Note. The statement of the problem is ambiguous since it is not clear whether the integer c is fixed. Solutions were submitted for both cases.

I. (c fixed.) *Solution by David Zeitlin, Minneapolis, Minnesota*. From the arithmetic-geometric inequality, we have

$$\frac{W}{c} = \frac{B_1 + B_2 + \cdots + B_c}{c} \geqq \sqrt[c]{B_1 B_2 \cdots B_c} \geqq \sqrt[c]{N}.$$

Thus, $W = c\sqrt[c]{N}$, if integral; otherwise, $W = [c\sqrt[c]{N}] + 1$.

II. (c not fixed.) *Solution by M. S. Klamkin, Ford Scientific Laboratory*. The dual of this problem is to find the largest number which can be obtained as the product of positive integers whose sum is $\leqq S$. This problem was proposed by Leo Moser and solved by L. Carlitz [Problem 125, *Pi Mu Epsilon Journal*, Fall, 1961]. If $P(S)$ denotes the maximum product, it was shown that

$$P(S) = \begin{cases} 3^m & \text{if } S = 3m, \\ 4 \cdot 3^{m-1} & \text{if } S = 3m + 1, \\ 2 \cdot 3^m & \text{if } S = 3m + 2. \end{cases}$$

Here S is partitioned into as many 3's as possible.

It now follows immediately that if $P(S) + 1 \leqq N \leqq P(S+1)$, then $W_{\min} = S + 1$ (the corresponding partition is not unique in general).

Also solved by M. T. Bird, Slobodan Ćuk & Jernej Polajnar (Yugoslavia), Michael Goldberg, M. G. Greening, (Australia), G. A. Heuer, T. F. Hughes, Douglas Lind, Henrik Meyer (Denmark), Norman Miller, E. F. Schmeichel, C. S. Venkataraman (India), and the proposer.

ADVANCED PROBLEMS

All solutions of Advanced Problems should be sent to J. Barlaz, Rutgers—The State University, New Brunswick, NJ 08903. To facilitate their consideration, solutions of Advanced Problems in this issue should be typed (with double spacing) on separate signed sheets and should be mailed before April 30, 1970. Contributors (in the United States) who desire acknowledgement of receipt of their solutions are asked to enclose self-addressed stamped postcards.

5707. *Proposed by W. A. Vasconcelos, Rutgers—The State University*

Let R be an integral domain and G a finite group. Assume the characteristic of R does not divide $|G|$. Prove that each R-derivation of $R[G]$ is inner.

5708. *Proposed by C. W. Avery, San Jose State College*

Let K be a finite extension of the field k, complete in a non-Archimedean valuation. Let $\overline{K}$ and $\bar{k}$ denote the residue class fields. Problem 16, p. 129 of P. J. McCarthy, *Algebraic Extensions of Fields* asserts that K is separable over k if $\overline{K}$ is separable over $\bar{k}$. Disprove.

5709. *Proposed by W. A. J. Luxemburg, California Institute of Technology*

For all $x>0$, determine

$$\lim_{n\to\infty} \frac{1}{(\sqrt{\pi})^n} \int_{D_n(x)} \cdots \int \exp[-(x_1^2 + \cdots + x_n^2)]dx_1 \cdots dx_n,$$

where

$$D_n(x) = \left\{(x_1, \cdots, x_n): \left|\frac{x_1}{1} + \frac{x_2}{\sqrt{2}} + \cdots + \frac{x_n}{\sqrt{n}}\right| \leqq x\right\}.$$

5710. *Proposed by R. E. Shafer, Lawrence Radiation Laboratory, Livermore, California*

It is well known that

$$[R^2 - 2Rr\cos\theta + r^2]^{-\nu} = \sum_{n=0}^{\infty} \frac{r^n}{R^{n+2\nu}} C_n^\nu(\cos\theta),$$

$|r| < |R|$. $\mathrm{Re}(\nu) > -1$, $\mathrm{Re}(\nu) \neq 0$. Find the set of functions $F_n(r, R)$ independent of θ such that

$$[R^2 - 2Rr\cos\theta + r^2]^{-\mu} = \sum_{n=0}^{\infty} F_n(r, R) C_n^\nu(\cos\theta), \quad \mathrm{Re}(\mu) > -1.$$

5711. *Proposed by Dan Marcus, York University, Toronto*

Let $A = (a_{m,n})_{m,n=1}^{\infty}$ be an infinite matrix of nonzero integers such that for each m, the set of prime divisors of numbers in the mth row is finite. Prove that the system of congruences $x_{m+n} \equiv a_{m,n} (\mathrm{mod}\ x_m)$ is solvable in primes.

5712. *Proposed by Dan Marcus, York University, Toronto*

Is it possible to topologize the integers in such a way that the connected sets are the sets of consecutive integers? Generalize to the lattice points of n-space.

5713. *Proposed by D. P. Geller, University of Michigan*

For any graph G with p points, q lines, and chromatic number χ, show

$$\chi \geqq p^2/(p^2 - 2q).$$

SOLUTIONS OF ADVANCED PROBLEMS

Multiplicative Solutions of Some Number Theoretic Equations

5653 [1969, 200]. *Proposed by Richard Stanley, Harvard University*

Find in each case the real-valued multiplicative number-theoretic function f which satisfies the stated condition. μ is the Möbius function, ϕ is the Euler totient function, $d(n)$ is the number of divisors of n, and $\sigma(n)$ is the sum of the