A Survey on Data Structures

Jeffrey Mei

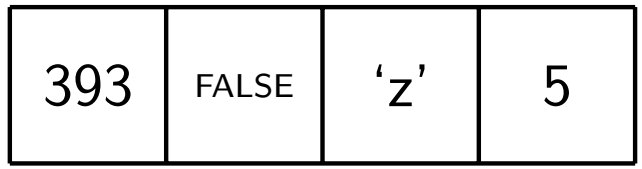
Introduction

- what are data structures?
  - data format for storage/organization
- why do we need data structures?
  - efficient storage
  - efficient retrieval
- needed for more complex tasks

Outline

- Arrays
- Linked Lists
  - Queues
  - Stacks
- Trees
  - Binary Search Trees
  - Heaps
- K-Dimensional Trees

Memory

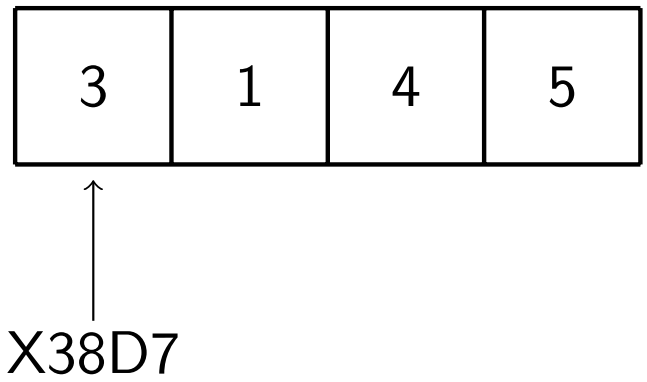


- memory addresses represented by hexadecimal code
- variables refer to addresses

Common Size of Data Types

- bool: 1 bit
- character: 8 bits (1 byte)
- integer: 32 bits (4 bytes)
- double: 64 bits (8 bytes)

Arrays



Array

- defined as an address in memory
- holds fixed number of values of single type
- all content is denoted as subsequent spaces from address
- $\mathrm{arr}\,[\mathrm{k}] = \mathrm{arr} + \mathrm{k} * \mathrm{DATATYPE\_SIZE}$

Restaurant Reservation Problem

- do not know how many values to store (e.g. logging events)
- concatenation: add elements to array
- not specifying space needed can be very inefficient

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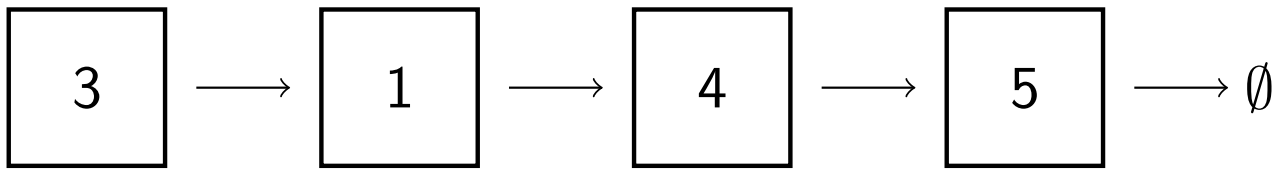
Arrays: why aren't they good enough?

- not dynamic (can use up too much/little space)
- unstructured search
- doesn't always make use of all contextual information (hierarchical, spatial, temporal, priority, etc.)

Linked Lists



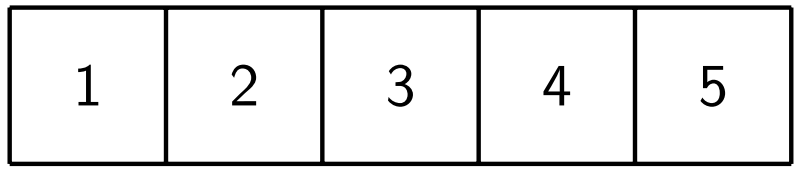
Each node contains:

1. data
2. next address

Overview:

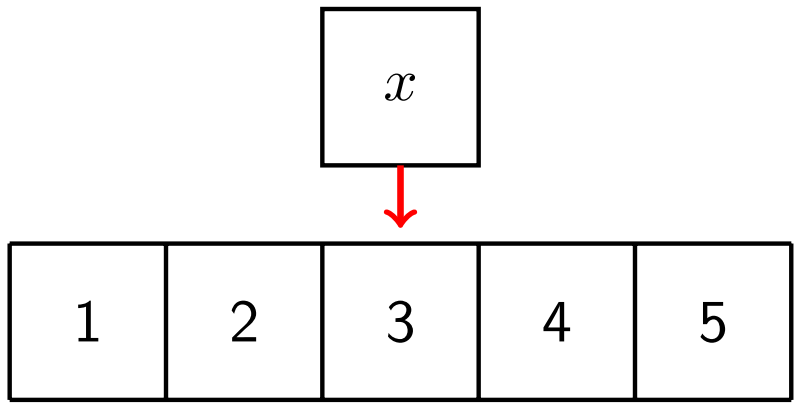
- does not rely on contiguous memory spaces
- building blocks of many data structures

Linked Lists: Insertion and Deletion



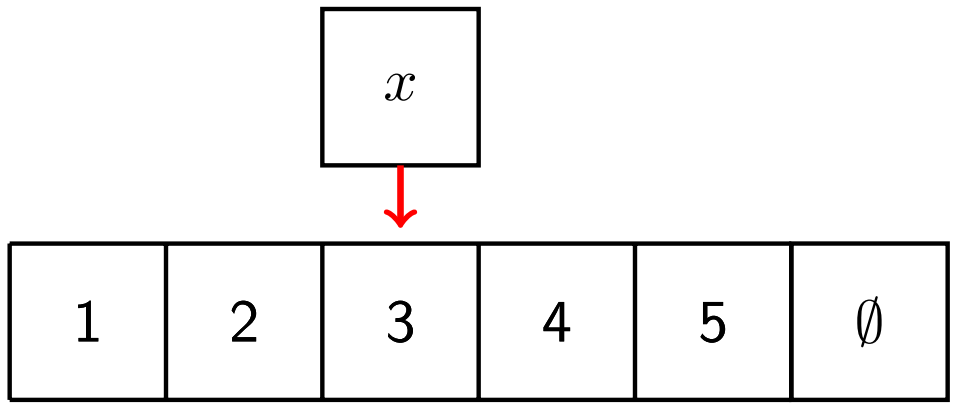
Insert value into middle of array

Linked Lists: Insertion and Deletion



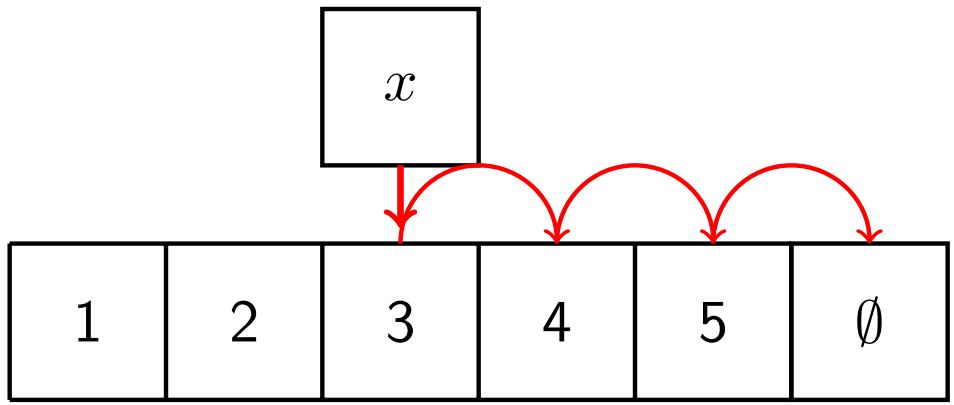
Cannot insert without overwriting

Linked Lists: Insertion and Deletion



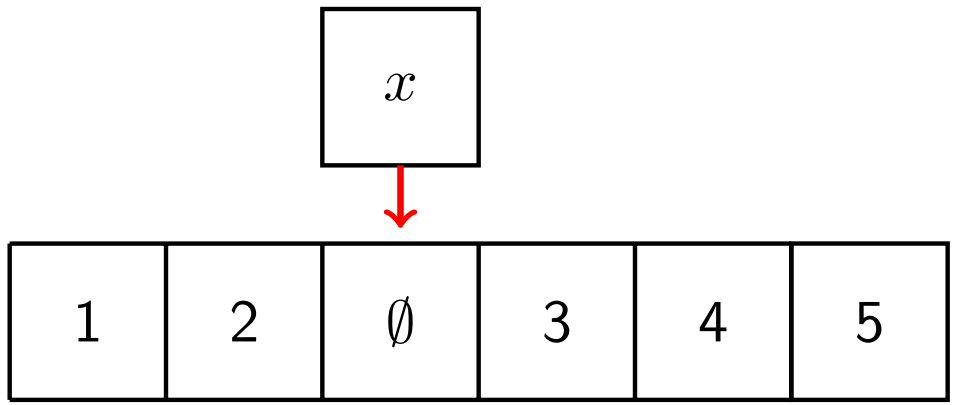
Must resize array

Linked Lists: Insertion and Deletion



Shuffle array

Linked Lists: Insertion and Deletion



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Linked Lists: Insertion and Deletion

Insert value

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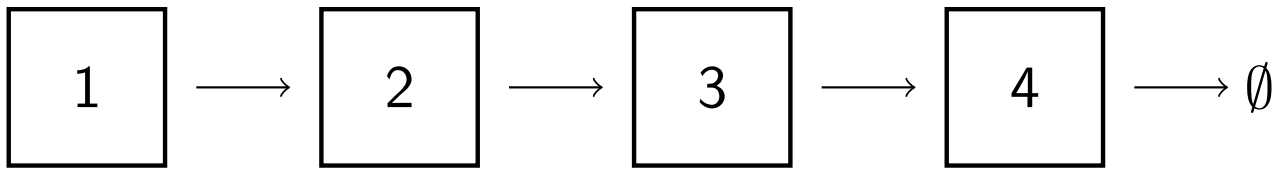
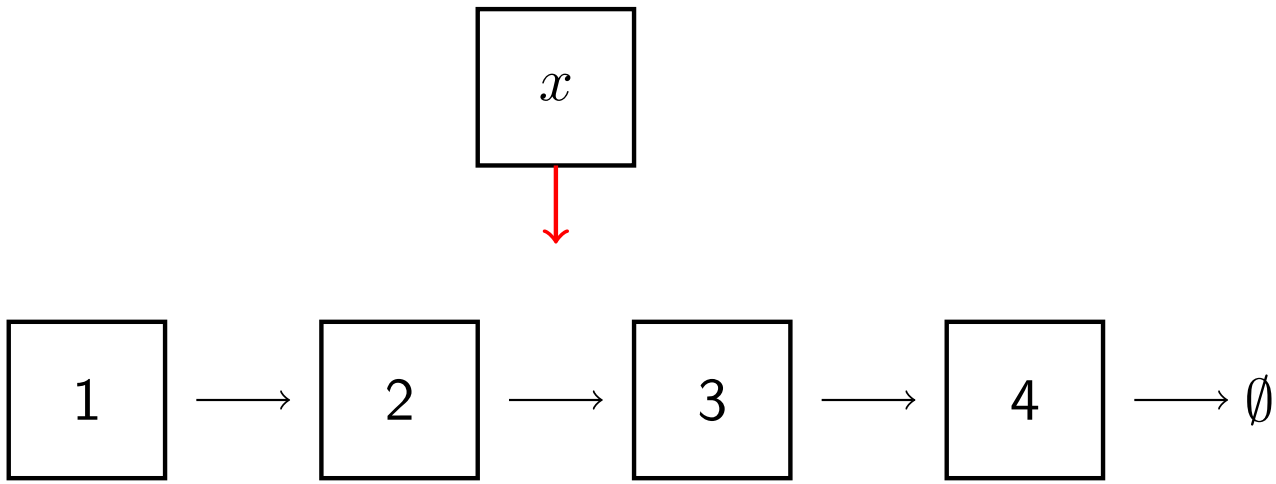


Insert value into middle of linked list (after 2 and before 3)

