Cavallaro, Jeffery
Math 275A
Homework #7 Rewrite

Theorem: 5.11

Every uncountable set in a 2^{nd} countable space has a limit point.

Proof. Assume that X is a 2^{nd} countable space and assume that $A \subset X$ such that A is uncountable. Now, ABC that A has no limit points. This means that for all $a \in A$ it is the case that there exists $U \in \mathcal{U}_a$ such that $U_a \cap A = \{a\}$ and hence every $a \in A$ is an isolated point. So assume that $x, y \in A$ such that $x \neq y$. There exists $U \in \mathcal{U}_x$ and $V \in \mathcal{U}_y$ such that $U \neq V$. So for any basis $\mathcal{B}$ of X, there exists $B_x \subset U$ and $B_y \subset V$ such that $B_x \neq B_y$. Thus, $a \mapsto B_a$ is injective and hence $\mathcal{B}$ is uncountable, contradicting the assumption that X is 2^{nd} countable.

Therefore A contains a limit point. ■