# Math-08 Homework #14 Solutions

Reading

- Text book chapter 4

Problems

Make sure that all sketches have all important points and asymptotes clearly marked.

1). List the transformations, find all intercepts, and sketch:

$$y = -2e^{x+1} + 5$$

Transformations:

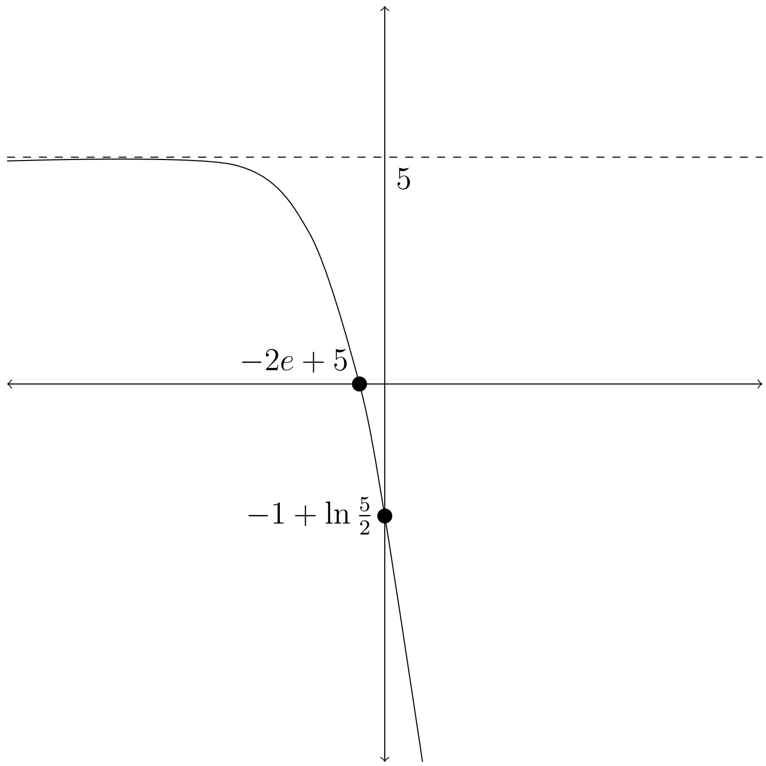
1) Start with $y = e^x$
2) Translate left 1
3) Scale by 2
4) Reflect across x-axis
5) Translate up 5

y-intercept:

$$y = -2e^{0+1} + 5 = -2e + 5 \approx -0.44$$

x-intercept:

$$\begin{aligned} 0 &= -2e^{x+1} + 5 \\ 2e^{x+1} &= 5 \\ e^{x+1} &= \frac{5}{2} \\ x+1 &= \ln\frac{5}{2} \\ x &= -1 + \ln\frac{5}{2} \\ x &\approx -0.1 \end{aligned}$$




2). List the transformations, find all intercepts, and sketch:

