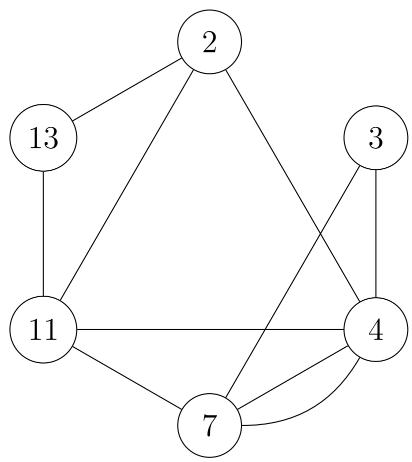
1.4: Multigraphs and Digraphs

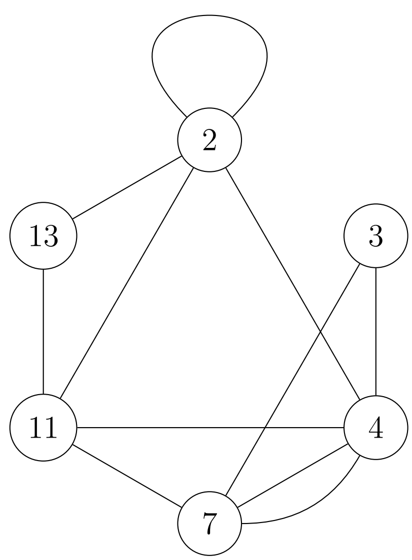
29. Let $S = \{2, 3, 4, 7, 11, 13\}$.

    (a) Construct the multigraph M whose vertex set is S and where ij is an edge for distinct elements i and j in S whenever $i + j$ and ij is an edge whenever $|i - j| \in S$. In other words, i and j are joined by two edges if both $i + j \in S$ and $|i - j| \in S$.

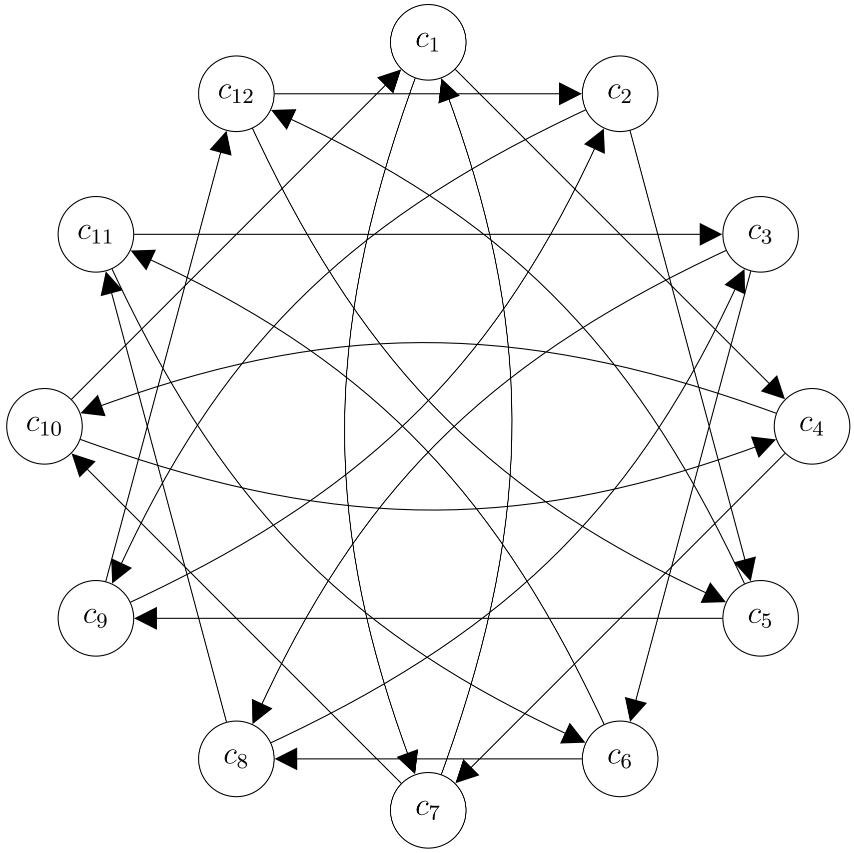
    

    (b) How are the problem and solution in (a) affected if we remove the word "distinct."

    This allows for loop edges, so add a loop on vertex 2.

