Math-19 Homework #4 Solutions

Problems

1). Rationalize the denominator and simplify completely. Your final answer should have no radicals and no negative exponents. Be careful to use absolute value where necessary.

$$\frac{xyz}{\sqrt[3]{x^2y^6z^4}}$$

First, make a table that shows what is needed to bring each factor in the radicand up to an exponent that is a power of 3:

factor	needed	final
x^2	x	x^3
y^6	1	y^6
z^4	z^2	z^6

Now apply all of the necessary factors and simplify:

$$\begin{aligned}
\frac{xyz}{\sqrt[3]{x^2y^6z^4}} &= \frac{xyz}{\sqrt[3]{x^2y^6z^4}} \cdot \frac{\sqrt[3]{xz^2}}{\sqrt[3]{xz^2}} \\
&= \frac{xyz\sqrt[3]{xz^2}}{\sqrt[3]{x^3y^6z^6}} \\
&= \frac{xyzx^{\frac{1}{3}}z^{\frac{2}{3}}}{xy^2z^2} \\
&= \frac{x^{\frac{4}{3}}yz^{\frac{5}{3}}}{xy^2z^2} \\
&= \frac{x^{\frac{1}{3}}z^{\frac{2}{3}}}{yz}
\end{aligned}$$

Now let's check to see if we need to apply any absolute values. Note that for x, y, or $z < 0$, the variable contributes a negative to the numerator and a positive to the denominator for a net negative. In the final result, each negative value contributes a net negative as well. Thus, no absolute values are needed.

2). Consider the quadratic equation $2x^2 + 5x - 7 = 0$.

a). Solve for x by completing the square.

$$\begin{aligned}
2x^2 + 5x &= 7 \\
2\left(x^2 + \frac{5}{2}x\right) &= 7 \\
\left(x^2 + \frac{5}{2}x\right) &= \frac{7}{2}
\end{aligned}$$

$b = \frac{5}{2}$ $\quad \frac{b}{2} = \frac{5}{4}$ $\quad \left(\frac{b}{2}\right)^2 = \frac{25}{16}$

$$\begin{aligned} x^2 + \frac{5}{2}x + \frac{25}{16} &= \frac{7}{2} + \frac{25}{16} \\ \left(x + \frac{5}{4}\right)^2 &= \frac{81}{16} \\ \left(x + \frac{5}{4}\right)^2 &= \frac{81}{16} \\ \left|x + \frac{5}{4}\right| &= \frac{9}{4} \\ x + \frac{5}{4} &= \pm\frac{9}{4} \\ x &= -\frac{5}{4} \pm \frac{9}{4} \\ x &= -\frac{7}{2}, 1 \end{aligned}$$

b). Solve for x using the quadratic formula.

$$\begin{aligned} x &= \frac{-5 \pm \sqrt{5^2 - 4(2)(-7)}}{2(2)} \\ &= \frac{-5 \pm \sqrt{25 + 56}}{4} \\ &= \frac{-5 \pm \sqrt{81}}{4} \\ &= \frac{-5 \pm 9}{4} \\ &= -\frac{7}{2}, 1 \end{aligned}$$

3). Solve for x:

$$\frac{5}{x+2} - \frac{x+5}{x-2} + \frac{28}{x^2-4} = 0$$

Multiple both sides by the LCD=$(x+2)(x-2)$:

$$\begin{aligned} 5(x-2) - (x+5)(x+2) + 28 &= 0 \\ 5x - 10 - (x^2 + 7x + 10) + 28 &= 0 \\ 5x + 18 - x^2 - 7x - 10 &= 0 \\ -x^2 - 2x + 8 &= 0 \\ x^2 + 2x - 8 &= 0 \\ (x+4)(x-2) &= 0 \\ x &= -4, 2 \end{aligned}$$

However, we see that $x = 2$ is an extraneous solution, and so: $x = -4$

4). Solve for x:

$$4(x+1)^{\frac{1}{2}} - 5(x+1)^{\frac{3}{2}} + (x+1)^{\frac{5}{2}} = 0$$

$$\begin{aligned}
(x+1)^{\frac{1}{2}}[4 - 5(x+1) + (x+1)^2] &= 0 \\
(x+1)^{\frac{1}{2}}[(x+1)^2 - 5(x+1) + 4] &= 0 \\
(x+1)^{\frac{1}{2}}[(x+1) - 4][(x+1) - 1] &= 0 \\
x(x+1)^{\frac{1}{2}}(x-3) &= 0 \\
x &= -1, 0, 3
\end{aligned}$$

5). A man stands atop a $256ft$ cliff with a ball.

a). How long does it take for the ball to hit the ground if he simply releases the ball?

$$\begin{aligned}
256 + 0t - 16t^2 &= 0 \\
-16(t^2 - 16) &= 0 \\
t^2 - 16 &= 0 \\
(t+4)(t-4) &= 0
\end{aligned}$$

This yields two solutions: $t = \pm 4$ seconds. We take the positive solution here: $t = 4$ seconds.

b). How long does it take for the ball to hit the ground if he throws the ball up with a velocity of $16ft/s$?

$$\begin{aligned}
256 + 16t - 16t^2 &= 0 \\
-16(t^2 - t - 16) &= 0 \\
t^2 - t - 16 &= 0
\end{aligned}$$

$$\begin{aligned}
t &= \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-16)}}{2(1)} \\
&= \frac{1 \pm \sqrt{1 + 64}}{2} \\
&= \frac{1 \pm \sqrt{65}}{2}
\end{aligned}$$

This yields two solutions: $t \approx -3.5$ seconds and $t \approx 4.5$ seconds. We take the positive solution here: $t \approx 4.5$; however, we save the negative solution for later.

c). How long does it take for the ball to hit the ground if he throws the ball down with a velocity of $16ft/s$? (Hint: no additional calculations are needed).

Note that in the previous problem the man threw the ball up at $+16ft/s$. The ball is going to travel up, slow down due to gravity, eventually stop, and then start falling. When the ball passes the man again it must be going at $-16ft/s$. Thus, the two problems are the same! Furthermore, the negative solution from the previous problem is the answer here: $t \approx 3.5$ seconds.

d). Assume that a lady is standing on the ground below the cliff and throws a ball up so that it passed the man on the cliff at a velocity of $16ft/s$. How long would it be before the ball hits the ground? (Hint: you already have all the information that you need).

Once again, this is the *same* problem. The roundtrip time is the sum of the previous two times: $t \approx 3.5 + 4.5 \approx 8.0$ seconds.