Math-71 Sections 9, 11, 12

Homework #10

Due: 4/30/2019 5:45pm

Reading

Read section 13.6.

Problem

You work for a large conglomerate of 100 associated companies. In order to avoid antitrust issues with the DOJ, you need to divide the 100 companies into 4 different groups such that:

1. Companies within the same group cannot do business with each other.
2. Companies in different groups can do business with each other.

What organization of the 100 companies into the 4 groups maximimizes the number of business opportunities?

a) Start by labeling the groups X, Y, Z, and W. Let $x =$ the number of companies assigned to group X and so on for the other groups. Construct an equation in x for the number of business opportunities for a company in group X — i.e., how many companies can that company do business with?

b) Now build an equation in x for the total number of business opportunities for all companies in group X.

c) Do likewise for the remaining groups and construct a function $f(x, y, z, w)$ that gives the total number of business opportunities across all the groups.

d) What is the constraint on x, y, z, and w?

e) Introduce a Lagrange multiplier λ and determine $f_x = \lambda g_x$, where g is the function constructed from the above constraint.

f) Do likewise for f_y, f_z, and f_w, and combine them with the constraint so that you have 5 equations in 5 unknowns.

g) Use substitution to determine a value for λ.

h) Substitute the value for λ into the other equations to determine the optimal distribution of companies into the 4 groups.