Linked Lists: Insertion and Deletion



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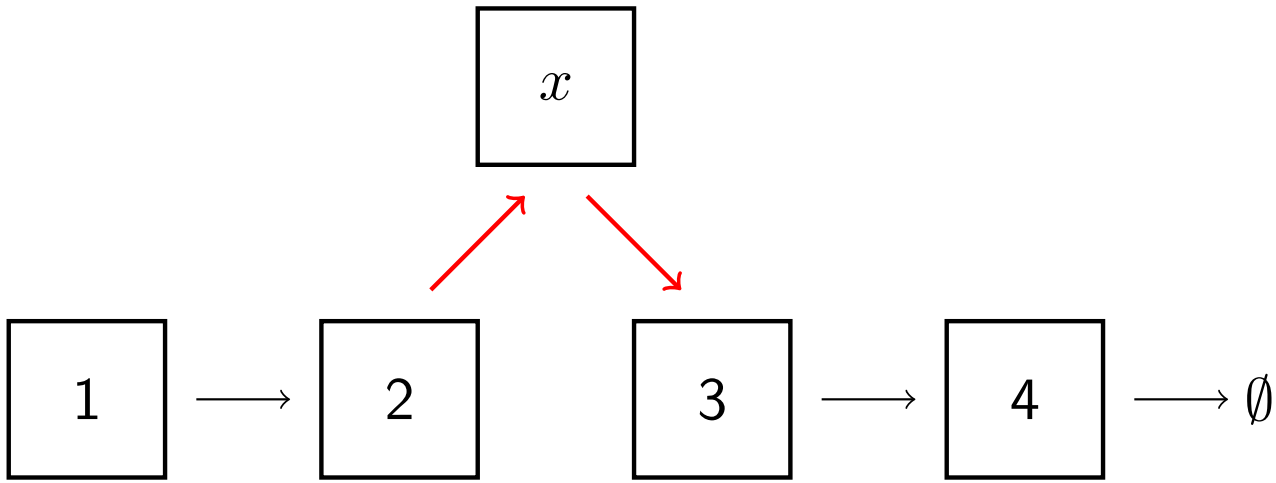


Change next address

Linked Lists: Variations

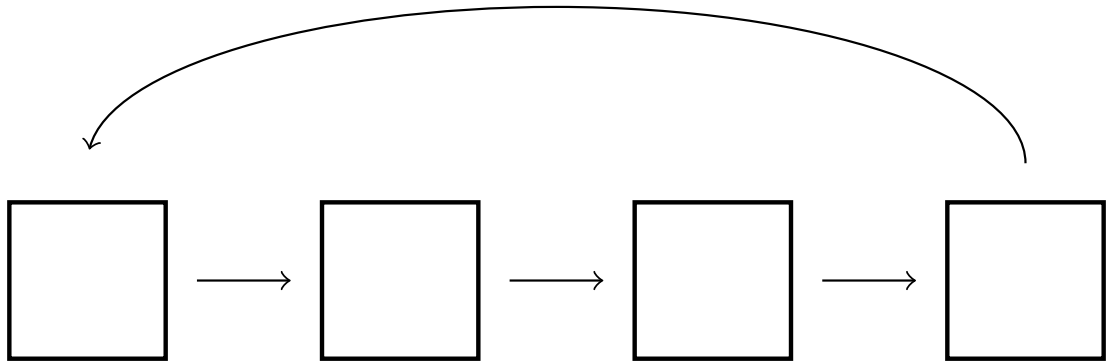


Figure: Circular Linked Lists

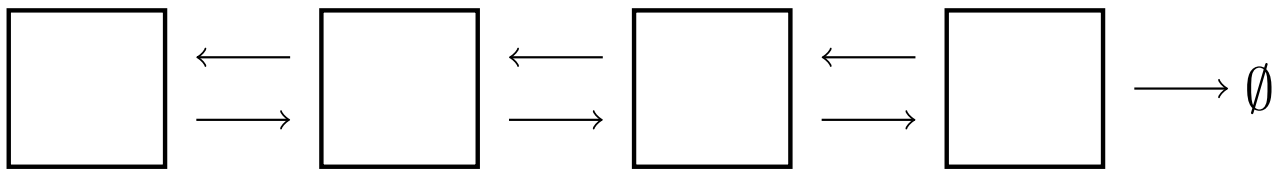


Figure: Doubly Linked Lists

Linked Lists: Variations

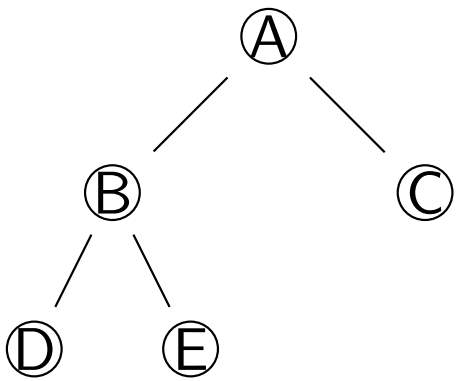


Figure: Trees

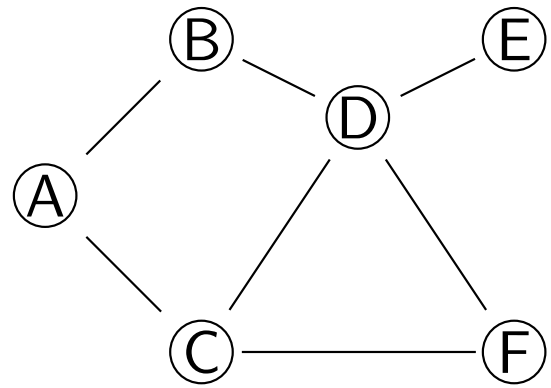


Figure: Graphs

Linked Lists: Overview

Better than arrays because...

- dynamic size
- faster insertion and deletion

Problems:

- no random access: must traverse through each node sequentially
- cannot move backwards

Queue

Overview:

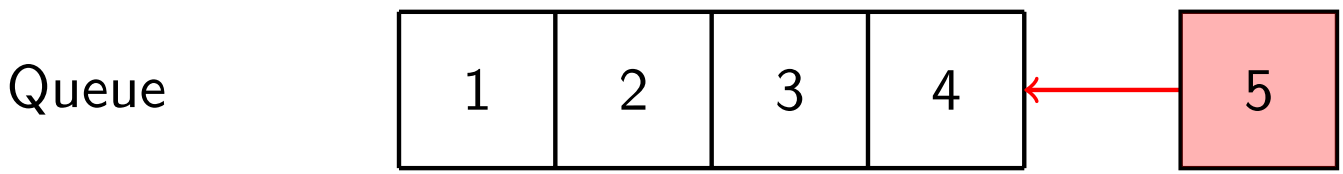
- exploits temporal information
- first in, first out (FIFO)
- dynamic array (size changes)

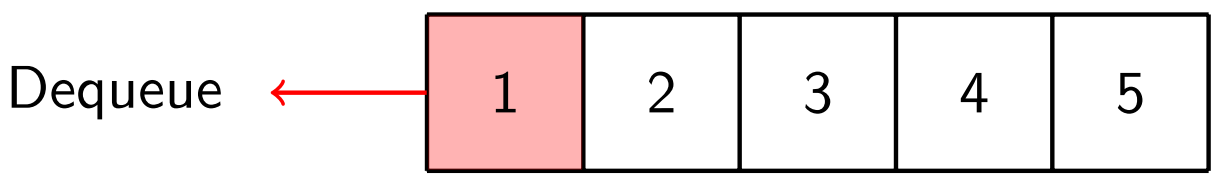
Operations:

- queue: add to rear
- dequeue: remove from front

Applications:

- task scheduling
- resource allocation





Stack

Overview:

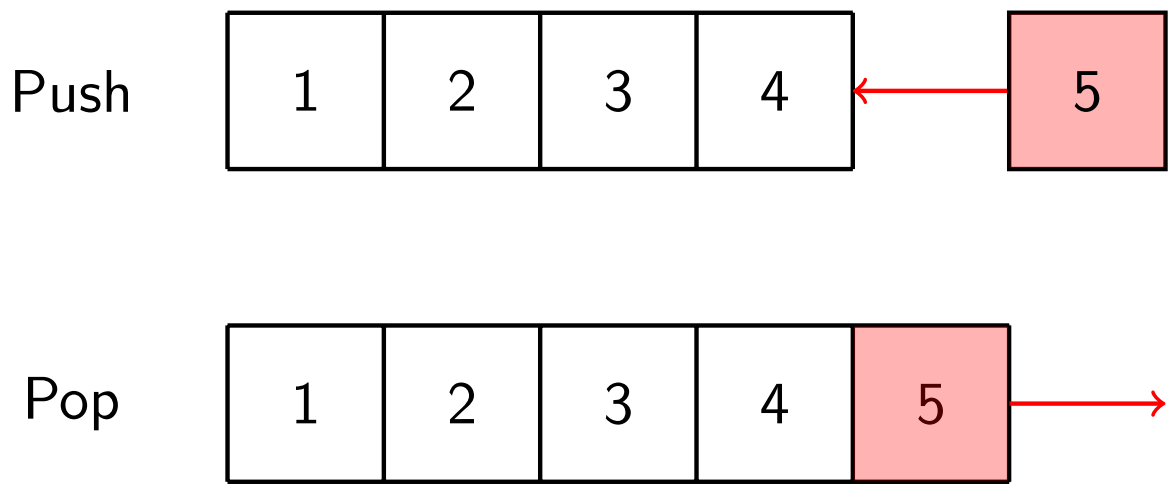
- exploits temporal information
- last in, first out (LIFO)
- dynamic array (size changes)

Operations:

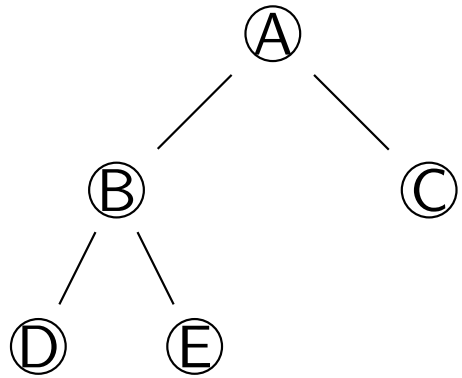
- push: add to stack
- pop: remove from stack

Applications:

- used for "undo" operations



Trees



Overview:

- exploits hierarchical structure

Applications:

- file systems
- expression trees
- game trees

Tree Applications

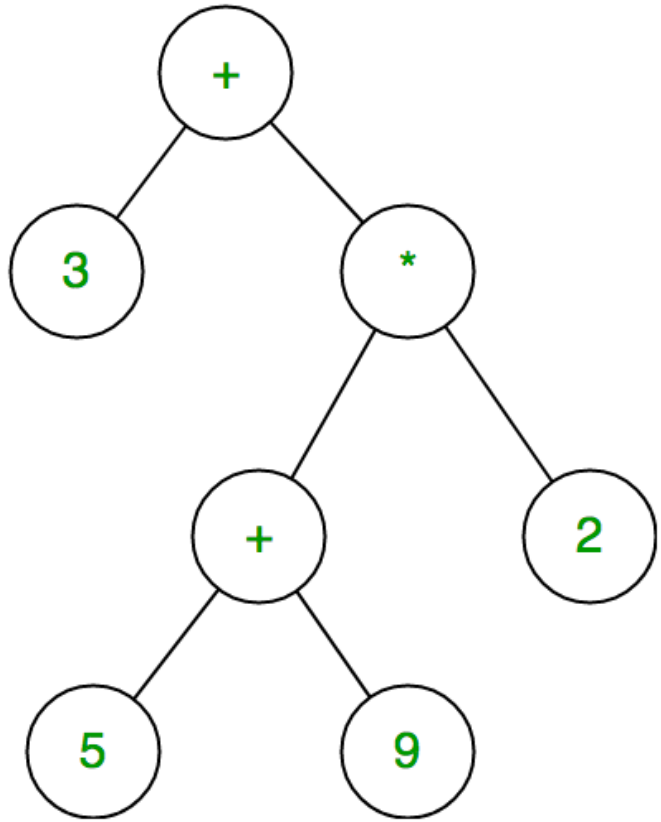


Figure: Expression Tree of $3 + ((5 + 9) * 2)$:
[https://www.geeksforgeeks.org/expression-tree/]

Tree Applications (2)

