【不可使用計算機】＊作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！！

請使用答案卡作答

For each problemset, if your answer is correct for all the questions in the problemset, you receive the full points; or otherwise your answer is incorrect for any of the questions in the problemset, you receive 0 point. If we place a mark †† next to a problemset number, each question in the problemset may have one or more answers. If a question has multiple answers, then a correct response needs to contain all of them.

1. †† (6%) Let G be a connected undirected simple graph of 6 nodes and 8 edges, and let T be a depth-first search (DFS) tree of G rooted at node r. It is known that r has degree 5 in G. Answer the following questions.

   - r may have degree ① in T.
     (A) 1 (B) 2 (C) 3 (D) 4 (E) 5
   - A node is an articulation point if and only if removing it (and edges through it) disconnects G. Is r always an articulation point in G for any possible G? ②
     (A) Yes (B) Not necessary

2. (6%) Let G be an n-node simple undirected graph. A k-coloring of G is to assign a value in $\{1, 2, \ldots, k\}$ to each node so that no two adjacent nodes have the same assigned value. Answer the following questions.

   - For $k = 2$, the best known algorithm has time complexity ③
     (A) $O(n)$ (B) $O(n^2)$ (C) $O(n^2 \log n)$ (D) $O(n^3)$ (E) superpolynomial time
   - For $k = 3$, the best known algorithm has time complexity ④
     (A) $O(n)$ (B) $O(n^2)$ (C) $O(n^2 \log n)$ (D) $O(n^3)$ (E) superpolynomial time
   - For $k = 4$, the best known algorithm has time complexity ⑤
     (A) $O(n)$ (B) $O(n^2)$ (C) $O(n^2 \log n)$ (D) $O(n^3)$ (E) superpolynomial time

3. †† (13%) A Hamiltonian cycle of a graph G is a simple cycle that visits all nodes in G. Suppose there exists an $O(n^7)$-time algorithm that decides HamC(G) for any n-node graph G.

   HamC(G)

   Input: a simple undirected graph G.

   Output: "true," if G has a Hamiltonian cycle; "false," otherwise.

   Complete Algorithm 1, an $O(n^7)$-time algorithm that uses HamC at most once to decide $\text{HamP}_{2\times3}$ for any n-node graph G, for any distinct nodes $a_1, a_2, x_1, x_2, x_3 \in G$.

   $\text{HamP}_{2\times3}(G = (V, E), a_1, a_2, x_1, x_2, x_3)$

   Input: a simple undirected graph $G = (V, E)$ that contains at least the five distinct nodes a_1, a_2, x_1, x_2, x_3.

   Output: "true," if G has a simple path of length $|V| - 4$ that starts at a_i for some $i \in \{1, 2\}$, visits every node in $V \setminus \{a_1, a_2, x_1, x_2, x_3\}$ exactly once, and finally stops at x_j for some $j \in \{1, 2, 3\}$; otherwise, "output false."

Algorithm 1: $\text{HamP}_{2\times 3}(G = (V, E), a_1, a_2, x_1, x_2, x_3)$

1 $U \leftarrow V \cup \{\ell_1, \ell_2, \ell_3, \ell_4\}$;

2 $F \leftarrow E$;

`/* Add some edges incident to node` ℓ_1 `to` F. `*/`

3 $F \leftarrow F \cup \{\underline{\quad\quad}\}$;

`/* Add some edges incident to node` ℓ_2 `to` F. `*/`

4 $F \leftarrow F \cup \{\underline{\quad\quad}\}$;

`/* Add some edges incident to node` ℓ_3 `to` F. `*/`

5 $F \leftarrow F \cup \{\underline{\quad\quad}\}$;

`/* Add some edges incident to node` ℓ_4 `to` F. `*/`

6 $F \leftarrow F \cup \{\underline{\quad\quad}\}$;

7 **return** $\text{HamC}(H = (U, F))$;

- Which of the following undirected edges shall be placed in the missing part ⑥ of Line 3?
(A) (ℓ_1, ℓ_2) (B) (ℓ_1, a_1) (C) (ℓ_1, a_2) (D) (ℓ_1, x_2)

