Math-19 Exam #1

Name: _____

This exam is closed book and notes. You may use a calculator; however, no cell phones or tablets are allowed. You are also allowed notes on both sides of a 3x5" note card. Show all work; there is no credit for guessed answers. All values should be exact unless you are specifically asked for an approximate value answer.

1. The following questions are related to the classifications of the real numbers:
   (a) Give an example of an integer that is not a natural number. _____

   (b) State the definition of the set of rational numbers.

   (c) State how we represent the set of irrational numbers using set difference notation.

   (d) True or false: 0 is a rational number. Explain your answer (hint: refer to the definition).

   (e) True or false: all fractions are rational numbers. If not then show a counterexample and explain why it makes the statement false.

2. Convert the number $2.1\overline{35}$ to rational form.

3. Shown below is a careful proof of the fact that $\forall a \in \mathbb{R}, a0 = 0$. Fill in the reason for each step. You may use either the codes or complete names for each rule. Note that substitution is assumed.

Assume $a \in \mathbb{R}$	
$a0 = a0$	
$a(0 + 0) = a0$	_____
$a0 + a0 = a0$	_____
$(a0 + a0) + (-a0) = a0 + (-a0)$	_____
$(a0 + a0) + (-a0) = 0$	_____
$a0 + (a0 + (-a0)) = 0$	_____
$a0 + 0 = 0$	_____
$a0 = 0$	_____

4. Logic problem.

   (a) Determine if each statement is either true or false:

   $P :=$ $\forall r, s \in \mathbb{Q}, r + s \in \mathbb{Q}$ _____

   $Q :=$ $0 \in (0, 5]$ _____

   $R :=$ The number π can be expressed with a repeating decimal _____

   (b) Show whether the following statement is true or false:

   P and Q or P and not R

5. Simplify. Your answer should have no negative exponents or compound fractions. (Hint: convert all of the radicals to fractional exponents and see if anything factors out).

$$\frac{x^{\frac{3}{2}} - 3\sqrt{x} - 4\sqrt{\frac{1}{x}}}{x^2 - 16}$$

6. Simplify. Your answer should have no negative exponents or compound fractions and should be left in factored form.

$$\frac{\frac{1}{x-1} - \frac{1}{x-3}}{\frac{1}{x+1}}$$

7. Solve for x by *completing the square*.

$$2x^2 + 4x - 3 = 0$$

8. Solve for x:

$$\frac{1}{x^3} + \frac{4}{x^2} + \frac{4}{x} = 0$$

9. Solve for x:

$$4|3x - 2| - 1 = 3(x + 1)$$

10. Muri is a shopkeeper that specializes in pickled vegetables. She has determined over the years that the best brine (salt solution) for pickling vegetables is 2 kg of salt per liter of water (2 kg/L). One day, she has her not-so-bright nephew helping her and he uses too much salt, resulting in a 5 kg/L solution. If her nephew made up 10 liters of the too-salty solution, how much pure water must he add to it to get the ideal 2 kg/L solution?