Please draw a following table on the first page of the answer sheets and fill in the answers accordingly.

1			6	(a)	(b)
2			7		
3			8		
4			9		
5			10		

1. (10%) A graph is bipartite if and only if it has no cycle of _________ length. A bipartite graph can be colored with _________ colors.

2. (10%) If $a_n = 3\,a_{n/2} + 1$ for $n \geq 2$ and $a_1 = 1$. Then $\log_2 a_n \approx$ _________$\times \log_2 n$.

3. (10%) There are _________ ways a binomial random walk, starting at the origin, returns to the origin the first time after exactly $2n$ steps.

4. (10%) With $0 \leq m, n$, $A =$_________ and $B =$_________ in
$$\sum_{k=m}^{n} \binom{k}{m} = \binom{A}{B}.$$

5. (10%) Let $(G, \circ)$ be a finite group with subgroup H. Prove that $|H|$ divides $|G|$.

6. (a) (5%) Suppose U and V are distinct six-dimensional subspaces of a vector space W and $\dim W = 8$. Find $\dim(U \cap V)$.

   (b) (5%) Let V be a vector space of all symmetric 2×2 matrices. Define a linear transformation $T: V \to P_2(R)$ by
$$T\left(\begin{bmatrix} a & b \\ b & c \end{bmatrix}\right) = (a-b) + (b-c)x + (c-a)x^2.$$
   Find rank(T).

7. (10%) Suppose $m(t) = t^r + a_{r-1}t^{r-1} + \cdots + a_1 t + a_0$ is the minimal polynomial of a nonsingular $n \times n$ matrix A. Find A^{-1}.

8. (10%) Solve for t if $det\left(\begin{bmatrix} 3-t & 2 & 0 & 0 & 0 \\ 1 & 4-t & 0 & 0 & 0 \\ 0 & 0 & 3-t & 1 & 0 \\ 0 & 0 & 1 & 3-t & 0 \\ 0 & 0 & 0 & 0 & 4-t \end{bmatrix}\right) = 0.$

For problems 9 and 10, your answer will be considered correct only if all the true statements are selected.

9. (10%) Which of the following statements are true?
   (a) Let A and B be $n \times n$ matrices. Assume that A is invertible and $B^3 = 0$. If AB=BA, then A+B is also invertible.
   (b) Let S be the set of all sequences in R^∞ which have exactly N non-zero elements where N is a known constant. S is a subspace of R^∞.
   (c) Any square matrix A can be represented as a sum of a symmetric matrix and a skew-symmetric matrix.
   (d) If A and B are similar, they have the same eigenvectors.
   (e) If $A, B \in R^{m \times n}$, rank $(A - B) \leq$ rank(A)−rank(B).

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10. (10%) Which of the following statements are true?

(a) Let $D: V \to V$ be defined by $D(f) = \frac{df}{dt}$, where V is the space of functions with basis $\{\sin t, \cos t\}$. The eigenvalues of D are ± 1.

(b) F is linear if $F: \boldsymbol{R}^3 \to \boldsymbol{R}^2$ is defined by $F(x, y, z) = (x + 1, y + z)$.

(c) Assume that $A \in M_{n \times n}(F)$ has two distinct eigenvalues, λ_1 and λ_2. If $\dim(E_{\lambda_1}) = n - 1$, A is diagonalizable.

(d) Let A, B and C be three $n \times n$ matrices such that for $k = 1, 2, \cdots, n$

$$c_{ik} = a_{ik} + b_{ik}, \text{ for some } i;$$
$$a_{jk} = b_{jk} = c_{jk}, \text{ for } j \neq i.$$

Then we have $det\mathrm{C} = det\mathrm{A} + det\mathrm{B}$.

(e) Let T be a diagonalizable linear operator on a finite-dimensional vector space, and let m be any positive integer. T and T^m are simultaneously diagonalizable.

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