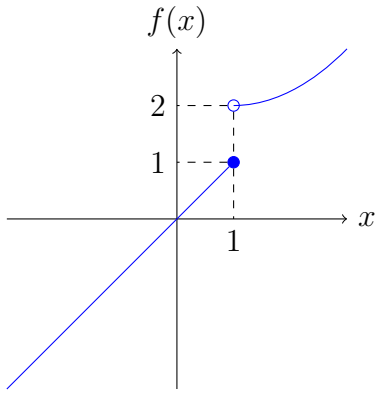
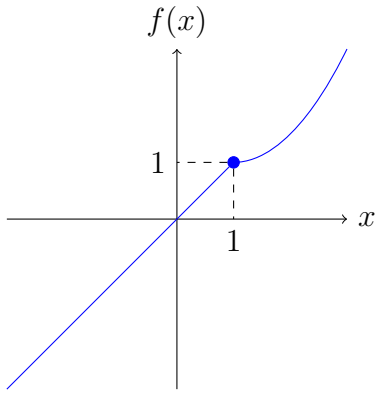
Limits

1. For each of the following functions, determine $\lim_{x \to 1} f(x)$ or state that the limit DNE.

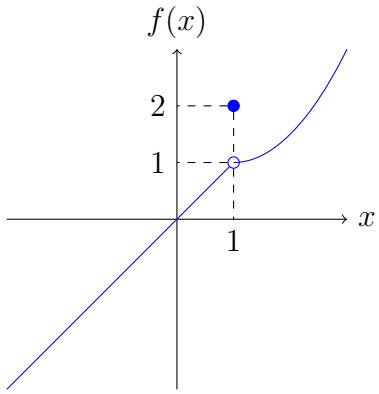
$$f(x) = \begin{cases} x, & x \leq 1 \\ (x-1)^2 + 2, & x > 1 \end{cases}$$



$$f(x) = \begin{cases} x, & x \leq 1 \\ (x-1)^2 + 1, & x > 1 \end{cases}$$

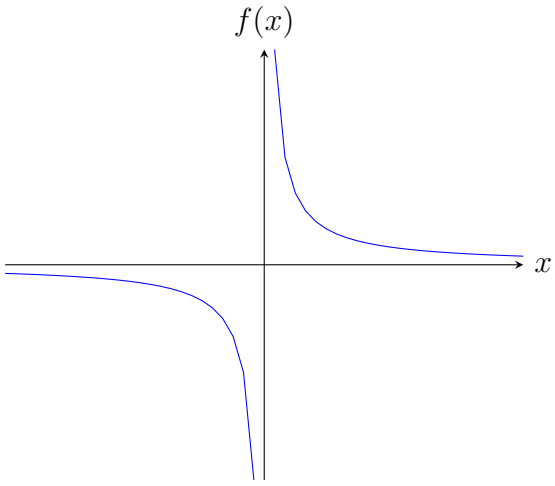


$$f(x) = \begin{cases} x, & x < 1 \\ 2, & x = 1 \\ (x-1)^2 + 1, & x > 1 \end{cases}$$



2. Let $f(x) = \dfrac{1}{x}$.

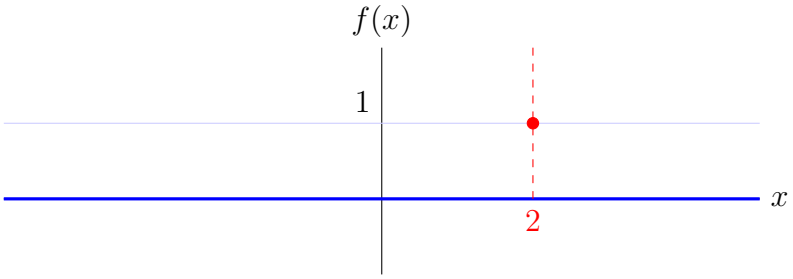
   Does $\lim_{x \to 0} f(x)$ exist?



3. Define $f(x)$ as follows:

$$f(x) = \begin{cases} 1, & x \in \mathbb{Q} \\ 0, & x \in \mathbb{R} - Q \end{cases}$$

   Does $\lim_{x \to 2} f(x)$ exist?



   (Hint: density of the reals)