Noetherian Rings

Definition: Noetherian

Let R be a ring. To say that R is *Noetherian* means for every ascending chain of ideals:

$$I_1 \subseteq I_2 \subseteq I_3 \subseteq \dots$$

there exists $k \in \mathbb{Z}^+$ such that $I_j = I_k$ for all $j \geq k$.

A non-Noetherian ring is a ring that contains a chain of ideals with proper containment.

Theorem

Let R be a Noetherian integer domain in which every irreducible is prime:

R is a UFD

Proof

Let $r \in R$ be non-zero and not a unit
If r is irreducible then done
Otherwise, $r = d_1 r_1$ where neither are units