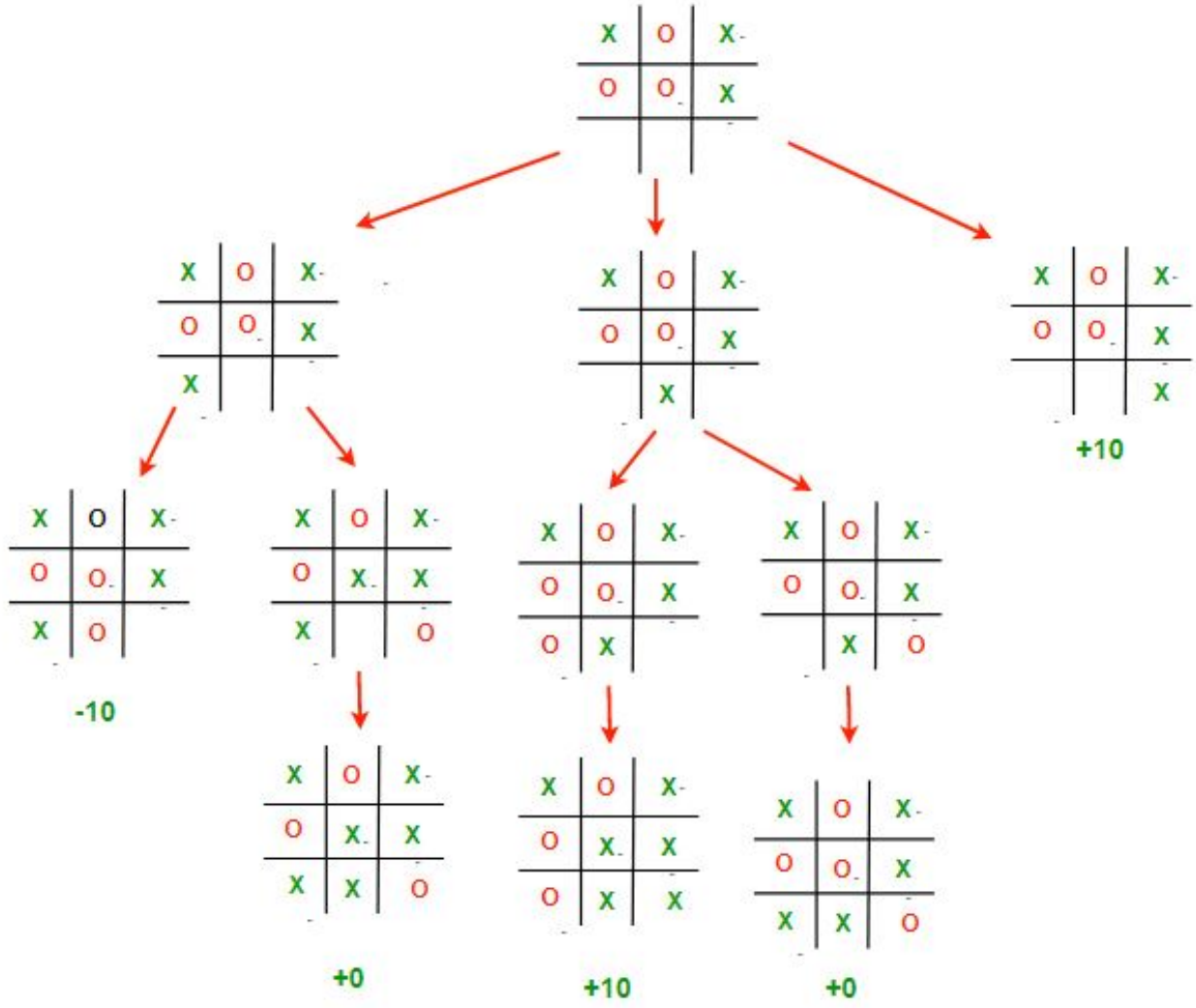
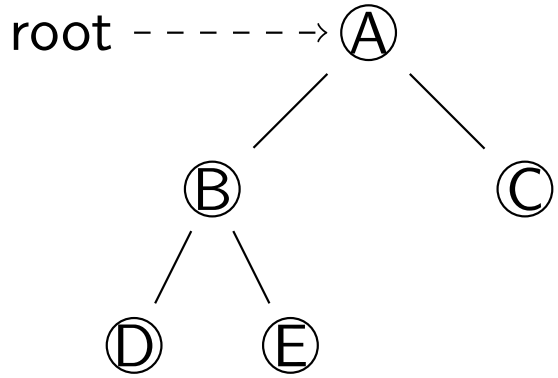


Figure: Monte Carlo Search Tree
[https://www.analyticsvidhya.com/blog/2019/01/monte-carlo-tree-search-introduction-algorithm-deepmind-alphago/]

Anatomy of a Tree



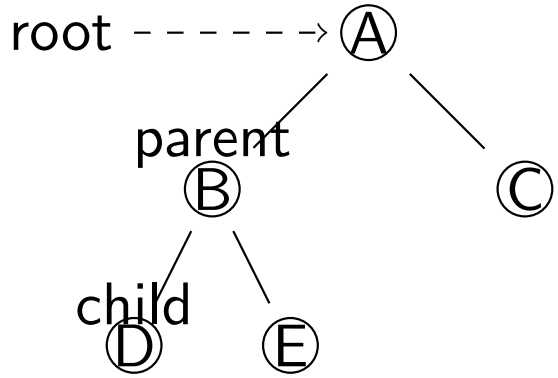
Parts:

- **root:** topmost node
- **parent:** node with child nodes
- **child:** node connected to parent
- **leaf:** node with no children
- **subtree:** all descendants from a node in a tree

Properties

- **complete:** every level but the last is entirely filled
- **balanced:** left and right subtrees do not differ than more than one

Anatomy of a Tree



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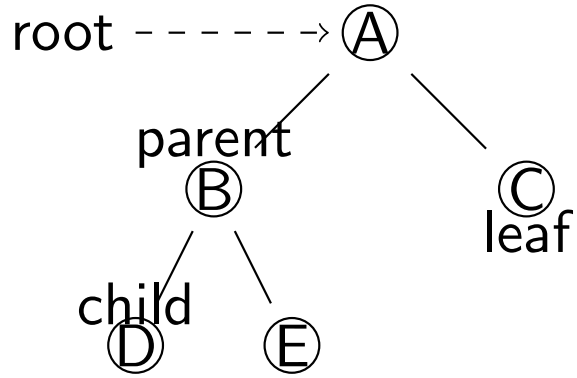

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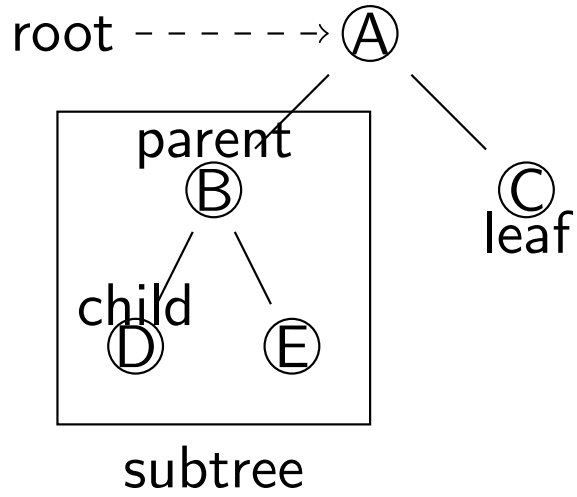

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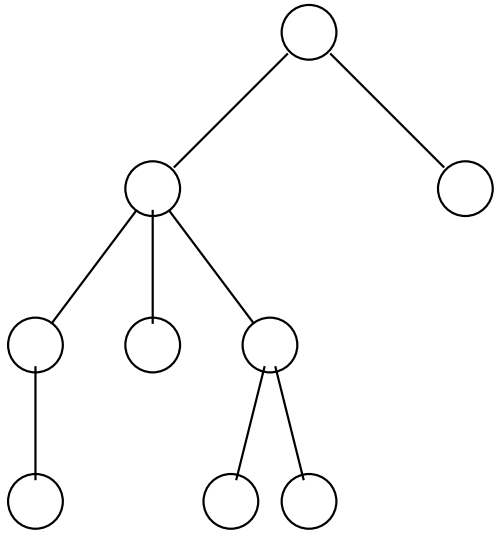
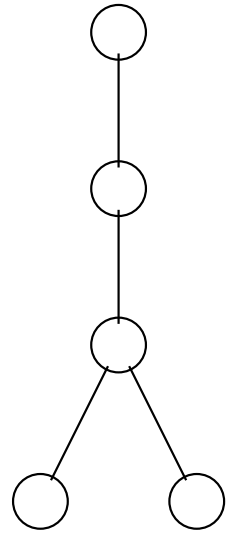
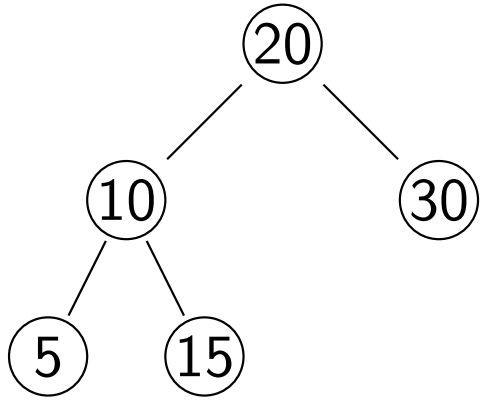
Trees





- trees can be fairly unstructured
  - no restriction on number of children
  - no ordering structure
- we will apply structure and go over some special properties

Binary Search Tree



Ordering Property

- each node has at most two children
- left children have values less than parent
- right children have values greater than parent

Overview:

- structure allows for efficient searching ($O(\log n)$)
- in-order traversal

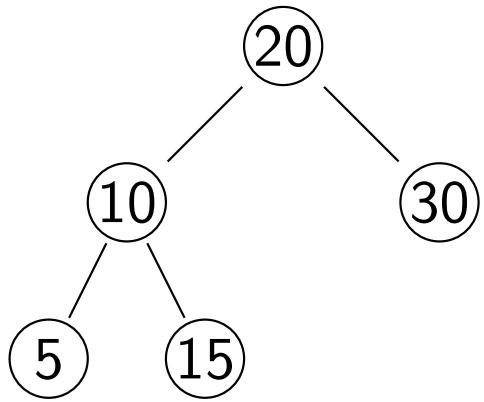
Heaps

Overview:

- min/max is always found at root
- complete binary tree
- last level filled left to right

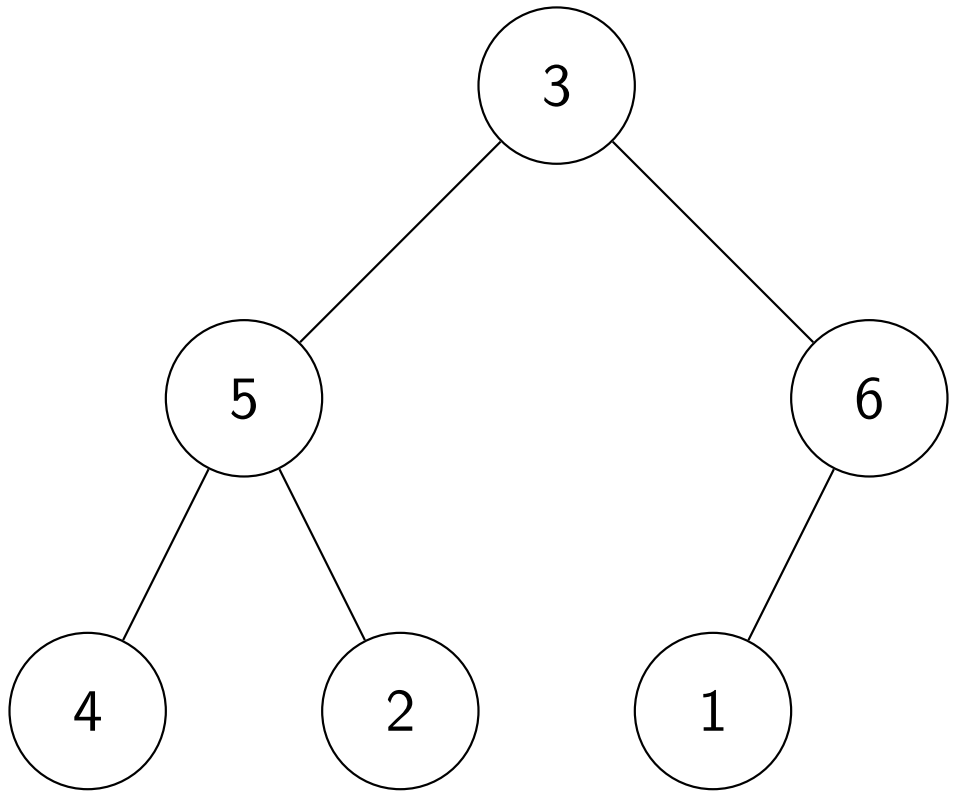
Applications:

