Math-19 Homework #2

1. We have fairly straightforward definitions for a^b when a and b are rational numbers. When b is an integer we have

   $$a^b = a \cdot a \cdot \ldots \cdot a$$

   where a is multiplied b times. When b is rational we have:

   $$a^{\frac{p}{q}} = \sqrt[q]{a^p}$$

   where $p, q \in \mathbb{Z}$. But what about when a or b are irrational? For example, what does $\pi^{\sqrt{2}}$ even mean? Once again, we turn to the notion of *approximate* values getting arbitrarily close to the *exact* value.

   Start by using your calculator to get an approximate value for $\pi^{\sqrt{2}}$. Then create a table as follows:

π	$\sqrt{2}$	$\pi^{\sqrt{2}}$
3	1	3
3.1	1.4	4.87423

   In other words, make the approximations of π and $\sqrt{2}$ finer and finer and see if the value approaches your value for $\pi^{\sqrt{2}}$ that you got directly from your calculator. Do up to 5 decimal places for each approximation.

2. A careful proof for that fact that a rational number plus a rational number is always a rational number may look something like this:

Let $q_1, q_2 \in \mathbb{Q}$	
$q_1 = \frac{a}{b}$ where $a, b \in \mathbb{Z}$ and $b \neq 0$	Definition of rationals
$q_2 = \frac{c}{d}$ where $c, d \in \mathbb{Z}$ and $d \neq 0$	Definition of rationals
$q_1 + q_2 = \frac{a}{b} + \frac{c}{d}$	Substitution
$q_1 + q_2 = \frac{ad+bc}{bd}$	Rule of fraction addition, Substitution
$ad + bc \in \mathbb{Z}$	Integers closed under addition and multiplication
$bd \in \mathbb{Z}$	Integers closed under multiplication
$b \neq 0$ and $d \neq 0$, so $bd \neq 0$	Property of 0
$\frac{ad+bc}{bd}$ is a rational number	Definition of rationals
Therefore, $q_1 + q_2 \in \mathbb{Q}$	

   Note that each step in the proof has a stated reason.

   In a similar manner, prove that a rational number plus an irrational number is always an irrational number. Start by chosing a rational number $q = \frac{a}{b}$ and an irrational number

i. Assume that their sum is another rational number — form an equation that represents this. Then see if that equation leads to some sort of contradiction. If is does, then the assumption that the sum is rational is incorrect, and thus the sum must be irrational.

3. Simplify completely. Your answer should have no negative exponents and please rationalize the denominator. Don't worry if the exponents get messy.

$$\frac{\sqrt[4]{\sqrt{75}+\sqrt{27}}}{\sqrt{4\sqrt{20}\sqrt[3]{54}}}$$

4. Simplify completely:

$$\frac{(9st)^{\frac{3}{2}}}{(27s^3t^{-4})^{\frac{2}{3}}}\left(\frac{3s^{-2}}{4t^{\frac{1}{3}}}\right)^{-\frac{3}{2}}$$

5. Simplify completely. Your answer should have no negative exponents but do not rationalize the denominator.

$$\frac{(x^2y^{\frac{1}{3}})^{\frac{1}{2}}}{(xy^3)^{\frac{1}{3}}}$$