

Math-19 Homework #3 Solutions

Problems

- 1). Simplify the following expression. Your answer should contain no radicals and no negative exponents. You may assume that $a, b, c > 0$.

$$\begin{aligned} \frac{a^2 b^{-3} \sqrt{abc^3}}{\sqrt[3]{a^{-2}} \sqrt{b^3 c}} &= \frac{a^2 b^{-3} a^{\frac{1}{2}} b^{\frac{1}{2}} c^{\frac{3}{2}}}{(a^{-2} b^{\frac{3}{2}} c^{\frac{1}{2}})^{\frac{1}{3}}} \\ &= \frac{a^{\frac{5}{2}} b^{-\frac{5}{2}} c^{\frac{3}{2}}}{a^{-\frac{2}{3}} b^{\frac{1}{2}} c^{\frac{1}{6}}} \\ &= a^{\frac{19}{6}} b^{-3} c^{\frac{4}{3}} \\ &= \frac{a^{\frac{19}{6}} c^{\frac{4}{3}}}{b^3} \end{aligned}$$

- 2). Determine whether each of the following statements is either correct, incorrect, or misleading. Explain why incorrect and misleading statements are incorrect or misleading.

a). $\sqrt{9} = \pm 3$

This is incorrect. $\sqrt{9}$ asks for the principle (positive) root only. Correct is: $\sqrt{9} = 3$.

b). $\left(x^{\frac{1}{2}}\right)^2 = |x|$

This is misleading. Since we have $x^{\frac{1}{2}}$, it is assumed that $x \geq 0$. Thus, the absolute value, although not hurting anything, is not really needed.

c). $(x^2)^{\frac{1}{2}} = x$

This is incorrect. The LHS is always positive; however, x may be positive or negative. Correct is: $(x^2)^{\frac{1}{2}} = |x|$.

d). $(x^3)^{\frac{1}{3}} = |x|$

This is incorrect. Odd powers preserve negativity. Thus, the LHS can be either positive or negative. Correct is: $(x^3)^{\frac{1}{3}} = x$.

- 3). Expand the following expression. Your answer should contain no radicals:

$$(xy^2 - z\sqrt{y})^2$$

$$\begin{aligned}
(xy^2 - z\sqrt{y})^2 &= (xy^2 - y^{\frac{1}{2}}z)^2 \\
&= (xy^2)^2 - 2(xy^2)(zy^{\frac{1}{2}}) + (y^{\frac{1}{2}}z)^2 \\
&= x^2y^4 - 2xy^{\frac{5}{2}}z + yz^2
\end{aligned}$$

4). Factor an xy^2 out of $x^2y - 2$.

$$x^2y - 2 = xy^2 \left(\frac{x^2y - 2}{xy^2} \right) = xy^2 \left(\frac{x^2y}{xy^2} - \frac{2}{xy^2} \right) = xy^2 \left(\frac{x}{y} - \frac{2}{xy^2} \right)$$

5). Simplify completely. Leave everything in factored form.

$$\begin{aligned}
\frac{2}{x-3} + \frac{4x}{x+3} - \frac{6}{x^2-9} \\
\frac{2}{x-3} + \frac{4x}{x+3} - \frac{6}{x^2-9} &= \frac{2(x+3) + 4x(x-3) - 6}{(x+3)(x-3)} \\
&= \frac{2x+6+4x^2-12x-6}{(x+3)(x-3)} \\
&= \frac{4x^2-10x}{(x+3)(x-3)} \\
&= \frac{2x(2x-5)}{(x+3)(x-3)}
\end{aligned}$$