3.6

1. $\infty, -\infty$

2. 2

3. $(-1, 0)$ $(2, 0)$

4. $r(0) = \frac{1 \cdot (-2)}{2 \cdot (-3)} = \frac{2}{6} = \frac{1}{3}$
   $(0, \frac{1}{3})$

5. $-2, 3$

6. 1

7. $r(x) = \frac{x^2 + x}{(x+1)(2x-4)}$
   $= \frac{x(x+1)}{2(x+1)(x-2)}$
   $r(x) = \frac{x}{2(x-2)}$

   a. False
   b. True.
   c. False
   d. True

8. True

44. $r(x) = \dfrac{2x+6}{-6x+3}$

$= \dfrac{2(x+3)}{-3(2x-1)}$

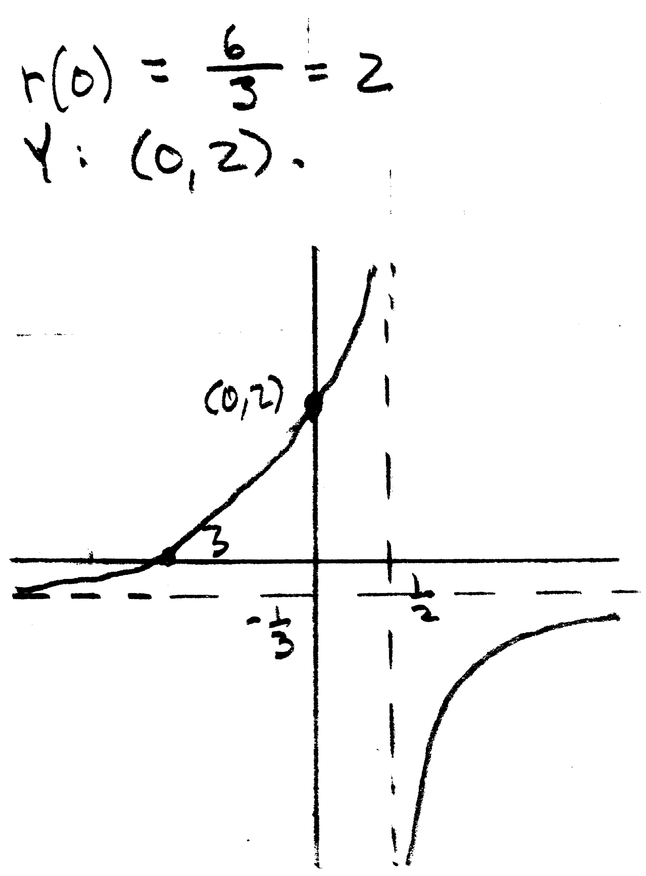
$= -\dfrac{2(x+3)}{3(2x-1)}$.

$$\begin{array}{r|l} & -\frac{1}{3} \\ \hline -6x+3 & 2x+6 \\ & \underline{2x-1} \\ & 7 \end{array}$$

$r(x) = -\frac{1}{3} - \frac{7}{6x-3}$

Z: -3
VA: $\frac{1}{2}$
HA: $-\frac{1}{3}$
$r(x) \to -\frac{1}{3}^-$ as $x \to \infty$
$r(x) \to -\frac{1}{3}^+$ as $x \to -\infty$

$r(0) = \frac{6}{3} = 2$
Y: $(0,2)$.



