Math-13 Sections 01 and 02

Homework #4 Solutions

Due: Midnight 9/25

1. Determine the following limit:

$$\begin{aligned}
\lim_{x\to\frac{1}{2}} \frac{2x^3+9x^2-5x}{4x^2-1} &= \lim_{x\to\frac{1}{2}} \frac{x(2x^2+9x-5)}{(2x+1)(2x-1)} \\
&= \lim_{x\to\frac{1}{2}} \frac{x\cancel{(2x-1)}(x+5))}{(2x+1)\cancel{(2x-1)}} \\
&= \lim_{x\to\frac{1}{2}} \frac{x(x+5)}{2x+1} \\
&= \frac{\frac{1}{2}\left(\frac{1}{2}+5\right)}{2\left(\frac{1}{2}\right)+1} \\
&= \frac{\frac{1}{2}\left(\frac{11}{2}\right)}{1+1} \\
&= \frac{\frac{11}{4}}{2} \\
&= \frac{11}{8}
\end{aligned}$$

2. Determine the following limit:

$$\begin{aligned}
\lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} &= \lim_{h\to 0} \left[\left(\frac{\sqrt{x+h}-\sqrt{x}}{h}\right)\left(\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}\right)\right] \\
&= \lim_{h\to 0} \frac{(\cancel{x}+h)-\cancel{x}}{h(\sqrt{x+h}+\sqrt{x})} \\
&= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h}+\sqrt{x})} \\
&= \lim_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}} \\
&= \frac{1}{\sqrt{x+0}+\sqrt{x}} \\
&= \frac{1}{\sqrt{x}+\sqrt{x}} \\
&= \frac{1}{2\sqrt{x}}
\end{aligned}$$