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A shape analysis and template matching of building features by the Fourier transform method

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ABSTRACT

Shape cognition and representation play an important role in spatial analysis because shape contains some characteristics of geographic phenomena that can be mined to discover hidden geographic principles. As a difficult cognition problem, the shape representation problem in GIS field has the properties of abstraction, indetermination and symbolization. How to use a model to represent shape cognition in our mental world and how to use a single number to compute the shape measure are interesting questions. In the image processing domain, there are many shape measure methods, but there are few proposals for corresponding vector data. This study aims to build a polygon shape measure and offers a Fourier transform-based method to compute the degree of shape similarity. The procedure first represents the boundary of the vector polygon shape as a periodic function, which is expanded in a Fourier descriptor series, and then, it obtains a set of coefficients that capture the shape information. Through the experiment on spatial shape match and shape query, the study shows that Fourier transform-based shape identification and template matching is consistent with human cognition.

1. Introduction

At human scales, the world is composed of objects, events, processes, and background environments. From the GIS perspective, we use spatial analysis to study the world by extracting spatial rules and geographic principles (Montello & Freundschuh, 1995). Spatial analysis, on the one hand, investigates the ontology properties of the spatial object as an existing entity; on the other hand, we must consider the thinking process in our brains as a cognition procedure. Spatial cognition, which addresses the cognition of spatial properties of the world, including the location, size, distance, direction, shape, pattern, movement and inter-object relations, has an important role to play in spatial analysis. However, in the GIS domain, the traditional spatial analysis concentrates on geometric, topologic or semantic information but pays little attention to the spatial cognition-related information.

Cognitive structures and processes are part of the mind, which emerges from a brain and nervous system inside of a body that exists in a social and physical world (Freksa, 1991). The result of spatial cognition is usually to act as a mental map, which represents a spatial shape and pattern in human memory. The spatial shape obtained by thinking and reasoning contains some characteristics of geographic phenomena, which can be mined to discover hidden geographic principles. The famous example in geo-science history is that of Alfred Wegener, who built the theory of continental drift, which was first driven by continental shape analysis. Wegener in 1912 first noticed that the shapes of the continents on either side of the Atlantic Ocean appeared to fit together, for example, Africa and South America. To some degree, shape is the result of the evolution of geographic entities and the interaction with phenomena in history. For example, the river shape and drainage pattern are related to hydrological and geological conditions in the natural environment. In human geography, the shapes of ancient buildings reflect the cultural characteristics of construction in the corresponding era. By shape identification and analysis, we can discover some special spatial characteristics and pattern principles that are hidden behind the objects.

Shape allows the prediction of many facts about an object and, in some situations, its effect exceeds other features, for example, size or location. We usually must find an object that matches the mental symbol in our memory by human reasoning, to express the "similar to" judgment. A shape-based spatial query usually expresses the recognition that one object is similar to another object or to two objects in their shape structure. For example, we want to extract some buildings that are "T" shaped or "U" shaped from a spatial database. The retrieved result is usually uncertain, depending on the human's emotion, background knowledge and perception abilities. The shape template and the degree to which two objects are similar vary with different people. The traditional

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spatial query, for example, the spatial SQL language (Shekhar & Chawla, 2002), can conduct only the query on geometric or topological measures and cannot involve the cognition measure. From the perspective of spatial communications, the SQL on a spatial query must extend to operations that can compare and extract shape information.

Because of the uncertainty of shape judgment, a spatial shape can be qualitatively compared only by such methods as fuzzy mathematics. In this process, the key task is to describe mathematically the shapes and to derive a similarity measurement to compare the shapes under the idea of fuzzy properties. In the multimedia and image processing domain, shape representation and measurement actively generate many methods and algorithms (Bengtsson & Eklundh, 1991; Hu, 1962; Jones & Ware, 1998; Latecki & Lakämper, 2002; Paul, Rosin, & Žunić, 2011; Žunić, Hirota, & Rosin, 2010). These methods aim at a region, boundary, and structure. For the description of global shape properties, there are geometric parameters, which include size, perimeter, convex perimeter, elongation, roughness and compactness. For polygon objects, the turn function or bend angle function based on contour points can be applied to measure a region shape with invariance to translation, rotation, and scale (Latecki & Lakämper, 2002). In these fields, the moment-based algorithm is an efficient method to represent the shape, aiming at the area boundary, which includes invariant moments, higher order moments, and generalized complex moments (Cho-Huak & Roland, 1988; Kim & Kim, 1998; Žunić et al., 2010).

In the image database field, the measurement of shape is conducted on the basis of a pixel or raster grid through the integration of a set of pixels to obtain a complete shape concept. However, in the GIS database, which mainly stores vector data, the shape measure directly faces individual geometric entities, such as lines, polygons, arcs, and point clusters. Thus, the shape measuring method in the GIS domain is different from that of image databases. It is easy to transform pixel data to the frequency domain representation, which usually acts as the basis of the shape measure. In an image database, the object boundary is usually represented as the chain code, which has changes along the boundary tracking or central angle that can be converted to the frequency domain.

In contrast to the image pixel data, this study aims at GIS vector data investigating the shape representation and studying the shape matching for building features. The applied algorithm is the Fourier transform, which has been widely used in image data analysis. However, in this study, the Fourier transform is based on the continuous function of vector data rather than the discrete pixel chain. The studied question behaves as template matching, which is controlled by the Fourier descriptor.

The remainder of this paper is organized as follows. Section 2 examines the characteristics of the region shape, taking the building features as an example. Section 3 presents the method of the Fourier transform on vector polygon data and discusses the formula for the shape measure. The template-based spatial shape matching is offered in Section 4 with experimental analysis. Section 5 discusses the characteristics of this method and concludes with proposals for future research.

2. Shape representation

Shape is probably the most important property that is perceived about objects (Palmer, 1999). Aiming at different content, the shape representation can be divided into three classes, namely, the boundary, the region and the structure objective. The classification is based on whether shape features are extracted from the contour only or are extracted from the whole shape region (Latecki, Lakämper, & Eckhardt, 2000; Zhang & Lu, 2004). The boundary shape of an object describes the complexity of the curve pattern, which is the extension trend in one dimension. Its comparison can be measured by smoothness, fitness, curvature and other computations. The region shape regards the object as a point set, a connected component and, in two dimensions, represents the pattern of object distribution and extension. The region shape can be described with vague terms such as elongated, round, or compact. The measures on the region shape usually include area, circularity (perimeter^2/area), eccentricity (length of major axis/length of minor axis), major axis orientation and blending energy. These geometric measures represent the shape in only a general way from the perspective of a quantitative measure that does not describe the shape from a cognition perspective. We cannot judge the similarity of two shapes by these measures. The structure shape regards the whole object as a compendium of different parts or components and studies the organization pattern. A given object can be mapped into a graph in which nodes correspond to divided pieces and arcs encode spatial relationships by skeleton conversion at reduced dimensions. Common methods of structure shape decomposition are based on polygonal approximation, curvature decomposition and curve fitting (Pavlidis, 1982). The present study will investigate the region shape in GIS data structures and will build a shape measure to quantitatively compare the similarity of two shapes.

Compared with other domains, such as image processing, computer vision and computer graphics, GIS addresses spatial objects with a vector representation at a larger spatial scale. The shape representation in GIS has the following properties.

2.1. Abstraction

According to Gestalt cognition principles, we first look at an object as a whole to obtain the complete sense of the region shape, and then, we look into the details that compose the shape. This approach implies that the shape representation should be abstracted first through a simplification, to obtain a generalized concept. In image processing, we use noise reduction methods to remove minor or deflection details. In the GIS field, the vector data can apply map generalization technology to obtain the abstracted structure (Brassel & Weibel, 1988; Wang & Muller, 1998), as shown in Fig. 1. Compared with the objective of the map representation from a large scale to a small scale, the generalization that aims at shape extraction need to be conducted at a large degree (Ai, Guo, & Liu, 2000; Li, Yan, & Ai, 2004). There is a special map cartogram called a value-by-area map (Dent, 1975) that depicts the attribute of a geographic object as the object's area. The abstraction makes the region size completely different from the original area, but the shape remains similar to the original. Another map, called the schematic map, is a linear abstraction of functional networks

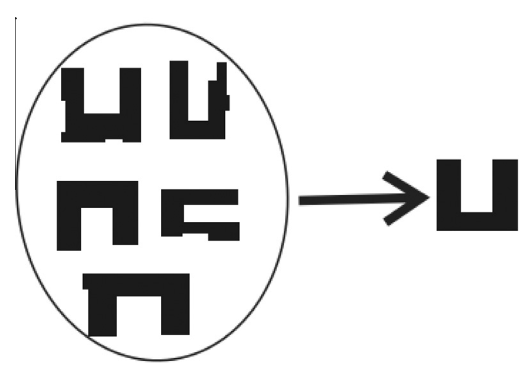


Fig. 1. The shape abstraction from different region scenes.

for subway, railway, or shipping lane representations through line simplifications, to represent the network structure and topological relationships (Avelar & Muller, 2000; Sergio, Mark, Steven, & Kreveld, 2001). The main shape of the network edge is maintained as well as possible in the simplification process. The spatial data handling examples above belong to the shape abstraction to reduce the noise, defects, arbitrary distortion and occlusion in the shape representation.

