國立中興大學

108學年度

碩士班考試入學招生

試　　題

學系：資訊科學與工程學系

甲組

科目名稱：基礎數學A

Part I Discrete Mathematics

A. Fill the blanks from ① to ⑨. (4 points each)

1. Assume that an automatic recognizer is used to distinguish boys from a group of people which consists of 10 boys and 8 girls. There are 12 persons recognized as boys by the recognizer. However, only 9 out of these 12 persons are actually boys, other 3 persons are girls. With these statistics, the precision and accuracy of this recognizer are ① and ② respectively. .
2. In the equation of $y_1 + y_2 + 5y_3 = 12$, there are 7 solutions of positive integers (that is, y_1, y_2, and y_3 are all positives) and ③ non-negative integers.
3. Translate the following 2 statements using logical symbols, such as $\forall$ and $\exists$, propositional variables, and logical operators. The logical expression for "There is no maximum integer" is ④ and the logical expression of "Every integer has a unique additive inverse" is ⑤ .
4. The recurrence relation of the number of moves required for Hanoi tower is a_k = ⑥ , where $a_1 = 1$, $a_2 = 3$.
5. Assume that there are 1 red ball and 2 blue balls in box 1, and 2 red balls and 3 blue balls in box 2. You choose one ball randomly. If you have selected a red ball, then the probability that you selected a ball from the 1st box is ⑦ .
6. The number of bit strings of length 10 having more 0s than 1s is ⑧, and the number of bit strings of length 10 having at least 3 1s is ⑨.

B. True or false (2 pts each for a correct answer and -1 point for a wrong answer)

1. Incidence matrix, for graph representation, is a symmetric matrix.
2. The cardinality of Q is the same as the cardinality of Z.
3. Among 100 people there are at least 9 who were born in the same month.
4. $(P(S), \subseteq)$ is a partially ordered set, where P(S) is a power set of S={1,2,4}.
5. "$\neg p \rightarrow q$" is logically equivalent to "$\neg(q \leftrightarrow p)$", where $\neg$ stands for "not"
6. There are 81 ways to put 4 distinguishable balls into 3 different boxes.
7. Traveling salesman problem is the problem to find an Euler circuit of least cost.

Part II Linear Algebra

1. Determine whether the set S is linear independent or dependent. (3% each)

(a) $S = \{(2,-1,4),(3,6,2),(2,10,-4)\}$ in R^3.

(b) $S = \{(2,1,1),(2,-1,3),(2,3,-1)\}$ in R^3

(c) $S = \{0, x, x^2\}$ in polynomial space P_2.

(d) $S = \{3+x+x^2,\ 2-x+5x^2, 4\text{-}x^2\}$ in polynomial space P_2.

(e) $S = \{(1+x)^2,\ x^2+2x, 3\}$ in polynomial space P_2.

2. Let matrix $A = \begin{bmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

(a) Find A^{-1}. (5%)

(b) Verify whether A is positive definite. (5%)

(c) Find a matrix P such that $P^{-1}AP$ is diagonal. (10%)

3. Let $T : R^2 \to R^3$ be defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 2x_2 \\ -x_1 \\ 0 \end{bmatrix}$$

(a) Find the matrix of T with respect to the bases $B = \{\mathbf{u}_1, \mathbf{u}_2\}$ and $B' = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, where

$\mathbf{u}_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ $\mathbf{u}_2 = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$ $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$ $\mathbf{v}_3 = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$. (10%)

(b) Use the matrix obtained in (a) to compute $T\left(\begin{bmatrix} 4 \\ 6 \end{bmatrix}\right)$. (5%)