

# Picard's Theorem

## Theorem

In every neighborhood of an essential singularity, an analytic function takes on every value (with one possible exception) an infinite number of times.

## Example

$$w = f(z) = e^{\frac{1}{z}}$$

$$f(z) = \sum_{k=0}^{\infty} \frac{1}{k!z^k}$$

Thus,  $z = 0$  is an essential singularity for  $f(z)$

Note that  $f(z) \neq 0$ ; this is the one exception

Let  $w_0 = e^{\frac{1}{z}}$

$$\log w_0 + 2k\pi i = \frac{1}{z}$$

$$z = \frac{1}{\log w_0 + 2k\pi i}$$

Thus, each  $w_0$  occurs at an infinite number of  $z$ .