Math-19 Homework #3 Solutions

Reading

Please read sections 1.8 through 1.12 and do all concept problems in the posted sections on web-assign.

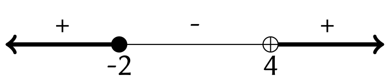
Problems

1). Solve for x. Remember, the answer should be a subset of the real numbers expressed in interval notation — not just single numbers.

$$\frac{x^{\frac{5}{2}} - 3x^{\frac{3}{2}} - 10x^{\frac{1}{2}}}{x^2 - 9x + 20} \geq 0$$

$$\begin{aligned}
\frac{x^{\frac{5}{2}} - 3x^{\frac{3}{2}} - 10x^{\frac{1}{2}}}{x^2 - 9x + 20} &\geq 0 \\
\frac{x^{\frac{1}{2}}(x^2 - 3x - 10)}{x^2 - 9x + 20} &\geq 0 \\
\frac{x^{\frac{1}{2}}(x-5)(x+2)}{(x-5)(x-4)} &\geq 0 \\
\frac{x^{\frac{1}{2}}(x+2)}{x-4} &\geq 0 \qquad x \neq 5
\end{aligned}$$

So far, we have $x \neq 5$. Also, the $x^{\frac{1}{2}}$ factor means $x \geq 0$. We need a sign table to resolve the sign of the remaining terms:



x	$x+2$	$x-4$	sign
-3	-	-	+
0	+	-	-
5	+	+	+

Taking the intersection of all three conditions we get: $(4,5) \bigcup (5,\infty)$. But note, because of the $\sqrt{x}$ factor, $x = 0$ works. So the final answer is: $\{0\} \bigcup (4,5) \bigcup (5,\infty)$.

2). We want a circle whose diameter is the line segment between the points $(5,4)$ and $(-3,-2)$. Using the distance and midpoint formulas:

a). Determine the center of the circle.

This is the midpoint of the diameter:

$$\left(\frac{5-3}{2}, \frac{4-2}{2}\right) = \left(\frac{2}{2}, \frac{2}{2}\right) = (1, 1)$$

b). Determine the radius of the circle.

First, determine the length of the diameter:

$$\begin{aligned} d &= \sqrt{(5+3)^2 + (4+2)^2} \\ &= \sqrt{8^2 + 6^2} \\ &= \sqrt{64 + 36} \\ &= \sqrt{100} \\ &= 10 \end{aligned}$$

The radius is half the diameter, so $r = 5$.

c). What is the equation of the circle in standard form?

$$(x-1)^2 + (y-1)^2 = 25$$

d). What is the equation of the circle in general form?

$$\begin{aligned} (x-1)^2 + (y-1)^2 &= 25 \\ (x^2 - 2x + 1) + (y^2 - 2y + 1) - 25 &= 0 \\ x^2 - 2x + y^2 - 2y - 23 &= 0 \end{aligned}$$

3). Find the equation of the line containing the diameter in question (2):

a). In point/slope form.

$$m = \frac{4+2}{5+3} = \frac{6}{8} = \frac{3}{4}$$

$$y - 4 = \frac{3}{4}(x - 5)$$

or

$$y + 2 = \frac{3}{4}(x + 3)$$

b). In slope-intercept form.

$$y - 4 = \frac{3}{4}(x - 5)$$

$$y - 4 = \frac{3}{4}x - \frac{15}{4}$$
$$y = \frac{3}{4}x - \frac{15}{4} + 4$$
$$y = \frac{3}{4}x + \frac{1}{4}$$

c). In general form.

$$y = \frac{3}{4}x + \frac{1}{4}$$
$$4y = 3x + 1$$
$$3x - 4y + 1 = 0$$

d). Find the equation of the line through the center of the circle and perpendicular to the line containing the stated diameter.

$$\frac{3}{4}m = -1$$
$$m = -\frac{4}{3}$$
$$y - 1 = -\frac{4}{3}(x - 1)$$

4). The amount of heat energy (Q) needed to change the temperature of an object (without going through a phase change like melting or boiling) is jointly proportional to the mass of the object (m) and the *change* in temperature (ΔT).

a). Write an equation that models this physical phenomenon. Use c for the constant of proportionality.

$$Q = cm\Delta T$$

b). The MKS unit for heat energy is the Joule (J). The constant of proportionality is specific to the substance being heated and is referred to as the *specific heat* of the substance. If Q is measured in Joules (J), m is measured in grams (g), and temperature is measured in Kelvin (K), what are the units of c?

$$Q = cm\Delta T$$
$$J = c(g)(K)$$
$$c = Jg^{-1}K^{-1}$$

c). In the lab, it is found that $41790J$ of heat energy raises the temperature of $1L$ of water by $10K$. What is the specific heat of water? (1L of water=1000g)

$$\begin{aligned} 41790J &= c(1L)\left(\frac{1000g}{1L}\right)(10K) \\ 41790J &= c(10000gK) \\ c &= 4.179Jg^{-1}K^{-1} \end{aligned}$$

You can actually look this up in any specific heat table on the internet or in your chemistry book.

5). Consider the equation:

$$y = x^2 + 2x - 5$$

For each of the parts below, use the graphing functions under the *math* (TI-89) or *calc* (TI-83/84) menus to find the answer and submit a screen-shot from your calculator that shows the correct answer.

a). Find the y-value when $x = 1.3$ using the *value* function.

b). Find the x-intercepts using the *zero* function.

c). Determine the minimum value using the *minimum* function.

d). Determine the x-values for $y = 5$ using the *intersect* function. Note that you will need to add something to your graph to do this. Also note that there are multiple answers.

e). Now graph the function $y = x^2 + 11$. Huh!? Nothing seems to appear! Why, and how can you fix this? Submit a screen shot that uses your fix.

See hw03-calc.pdf