Math-19 Homework #5 Solutions

Reading

Please read sections 3.1 through 3.7, omitting 3.5, and do all concept problems in the posted sections on webassign.

Problems

1). Let $f(x) = 1 - 6x - x^2$.

a). Convert the parabola to standard form.

$$\begin{aligned} f(x) &= 1 - 6x - x^2 \\ &= -(x^2 + 6x) + 1 \\ &= -(x^2 + 6x + 9) + 1 + 9 \\ &= -(x+3)^2 + 10 \end{aligned}$$

b). What are the coordinates of the vertex?

$(-3, 10)$

c). What are the x-intercepts (if any)?

$$\begin{aligned} -(x+3)^2 + 10 &= 0 \\ (x+3)^2 &= 10 \\ x + 3 &= \pm\sqrt{10} \\ x &= -3 \pm \sqrt{10} \end{aligned}$$

$(-3 \pm \sqrt{10}, 0)$

d). What are the y-intercepts (if any)?

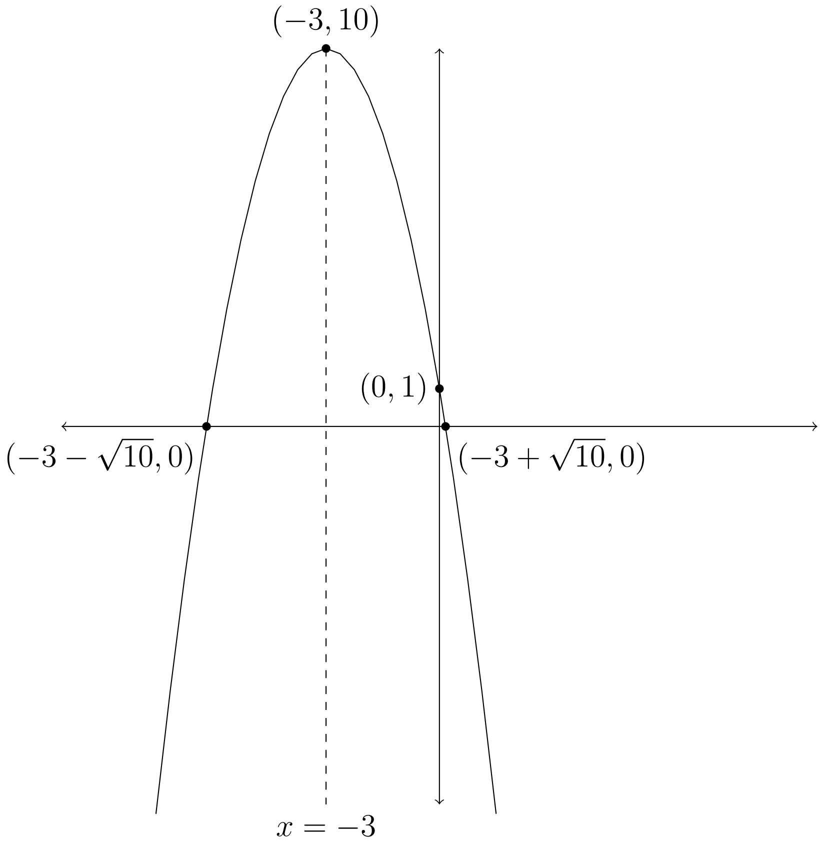
$f(0) = 1 - 6(0) - 0^2 = 1$

$(0, 1)$

e). What is the axis of symmetry?

$x = -3$

f). Sketch the parabola. Be sure to label the vertex and all intercepts!



g). What is the domain?

$\mathbb{R}$

h). What is the range?

$(-\infty, 10]$

2). Let $p(x) = 10 - 49x + 80x^2 - 53x^3 + 20x^4 - 4x^5$. Factor completely. Use the results of the factoring and any y-intercepts to sketch the function. Be sure to label all intercepts!

