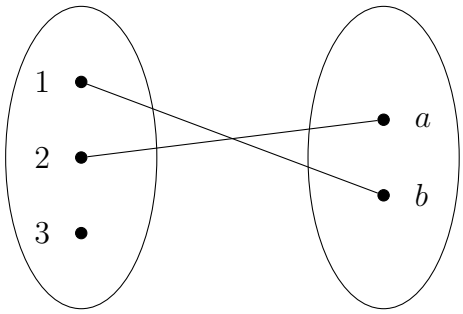
Math-42 Worksheet #11

Functions

1. Determine if each of the following is a function. If so then identify the domain and range. If not then indicate why not.

   (a) $\{(1, a), (2, b), (3, a), (4, b)\}$

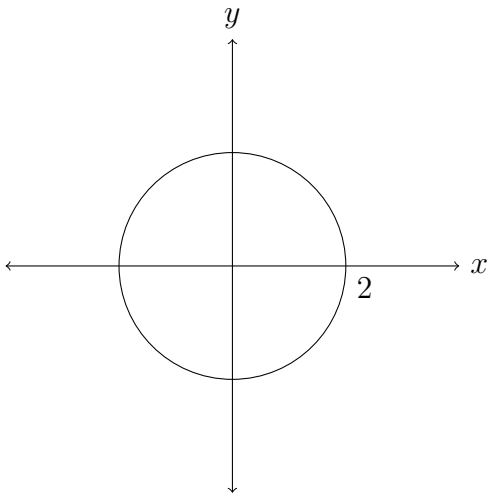
   (b)



   (c) $f(x) = \sqrt{4 - x^2}$

   (d) $f(x) = \frac{1}{x}$

   (e)



2. Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^3$.

   (a) Prove that $f(x)$ is injective (one-to-one) using a standard injection proof.

   (b) Prove that $f(x)$ is surjective (onto) using a standard surjection proof.

(c) Prove that $f(x)$ is bijective. (Hint: only one line is required).

3. Let $g : A \to B$ and $f : B \to C$. Prove each of the following:
   (a) If f and g are injective then $f \circ g$ is injective.
   (b) If f and g are surjective then $f \circ g$ is surjective.
   (c) If $f \circ g$ is surjective then f is surjective.
   (d) If $f \circ g$ is injective then g is injective.

4. Let $g : A \to B$ and $f : B \to C$. Use a function diagram to provide a counterexample to each of the following false propositions:
   (a) If $f \circ g$ is surjective then g is surjective.
   (b) If $f \circ g$ is injective then f is injective.

5. Let $f : A \to B$ and $S, T \subseteq A$. Prove that $f(S \cup T) = f(S) \cup f(T)$.

6. Let $f : A \to B$ and $S, T \subseteq A$:
   (a) Prove that $f(S \cap T) \subseteq f(S) \cap f(T)$
   (b) Which step in your proof is non-reversible, thus precluding equality, and why?
   (c) What is required of f so that equality holds and why?

7. Let $f : A \to B$ and $S, T \subseteq B$. Prove each of the following:
   (a) $f^{-1}(S \cup T) = f^{-1}(S) \cup f^{-1}(T)$
   (b) $f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$

8. Let $f : A \to B$ and $S \subset A$. Draw a function diagram to demonstrate why $f^{-1}(f(S)) \supseteq S$. What is required of f so that equality holds?

9. Let $f : A \to B$ and $T \subset A$. Draw a function diagram to demonstrate why $f(f^{-1}(T)) \subseteq T$. What is required of f so that equality holds?

10. Evaluate the following:

    (a) $\lfloor \pi \rfloor$

    (b) $\lceil \pi \rceil$

    (c) $\lfloor -\pi \rfloor$

    (d) $\lceil -\pi \rceil$