Math-19 Homework #9 Solutions

Reading

Please read sections 7.1-7.5, then do all concept problems in the posted sections on webassign.

Problems

1). Prove that the following is an identity:

$$\frac{1}{\csc x + \cot x} + \frac{1}{\csc x - \cot x} = 2\csc x$$

$$\begin{aligned}
\frac{1}{\csc x + \cot x} + \frac{1}{\csc x - \cot x} &= \frac{(\csc x - \cot x) + (\csc x + \cot x)}{\csc^2 x - \cot^2 x} \\
&= \frac{2\csc x}{(1 + \cot^2 x) - cot^2 x} \\
&= \frac{2\csc x}{1} \\
&= 2\csc x
\end{aligned}$$

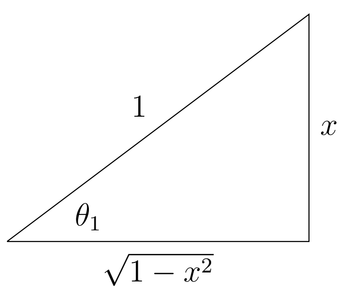
2). : Write the following as a function of x with no trig functions:

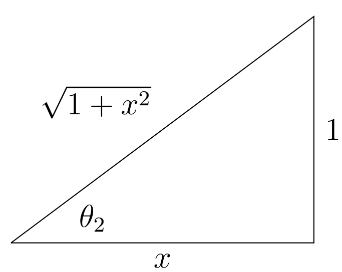
$$\sin\left(\sec^{-1}\frac{1}{\sqrt{1-x^2}} + \csc^{-1}\sqrt{1+x^2}\right)$$

Let:

$$\theta_1 = \sec^{-1}\frac{1}{\sqrt{1-x^2}}$$

$$\theta_2 = \csc^{-1}\sqrt{1+x^2}$$





$$
\begin{aligned}
\sin(\theta_1+\theta_2) &= \sin\theta_1\cos\theta_2+\cos\theta_1\sin\theta_2\\
&= \left(\frac{x}{1}\right)\left(\frac{x}{\sqrt{1+x^2}}\right)+\left(\frac{\sqrt{1-x^2}}{1}\right)\left(\frac{1}{\sqrt{1+x^2}}\right)\\
&= \frac{x^2+\sqrt{1-x^2}}{\sqrt{1+x^2}}
\end{aligned}
$$

3). Write the following as a single sine function. Note that you can use approximate values (i.e., your calculator) for the various coefficient and angle calculation. Use 4 decimal places.

$$\cos\left(x+\frac{\pi}{3}\right)+\sin\left(x-\frac{\pi}{4}\right)$$

$$
\begin{aligned}
\cos\left(x+\frac{\pi}{3}\right)+\sin\left(x-\frac{\pi}{4}\right) &= (\cos x\cos\frac{\pi}{3}-\sin x\sin\frac{\pi}{3})+(\sin x\cos\frac{\pi}{4}-\cos x\sin\frac{\pi}{4})\\
&= \frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x+\frac{1}{\sqrt{2}}\sin x-\frac{1}{\sqrt{2}}\cos x\\
&= \left(\frac{1}{\sqrt{2}}-\frac{\sqrt{3}}{2}\right)\sin x+\left(\frac{1}{2}-\frac{1}{\sqrt{2}}\right)\cos x\\
&= -0.1589\sin x-0.2071\cos x
\end{aligned}
$$

Now, let $A=\sqrt{(-0.1589)^2+(-0.2071)^2}=0.2610$

$$
\begin{aligned}
\cos\left(x+\frac{\pi}{3}\right)+\sin\left(x-\frac{\pi}{4}\right) &= A\left[\frac{-0.1589}{A}\sin x+\frac{-0.2071}{A}\cos x\right]\\
\cos\phi &= \frac{-0.1589}{A}\\
\sin\phi &= \frac{-0.2071}{A}\\
\cos\left(x+\frac{\pi}{3}\right)+\sin\left(x-\frac{\pi}{4}\right) &= A\left[\sin x\cos\phi+\cos x\sin\phi\right]\\
&= A\sin x+\phi\\
&= 0.2610\sin\left[x+\tan^{-1}\frac{(-0.2071)}{(-0.1589)}\right]\\
&= 0.2610\sin(x+0.9163+\pi)\\
&= 0.2610\sin(x+4.0579)\\
&= 0.2610\sin(x+232.5^\circ)
\end{aligned}
$$

Be careful! Your calculator is going to give you the angle in the reduced domain, in this case QI; however, the actual angle is in QIII. So, we must add π to the value you get from your calculator.

4). Find *all* possible solutions for x:

$$2\sin^2 x + (\sqrt{3} - 4)\sin x - 2\sqrt{3} = 0$$

$$(2\sin x + \sqrt{3})(\sin x - 2) = 0$$

$\sin x = 2$ has no solutions

$\sin x = -\frac{\sqrt{3}}{2}$

This occurs in QIII and QIV with a reference angle of $\frac{\pi}{3}$

$$x = -\frac{\pi}{3} + 2\pi k$$

or

$$x = -\frac{2\pi}{3} + 2\pi k$$

5). Find *all* possible solutions for x:

$$\sin 2x + cosx = 0$$

$$\begin{aligned} \sin 2x + cosx &= 0 \\ 2\sin x\cos x + \cos x &= 0 \\ \cos x(2\sin x + 1) &= 0 \end{aligned}$$

$\cos x = 0$ or $\sin x = -\frac{1}{2}$

$$x = \frac{\pi}{2} + \pi k$$

or

$$x = -\frac{\pi}{6} + 2\pi k$$

or

$$x = -\frac{5\pi}{6} + 2\pi k$$