

Math-19 Final Exam

Name: _____

This exam is closed book and notes. You may use a calculator; however, no cell phones or tablets are allowed. You are also allowed notes on both sides of a 3x5" note card. Show *all* work; there is **no** credit for guessed answers. All values should be exact unless you are specifically asked for an approximate value answer. In particular, you may leave answers to trig questions in terms of π .

- 1). You open up a savings account at your local bank on January 1st with \$1000 from your current paycheck. The account pays 1.5% annual interest compounded monthly. You set up an auto-deposit from your paycheck that will deposit an additional \$1000 to the account on the first of every month, starting with your next (February) paycheck. On March 1st you withdraw \$1500 from the account to help with the down payment on a new car.
 - a). Who is the lender?
 - b). Who is the borrower?
 - c). Construct a polynomial in $(1 + \frac{r}{n})$ that describes the account activity up through May 31st (i.e., after 4 months).
 - d). What is the account balance on May 31st?
 - e). What would be the account balance if the only transaction was the initial \$1000 deposit and the account compounds continually?

2). Consider the function $f(x) = 5 - e^{-x}$.

a). What are the x-intercepts (if any)?

b). What are the y-intercepts (if any)?

c). What is the end behavior as $x \rightarrow \infty$? If it is asymptotic, be sure to state what the asymptote is, whether it is horizontal or vertical, and if the function approaches it from above or below.

d). Repeat for $x \rightarrow -\infty$.

e). Sketch the graph of the function. Be sure to label all intercepts and asymptotes and show the proper end behavior. Any attempt to simply plot points results in zero credit.

f). What is the domain?

g). What is the range?

3). Consider $y = \log_a x$.

a). What is the corresponding exponential equation?

b). You need to calculate $y = \log_5 100$, but your calculator can only do common and natural logarithms. Derive a change-of-base formula to compute \log_5 values using your natural logarithm key.

c). Use your formula to calculate $\log_5 100$. Round your answer to 4 decimal places.

d). If you accidentally used your common log key instead of your natural log key in the calculation, would you get a different answer? Why or why not?

- 4). Let $\log_a 10 = 2.0859$, $\log_a 5 = 1.4650$, and $\log_a 30 = 3.0959$. Calculate $\log_a 60$ without knowing the base a . Determining a and then calculating directly receives zero credit - you must use the log rules!

- 5). Solve for x , leaving your answer as an exact value. Points will be deducted for approximate (decimal) answers!

$$\frac{10}{1 - e^{-x}} = 2$$

- 6). Solve for x , leaving your answer as an exact value. Points will be deducted for approximate (decimal) answers!

$$\log_5(x + 1) - \log_5(x - 1) = 2$$

- 7). Some archaeologists are digging at what appears to be a pre-Columbian human campsite in California. They find some animal bones with human teeth marks on them. Upon carbon-14 analysis, it is found that the bones have 75% of their original C_{14} . About how old are the bones, and hence the campsite? The half-life of C_{14} is 5730 years.

- 8). Let $\tan x = -\frac{8}{15}$ with x in Quadrant II.

a). Find $\sin 2x$.

b). Find $\cos 2x$.

9). Rewrite in terms of x and y without trig functions: $\cos(\sin^{-1} x + \tan^{-1} y)$.

10). Find all possible solutions for θ :

$$2 \sin^2 \theta + (\sqrt{3} - 4) \sin \theta - 2\sqrt{3} = 0$$