Math-19 Homework #14 Solutions

Problems

1). A blu-ray disk has a diameter of 12 cm. The track of recorded information on the disk spirals in from the outside toward the center. A blu-ray player must make sure that the track passing under the optical reader stays at a constant linear speed, so the motor will vary from 200 to 500 rpm, depending on where the disk is being read.

a). Which angular speed is used for the part of the track on the outer rim of the disk? Why?

For constant angular speed, a point on the outer rim is going to have a faster linear speed than a point closer to the center. The equation $v = r\omega$ tells us this. Thus, the player motor must start slow at the rim and speed up as the reader moves along the spiral toward the center. Therefore, the lowest speed of 200 rpm is used when reading from near the rim.

b). What is the linear speed of the outer track in cm/s?

$$v = r\omega = 6\,\text{cm} \cdot \frac{200\,\text{rev}}{1\,\text{min}} \cdot \frac{2\pi}{1\,\text{rev}} \cdot \frac{1\,\text{min}}{60\,\text{sec}} = 40\pi\,\text{cm/s} \approx 126\,\text{cm/s}$$

c). What is the distance from the center of the disk to the innermost part of the track?

Since the linear speed is constant, we use the highest angular speed and solve for the radius. Note that we can keep the original units because they will cancel properly.

$$\begin{aligned} \omega_1 &= \omega_2 \\ 6\,\text{cm} \cdot 200\,\text{rpm} &= r \cdot 500\,\text{rpm} \\ r &= 6\,\text{cm}\left(\frac{200\,\text{rpm}}{500\,\text{rpm}}\right) \\ r &= 2.4\,\text{cm} \end{aligned}$$

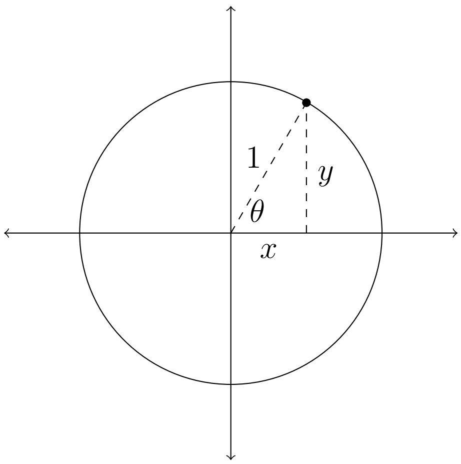
2). Determine a positive and a negative coterminal angle for the angle $\dfrac{2\pi}{3}$.

Coterminal angles are multiples of 2π in either direction. For example:

$$\frac{2\pi}{3} + 2\pi = \frac{2\pi}{3} + \frac{6\pi}{3} = \frac{8\pi}{3}$$

$$\frac{2\pi}{3} - 2\pi = \frac{2\pi}{3} - \frac{6\pi}{3} = -\frac{4\pi}{3}$$

3). Use a sketch of the unit circle to show why $\sin^2\theta + \cos^2\theta = 1$, and then use that formula to prove the other two forms of the pythagrean identity.



$$x^2 + y^2 = 1$$
$$x = 1 \cdot \cos\theta = \cos\theta$$
$$y = 1 \cdot \sin\theta = \sin\theta$$
$$\sin^2\theta + \cos^2\theta = x^2 + y^2 = 1$$

Divide both sides by $\cos^2\theta$ to get:

$$\tan^2\theta + 1 = \sec^2\theta$$

Divide both sides by $sin^2\theta$ to get:

$$1 + \cot^2\theta = \csc^2\theta$$

4). Consider the following sinusoidal function:

$$f(x) = -3\sin\frac{\pi}{2}(x-1)$$

a). What is the amplitude?

$$A = |-3| = 3$$

b). What is the period?

$$P = \frac{2\pi}{\frac{\pi}{2}} = 4$$

c). What is b (the horizontal translation)?

1 unit to the right

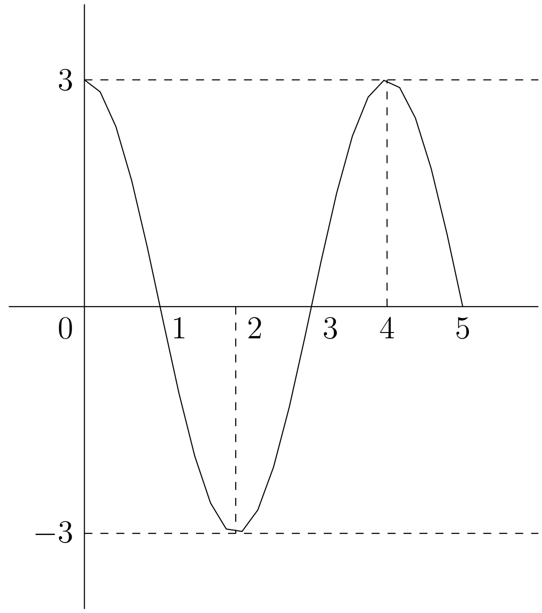
d). What is ϕ (the phase angle)?

$$\phi = \frac{\pi}{2}(-1) = -\frac{\pi}{2}$$

e). Is the phase angle leading or lagging?

Lagging

f). Sketch the graph from $[0, b + \text{period}]$, i.e., one full period starting from the horizontal shift point, and then extended back to 0. You must clearly show the amplitude and the x values for each zero/min/max.



g). Looking at your sketch, what is an equivalent function in terms of $\cos$? (Hint: try to find where a cos graph overlays your graph)

$$y = 3\cos\frac{\pi}{2}x$$