

Homework #3 Solutions

Problem

Consider the function: $f(x) = \sqrt{x}$.

1. Calculate $f'(x)$ using the definition of the derivative.

$$\begin{aligned}
 f'(t) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \\
 &= \lim_{h \rightarrow 0} \left(\frac{\sqrt{x+h} - \sqrt{x}}{h} \right) \left(\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \right) \\
 &= \lim_{h \rightarrow 0} \frac{(x+h) - (x)}{h(\sqrt{x+h} + \sqrt{x})} \\
 &= \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})} \\
 &= \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} \\
 &= \frac{1}{\sqrt{x+0} + \sqrt{x}} \\
 &= \frac{1}{\sqrt{x} + \sqrt{x}} \\
 &= \frac{1}{2\sqrt{x}}
 \end{aligned}$$

2. Calculate $f'(x)$ using some other method.

Make sure that your two final answers match exactly.

$$f(x) = \sqrt{x} = x^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2x^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}}$$