國立中央大學 109 學年度碩士班考試入學試題

所別： 資工類　　共 5 頁　第 1 頁

科目： 離散數學與線性代數

本科考試禁用計算器　　＊請在答案卷(卡)內作答

一、1～30 題，每一題答對給 1 分、答錯倒扣 1 分。

True or False.

A, B, and C are matrices, r is a scalar

1. $AB = BA$. (A) True. (B) False.
2. $AB = AC \Rightarrow B = C$. (A) True. (B) False.
3. $AB = 0 \Rightarrow A = 0$ or $B = 0$. (A) True. (B) False.
4. $(AB)^T = A^T B^T$. (A) True. (B) False.
5. $(rA)^{-1} = rA^{-1}$. (A) True. (B) False.
6. $\det(A + B) = \det A + \det B$. (A) True. (B) False.
7. $\det(AB) = \det A \det B$. (A) True. (B) False.
8. $\det(AB) = \det(BA)$. (A) True. (B) False.
9. $\det(rA) = r \det A$. (A) True. (B) False.
10. $\det(A^{-1}) = (-1) \det A$. (A) True. (B) False.

$Ax = b$ is a consistent linear system.

11. If $b \neq 0$, the solution set may be a subspace. (A) True. (B) False.
12. If $b \neq 0$, the solution set is not a subspace. (A) True. (B) False.
13. If $b = 0$, the solution set may be or may not be a subspace. (A) True. (B) False.
14. If $b = 0$, the solution set is not a subspace. (A) True. (B) False.
15. If $b = 0$, the solution set is exactly a subspace. (A) True. (B) False.

A and B are two invertible matrices. If A is similar to B.

16. $\det(A) = \det(B)$. (A) True. (B) False.
17. A^2 is similar to B^2. (A) True. (B) False.
18. A^T is similar to B^T. (A) True. (B) False.
19. A^{-1} is similar to B^{-1}. (A) True. (B) False.
20. AB is similar to BA. (A) True. (B) False.

If $n \times n$ matrix A is diagonalizable, then

21. A has n distinct eigenvalues. (A) True. (B) False.
22. A has n linearly independent columns. (A) True. (B) False.
23. A^T and A^{-1} are all diagonalizable. (A) True. (B) False.
24. A has no zero eigenvalues. (A) True. (B) False.
25. A has n linearly independent eigenspaces. (A) True. (B) False.

參考用

注意：背面有試題

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Let A be an $m \times n$ matrix with orthogonal columns.
If W is a subset of R^n and $W^{\perp}$ is the orthogonal complement of W.

26. **Nul** A is the orthogonal complement of **Row** A. (A) True. (B) False.
27. $W^{\perp}$ is always a subspace. (A) True. (B) False.
28. $(W^{\perp})^{\perp} = W$. (A) True. (B) False.
29. If v and w is orthogonal, then Av and Aw is orthogonal. (A) True. (B) False.
30. $W \cup W^{\perp} = R^n$. (A) True. (B) False.

二、31～40題，每一題答對給2分、答錯倒扣2分。

True or False.

A is a linear-transformation matrix in the standard coordinate system.
If the coordinate system is changed to β coordinate system,
to find the transformation matrix (called the β-matrix) relative to β for
the transformation $x \mapsto Ax$, where $\beta = \{b_1, b_2, b_3\}$,

$$A = \begin{bmatrix} -7 & -48 & -16 \\ 1 & 14 & 6 \\ -3 & -45 & -19 \end{bmatrix}, \; b_1 = \begin{Bmatrix} -3 \\ 1 \\ -3 \end{Bmatrix}, \; b_2 = \begin{Bmatrix} -2 \\ 1 \\ -3 \end{Bmatrix}, \; b_3 = \begin{Bmatrix} 3 \\ -1 \\ 0 \end{Bmatrix}.$$

31. 0, -1 are in the β-matrix. (A) True. (B) False.
32. 1, -3 are in the β-matrix. (A) True. (B) False.
33. -2, -4 are in the β-matrix. (A) True. (B) False.
34. -6, -7 are in the β-matrix. (A) True. (B) False.
35. 8, -9 are in the β-matrix. (A) True. (B) False.

Find a ***QR*** factorization of matrix $\begin{bmatrix} 1 & 3 & 5 \\ 1 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 3 & 3 \end{bmatrix}$.

36. 1 is **not** in R matrix. (A) True. (B) False.
37. 3 is **not** in R matrix. (A) True. (B) False.
38. 5 is **not** in R matrix. (A) True. (B) False.
39. 7 is **not** in R matrix. (A) True. (B) False.
40. 9 is **not** in R matrix. (A) True. (B) False.

