

# Eschelon Forms

## Definition

A *non-zero row (column)* in a matrix is a row (column) that contains at least one non-zero entry.

The *leading entry* of a non-zero row is the leftmost non-zero entry.

## Definition

A matrix is said to be in *row echelon form* (REF) when it has the following three properties:

- 1). All non-zero rows precede all zero rows.
- 2). Each leading entry of a non-zero row occurs in a column to the right of the leading entries for all preceding rows.
- 3). All entries in a column below a leading entry are zero.

A matrix is said to be in *reduced row echelon form* (RREF) when it is in row echelon form and has the following two additional properties:

- 1). The leading entry in each non-zero row equals 1.
- 2). Each leading entry is the only non-zero entry in its column.

## Example

$$\begin{bmatrix} 2 & 0 & 4 & -2 \\ 0 & 1 & 4 & 5 \\ 0 & 0 & 0 & -3 \end{bmatrix}$$

row echelon  
form

$$\begin{bmatrix} -1 & 4 & 8 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

not in  
row echelon  
form

$$\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

row echelon  
form

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

reduced  
row echelon  
form

## Theorem

Each matrix is row equivalent to exactly one reduced row echelon form.