$$y = 3\ln(x-2) + 1$$

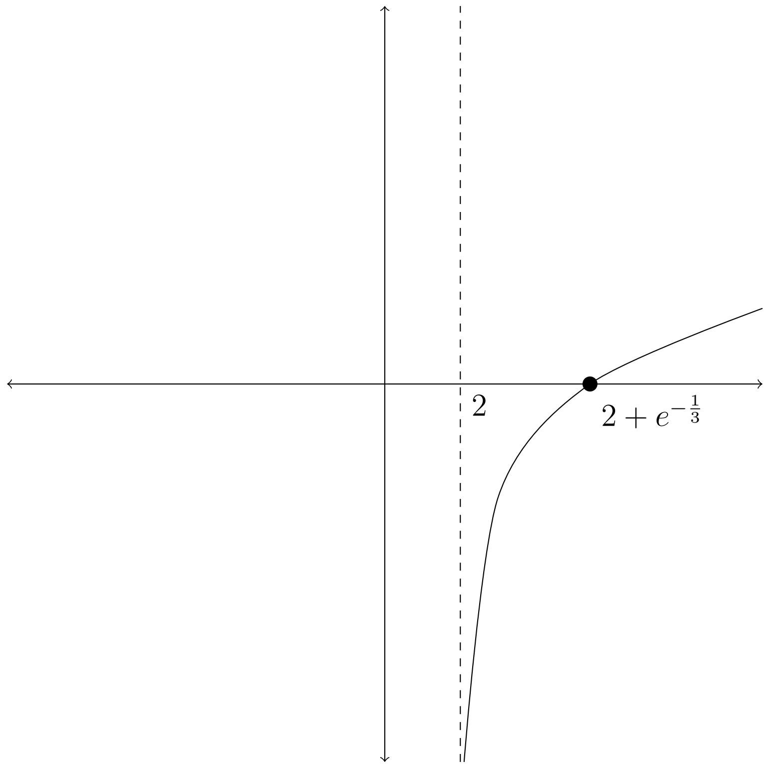
Transformations:

1) Start with $y = \ln x$
2) Translate right 2
3) Scale by 3
4) Translate up 1

y-intercept: none

x-intercept:

$$\begin{aligned} 0 &= 3\ln(x-2)+1 \\ 3\ln(x-2) &= -1 \\ \ln(x-2) &= -\frac{1}{3} \\ x-2 &= e^{-\frac{1}{3}} \\ x &= 2+e^{-\frac{1}{3}} \\ x &\approx 2.7 \end{aligned}$$




3). Given:

$$\log_b 2 = 0.6931$$

$$\log_b 3 = 1.0986$$

$$\log_b 5 = 1.6094$$

find $\log_b\left(\frac{75}{4}\right)$. You must use each one of the given values, you are not allowed to determine the value of $b$, and you must show exactly how you obtained the answer.

$$\begin{aligned}
\log_b\left(\frac{75}{4}\right) &= \log_b\left(\frac{3\cdot 5^2}{2^2}\right)\\
&= \log_b 3 + \log_b 5^2 - \log_b 2^2\\
&= \log_b 3 + 2\log_b 5 - 2\log_b 2\\
&= 1.0986 + 2(1.6094) - 2(0.6931)\\
&= 2.9312
\end{aligned}$$

4). Consider the equation: $y = \log_a x$

a). Derive the change of base formula for some arbitrary base $b$.

$$\begin{aligned} y &= \log_a x \\ a^y &= x \\ \log_b a^y &= \log_b x \\ y \log_b a &= \log_b x \\ y &= \frac{\log_b x}{\log_b a} \end{aligned}$$

b). Use your formula with $b = e$ and your calculator to compute $\log_7 100$.

$$\log_7 100 = \frac{\ln 100}{\ln 7} = 2.3666$$

You can check this by:

$$7^{2.3666} \approx 100$$

c). Assume that you made a mistake and used the common log key instead of the natural log key in the above calculation. Would you get a different answer? Why or why not?

You would get the same answer because the change-of-base formula is independent of the base selected.

5). Researchers tend to prefer exponential (base $e$) equations. For example, the normal equation for the radioactive decay of Carbon-14, which has a half-life of 5730 years, would be:

$$A = A_0 \cdot 2^{-\frac{t}{5730}}$$

But the preferred exponential equations is:

$$A = A_o e^{-\frac{t}{a}}$$

Solve for $a$, rounding to the nearest integer value.

$$\begin{aligned} 2^{-\frac{t}{5730}} &= e^{-\frac{t}{a}} \\ \ln 2^{-\frac{t}{5730}} &= \ln e^{-\frac{t}{a}} \\ -\frac{t}{a} &= -\frac{t}{5730} \ln 2 \\ \frac{1}{a} &= \frac{1}{5730} \ln 2 \\ a &= \frac{5730}{\ln 2} \\ a &\approx 8267 \end{aligned}$$