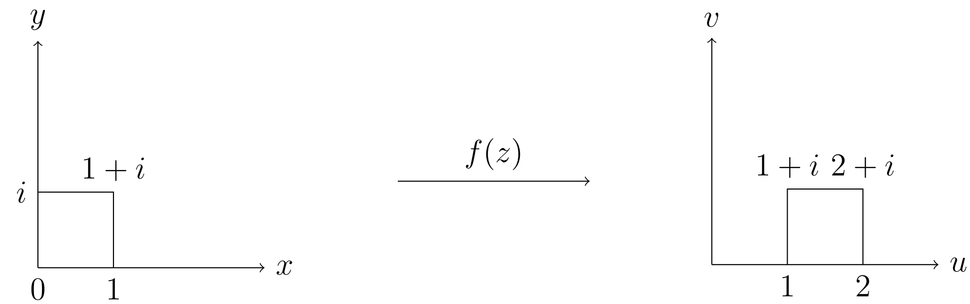
Mappings

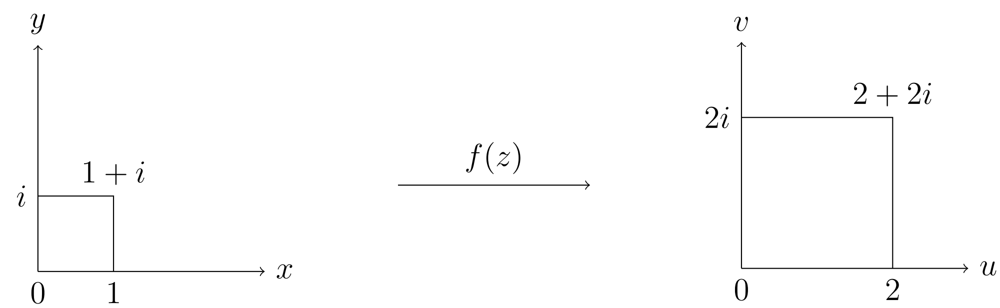
Example: Translation

$w = f(z) = z + 1$



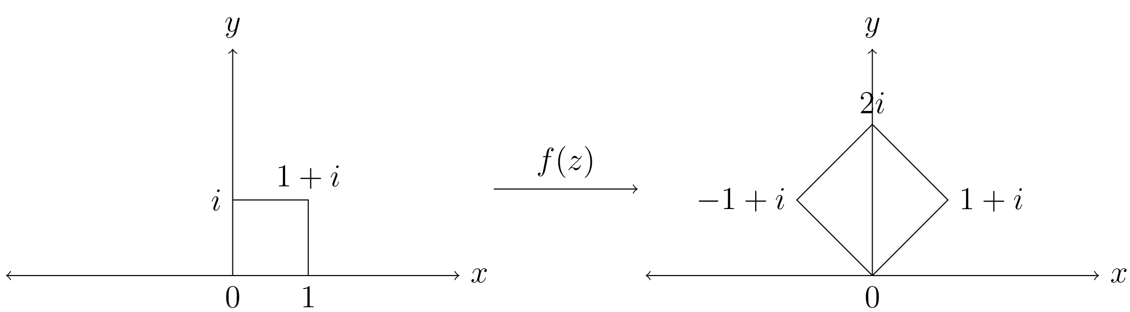
Example: Scaling

$w = f(z) = 2z$



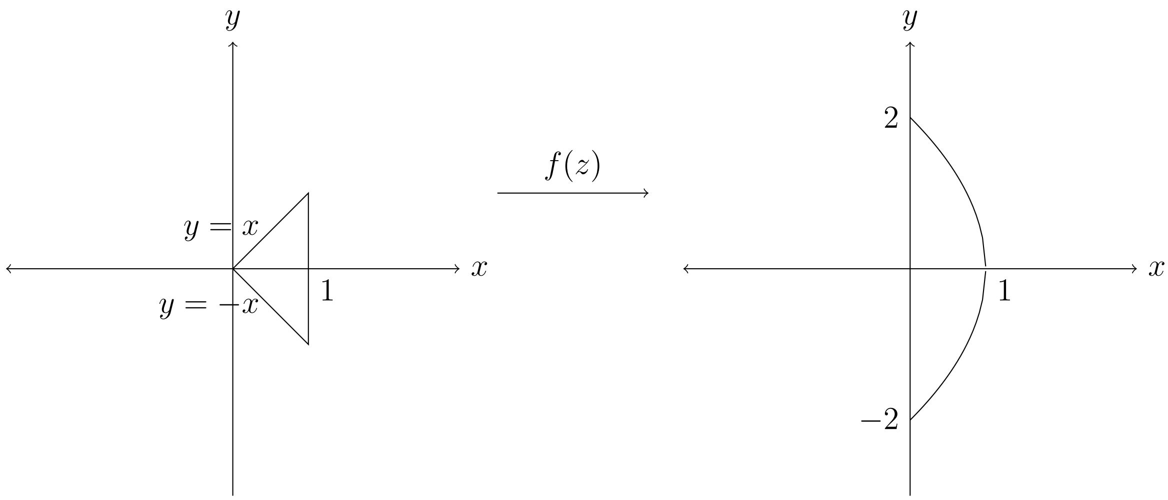
Example: Rotation

$w = f(z) = (1 + i)z = (\sqrt{2}e^{i\frac{\pi}{4}})z$



Example

$w = f(z) = z^2 = (x^2 - y^2) + i2xy$



Along $y = x$ from $x = 0$ to 1:
$u = 0$ and $v = 2x^2$

Along $y = -x$ from $x = 0$ to 1:
$u = 0$ and $v = -2x^2$

Along $x = 1$:
$u = 1 - y^2$ and $v = 2y$
$u = 1 - (\frac{v}{2})^2$
$4u = 4 - v^2$
$v^2 = -4(u - 1)$

Example

$w = f(z) = \frac{1}{z} = \frac{x}{x^2+y^2} - i\frac{y}{x^2+y^2}$

$u = \frac{x}{x^2+y^2}$ and $v = -\frac{y}{x^2+y^2}$
But since $z = \frac{1}{w}$:
$x = \frac{u}{u^2+v^2}$ and $y = -\frac{v}{u^2+v^2}$

$|w| = \frac{1}{|z|}$
So the circle $|z| = r$ maps to the circle $|w| = \frac{1}{r}$

$|w|^2 = \frac{1}{|z|^2}$
$u^2 + v^2 = \frac{1}{x^2+y^2}$
$x^2 + y^2 = \frac{1}{u^2+v^2}$

Consider the circle $a(x^2+y^2)+bx+cy+d=0$ and apply the mapping:

$$a(\frac{1}{u^2+v^2})+b(\frac{u}{u^2+v^2})+c(-\frac{v}{u^2+v^2})+d=0$$

$$d(u^2+v^2)+bu-cv+a=0$$

So as long as $a, d \neq 0$, a circle is mapped to a circle.