Domain: $\mathbb{R} - \{\frac{1}{2}\}$
Range: $\mathbb{R} - \{-\frac{1}{3}\}$

48. $r(x) = \dfrac{x^2+2x+3}{2x^2+4x+2}$

$b^2-4ac = 2^2-4(1)(3) = 4-12 = -8 < 0$

$r(x) = \dfrac{x^2+2x+3}{2(x^2+2x+1)}$

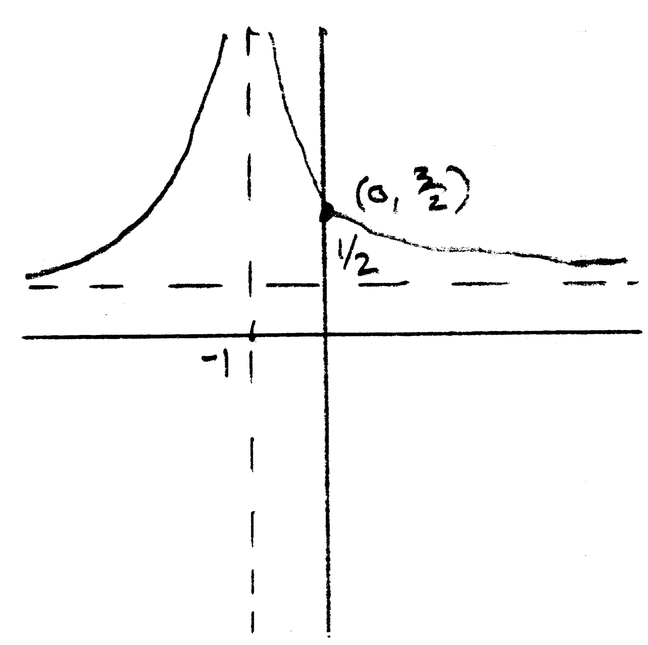
$r(x) = \dfrac{x^2+2x+3}{2(x+1)^2}$

$$\begin{array}{r|l} & \frac{1}{2} \\ \hline 2x^2+4x+2 & x^2+2x+3 \\ & \underline{x^2+2x+1} \\ & 2 \end{array}$$

$r(x) = \frac{1}{2} + \frac{2}{2x^2+4x+2}$
$= \frac{1}{2} + \frac{1}{x^2+2x+1}$
$r(x) = \frac{1}{2} + \frac{1}{(x+1)^2}$

Z: none
VA: -1
HA: $\frac{1}{2}$
$r(x) \to \frac{1}{2}^+$ as $x \to \infty$
$r(x) \to \frac{1}{2}^+$ as $x \to -\infty$

$r(0) = \frac{3}{2}$
Y: $(0, \frac{3}{2})$

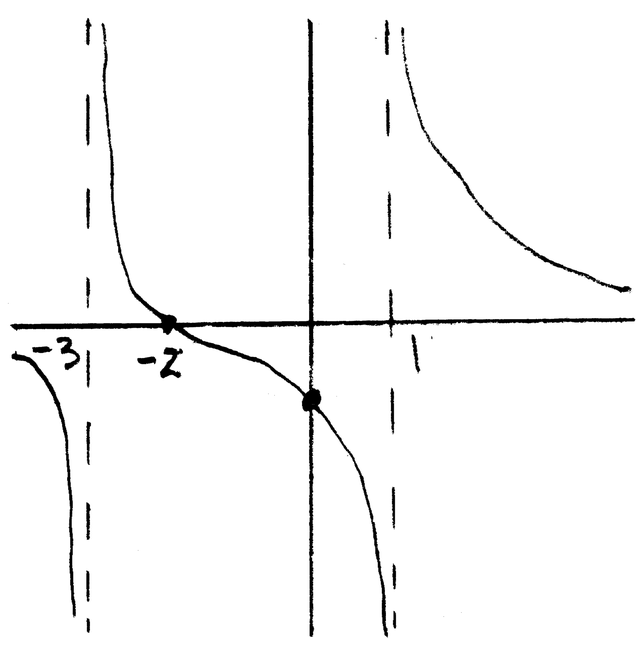


Domain: $\mathbb{R} - \{-1\}$
Range: $(\frac{1}{2}, \infty)$

52. $s(x) = \dfrac{(x+2)}{(x+3)(x-1)}$

Z: -2
VA: $-3, 1$
HA: 0
$s(x) \to 0^+$ as $x \to \infty$
$s(x) \to 0^-$ as $x \to -\infty$