- Which of the following undirected edges shall be placed in the missing part ⑦ of Line 4?
(A) (ℓ_2, a_1) (B) (ℓ_2, a_2) (C) (ℓ_2, x_2) (D) (ℓ_2, x_3)

- Which of the following undirected edges shall be placed in the missing part ⑧ of Line 5?
(A) (ℓ_3, ℓ_1) (B) (ℓ_3, ℓ_2) (C) (ℓ_3, x_1) (D) (ℓ_3, x_2)

- Which of the following undirected edges shall be placed in the missing part ⑨ of Line 6?
(A) (ℓ_4, ℓ_3) (B) (ℓ_4, x_1) (C) (ℓ_4, x_2) (D) (ℓ_4, x_3)

4. †† (6%) Answer the following two questions about *B*-trees.

- Let t be the minimum degree of a *B*-tree. Which of the following statements are (or is) TRUE? ⑩ . (A) Every node can contain at most $2t$ keys. (B) The depth of every leaf can be different. (C) There exists one node which may have less than t children. (D) Every node has at least $t - 1$ keys. (E) If $n \geq 1$, then for any n-key B-tree of height h and minimum degree $t \geq 2$, the condition $\log_t(2t^h) \leq \log_t(n + 1)$ is true.

- When a new key is added to a B-tree, which of the following statements are (or is) TRUE? ⑪ . (A) If the key is inserted into a full node, the full node is split. Then the key must be placed in a new node which is the parent node of the nodes that are split from the full node. (B) The root node can be split even if it is not full. (C) When a full node is split, it is split around the median key. (D) The height of the B-tree increases if and only if the root node is split. (E) After a full node is split, it is impossible that another node is split too.

國立交通大學 109 學年度碩士班考試入學招生試題

科目：資料結構與演算法(1101)　　　　考試日期：109 年 2 月 4 日 第 1 節

系所班別：資訊聯招　　　　

【不可使用計算機】＊作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！！

5. (7%) G is a weighted graph (Figure 1).

- The minimum spanning tree (MST) of G is ⑫ B.

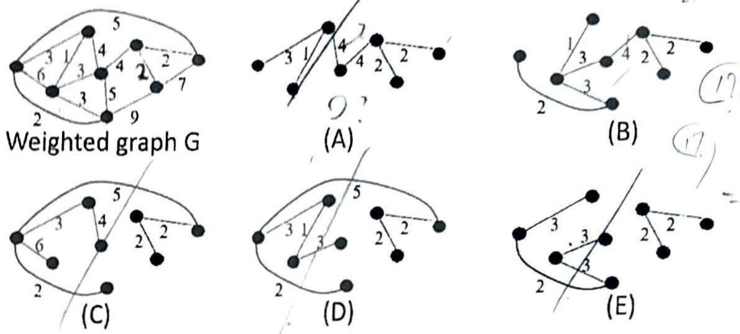


Figure 1: The numbers are weights of the edges.

- A new graph can be obtained after the weight of an edge of graph G is changed to 1 so that the weight of the MST of the new graph has the lowest weight among all the possible resulting graphs. The weight of the MST of the new graph is ⑬ .
(A) 17 (B) 18 (C) 13 (D) 14 (E) 15

- A new graph can be obtained after an edge of graph G is deleted so that the weight of the MST of the new graph has the highest weight among all the possible resulting graphs. The weight of the MST of the new graph is ⑭ A.
(A) 22 (B) 23 (C) 19 (D) 20 (E) 21

6. (8%) G is a directed weighted graph (Figure 2). Some edges of G have negative weights. If there is no specification, an edge or a node can be visited for more than one time. Determine a path with the lowest weight from node A to node F under three different conditions.

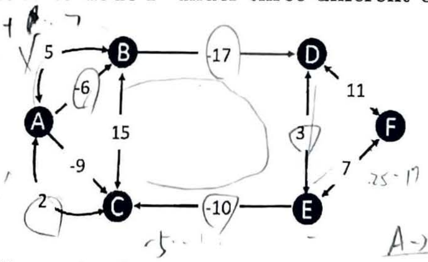


Figure 2: The arrows indicate the directions. The numbers on the edges are weights.

