Homework # 7

Reading

- I've posted an excerpt from G. Lawler's book on stochastic processes relating to Poisson processes. Take a look, it covers similar ideas as mentioned in the lecture.
- Section 11.3 of Bishop discusses Gibbs sampling.

1. (This problem is related to exercise 4.1 in chapter 2 of the book Stochastic Simulation by Asmussen and Glynn). The Gompertz-Makecham distribution is used as a lifetime distribution by life insurance companies. To explain the distribution, let $X(t)$ be a Poisson process with rate $\lambda(t) = a + be^{ct}$ where $a = 5 \cdot 10^{-4}$, $b = 7.5858 \cdot 10^{-5}$, $c = \log(1.09144)$. If a person is, say, 45 years old, then the first jump time of $X(t)$ after $t = 45$ is used as a model for the person's age when they die.
   (a) As a warm-up, write a function that samples the jump times of a Poisson process with constant rate λ that occur prior to some specified time t.
   (b) Write a sampler that generates the Poisson process $X(t)$ up to $t = 100$. Use the accept-reject algorithm that we talked about in class based on a constant rate Poisson process. Show that the accept-reject algorithm is valid. (Hint: to show the validity of the accept-reject algorithm show that it will sample a Poisson process with a jump in $[t, t + \Delta t]$ with probability approximately $\lambda(t)\Delta t$. We did this in class.)
   (c) Derive a formula for the cdf of the age upon death of a 60 year old. (Don't just quote the result I presented in class, derive it yourself. Use the idea of splitting up time into small time intervals and taking a limit as those intervals go to 0 in size.)
   (d) Compute the probability a 60 year old lives to be 90 in two ways
      i. Use your languages integrate function to evaluate the formula for this probability based on your cdf in (c).

    ii. Use a Monte Carlo integration approach and your sampler from (b).

2. Consider a Poisson process with rate $\lambda(t) = 1/\sqrt{t}$. Notice that $\lambda(t)$ is unbounded. Calculate the pdf of the first jump time.

3. (a) Redo problem 4b of homework 4, but this time use a Gibbs sampler to sample from Y rather than a Metropolis-Hastings sampler.

   (b) Explain why you cannot use a Gibbs sampler based on switching a single coordinate (i.e. die role) to sample die rolls for problem 2 of homework 6.

   (c) Construct a Gibbs sampler to sample die rolls for problem 2 of homework 6 by choosing two dice, computing their marginal distribution given all other die rolls, and sampling from the marginal to produce the Markov chain update. Such a sampler is not a Gibbs sampler in the sense that it considers one coordinate at a time, but it is a Gibbs sampler in the sense that it uses marginal distributions to update the chain.