Math-19 Homework #6 Solutions

Problems

1). Solve for x. For full credit you must include a graph, a test point table or a list of multiplicity decisions, and the final answer in interval notation.

$$\frac{(6-x-x^2)(3-x)^2}{(x-2)(x^2+3x-5)} \geq 0$$

First, lets factor and clean this up a bit:

$$\frac{-(x^2+x-6)(x-3)^2}{(x-2)(x^2+3x-5)} \geq 0$$

$$\frac{(x^2+x-6)(x-3)^2}{(x-2)(x^2+3x-5)} \leq 0$$

$$\frac{(x+3)(x-2)(x-3)^2}{(x-2)(x^2+3x-5)} \leq 0$$

$\dfrac{(x+3)(x-3)^2}{x^2+3x-5} \leq 0$, but $x \neq 2$

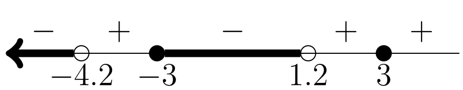
The quadratic in the denominator doesn't factor nicely, so we need to use the quadratic formula:

$$x = \frac{-3 \pm \sqrt{29}}{2}$$

So, the final form is:

$\dfrac{(x+3)(x-3)^2}{[x-(\frac{-3-\sqrt{29}}{2})][x-(\frac{-3+\sqrt{29}}{2})]} \leq 0$, but $x \neq 2$

The poles at approximately -4.2 and 1.2, so we can now set up our number line and use multiplicity to determine the sign changes:



Note that we do not change sign across the zero with the even multiplicity at $x = 3$.

$$x \in (-\infty, \frac{-3-\sqrt{29}}{2}) \cup [-3, \frac{-3+\sqrt{29}}{2}) \cup \{3\}$$

Note that it turns out that $x = 2$ is in an already rejected interval, so we don't need to make a hole for it.

2). We want a circle whose diameter is the line segment between the points $(5, 4)$ and $(-3, -2)$. Using the distance and midpoint formulas:

a). Determine the center of the circle.

The center of the circle will be at the midpoint of the two diameter points:

$x = \dfrac{5-3}{2} = 1$ and $y = \dfrac{4-2}{2} = 1$

$C(1, 1)$

b). Determine the radius of the circle.

The length of the radius can be calculated using the distance formula to find the distance between the center of the circle and one of its diameter points:

$r = \sqrt{(1-5)^2 + (1-4)^2} = \sqrt{(-4)^2 + (-3)^2} = 5$

Just to check, let's calculate this using the other point:

$r = \sqrt{(1+3)^2 + (1+2)^2} = \sqrt{4^2 + 3^2} = 5$

As expected, we get the same answer.

$r = 5$

c). What is the equation of the circle in standard form?

$(x-1)^2 + (y-1)^2 = 25$

d). What is the equation of the circle in general form?

$x^2 - 2x + 1 + y^2 - 2y + 1 = 25$

$x^2 + y^2 - 2x - 2y - 23 = 0$

3). Find the equation of the line containing the diameter in question (2):

$m = \dfrac{4+2}{5+3} = \dfrac{6}{8} = \dfrac{3}{4}$

a). In point/slope form.

$y - 4 = \dfrac{3}{4}(x-5)$ or $y + 2 = \dfrac{3}{4}(x+3)$

b). In slope-intercept form.

$y - 4 = \dfrac{3}{4}x - \dfrac{15}{4}$

$y = \dfrac{3}{4}x + \dfrac{1}{4}$

c). In general form.

$\dfrac{3}{4}x - y + \dfrac{1}{4} = 0$

d). Find the equation of the line through the center of the circle and perpendicular to the line containing the stated diameter.

$m_\perp = -\frac{4}{3}$

$y - 1 = -\frac{4}{3}(x - 1)$

Since I didn't ask for a particular form, you can leave it like this, or convert to y-intercept form:

$y - 1 = -\frac{4}{3}x + \frac{4}{3}$

$y = -\frac{4}{3}x + \frac{7}{3}$

4). The amount of heat energy (Q) needed to change the temperature of an object (without going through a phase change like melting or boiling) is jointly proportional to the mass of the object (m) and the *change* in temperature (ΔT).

a). Write an equation that models this physical phenomenon. Use c for the constant of proportionality.

$$Q = cm\Delta T$$

b). The MKS unit for heat energy is the Joule (J). The constant of proportionality is specific to the substance being heated and is referred to as the *specific heat* of the substance. If Q is measured in Joules (J), m is measured in grams (g), and temperature is measured in Kelvin (K), what are the units of c?

$$J = \frac{J}{gK}gK$$

So the units of c are J/gK.

c). In the lab, it is found that $41790J$ of heat energy raises the temperature of $1L$ of water by $10K$. What is the specific heat of water? (1L of water=1000g)