- priority queue: parallel computing
- shortest path search



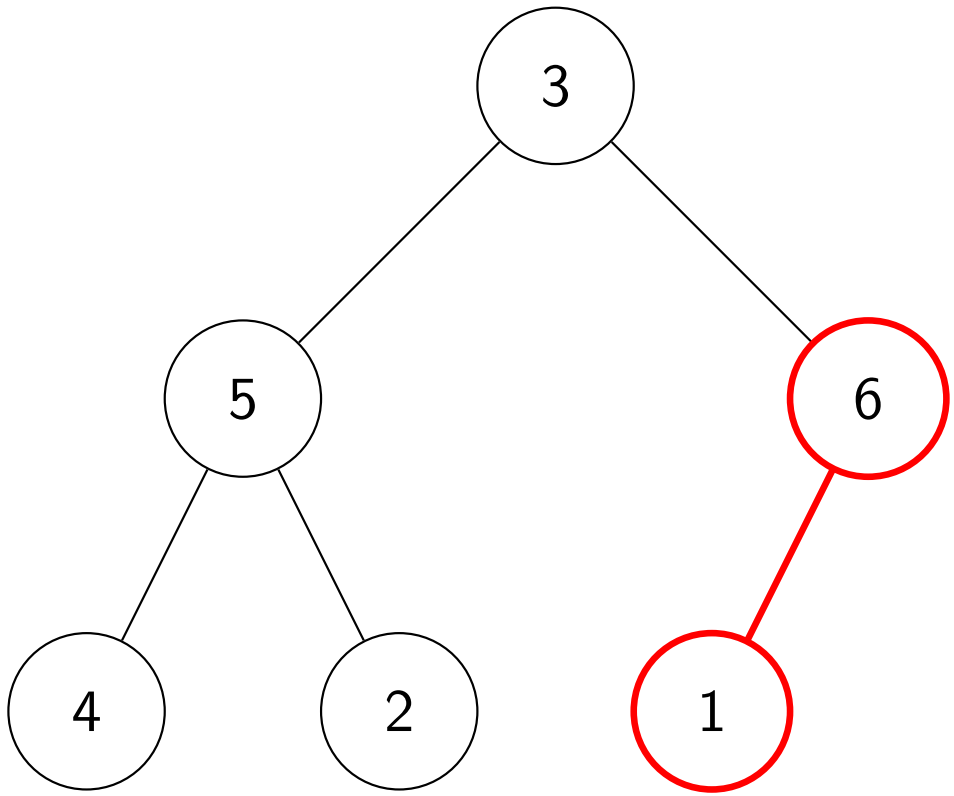
Heapify (Min Heap)

x = [3, 5, 6, 4, 2, 1]
Build complete binary tree



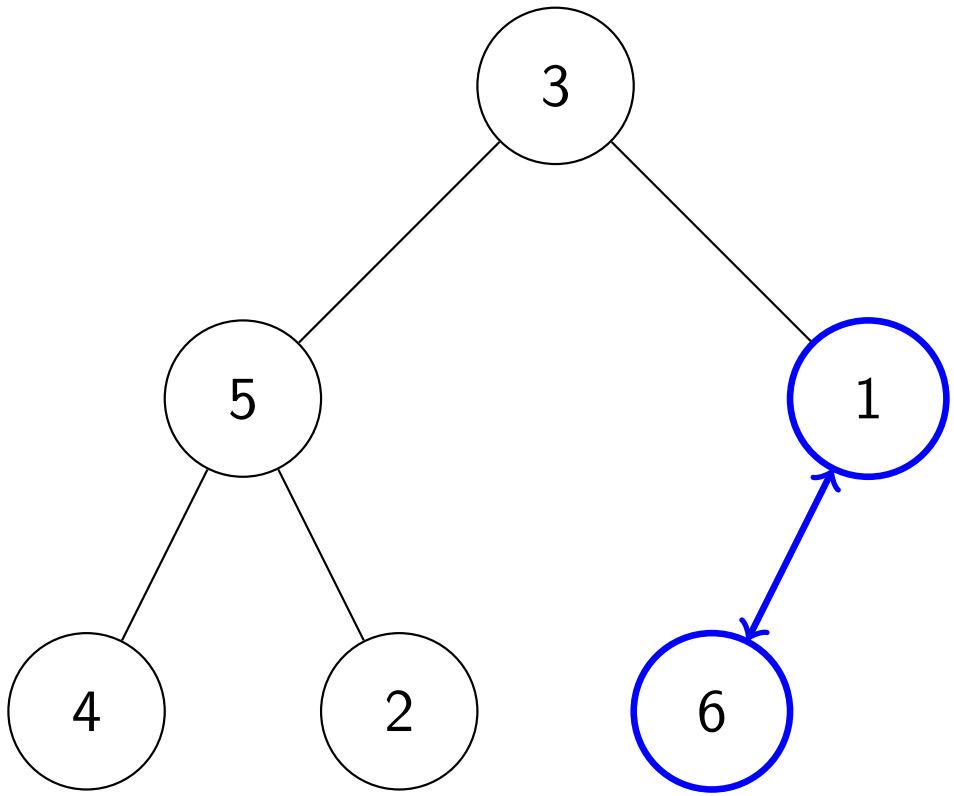
Heapify (Min Heap)

Start at last parent



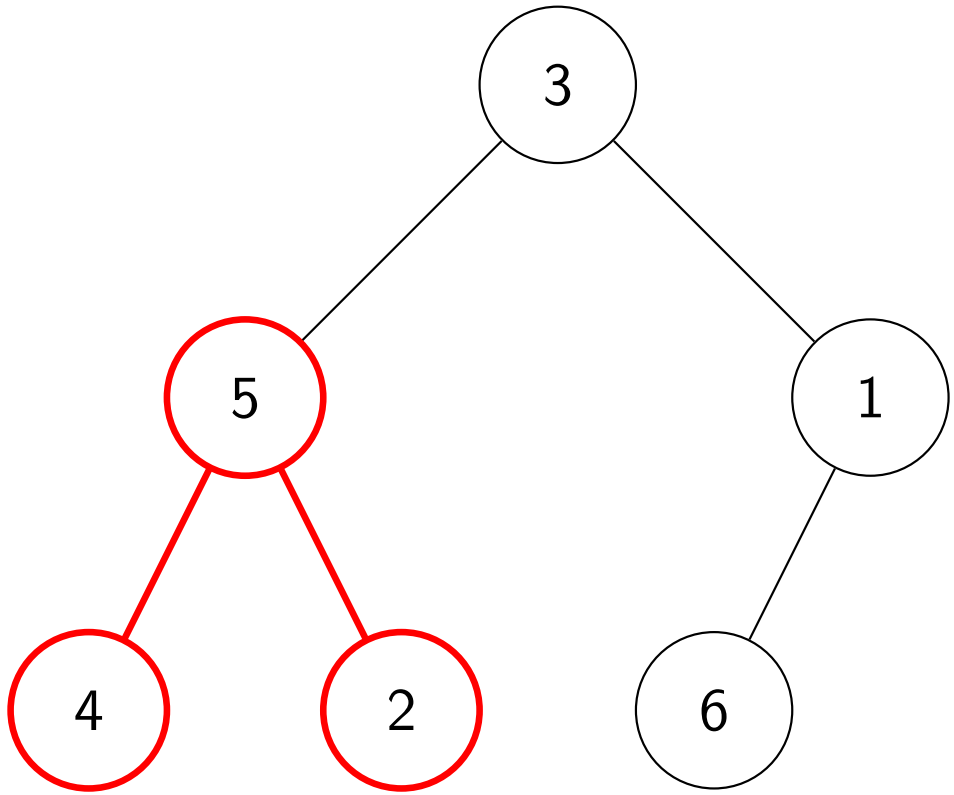
Heapify (Min Heap)

Swap



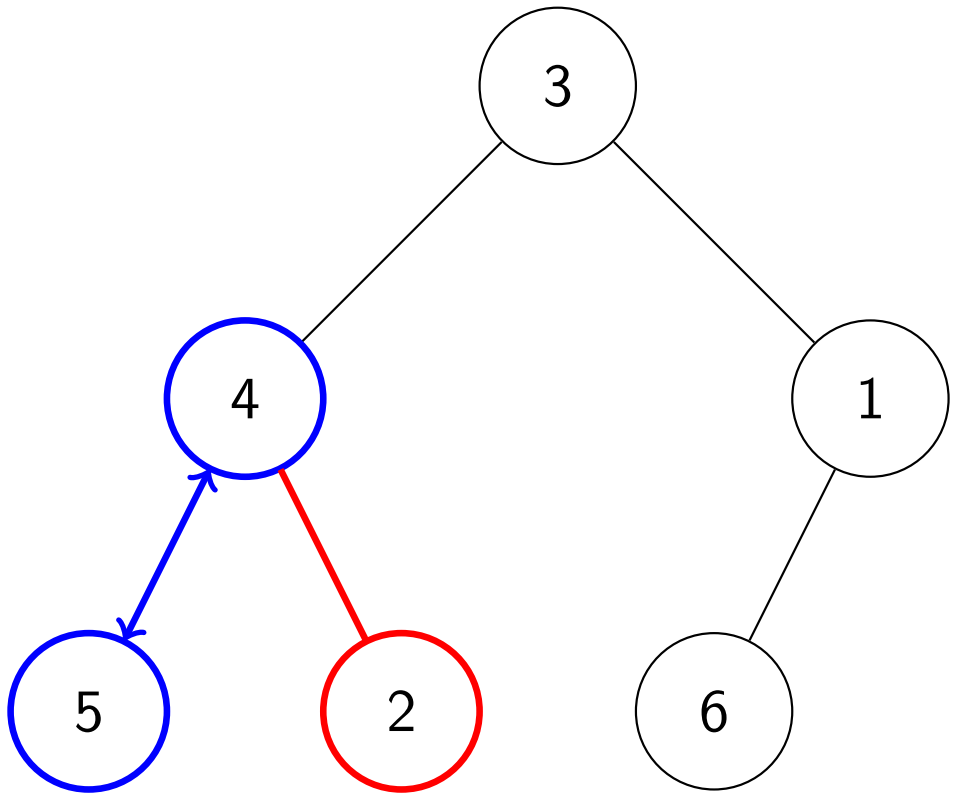
Heapify (Min Heap)

Move to next subtree



Heapify (Min Heap)

Swap left child



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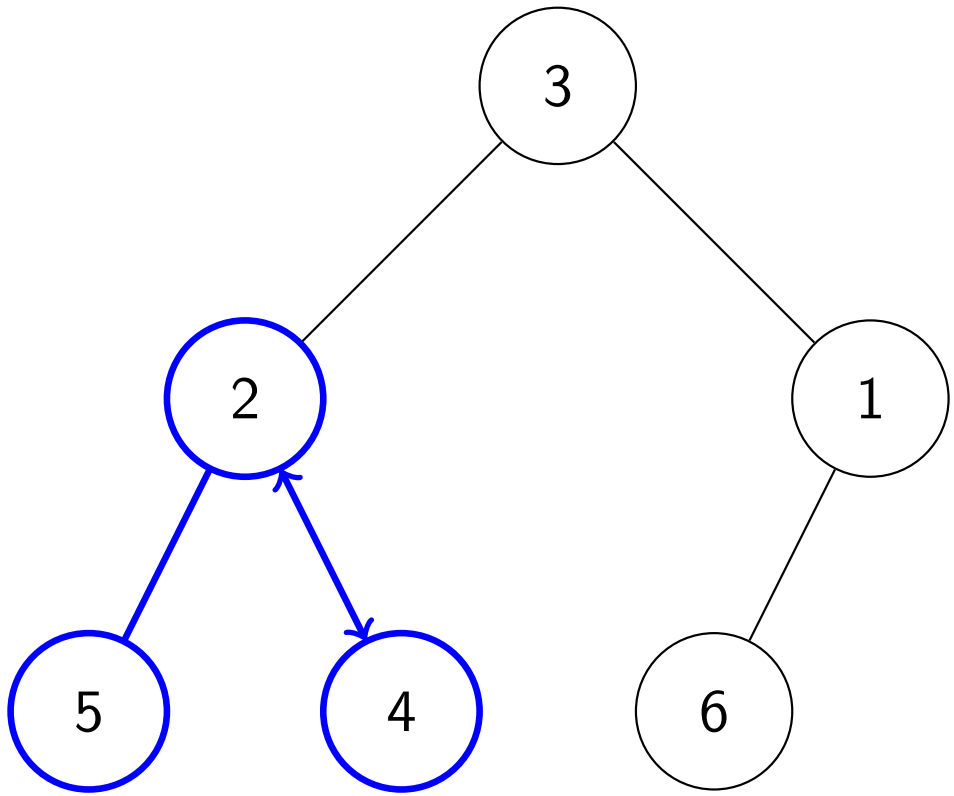