$s(0) = \dfrac{2}{3(-1)} = -\dfrac{2}{3}$
Y: $(0, -\frac{2}{3})$



Domain: $\mathbb{R} - \{-3, 1\}$
Range: $\mathbb{R}$

56. $r(x) = \dfrac{3x^2+6}{x^2-2x-3}$

$= \dfrac{3(x^2+2)}{(x+1)(x-3)}$

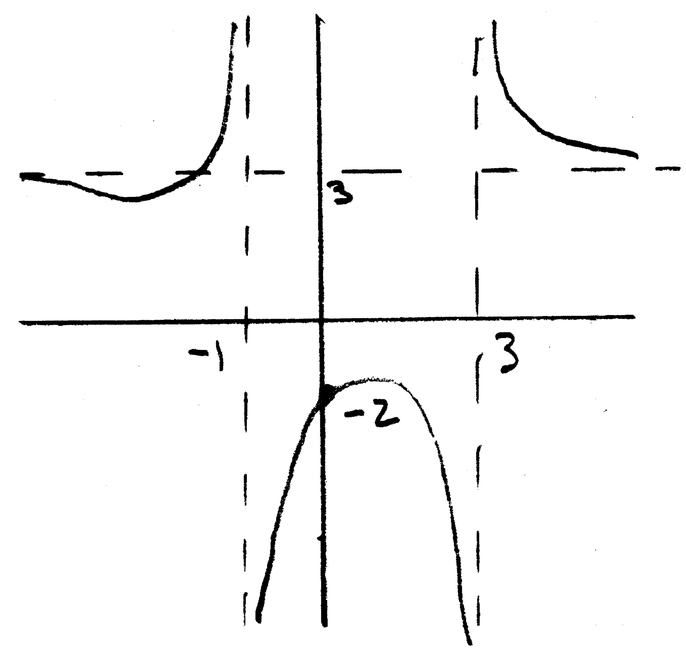
$$\begin{array}{r|l} & \quad 3 \\ \hline x^2-2x-3 & 3x^2+0x+6 \\ & \underline{3x^2-6x-9} \\ & \qquad 6x+15 \end{array}$$

$r(x) = 3 + \dfrac{6x+15}{x^2-2x-3}$

56. (Continued).

Z: none
VA: $-1, 3$
HA: 3
$s(x) \to 3^+$ as $x \to \infty$
$s(x) \to 3^-$ as $x \to -\infty$

$s(0) = \dfrac{6}{-3} = -2$
Y: $(0, -2)$



Domain: $\mathbb{R} - \{-1, 3\}$

Calculator: LMax: $(0.4, -1.8)$
LMin: $(-5.4, 2.5)$

Range: $(-\infty, -1.8] \cup [2.5, \infty)$

60. $r(x) = \dfrac{4x^2}{x^2-2x-3}$

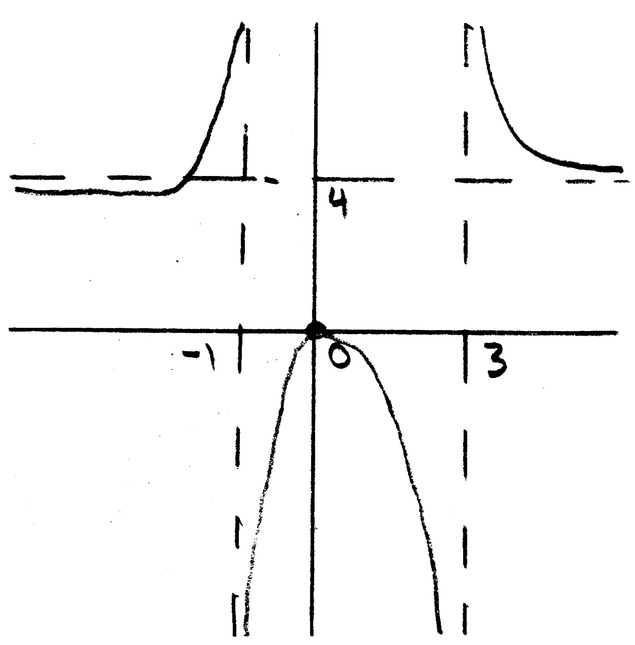
$= \dfrac{4x^2}{(x+1)(x-3)}$

$$\begin{array}{r|l} & 4 \\ \hline x^2-2x-3 & 4x^2+0x+0 \\ & \underline{4x^2-8x-12} \\ & 8x+12 \end{array}$$