$p = \pm 1, \pm 2, \pm 5, \pm 10$

$q = \pm 1, \pm 2, \pm 4$

$p(1) = 4 \neq 0$

$p(-1) = 216 \neq 0$

$p(2) = 0$

$$
\begin{array}{r}
-4x^4+12x^3-29x^2+22x-5 \\
x-2\overline{\big)-4x^5+20x^4-53x^3+80x^2-49x+10} \\
\underline{4x^5-8x^4} \\
12x^4-53x^3 \\
\underline{-12x^4+24x^3} \\
-29x^3+80x^2 \\
\underline{29x^3-58x^2} \\
22x^2-49x \\
\underline{-22x^2+44x} \\
-5x+10 \\
\underline{5x-10} \\
0
\end{array}
$$

$$p(x) = (x-2)q(x) = (x-2)(-4x^4+12x^3-29x^2+22x-5)$$

$$p = \pm 1, \pm 5$$

$$q = \pm 1, \pm 2, \pm 4$$

We already know ± 1 doesn't work.

$$q(5) = -1620 \neq 0$$

$$q(-5) = -4840 \neq 0$$

$$q\left(\frac{1}{2}\right) = 0$$

The factor is $\left(x-\frac{1}{2}\right)$; however, we can factor out the $\frac{1}{2}$ and use $(2x-1)$.

$$
\begin{array}{r}
-2x^3+5x^2-12x+5 \\
2x-1\overline{\big)-4x^4+12x^3-29x^2+22x-5} \\
\underline{4x^4-2x^3} \\
10x^3-29x^2 \\
\underline{-10x^3+5x^2} \\
-24x^2+22x \\
\underline{24x^2-12x} \\
10x-5 \\
\underline{-10x+5} \\
0
\end{array}
$$

$$p(x) = (x-2)(2x-1)r(x) = (x-2)(2x-1)(-2x^3+5x^2-12x+5)$$

$$p = \pm 1, \pm 5$$

$$q = \pm 1, \pm 2$$

We already know that ± 1 and ± 5 don't work, but we need to retry $\frac{1}{2}$, just in case it is a repeated root.

$$r\left(\frac{1}{2}\right) = 0$$

$$\begin{array}{r|l} & -x^2 + 2x - 5 \\ \hline 2x-1 & -2x^3 + 5x^2 - 12x + 5 \\ & \underline{2x^3 - x^2} \\ & \quad 4x^2 - 12x \\ & \quad \underline{-4x^2 + 2x} \\ & \qquad -10x + 5 \\ & \qquad \underline{10x - 5} \\ & \qquad\qquad 0 \end{array}$$

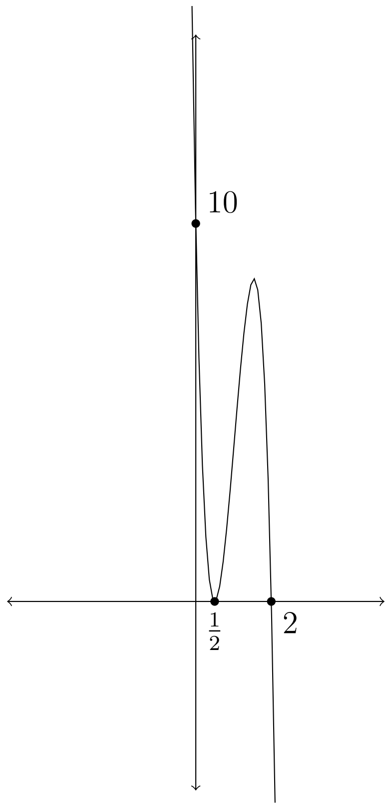
$$p(x) = (x-2)(2x-1)^2(-x^2+2x-5) = -(x-2)(2x-1)^2(x^2-2x+5)$$

Notice that the leading coefficient will be correct:

$$-x \cdot (2x)^2 \cdot x^2 = -4x^5$$

Also note that the final quadratic does not factor in the reals. So, we use our calculator and find that this quadratic doesn't introduce any more humps (see graph for next problem).

So, we have x-intercepts at $(1/2, 0)$ and $(2, 0)$ and a y-intercept at (0,10).



3). For the previous problem, graph the function using your calculator. You will need to play with the window a bit so that you can see the important detail. Determine all local maxima/minima and attach screenshots.

We already know that we have a local minimum at $(1/2, 0)$. For the remaining local maximum, see the graph.

4). Let:

$$f(x) = \frac{(2x^2+2x-4)(x+3)}{x^4+4x^3+3x^2}$$

a). What are the zeros?

$$\begin{aligned} f(x) &= \frac{(2x^2+2x-4)(x+3)}{x^4+4x^3+3x^2} \\ &= \frac{2(x^2+x-2)(x+3)}{x^2(x^2+4x+3)} \\ &= \frac{2(x+2)(x-1)(x+3)}{x^2(x+1)(x+3)} \\ &= \frac{2(x+2)(x-1)}{x^2(x+1)} \qquad x \neq -3 \end{aligned}$$

$(-2, 0), (1, 0)$

b). What are the vertical asymptotes (if any)?

$$x = -1, x = 0$$

c). What are the horizontal asymptotes (if any)?

