Diagonal Matrices

Definition

To say that a square matrix A is *diagonal* means:

$$i \neq j \implies A_{ij} = 0$$

Thus, all entries not on the main diagonal must be 0.

Note that a zero matrix is a diagonal matrix.

Theorem

Let $D_n(\mathbb{F})$ be the set of $n \times n$ diagonal matrices over a field $\mathbb{F}$:

$D_n(\mathbb{F})$ is a subspace of $M_n(\mathbb{F})$.

Proof

Clearly, $I_n \in D_n(\mathbb{F})$

Assume $A, B \in D_n(\mathbb{F})$
Assume $i \neq j$
$(A + B)_{ij} = A_{ij} + B_{ij} = 0 + 0 = 0$
Thus, $A + B \in D_n(\mathbb{F})$

Therefore, $D_n(\mathbb{F})$ is closed under matrix addition.

Assume $c \in \mathbb{F}$
$(cA)_{ij} = cA_{ij} = c \cdot 0 = 0$
Thus, $cA \in D_n(\mathbb{F})$

Therefore, $D_n(\mathbb{F})$ is closed under scalar multiplication.

Therefore, by the subspace test, $D_n(\mathbb{F})$ is a subspace of $M_n(\mathbb{F})$.