

CS 415 Project 1

Keegan Donley, Jeff Hultman

0.1 Task 1

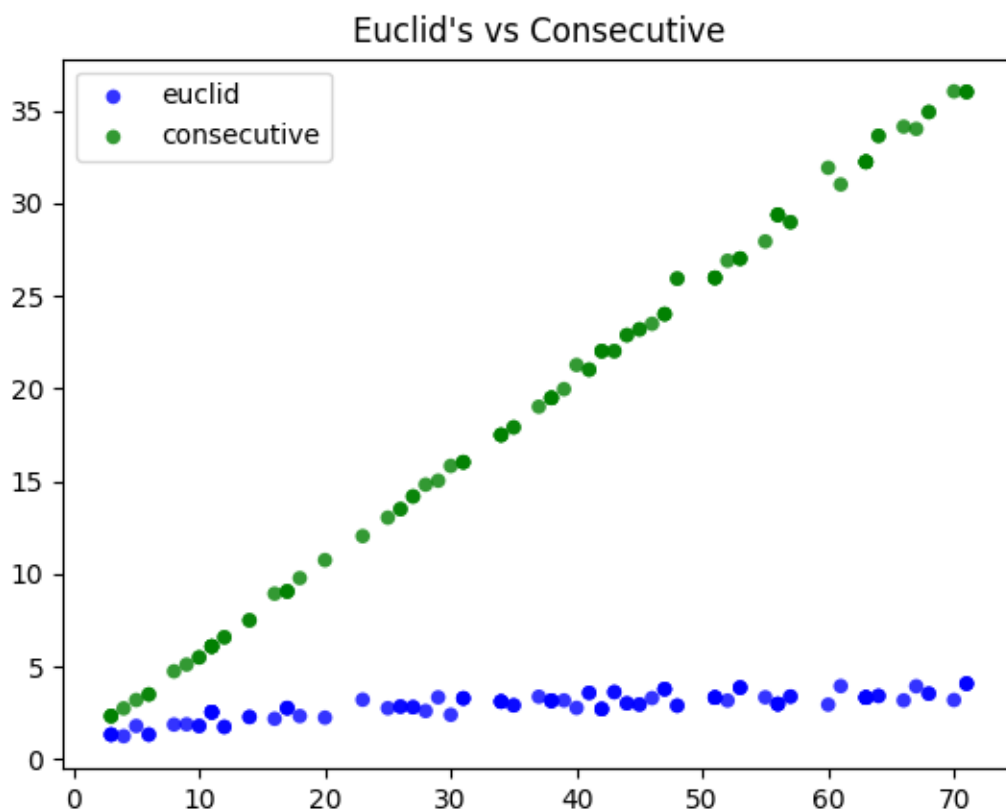
Average-case efficiency of Euclid's algorithm and consecutive integer checking algorithm To test the average-case efficiency of these algorithms, we generate 100 values of n from 1 to 70, then count the number of operations needed to calculate the average GCD for n using Euclid's algorithm and consecutive integer checking.

To calculate the average number of operations for n , we take the average number of operations for:

$\text{euclidGCD}(n, 1), \text{euclidGCD}(n, 2), \dots, \text{euclidGCD}(n, n)$

and

$\text{consecutiveGCD}(n, 1), \text{consecutiveGCD}(n, 2), \dots, \text{consecutiveGCD}(n, n)$



Euclid $\theta(\log n)$

In the average case, Euclid's algorithm runs in $\theta(\log n)$ time and took less than 5 modulo divisions.

Consecutive Integer $\theta(n)$

The empirical testing shows the consecutive integer testing results to be nearly perfectly linear.