- Condition One: The same node is visited a maximum of two times. The weight of the path under Condition One is ⑮ C.
(A) -12 (B) -13 (C) -29 (D) -41 (E) -43

- Condition Two: The same node is visited at most once. The weight of the path under Condition Two is ⑯ D
(A) -10 (B) -11 (C) -12 (D) -13 (E) -17

- Condition Three: The same edge is visited a maximum of two times. The weight of the path under Condition Three is ⑰ B.
(A) -50 (B) -48 (C) -45 (D) -44 (E) -37

【不可使用計算機】*作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！！

7. (4%) The following questions are about disjoint sets (linked list representation) and disjoint-set forests (forest representation).

What is the purpose of union by rank? __(18)__. B

What is the purpose of path compression? __(19)__. D

What is the purpose of weighted-union heuristic? __(20)__ A.

(A) Append the smaller list onto the longer list, with ties broken arbitrarily. (B) Make the root of a disjoint-set forest with smaller rank point to the root of another disjoint-set forest with larger rank. (C) Make each node of a tree in the disjoint-set forest point to the tree root during a certain operation. (D) Flatten a disjoint-set forest. (E) Union two disjoint-set forests into a new forest and set their smallest element as the representative of the new forest.

8. †† (6%) Consider the formula $a_n + b_n\sqrt{3} = (1+2\sqrt{3})^n$, where n is a positive integer.

Algorithm 2: : $P(n)$

```
if n == 1 then
    return (1, 2)
else
    (a, b) = P(n − 1)
    return (a + 6b, 2a + b)
end if
```

$a_1 = 1$, $b_1 = 2$ $a_2 = 13$, $b_2 = 4$ $a_3 = 37$, $b_3 = 30$ $a_4 = 217$, $b_4 = 104$

$(13+3\sqrt{3})(1+2\sqrt{3})$

B C D

- __(21)__ Which of the following statements are (or is) TRUE?
  (A) $a_2 = 1$. ✗
  (B) $b_3 = 30$. ✓
  (C) $a_5 = a_4 + 6b_4$. ✓
  (D) $b_5 = 2a_4 + 6b_4$. ✓
  (E) For some finite n, a_n is not integeral. ✗

$T(n) = T(n-1)$

D
- Algorithm 2 has running time __(22)__ in the worst case (as close as possible).
  (A) $O(\log n)$ (B) $O(n)$ (C) $O(n \log n)$ (D) $O(n^2)$ (E) $O(\log^2 n)$

B D
- __(23)__ Which of the following statements are (or is) TRUE?
  (A) Algorithm 2 is the most efficient algorithm to compute a_n and b_n.
  (B) a_n and b_n can be computed in $O(\log n)$ time. ✓
  (C) $(1+2\sqrt{3})^n$ is not integral. Thus, there is no integral solution for a_n and b_n. ✗
  (D) If n is a positive even integer, then $a_n = a_{n/2}^2 + 3b_{n/2}^2$. ✓
  (E) If n is odd, then $b_n = 2a_{\lfloor n/2 \rfloor}b_{\lfloor n/2 \rfloor}$. ✗

$a_2 = a_1^2 + 3b^2$

$b_3 = 2a_1 b_1$

$169 + 3\times 2^2$

9. †† (9%) Let $A = \langle a_1, a_2, \ldots, a_n \rangle$ be a sequence of positive integers. We are interested in the subsequence of A: $\langle a_{i_1}, a_{i_2}, \ldots, a_{i_k} \rangle$, such that for any $1 < j \le k$, $a_{i_1} + a_{i_2} + \cdots + a_{i_{j-1}} \le 6a_{i_j}$, where $1 \le i_1 < i_2 < i_3 < \cdots < i_k \le n$. We call this the *Prefix-Sum condition*. Given a sequence A, we want to find the largest possible k.

