San José State University
Fall 2015
Math-8: College Algebra
Section 03: MW noon–1:15pm
Section 05: MW 4:30–5:45pm

Quiz #13 (Solutions)

When a function $f(x)$ has an inverse, we denote the inverse function as $f^{-1}(x)$. Note that this should note be confused with $\frac{1}{f(x)}$, which we would denote by $[f(x)]^{-1}$.

Consider the function $f(x) = x^2 - 4x + 3$.

1. Put $f(x)$ in standard form and sketch the graph.
First, complete the square. Many of you insist on using the $x = -\frac{b}{2a}$ method. Please learn how to complete the square!

$$f(x) = (x^2 - 4x + 4) + 3 - 4 = (x - 2)^2 - 1$$

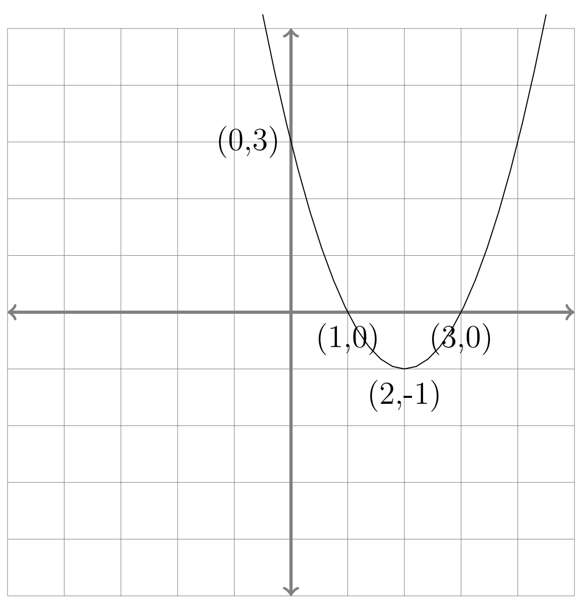
So the vertex is $(2, -1)$. Next, we can factor to find the x-intercepts:

$$f(x) = (x - 1)(x - 3)$$

and thus the x-intercepts are $(1, 0)$ and $(3, 0)$. Please always write these as points and not just as $x = 1, 3$. Finally, for the y-intercept, we set $x = 0$:

$$f(0) = (0 - 1)(0 - 3) = (-1)(-3) = 3$$

So the y-intercept is $(0, 3)$. Putting all of this together, we sketch the graph as follows:



2. What is the domain of $f(x)$?

Domain=$(-\infty, \infty)$ (all real numbers).

3. By looking at the graph, does $f(x)$ have an inverse? Why or why not?

No, it fails the horizontal line test.

4. How can the domain of $f(x)$ be adjusted such that it does have an inverse?

We need to adjust the domain so that it can pass the horizontal line test. Note that the axis of symmetry is $x = 2$, i.e., the y values repeat on either side of the axis. So, we pick one side. Correct answers here are $(-\infty, 2]$ or $[2, \infty)$.

5. Using the adjustment you found in (4), find $f^{-1}(x)$.
Setting $f(x) = y$ and solving for x we get:

$$\begin{aligned} y &= (x-2)^2 - 1 \\ y+1 &= (x-2)^2 \\ (x-2) &= \pm\sqrt{y+1} \\ x &= 2 \pm \sqrt{y+1} \end{aligned}$$

But this is not a function because of the plus/minus. The sign to select must match your choice of domain in (4):

$(-\infty, 2]$	$x = 2 - \sqrt{y+1}$
$[2, \infty)$	$x = 2 + \sqrt{y+1}$