Math-19 Homework #4 Solutions

Reading

Please read sections 2.1 through 2.7 and do all concept problems in the posted sections on web-assign.

Problems

State all domains in interval notation!

1). Consider the function:

$$y = -\sqrt[3]{x-5} + 1$$

a). List the transformations, starting from a basic function.

i. Start with the standard function $y = \sqrt[3]{x}$.

ii. Translate right by 5.

iii. Reflect across the x-axis.

iv. Translate up by 1.

b). Determine any x-intercepts.

$$\begin{aligned} -\sqrt[3]{x-5} + 1 &= 0 \\ \sqrt[3]{x-5} &= 1 \\ (\sqrt[3]{x-5})^3 &= 1^3 \\ x - 5 &= 1 \\ x &= 6 \end{aligned}$$

So there is an x-intercept at $(6, 0)$.

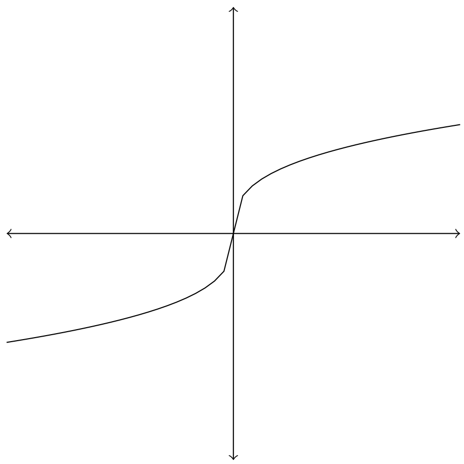
c). Determine any y-intercepts.

$$-\sqrt[3]{0-5} + 1 = -\sqrt[3]{-5} + 1 = \sqrt[3]{5} + 1$$

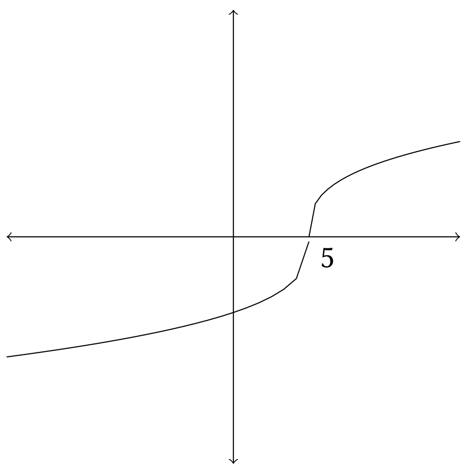
So there is a y-intercept at $(0, \sqrt[3]{5} + 1)$.

d). Sketch a graph of the function.

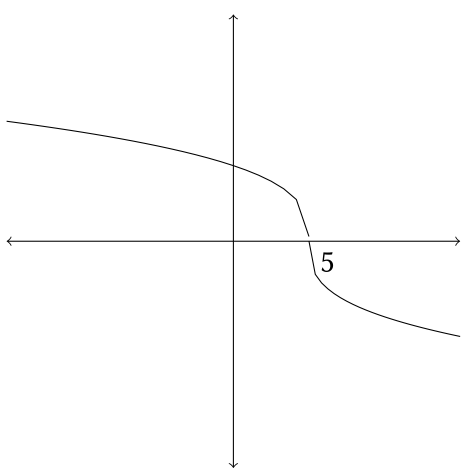
i. Start with the standard function $y = \sqrt[3]{x}$.



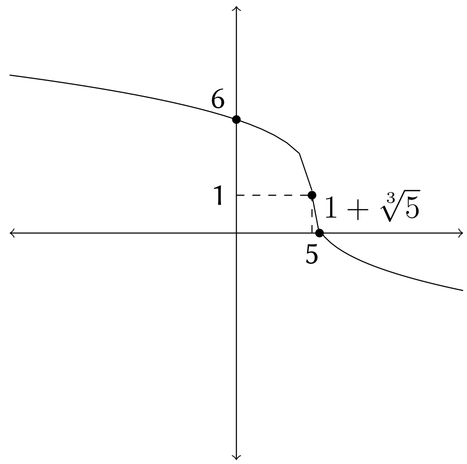
ii. Translate right by 5.



iii. Reflect across the x-axis.



iv. Translate up by 1.



e). Determine the domain of the function.

Domain: $\mathbb{R}$

f). Determine the range of the function.

Range: $\mathbb{R}$

g). On which intervals is the function increasing?

Increasing: none!

h). On which intervals is the function decreasing?

Decreasing: $\mathbb{R}$

2). Let:

$f(x) = \sqrt{x}(x+1)$

$g(x) = \sqrt{x}$

a). Determine $f + g$ and state the domain.

$$(f+g)(x) = \sqrt{x}(x+1) + \sqrt{x} = \sqrt{x}(x+2)$$

Domain: $[0, \infty)$

b). Determine fg and state the domain.

$$(fg)(x) = \sqrt{x}(x+1)\sqrt{x} = x(x+1)$$

Domain: $[0, \infty)$

c). Determine $\frac{f}{g}$ and state the domain.

$$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x}(x+1)}{\sqrt{x}} = x+1$$

Domain: $(0, \infty)$

d). Determine $\frac{f}{f}$ and state the domain.

$$\left(\frac{f}{f}\right)(x) = 1$$

Domain: $(0, \infty)$

3). Let:

$$h(x) = \sqrt[3]{\frac{x+1}{x-1}} - 5$$

Find a suitable $f(x)$ and $g(x)$ such that $h = f \circ g$. Remember, neither is allowed to be just x. Be careful to correctly determine which is the inner function and which is the outer function.

One possible solution is:

$$f(x) = \sqrt[3]{x} - 5$$

$$g(x) = \frac{x+1}{x-1}$$

This also works:

$$f(x) = x - 5$$

$$g(x) = \sqrt[3]{\frac{x+1}{x-1}}$$

A bit more complicated is:

$$f(x) = \sqrt[3]{\frac{x}{x-1}} - 5$$

$$g(x) = x + 1$$

4). Let:

$$f(x) = \frac{1}{x}$$

Compute the difference quotient $\frac{f(x+h)-f(x)}{h}$

$$\begin{aligned}
\frac{f(x+h)-f(x)}{h} &= \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \\
&= \frac{x-(x+h)}{x(x+h)} \cdot \frac{1}{h} \\
&= \frac{-h}{x(x+h)} \cdot \frac{1}{h} \\
&= -\frac{1}{x(x+h)}
\end{aligned}$$

5). A certain chemical reaction proceeds at a linear pace with 4kg of product being produced every 30 seconds. At the start of the reaction there was already 2kg of product existing.

a). Express the amount of product at time t (starting at $t = 0$) by a linear equation: $p(t) = At + B$.

$$p(t) = \frac{4}{30}t + 2 = \frac{2}{15}t + 2$$

b). What does A represent?

The constant rate of the creation of product.

c). What does B represent?

The initial amount of product (at $t = 0$).

d). How much product is there after 15 seconds?

$$p(15) = \frac{2}{15}(15) + 2 = 2 + 2 = 4$$

After 15 seconds there will be 4kg of product.