

Math-8 Exam #4

Name: _____

This is a take-home exam. It is due on Wednesday morning, 05 July, at 8:00am (start of class). It is open book and you may use your class notes, lab sheets, and a graphing calculator. No other printed material, electronics, or web-searching is allowed. You must do this exam by yourself, without any help from or discussion with anyone else. Show all work; there is no credit for guessed answers. All answers should be in exact values, unless you are specifically asked for an approximate value.

- 1). Let $f(x) = x^2$ and $g(x) = x^2(x - 1)$. Evaluate each of the following, simplify where appropriate, and state the domain of each simplified expression:

a). $(f + g)(x)$

b). $(fg)(x)$

c). $\left(\frac{f}{g}\right)(x)$

d). $(f \circ g)(x)$

e). $(g \circ f)(x)$

2). Consider the following function:

$$h(x) = \sqrt{x-1} + (x-1)^2 + 2$$

Find an $f(x)$ and a $g(x)$ such that $h(x) = (f \circ g)(x)$. Neither of your functions is allowed to be just x .

3). The freezing point of water is 0°C , which is 32°F . The boiling point of water is 100°C , which is 212°F . Let y = degrees Celsius and x = degrees Fahrenheit:

a). Derive the linear function $y = f(x)$ to convert from Fahrenheit to Celsius.

b). Derive the linear function $x = g(y)$ to convert from Celsius to Fahrenheit.

c). Prove that f and g are inverse functions.

- 4). Evaluate the difference quotient for $f(x) = x^3$. Your answer should be simplified appropriately.

5). Consider the parabolic function:

$$y = -x^2 + 3x - 5$$

a). Convert the general form to standard form by completing the square.

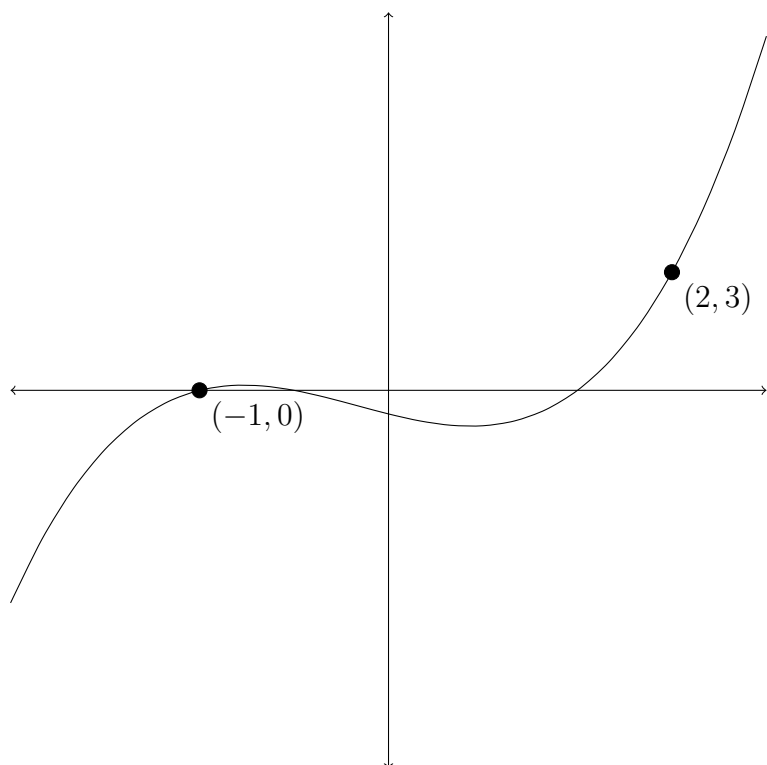
b). What are the coordinates of the vertex?

c). What are the x -intercepts (if any)?

d). What are the y -intercepts (if any)?

e). Sketch the graph. Make sure that you label all important points.

- 6). Consider the following graph of some polynomial function whose equation is unknown but is known to pass through the labeled points:



a). What is the remainder when $f(x)$ is divided by $x - 2$?

b). Explain why you know that $f(x)$ is divisible by $x + 1$.

- 7). Divide $x^4 - 2x^2 + x - 3$ by $x^2 + 1$ using long division and state the answer in division-algorithm form.

8). Consider the polynomial function:

$$y = x^7 - 9x^5 - 4x^4 + 12x^3$$

Factor completely. For full credit you must show how you obtained candidate roots, how you selected the actual roots, and then how you decomposed the function into linear factors using long (or synthetic) division.

9). Sketch the graph for the function in (8). You must label all x and y intercepts and indicate the behavior at each zero using either a sign table or a list of multiplicity decisions. Be sure to show the correct shape at each zero.

10). Using a calculator, identify all minima and maxima on the graph in (9). There are four of them.