Lab 8: Factoring and Sketching Polynomials

In class we learned how to factor a polynomial:

1). Use the rational root test to identify a set of candidate roots/zeros.

2). Use the remainder (factor) theorem to identify actual roots/zeros from the set of candidates.

3). Use long division to decompose the polynomial into linear factors.

The examples we did in class all had the leading coefficient $a_n = 1$ and the constant coefficient $a_0 \neq 0$. Lets examine a case where neither of these two things are true. Consider the polynomial:

$$y = -2x^5 + 3x^4 + 2x^3 - 3x^2$$

Note that $a_n = -2$ and $a_0 = 0$.

Step one is to simply divide out a -1. Do so now:

$$y =$$

Now you have something of the form $y = -\left(p(x)\right)$, where the leading coefficient of $p(x)$ is now positive.

Step two is to factor out some power of x so that the resulting polynomial has a non-zero constant term. Do so now:

$$y =$$

Your answer should now look somthing like: $y = -x^k\left(q(x)\right)$ for some integer k and some cubic polynomial $q(x)$ with an $a_0 \neq 0$.

Step three is to apply the rational root test to the resulting $q(x)$ in order to find candidate roots/zeros. Let p represent all of the factors of a_0:

$$p :$$

Let q represent all of the factors of a_n:

$$q :$$

Now determine the set of candidate roots by making all possible combinations of $\frac{p}{q}$:

$$\frac{p}{q} :$$

If you did this correctly then you have some fractional roots, since $a_n \neq 1$. One of these should be $\frac{3}{2}$. Apply the remainder/factor test to verify that this is in fact a root/zero of the polynomial $q(x)$:

$$q\left(\frac{3}{2}\right) =$$

Step four is to use long division to factor the $\left(x - \frac{3}{2}\right)$ out of $q(x)$. But this would be a mess due to the fraction, so let's think about an easier way. If it is true that $\left(x - \frac{3}{2}\right)$ divides $q(x)$, then we would have the following:

$$y = -x^k \left(x - \frac{3}{2}\right) q_2(x)$$

for some quadratic polynomial $q_2(x)$. Now, if we factor a $\frac{1}{2}$ out of the linear factor, we have:

$$y = -\frac{1}{2}x^k(2x - 3)q_2(x)$$

Make sure that you understand how to factor out the fractional value! Thus, if $\left(x - \frac{3}{2}\right)$ divides the original $q(x)$ then it must be the case that $(2x - 3)$ also divides the original $q(x)$. Make sure that you understand this! So instead of doing long division with $\left(x - \frac{3}{2}\right)$, we can do so with the much easier $(2x - 3)$. Do that now:

Using the result to rewrite the original polynomial in the form $y = -\frac{1}{2}x^k(2x-3)q_2(x)$, where $q_2(x)$ is some quadratic polynomial:

$y =$

The $q_2(x)$ should be factorable by inspection. Write the final factored polynomial:

$y =$

Now we are ready to sketch. Plot the zeros and y-intercept, determine the end behavior, and use multiplicity to sketch the graph:

Your graph should have two local minima and two local maxima. Use your calculator to determine where these four points occur:

$min_1 =$

$min_2 =$

$max_1 =$

$max_2 =$

Remember that these are points, so they should be stated in (x, y) form.