Subbases

Definition: Subbasis

Let $(X, \mathscr{T})$ be a topological space and let $\mathcal{S} \subset 2^X$. To say that $\mathcal{S}$ is a *subbasis* for $\mathscr{T}$ means that the set $\mathcal{B}$ consisting of all finite intersections of subsets of $\mathcal{S}$ is a basis for $\mathscr{T}$.

Theorem

Let $\mathscr{T}$ be the standard topology on $\mathbb{R}$ and let:

$$\mathcal{S} = \{(-\infty, b) \,|\, b \in \mathbb{R}\} \cup \{(a, \infty) \,|\, a \in \mathbb{R}\}$$

$\mathcal{S}$ is a subbasis for $\mathscr{T}$.

Proof. Since all of the sets in S are open, all finite intersections are also open. In particular, for all $a, b \in \mathbb{R}$ such that $a < b$:

$$(-\infty, b) \cap (a, \infty) = (a, b)$$

But the (a, b) are known to be a basis of $\mathscr{T}$. Furthermore, adding more open sets to a basis just results in a finer basis.

Therefore, $\mathcal{S}$ is a subbasis of $\mathscr{T}$. ∎