Answers

1. a. $\mu_X = n\mu = 40, \sigma_X = \sqrt{n}\sigma = 2$

   b. $\mu_Y = \mu = 10, \sigma_Y = \frac{\sigma}{\sqrt{n}} = \frac{1}{2}$

   c. By the CLT, $\bar{X} \overset{\text{approx}}{\sim} N(\mu = 10, \frac{\sigma^2}{n} = \frac{1}{36})$. Thus, $P(\bar{X} < 9.8) \approx P(Z < \frac{9.8-10}{1/\sqrt{36}}) = P(Z < -1.2) = 0.115$

2. a. $\hat{\theta}_1 = \min X_i$ and $\hat{\theta}_2 = 2\bar{X} - 1$.

   b. $\hat{\theta}_1$ is biased (because it is always overestimation) but $\hat{\theta}_2$ is unbiased (because $\bar{X}$ is unbiased for estimating $\frac{\theta+1}{2}$).

3. a. 66.2, don't know whether it underestimates or overestimates

   b. $\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 66.2 \pm 1.96 \cdot \frac{4.1}{\sqrt{400}} = 66.2 \pm 0.40$

   c. We don't know as the value of μ is unknown.

   d. Yes, because $\bar{x}$ is the center of the interval.

   e. 1600

4. 95% confidence interval is $\bar{x} \pm t_{\alpha/2}(n-1)\frac{s}{\sqrt{n}} = 66.2 \pm 1.966 \cdot \frac{5.0}{\sqrt{400}} = 66.2 \pm 0.49$. The other answers remain the same.

5. – **Question 3 setting**: At level 5%, the rejection region of the test

$$|\bar{x} - 66.8| > z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 0.4$$

   Since $|66.2 - 66.8| > 0.4$, we can reject $H_0 : \mu = 66.8$ (at level 5%) and correspondingly accept the alternative ($H_1 : \mu \neq 66.8$) that population mean is not 66.8 inches..

   Alternatively, we can compute the p-value of the observed sample mean 66.2 (not required):

$$2P(\bar{X} < 66.2) = 2P\left(\frac{\bar{X} - 66.8}{4.1/\sqrt{400}} < \frac{66.2 - 66.8}{4.1/\sqrt{400}}\right) = 2P(Z < -2.93) = 2 \cdot .0017 = .0034$$

   It is (strongly) significant at level 5%, and thus we can reject the null hypothesis and correspondingly conclude that population mean is not 66.8 inches.

   – **Question 4 setting**: At level 5%, the rejection region of the test

$$|\bar{x} - 66.8| > t_{\alpha/2}(n-1)\frac{s}{\sqrt{n}} = 0.49$$

   Since $|66.2 - 66.8| > 0.49$, we can reject $H_0 : \mu = 66.8$ (at level 5%) and correspondingly accept the alternative ($H_1 : \mu \neq 66.8$) that population mean is not 66.8 inches..

   Alternatively, we can compute the p-value of the observed sample mean 66.2 (not required):

$$2P(\bar{X} < 66.2) = 2P(\frac{\bar{X} - 66.8}{5/\sqrt{400}} < \frac{66.2 - 66.8}{5/\sqrt{400}}) = 2P(t(399) < -2.4) = 2 \cdot .0084 = .0168$$

   It is also significant at level 5%, and thus we can reject the null hypothesis and correspondingly conclude that population mean is not 66.8 inches.