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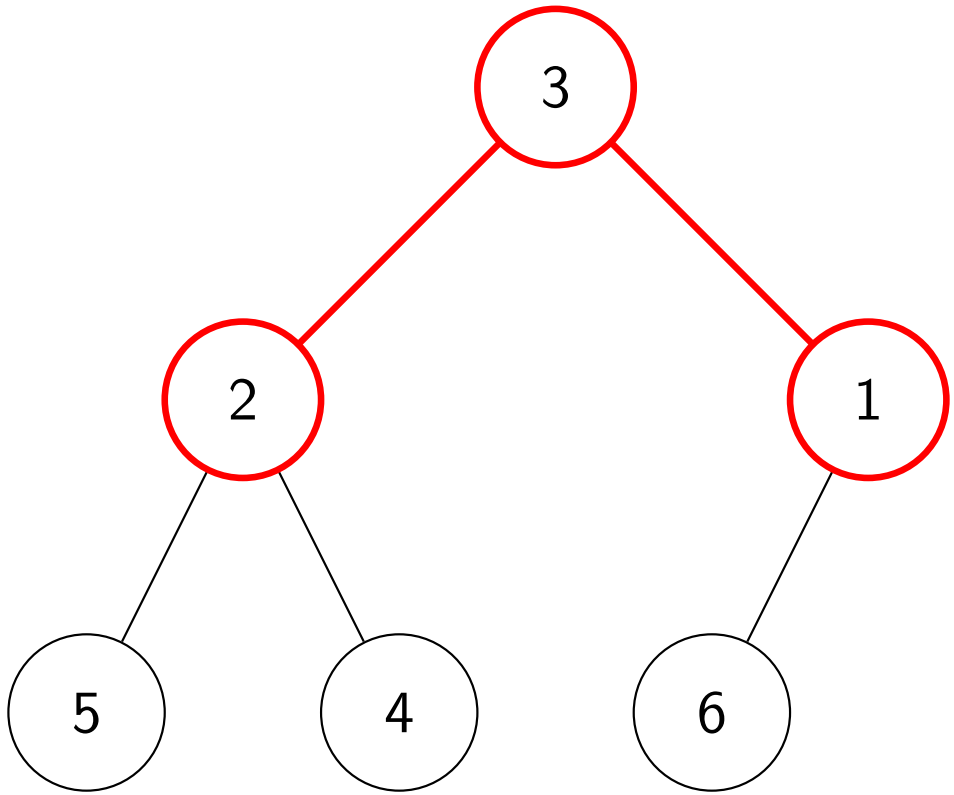
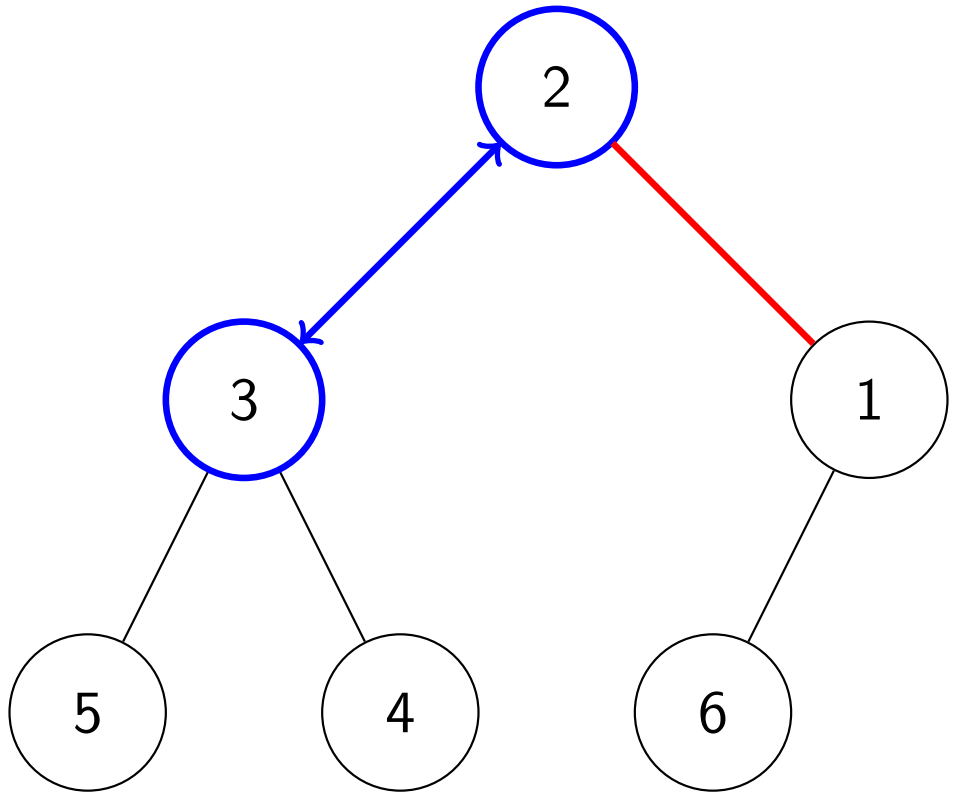


Heapify (Min Heap)

Check left child



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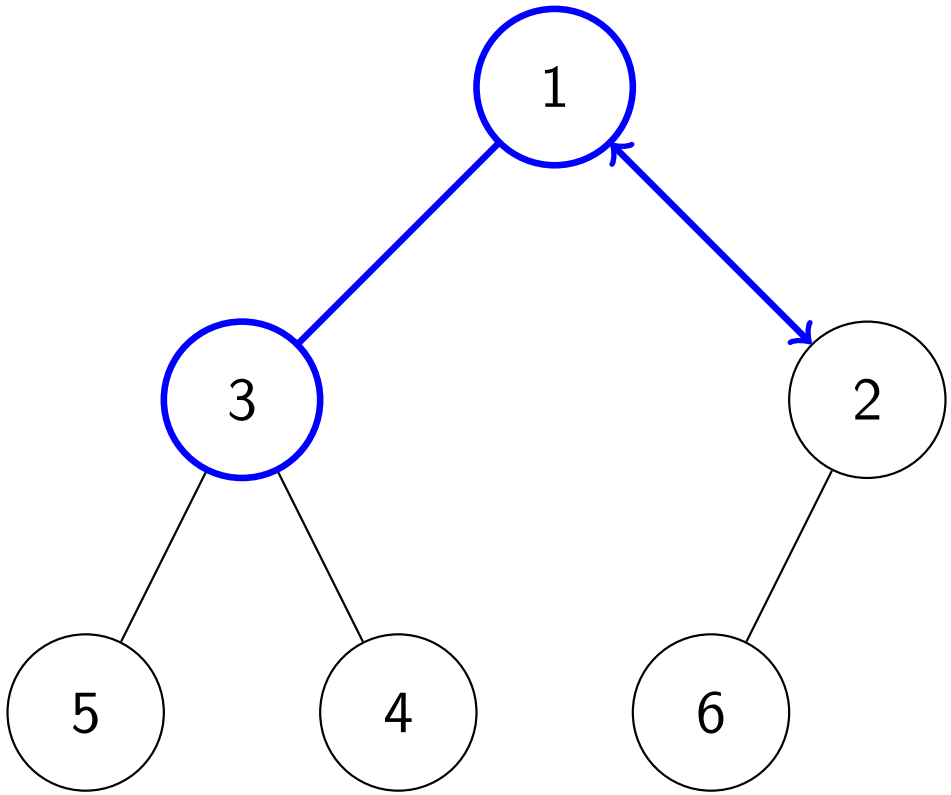

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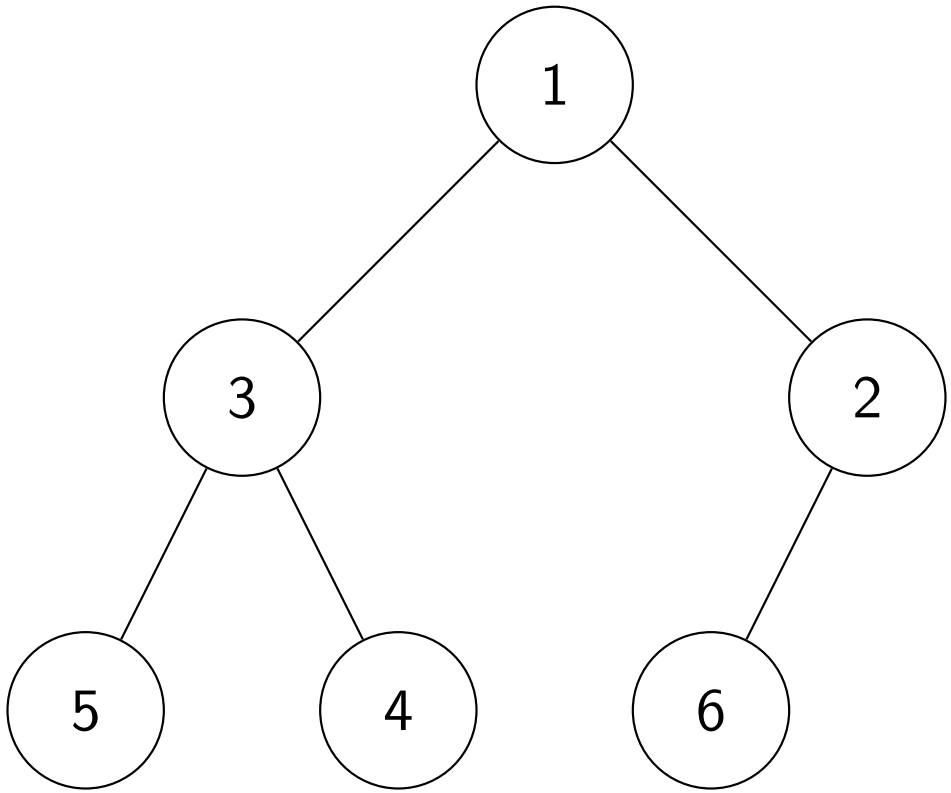
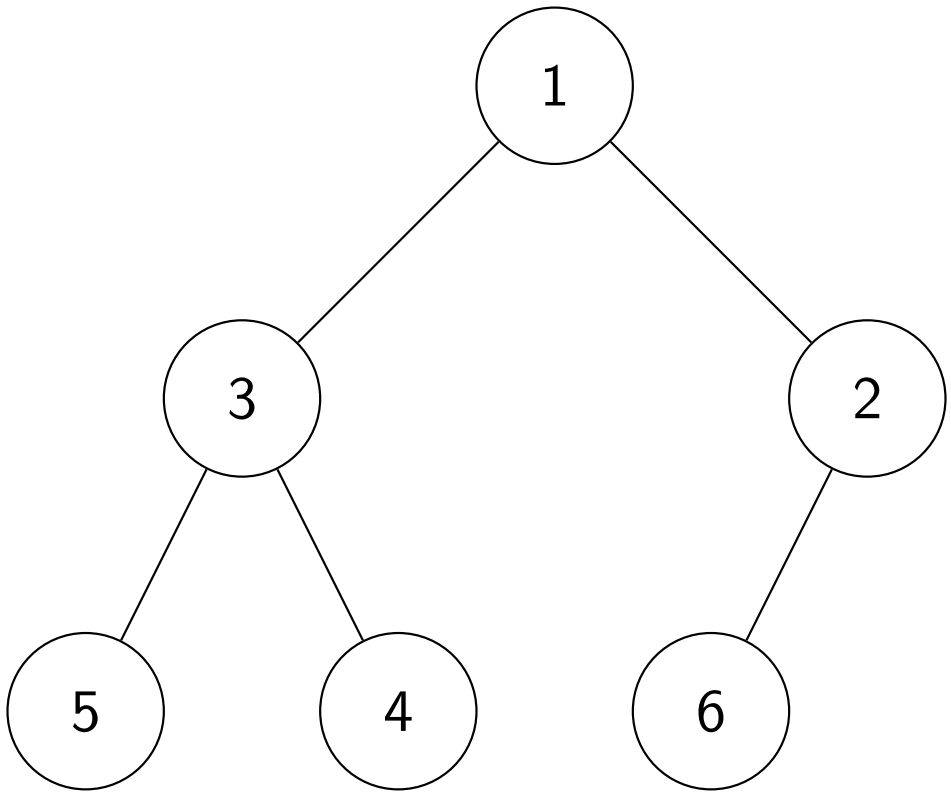


Heap: Insert

insert 0 to heap



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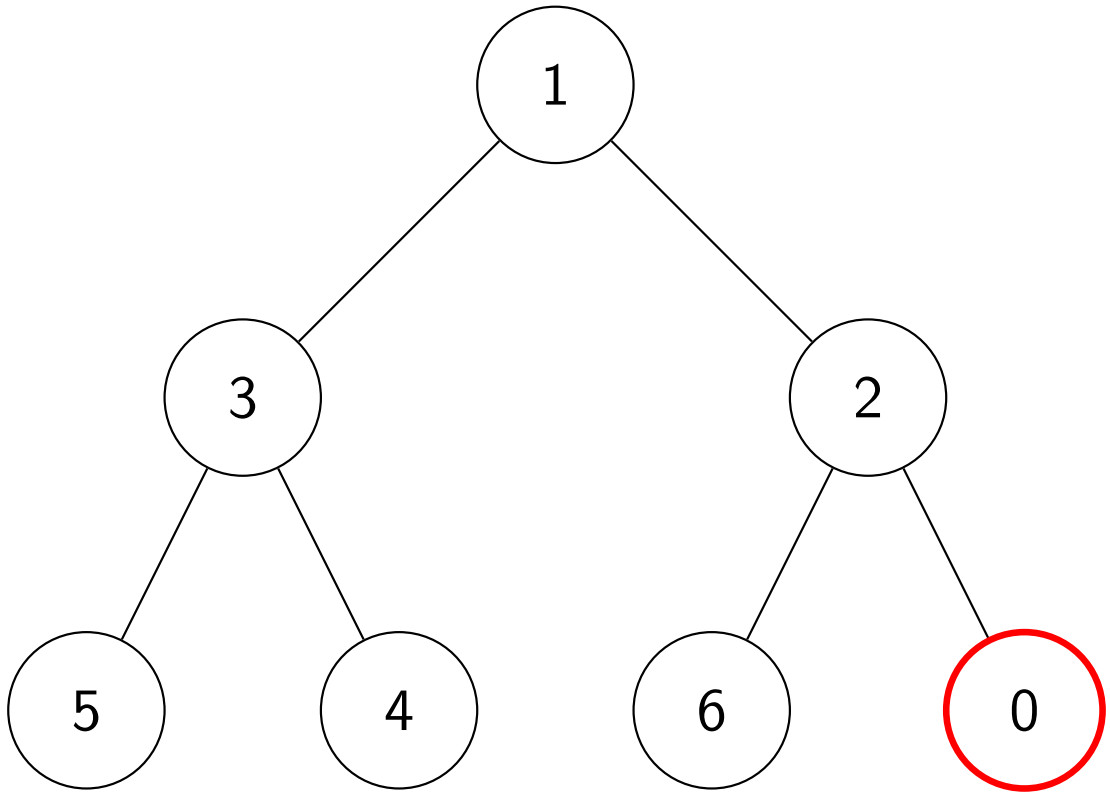