For example, given 9 positive integers: 10, 10, 10, 10, 10, 10, 10, 10, and 100, we can select up to 8 integers, 10, 10, 10, 10, 10, 10, 10, and 100, to satisfy the above condition.

Consider the input: 2, 5, 3, 6, 2, 1, 2.

- ___(24)___ What is the largest k?
  (A) 6 (B) 4 (C) 3 (D) 7 (E) 5
- ___(25)___ Consider the sequence $\langle 1, 1, 1, 1, 1, 1, w, 2, 2, 2, 3, 3, 4, 4, x, y \rangle$ that satisfies the above condition. Which of the following statements are (or is) TRUE?
  (A) $x \ge 5$.
  (B) $y > 5$.
  (C) The smallest possible w is 2.
  (D) There is a possible solution with $w + x + y = 12$.
  (E) This sequence can be extended to be an infinite sequence that satisfies the above condition.
- ___(26)___ Denote $D(i, s)$ to be the maximum number of elements that can be selected from the first i integers in A with a sum at most s, and satisfy the Prefix-Sum condition. Which of the following statements are (or is) TRUE?
  (A) This problem can be solved with Dynamic Programming.
  (B) $D(i, s) = \max(D(i-1, s), D(i-1, s-a_i) + 1)$.
  (C) $D(i, s) = \max(D(i-1, s), D(i-1, \min(s - a_i, 6a_i)) + 1)$.
  (D) The answer is $D(n, 6a_n)$.
  (E) The time complexity to find the answer is $O(n \max_{i \in \{1,2,\cdots,n\}} \{a_i\})$.

10. †† (5 %) Which of the following statements are (or is) TRUE? ___(27)___

(A) Let α be the load factor of a hash table. If the collisions are resolved by chaining in the hash table, then both of the successful search and the unsuccessful search take expected time $O(1 + \alpha)$, under the assumption of simple uniform hashing. (B) The asymptotic time complexity of searching a key in a heap structure is $O(\log n)$, where n is the number of elements in the structure. (C) Suppose that a hash table has m slots. The table can store at most m keys if collision is resolved by chaining. (D) An AVL tree is a binary search tree. (E) In a red-black tree, a node has two children and one of the children is red. The node must be black.

11. (5%) What is the asymptotic time complexity for the following algorithms to sort n elements in a descending order?

- BUBBLESORT has running time complexity ___(28)___ in the worst case.
  (A) $O(\log n)$ (B) $O(n \log n)$ (C) $O(n)$ (D) $O(n^2)$ (E) $O(1)$
- HEAPSORT has running time complexity ___(29)___ in the worst case.
  (A) $O(\log n)$ (B) $O(n \log n)$ (C) $O(n)$ (D) $O(\sqrt{n})$ (E) $O(1)$

12. †† (10%) The notation $i = 1..N$ means that i is inside the interval $[1, N]$. Consider a two-processor computer system and a large program, which consists of N modules indexed from 1 to N. Let a_i and b_i be the cost of running module i ($i = 1..N$) on processor 1 and 2, respectively. Assigning two modules to different processors incurs costs due to inter-processor communication. Each module can be assigned to one processor only. Let $c_{i,j}$ be the communication cost if modules i and j are assigned to different processors. We want to assign the modules on the two processors to minimize the total cost.

Table 1: Cost on processors

i	1	2	3	4
a_i	6	5	10	4
b_i	4	10	3	8

- ㉚ Consider the above table with $N = 4$ and let the communication cost be: $c_{1,2} = 5, c_{2,3} = 6, c_{2,4} = 2, c_{3,4} = 1$ and the rest have 0 communication cost. Let X be the best answer. Which of the following statements are (or is) TRUE?
  (A) $X \bmod 10 = 4$
  (B) $\lfloor X/10 \rfloor = 1$
  (C) $X \leq 25$
  (D) $X \bmod 2 = 0$
  (E) None of the above is true.