The numerator is degree=2 and the denominator is degree=3.

$2 < 3$

So, there is a horizontal asymptote at $y = 0$ (the x-axis).

d). What is the end behavior as $x \to \infty$?

$$f(x) \to \frac{2 \cdot + \cdot +}{+ \cdot +} \to +$$

$$f(x) \to 0^+$$

e). What is the end behavior as $x \to -\infty$?

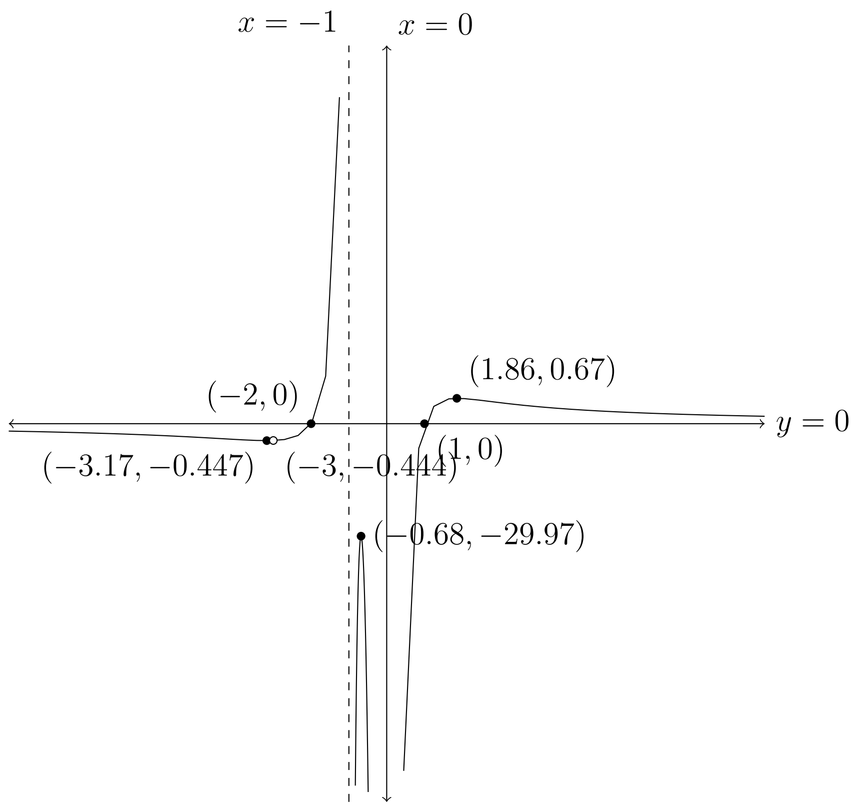
$$f(x) \to \frac{2 \cdot - \cdot -}{+ \cdot -} \to -$$

$$f(x) \to 0^-$$

f). What are the y-intercepts (if any)?

none ($x = 0$ is a vertical asymptote).

g). Sketch the graph. Be sure to label all intercepts, asymptotes, and local extrema and show the proper end behavior. Note that you may need to use a calculator to determine the local extrema.



h). What is the domain?

$$(-\infty, -3) \cup (-3, -1) \cup (-1, 0) \cup (0, \infty)$$

i). What is the range?

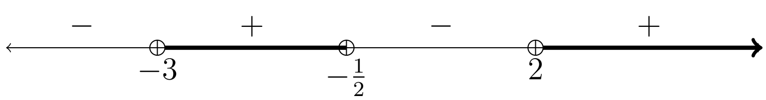
$$\mathbb{R}$$

5). Solve for x:

$$\frac{x+2}{x+3} < \frac{x-1}{x-2}$$

Remember to state the result in interval notation.

$$
\begin{aligned}
\frac{x+2}{x+3} - \frac{x-1}{x-2} &< 0 \\
\frac{x+2}{x+3} - \frac{x-1}{x-2} &< 0 \\
\frac{(x+2)(x-2) - (x+3)(x-1)}{(x+3)(x-2)} &< 0 \\
\frac{(x^2-4) - (x^2+2x-3)}{(x+3)(x-2)} &< 0 \\
\frac{-2x-1}{(x+3)(x-2)} &< 0 \\
\frac{2x+1}{(x+3)(x-2)} &> 0
\end{aligned}
$$



$$(-3, -\frac{1}{2}) \cup (2, \infty)$$