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Math 275A
Homework #0

3.1

Define two points (x_0, y_0) and (x_1, y_1) of the plane to be equivalent if:

$$y_0 - x_0^2 = y_1 - x_1^2$$

Check that this is an equivalence relation and describe the equivalence classes.

(R) Assume $(x_0, y_0) \in \mathbb{R}^2$.

$$y_0 - x_0^2 = y_0 - x_0^2$$

$$\therefore (x_0, y_0) \sim (x_0, y_0)$$

(S) Assume $(x_0, y_0) \sim (x_1, y_1)$.

$$y_0 - x_0^2 = y_1 - x_1^2$$
$$y_1 - x_1^2 = y_0 - x_0^2$$

$$\therefore (x_1, y_1) \sim (x_0, y_0)$$

(T) Assume $(x_0, y_0) \sim (x_1, y_1)$ and $(x_1, y_1) \sim (x_2, y_2)$.

$y_0 - x_0^2 = y_1 - x_1^2$ and $y_1 - x_1^2 = y_2 - x_2^2$
$y_0 - x_0^2 = y_2 - x_2^2$

$$\therefore (x_0, y_0) \sim (x_2, y_2)$$

The equivalence classes are the parabolas $y = x^2 + c$.

3.3

Here is a "proof" that every relation $\sim$ that is both symmetric and transitive is also reflexive: "Since $\sim$ is symmetric, $a \sim b \implies b \sim a$. Since $\sim$ is transitive, $a \sim b$ and $b \sim a \implies a \sim$, as desired. Find the flaw in this argument.

Suppose a is not related to any b? For example, consider the set $X = \{1, 2, 3\}$ and define a relation on X as follows:

$$\{(2, 2), (2, 3), (3, 2), (3, 3)\}$$

There no way to show $1 \sim 1$ without an explicit statement of reflexivity.

3.4

Let $f : A \to B$ be a surjective function. Let us define a relation on A by setting $a_0 \sim a_1$ if:

$f(a_0) = f(a_1)$

(a) Show that this is an equivalence relation.

(R) Assume $a_0 \in A$.

f is well-defined.
And so $f(a_0) = f(a_0)$

$\therefore a_0 \sim a_0$

(S) Assume $a_0 \sim a_1$.

$f(a_0) = f(a_1)$
$f(a_1) = f(a_0)$

$\therefore a_1 \sim a_0$

(T) Assume $a_0 \sim a_1$ and $a_1 \sim a_2$.

$f(a_0) = f(a_1)$ and $f(a_1) = f(a_2)$
$f(a_0) = f(a_2)$

$\therefore a_0 \sim a_2$

(b) Let A^* be the set of equivalence classes. Show that there is a bijective correspondence of A^* with B.

Define $g : A^* \to B$ by $g(a^*) = b$ where $\forall\, a \in a^*, f(a) = b$.

Assume $g(a_0^*) = g(a_1^*)$.
Assume $a_0 \in a_0^*$ and $a_1 \in a_1^*$.
$f(a_0) = f(a_1)$ and so $a_0 \sim a_1$.

$\therefore a_0^* = a_1^*$ and thus g is injective.

Assume $b \in B$.
Since f is surjective, $\exists a \in A, f(a) = b$.
But $a \in a^*$.

$\therefore g(a^*) = b$ and thus g is surjective.

$\therefore g$ is bijective.

3.6

3.9

3.10

3.12