Math-42 Worksheet #1

Propositional Logic

1. Determine whether or not the following sentences are statements of propositions. If a sentence is a proposition then determine whether it is true or false. If a sentence is not a proposition then indicate whether it is subjective or ambiguous.
   (a) Microsoft Windows is an operating system.
   (b) Linux is an operating system.
   (c) Linux is a better operating system than Windows.
   (d) $1 + 1 = 3$
   (e) $1{,}000{,}000$ is a big number.
   (f) $x + 1 = 4$
   (g) The letter 'y' is a vowel.
   (h) 0 is an even integer.
   (i) 0 is a rational number.
   (j) π is a rational number.

2. Negate the following propositions. State whether the original or negated proposition is true.
   (a) It is raining in San Jose.
   (b) 7 is a prime number.
   (c) $1 + 1 = 3$
   (d) $2 > 3$
   (e) 0 is an even number.

3. Consider the following propositions:

   $p \coloneqq$ Skiing in Lake Tahoe is fun.
   $q \coloneqq$ Driving to Lake Tahoe is boring.
   $r \coloneqq$ Ann hates going to Lake Tahoe.

   Represent the following statements using logical operators:

(a) Skiing in Lake Tahoe is fun but it is boring to drive there.

(b) If driving to Lake Tahoe is fun then Ann likes going there.

(c) Driving to Lake Tahoe is entertaining and skiing there is fun.

(d) Ann likes going to Lake Tahoe if and only if driving to there is not boring and skiing there is fun.

(e) Driving to or skiing in Lake Tahoe is boring.

4. Rewrite each implication in $p \longrightarrow q$ form:

   (a) There are clouds in the sky if it is raining.

   (b) You will pass the exam only if you receive a score of 70 or better.

   (c) x is an integer is sufficient to conclude that x is a rational number.

   (d) x is an integer is a necessary condition for x to be a rational number.

   (e) x is irrational unless x can be written as a ratio of integers $\frac{p}{q}$ where $q \neq 0$.

5. Determine whether the following implications are true or false.

   (a) If 0 is an even number then $\sqrt{2}$ is rational.

   (b) If 0 is an even number then $\sqrt{2}$ is irrational.

   (c) If 0 is an odd number then $\sqrt{2}$ is rational.

   (d) If 0 is an odd number then $\sqrt{2}$ is irrational.

6. Consider the proposition: if $x = \sqrt{2}$ then x is a irrational number.

   (a) Construct the inverse, converse, and contrapositive.

   (b) Of the four forms of the implication, which are true and which are false?

7. Determine whether the following equivalences are true or false.

   (a) Sacramento is the capital of California if and only if Carson City is the capital of Nevada.

   (b) x is an even integer is equivalent to there exists an integer k such that $x = 2k$.

   (c) Pigs can grow wings and fly iff the moon is made of green cheese.

(d) An integer is composite if and only if it is not prime.

(e) An integer x is a perfect square iff there exists an integer k such that $x = k + k$.

8. Let p, q, and r be propositions and consider the compound proposition:

$$p \longrightarrow p \wedge \neg q \longleftrightarrow q \vee r$$

(a) Use parentheses to indicate the correct order of operation with respect to operator precedence.

(b) Construct a truth table for this 3-variable proposition. Be sure to show each intermediary result and then the final result.

(c) Consider the following propositions:

$p \coloneqq \sqrt{2}$ is a rational number.
$q \coloneqq 0$ is an even number.
$r \coloneqq x^2 = 1 \longrightarrow x = 1$

Using your truth table, indicate whether the compound proposition is true or false. Be sure to clearly indicate the row that gives you the correct answer.