

Math-08 Homework #4 Solutions

Reading

- Text book section 0.5, 0.6, and 0.7

Problems

- 1). Expand the following. Your answer should contain *no* radicals.

$$\begin{aligned}(4x^2 - 3y)(x^{\frac{1}{3}} + 2\sqrt{y}) &= (4x^2 - 3y)(x^{\frac{1}{3}} + 2y^{\frac{1}{2}}) \\&= (4x^2)(x^{\frac{1}{3}}) + (4x^2)(2y^{\frac{1}{2}}) + (-3y)(x^{\frac{1}{3}}) + (-3y)(2y^{\frac{1}{2}}) \\&= 4x^{\frac{7}{3}} + 8x^2y^{\frac{1}{2}} - 3x^{\frac{1}{3}}y - 6y^{\frac{3}{2}}\end{aligned}$$

- 2). Explain why the following is incorrect, and then state what it should be:

$$(2x + 3y)^2 = 4x^2 + 9y^2$$

Never distribute an exponent across addition! This needs to be FOIL'ed:

$$(2x + 3y)^2 = (2x)^2 + 2(2x)(3y) + (3y)^2 = 4x^2 + 12xy + 9y^2$$

- 3). Factor the following. Your answer should contain *no* negative exponents:

$$\begin{aligned}x^{\frac{5}{3}} - 2x^{\frac{2}{3}} + x^{-\frac{1}{3}} &= x^{-\frac{1}{3}} \left(x^{\frac{5}{3} + \frac{1}{3}} - 2x^{\frac{2}{3} + \frac{1}{3}} + 1 \right) \\&= x^{-\frac{1}{3}} (x^2 - 2x + 1) \\&= x^{-\frac{1}{3}} (x - 1)^2 \\&= \frac{(x - 1)^2}{x^{\frac{1}{3}}} \\&= \frac{(x - 1)^2}{\sqrt[3]{x}}\end{aligned}$$

- 4). Consider:

$$\frac{x+1}{x+2} \left(\frac{1}{x} + \frac{2}{x+1} - \frac{x-5}{x-2} \right)$$

- a). Combine into a single, simplified rational expression.

Work on the inside first. We need a common denominator, which will be $x(x+1)(x-2)$. We adjust each numerator for the common denominator by multiplying by the missing factors:

$$\frac{x+1}{x+2} \left[\frac{(x+1)(x-2) + 2x(x-2) - x(x-5)(x+1)}{x(x+1)(x-2)} \right]$$

Now, do the necessary FOIL'ing in the numerator. Be careful of the minus sign in the last term!

$$\frac{x+1}{x+2} \left[\frac{(x^2 - x - 2) + (2x^2 - 4x) - (x^3 - 4x^2 - 5x)}{x(x+1)(x-2)} \right]$$

Now, combine like terms in the numerator:

$$\frac{x+1}{x+2} \left[\frac{-x^3 + 7x^2 - 2}{x(x+1)(x-2)} \right]$$

We can now do the multiplication. Note that the $(x+1)$ factor cancels; however, we make a little note to ourselves that $x \neq -1$ for when we determine the domain at the end:

$$\frac{-x^3 + 7x^2 - 2}{x(x+1)(x-2)(x+2)}$$

We can leave it like this. Don't bother to multiply out the denominator - the factored form gives us more information. One optional step is to factor out the negative in the numerator:

$$-\frac{x^3 - 7x^2 + 2}{x(x-2)(x+2)}$$

b). State the domain.

A zero denominator is not allowed:

$$x(x-2)(x+2) \neq 0$$

We use the property of zero that says if you multiply a bunch of factors and the result is non-zero, then none of the factors is zero:

$$x \neq 0$$

$$x-2 \neq 0$$

$$x+2 \neq 0$$

Which results in $x \neq 0, \pm 2$. But don't forget, we eliminated $x = -1$ earlier as well. So the final domain is:

$$\{x \in \mathbb{R} \mid x \neq 0, -1, \pm 2\}$$