Math-08 Homework #7 Solutions

Reading

- Text book section 1.5

Problems

1). Solve for x (Hint: quadratic-like?)

$$x + 2\sqrt{x} - 15 = 0$$

$$\begin{aligned} (\sqrt{x})^2 + 2(\sqrt{x}) - 15 &= 0 \\ (\sqrt{x}+5)(\sqrt{x}-3) &= 0 \end{aligned}$$

$\sqrt{x}+5=0$	$\sqrt{x}-3=0$
$\sqrt{x}=-5$	$\sqrt{x}=3$
no solution	$x=9$

So $x = 9$. As a sanity check, we see that we can indeed plug 9 into the original equation (it is in the domain) and see that it works.

2). Solve for x (Hint: there should be only two solutions, not four)

$$2\left|2x+3\right| - 6 = 3\left|x\right| + 1$$

Since one of the absolute values is a term in an expression, we need to do some work. Start by isolating one of the absolute values and then taking the plus/minus:

$$2\left|2x+3\right| = 3\left|x\right| + 7$$

$$\left|2x+3\right| = \frac{3}{2}\left|x\right| + \frac{7}{2}$$

$$2x+3 = \pm\left(\frac{3}{2}\left|x\right| + \frac{7}{2}\right)$$

This results in two separate equations:

$$\begin{aligned} 2x+3 &= \frac{3}{2}\left|x\right| + \frac{7}{2} \\ \frac{3}{2}\left|x\right| &= 2x - \frac{1}{2} \\ \left|x\right| &= \frac{4}{3}x - \frac{1}{3} \end{aligned}$$

$$\begin{aligned} 2x+3 &= -\left(\frac{3}{2}\left|x\right| + \frac{7}{2}\right) \\ 2x+3 &= -\frac{3}{2}\left|x\right| - \frac{7}{2} \\ \frac{3}{2}\left|x\right| &= -2x - \frac{13}{2} \\ \left|x\right| &= -\frac{4}{3}x - \frac{13}{3} \end{aligned}$$

Each of these now gives rise to two equations. Start with the first:

$$x = \pm\left(\frac{4}{3}x - \frac{1}{3}\right)$$

$$\begin{aligned} x &= \frac{4}{3}x - \frac{1}{3} \\ \frac{1}{3}x &= \frac{1}{3} \\ x &= 1 \end{aligned}$$

$$\begin{aligned} x &= -\left(\frac{4}{3}x - \frac{1}{3}\right) \\ x &= -\frac{4}{3}x + \frac{1}{3} \\ \frac{7}{3}x &= \frac{1}{3} \\ x &= \frac{1}{7} \end{aligned}$$

And now the second:

$$x = \pm\left(-\frac{4}{3}x - \frac{13}{3}\right)$$

$$\begin{aligned} x &= -\frac{4}{3}x - \frac{13}{3} \\ \frac{7}{3}x &= -\frac{13}{3} \\ x &= -\frac{13}{7} \end{aligned}$$

$$\begin{aligned} x &= -\left(-\frac{4}{3}x - \frac{13}{3}\right) \\ x &= \frac{4}{3}x + \frac{13}{3} \\ \frac{1}{3}x &= -\frac{13}{3} \\ x &= -13 \end{aligned}$$

So we have four candidates:

$$x = -13, -\frac{13}{7}, \frac{1}{7}, 1$$

But absolute value equations are tricksy hobbits. We need to make sure that our found candidates are actual solutions:

$$\begin{aligned} 2\left|2(-13)+3\right| - 6 &\stackrel{?}{=} 3\left|-13\right| + 1 \\ 2\left|-26+3\right| - 6 &\stackrel{?}{=} 3(13) + 1 \\ 2\left|-23\right| - 6 &\stackrel{?}{=} 39 + 1 \\ 2(23) - 6 &\stackrel{?}{=} 40 \\ 46 - 6 &\stackrel{?}{=} 40 \\ 40 &= 40 \end{aligned}$$

$$
\begin{aligned}
2\left|2\left(-\frac{13}{7}\right)+3\right|-6 &\stackrel{?}{=} 3\left|-\frac{13}{7}\right|+1 \\
2\left|-\frac{26}{7}+3\right|-6 &\stackrel{?}{=} 3\left(\frac{13}{7}\right)+1 \\
2\left|-\frac{5}{7}\right|-6 &\stackrel{?}{=} \frac{39}{7}+1 \\
2\left(\frac{5}{7}\right)-6 &\stackrel{?}{=} \frac{46}{7} \\
\frac{10}{7}-6 &\stackrel{?}{=} \frac{46}{7} \\
-\frac{32}{7} &\neq \frac{46}{7}
\end{aligned}
$$

$$
\begin{aligned}
2\left|2\left(\frac{1}{7}\right)+3\right|-6 &\stackrel{?}{=} 3\left|\frac{1}{7}\right|+1 \\
2\left|\frac{2}{7}+3\right|-6 &\stackrel{?}{=} 3\left(\frac{1}{7}\right)+1 \\
2\left|\frac{23}{7}\right|-6 &\stackrel{?}{=} \frac{3}{7}+1 \\
2\left(\frac{23}{7}\right)-6 &\stackrel{?}{=} \frac{10}{7} \\
\frac{46}{7}-6 &\stackrel{?}{=} \frac{10}{7} \\
\frac{4}{7} &\neq \frac{10}{7}
\end{aligned}
$$

$$
\begin{aligned}
2\left|2(1)+3\right|-6 &\stackrel{?}{=} 3\left|1\right|+1 \\
2\left|2+3\right|-6 &\stackrel{?}{=} 3(1)+1 \\
2\left|5\right|-6 &\stackrel{?}{=} 3+1 \\
2(5)-6 &\stackrel{?}{=} 4 \\
10-6 &\stackrel{?}{=} 4 \\
4 &= 4
\end{aligned}
$$

So, out of the four candidates, only two of them work: extraneous):

$$x = -13, 1$$

The other two are extraneous.

3). Solve for x

a). $(x+1)^{\frac{2}{3}} = 9$

$$|x+1| = 9^{\frac{3}{2}} = 27$$

$$x+1 = \pm 27$$

$$x = -28, 26$$

b). $(x+1)^{\frac{2}{3}} = -9$

$$|x+1| = (-9)^{\frac{3}{2}}$$

no solution, since we cannot take the square root of a negative number.

c). $(x+1)^{\frac{3}{2}} = 27$

$$x+1 = 27^{\frac{2}{3}} = 9$$

$$x = 8$$

d). $(x+1)^{\frac{3}{2}} = -27$

no solution, since the principle value of a square root can never be negative.

4). Consider $x^4 - 81 = 0$

a). Solve for x

$$(x^2+9)(x^2-9) = 0$$

$$(x^2+9)(x+3)(x-3) = 0$$

$$x = \pm 3$$

b). This is a degree-4 polynomial, so there is a maximum of four possible solutions. You should have found only two. Why are there only two?

The $x^2 + 9$ factor is irreducible in $\mathbb{R}$. To see this, try to solve $x^2 + 9 = 0$ using the quadratic equation - the discriminant is < 0. This wipes out two of the possible solutions.