Math-42 Worksheet #6

Rules of Inference

1. Identify each rule of inference, or state that an argument is a fallacy:
   (a) $n \in \mathbb{N}$ or $n < 0$. $n \notin \mathbb{N}$ or $n = 1$. Therefore $n < 0$ or $n = 1$.
   (b) If n is odd then n^2 is odd. n is odd. Therefore n^2 is odd.
   (c) For all $n \in \mathbb{Z}$, if n is even then there exists $k \in \mathbb{Z}$ such that $n = 2k$. 100 is even. Therefore there exists $k \in \mathbb{Z}$ such that $100 = 2k$.
   (d) $n \in \mathbb{N}$ and $n \in \mathbb{Z}$. Therefore $n \in \mathbb{Z}$.
   (e) $a < x$. $x < b$. Therefore $a < x < b$.
   (f) If $n < n^2$ then $n \neq 1$. If $n \neq 1$ then $n + 5 \neq 6$. Therefore if $n < n^2$ then $n + 5 \neq 6$.
   (g) If $n = 1$ then $n^2 = 1$. $n^2 = 1$. Therefore $n = 1$.
   (h) If n is even then n^2 is even. n^2 is odd. Therefore n is odd.
   (i) $a \leq b$. $a \neq b$. Therefore $a < b$.
   (j) $a < b$. Therefore $a \leq b$.
   (k) If $n = 1$ then $n^2 = 1$. $n \neq 1$. Therefore $n^2 \neq 1$.
   (l) For all $n \in \mathbb{Z}$, if n is even then n^2 is even. $225 = 15^2$ is odd. Therefore 15 is odd.

2. Consider the following two predicates:

$$C(x) \coloneqq x \text{ is an calculus problem}$$
$$H(x) \coloneqq x \text{ is a hard problem}$$

   and the following premises:

   - If x is a calculus problem then it is hard.
   - Integration is a calculus problem.
   - Solving a linear equation in one variable is not hard.
   - Solving a system of 100 linear equation in 100 unknowns is not a calculus problem.

   (a) Write each premise as a logical expression.
   (b) What conclusions can you make from the premises?

(c) Which premise does not lead to any conclusion and why?

3. Consider the following three predicates:

$$Z(x) \coloneqq x \text{ is an integer}$$
$$O(x) \coloneqq x \text{ is odd}$$
$$E(x) \coloneqq x \text{ is even}$$

and the following premises:

- If x is an integer then it is either even or odd.
- a is an integer or b is an integer.
- a is neither even nor odd.
- b is not even.

(a) Write each premise as a logical expression.

(b) What three conclusions can you make from the premises?

(c) What is the rule of inference used for each conclusion?

4. One of the most important uses of rules of inference is modus ponens (or modus tolens) on a definition. A definition is an equivalence: $p \longleftrightarrow q$, meaning p is the same thing as q. Thus, if p is true then we can conclude that q is true and if q is true then we can conclude that p is true. Also, by modus tolens, if q is false then p is false and if p is false then q is false. Some common ways to state definitions are:

- $p \longleftrightarrow q$
- p if and only if q
- p iff q
- To say that p means q
- If p then q.

Note that in the last case a simple implication is used; however, if it is understood that what is being stated is a definition then an equivalence is assumed.

So consider the following definition of a rational number:

$$x \in \mathbb{Q} \longleftrightarrow \exists p, q \in \mathbb{Z}, q \neq 0 \wedge x = \frac{p}{q}$$

Use modus pollens or modus tollens to make conclusions from the following premises:

(a) 0.5 is a rational number.

(b) $0.5 = \frac{1}{2}$

(c) π is an irrational number.

(d) $\forall\, p, q \in \mathbb{Z}, \sqrt{2} \neq \frac{p}{q}$

5. Consider the following argument:

   - For all candy, if the candy contains chocolate then it is good.
   - A Hersey bar contains chocolate.
   - A Hershey bar is good.
   - There exists a good candy.

   (a) Convert the propositions in the argument to logical expressions.

   (b) If the first two propositions are premises, what are the rules of inference that result in the third and fourth propositions?

6. Consider the following argument:

$$\exists\, x(P(x) \wedge Q(x))$$
$$\forall\, x(Q(x) \rightarrow R(x))$$
$$P(a) \wedge Q(a)$$
$$P(a)$$
$$Q(a)$$
$$R(a)$$
$$P(a) \wedge R(a)$$
$$\exists\, x(P(x) \wedge R(x))$$

   Assuming that the first two lines are premises, what are the rules of inference that result in each of the remaining lines.