Math-13 Sections 01, 02

Exam #2

Due: 11/8/2020 at 11:59pm

This exam is open book and notes. You may use a calculator. No collaboration or other web access is allowed. All answers must be in exact form unless stated otherwise (i.e., no decimal answers allowed). You *must* show all work and that work must be logical and complete; there is *no* credit for guessed answers or answers without supporting work.

You must work the exam problems, in order, on separate sheets of paper; camscan your results into a single PDF file; and then submit your PDF file back to Moodle (just like the written home-works). This *must* be done by the deadline; late exams, multiple page or non-PDF submissions, and exams sent by email will not be accepted.

Good luck!

1. What are the three characterizations of the derivative that were discussed in class?

2. Let $f(x)$ be continuous on $[1, 5]$ and differentiable on $(1, 5)$. State the theorem that supports the following conclusions:

   (a) If $f(1) = 1$ and $f(5) = 9$ then there exists some $c \in (1, 5)$ such that $f'(c) = 2$.

   (b) If $f(1) = -1$ and $f(5) = 1$ then there exists some $c \in [1, 5]$ such that $f(c) = 0$.

   (c) $f(x)$ has an absolute minimum and an absolute maximum on $[1, 5]$.

   (d) If $f(1) = 1$ and $f(5) = 1$ then there exists some $c \in (1, 5)$ such that $f'(c) = 0$.

3. Let $f(x) = \sqrt{x^2 + 1}$.

   (a) Using the definition of the derivative (not the formulas!), determine $f'(x)$.

   (b) Using the chain rule, determine $f'(x)$.

4. Let $f(t) = \frac{1}{3}t^3 - \frac{1}{4}t^2 + 3t - 100$. Determine $f'(t)$.

5. Let $f(x) = 2x^2 - 3x + 5$.

    (a) Determine the equation of the tangent line at $x = 2$.

    (b) Determine the equation of the normal line at $x = 2$.

6. Let $f(x) = (2x + 3)^2\sqrt{x^2 + 1}$. Determine $f'(x)$. Your answer must be fully simplified for full credit.

7. Let $f(x) = \dfrac{x^2 + 3x - 2}{2x - 1}$. Using the quotient rule, determine $f'(x)$. Your answer must be fully simplified for full credit.

8. Let $y^2 + y - 5x^2 = 100$. Determine y'.

9. A home-made model rocket is launched from the ground at an initial speed of 256 ft/s. The height h of the rocket (in feet) at time t (in seconds) is given by $h(t) = 256t - 16t^2$. What is the maximum height of the rocket and how long does it take to achieve that height?

10. Let $f(x) = x^3 + x^2 - 4x - 4$.

    (a) Using the rational roots theorem, completely factor $f(x)$. If you don't think that you can do this then you can ask Wolfram Alpha to help you; however, you will receive no credit for this part.

    (b) What are the critical points of $f(x)$?

    (c) What is the y-intercept of $f(x)$?

    (d) What are the critical points of $f'(x)$?

    (e) Using the first derivative test, determine the relative extrema of $f(x)$.

    (f) Using the second derivative test, verify the relative extrema of $f(x)$.

    (g) What are the critical points of $f''(x)$?

    (h) Using the second derivative, prove that the critical point of $f''(x)$ is a point of inflection.

    (i) What is the end behavior of $f(x)$?

(j) Sketch $f(x)$. For full credit, all intercepts, extrema, and points of inflection must be labeled with their coordinate values.