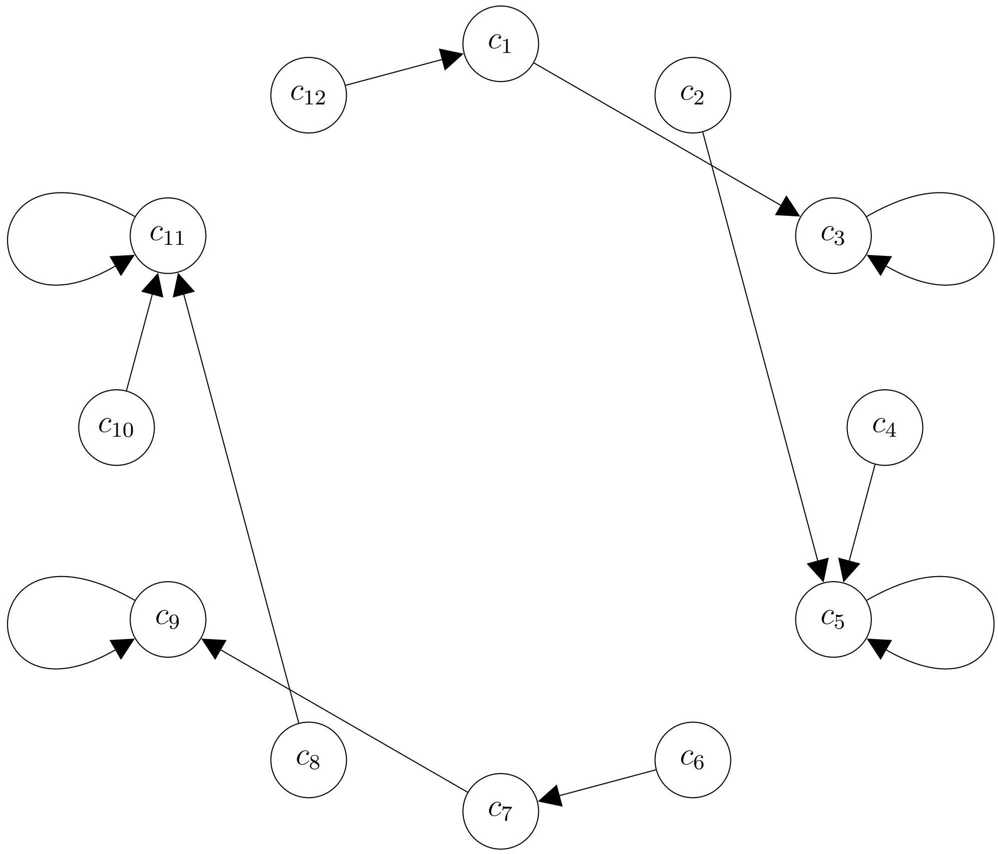
    

30. Consider the twelve configurations $c_i, 1 \le i \le 12$, in Figure 1.38. Draw the digraph D, where $V(D) = \{c_1, c_2, \ldots, c_{12}\}$ and where (c_i, c_j) is a directed edge of D if it is possible to obtain c_j by rotating the configuration c_i either $90°$ or $180°$ clockwise about the midpoint of the checkerboard.



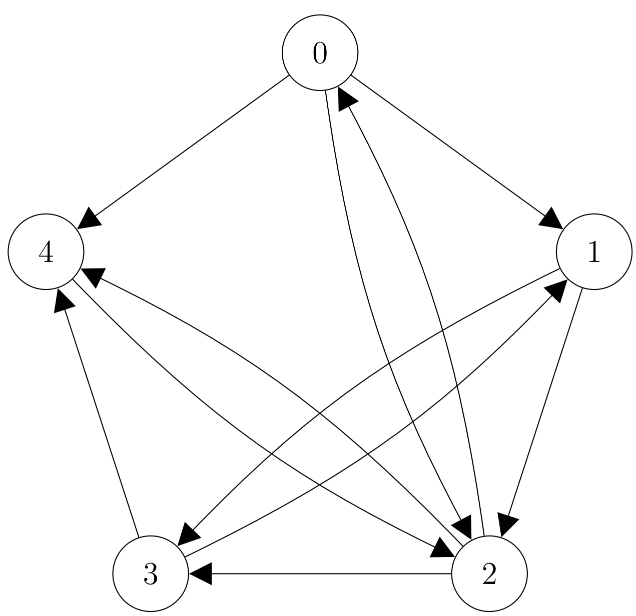
31. Using the twelve configurations in Figure 1.38, define a transformation different from the one described in Exercise 1.30 which can be modeled by a digraph but not by a graph.

    First, move any coin in the upper row to the right (if possible), and then move the leftmost coin in the lower row up (if possible).



32. Let S and A be two finite nonempty sets of integers. Define a digraph D with $V(D) = A$, where (x, y) is an arc of D if $x \neq y$ and $y - x \in S$.

    (a) Draw the digraph D for $A = \{0, 1, 2, 3, 4\}$ and $S = \{-2, 1, 2, 4\}$.



(b) What can be said about D if A and S consist only of odd integers?

D will be empty because the difference of two odds is always even.

