The Lowest Common Denominator Method

When we want to add or subtract two fractions with different denominators, we need to modify one or more of the fractions so that we have a common denominator. We will then be able to use the rule:

$$\frac{a}{c} \pm \frac{b}{c} = \frac{a \pm b}{c}$$

We could just multiple the denominators by each other in order to achieve such a common denominator, but that typically leads to large, error-prone numbers— especially when there are more than two fractions in our expression. Instead, we want to pick the lowest common multiple (LCM) of all the denominators.

Let's take the example problem:

$$\frac{3}{10} + \frac{7}{12}\frac{3}{5}$$

1. Perform a prime factorization of each denominator.

$10 = 2 \cdot 5$
$12 = 2 \cdot 6 = 2 \cdot 2 \cdot 3 = 2^2 \cdot 3$
$5 = 5$ (already prime!)

2. Take the highest power of each prime across all the factorizations.

From 2^1, 2^2 get 2^2.
From 3^1 we get just 3.
From 5^1 and 5^1 we get 5.

So the LCM of the denominators, i.e., the LCD, is $2^2 \cdot 3 \cdot 5 = 60$.

3. Multiply each fraction above and below the achieve the common denominator and then perform the operation.

$$\begin{aligned}
\frac{3 \cdot 6}{10 \cdot 6} + \frac{7 \cdot 5}{12 \cdot 5} - \frac{3 \cdot 12}{5 \cdot 12} &= \frac{18}{60} + \frac{35}{60} - \frac{36}{60} \\
&= \frac{18 + 35 - 26}{60} \\
&= \frac{27}{60}
\end{aligned}$$

4. Use prime factorizations on the numerator and denominator to simply the result.

$27 = 3 \cdot 9 = 3 \cdot 3 \cdot 3 = 3^3$
$60 = 2^2 \cdot 3 \cdot 5$.

$$\begin{aligned}
\frac{27}{60} &= \frac{3^3}{2^2 \cdot 3 \cdot 5} \\
&= \frac{3^2}{2^2 \cdot 5} \\
&= \frac{9}{4 \cdot 5} \\
&= \frac{9}{20}
\end{aligned}$$