國立中興大學

110 學年度

碩士班考試入學招生

試　題

學系：資訊科學與工程學系

甲組

科目名稱：基礎數學A

PART I

一. 選擇題(單選或多重選擇，每題二分)

1. Let A={a,b,c} B=P(A), the power set of A. Which of the following properties is correct?
   (a) $\phi \subseteq B$ (b) $\phi \in B$ (c) $A \in B$ (d) $A \subseteq B$ (note: ϕ is the empty set)
2. Let R be an equivalence relation on a set A. Both a and b are elements of A and are equivalent. [a] stands for the equivalence class of a. Which of the followings is correct?
   (a) aRb (b) bRa (c) [a] = [b] (d) $[a] \cap [b] \neq \phi$
3. Which of the following propositions is equivalent to "$p \rightarrow q$"?
   (a) $\neg q \rightarrow \neg p$ (b) $\neg p \vee \neg q$ (c) $\neg(q \wedge p)$ (d) $\neg p \vee q$
4. Which of the following numbers is a primitive root in $\mathbf{Z}_{13}$?
   (a) 2 (b) 3 (c) 5 (d) 7
5. Which of the following numbers is prime?
   (a) $2^6 - 1$ (b) $2^7 - 1$ (c) $2^8 - 1$ (d) $2^9 - 1$
6. What is the value of the postfix expression "7 2 3 * - 4 ^ 9 3 / +", where ^ stands for exponentiation?
   (a) 3 (b) 4 (c) 8 (d) 12
7. Let T be a full m-ary tree, which a node has either 0 or m child nodes, with n vertices. If m=3 and n=100, which of the following statements about this tree is correct?
   (a) there are 98 edges (b) there are 33 internal vertices (c) the height of T is 4 (d) there are 66 leaf nodes
8. Which of following sets is countable?
   (a) (0,1) (b) Z^+ (c) {N,Z,Q} (d) P(N), the power set of natural numbers
9. Let P(n) be a propositional function. Which of the following statements is enough to verify P(n) is true for all positive integers n?
   (a) P(k) is true for large k (b) $P(1) \wedge [\forall k>1\ (P(k) \rightarrow P(k+1))]$ is true (c) $P(1) \wedge [\forall k>1\ (P(1) \wedge \ldots \wedge P(k) \rightarrow P(k+1))]$ is true (d) $P(1) \wedge P(n)$ is true for some n >1
10. Which of the following graphs is planar?
    (a) complete, K_4 (b) 3-cube, Q_3 (c) complete bipartite, $K_{3,3}$ (d) wheel, W_4

二. 是非題(每題二分，答錯倒扣一分)

1. The incidence matrix for representation of any simple graph is a symmetric matrix.
2. The cardinality of Q is the same as the cardinality of Z.
3. Among 100 people there are at least 9 who were born in the same month.
4. $(P(S), \subseteq)$ is a partially ordered set, where P(S) is a power set of S={1,2,4}.
5. "$\neg p \rightarrow q$" is logically equivalent to "$\neg(q \leftrightarrow p)$", where $\neg$ stands for "not".
6. There are 81 ways to put 4 distinguishable balls into 3 different boxes.
7. Traveling salesman problem is the problem to find a Euler circuit of least cost.
8. There are 1250 positive integers less than 100000 having the sum of their digits equal to 12.
9. The least number of colors needed for coloring a planar graph is no longer than 5.
10. A simple weighted graph is connected if and only if it has a minimum spanning tree.

三. 計算題(每題五分)

1. Over the set of {1,2,3,4,5,6}, what is the next permutation in lexicographic order after 326541?
2. How many one-to-one functions are there from a set with 5 elements to a set with 7 elements?

PART II

一. 是非題(每題二分，答錯倒扣一分)

1. A square matrix A is called skew-symmetric if $A^t = -A$. If B is a square matrix, then $B - B^t$ is skew-symmetric.
2. If $a, b, c, \cdots, i \in R$, then $\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \neq \det\begin{bmatrix} a+d & b+e & c+f \\ d & e & f \\ g & h & i \end{bmatrix}$.
3. $p_1 = 6 - x^2$ and $p_2 = 1 + x + 4x^2$ are linear independent in P_2.
4. The set of vectors $(1,6,4)$, $(2,4,-1)$ and $(-1,2,5)$ is a basis for a vector space R^3.
5. Let $T: R^2 \to R^2$ be the rotation of R^2 through the angle $\pi/4$. T has rank=2 and nullity=1.
6. If A and B are similar matrices, then $\det(A) = det(B)$.
7. A square matrix $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$ is diagonalizable.
8. If A, B and C are three square matrices, then $\mathrm{Trace}(ABC) = Trace(ACB)$.
9. If A is an $n \times n$ matrix, then the sum of the multiplicities of the eigenvalues of A equals n.
10. The matrix $A = \begin{bmatrix} 2 & -1 & -3 \\ -1 & 2 & 4 \\ -3 & 4 & 9 \end{bmatrix}$ is not positive definite.

二. 選擇題(單選或多重選擇，每題四分)

1. Which of the following are elementary matrices?

   (a) $\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$

2. Which of the following are linear combinations of $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 2 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & -2 \\ 0 & -2 \end{bmatrix}$?

   (a) $\begin{bmatrix} 6 & 3 \\ 0 & 8 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 7 \\ 5 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 6 & -1 \\ -8 & -8 \end{bmatrix}$

3. Which of the following are Hermitian matrices?

   (a) $\begin{bmatrix} 1 & 1+i \\ 1-i & -3 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & i \\ i & 2 \end{bmatrix}$ (c) $\begin{bmatrix} i & i \\ -i & i \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

4. Which of the following are the singular values of $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$?

   (a) 1 (b) $\sqrt{2}$ (c) $\sqrt{3}$ (d) $\sqrt{6}$

5. Suppose that $T: R^2 \to R^2$ is a linear transformation such that $T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $T\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$. $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) + T\left(\begin{bmatrix} 0 \\ -1 \end{bmatrix}\right) = ?$

   (a) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ (b) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ (c) $\begin{bmatrix} 4 \\ -1 \end{bmatrix}$ (d) $\begin{bmatrix} 6 \\ 2 \end{bmatrix}$

三. 計算題(十分)

1. Let $A = \begin{bmatrix} 2 & 1 & -1 \\ -2 & -1 & 2 \\ 2 & 1 & 0 \end{bmatrix}$, express A in the form $A = LDU$, where L is lower triangular with 1's along the main diagonal, U is upper triangular, and D is a diagonal matrix.