$r(x) = 4 + \dfrac{8x+12}{x^2-2x-3}$

Z: 0
VA: $-1, 3$
HA: 4
$r(x) \to 4^+$ as $x \to \infty$
$r(x) \to 4^-$ as $x \to -\infty$

$r(0) = 0$
Y: $(0,0)$



Domain: $\mathbb{R} - \{-1, 3\}$

Calculator: LMin: $(-3, 3)$

Range: $(-\infty, 0] \cup [3, \infty)$

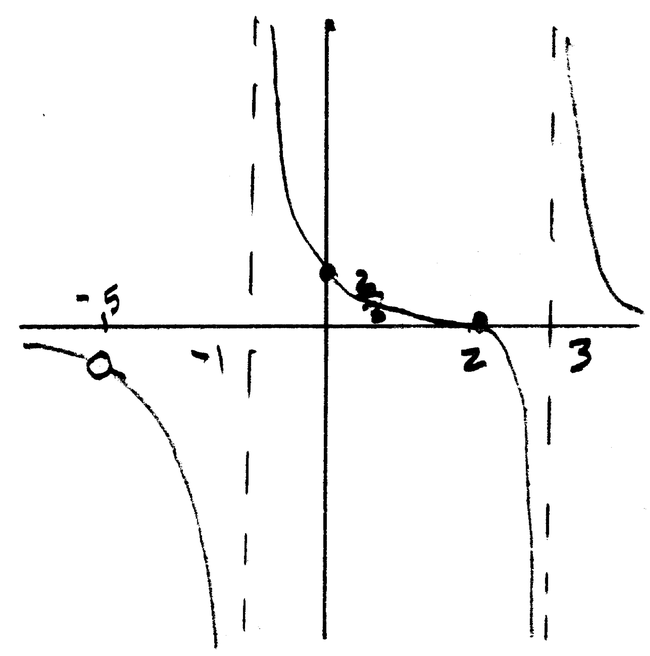
64. $r(x) = \dfrac{x^2+3x-10}{(x+5)(x+1)(x-3)}$

$= \dfrac{\cancel{(x+5)}(x-2)}{\cancel{(x+5)}(x+1)(x-3)}$

$r(x) = \dfrac{(x-2)}{(x+1)(x-3)} \qquad x \neq -5$

Z: 2
VA: $-1, 3$
HA: 0
$r(x) \to 0^+$ as $x \to \infty$
$r(x) \to 0^-$ as $x \to -\infty$

$r(0) = \dfrac{(-2)}{1(-3)} = \dfrac{2}{3}$
Y: $(0, \frac{2}{3})$



Domain: $\mathbb{R} - \{-5, -1, 3\}$
Range: $\mathbb{R}$.

