

國立中興大學

108 學年度

碩士班考試入學招生

試 題

學系：資訊科學與工程學系

乙組

科目名稱：基礎數學 B

國立中興大學108學年度碩士班招生考試試題

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本科目不得使用計算機

本科目試題共2頁

Discrete Mathematics

1. (10%) Find all solutions to the system of congruences  $x \equiv 1(\text{mod}2)$ ,  $x \equiv 2(\text{mod}3)$ ,  $x \equiv 3(\text{mod}5)$ , and  $x \equiv 4(\text{mod}11)$ .
2. (10%) How many bit strings of length 10 contain at least four 1s?
3. (10%) Solve the following recurrence relation together with the initial conditions given.  
$$a_n = 5a_{n-1} - 6a_{n-2} \text{ for } n \geq 2, a_0 = 1, a_1 = 0.$$
4. (10%) Which of these relations on the set  $\{1, 2, 3, 4\}$  are antisymmetric and transitive?
  - (a)  $\{(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$
  - (b)  $\{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$
  - (c)  $\{(2, 4), (4, 2)\}$
  - (d)  $\{(1, 2), (2, 3), (3, 4)\}$
  - (e)  $\{(1, 1), (2, 2), (3, 3), (4, 4)\}$
5. (10%) Use a K-map to simplify the following sum-of-products expansion.  
$$wx\bar{y}\bar{z} + w\bar{x}yz + w\bar{x}y\bar{z} + w\bar{x}\bar{y}z + \bar{w}x\bar{y}\bar{z} + \bar{w}\bar{x}y\bar{z} + \bar{w}\bar{x}\bar{y}z$$

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Part II Linear Algebra

1. Let  $A$  be a  $4 \times 3$  matrix of rank 3.
  - (a) Give a simple example of such a matrix  $A$ . Justify your answer. (5 %)
  - (b) What is the dimension of the null space of  $A$ ? (3 %)
  - (c) What is the dimension of the column space of  $A$ ? (3 %)
2. Find the point  $Q$  on the line  $y = x/3$  that is closest to the point  $(1, 4)$ . (6 %)
3. Let  $\{u_1, u_2, u_3\}$  be an orthonormal basis for an inner product space. If  $x = c_1u_1 + c_2u_2 + c_3u_3$  is a vector with properties  $\|x\| = 5$ ,  $\langle x, u_1 \rangle = 4$  and  $x \perp u_2$ , then what are the possible values of  $c_1, c_2, c_3$ ? (9 %)
4. Suppose  $A$  is a  $3 \times 3$  matrix with eigenvalues  $\lambda_1 = 1$  and  $\lambda_2 = 2$ . Suppose also that  $A - I$  has rank one.
  - (a) Which eigenvalue of  $A$  is repeated? (3 %) Justify your answer. (5 %)
  - (b) Give a simple matrix  $B$  which is symmetric and similar to  $A$ . Justify your answer. (6 %)
5. Consider the vector space of polynomials of the form  $p(x) = ax^3 + bx^2 + cx + d$ . Are the following subspaces? Explain briefly.
  - (a) Those  $p(x)$  for which  $p(1) = 0$ . (3 %)
  - (b) Those  $p(x)$  for which  $p(0) = 1$ . (3 %)
  - (c) Those  $p(x)$  for which  $a + b = c + d$ . (4 %)