Math-42 Worksheet #9

Sets

1. Identify whether each of the following sets are well-defined:
   (a) The integers.
   (b) The real numbers between 1 and 100, inclusive.
   (c) Large numbers.
   (d) People that Mary knows.
   (e) Restaurants close to Mary's house.

2. Make sure that you can identify the following special subsets of the real numbers:
   (a) $\mathbb{N}$
   (b) $\mathbb{Z}$
   (c) $\mathbb{Q}$
   (d) $\mathbb{R}$
   (e) $\mathbb{R} - \mathbb{Q}$

3. Use real number line graphs to represent the following sets:
   (a) $\{x \in \mathbb{R} \,|\, -2 < x < 2\}$
   (b) $\{x \in \mathbb{Z} \,|\, -2 < x < 2\}$
   (c) $\{x \in \mathbb{N} \,|\, -2 < x < 2\}$

4. Write each of the following sets using interval notation:
   (a) $\{x \in \mathbb{R} \,|\, -\pi < x \leq 2\pi\}$
   (b) $\{x \in \mathbb{R} \,|\, x > 3\}$
   (c) $\{x \in \mathbb{R} \,|\, x \leq -1\}$
   (d) $\mathbb{R}$

5. Consider the following sets:

$$A = \{1, 2, 3, 4\}$$
$$B = \{3, 1, 2, 3\}$$
$$C = \{1, 2, 3, 4, 1\}$$
$$D = \{1, 2, 4\}$$
$$E = \{1, 2, 3, 4, 5\}$$
$$F = \{1, 2, 3, 4, \{5\}\}$$

Determine if the following propositions are true or false and give a reasons for each answer:

(a) $A = A$

(b) $A \subseteq A$

(c) $A \subset A$

(d) $A \in A$

(e) $A = B$

(f) $B = A$

(g) $A \subseteq B$

(h) $A \subset B$

(i) $A \in B$

(j) $A = C$

(k) $A = D$

(l) $D = A$

(m) $A \subset D$

(n) $D \subseteq A$

(o) $D \subset A$

(p) $A \subset E$

(q) $A \subseteq E$

(r) $E \subseteq A$

(s) $E \subset F$

(t) $3 \in E$

(u) $0 \in E$

(v) $5 \in E$

(w) $5 \in F$

(x) $\{5\} \in E$

(y) $\{5\} \in F$

(z) $\{1, 2, 3\} \in F$

6. Determine the cardinality of each of the sets in the previous problem.

7. Determine if the following propositions are true or false:

   (a) $\emptyset = \{\}$

   (b) $\emptyset = \{\emptyset\}$

   (c) $\emptyset \in \{\emptyset\}$

   (d) $\emptyset \in \{\{\emptyset\}\}$

   (e) $\emptyset \subseteq \{\}$

   (f) $\emptyset \subset \{\}$

   (g) $\emptyset = \{1, 2, 3\}$

   (h) $\emptyset \subset \{1, 2, 3\}$

   (i) $\{1, 2, 3\} \subset \emptyset$

   (j) $\emptyset \in \{1, 2, 3\}$

8. Determine if the following sets are finite or infinite:

   (a) $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

   (b) $\{1, 2, 3, 4, 5, \ldots, 1000000000\}$

   (c) $\{1, 2, 3, \ldots\}$

   (d) $\{\ldots, -2, -1, 0, 1, 2, 3\}$

   (e) $\mathbb{N}$

   (f) $\{x \in \mathbb{R} \,|\, 0 \leq x \leq 0.0001\}$

   (g) $\{x \in \mathbb{N} \,|\, 0 < x < 100\}$

   (h) $\emptyset$

9. Let $A = \{1, 2, a, z\}$. Determine $\mathcal{P}(A)$.

10. Let $A = \{1, -10, \pi\}$ and $B = \{a, z\}$. Determine $A \times B$.