### 國立中與大學

109 學年度 碩士班考試入學招生

# 試

## 題

學系:資訊科學與工程學系 乙組

科目名稱:基礎數學B

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#### 本科目不得使用計算機

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#### **Part I: Discrete Mathematics**

- 1. (20%) Find the number of ways to give 5 identical red pens and 4 identical blue pens to 3 children under the following conditions. (**Hint**: You can assume 3 children to be A, B, C, then discuss how to allocate red pens and blue pens to them respectively.)
  - (a) (10%) Any combination of pens for the three children.
  - (b) (10%) If one specific child needs to be assigned at least one pen (either red or blue).
- 2. (10%) Find the number of ways to divide 10 students into 4 groups [G1, G2, G3, G4] so that two of the four groups contain 3 students and the other two groups contain 2 students.
- 3. (10%) Consider the congruence equation  $7x \equiv 1 \pmod{9}$ , find the general solution of x
- 4. (10%) Solve the following Boolean algebra.
  - (a) (5%) Express E(x, y, z) = x(xy + y' + x'y) as its complete sum-of-products form (Hint: needs to consider z terms)
  - (b) (5%) Find the Boolean expression E = E(x, y, z) corresponding to the truth tablet T(E) = 01001001

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#### Part II Linear Algebra

- 1. Define a sequence of numbers in the following way:  $G_0 = 0$ ,  $G_1 = 1/2$ . and  $\forall k \geq 0$ ,  $G_{k+2} = (G_{k+1} + G_k)/2$ .
  - (a) (4 %) What are the values of  $G_4$  and  $G_5$ ?
  - (b) (5 %) Give a matrix A such that

$$A \left[ \begin{array}{c} G_{k+1} \\ G_k \end{array} \right] = \left[ \begin{array}{c} G_{k+2} \\ G_{k+1} \end{array} \right].$$

- (c) (5 %) Find an explicit formula for  $G_k$ .
- (d) (6 %) What is the limit of  $G_k$  as  $k \to \infty$ ?
- 2. Suppose A is 3 by 4, and Ax = 0 has exactly 2 special solutions:

$$\mathbf{x}_1 = \begin{bmatrix} \mathbf{1} \\ \mathbf{1} \\ \mathbf{1} \\ 0 \end{bmatrix}$$
, and  $\mathbf{x}_2 = \begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \end{bmatrix}$ 

- (a) (6 %) Find the reduced row echelon form of A.
- (b) (3 %) What are dimensions of Col(A), Row(A), and Null(A)? (6 %) Justify your answers.
- 3. Let H be a plane and y be a given point in  $R^3$ 
  - (a) Let p be the orthogonal projection of y onto the plane H. Show that p is the point in H that is closest to y. (6%)

(b) Let 
$$H = \{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} | x_1 + 2x_2 + x_3 = 4 \}$$
 and  $y = \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix}$ . Find  $p$ . (4 %)

(c) (5 %)

Find the distance between y and p.