2.2. Symbolization

For a region shape description, we usually establish the association between the recognized object and the template pattern in our memory, which is familiar to us, according to our knowledge and experience. This template is used as a symbol to represent the region shape (Rainsford & Mackaness, 2002). The shape template could be the text letter, the Chinese text, a familiar animal, the goods in daily life, and other entities. We usually say that the territory of China is similar to a large rooster and Italy is similar to a boot. This symbolization makes the shape representation easy to understand and communicate.

2.3. Indetermination

Shape representation relies on the cognition of humans who have different background knowledge, intentional interests and emotions. Thus, for the same region object, different people could recognize different shapes. The indetermination of region shape recognition makes the shape query uncertain in template selection and the degree to which the decision about two objects are similar. The same building could be identified as L, U or V shapes by different people. The shape-based spatial query should belong to a fuzzy query class. For a measurement of shape similarity, the qualitative method divides the representation into such classes as *very strongly similar*, *strongly similar*, *moderately similar and weakly similar*. For this consideration, the shape comparison can apply fuzzy mathematics to define the membership function for the fuzzy term "similar"(Ai, Shuai, & Li, 2008).

Considering the above properties, we can find that shape is a very difficult concept to capture mathematically and with consistency. Determining how to use a model to represent the shape sense in our mental world and by a single number to compute the shape measure is not easy. The desired property of a similarity function, C, should be a metric that has the characteristics of self-identity, positivity, symmetry and triangle inequality (Basri, Costa, Geiger, & Jacobs, 1998), and additionally, the similarity function should be continuous and invariant to geometric transformation(Latecki et al., 2000). Many algorithms have been developed to measure shape similarity, and these algorithms are aimed at different situations and requirements. Among them, the moment-based and Fourier transform-based methods play important roles under the requirements of shape similarity stated above. These algorithms combine information across an entire object rather than providing information at only a single boundary point. They capture some of the global properties that are missing from many purely contour-based representations, such as the overall orientation, elongation, and others. The next sections will discuss the region shape measure by Fourier transform, considering the vector data structure rather than chain coding.

3. Shape measure by Fourier transform

Fourier transform is a well-known data analysis method for the frequency domain and is broadly applied in image processing, such as in shape representation. Many Fourier transform methods have been reported in the literature, including using a Fourier descriptor for shape analysis and character recognition (Persoon & Fu, 1977), shape classification (Kauppinen, Seppanen, & Pietikainen, 1995), and shape retrieval (Lu & Sajjanhar, 1999). In these methods, different shape signatures have been exploited to obtain the Fourier descriptor. The basic idea of shape representation by Fourier transform is to describe the shape in terms of its spatial frequency content. This process first represents the boundary of the shape as a periodic function, which is expanded in a Fourier descriptor series, and then, it obtains a set of coefficients that captures the shape information (Arkin, Chew, Huttenlocher, Kedem, & Mitchell, 1991). A Fourier descriptor begins by tracking the region boundary, which can be represented as two data structures, namely the chain code in grid representation and consecutive points in vector representation. Both ways meet the important condition in shape representation, namely, to maintain invariance during region translation, rotation, and scaling. We will apply the vector data structure-based method because GIS mainly deals with vector data.

3.1. Fourier descriptor on vector polygon

Usually, we convert 2D areas or boundaries into 1D function by using a shape signature to represent the shape. There are four shape signatures, namely, central distance, complex coordinates (position function), curvature and cumulative angular function (Persoon & Fu, 1977). Here, we consider the position-based shape signature by the Fourier descriptor method.

The boundary is represented as a set of connection points. Each point k is regarded as a complex number pair by treating the x-axis as the real axis and the y-axis as the imaginary axis. Thus, a given point, which is a function of the arc length s, can be represented as

$$U(s) = x(s) + iy(s),$$

where s is the distance of the arc path between the given point and the reference original point, for example, b_0 is shown in Fig. 2, and i is $\sqrt{-1}$. The representation treats the plane of the region as an Argand diagram, which reduces a 2D problem to a 1D problem.

Let the region perimeter be Z; then, $U(s)$ is a periodic function with

$$U(s+Z) = U(s), \quad 0 \leqslant s < Z.$$

Let $t = \frac{2\pi s}{Z}$; then, the equation is modified to

$$U(t) = x(t) + iy(t), \quad 0 \leqslant t < 2\pi,$$

a periodic function with the period 2π, and its Fourier expansion given by

$$U(t) = \sum_{n=-\infty}^{+\infty} p_n e^{-int}, \quad 0 \leqslant t < 2\pi.$$

The coefficients of the Fourier transform are

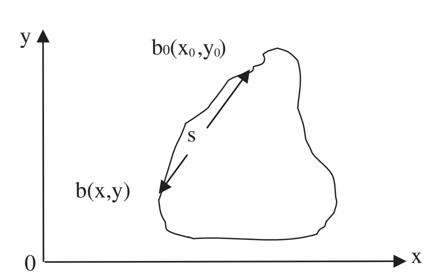


Fig. 2. The boundary representation in a complex plane.

$$p_n = \frac{1}{2\pi}\int_0^{2\pi} U(t)e^{-int}dt, \quad n = 0, \pm 1, \pm 2, \ldots.$$

Suppose that the region boundary has M points; then, the boundary can be regarded as a consecutive accumulation of $M - 1$ segments. Let s_k be the length of the arc from point k to the reference point around the boundary

$$s_k = \left\{ \begin{array}{ll} 0, & k = 0; \\ \sum_{\lambda=0}^{k-1}\sqrt{(x_{\lambda+1} - x_\lambda)^2 + (y_{\lambda+1} - y_\lambda)^2}, & k = 1, 2, 3 \ldots M-1 \end{array} \right\}.$$

Assume that the change point $(x(s), y(s))$ is located between point k and point $(k + 1)$. Let γ be the distance between point $(x(s), y(s))$ and point k, then

$$\gamma = \sqrt{(x_{(s)} - x_k)^2 + (y_{(s)} - y_k)^2}, \quad k = 1, 2, 3 \ldots, M-1.$$

The accumulated distance is $s = s_k + \gamma$. The segment length between point k and point $k + 1$ is

$$l_k = \sqrt{(x_{k+1} - x_k)^2 + (y_{k+1} - y_k)^2}; \text{ then the length rate } D_k = (x_{k+1} - x_k)/l_k.$$

Construct a linear function across point k and point $k + 1$ and apply the integral with a segment length that ranges from 0 to l_k. Then, the coefficients P_n are represented as

$$P_n = \frac{1}{Z}\int_0^Z U(s)e^{-i\frac{2\pi ns}{Z}}ds = \frac{1}{Z}\sum_{k=0}^{M-1}\int_0^{l_k} U(s_k + \gamma)e^{-i\frac{2\pi n(s_k+\gamma)}{Z}}d\gamma = \frac{1}{Z}\sum_{k=0}^{M-1}(a_k + ib_k),$$

where a_k, b_k are expressed as

$$a_k = \int_0^{l_k}[(x_k + \gamma D_k)\cos(-\tfrac{2\pi n(s_k+\gamma)}{Z}) - (y_k + \gamma D_k)\sin(-\tfrac{2\pi n(s_k+\gamma)}{Z})]d\gamma$$
$$b_k = \int_0^{l_k}[(x_k + \gamma D_k)\sin(-\tfrac{2\pi n(s_k+\gamma)}{Z}) + (y_k + \gamma D_k)\cos(-\tfrac{2\pi n(s_k+\gamma)}{Z})]d\gamma.$$

