Math-42 Worksheet #3

Propositional Equivalences

1. Using truth tables, show that implication is left distributive:

$$p \to (q \wedge r) \equiv (p \to q) \wedge (p \to r)$$
$$p \to (q \vee r) \equiv (p \to q) \vee (p \to r)$$

2. Using truth tables, show that implication is *not* right distributive.

3. What are the correct equivalences for right distributivity?

4. Prove that $\neg(p \to q) \equiv p \wedge \neg q$ using:
   (a) A truth table.
   (b) Logical equivalences (hint: implication equivalence and deMorgan).

5. Negate each of the following compound propositions:
   (a) Jack goes to school and he gets good grades.
   (b) If Jack goes to school then he will get good grades.
   (c) If Jack goes to school and gets good grades then he will get a good job.
   (d) Jack goes to school or gets a good job.

6. Prove that $[\neg p \wedge (p \vee q)] \to q$ is a tautology using:
   (a) A truth table.
   (b) Logical equivalences.

7. Prove that $p \oplus q$ is equivalent to $\neg(p \leftrightarrow q)$ using:

   (a) A truth table.

   (b) Logical equivalences.

8. Consider the following logical equation

   $$(p \wedge q) \wedge (p \leftrightarrow r) \wedge (r \vee p \rightarrow q) \wedge (s \rightarrow \neg r) \wedge s$$

   Without using a truth table (and you wouldn't want to), determine whether the logical equation is satisfiable.