Math-42 Worksheet #16

## Mathematical Induction

1. In a previous exercise we proved the following identity:

   $$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

   By starting with the telescoping sum:

   $$\sum_{k=1}^{n}[(k+1)^3 - k^3]$$

   Proof by induction doesn't yield new results, it just confirms conjectures or things proved using other techniques. Use proof by induction to confirm this identity.

2. What is wrong with this proof by induction used to supposedly prove that for all $n \in \mathbb{N}$, $n+1 < n$?

   **Inductive Hypothesis:** Assume that $n+1 < n$.

   **Inductive Step:** Consider $n+1$:

   $$(n+1)+1 < n+1$$

3. Prove by induction: The sum of the first $n$ positive odd integers is $n^2$.

4. Prove by induction: For all $n \in \mathbb{N}$, $n < n+1$.

5. Prove by induction: For all positive integers greater than $1$, $n! < n^n$. (Hint: you will need to use the result from the previous problem).

6. Assuming DeMorgan's Theorem for two sets:

   $$\overline{A \cup B} = \bar{A} \cap \bar{B}$$

Prove the general case using induction:

$$\overline{\bigcup_{k=1}^{n} A_k} = \bigcap_{k=1}^{n} \overline{A_k}$$

7. Prove by induction: For all positive integers greater than or equal to $3$, $n^2 - 7n + 12$ is non-negative.

8. Prove by induction: For all $n \in \mathbb{N}$, $21 \mid (4^{n+1} + 5^{2n-1})$.

9. Prove by induction: For all $n \in \mathbb{N}$, $6|(n^3 - n)$ (Hint: proof by cases may be useful here.)

10. Consider the proposition that for all positive integers greater than or equal to $3$, $2n + 1 < n^2$.
    (a) Prove this proposition without using induction.
    (b) Prove this proposition using induction.