

## Math-71 Sections 9, 11, 12

### Homework #4 Solutions

#### Problem

You are the finance manager for a company that just had a great year. Last year's income statement and this year's expectations indicate that the company has a surplus of cash. You decide to invest \$100,000 of this cash in a 5 year CD that compounds monthly. The total amount of the investment after the 5 years is given by:

$$A(r) = 100000 \left(1 + \frac{r}{12}\right)^{60}$$

where  $r$  is the annual interest rate. Assuming that the interest rate is 3% ( $r = 0.03$ ):

1. What is the total amount of the investment after 5 years?

$$A(0.03) = 100000 \left(1 + \frac{0.03}{12}\right)^{60} = \$116161.68$$

2. How fast is the amount growing with respect to  $r$ , in dollars per percent?

Getting the correct units on this is tricky. Here are two methods. I prefer the second method.

- (a) Using the chain rule:

$$A'(r) = 100000 \left[ 60 \left(1 + \frac{r}{12}\right)^{59} \right] \left( \frac{1}{12} \right) = 500000 \left(1 + \frac{r}{12}\right)^{59}$$

$$A'(0.03) = 500000 \left(1 + \frac{0.03}{12}\right)^{59} = \$579359.99 \text{ per percent}/100 = 579359.99\$100/\text{percent}$$

Note the weird units: percent/100. This is because the equation for  $A(r)$  needs  $r$  to be a fractional number — i.e., percent/100. To get rid of this weird unit, we need to divide the final answer by 100:

$$A'(0.03) = \$5793.60/\text{percent}$$

- (b) The first method is a bit of a kludge. A much better way is to adjust the original  $A(r)$  so that it accepts whole percent values:

$$A(r) = 100000 \left(1 + \frac{r/100}{12}\right)^{60} = 100000 \left(1 + \frac{r}{1200}\right)^{60}$$

Now:

$$A'(r) = 100000 \left[ 60 \left(1 + \frac{r}{1200}\right)^{59} \right] \left( \frac{1}{1200} \right) = 5000 \left(1 + \frac{r}{1200}\right)^{59}$$

$$A'(0.03) = 5000 \left(1 + \frac{0.03}{1200}\right)^{59} = \$5793.60 \text{ per percent}$$