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Math 275A
Homework #3 Rewrite

Theorem: 2.31

Let $(\mathbb{R}^n, \mathscr{T})$ be the standard topology, $A \subset \mathbb{R}^n$, and $p \in X$ be a limit point of A. There exists a sequence of points in A that converge to p.

Proof. Let $U_i = B\left(p, \epsilon_i\right)$ where $\epsilon_i = \frac{1}{i}$ for $i \in \mathbb{N}$. Note that $\epsilon_i = \frac{1}{i} \to 0$ as $i \to \infty$. Also note that $U_i \cap A \neq \emptyset$ because p is a limit point of A, so select $x_i \in U_i \cap A$. Thus, all of the $x_i \in A$.

Claim: $(x_i)_{i \in \mathbb{N}}$ is a sequence in A converging to p.

Assume $U \in \mathcal{U}_p$. Then there exists some $\epsilon > 0$ such that $B(p, \epsilon) \subset U$. Since the $\epsilon_i \to 0$, there exists some $\epsilon_N < \epsilon$. Assume $i > N$. This means that $e_i < e_N < e$ and so $x_i \in U_i \subset U_N \subset U$ and therefore $x_i \in U$. ∎