68. $r(x) = \dfrac{x^2+4x-5}{x^3+7x^2+10x}$

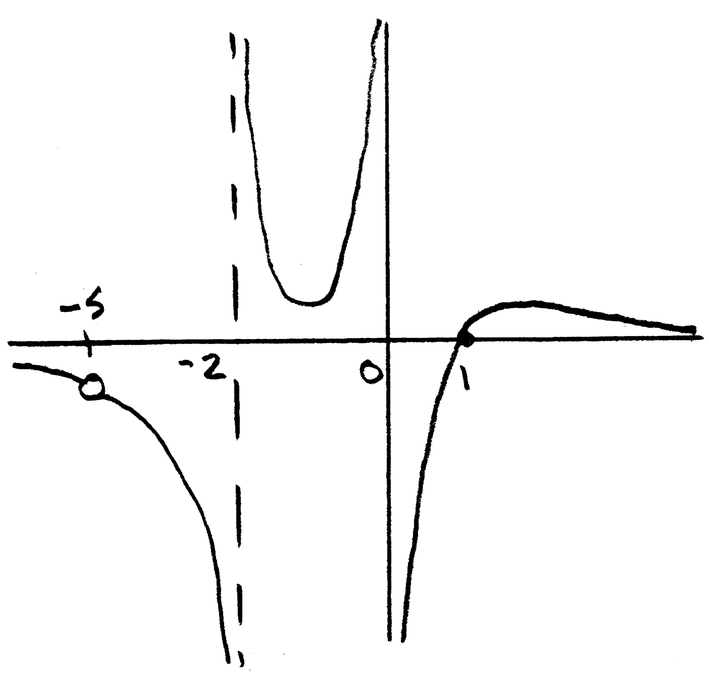
$= \dfrac{(x+5)(x-1)}{x(x^2+7x+10)}$

$= \dfrac{\cancel{(x+5)}(x-1)}{x\cancel{(x+5)}(x+2)}$

$r(x) = \dfrac{(x-1)}{x(x+2)} \qquad x \neq -5$

Z: 1
VA: 0, −2
HA: 0
$r(x) \to 0^+$ as $x \to \infty$
$r(x) \to 0^-$ as $x \to -\infty$

Y: none.



Domain: $\mathbb{R} - \{-5, -2, 0\}$

Calculator: LMin: (−0.7, 1.9).
LMax: (2.7, 0.13)

Range: $(-\infty, 0.13] \cup [1.9, \infty)$

70. $r(x) = \dfrac{x^2+2x}{x-1}$ (Not assigned)

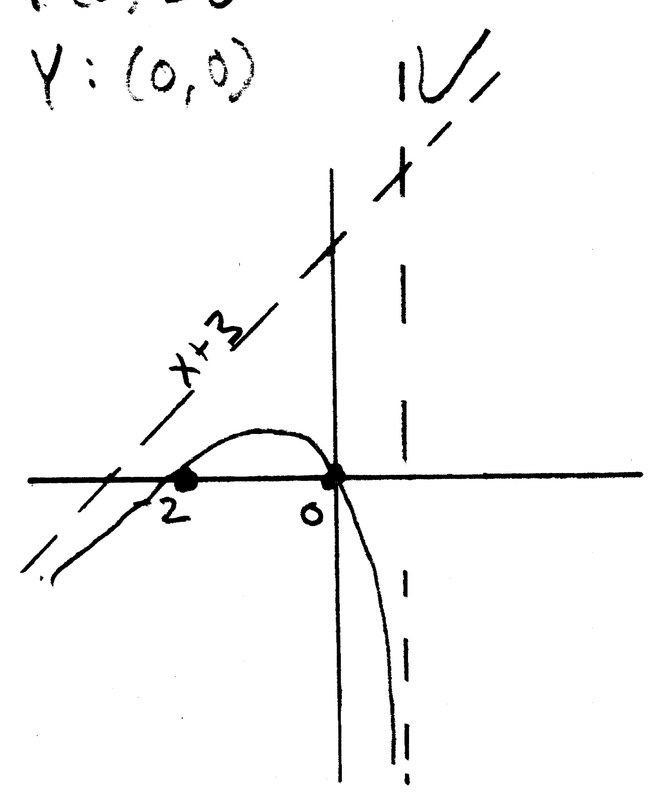
$r(x) = \dfrac{x(x+2)}{x-1}$

$$\begin{array}{r|l} & x+3 \\ \hline x-1 & x^2+2x+0 \\ & \underline{x^2-x} \\ & \quad 3x+0 \\ & \quad \underline{3x-3} \\ & \qquad 3 \end{array}$$

$r(x) = x + 3 + \dfrac{3}{x-1}$

Z: 0, −2
VA: 1
SA: $x+3$
$r(x) \to (x+3)^+$ as $x \to \infty$
$r(x) \to (x+3)^-$ as $x \to -\infty$

$r(0) = 0$
Y: (0, 0)



