Operations

1). Union

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

2). Intersection

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

3). Difference

$$A - B = \{x \mid x \in A \text{ and } x \notin B\}$$

4). Cartesian Product

$$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$$

5). Power Set

$$\mathcal{P}(A) = \{X \mid X \subseteq A\}$$

6). Cardinality

$$|A| = \text{the (possibly infinite) number of elements in } A$$

Example

$A = \{1, 2, 3\} \quad B = \{3, 4\}$

$A \cup B = \{1, 2, 3, 4\}$	$\lvert A \cup B \rvert = 4$
$A \cap B = \{3\}$	$\lvert A \cap B \rvert = 4$
$A - B = \{1, 2\}$	$\lvert A - B \rvert = 2$
$A \times B = \{(1,3), (1,4), (2,3), (2,4), (3,3), (3,4)\}$	$\lvert A \times B \rvert = \lvert A \rvert \lvert B \rvert = 3 \cdot 2 = 6$
$\mathcal{P}(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$	$\lvert \mathcal{P}(A) \rvert = 2^{\lvert A \rvert} = 2^3 = 8$

Theorem

For all sets A and B:

1). $|A \times B| = |A|\,|B|$

2). $|\mathcal{P}(A)| = 2^{|A|}$