Math-42 Sections 01, 02, 05

Homework #3 Solutions

Problem

Prove the following logical equivalence using the definition of implication and the rules in Table 6 on page 29 in the text:

$$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$$

Indicate the name of the rule that you use in each step. Do NOT skip any steps. In particular, if you need to rearrange order or precedence then include commutative or associative steps as necessary. In order words, each step must exactly match a rule from Table 6.

$(p \to r) \vee (q \to r)$	
$(\bar{p} \vee r) \vee (\bar{q} \vee r)$	Equivalence of implication
$\bar{p} \vee (r \vee (\bar{q} \vee r))$	Associativity
$\bar{p} \vee (r \vee (r \vee \bar{q}))$	Commutativity
$\bar{p} \vee ((r \vee r) \vee \bar{q})$	Associativity
$\bar{p} \vee (r \vee \bar{q})$	Idempotence
$\bar{p} \vee (\bar{q} \vee r)$	Commutativity
$(\bar{p} \vee \bar{q}) \vee r$	Associativity
$\overline{p \wedge q} \vee r$	DeMorgan
$(p \wedge q) \to r$	Equivalence of implication