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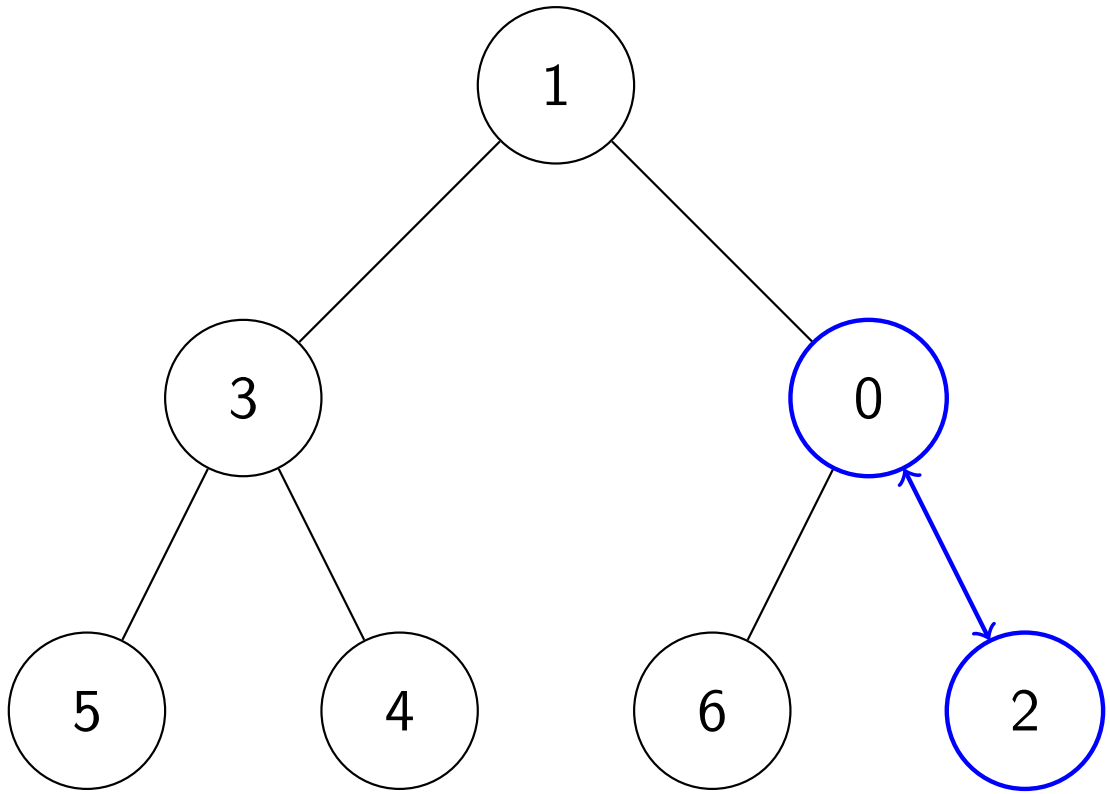

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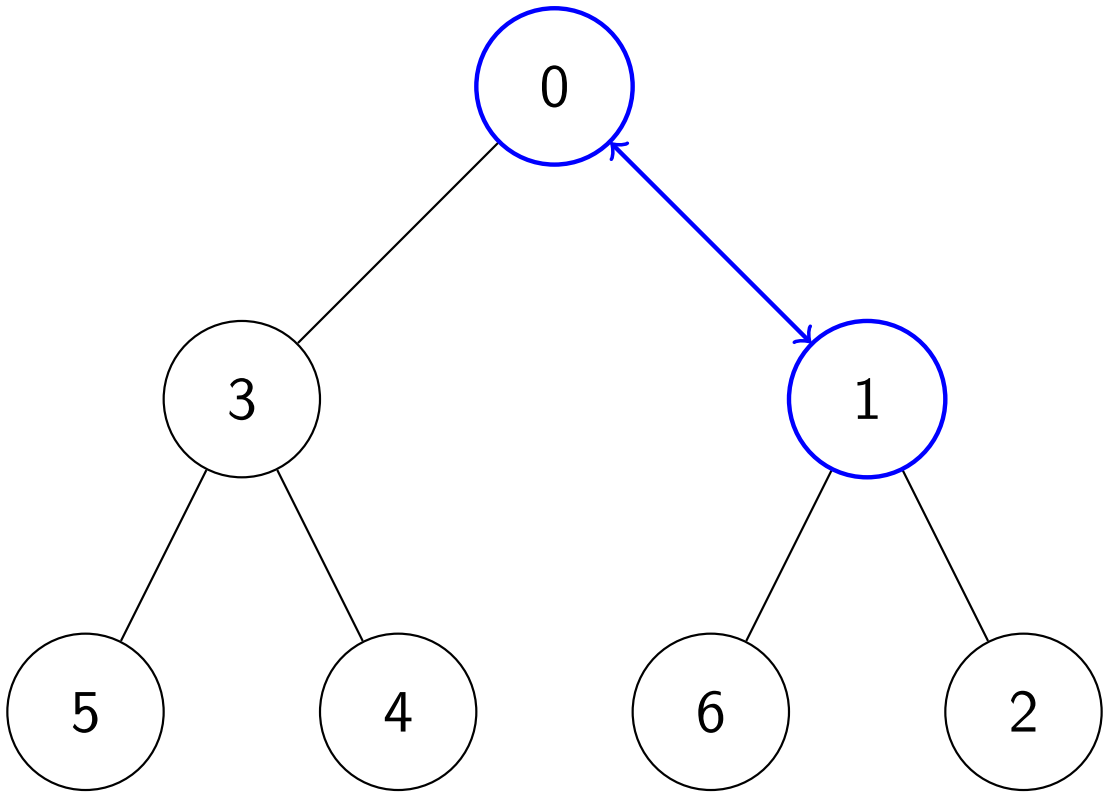
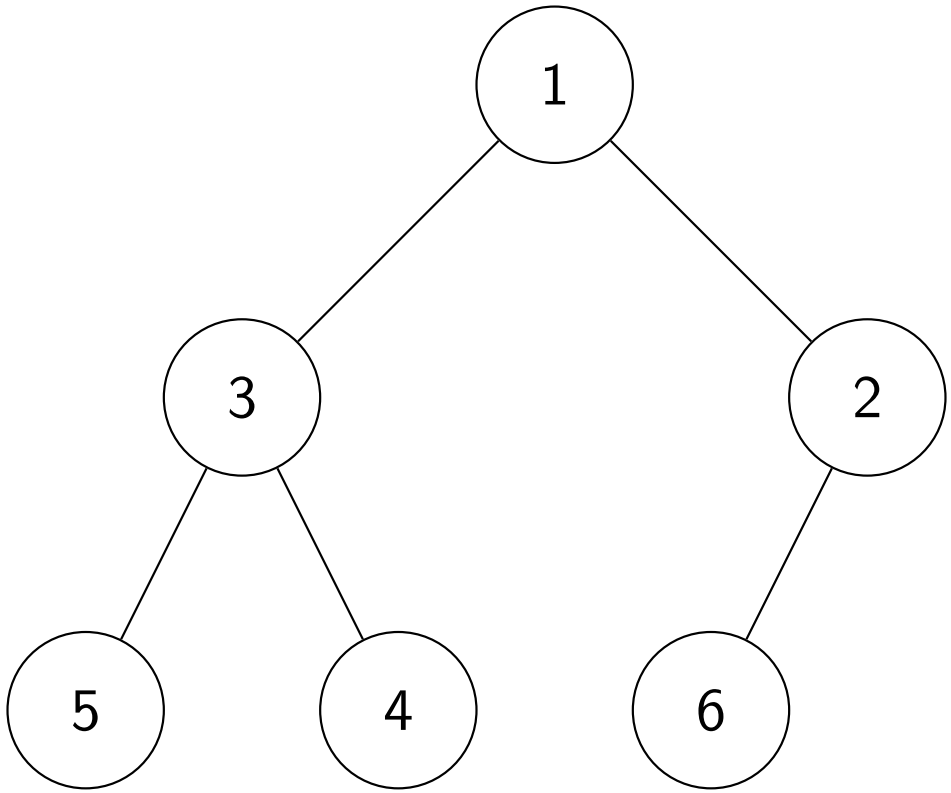


Heap: Delete

Delete root



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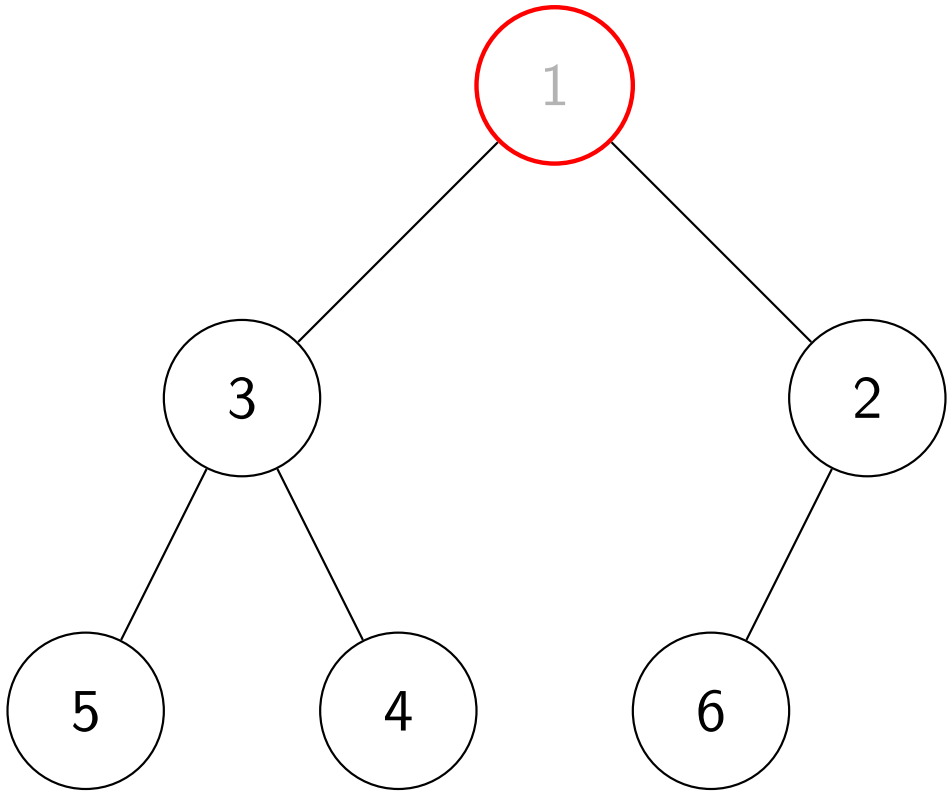
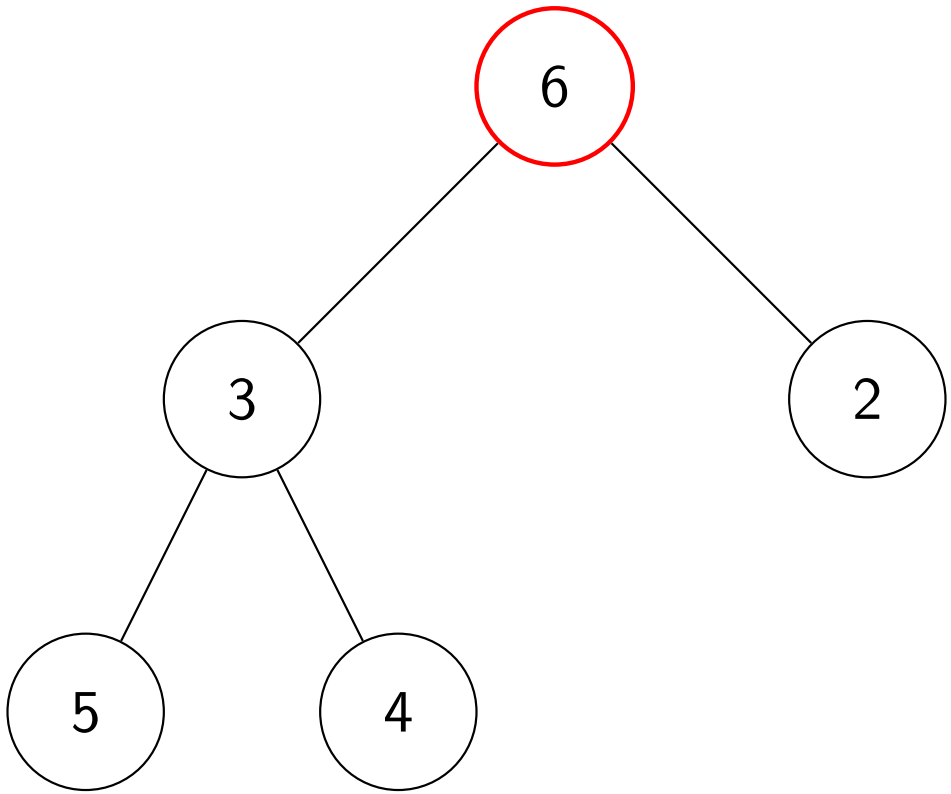