Furthermore, expanding the above formula finally gives the expression of the coefficients as

$$\left\{ \begin{array}{l} \left. \begin{array}{l} A_n = \frac{1}{Z}\sum_{k=0}^{M-1}\frac{l_k}{2}(x_{k+1} + x_k) \\ B_n = \frac{1}{Z}\sum_{k=0}^{M-1}\frac{l_k}{2}(y_{k+1} + y_k) \end{array} \right\} n = 0; \\ \left. \begin{array}{l} A_n = \frac{1}{2\pi n}\sum_{k=0}^{M-1}\left\{x_k \sin\alpha - x_{k+1}\sin\beta + y_k\cos\alpha - y_{k+1}\cos\beta + \frac{Z}{2\pi n l_k}[(y_{k+1} - y_k)\times(\sin\alpha - \sin\beta) - (x_{k+1} - x_k)\times(\cos\alpha - \cos\beta)]\right\} \\ B_n = -\frac{1}{2\pi n}\sum_{k=0}^{M-1}\left\{x_k\cos\alpha - x_{k+1}\cos\beta - y_k\sin\alpha + y_{k+1}\sin\beta + \frac{Z}{2\pi n l_k}[(y_{k+1} - y_k)\times(\cos\alpha - \cos\beta) + (x_{k+1} - x_k)\times(\sin\alpha - \sin\beta)]\right\} \\ \text{where} : \alpha = -\frac{2\pi n s_k}{Z}, \quad \beta = -\frac{2\pi n s_{k+1}}{Z} \end{array} \right\} n \neq 0 \end{array} \right.$$

$P_n = A_n + iB_n$, where A_n, B_n are expressed as

In the Fourier descriptor above, we use the accumulation of $M - 1$ boundary segments to expand the coefficient formula, and for each segment, the integral of a continuous linear function is applied. Traditionally, the boundary is divided into chain codes or is partitioned as a series of segments by a determinate step, and then, the formula is expanded by the discrete integral method. Obviously, the method in this study has a higher accuracy due to the continuous integral.

The magnitude of the coefficient, P_n, has both rotation and translation invariance. A new coefficient $d(n)$ is defined to normalize the coefficient, P_n; the new coefficient is obtained by dividing the magnitude values of all of the other descriptors by the magnitude value of the second descriptor, thus

$$d(n) = \frac{|p_n|}{|p_1|}, \quad n = 1, 2, \ldots, M-1.$$

The descriptors also have invariance to scale. The series of coefficients $d(n)$ is usually called the shape parameters because they capture the main shape information.

3.2. Shape measure

According to the characteristics of the Fourier transform, the Fourier descriptor approximates the original region at different accuracies, and the coefficients capture the shape information with invariance to translation, rotation, and scale. The coefficient, P_n, at different orders, n, represents different frequency content of the region. The frequency domain at lower orders corresponds to more significant shape components. Because the shape is an abstracted representation, we can use a subset of low order coefficients to represent the overall features of the shape. The very high frequency information describes the small details of the shape. The high-order coefficients are not important in shape discrimination and, therefore, can be ignored.

Based on the coefficients $d(n)$, the feature vector to index the shape is:

$$f = [d(1), d(2), d(3), \ldots d(N-1)].$$

Because both features are normalized with respect to translation, rotation, and scale, for two model shapes indexed by the Fourier descriptor feature f_i and f_j, the Euclidean distance between these two feature vectors can be used as the following similarity measure

$$dis = \sqrt{\sum_{k=1}^{N}|d_i(k) - d_j(k)|^2},$$

where N is the truncation number of harmonics that is needed to index the shape.

Given a region, the truncation number, N, is determined by the degree that the Fourier descriptor approximates the original region. Assume that the original region is C_o; the approximated region by the Fourier descriptor is C_a, and the intersection of C_o and C_a is $C_{o\cap a}$. We define a parameter approximation degree, which is computed as follows

$$A\_degree = area(C_{o \cap a})/area(C_o).$$

The parameter, *A_degree*, ranges from 0 to 1. The closer the value is to 1, the higher the accuracy of the approximation. Given a determinate value for *A_degree*, for example, 0.85, different shapes require different order expansions of the Fourier descriptor to obtain sufficient accuracy. According to the characteristics of the coefficients P_n at different orders (frequency domain), the approximation needs fewer orders to reach a given accuracy if the region shape intends to meet the following conditions:

The region is close to a circle (with a high degree of compactness).

The region shape is close to convex (with a high convex degree).

The boundary is smooth with few angular arches on the boundary.

Fig. 3 shows that, from the experiments, the Fourier descriptor of the different shape types has the above aspects. A shape with a complex boundary needs more orders of expansions of the Fourier descriptor to access the approximation accuracy compared with a shape that has a smooth boundary. The concave polygon has the same trend as the convex polygon.

For two given regions, the shape measure should pre-determine the measure accuracy according to the above approximation degree to control the expansion order. Let n_i be the stop order when the expansion satisfies only the approximation accuracy. The larger of the two computed expansion orders $\{n_1, n_2\}$ is selected as the truncation order number of harmonics needed to index the shape in the computation of the shape similarity distance. This approach means that the shape measure needs both of the original regions mapped to Fourier descriptor representations at a sufficiently high accuracy. To compute the shape similarity distance, two vectors, $f_i = [d_i(1), d_i(2), d_i(3), \ldots, d_i(n_1 - 1)]$, $f_j = [d_j(1), d_j(2), d_j(3), \ldots, d_j(n_2 - 1)]$, must be matched with the same dimension. Then, the large stop order is selected as the truncation order number.

3.3. Experiments

We select building polygons as experimental data and apply the above Fourier transform method to compute the degree of the building shape similarity. The process first represents the boundary of the building shape as a periodic function, which is expanded in a Fourier descriptor series and then obtains a set of coefficients that capture the shape information to compute the shape measure distance. Part of the experimental data is shown in Fig. 4. The regular geometric shapes are selected as template shapes, and the building polygons to be compared are selected as candidate shapes.

For each pair that is composed of a candidate polygon and a template polygon, we compute the shape measure distance. Then, a matrix of shape measure distances between the template and each candidate building is obtained, as shown in Table 1. The element value describes the shape similarity degree between the corresponding column template and the corresponding row candidate building. Considering that the shape is the building representation that is close to orthogonal with long edge characteristics, the shape expansion by Fourier transform requires more orders. In this experiment, we expand the building shape to order 20, and we find that the area cover between the approximation and the original building reaches 97%. Based on the shape measure matrix, scanning each row allows us to find the optimal template to match the investigated building. Among the 20 elements of the building shape, 17 shapes are correctly judged as being similar to the templates, which is consistent with manual cognition. The recognition correctness is 85%. In general, the shape recognition result is consistent with human cognition.

3.4. Comparison with the turning tangent-based method

For the polygon shape measure, a very similar method is the turning tangent-based method that is performed using the L_2-

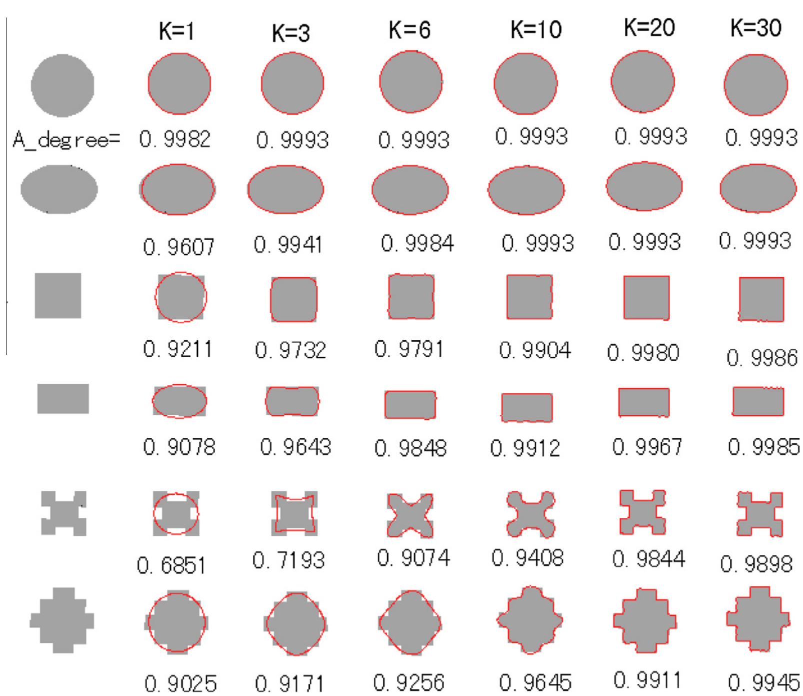


Fig. 3. The polygon approximation by Fourier descriptors for different shape types at different expansion orders.

