Math-42 Worksheet #21

Binomial Coefficients and Identities

1. Expand: $(x+y)^6$

2. Expand: $(2x+3y)^4$

3. Expand: $(x^2-2y)^3$

4. What is the x^4y^6 coefficient of $(x+y)^{10}$.

5. What is the x^6y^4 coefficient of $(x+y)^{10}$.

6. What is the x^4y^6 coefficient of $(x-2y)^{10}$.

7. We know that $2^n = \sum_{k=1}^{n} \binom{n}{k}$. Construct similar expressions for 3^n and 4^n. Can you generalize this for m^n?

8. Prove Pascal's Identity analytically (i.e., by manipulating the factorials).

9. Construct a clear committee selection argument for Pascal's Identity.

10. For this last exercise we will prove the binomial theorem using induction:

$$(x+y)^n = \sum_{k=1}^{n} \binom{n}{k} x^{n-k} y^k$$

(a) Start with the base case for $n = 0$.

(b) Write the inductive hypothesis.

(c) Rewrite $(x+y)^{n+1}$, apply the inductive hypothesis, and then distribute.

(d) Bring the extra x and y into their respective sums (since they are not dependent on the index).

(e) Move the index on the sum with the extra y from 0 to 1.

(f) Pull out the $k = 0$ term from the sum with the extra x.

(g) Combine the two sums.

(h) Apply Pascal's Identity.

(i) Fold the $k = 0$ term back into the sum.