Math-19 Homework #8 Solutions

Reading

Please read sections 5.4-5.6 and 6.4-6.6, then do all concept problems in the posted sections on webassign.

Problems

1). Consider the function:

$$f(x) = 2\tan(4\pi x - \pi) + 1 = 2\tan 4\pi\left(x - \frac{1}{4}\right) + 1$$

a). What is the period P?

$$P = \frac{\pi}{4\pi} = \frac{1}{4}$$

b). What is the horizontal translation b?

$\frac{1}{4}$ to the right

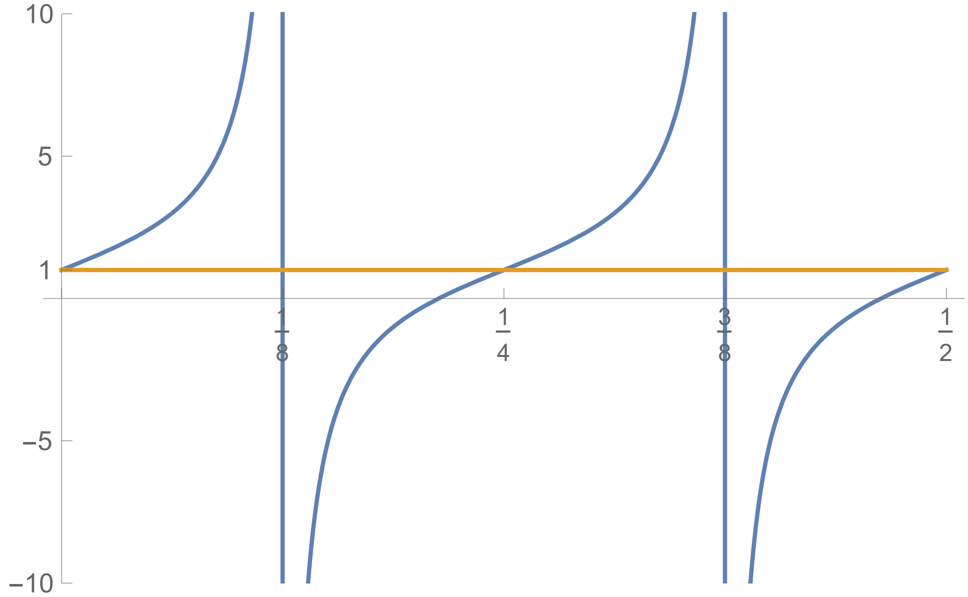
c). What is the phase angle ϕ?

$$\pi = -\pi$$

d). What is the y-intercept?

$$f(0) = 2\tan(-\pi) + 1 = 0 + 1 = 1$$

e). Sketch one cycle of the graph in the interval $(b, b + P)$ and then extend the sketch back to the y-intercept.



2). Solve for x:

$$\tan\left(3x+\frac{\pi}{2}\right)\sin(2\pi x)\cos(6x+\pi)=0$$

Hint: be careful about domain!

Each of the factors results in a set of solutions:

$$\begin{aligned}\tan\left(3x+\frac{\pi}{2}\right) &= 0\\ 3x+\frac{\pi}{2} &= k\pi\\ 3x &= -\frac{\pi}{2}+k\pi\\ x &= -\frac{\pi}{6}+k\frac{\pi}{3}\end{aligned}$$

$$\begin{aligned}\sin(2\pi x) &= 0\\ 2\pi x &= k\pi\\ x &= \frac{k}{2}\end{aligned}$$

$$\begin{aligned}
\cos(6x+\pi) &= 0 \\
6x+\pi &= \frac{\pi}{2}+k\pi \\
6x &= -\frac{\pi}{2}+k\pi \\
x &= -\frac{\pi}{12}+k\frac{\pi}{6}
\end{aligned}$$

As a final check, we make sure that none of the solutions violate the domain of the tangent function. Since none of the solutions to the sine or cosine parts land on a vertical asymptote, all of the solutions are OK.

3). Two $1kg$ masses are each suspended on a spring with $k=\pi^2$ and are stretched downward by 2 units. The first spring is released at $t=0$. The second spring is released at $t=3$.

a). Find $f_1(t)$ for the first mass.

$$f_1(t) = 2\cos\left(\sqrt{\frac{\pi^2}{1}}t\right) = 2\cos\pi t$$

b). Find $f_2(t)$ for the second mass.

$$f_2(t) = 2\cos\pi(t-3) = 2\cos(\pi t - 3\pi)$$

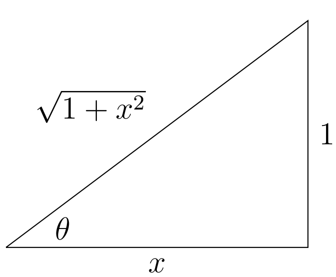
c). What is the phase difference between the two masses?

The phase angle in the previous part is 3π; however, since the period is only 2π, we can state the phase angle as $3\pi - 2\pi = \pi$ or 180°.

4). Evaluate:

$$\cot\left(\cos^{-1}\frac{x}{\sqrt{1+x^2}}\right)$$

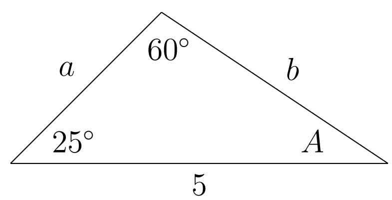
First, we repeat to ourselves, “the angle whose cosine is ....” Next, we draw a right triangle corresponding to our angle:



Note that by using the Pythagorean theorem we determine that the length of the opposite side is 1. We now take the cotangent of our angle:

$$\cot\left(\cos^{-1}\frac{x}{\sqrt{1+x^2}}\right) = \frac{x}{1} = x$$

5). Consider the following triangle:



a). Determine A.

$$A = 180^\circ - (60^\circ + 25^\circ) = 180^\circ - 85^\circ = 95^\circ$$

b). Determine a.

$$\frac{\sin 95^\circ}{a} = \frac{\sin 60^\circ}{5}$$

$$a = \frac{5\sin 95^\circ}{\sin 60^\circ}$$

$$a = 5.75$$

c). Determine b.

$$\frac{\sin 25^\circ}{b} = \frac{\sin 60^\circ}{5}$$

$$b = \frac{5\sin 25^\circ}{\sin 60^\circ}$$

$$b = 2.44$$

d). Using Heron's Formula, determine the area of the triangle.

$$s = \frac{5 + 5.75 + 2.44}{2} = 6.6$$

$$A = \sqrt{6.6(6.6-5)(6.6-5.75)(6.6-2.44)} = 6.11$$