Math-08 Homework #10 Solutions

<u>Reading</u>

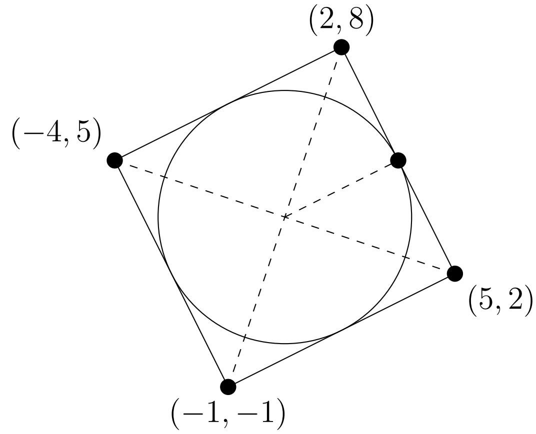
- Text book section 2.1 to 2.3

<u>Problems</u>

1). The corners of a square are given by the following coordinates:

$$(-1,-1), (-4,5), (2,8), (5,2)$$

a). Determine the equation of the circle inscribed inside the square.



The center of the circle is the midpoint on either of the diagonals. Let's pick the diagonal from $(-1,-1)$ to $(2,8)$:

$$x = \frac{-1+2}{2} = \frac{1}{2}$$

$$y = \frac{-1+8}{2} = \frac{7}{2}$$

So the center is at $\left(\frac{1}{2}, \frac{7}{2}\right)$

Since the circle touches the square halfway along each edge, we can find another point on the circle by taking the midpoint between any two adjacent points. Let's pick $(2,8)$ and $(5,2)$:

$$x = \frac{2+5}{2} = \frac{7}{2}$$

$$y = \frac{8+2}{2} = 5$$

So the point of intersection is $\left(\frac{7}{2}, 5\right)$

The radius is the distance between the center and the found point:

$$\begin{aligned}
r^2 &= \left(\frac{7}{2} - \frac{1}{2}\right)^2 + \left(5 - \frac{7}{2}\right) \\
&= 3^2 - \left(\frac{3}{2}\right)^2 \\
&= 9 + \frac{9}{4} \\
&= \frac{45}{4}
\end{aligned}$$

So, the equation for the circle is:

$$\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{7}{2}\right)^2 = \frac{45}{4}$$

b). Determine the equation of the line parallel to the side from $(-1,-1)$ to $(5,2)$ and through the center of the circle.

Parallel lines have the same slope. The slope of the line connecting the two points on the square is:

$$m = \frac{2+1}{5+1} = \frac{3}{6} = \frac{1}{2}$$

Since we have a slope and a point (the center of the circle), simply plug values into the point-slope form:

$$y - \frac{7}{2} = \frac{1}{2}\left(x - \frac{1}{2}\right)$$

This answer is good enough, but if you really want to see the y-intercept form:

$$y - \frac{7}{2} = \frac{1}{2}\left(x - \frac{1}{2}\right) \tag{1}$$

$$y - \frac{7}{2} = \frac{1}{2}x - \frac{1}{4} \tag{2}$$

$$y = \frac{1}{2}x + \frac{13}{4} \tag{3}$$

$$\tag{4}$$

c). Determine the equation of the line perpendicular to the side from $(-1,-1)$ to $(5,2)$ and through the center of the circle.

The slope of the perpendicular line is the negative reciprical:

$$y - \frac{7}{2} = -2\left(x - \frac{1}{2}\right)$$

Once again, this answer is good enough, but if you really want to see the y-intercept form:

$$\begin{aligned} y - \frac{7}{2} &= -2\left(x - \frac{1}{2}\right) && (5)\\ y - \frac{7}{2} &= -2x + 1 && (6)\\ y &= -2x + \frac{9}{2} && (7)\\ & && (8) \end{aligned}$$

2). Consider the line through the points (1,5) and (1,-1).

a). Determine the equation of the line.

Note that the x values are the same, so this is a vertical line:

$$x = 1$$

b). Determine the equation of the line parallel to the first line and through the point (-2,-2).

We want another vertical line:

$$x = -2$$

c). Determine the equation of the line perpendicular to the first line and through the point (-2,-2).

This time, we want a horizonal line:

$$y = -2$$

3). An object moving in a straight line at constant velocity has its equation of motion given by: $s = s_0 + v_0 t$, where s is the position at time t, s_0 is the initial position, and v_0 is the constant speed.

a). What are the slope and y-intercept for this linear model?

The slope represents the rate of change, which in this case is the velocity v_0.

The y-intercept is the value at time $t = 0$, which is s_0.

b). An object is moving at 10 ft/s. At time 5 seconds the object is at position $s = 60$ feet. What is the initial position s_0?

$$
\begin{aligned}
s &= s_0 + v_0 t \\
60 &= s_0 + 10(5) \\
60 &= s_0 + 50 \\
s_0 = 10 &
\end{aligned}
$$

The initial position is 10 feet.

4). A manufacturing firm buys a new machine for \$150,000. After the machine is fully depreciated, it will have a salvage value of \$5,000. Assuming a 15-year straight-line depreciation model, what will be the value of the machine after 10 years?

The depreciable amount is the purchase price minus the salvage price, or:

$$150,000 - 5,000 = 145,000$$

This must be depreciated over 15 years, so the yearly depreciation is:

$$\frac{145,000}{15}$$

So the straight-line depreciation equation for the value of the asset after t years is:

$$y = 150,000 - \frac{145,000}{15} t$$

After 10 years, the value is:

$$y = 150,000 - \frac{145,000}{15}(10) = 53,333.33$$

So, the value of the asset after 10 years of depreciation is \$53,333.33