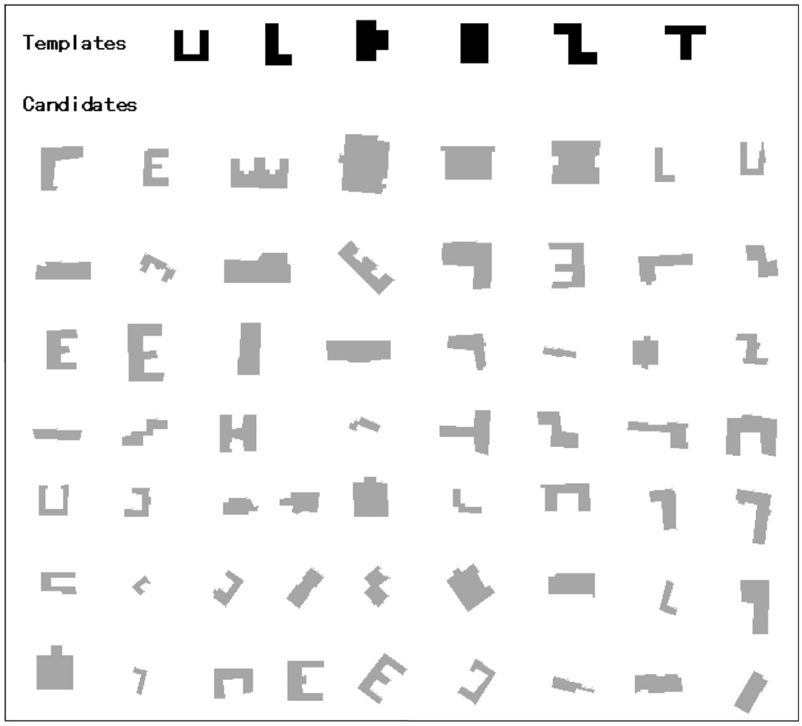


Fig. 4. The template buildings (on the top) and the object buildings to be matched in the experiment.

norm distance (Arkin et al., 1991; Latecki & Lakämper, 2002). To compare the difference in the polygon shape measures, we perform some experiments that aim at the same polygon data.

In the turning tangent-based shape measure (Arkin et al., 1991), the representation of the boundary of a simple polygon A is given the turning function $\theta_A(s)$. The function $\theta_A(s)$ measures the angle of the counterclockwise tangent as a function of the arc length s, which is measured from some reference point O on A's boundary. After two polygons, A and B, are converted to the representations of the turning function, respectively, $\theta_A(s)$ and $\theta_B(s)$, the degree to which A and B are similar ismeasured by the distance between the two turning functions. The shape distance between polygon A and B adopts the L_2 distance and is computed by the integration of the difference between the two turning functions from 0 to 1, as follows:

$$\left|d(A,B) = \int_0^1 |\theta_A(s) - \theta_B(s)|^2 ds\right|.$$

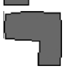
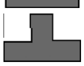
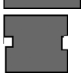
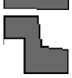
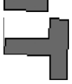
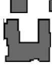
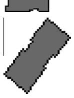
The distance $d(A,B)$ represents the area between two surrounding function lines in the geometry. This approach implies that the more similar the two turning functions are, the smaller the surrounding area is, and the smaller the distance $d(A,B)$. The distance $d(A,B)$ is sensitive to both the rotation of polygon A or B and the choice of the reference point. This method adopts the minimum $d(A,B)$ as the shape distance by rotating the object polygon and shifting the reference point on the polygon boundary. It can be proved that the minimum$d$ (A,B) exists; the computational approach is found in (Arkin et al., 1991).

For the same building polygon data as shown in the experiment above, we compute the shape distance by the turning tangent-based method and obtain the same shape measure matrix as shown in Table 2. In the same way, the smallest distance value has been marked to express the most similar shape. Among the 20 elements of the building shape, 16 shapes are correctly recognized as being similar to templates that are consistent with human cognition. The recognition correctness of the turning tangent method is 80%. To analyze two shape measure methods, we cannot simply compare the shape similarity according to two distance values because the meaning of the distance in the two different methods is not quantitatively comparable. However, we can compare the result in order scale. For the same templates, the candidate polygons can be ranked in an increasing sequence by the distance value for two shape measure methods. The smallest distance value corresponds to the most similar shapes among the candidate polygons for determining the template. The other values, in increasing order, represent similar shapes at the second and third levels and so on. Then, with human cognition, we can judge whether the ranked order is reasonable or not, and furthermore, for two ordered sequences of two shape measure methods, we can judge which method is better.

The ranked order is shown in Table 3, and the number represents the order number. From the ordered set of shape similarities, we can usually judge the difference between two shape measures. In Table 3, some candidates, such as O_3, O_5, have a similar serial order, which means that two measures have the same effect. Other candidates, such as O_7 and O_{20}, have very different serial orders, which mean that the two measures have a deviating effect. From the experiment, it is usually the case that the simpler the shape is (for example, a rectangle), the more different the two measures. A complex shape, such as a U-shaped polygon, can obtain a similar shape order compared with certain templates. When only considering the most similar shapes of the two measures (i.e., order 1, which corresponds to the smallest shape distance), we find that, among 20 candidates, there are 14 elements that are the same for the two measure methods. In the experiment, the correctness degree of the Fourier transform method is slightly higher than that of the turning tangent method when considering the most similar shape identification compared with manual cognition. However, a further conclusion needs to be investigated with more experiments in the future.

Through theory analysis and experimental comparison, we find two shape measure methods that have the following properties:

Table 1
The matrix of shape measure distance between the template and candidate buildings, and the most similar recognition result, by Fourier transform method.

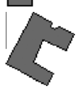
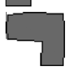
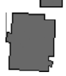
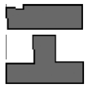
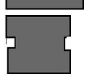
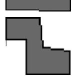
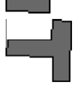
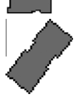
Candidates	Templates						
							Recognition correct?
	**0.3258**	0.6153	0.992	0.5403	0.3968	8.2265	Yes
	0.7036	**0.2905**	0.8201	1.0441	0.8068	7.8672	Yes
	**0.3345**	0.7025	0.9702	0.7051	0.796	7.9588	Yes
	1.0378	0.4655	1.3074	0.6332	**0.2548**	8.5714	**No**
	0.7214	**0.446**	0.7311	1.1553	0.9411	7.7344	Yes
	0.9181	0.9957	1.2989	**0.5573**	0.7733	8.4426	Yes
	1.1381	0.4913	1.3173	0.7783	**0.3955**	8.5862	**No**
	0.594	0.768	**0.5485**	1.3516	1.2346	7.4158	Yes
	1.9032	2.2739	2.4687	**1.6677**	2.0188	8.705	Yes
	0.8111	**0.1496**	0.9877	0.8947	0.592	8.1389	Yes
	**1.2508**	1.9396	1.6209	2.0341	2.1737	6.8424	Yes
	0.8929	0.3933	1.1778	0.5447	**0.2214**	8.455	Yes
	**0.546**	1.213	1.0503	1.4231	1.4911	8.2265	Yes
	7.8672	1.6826	1.4768	2.3161	2.1876	**1.2559**	Yes
	0.6775	0.4114	1.0704	0.4629	**0.343**′	7.9588	**No**
	**0.6907**	1.3421	1.2248	1.2796	1.4611	8.5714	Yes
	1.0673	0.4728	1.3005	**0.293**	0.6713	7.7344	Yes
	**0.4229**	0.4366	0.9249	0.6113	0.5772	8.4426	Yes
	0.8239	**0.1583**	0.9675	0.9294	0.6263	8.1102	Yes
	1.0793	0.4895	1.2868	0.6941	**0.3303**	8.5682	Yes

(1) Both methods can be applied to identify shape similarities that become close effects for simple shapes. Both methods are invariant to shape rotation, shifting and scale transformation. (2) The turning tangent method is more sensitive than the Fourier transform method. The Fourier transform method first expands the shape to some order, to reduce the detailed shape components, and then, it computes the vector distance at a coarse shape approximation. The turning tangent method directly computes the distance based on the original point turning around the boundary. Without the approximation, the process makes the turning tangent method sensitive to sharp changes in the boundary. For a natural object, such as a soil, lake or vegetation polygon with a much smaller boundary change, the Fourier transform method obtains a better effect. (3) In terms of the computational costs, the Fourier transform method is more complex. The Fourier expansion is a complex process, especially when expanding to more orders.

4. Applications of the shape measure

In this section, we discuss the applications of the Fourier transform-based shape measures to match and retrieve objects from a spatial database. We select the building feature for the experiment. The method works for any vector polygon; however, from the perspective of the shape measure applications, the building feature plays a more important role than other polygon features, such as vegetation, lake or land use. We are interested in extracting a building with a special shape, using a shape template to describe

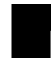
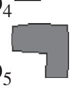
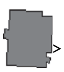
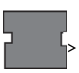
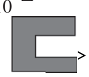
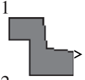
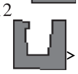
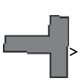
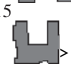
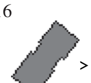
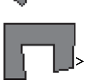
Table 2
The matrix of shape measure distance between the template and candidate buildings, and the most similar recognition result, by turning tangent method.