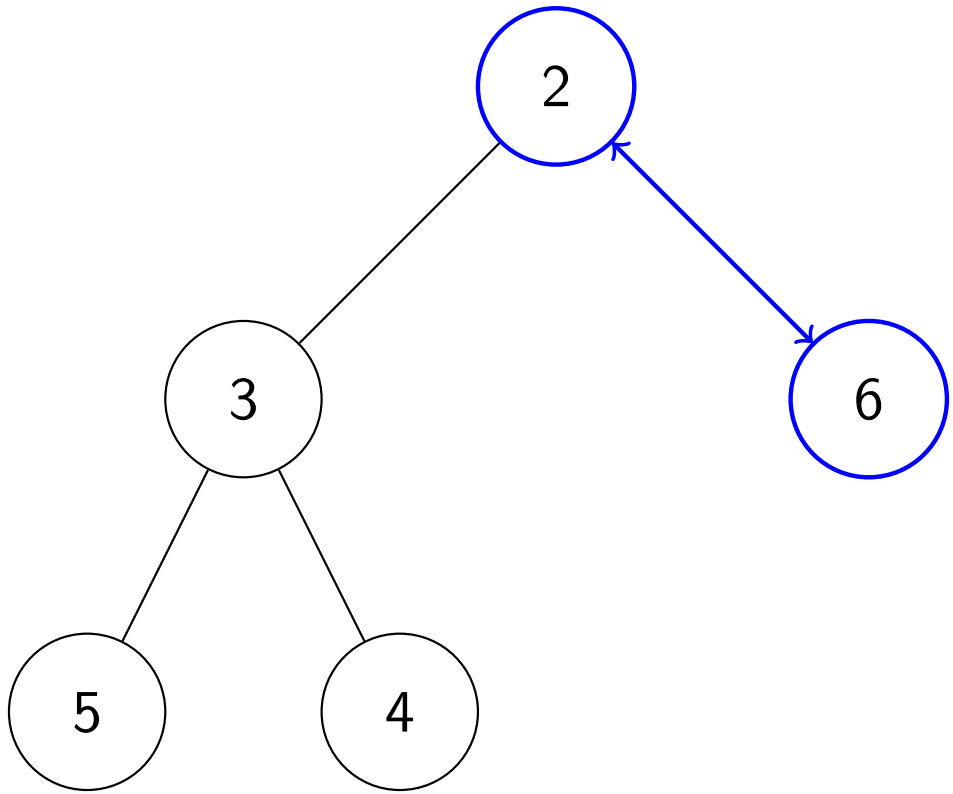
Heap: Delete

Replace with last node



Heap: Delete

Heapify smaller child



Heaps: Overview

- Build Heap: $O(n)$
- Insertion: $O(\log n)$
- Deletion: $O(\log n)$
- Fast to find min value
- Fast to reorganize heap after insertion/deletion

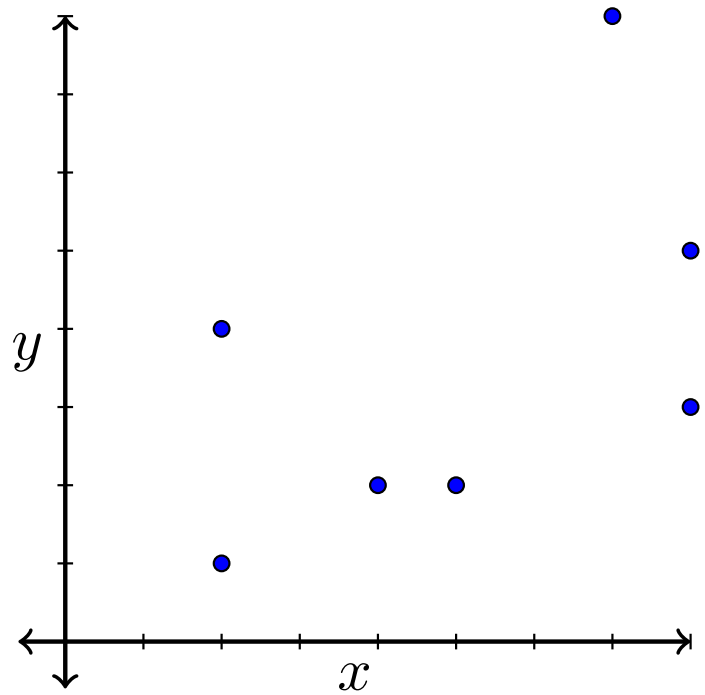
K-Dimensional Trees

Overview

- encodes spatial information
- efficient search for data points
- used for k-nearest neighbors

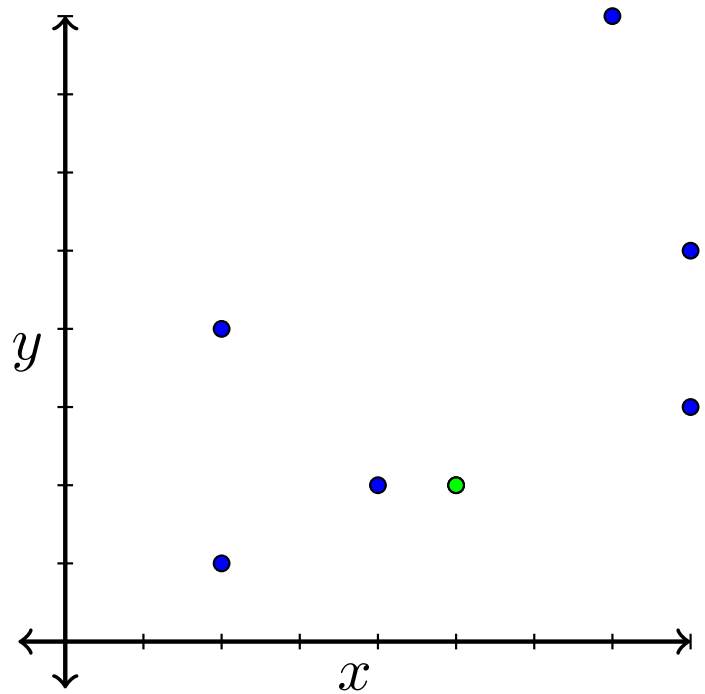
K-Dimensional Trees

Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)



K-Dimensional Trees

Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)

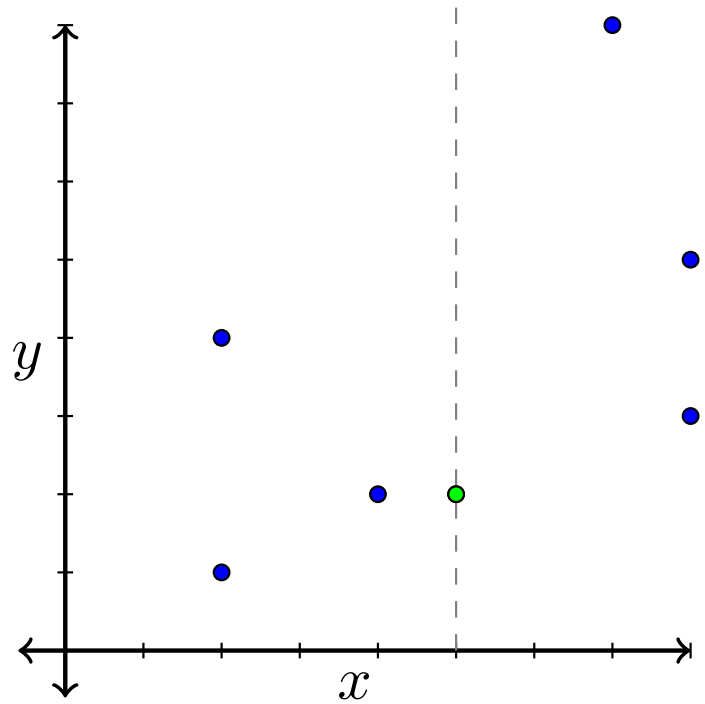


K-Dimensional Trees

Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)

make root

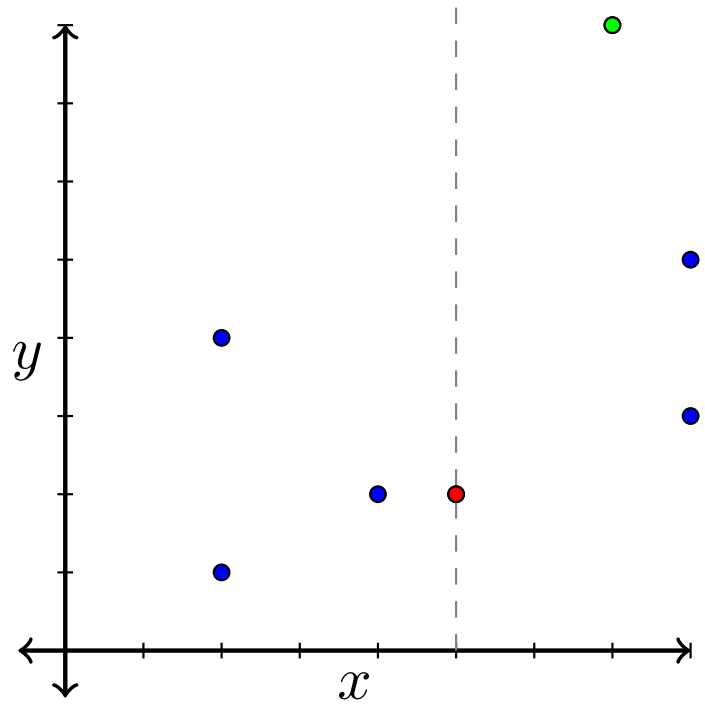
x ..... $(5, 2)$ .....



K-Dimensional Trees

Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)

x ..... $(5, 2)$ .....

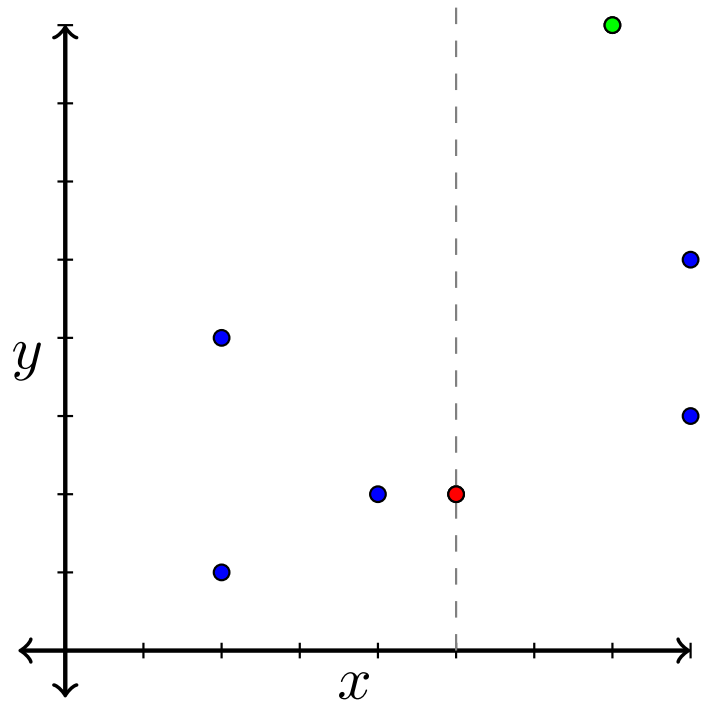


K-Dimensional Trees

Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)

$$(5, 2) \leq (7, 8)$$

x ..... $(5, 2)$ .....