- ㉛ We can model this problem with an undirected graph $G = (V, E)$, where $V = \{p_1, p_2, v_1, \ldots, v_N\}$ and $E = \{(p_1, v_i), (v_i, p_2) : i = 1..N\} \cup \{(v_i, v_j) : c_{i,j} > 0\}$. For $i = 1..N$, the weight for edge (p_1, v_i) is a_i and b_i for (v_i, p_2). The weight for $(v_i, v_j) \in E$ is $c_{i,j}$. Which of the following statements are (or is) TRUE?
  (A) There are 2^N possible ways to assign the modules to the two processors.
  (B) Let A_i be the set of modules assigned to processor i, for $i \in \{1, 2\}$. Then the cost is $\sum_{j \in A_1} a_j + \sum_{j \in A_2} b_j + \sum_{(v_i, v_j) \in A_1 \times A_2} c_{i,j}$.
  (C) The cost relates to a cut of the graph $G = (V, E)$ with p_1 and p_2 on the same side.
  (D) The minimum cost relates to a maximum flow from a proper source to a sink.
  (E) Let A_i be the set of modules assigned to processor i, for $i \in \{1, 2\}$. Then $(\{p_1\} \cup A_1, \{p_2\} \cup A_2)$ is a cut.

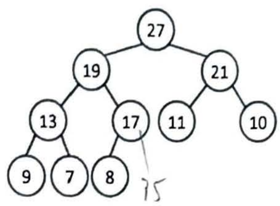
- ㉜ Which of the following statements are (or is) TRUE?
  (A) The above $G = (V, E)$ is a bipartite graph.
  (B) The minimum cost can be found with the Prim algorithm.
  (C) The minimum cost can be found with the Dijkstra algorithm.
  (D) The minimum cost can be found with the Edmonds-Karp algorithm.
  (E) The minimum cost can be found with the Ford-Fulkerson algorithm.

【不可使用計算機】＊作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！！

13. (5%) A key with value 35 is inserted into a maximum heap which is shown below. After that, two methods are adopted to traverse the heap.

What are the outputs by the post-order traversal? ㉝ C

What are the outputs by the in-order traversal? ㉞ E



(A) 35, 27, 21, 13, 19, 11, 10, 9, 7, 8, 17
(B) 35, 27, 13, 9, 7, 19, 8, 17, 21, 11, 10
(C) 9, 7, 13, 8, 17, 19, 27, 11, 10, 21, 35 → post
(D) 9, 7, 13, 8, 17, 19, 27, 35, 11, 10, 21
(E) 9, 13, 7, 27, 8, 19, 17, 35, 11, 21, 10 inorder

14. †† (5%) Consider a binary search tree which stores distinct elements. The keys in the left subtree of a node are smaller than those in the right subtree of the node (if any). Denote NIL as a null pointer. p and $right$ are the pointers directing to a node's parent node and the right child node, respectively. Function $Tree\_Minimum(n)$ returns the minimum node (i.e., the node that has the smallest key) of a sub-tree rooted at node n. Assume that x is a node in the binary search tree. The following code fragment is executed. After that, y is not equal to NIL.

if $right[x] \neq NIL$
  then $y = Tree\_Minimum(right[x])$; return
$y = p[x]$
while $y \neq NIL$ and $x = right[y]$
  do $x = y$ and $y = p[y]$
return;

㉟ B,E Which of the following statements are (or is) TRUE?

(A) y must be the successor node of x. (B) y must be the predecessor node of x. (C) x must be the maximum node of a sub-tree rooted at node y. (D) y must be the minimum node of a sub-tree rooted at node x. (E) y must be the parent node of x.

15. (5%) L is a circular linked list. Initially, L is empty. The elements 3, 4, 5, 9, and 7 are appended to L one by one. Assume that x is a node of L. Denote next[x] and prev[x] as the next and previous nodes of x, respectively. value[x] is the element stored in x. Given that x stores element 7. Thus, value[next[x]] and value[prev[x]] are 3 and 9, respectively.

㊱ C What is value[x] after the following code fragment is executed?

$n = 1$
while $n < 2020$
  do x = next[next[x]]
    $n = n + 1$
x = prev[prev[prev[x]]]

(A) 3 (B) 4 (C) 5 (D) 9 (E) 7