Candidates	Templates						
							Recognition correct ?
	1.256	1.0213	1.3193	**<u>0.8448</u>**	0.9325	1.450	**No**
	1.3914	1.0915	0.9977	1.0022	1.1459	**<u>0.8233</u>**	**No**
	**<u>0.4625</u>**	1.3807	1.1616	1.0519	0.9312	1.2634	Yes
	1.2305	0.4468	0.6542	**<u>0.2983</u>**	0.4384	0.8643	Yes
	0.8928	**<u>0.5464</u>**	0.6206	0.7314	0.8659	0.5838	Yes
	1.7649	0.8257	0.6388	**<u>0.4046</u>**	0.7786	1.0734	Yes
	1.3601	**<u>0.456</u>**	0.6397	0.5002	0.7965	0.9982	**No**
	1.5647	0.8048	**<u>0.5681</u>**	0.7523	1.1984	0.5879	Yes
	1.6688	0.8859	0.8632	**<u>0.3756</u>**	0.7456	1.1708	Yes
	1.4553	**<u>0.7257</u>**	1.0351	0.8943	1.1473	0.7734	Yes
	**<u>0.2074</u>**	1.872	1.5303	1.5121	1.3215	1.44	Yes
	1.3179	0.5561	1.016	0.7174	**<u>0.4269</u>**	0.78	Yes
	**<u>0.1437</u>**	1.1803	1.7737	1.8629	1.953	1.5916	Yes
	1.2863	0.8408	0.7752	0.9327	1.1112	**<u>0.2599</u>**	Yes
	**<u>0.5286</u>**	0.6538	1.3332	1.0053	0.7865	1.3657	Yes
	**<u>0.4887</u>**	1.5374	1.2589	1.2401	1.1458	1.2809	Yes
	1.3091	0.5207	0.7468	**<u>0.4452</u>**	0.5743	1.0284	Yes
	**<u>0.2679</u>**	0.5774	1.4064	1.2773	1.1351	1.4852	Yes
	0.6967	**<u>0.5586</u>**	0.8465	0.8158	1.0004	0.7538	Yes
	1.1641	0.7038	0.8668	**<u>0.5631</u>**	1.0468	1.1247	**No**

the construction, to compare two house shapes. Therefore, from the viewpoint of a practical requirement, we select a building feature to conduct the shape query and shape matching. Buildings have special properties in their spatial distribution, shape structure, and Gestalt nature compared with natural features. The building shape, to some degree, reflects the construction characteristics of the appropriate historical time and is consistent with special national cultural features.

Building shape matching is an important decision in applications such as spatial query (Ai et al., 2008), map generalization (Anders, Sester, & Fritsch, 1999; Rainsford & Mackaness, 2002; Regnauld, 1996), and spatial data mining (Lui, Thiruvenkadam, Wang, Thompson, & Chan, 2010; van der Werff & van der Meer, 2008). The building generalization of urban maps needs to identify either the shape of an individual building or the shape of a building cluster and then simplify the polygon or amalgamate polygon cluster under the condition of main shape preservation. Both individual building simplification and building cluster aggregation are active in the map generalization field and have many methods and algorithms. From the point of view of legibility, Regnauld, Edwardes, and Barrault (1999) discuss three operations for building simplification: detail removal, squaring and local enlargement. Lee (1999) presents some ideas on single building simplifications that focus on the shape maintenance. Based on the divide-and-conquer idea, Ai, Li, and Liu (2005) developed an algorithm to simplify building polygons through separating a building into a multiple hierarchical organization of rectangular elements. For building cluster aggregation, Regnauld (1996) developed a method to classify building groups by applying the MST model in graph theory. According to the continuous Gestalt characteristics of the building distribution, Steiniger (2006) developed a method to extract the building pattern and shape by the aggregation of the buildings within a buffer

Table 3
The comparison between two methods (F-Model: Fourier transform method; T-Model: Turning tangent based method).

Candidates		Templates						
								Recognition correct?
 O_1	F-Model	1	4	5	3	2	6	Yes
	T-Model	4	3	5	1	2	6	No
 O_2	F-Model	2	1	4	5	3	6	Yes
	T-Model	6	4	2	3	5	1	No
 O_3	F-Model	1	2	5	3	4	6	Yes
	T-Model	1	6	4	3	2	5	Yes
 O_4	F-Model	4	2	5	3	1	6	No
	T-Model	6	3	4	1	2	5	Yes
 O_5	F-Model	2	1	3	5	4	6	Yes
	T-Model	5	1	3	4	5	2	Yes
 O_6	F-Model	3	4	5	1	2	6	Yes
	T-Model	6	4	2	1	3	5	Yes
 O_7	F-Model	4	2	5	3	1	6	No
	T-Model	6	1	3	2	4	5	No
 O_8	F-Model	2	3	1	5	4	6	Yes
	T-Model	6	4	1	3	5	2	Yes
 O_9	F-Model	2	4	5	1	3	6	Yes
	T-Model	6	4	3	1	2	5	Yes
 O_{10}	F-Model	3	1	5	4	2	6	Yes
	T-Model	6	1	4	3	5	2	Yes
 O_{11}	F-Model	1	3	2	4	5	6	Yes
	T-Model	1	6	5	4	2	3	Yes
 O_{12}	F-Model	4	2	5	3	1	6	Yes
	T-Model	6	2	5	3	1	4	Yes
 O_{13}	F-Model	1	3	2	4	5	6	Yes
	T-Model	1	2	4	5	6	3	Yes
 O_{14}	F-Model	6	3	2	5	4	1	Yes
	T-Model	6	3	2	4	5	1	Yes
 O_{15}	F-Model	4	2	5	3	1	6	No
	T-Model	1	2	5	4	3	6	Yes
 O_{16}	F-Model	1	4	2	3	5	6	Yes
	T-Model	1	6	4	3	2	5	Yes
 O_{17}	F-Model	4	2	5	1	3	6	Yes
	T-Model	6	2	4	1	3	5	Yes
 O_{18}	F-Model	1	2	5	4	3	6	Yes
	T-Model	1	2	5	4	3	6	Yes
 O_{19}	F-Model	3	1	5	4	2	6	Yes
	T-Model	2	1	5	4	6	3	Yes
 O_{20}	F-Model	4	2	5	3	1	6	Yes
	T-Model	6	2	3	1	4	4	No

zone. All of these studies consider shape matching to be an important property in building generalization. However, how to quantitatively identify the similarity between candidate objects and the template through a geometric algorithm is a difficult question that requires an effective shape measuring method. None of the studies above, which are in the map generalization field, present the shape-matching approach.

4.1. Shape matching

We use the matrix of shape distances in Section 3.3 to find the matched template buildings for each candidate building polygon. We scan each row and, by the minimum distance, find the optimal matched template building. From a visual comparison by human judgment, we find that the recognition result is consistent with manual cognition in detecting the optimal template.

Because the shape measure by the Fourier transform is invariant to translation, rotation, and scale, the way in which the template matches the object in geometric characteristics must account for the change in the location, direction and size. This approach implies that once a matched template is found, the candidate building needs some geometric operations to cover the template in the best way for a geometric position. First, the template is scaled in size to force its MBR (Minimum Bounding Rectangle) to coincide with the MBR of the candidate object, as shown in Fig. 5. The scale transform in the x-axis and y-axis could be different in terms of the scale ratio. The template is then moved to the position of the object building to be matched, and the two MBRs are coincident. There are four cases in which the MBR edge coincides when considering the rotation direction, as illustrated in Fig. 6. For each case, we compute the approximation degree to find the optimal match direction. Through the above geometric processes, we finish the matching between the candidate and the template building. Fig. 7 is the illustration of the experiment result for the building candidate data in Fig. 4. The digit describes the approximation degree. The approximation accuracy is high if the template recognition is correct in terms of the shape measure; otherwise, the match is not satisfactory.

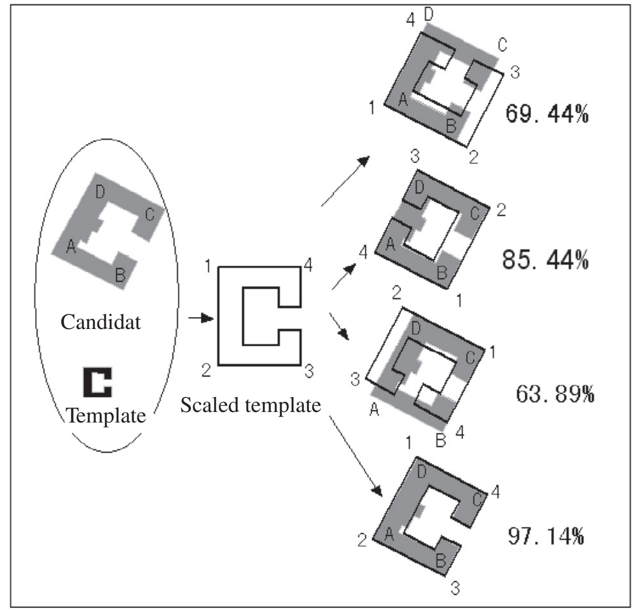


Fig. 6. The matching between template and object building by MBR coincidence with 4 cases.

