Math-08 Homework #1

Reading

- If necessary, skim through the documents in the resources module on canvas dealing with mathematical logic, sets, rational numbers, and prime factorization. The documents have more information than you will need; however, if you have something in your notes that you do not understand then you can find more description in the documents.
- Text book sections 0.1 and 0.2.

Problems

1). Let:

$$\begin{aligned} P &:= 0 \text{ is a positive number} \\ Q &:= 0 \text{ is a rational number} \end{aligned}$$

Determine whether the following are true or false:

a). P

b). Q

c). not P

d). not Q

e). P and Q

f). P or Q

2). Decimal to rational form conversion.

a). Convert $0.14\overline{23}$ to rational form.

b). Show that $0.\overline{1} = \frac{1}{9}$.

c). If this is so, then $\frac{2}{9}$ should equal $0.\overline{2}$, $\frac{3}{9}$ should equal $0.\overline{3}$, and so on until $\frac{8}{9}$ should equal $0.\overline{8}$. So, what do you think that $0.\overline{9}$ should equal?

d). Show that this is so by converting $0.\overline{9}$ to rational form.

e). Take a guess at what $25.3\overline{9}$ equals.

3). Rational numbers and closure.

a). Write down the definition of $\mathbb{Q}$ using setbuilder notation.

b). Prove that $\mathbb{Q}$ is closed under addition (Hint: Assume that two numbers are in $\mathbb{Q}$, use the definition to express them as a ratio of integers, then add then and show why the result must be rational).

c). Prove that $\mathbb{Q}$ is closed under multiplication (Hint: same as above, but multiply the two numbers).

d). Give a counterexample showing that $\mathbb{R} - \mathbb{Q}$ is not closed under addition.

e). Give a counterexample showing that $\mathbb{R} - \mathbb{Q}$ is not closed under multiplication.

4). Let:

$$\begin{aligned} A &= \text{the set of all positive real numbers} \\ B &= \text{the set of real numbers between -3 (exclusive) and 3 (inclusive)} \end{aligned}$$

a). Graph each set on the real number line.

b). Represent each set using set-builder notation.

c). Represent each set using interval notation.

d). Graph $A \cup B$ and represent it in interval notation.

e). Graph $A \cap B$ and represent it in interval notation.