三、41～50題，每題5分，單選題，答錯不倒扣

41. Four friends have been identified as suspects for an unauthorized access into a computer system. They have made statements to the investigating authorities. 小英 said "神掌 did it." 韓總 said "I did not do it." 神掌 said "柯 P did it." 柯 P said "神掌 lied when he said that I did it." If the authorities also know that exactly one is lying, who did it?
    A. 小英.
    B. 韓總.
    C. 神掌.
    D. 柯 P.
    E. 郭董.

42. Which of the following statement is most inappropriate?
    A. There exists a bijection function from N to Z.
    B. There exists a bijection function from Z to Q.
    C. There exists a bijection function from Q to R.
    D. There exists a bijection function from N to Q.
    E. There exists a bijection function from Z to N.

43. Which of the following statement is most inappropriate?
    A. $3^{302} \bmod 5 = 4$.
    B. $3^{302} \bmod 7 = 2$.
    C. $3^{302} \bmod 11 = 9$.
    D. $3^{302} \bmod 385 = 9$.
    E. None of the above.

44. A binary relation R on a set of 3 elements is represented by the matrix $\begin{bmatrix}0 & 1 & 0\\0 & 1 & 1\\0 & 0 & 1\end{bmatrix}$.

    Which of the following statement is most inappropriate?
    A. R is antisymmetric.
    B. The reflexive closure of R is $\begin{bmatrix}1 & 1 & 0\\0 & 1 & 1\\0 & 0 & 1\end{bmatrix}$.
    C. The symmetric closure of R is $\begin{bmatrix}1 & 1 & 0\\0 & 1 & 1\\0 & 0 & 1\end{bmatrix}$.
    D. The transitive closure of R is $\begin{bmatrix}0 & 1 & 1\\0 & 1 & 1\\0 & 0 & 1\end{bmatrix}$.
    E. R is an equivalent relation.

45. Two undirected graphs, $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, are isomorphic if
A. $|V_1| = |V_2|$.
B. $|E_1| = |E_2|$.
C. The number of vertices with any given degree is the same in both G_1 and G_2.
D. For every subgraph of G_1, there is a subgraph of G_2 that is isomorphic to it.
E. All of the above must hold.

46. Which statement **cannot** be supported by Gödel's incompleteness theorem?
a) "Graph isomorphism problem is NP-hard in general".
b) "No single Artificial Intelligent algorithm can solve all humans' problems".
c) "The cloud computing platform is not capable enough to solve all big-data tasks".
d) "There are problems which cannot be calculated by Turing machine".
e) "We cannot expect to construct a large enough Database to support analyzing all aspects of social networks".

47. Which operator or function is associative?
a) Set difference $(-)$ on sets.
b) Division $(/)$ on numbers.
c) Implication $(\rightarrow)$ on propositions.
d) Greatest common divisor(GCD) on integers.
e) Cartesian product $(\times)$ on sets.

48. We will evaluate the time complexity of a recursive algorithm A with input of ***n*** items. Algorithm *A* works as follow: when input size is *m*, the algorithm will first use $\theta(\sqrt{m})$ steps to prepare and divide the input into 4 roughly equal-size subsets; for each size $m/4$ subset, recursively call *A*; finally it use $\theta(\sqrt{m})$ steps to merge all 4 partial results to get the final solution. What is the time complexity for this algorithm?
a) $\theta(n)$ b) $\theta(n\log n)$ c) $\theta(n^2)$ d) $\theta(\sqrt{n})$ e) $\theta(\sqrt{n}\log n)$

49. A restaurant provides different ways to accept reservation of ordered seats. For any individual (1 person), he/she can make reservation by email or through the web page. For any party of 2 persons, they can use phone, email, or web page to reserve. How many different ways for this restaurant to make reservation of ***n*** ordered seats?

a) $\frac{5}{12}3^n + \frac{3}{4}(-1)^{n+1}$ b) $\frac{1}{12}(-3)^n + \frac{9}{4}$ c) $3^n + (-1)^n$ d) $\frac{3}{2}2^n + (-1)^n$

e) $\frac{3}{4}3^n + \frac{1}{4}(-1)^n$

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50. We define that a person is "happy" if he is liked by at least 3 other people. Suppose in a group of 10 people, everyone likes at least 4 other persons. What following conclusion can be reached?

a) At least 4 persons are happy.

b) At most 6 persons are not happy.

c) It is impossible that no one is happy.

d) "like" cannot be a symmetric relation.

e) For each person, the number of persons who like him cannot be all different among 10 people.