4.2. Shape query

The shape-based spatial query attempts to retrieve objects, such as the pre-determined template building from the spatial database, which can be formed from the following SQL statement:

$$Select\{O_i\}From\ DataBase\ Where\ O_i.shape\ LIKE\langle Template\rangle.$$

The operation *LIKE* (meaning 'similar to') is computed by a shape match such as the Fourier transform method. The ⟨*Template*⟩ buildings in the spatial database act as certain typical simple buildings, which look similar to the shape of a letter, a special geometric construction, or another shape. Considering the uncertainty in the shape similarity judgment, the above SQL statement should be enhanced as

$$Select\{O_i\}From\ DataBase\ Where\ O_i.shape\ LIKE\langle Template\rangle\ At\_degree\langle c_i\rangle.$$

The added term *At_degree* $\langle c_i\rangle$ describes the shape similarity to different extents. This query is similar to the concept of fuzzy mathematics; thus, we use a membership function to represent the degree of *LIKE*, which is defined on the basis of the shape similarity distance.

The shape similarity distance between a given template building and other objects to be queried is difficult to normalize. The range of shape distance values depends on the shape complexity and the template form. For different template buildings, the shape similar distance varies substantially. Thus, we cannot define a uniform membership function for all of the template buildings. Instead, one template building applies to one membership function.

To allow the shape-based query to be consistent with human cognition, the quantitative representation of the shape similarity distance must be transferred to a qualitative representation that correctly reflects the spatial cognition in our mental world. For the degree to which two shapes are alike, we distinguish four classes, namely, *very strongly similar*, *strongly similar*, *moderately similar and weakly similar*, which are denoted as c_1, c_2, c_3, c_4, respectively.

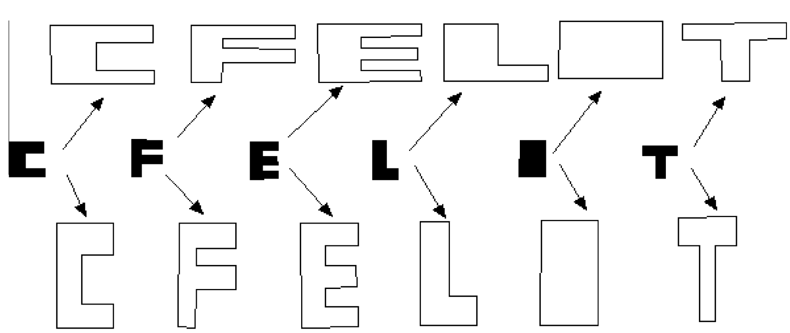


Fig. 5. The template building and the scale transformation to match the object size.

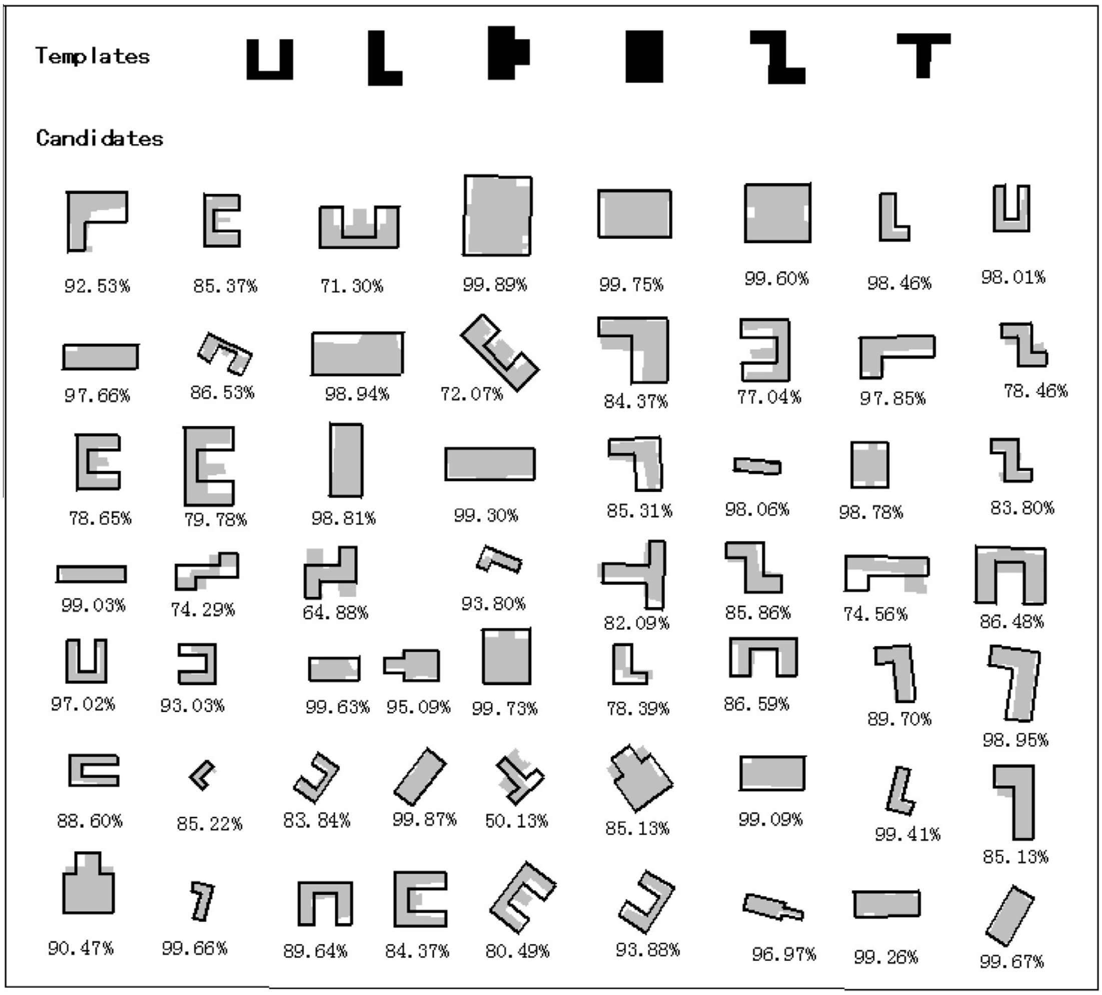


Fig. 7. The illustration of template overlapping over object buildings and the approximation accuracy.

The membership degree of the shape similarity can be, for example, 0.8, 0.6, 0.4, 0.2, respectively, which corresponds to the four fuzzy terms above.

The cognition experiment determines the relationship between c_i and the shape similarity measure. First, a series of building samples with reference to the template building is constructed. Based on the order of the shape similarity distance from small to large, the experiment participants are allowed to judge the similarity degree and divide the linear order into four stages. To make the building shape easily comparable, we allow the shape to change progressively, accessing different objectives based on the idea of a conceptual neighborhood in a cognition experiment domain. The sample data are organized in a radial structure, as shown in Fig. 8. The judgments from different participants are summarized to determine the value of c_i. Because of the limitation in the topic focus, the detailed cognition experiment is omitted here; we only list the results of the cognition experiment for an example of a "T"-shaped template building. The qualitative representation of the shape similarity can be defined according to the following function:

$$\text{Similarity} = \begin{cases} C_1(\text{very strongly}) & 0 \leqslant \text{dis} < 0.4; \\ C_2(\text{strongly}) & 0.4 \leqslant \text{dis} < 0.7; \\ C_3(\text{moderately}) & 0.7 \leqslant \text{dis} < 1.0; \\ C_4(\text{weakly}) & 1.0 \leqslant \text{dis} < 1.2. \end{cases}$$

Based on the function of the qualitative shape similarity representation and the template building, the SQL query can, in fact, be performed by an arithmetic comparison of the shape similarity distance to obtain different similar objects. Fig. 9 shows the result of the shape-based query for a set of building features to match a "T"-shaped template. Four groups of shaded polygons correspond to the retrieved buildings, which are *very strongly*, *strongly*, *moderately* and *weakly* similar to the template.

