Bernoulli Distribution

Definition: Bernoulli Trial

To say that an experiment is a *Bernoulli trial* means that:

1. There is only one trial.
2. There are only two possible outcomes: success (S) or failure (F).
3. The probability of getting a success is some number p.

Definition: Indicator Variable

To say that a random variable $X$is an *indicator* variable means that it has only two possible values:

- $X = 1$ (success)
- $X = 0$ (failure)

Definition: Bernoulli Distribution

To say that a random variable X has a *Bernoulli* distribution with parameter p, denoted:

$$X \sim \mathrm{Bernoulli}(p)$$

means that X is an indicator variable for a Bernoulli trial with probability p for success.

Examples: Bernoulli Distributions

1. Flip a fair coin: $X = 1$ (heads) or $X = 0$ (tails).

$$X \sim \mathrm{Bernoulli}\left(\frac{1}{2}\right)$$

2. Randomly select a ball from an urn that has 10 red balls and 20 green balls: $Y = 1$ (ball is red) or $Y = 0$ (otherwise).

$$Y \sim \mathrm{Bernoulli}\left(\frac{1}{3}\right)$$

3. Randomly select an individual from a population, 40% of whiuch have a certain characteristic: $Z = 1$ (the selected person has the characteristic) or $Z = 0$ (otherwise).

$$Z \sim \mathrm{Bernoulli}\left(0.4\right)$$

Theorem

Let X be a random variable with a Bernoulli distribution with parameter p:

- $f_X(x) = \begin{cases} p^x(1-p)^{1-x} & x = 0, 1 \\ 0 & \text{otherwise} \end{cases}$
- $E(X) = p$
- $V(X) = p(1-p)$

Proof. The probability for success is p (given). Thus, since there are only two possible outcomes, the probability for failure is $1-p$. Now check to see that candidate the pmf provides the proper results:

$$P(X=0) = p^0(1-p)^{1-0} = 1-p$$
$$P(X=1) = p^1(1-p)^{1-1} = p$$

$$E(X) = 0(p-1) + 1 \cdot p = p$$

$$E(X^2) = 0^2(1-p) + 1^2 \cdot p = p$$

$$V(X) = E(X^2) - E(X)^2 = p - p^2 = p(1-p)$$

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Example

Flip a fair coin ($p = \frac{1}{2}$):

$$X \sim \text{Bernoulli}\left(\frac{1}{2}\right)$$

$$E(X) = p = \frac{1}{2}$$

$$V(X) = p(1-p) = \frac{1}{2}\left(1 - \frac{1}{2}\right) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$\sigma = \sqrt{\frac{1}{4}} = \frac{1}{2}$$