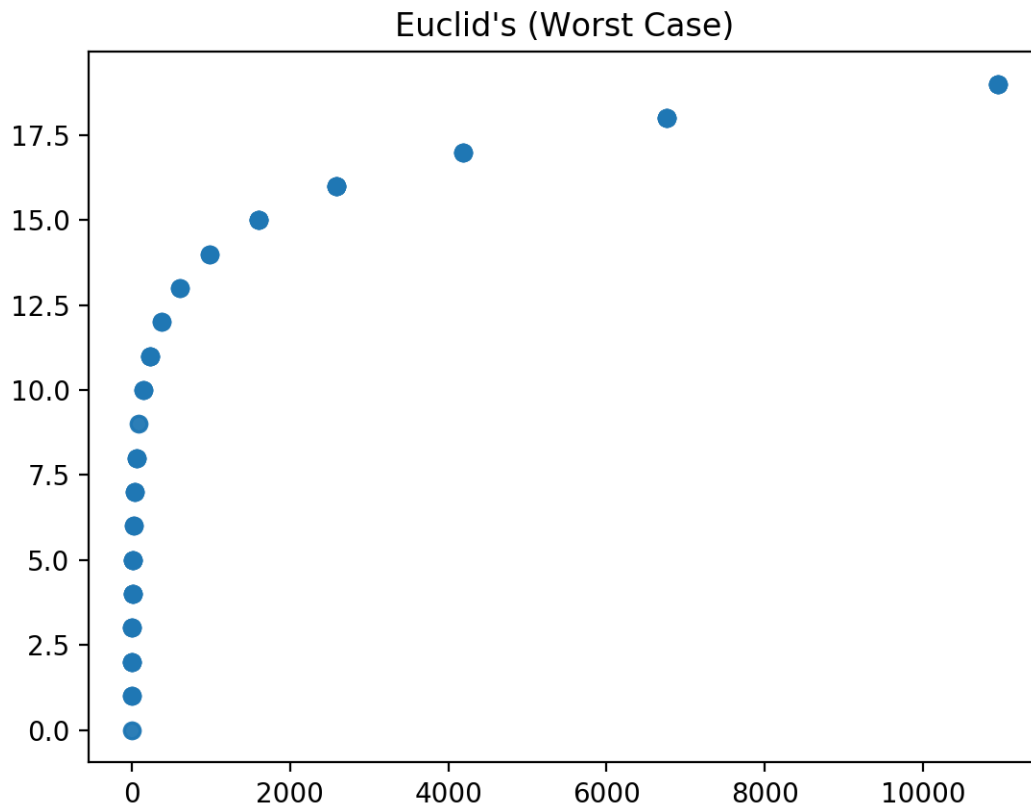
0.2 Task 2

Worst-case efficiency of Euclid's algorithm The worst case for Euclid's algorithm occurs when two consecutive integers from the Fibonacci sequence are used as m and n . To test the efficiency, we generate 200 values for k between 1 and 200. k is an index in the Fibonacci sequence, where $m = k + 1$ and $n = k$.

For example, based on the start of the Fibonacci sequence: [0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55]

$k = 5 \Rightarrow \text{gcd}(8, 5)$

$k = 8 \Rightarrow \text{gcd}(34, 21)$

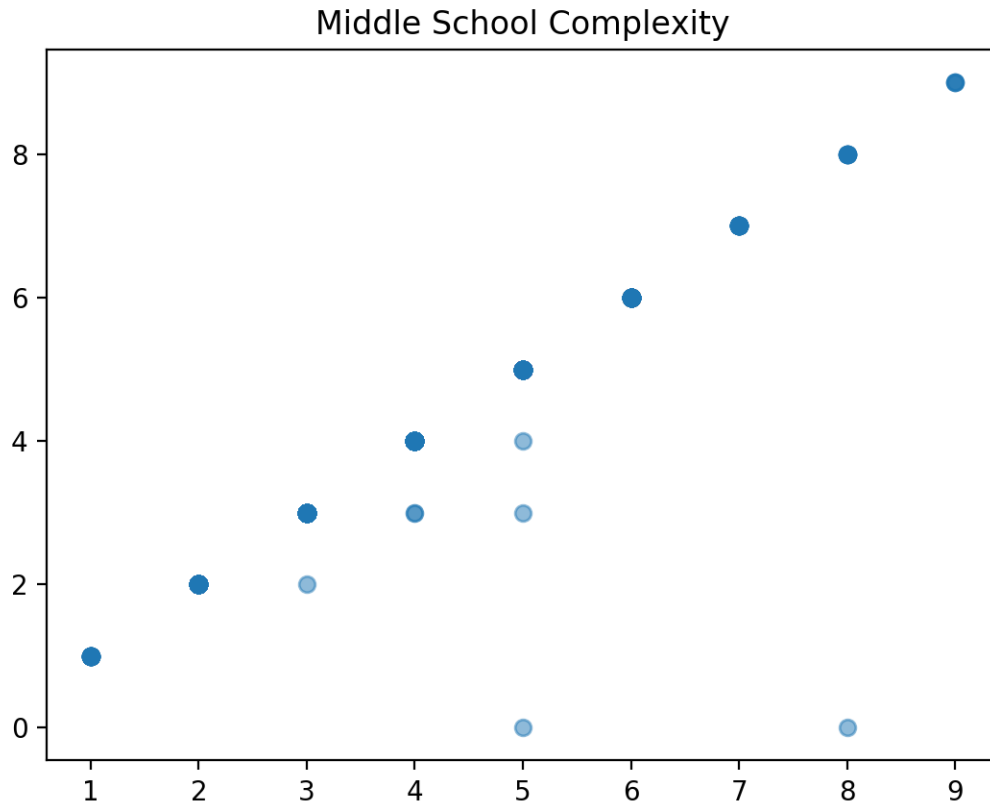


Because the gcd of two consecutive Fibonacci numbers is always 1, the complexity in the worst case for Euclid's algorithm is $\theta(\log n)$. We saw a trend that represents a complexity of $\theta(\log n)$ in the average case as well, however the number of modulo divisions tended to be lower than how many were needed in the worst case.

0.3 Task 3

The "middle-school procedure" To evaluate the complexity of the middle school method of calculating GCD, we generated 1000 pairs of integers from 1 - 999 as m and n . When plotted, the results show that the middle school method is $\theta(n)$.

On the x axis, we plotted $\max(|\text{prime factors in } m|, |\text{prime factors in } n|)$ against the y axis - the number of operations needed to compute the GCD. The basic operation is the comparison of prime numbers.



The evidence shows the complexity to be approximately linear, but there are some outliers where both m and n had prime factors, but none were common and the gcd was determined to be 1.