To judge the effect of the shape query by the Fourier transform method, we make a comparison to human recognition for the same shape objects. We select 15 undergraduate students taking a GIS major to conduct the cognition experiment. The participants are asked to give an order number to describe the shape similarity to the T-shaped template for every building among the experimental data. The order number must be from 0 to 4, with number 4 representing very strong similarity, 3 strong similarity, 2 moderate similarity, 1 weak similarity and 0 no similarity at all. We compute the average value of 15 recognition order numbers, obtaining the human recognition result as shown in Fig. 10. To judge the difference in the shape recognition between the shape queried result by Fourier transform in Fig. 9 and the human cognition result in Fig. 10, we apply Tversky's (1977) method to measure the semantic distance. Two sets of buildings in shape recognition can be represented as $A = \{a_1, a_2, \ldots, a_n\}$ and $B = \{b_1, b_2, \ldots, b_m\}$, where a_i, b_i represents the building ID number that corresponds to the

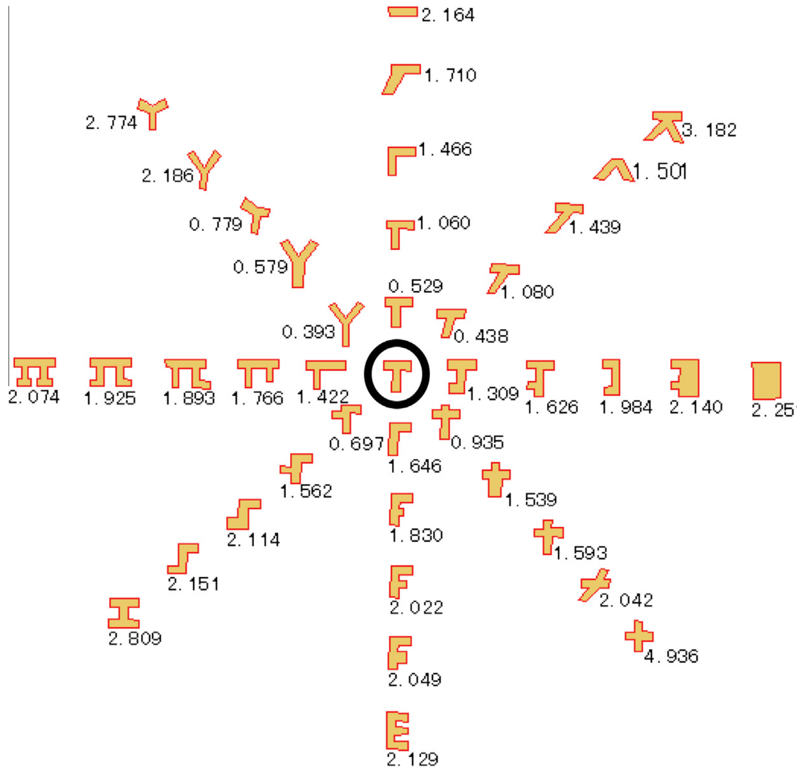


Fig. 8. The shape similarity distance referenced to the central ***T*** shaped template.

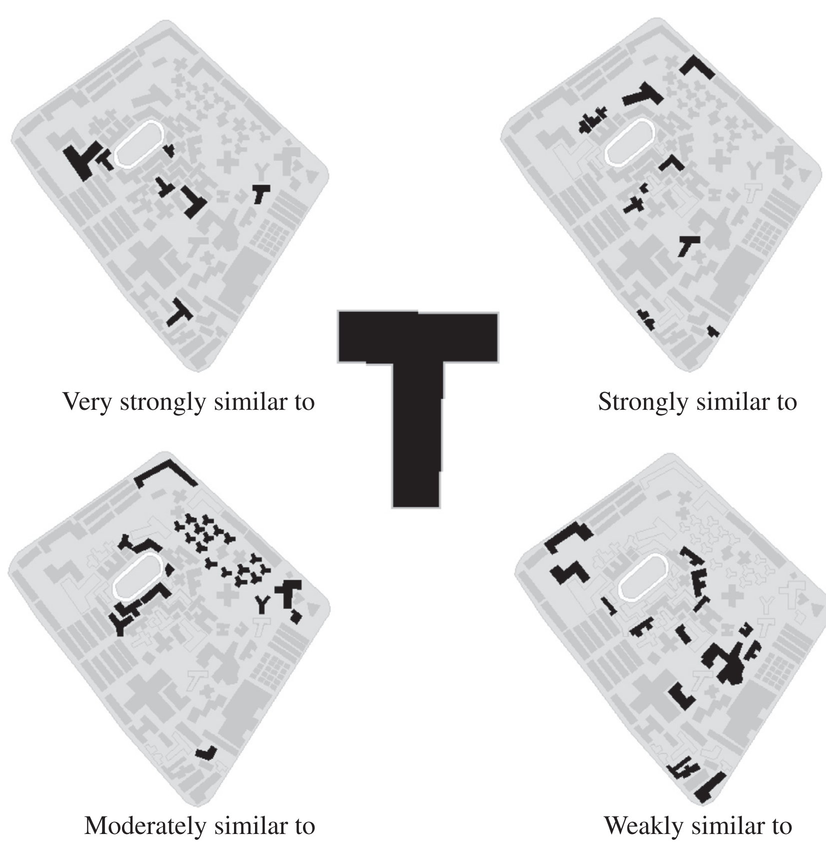


Fig. 9. The shape based query extracts the buildings similar to ***T*** shape to different extent.

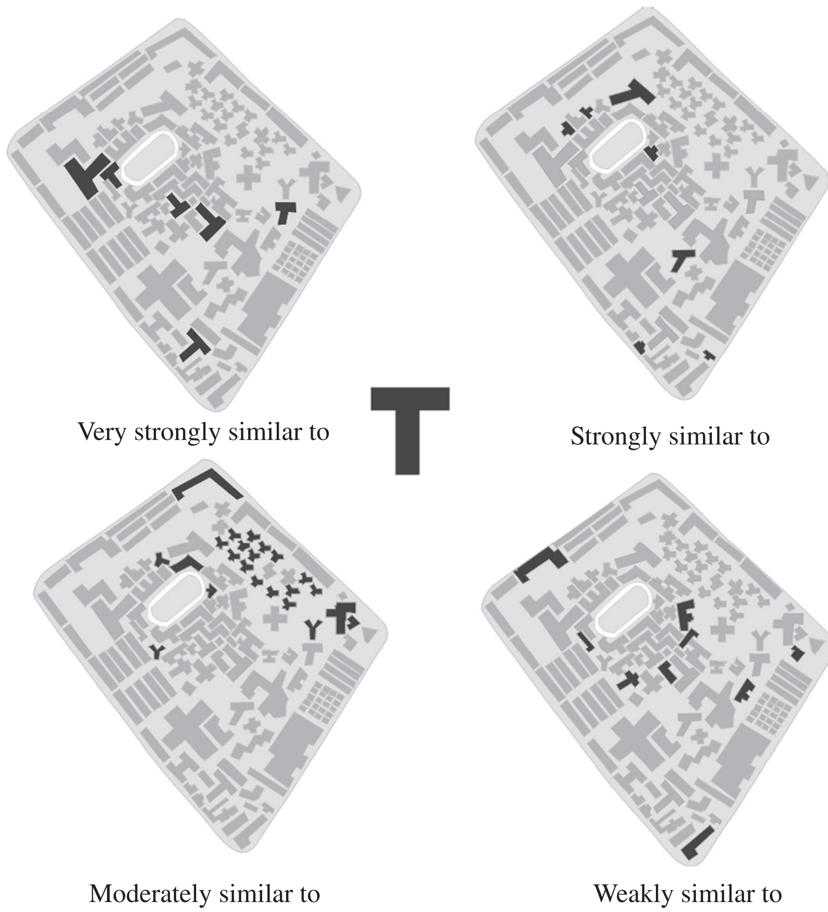


Fig. 10. Human cognition results in judging the building shapes similar to ***T*** shape to different extent.

queried building result. We use the following equation (Tversky, 1977),

$$S(a,b) = \frac{|A \cap B|}{||A \cap B| + \alpha(a,b)|A/B| + (1-\alpha(a,b))|B/A||},$$

to compute the semantic similarity between the two sets A and B, where $\alpha\,(a,b)$ represents the weight and gives the value 0.5 without loss of generality. The experiment obtains the semantic similarity between the two recognitions, which is 92.3% for very strong similarity, 63.2% for strong similarity, 88% for moderate similarity and 60.9% for weak similarity. The results show that the shape recognition by this method is close to human cognition. Among these results, the recognition of very strong similarity and moderate similarity have high similarity to human cognition. However, the weak similarity recognition is quite different. Many of the building objects are recognized as weakly similar to a T shape, but they are identified by human cognition as having nothing to do with the T shape. This finding means that the fuzzy membership value must be adjusted to remove some of the recognized objects in the weak similarity query. The levels of the shape recognition depend on the amount of shape data and the shape difference distribution. For the experiment data with a small number of T-shaped cases, distinguishing 4 levels of shape recognition is too detailed.

