Eigenvalues of Positive Operators

Theorem

Let A be a positive operator on a Hilbert space H:

λ is an eigenvalue of $A \implies \lambda \geq 0$.

Proof

Assume λ is an eigenvalue of A.
$\exists\, \vec{x} \in H$ such that $\vec{x} \neq \vec{0}$. $A \geq 0 \implies \langle A\vec{x}, \vec{x} \rangle \geq 0$

$\langle A\vec{x}, \vec{x} \rangle = \langle \lambda\vec{x}, \vec{x} \rangle = \lambda \left\| \vec{x} \right\| \geq 0$

$\therefore \lambda \geq 0$