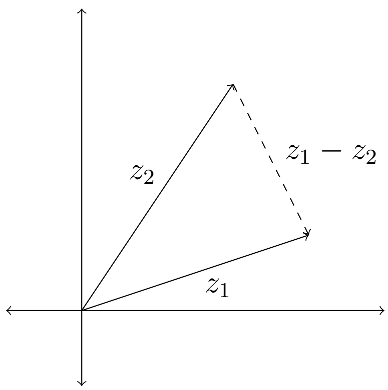
Metric

$\forall z_1, z_2 \in \mathbb{C}, |z_1 - z_2|$ is the *distance* between z_1 and z_2 in the complex plane:



$$\begin{aligned} dist(z_1, z_2) &= |z_1 - z_2| \\ &= |(x_1 + iy_1) - (x_2 + iy_2)| \\ &= |(x_1 - x_2) + i(y_1 - y_2)| \\ &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \end{aligned}$$

<u>Theorem</u>

$dist(z_1, z_2) = |z_1 - z_2|$ is a metric for $\mathbb{C}$.

<u>Proof</u>

1). positive-definite Assume $z_1, z_2 \in \mathbb{C}$

$$|z_1 - z_2| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \geq 0$$

$$\begin{aligned} |z_1 - z_2| = 0 \quad &\iff \quad \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = 0 \\ &\iff \quad x_1 - x_2 = 0 \text{ and } y_1 - y_2 = 0 \\ &\iff \quad x_1 = x_2 \text{ and } y_1 = y_2 \\ &\iff \quad z_1 = z_2 \end{aligned}$$

2). symmetry Assume $z_1, z_2 \in \mathbb{C}$

$$\begin{aligned} |z_1 - z_2| &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= |z_2 - z_1| \end{aligned}$$

3). sub-additivity Assume $z_1, z_2, z_3 \in \mathbb{C}$

$$|z_1 - z_3| = |(z_1 - z_2) + (z_2 - z_3)| \leq |z_1 - z_2| + |z_2 - z_3|$$