

Directed Graphs

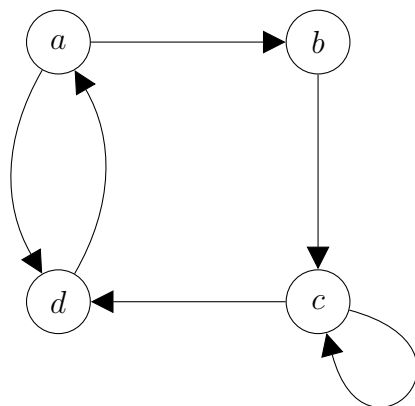
Definition: Directed Graph

A *directed graph (digraph)* $D = (V, E, \dots)$ is a graph with a non-empty and finite set of vertices $V(D)$ and a possibly empty and finite set of *directed edges* $E(D)$ such that each directed edge is represented by an ordered pair of vertices from $V(D)$:

$$E(D) \subseteq V(D) \times V(D)$$

Example

Digraphs are often portrayed visually using filled or labeled circles for the vertices and directed lines from the first endpoint in the ordered pair to the second endpoint in the ordered pair.



$$V = V(D) = \{a, b, c, d\}$$
$$E = E(D) = \{(a, b), (a, d), (b, c), (c, d), (d, a), (c, c)\}$$

Notation

The directed edge (u, v) is usually represented by just uv .

Note that directed multigraphs (multidigraphs) are also possible by adding an endpoint function:

$$\mathcal{E} : E(D) \rightarrow V(D) \times V(D)$$

to the digraph tuple.

Definition: Adjacent Vertices

Let D be a digraph such that $u, v \in V(D)$ and $e = uv \in E(D)$:

- u is said to be *adjacent to* v .
- v is said to be *adjacent from* u .

The directed edge e is said to be *incident* to u and v .

Definition: Orientation

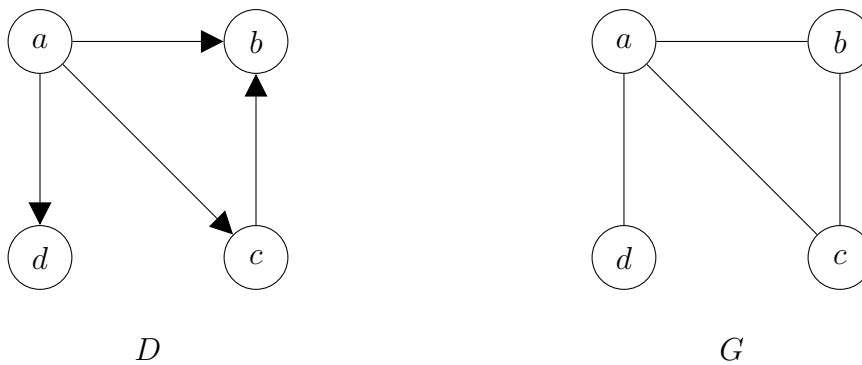
Let D be a digraph. To say that D is an *oriented* graph means that D has no loops and each pair of vertices in D has at most one directed edge between them:

$$\forall u, v \in V(D), uv \in E(D) \implies vu \notin E(D)$$

Furthermore, D is said to be an *orientation* for the underlying simple graph.

Example

D is an oriented digraph and is an orientation of the simple graph G :



Note that the digraph in the first example is not an oriented digraph because $cc \in E(D)$ and $ad, da \in E(D)$.