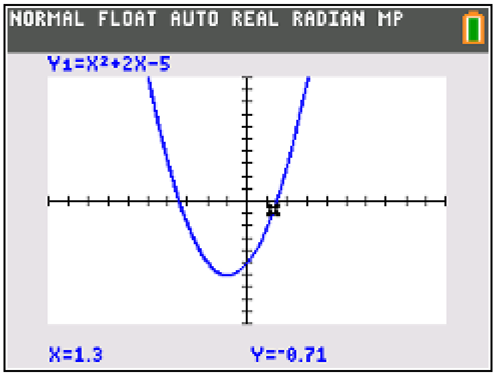
$$c = \frac{Q}{m\Delta T} = \frac{41\,790\ \mathrm{J}}{1000\ \mathrm{g} \cdot 10\ \mathrm{K}} = 4.1790\ \mathrm{J/gK}$$

5). Consider the equation:

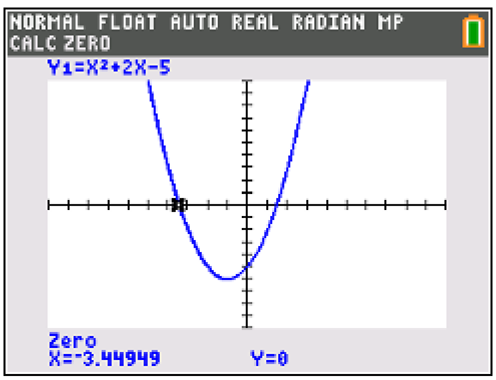
$$y = x^2 + 2x - 5$$

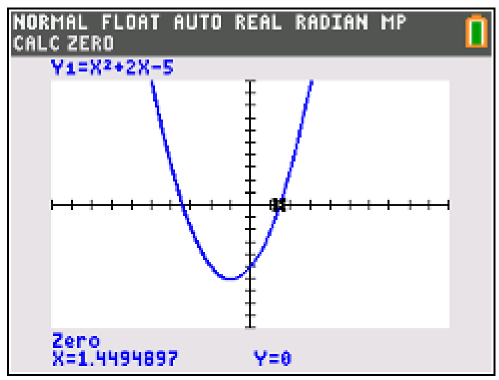
For each of the parts below, use the graphing functions under the *math* (TI-89) or *calc* (TI-83/84) menus to find the answer and submit a screen-shot from your calculator that shows the correct answer.

a). Find the y-value when $x = 1.3$ using the *value* function.

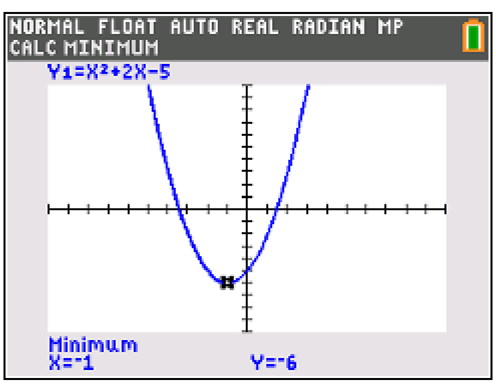


b). Find the x-intercepts using the *zero* function.

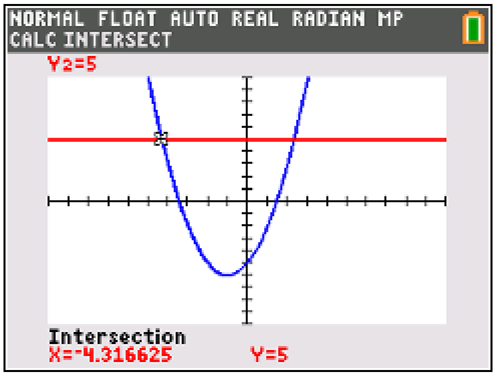


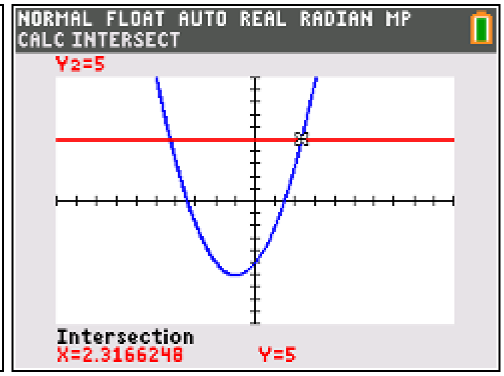


c). Determine the minimum value using the *minimum* function.



d). Determine the x-values for $y = 5$ using the *intersect* function. Note that you will need to add something to your graph to do this. Also note that there are multiple answers.





e). Now graph the function $y = x^2 + 11$. Huh!? Nothing seems to appear! Why, and how can you fix this? Submit a screen shot that uses your fix.