

San José State University  
Fall 2015  
Math-8: College Algebra  
Section 03: MW noon–1:15pm  
Section 05: MW 4:30–5:45pm

Quiz #7

Closed book and notes. No calculator allowed. All work must be shown for full credit.

1. The map of a town is laid out on a grid. The police station and the fire station are both located on Main Street, with the police station at the position (3, -7) and the fire station at the position (-2, 5).

a. How far is it between the two stations?

$$\begin{aligned}d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\&= \sqrt{(3 - (-2))^2 + (-7 - 5)^2} \\&= \sqrt{5^2 + (-12)^2} \\&= \sqrt{25 + 144} \\&= \sqrt{169} \\d &= 13\end{aligned}$$

b. City Hall is also located on Main Street, exactly halfway between the two stations. What is the position of City Hall?

$$\left(\frac{3 - 2}{2}, \frac{-7 + 5}{2}\right) = \left(\frac{1}{2}, \frac{-2}{2}\right) = \left(\frac{1}{2}, -1\right)$$

2. Water boils at  $212^{\circ}F$  and  $100^{\circ}C$ . It freezes at  $32^{\circ}F$  and  $0^{\circ}C$ . Let  $y$  be degrees Fahrenheit and  $x$  be degrees Celsius.

a. Find an equation for converting from Celsius to Fahrenheit expressed in slope-intercept form.

We need to build a line from the points  $(100, 212)$  and  $(0, 32)$ . First, we calculate the slope:

$$m = \frac{212 - 32}{100 - 0} = \frac{180}{100} = \frac{9}{5}$$

Next, we notice that the point  $(0, 32)$  gives us a  $y$ -intercept of 32. Thus, the equation of the line in slope-intercept form is:

$$y = \frac{9}{5}x + 32$$

b. Express the equation in point-slope form using the boiling point.

Using the boiling point of  $(100, 212)$ , we have:

$$y - 212 = \frac{9}{5}(x - 100)$$

c. Express the equation in general form.

Start with the slope-intercept form and do the algebra:

$$\begin{aligned} y &= \frac{9}{5}x + 32 \\ \frac{9}{5}x - y + 32 &= 0 \\ 9x - 5y + 160 &= 0 \end{aligned}$$

Note that multiplying by 5 in the last step is optional, but gets rid of the ugly fraction.

3. Consider the equation  $y = \frac{8}{x^2-2}$ .

a. Determine any x-intercept(s).

$$0 = \frac{8}{x^2 - 2}$$

Note that it is not possible to divide 8 by some number and get 0. Thus, there are no  $x$ -intercepts.

b. Determine any y-intercept(s).

$$y = \frac{8}{0^2 - 2} = \frac{8}{0 - 2} = \frac{8}{-2} = -4$$

Thus, there is a  $y$ -intercept at  $(0, -4)$ .

c. Determine any discontinuities.

Discontinuities occur when the denominator is 0. Thus:

$$\begin{aligned}x^2 - 2 &= 0 \\x^2 &= 2 \\x &= \pm\sqrt{2}\end{aligned}$$

d. Determine any symmetries by plugging in  $(-x)$  and/or  $(-y)$  appropriately. Be sure to correctly name any discovered symmetry.

Check for y-axis symmetry:

$$y = \frac{8}{(-x)^2 - 2}y = \frac{8}{x^2 - 2}$$

Yes. Now check for x-axis symmetry:

$$(-y) = \frac{8}{x^2 - 2}y = -\frac{8}{x^2 - 2}$$

No. Now check for origin symmetry:

$$(-y) = \frac{8}{(-x)^2 - 2}y = -\frac{8}{x^2 - 2}$$

No. So we only have  $y$ -axis symmetry.

e. Sketch the graph, showing all of the features found above.

In order to sketch this graph, we first plot the  $y$ -intercept and where the discontinuities occur. We must then determine how the graph behaves on each side of each discontinuity. Note that since there are no  $x$ -intercepts, the graph must stay negative between the two discontinuities, and can only possibly go positive across a discontinuity.

Start with the  $x = +\sqrt{2}$  case. If we move a little to the right the value is positive. If we move a little to the right it is negative. This is mirrored at the other discontinuity due to  $y$ -axis symmetry. We also note that the graph has  $y = 0$  as an asymptote. The final graph is thus as follows: