Math-42 Sections 01, 02, 05

Homework #1 Solutions

Problem

1. Let p, q, and r be propositions and consider the compound proposition:

   $$p \to p \wedge \neg q \leftrightarrow q \vee r$$

   Construct a truth table for this 3-variable proposition. Be sure to show each intermediary result and then the final result. Be wary of operator precedence!

   First, lets clarify the proper precedence using parentheses:

   $$(p \to (p \wedge \neg q)) \leftrightarrow (q \vee r)$$

   Now construct the truth table:

p	q	r	$\neg q$	$p \wedge \neg q$	$q \vee r$	$p \to (p \wedge \neg q)$	$(p \to (p \wedge \neg q)) \leftrightarrow (q \vee r)$
F	F	F	T	F	F	T	F
F	F	T	T	F	T	T	T
F	T	F	F	F	T	T	T
F	T	T	F	F	T	T	T
T	F	F	T	T	F	T	F
T	F	T	T	T	T	T	T
T	T	F	F	F	T	F	F
T	T	T	F	F	T	F	F

2. Consider the following propositions:

   $p \coloneqq \sqrt{2}$ is a rational number.
   $q \coloneqq 0$ is an even number.
   $r \coloneqq x^2 = 1 \to x = 1$

   Using your truth table from the first problem, indicate whether the compound proposition is true or false. Be sure to clearly indicate the row that gives you the proper answer.

   These propositions correspond to the FTF row (row 3), so the proposition is true.