Index Sets

Definition

Let I be a set. The collection of sets given by:

$$\{A_i \mid i \in I\}$$

is called an indexed family of sets and I is called an index set.

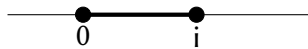Note that I may be infinite.

Operations

1). Union

$$\bigcup_{i \in I} A_i = \{a \mid \exists i \in I, a \in A_i\}$$
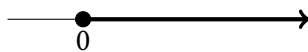
2). Intersection

$$\bigcap_{i \in I} A_i = \{a \mid \forall i \in I, a \in A_i\}$$

Example

Let $I = Z^+$ and $S_i = [0, i] = \{x \in \mathbb{R} \mid 0 \leq x \leq i\}$

$$S = \bigcup_{i \in I} S_i = [0, \infty) = \{x \in \mathbb{R} \mid x \geq 0\}$$

$$T = \bigcap_{i \in I} S_i = [0, 1] = \{x \in \mathbb{R} \mid 0 \leq x \leq 1\}$$

$$S - T = (1, \infty) = \{x \in \mathbb{R} \mid x > 1\}$$