

## Homework #8 Solutions

**Problem**

Consider the function:

$$f(x) = \ln \left( \frac{3x^2 e^{x^2} \sqrt{x^2 + 1}}{2 \sqrt[3]{(x-1)^2} \sqrt{e^{2x+1}}} \right)$$

Find  $f'(x)$ .

DO NOT try to use the quotient and product rules here. It will take you FOREVER and you will probably make a mistake. First, get rid of the radicals:

$$f(x) = \ln \left( \frac{3x^2 e^{x^2} (x^2 + 1)^{\frac{1}{2}}}{2(x-1)^{\frac{2}{3}} e^{\frac{2x+1}{2}}} \right)$$

Now use log expansion and the e/ln inverse relationship:

$$\begin{aligned} f(x) &= \ln(3) + \ln(x^2) + \ln(e^{x^2}) + \ln(x^2 + 1)^{\frac{1}{2}} - \ln(2) - \ln(x-1)^{\frac{2}{3}} - \ln\left(e^{\frac{2x+1}{2}}\right) \\ &= \ln(3) + 2\ln(x) + x^2 + \frac{1}{2}\ln(x^2 + 1) - \ln(2) - \frac{2}{3}\ln(x-1) - \frac{2x+1}{2} \end{aligned}$$

Now differentiate:

$$\begin{aligned} f'(x) &= 0 + \frac{2}{x} + 2x + \frac{2x}{2(x^2 + 1)} - 0 - \frac{2}{3(x-1)} - \frac{1}{2}(2) \\ &= \frac{2}{x} + 2x + \frac{x}{x^2 + 1} - \frac{2}{3(x-1)} - 1 \end{aligned}$$