1. BS formular:

$$\begin{cases} \frac{\partial c}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 c}{\partial S^2} + rS\frac{\partial c}{\partial S} - rC = 0 \\ C(S,T) = \max(S-K, 0) \\ C(0,t) = 0 \\ C(S,t) \approx S \quad \text{while} \quad S \to \infty \end{cases}$$

$$\begin{cases} C = S_0 N(d_1) - Ke^{-r(T-t)}N(d_2) \\ P = Ke^{-rt}N(-d_2) - SN(-d_1) \\ C + Ke^{-rt} = P + S \\ d_1 = \frac{\ln(S/K) + (r+\frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \\ d_2 = d_1 - \sigma\sqrt{T-t} \end{cases}$$

Greek: S, r, T 对 C 的敏感程度

$\frac{\partial C}{\partial S}$ $\frac{\partial C}{\partial \sigma}$

vol ↑ → delta ↑

看指的行情对实际的 vol 有个预估

$$C = S_0 N(d_1) - Ke^{-rt}N(d_2)$$

分析 $N(d_1)$ 与 $N(d_2)$ 的含义:

$$N(d_1) = \frac{\partial c}{\partial S} = \Delta$$

$N(d_2)$ 表示看涨期权行权概率

2. 由 BS → Greek

预测 vol 看 open interest 回增仓

Delta: 资产价格相对于标的资产价格变化的敏感程度

$$\frac{\partial c}{\partial S} = N(d_1), \quad \frac{\partial P}{\partial S} = -N(-d_1) = N(d_1) - 1$$

Gamma: Delta 相对价格的敏感度

$$\Gamma = \frac{\partial^2 c}{\partial S^2} = \frac{\partial^2 P}{\partial S^2} = N\left(\frac{\ln S_0/K + (r+\frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}\right)$$

$$= \underset{\text{pdf}}{n(d_1)} \cdot \frac{1}{S_0\sigma\sqrt{T-t}} > 0 \quad (\text{关于 } S \text{ 下凸})$$

Theta: 期权价值随时间不断减少的比例

$$\frac{\partial C}{\partial T} = S_0 \cdot n(d_1)\left(-\frac{\ln(S_0/K)}{\sigma}\frac{1}{2}(T-t)^{-\frac{3}{2}} + \frac{(r+\frac{1}{2}\sigma^2)}{\sigma}\frac{1}{2}(T-t)^{-\frac{1}{2}}\right) \text{①}$$

$$+ rK\cdot e^{-r(T-t)}N(d_2) - Ke^{-r(T-t)}n(d_2)\left(-\frac{\ln(S_0/K)}{\sigma}\frac{1}{2}(T-t)^{-\frac{3}{2}} + \frac{(r+\frac{1}{2}\sigma^2)}{\sigma}\frac{1}{2}(T-t)^{-\frac{1}{2}} - \frac{1}{2}\sigma(T-t)^{-\frac{1}{2}}\right) \text{②}$$

$$\text{①} = S_0 n(d_1)\frac{(T-t)^{-1}}{2}\left(-\frac{\ln(S_0/K)}{\sigma}(T-t)^{-\frac{1}{2}} + \frac{(r+\frac{1}{2}\sigma^2)}{\sigma}(T-t)^{\frac{1}{2}}\right)$$

$$\text{②} = rK\cdot e^{-r(T-t)}N(d_2) - Ke^{-r(T-t)}n(d_2)\frac{(T-t)^{-1}}{2}\left(-\frac{\ln(S_0/K)}{\sigma}(T-t)^{-\frac{1}{2}} + \frac{(r+\frac{1}{2}\sigma^2)}{\sigma}\frac{1}{2}(T-t)^{\frac{1}{2}} - \sigma(T-t)^{\frac{1}{2}}\right)$$

$$n(d_2) = n(d_1)\cdot e^{\frac{d_1\sqrt{T-t}}{\sigma}} \cdot e^{\frac{-(T-t)}{2}} = n(d_1)\cdot e^{\frac{\ln(S/K) + (r+\frac{1}{2}\sigma^2)(T-t)}{\sigma^2} - \frac{T-t}{2}}$$

