Calculation of Boundary Condition in the Migdal-Kadanoff Spin Glass and Size Chaos

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Abstract here

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Boundary Condition Calculation —

$$K' = K_1 + K_2 \tag{1}$$

$$K' = \frac{1}{2}\ln\Big(\frac{\cosh(K_1+K_2)}{\cosh(K_1-K_2)}\Big) \tag{2}$$

$$K' = \frac{1}{2}\ln\Big(\frac{\cosh(K_1+K_2+K_3+K_4+K_5+K_6+K_7+K_8)}{\cosh(K_1+K_2+K_3+K_4-K_5-K_6-K_7-K_8)}\Big) \tag{3}$$

$$K_{BC} = 3K_1 + (4K_1 : (3K_2 + 4K_2 : (3K_3 + 4K_3 : (3K_4 + 4K_4 : (3K_5 + ...))))) \tag{4}$$

Size Chaos —

$$F(n) = |\tanh(K_n) - \tanh(K_{\text{inf}})| \tag{5}$$

Necklace Analytics —
Let B_0 be the boundary condition for a finite subsystem through a supersystem one level larger. Let h be the largest level of the supersystem. To calculate the BC through a larger supersystem by one level we can do a replacement on the elements of B_0 as followed:

$$B_0 = (n-1)\cdot K_h \to B_0 = (n-1)\cdot K_h + n\cdot K_h : (n-1)\cdot K_{h+1} \tag{6}$$

Let G, G', and G'' be values drawn from the 0t distribution of the necklace MKRG distribution scaled to variance 1. The general replacement equation above can be rewritten as:

$$\sqrt{n}G \to \sqrt{n}G + \sqrt{n}G' : \sqrt{n}\tau G'') \tag{7}$$

Note that $n-1 = n$ when $n >> 1$ and τ is the scaling value for the necklace MKRG. This can be simplified to:

$$G \to G + sgn(GG)G \tag{8}$$

Diamond Analytics —
Let B_0 and h be the same as above.

$$B_0 = K_0 : (n-1)\cdot(K_0 : K_0) \tag{9}$$

The replacement rule is:

$$B_0 = (n-1)\cdot(K_h : K_h) \to B_0 = (n-1)\cdot(K_h : K_h) + K_{h+1} : ((n-1)\cdot(K \tag{10}$$

Again G, G', G'', G''', G'''' are drawn from the diamond MKRG distribution scaled to variance 1.

$$\sqrt{n}(G : G') \to \sqrt{n}(G : G') + \tau\sqrt{n}(G'' : (\sqrt{n}G''' : G'''')) \tag{11}$$

This reduces to:

$$G : G' \to G : G' + \tau[G'' : (\sqrt{n}G''' : G'''')] \tag{12}$$

$$H \equiv G : G' \tag{13}$$

$$H \to H + \tau[G : \sqrt{n}H'] \tag{14}$$

$$X \equiv \frac{G : G'}{\sqrt{VAR(G : G')}} = \frac{H}{\tau} \tag{15}$$

$$X\tau \to X\tau + \tau(G : \sqrt{n}\tau X') \tag{16}$$

$$X \to X + (G : \sqrt{n}\tau X') \tag{17}$$

$$X \to X + sgn(GX')G \tag{18}$$

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