Math-1003b Homework #5 Solutions

Reading

- Sections 8.1-8.4

Problems

1). Let $f(x) = x^2 + 2x - 5$. Evaluate:

$$\frac{f(x+h) - f(x)}{h}$$

Remember, to evaluate a function at a particular value, even if that value is specified in terms of the same or other variables, simply replace each occurrence of the variable in the original equation with the target value:

$$f(x+h) = (x+h)^2 + 2(x+h) - 5 = x^2 + 2xh + h^2 + 2x + 2h - 5$$

Now, putting it altogether (remember to use parentheses during substitution!):

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{(x^2 + 2xh + h^2 + 2x + 2h - 5) - (x^2 + 2x - 5)}{h} \\ &= \frac{2xh + h^2 + 2h}{h} \\ &= 2x + h + 2 \end{aligned}$$

Notice that only terms with an h were left in the numerator, to be canceled by the denominator! This is a very important form, called the *difference quotient* form, in calculus. In fact, those of you who go on to take calculus (and I sincerely hope that some of you do), will encounter this form in the first week.

2). Let $f(x) = 2x+5$ and $g(x) = x^2$. For each of the following, evaluate and state the domain.

a). $f + g$

$(f+g)(x) = f(x) + g(x) = (2x+5) + (x^2) = x^2 + 2x + 5$

Domain: $\mathbb{R}$

b). fg

$(fg)(x) = f(x)g(x) = (2x+5)(x^2) = 2x^3 + 5x^2$

Domain: $\mathbb{R}$

c). $\frac{f}{g}$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{2x+5}{x^2}$

Since there is a possibility of a zero denominator, $x \neq 0$. So, stating domain in set-builder and interval form:

Domain: $\{x \in \mathbb{R} \mid x \neq 0\} = (-\infty, 0) \cup (0, \infty)$

d). $\frac{f}{f}$

$\left(\frac{f}{f}\right)(x) = \frac{f(x)}{f(x)} = \frac{2x+5}{2x+5} = 1$

Since the result is a constant function, it looks like there are no limitations on x; however, we need to honor the original form, where $x \neq -\frac{5}{2}$:

Domain: $\{x \in \mathbb{R} \mid x \neq -\frac{5}{2}\} = (-\infty, -\frac{5}{2}) \cup (-\frac{5}{2}, \infty)$

e). $f \circ g$

Remember that the *inner* function goes first:

$(f \circ g)(x) = f(g(x)) = f(x^2) = 2x^2 + 5$

Domain: $\mathbb{R}$

Make sure that you also understand the following:

$(g \circ f)(x) = g(f(x)) = g(2x + 5) = (2x + 5)^2$