$$= n(d_1)\frac{S}{K}e^{r(T-t)}$$

$$② = rk\cdot e^{-r(T-t)} - ke^{-r(T-t)}n(d_2)\frac{(T-t)^{-1}}{2}\left(-\frac{\ln(S_0/K)}{\sigma}(T-t)^{-\frac{1}{2}} + \frac{(r+\frac{1}{2}\sigma^2)}{\sigma}\frac{1}{2}(T-t)^{\frac{1}{2}} - \sigma(T-t)^{\frac{1}{2}}\right)$$

$$= rk\cdot e^{-r(T-t)} - S_0 n(d_1)\frac{(T-t)^{-1}}{2}\left(-\frac{\ln(S_0/K)}{\sigma}(T-t)^{-\frac{1}{2}} + \frac{(r+\frac{1}{2}\sigma^2)}{\sigma}\frac{1}{2}(T-t)^{\frac{1}{2}} - \sigma(T-t)^{\frac{1}{2}}\right)$$

$$①+② = rke^{-r(T-t)}N(d_2) + S_0 n(d_1)(T-t)^{-1}\frac{1}{2}\sigma(T-t)^{\frac{1}{2}}$$

$$= rke^{-r(T-t)}N(d_2) + \frac{S_0\sigma}{2\sqrt{T-t}}n(d_1)$$

Call: $\text{Theta} = -\frac{\partial c}{\partial T} = -rke^{-r(T-t)}N(d_2) - \frac{S_0\sigma}{2\sqrt{T-t}}n(d_1)$

put: $\text{Theta} = -\frac{\partial P}{\partial T} = rke^{-r(T-t)}N(-d_2) - \frac{S_0\sigma}{2\sqrt{T-t}}n(d_1)$

Vega：期权价值相对波动变化的敏感程度

$$\frac{\partial c}{\partial \sigma} = S_0\cdot n(d_1)\cdot\left[-\frac{\ln(S_0/K)}{\sqrt{T-t}}\frac{1}{\sigma^2} - r\sqrt{T-t}\frac{1}{\sigma^2} + \frac{1}{2}\sqrt{T-t}\right]$$

$$- Ke^{-r(T-t)}n(d_2)\left[-\frac{\ln(S_0/K)}{\sqrt{T-t}}\frac{1}{\sigma^2} - r\sqrt{T-t}\frac{1}{\sigma^2} - \frac{1}{2}\sqrt{T-t}\right]$$

$$\because n(d_2) = n(d_1)\frac{S_0}{K}e^{r(T-t)}$$

$$\therefore \frac{\partial c}{\partial \sigma} = S_0 n(d_1)\sqrt{T-t}$$

$$\therefore V = \frac{\partial c}{\partial \sigma} = \frac{\partial P}{\partial \sigma} = S_0 n(d_1)\sqrt{T-t} = \Gamma S_0^2 (T-t)\sigma$$

Rho：期权价值相对于无风险利率变化的敏感程度

Call: $\rho = \frac{\partial c}{\partial r} = S_0\cdot n(d_1)\frac{\sqrt{T-t}}{\sigma} + (T-t)ke^{-r(T-t)}N(d_2) - ke^{-r(T-t)}n(d_2)\cdot\frac{\sqrt{T-t}}{\sigma}$

$$= (T-t)ke^{-r(T-t)}N(d_2)$$

put: $\rho = \frac{\partial P}{\partial r} = -kTe^{-rT}N(-d_2)$

2. Greek 的理解

① price 向 payoff 收敛

② delta 是 price 的斜率

Delta 对冲：卖出一份看涨 → 持有Δ份的股票

③ gamma 看 price 的凸凹性 → Gamma 对冲

④ vega 看 window-widen 的 price 变化

⑤ vega 看 price 向 payoff 的收敛方向.

$C = C(S, K, r, t, \sigma)$

3、pnl分解:

由 Black-Schole 微分方程:

$$\frac{\partial C}{\partial t} + rS\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} = rC$$

∴ Taylor Expansion

$$dC = \frac{\partial C}{\partial S}\Delta S + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}\Delta S^2 + \frac{\partial C}{\partial T}\Delta T + \frac{\partial C}{\partial \sigma}\Delta\sigma + \varepsilon$$

$$= \underbrace{\text{delta}\,\Delta S + \frac{1}{2}\text{gamma}\,\Delta S^2 + \text{theta}\,\Delta T + \text{Rho}\,\Delta\sigma + \varepsilon}_{\text{分解.}}$$