

Homework #1 Solutions

Problem

Consider the function: $f(x) = x^2$. Calculate the following limit:

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

First note that if we try substitution with this form we get the indeterminate form $\frac{0}{0}$. So, we need to do some algebra first:

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} &= \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h} \\ &= \lim_{h \rightarrow 0} \frac{2xh + h^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cancel{h}(2x + h)}{\cancel{h}} \\ &= \lim_{h \rightarrow 0} (2x + h) \\ &= 2x + 0 \\ &= 2x \end{aligned}$$

This is how the solution should be presented, with no shortcuts. Each line is a complete statement. Some students insist on syntactic shortcuts that don't make sense, like omitting the limit operator until the end. When you do this you are in danger of me not being able to follow what you are doing. If I have to go back and reread your solutions 3 times to try and figure out what is going on then I am just going to assume that it is wrong and I will not entertain verbal explanations after the fact on why it is right — i.e., no video replay in this game.

Also please note that some students insist on continuing to write the limit operator after they have done the substitution:

$$\begin{aligned} \lim_{h \rightarrow 0} (2x + 0) \\ \lim_{h \rightarrow 0} 2x \end{aligned}$$

This is incorrect. I didn't take off points this time, but I will in the future.

One final note, please submit your homework on a separate piece of college rule or graph paper — not the original problem sheet. Remove any ripped edges. Finally, do not add extra notes or scratched out work on your submission. Pencil is best.