EXAM 1

Math 161a: Appl. Prob. & Stats.
Instructor: Guangliang Chen
San Jose State University
Spring 2018

You have 75 minutes.

No books, but you are allowed to use a flash-card (provided by the instructor) as cheat sheet.

Please write legibly (unrecognizable work will receive zero credit).

You must show all necessary steps to receive full credit.

Good luck!

Name: _____

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____

"I have adhered to the SJSU Academic Integrity Policy in completing this exam."

Signature: _____

Date: _____

Total score: _____ (/50 points)

1. *(9 pts)* A small class has 4 boys and 5 girls.

   (a) In how many different ways can you arrange them along a line? What if the students of each sex must stand together?

   (b) In how many different ways can you select 2 boys and 2 girls to form a team of size 4 to work on some project?

2. *(10 pts)* A poker hand of 5 cards is drawn from an ordinary deck of 52 cards at random. Consider the following two events: A = {All hearts}, B = {5 consecutive numbers} (i.e., $x, x+1, x+2, x+3, x+4$ where x represents the face value; Ace can only be used as 14).

   (a) Determine $P(B)$.

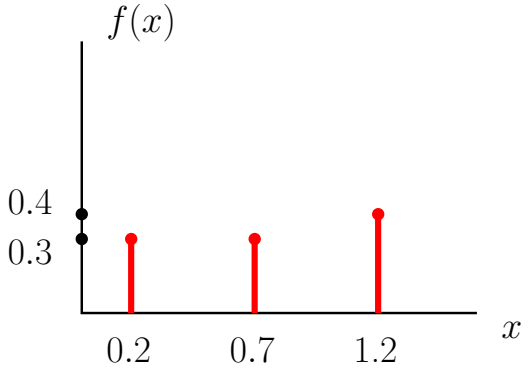
   (b) Find $P(B \mid A)$. Are the events A and B independent?

3. *(10 pts)* Suppose that 55% of the defendants are truly guilty. Suppose also that juries vote a guilty person innocent with probability 0.2 whereas the probability that a jury votes an innocent person guilty is 0.1.

   (a) Find the probability that a defendant is convicted.

   (b) What percentage of convicted defendants are actually innocent?

4. *(11 pts)* The distribution of a random variable X is displayed in the following plot:



(a) What is the range of X?

(b) Find the following probabilities:

$$P(X = 0.3) = \qquad P(X \leq 0.3) =$$
$$P(X = 0.7) = \qquad P(X \leq 0.7) =$$

(c) Plot the cumulative distribution function (cdf) of X as a graph, to the right of the given graph. Make sure you mark everything clearly.

(d) What are the expected value and standard deviation of X?

(e) What is $\text{Exp}(2X - 3)$?

5. *(10 pts)* Toss two fair dice independently and let Y be the smaller of the two numbers. Find the pmf of Y.

6. *(5 pts)* **Extra credit question**. *Your score earned for this question will be posted separately on Canvas under extra credit assignments.*

   Consider the experiment of independently tossing two different dice with probabilities of getting heads equal to 0.5 and 0.6 respectively, and let X denote their sum. Find the expected value and variance of X.