Weyr Indices

It has been proven that every matrix $A \in M_n$ is similar to a Jordan matrix J_A, but how does one find such a J_A?.

Definition: Weyr Index

Let $A \in M_n$ and let $A \sim J_A$. The *Weyr index* denoted $b_i(\lambda)$ indicates the number of $i \times i$ Jordan blocks with respect to $\lambda \in \sigma(A)$ in J_A.

Example

$J_A = J_3(3)$

$\sigma(A) = \{3\}$

$b_1(3) = 0$
$b_2(3) = 0$
$b_3(3) = 1$

$$J_A = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$

$J_B = J_2(3) \oplus J_1(3)$

$\sigma(B) = \{3\}$

$b_1(3) = 1$
$b_2(3) = 1$
$b_3(3) = 0$

$$J_B = \left[\begin{array}{cc|c} 3 & 1 & 0 \\ 0 & 3 & 0 \\ \hline 0 & 0 & 3 \end{array}\right]$$

Example

$J_C = J_1(2) \oplus J_2(2) \oplus J_2(2) + J_4(3)$

$\sigma(C) = \{2, 3\} \qquad n = 9$

	$\lambda = 2$	$\lambda = 3$
b_1	1	0
b_2	2	0
b_3	0	0
b_4	0	1
b_5	0	0
b_6	0	0
b_7	0	0
b_8	0	0
b_9	0	0

$$\left[\begin{array}{c|cc|cc|cccc} 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 3 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 \end{array}\right]$$