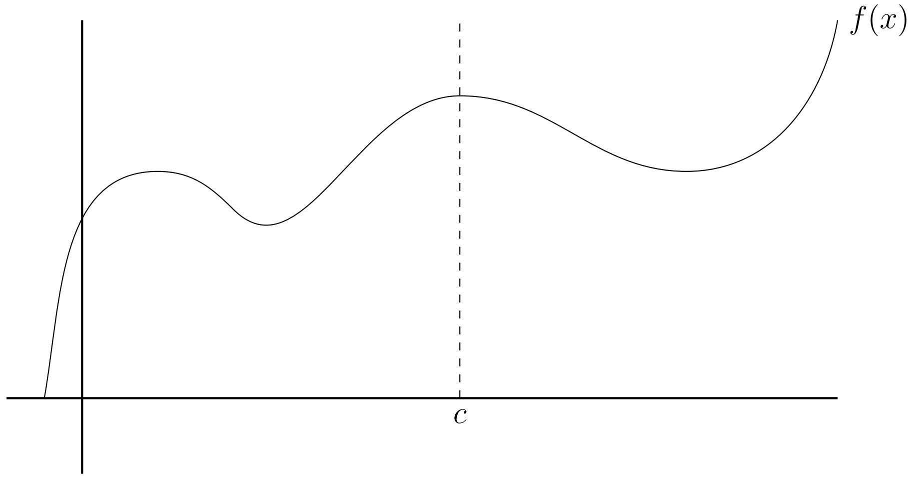
Limit of a Function

What is the behavior of a function $f(x)$ as x gets arbitrarily close to some value c?

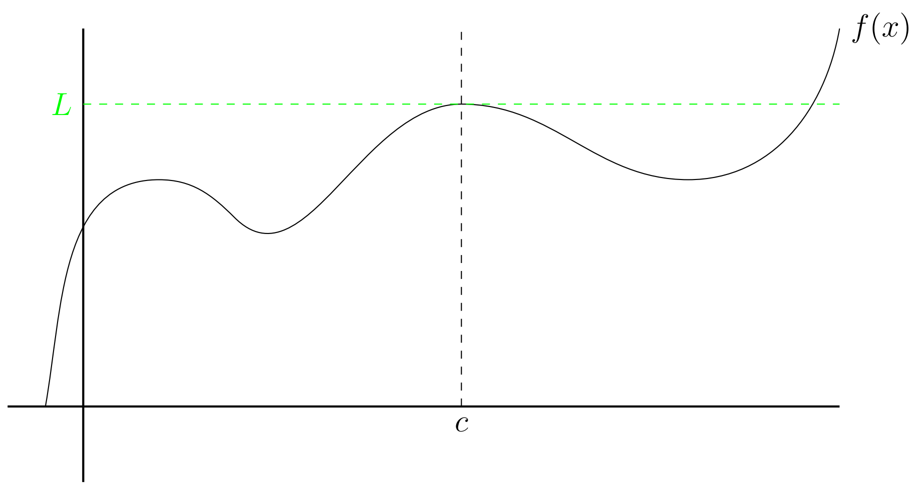


In particular, does $f(x)$ get arbitrarily close to some value L as x gets arbitrarily close to c from both directions (left and right), regardless of whether or not c is in the domain of $f(x)$? If so, then we call L the *limit* of $f(x)$ as x approaches c:

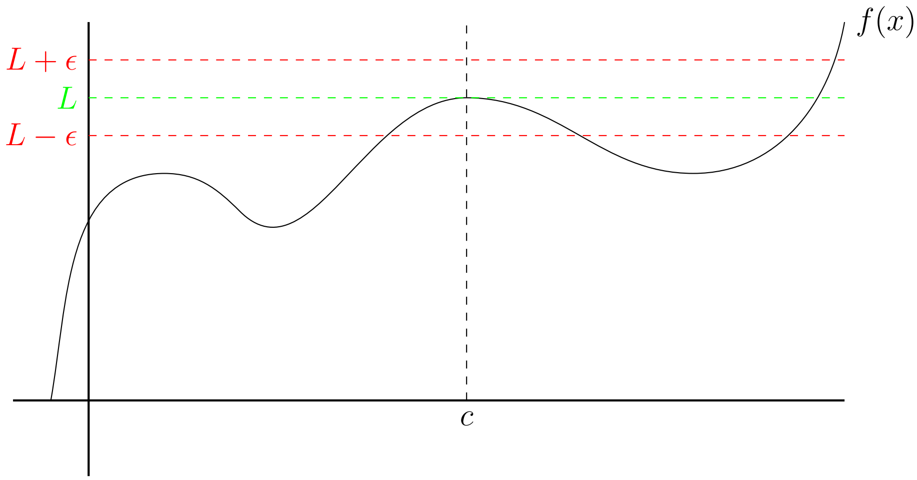
$$\lim_{x \to c} f(x) = L$$

This is essentially the book definition; however, we need something a little more analytical to see what this really means:

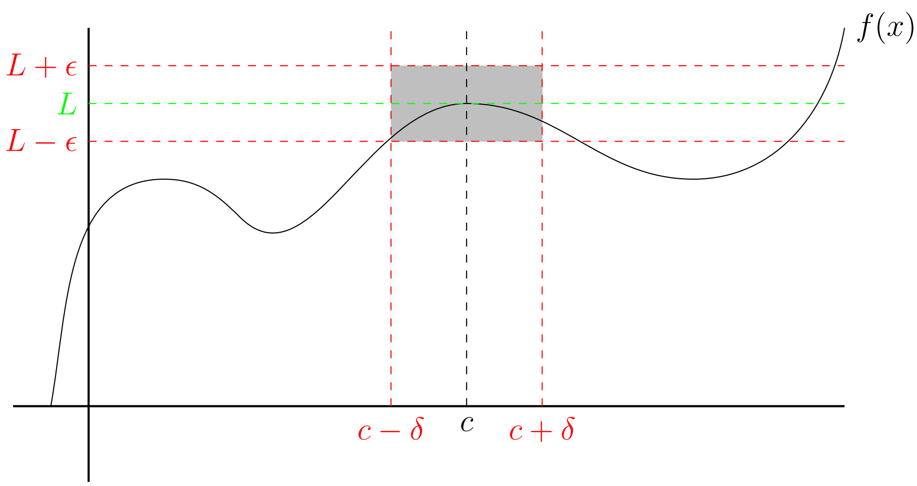
1. Assume that we know that $f(x) = L$.



2. Assume $\epsilon > 0$:



3. Select some $\delta > 0$ so that for all $|x - c| < \delta$ all the $f(x$ values are contained in the resulting box:



Note that as $\epsilon \to 0$, this forces $\delta \to 0$ and the box converges on the limit L. This happens regardless of whether $x = c$ is in the domain of $f(x)$ or not, since $\delta > 0$ and thus $x \neq c$.

The functions that we are going to look at are fairly well-behaved and so finding limits by visualization and a few simple rules will be sufficient.

Examples

Let's start with some visual examples: p549 #1–4.

But where might there fail to be some limit of a function as x approaches some point c?

1. Gaps

$$f(x) = \frac{x}{|x|}, c = 0$$

2. Breaks

$$f(x) = \frac{1}{x}, c = 0$$

You can always find an ϵ for which no suitable δ exists.

Examples

p552 #66–70.

Now, lets see how to analytically find limits for some of our basic functions (Section 2.5 p215).