrt{5}\cdot 3\sqrt[3]{2})^{1/2} \\
&= (2^3\cdot 5^{1/2}\cdot 3\cdot 2^{1/3})^{1/2} \\
&= (2^{10/3}3^1 5^{1/2})^{1/2} \\
&= 2^{5/3}3^{1/2}5^{1/4}
\end{aligned}$$

Finally, putting it all together and rationalizing:

$$\begin{aligned}
\frac{\sqrt[4]{\sqrt{75}+\sqrt{27}}}{\sqrt{4\sqrt{20}\sqrt[3]{54}}} &= \frac{2^{3/4}3^{1/8}}{2^{5/3}3^{1/2}5^{1/4}} \\
&= 2^{(3/4-5/3)}3^{(1/8-1/2)}5^{(0-1/4)} \\
&= 2^{-11/12}3^{-3/8}5^{-1/4} \\
&= \frac{1}{2^{11/12}3^{3/8}5^{1/4}} \\
&= \left(\frac{1}{2^{11/12}3^{3/8}5^{1/4}}\right)\left(\frac{2^{1/12}3^{5/8}5^{3/4}}{2^{1/12}3^{5/8}5^{3/4}}\right) \\
&= \frac{2^{1/12}3^{5/8}5^{3/4}}{2\cdot 3\cdot 5} \\
&= \frac{2^{1/12}3^{5/8}5^{3/4}}{30}
\end{aligned}$$

2). A student writes the following statements. Determine if each is either correct or incorrect (or misleading). Explain why incorrect statements are incorrect.

a). $\sqrt{9} = \pm 3$

Incorrect. $\sqrt{9}$ is asking for the principle root: $+3$.

b). $\left(x^{\frac{1}{2}}\right)^2 = |x|$

Misleading. $x^{1/2}$ implies that $x >= 0$, so the absolute value is extraneous.

c). $(x^2)^{\frac{1}{2}} = x$

Very incorrect! $(x^2)^{\frac{1}{2}}$ is always ≥ 0, so we need $|x|$ here, just in case $x < 0$!

d). $(x^3)^{\frac{1}{3}} = |x|$

Incorrect. We don't use the absolute value with odd roots. Plug in $x = -1$ and see why this is wrong.

3). Solve for x by *completing the square*.

$$2x^2 + 4x - 3 = 0$$

$$\begin{aligned} 2x^2 + 4x - 3 &= 0 \\ 2x^2 + 4x &= 3 \\ x^2 + 2x &= \frac{3}{2} \\ x^2 + 2x + 1 &= \frac{3}{2} + 1 \\ (x+1)^2 &= \frac{5}{2} \\ x + 1 &= \pm\sqrt{\frac{5}{2}} \\ x &= -1 \pm \sqrt{\frac{5}{2}} \end{aligned}$$

4). A man stands atop a $256ft$ cliff with a ball.

a). How long does it take for the ball to hit the ground if he simply releases the ball?

$$\begin{aligned} 256 + 0t - 16t^2 &= 0 \\ 16(16 - t^2) &= 0 \\ 16 - t^2 &= 0 \\ (4+t)(4-t) &= 0 \end{aligned}$$

This yields two solutions: $t = \pm 4$ seconds. We take the positive solution here: $t = 4$ seconds.

b). How long does it take for the ball to hit the ground if he throws the ball up with a velocity of $16ft/s$?

$$\begin{aligned} 256 + 16t - 16t^2 &= 0 \\ -16(t^2 - t - 16) &= 0 \\ t^2 - t - 16 &= 0 \end{aligned}$$

$$\begin{aligned} t &= \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-16)}}{2(1)} \\ &= \frac{1 \pm \sqrt{1 + 64}}{2} \\ &= \frac{1 \pm \sqrt{65}}{2} \end{aligned}$$

This yields two solutions: $t \approx -3.5$ seconds and $t \approx 4.5$ seconds. We take the positive solution here: $t \approx 4.5$; however, we save the negative solution for later.

c). How long does it take for the ball to hit the ground if he throws the ball down with a velocity of $16ft/s$? (Hint: no additional calculations are needed).

Note that in the previous problem the man threw the ball up at $+16ft/s$. The ball is going to travel up, slow down due to gravity, eventually stop, and then start falling. When the ball passes the man again it must be going at $-16ft/s$. Thus, the two problems are the same! Furthermore, the negative solution from the previous problem is the answer here: $t \approx 3.5$ seconds.

d). Assume that a lady is standing on the ground below the cliff and throws a ball up so that it passed the man on the cliff at a velocity of $16ft/s$. How long would it be before the ball hits the ground? (Hint: you already have all the information that you need).

Once again, this is the *same* problem. The roundtrip time is the sum of the previous two times: $t \approx 3.5 + 4.5 \approx 8.0$ seconds.

5). Muri is a shopkeeper that specializes in pickled vegetables. She has determined over the years that the best brine (salt solution) for pickling vegetables is 2 kg of salt per liter of water (2 kg/L). One day, she has her not-so-bright nephew helping her and he uses too much salt, resulting in a 5 kg/L solution. If her nephew made up 10 liters of the too-salty solution, how much pure water must he add to it to get the ideal 2 kg/L solution? For full credit, show the mixture equation and the appropriate values for each concentration and volume value in the equation.

$$c_1v_1 + c_2v_2 = c_3(v_1 + v_2)$$

$c_1 =$ initial salt concentration $= 5kg/L$
$v_1 =$ original water volume $= 10L$
$c_2 =$ salt concentration of pure water $= 0kg/L$
$v_2 =$ amount of water to be added $= x$ (unknown)
$c_3 =$ desired salt concentration $= 2kg/L$

$$\begin{aligned} 5(10) + 0x &= 2(10 + x) \\ 50 + 0 &= 20 + 2x \\ 2x &= 30 \\ x &= 15 \end{aligned}$$

So, 15 L of pure water must be added to achieve the desired concentration.