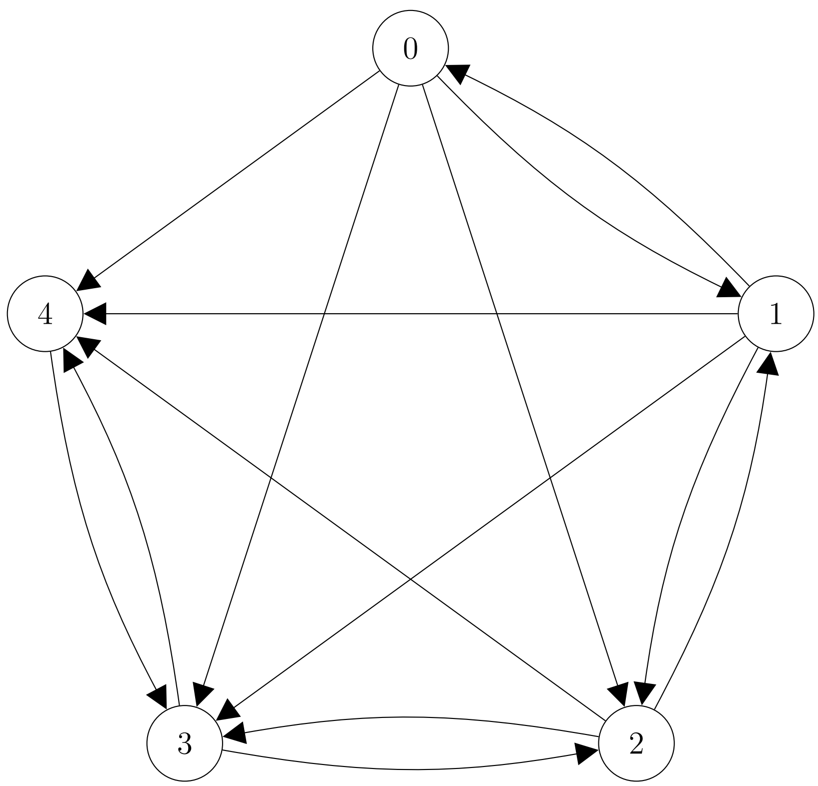
(c) How can the question in (b) be generalized?

D will be empty whenever S is all odd integers and A is either all odd or all even integers.

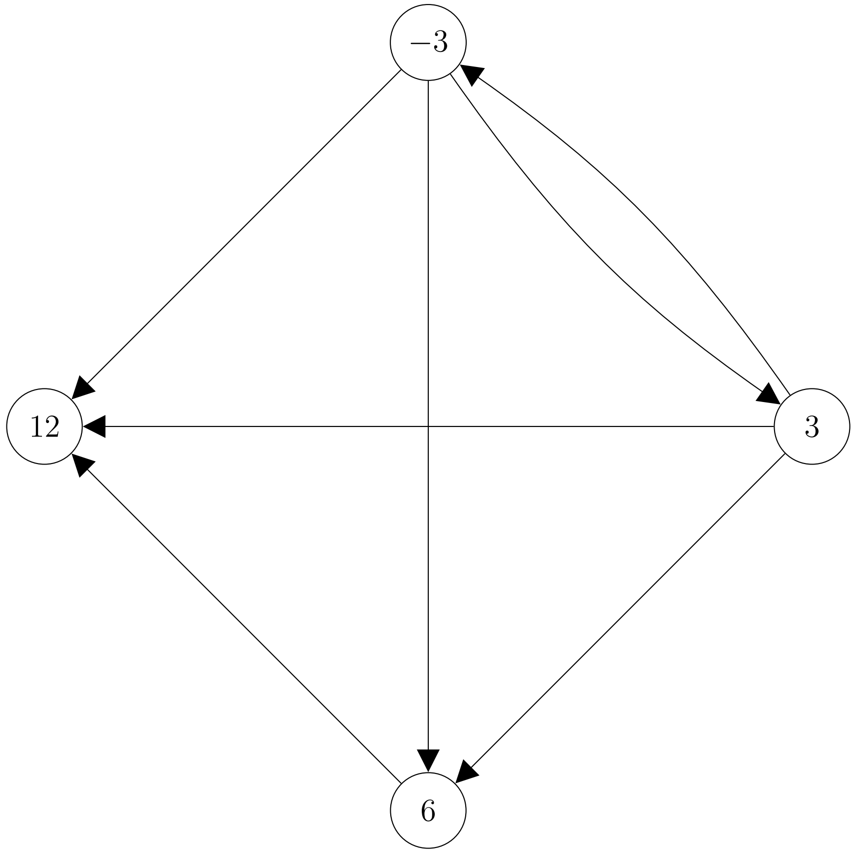
(d) If $|A| = |S| = 5$, how large can the size of D be?

$m = 14$

For example: $A = \{0, 1, 2, 3, 4\}$ and $S = \{-1, 1, 2, 3, 4\}$.



33. A digraph D has vertex set $\{-3, 3, 6, 12\}$ and $(i, j) \in D$ if $i \neq j$ and $i \mid j$, that is, j is a multiple of i. Draw the digraph D.



−3
12
3
6