72. $r(x) = \dfrac{3x - x^2}{2x-2}$

$= \dfrac{-x(x-3)}{2(x-2)}$

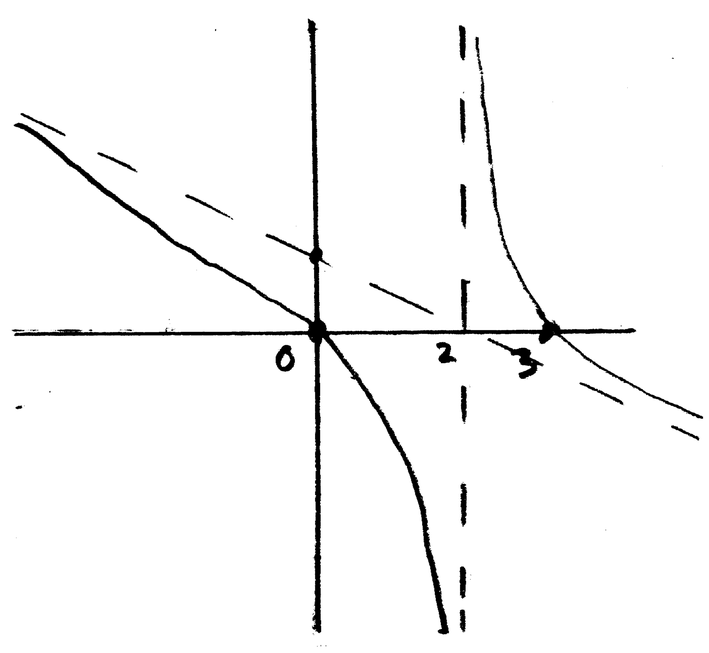
$r(x) = -\dfrac{x(x-3)}{2(x-2)}$

$$\begin{array}{r|l} & -\frac{1}{2}x + 1 \\ \hline 2x-2 & -x^2 + 3x + 0 \\ & -x^2 + x \\ \hline & 2x + 0 \\ & 2x - 2 \\ \hline & 2 \end{array}$$

$r(x) = -\frac{1}{2}x + 1 + \frac{1}{x-2}$

Z: 0, 3
VA: 2
SA: $-\frac{1}{2}x + 1$
$r(x) \to \left(-\frac{1}{2}x+1\right)^+$ as $x \to \infty$
$r(x) \to \left(-\frac{1}{2}x+1\right)^-$ as $x \to -\infty$

$r(0) = 0$
Y: (0,0).

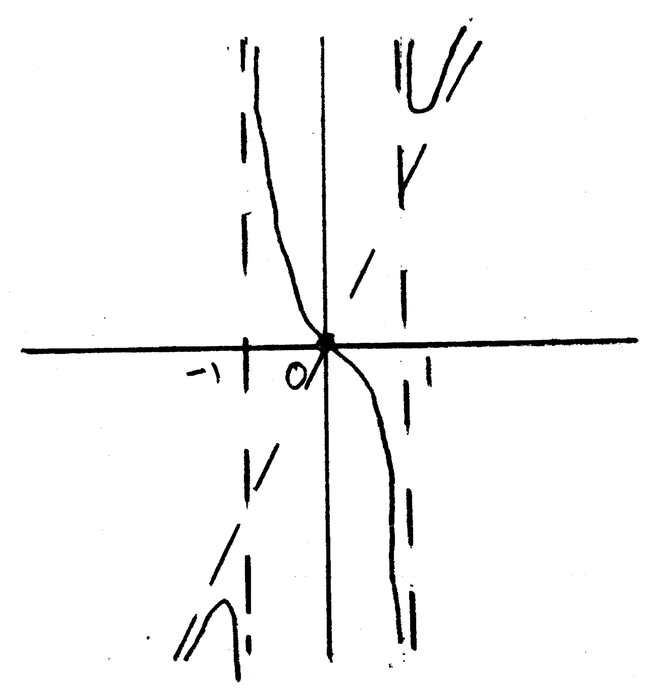


76. $r(x) = \dfrac{2x^3 + 2x}{x^2 - 1}$

$r(x) = \dfrac{2x(x^2+1)}{(x+1)(x-1)}$

$$\begin{array}{r|l} & 2x \\ \hline x^2 + 0x - 1 & 2x^3 + 0x^2 + 2x + 0 \\ & 2x^3 + 0x^2 - 2x \\ \hline & 4x \end{array}$$

$r(x) = 2x + \dfrac{4x}{x^2-1}$

Z: 0
VA: ± 1
SA: $2x$
$r(x) \to 2x^+$ as $x \to \infty$
$r(x) \to 2x^-$ as $x \to -\infty$

$r(0) = 0$
Y: (0,0).