This method of shape querying by Fourier transform can work for any shape template to retrieve similar shapes to different extents. However, for templates with different shapes, the shape similarity membership must be predefined by the shape training. We take another template, the C shape, while conducting the same experiment. We construct the similarity membership according to the shape distance in Fig. 11. The retrieved result is shown in Fig. 12 for C-shape detection. Considering the case that the experiment data contains a large number of T-shaped buildings but a small number of C-shaped buildings, we distinguish only two types of similarity: strongly similar and moderately similar.

5. Conclusions

As a significant feature in spatial cognition, shape plays an important role in spatial data handling. Shape-based matching and querying belong to data handling at a high level, with the properties of abstraction, indetermination and symbolization, and are supported by the knowledge of spatial cognition. It is difficult to establish a shape measure model by a quantitative method to represent the qualitative characteristics of shape cognition. We cannot anticipate the design of a uniform query operation nor can we obtain a determined result by the shape query because of the uncertainty in shape description, measurement, and mathematical modeling. In the image process domain, there are many shape measurement methods, but few are for vector data. The shape measure should be invariant to location, rotation, and scale size in the mathematical model. The Fourier transform method satisfies this property by expanding the shape representation as a coefficient vector, which allows the vector distance to be applied to compute the shape similarity. This study shows that for special area object building features that have shapes that are not too complex, the Fourier transform can capture the main shape information and correctly measure the shape similarity. The experiment on real building data demonstrates this point through the comparison with other similar shape measure methods.

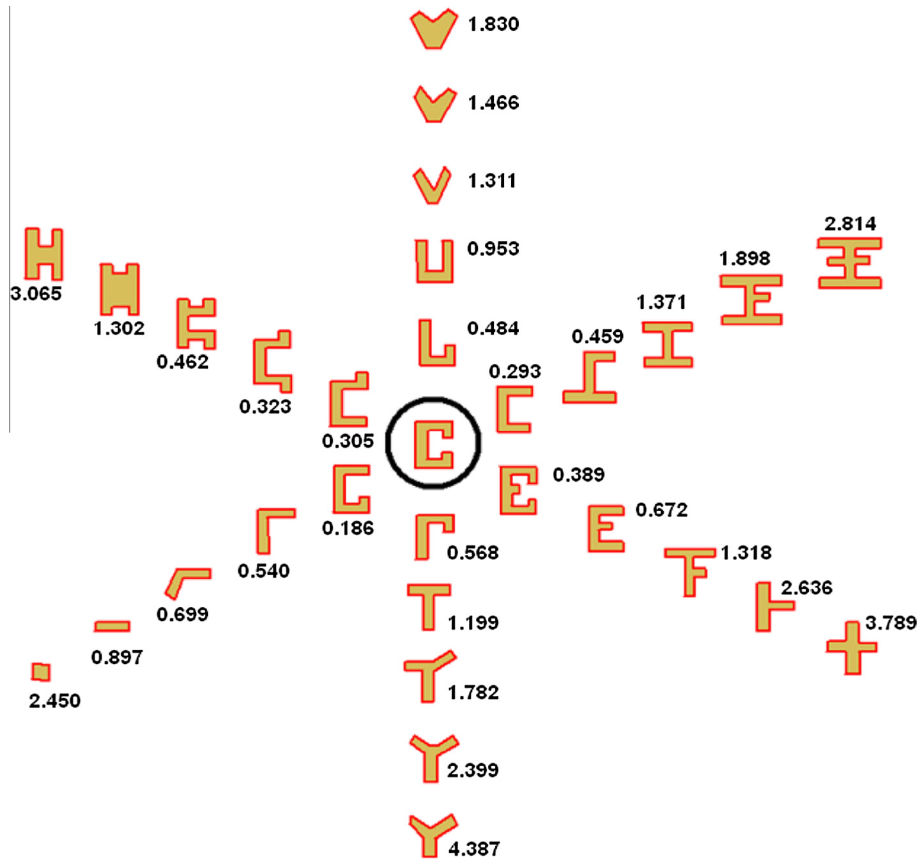


Fig. 11. The shape similarity distance referenced to the central ***C*** shaped template.

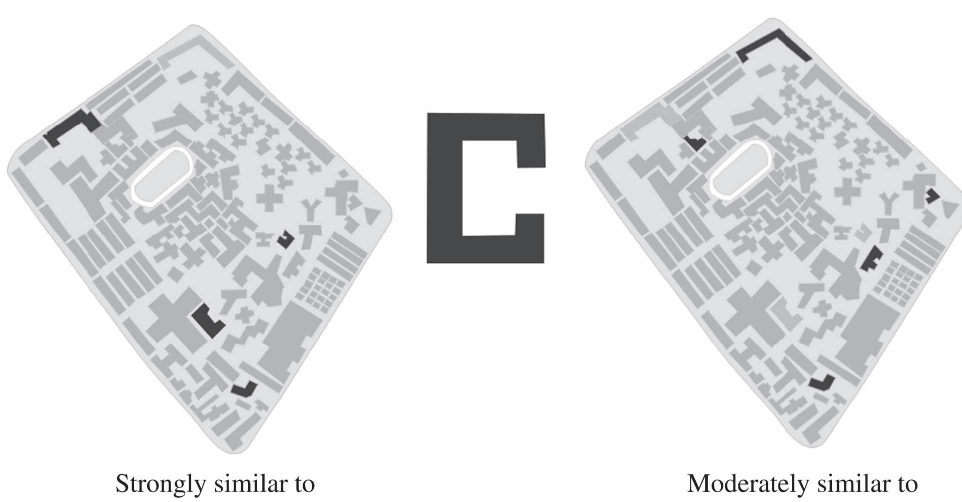


Fig. 12. The shape based query extracts the buildings *strongly* and *moderately* similar to the template ***C*** shape respectively.

This study applies the Fourier transform method in building shape matching and querying. The polygon building is first approximated by a Fourier descriptor series and then by a distance comparison of coefficient vectors to find the similarity degree between candidate buildings and the given template. Because the shape similarity distance is not normalized to all of the templates, we develop the similarity membership function for each template, and through cognition experiments, we establish the classes of shape similarity as *very strongly*, *strongly*, *moderately*, and *weakly* similar. Through experiments on buildings, the shape-based query can extract different groups of buildings in a manner that is consistent with human cognition.

Because of the shortcomings of the Fourier transform in shape representation, in the experiment we find that the ''LIKE'' operation in the shape query statement is sensitive to the shape structure and is not adaptable to all shapes. For example, if the polygon is too concave, such as a star shape, or if the boundary is too complex with too many angular sections, the Fourier descriptor cannot capture the main shape information and the boundaries of the approximated polygon could intersect, as illustrated in Fig. 13.

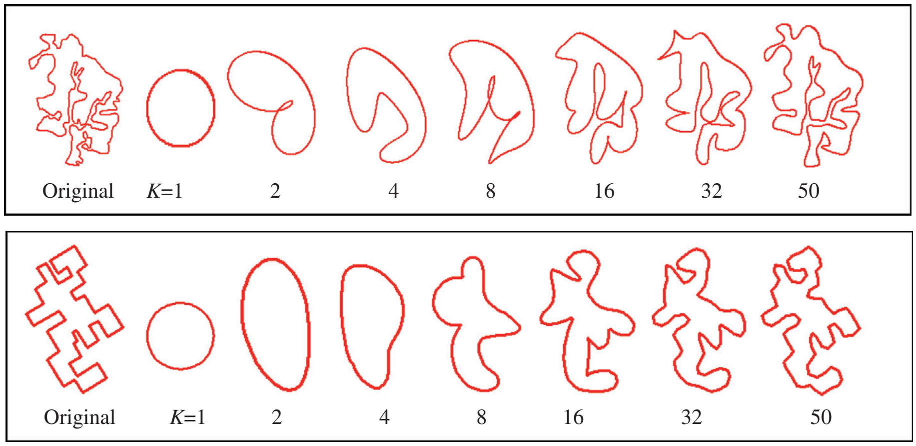


Fig. 13. Two examples of complex polygon and the shape approximations by Fourier expand at different order *K*.

For the shape signature, there are different methods in the Fourier transform, including central distance, complex coordinates (position function), curvature, and cumulative angular function, each of which is adaptable to different situations. In this study, we consider only the position-based method and, in the future, other methods must be tested and comparisons made to enhance the Fourier transform-based shape measure.

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