K-Dimensional Trees

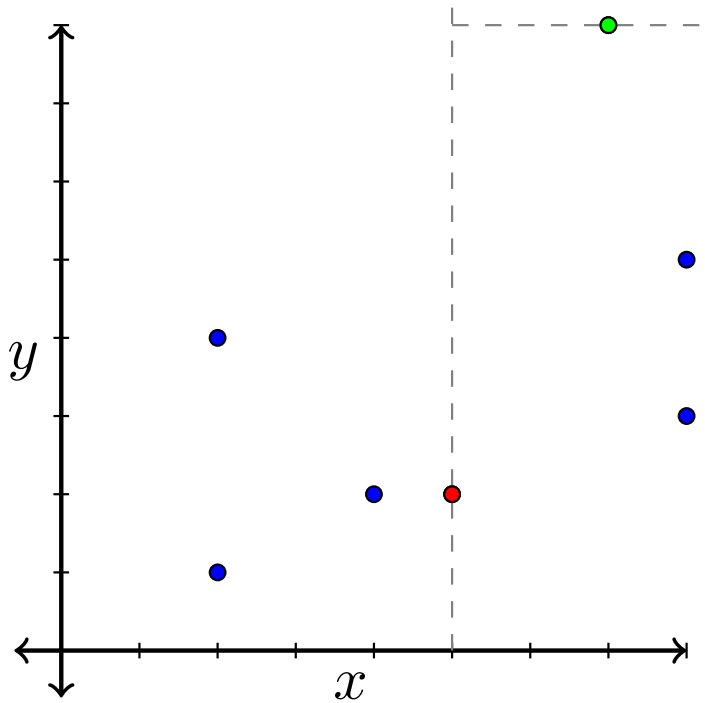
Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)

$$(5, 2) \leq (7, 8)$$

x ..... $(5, 2)$ .....

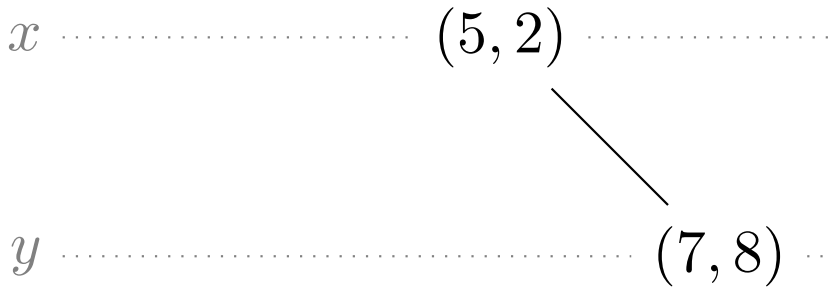
make child

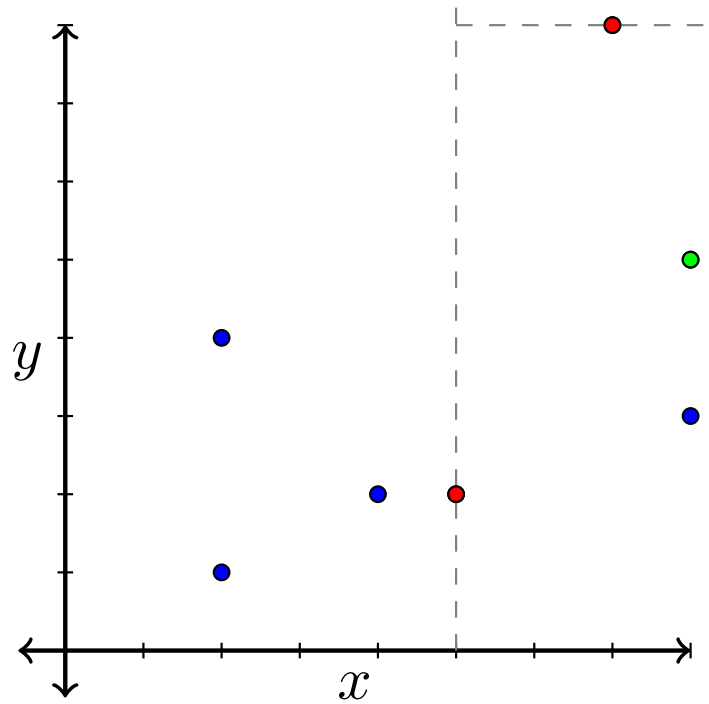
y ..... $(7, 8)$ ..



K-Dimensional Trees

Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)

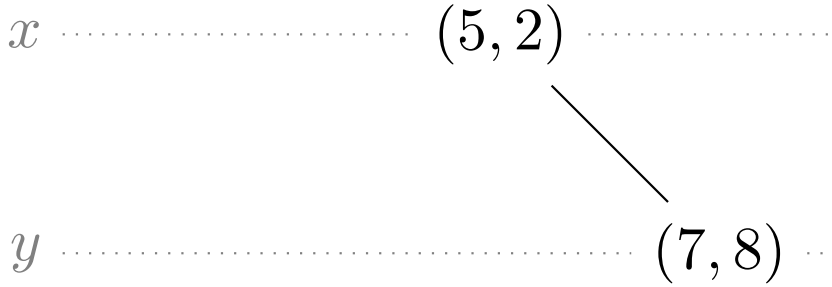


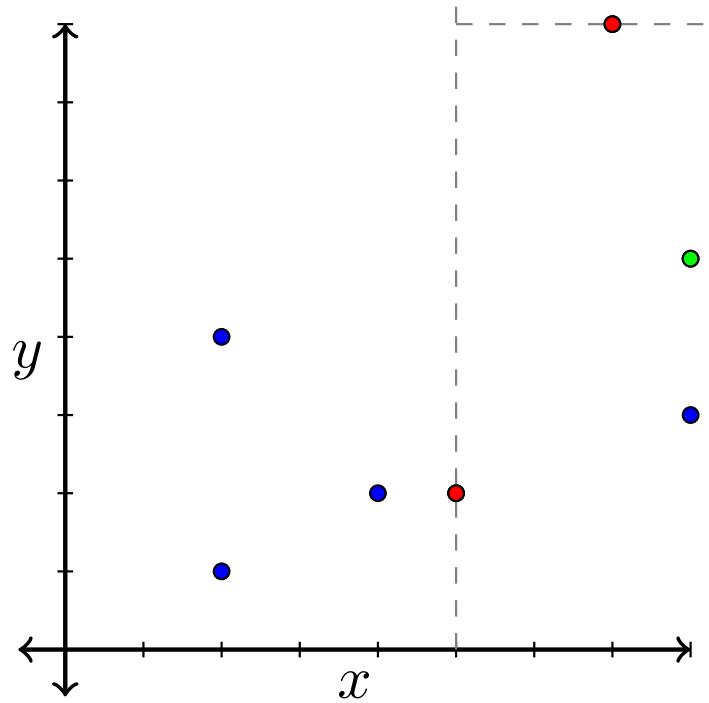


K-Dimensional Trees

Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)

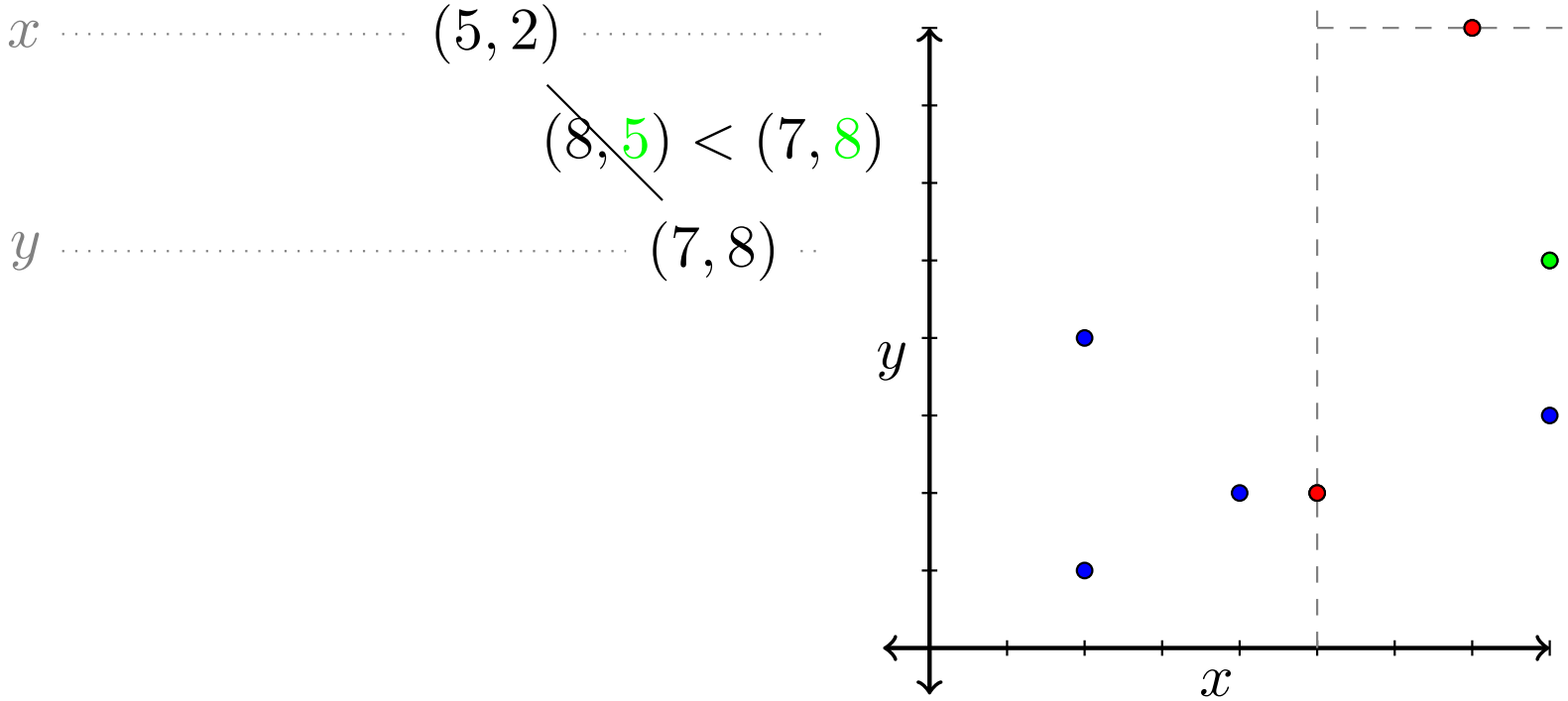
$$(5, 2) \leq (8, 5)$$





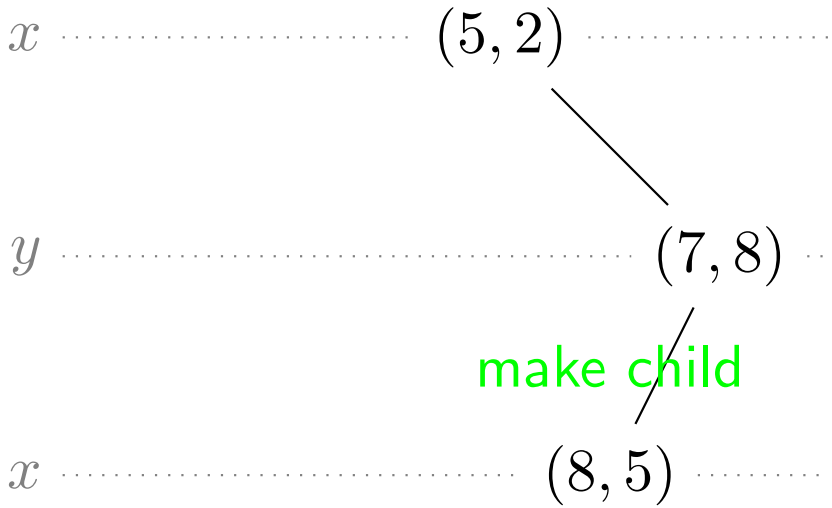
K-Dimensional Trees

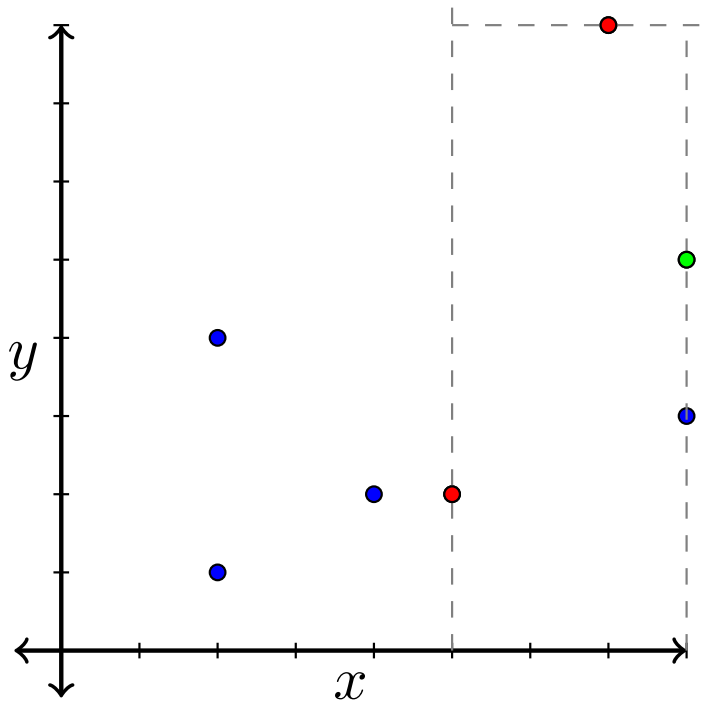
Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)



K-Dimensional Trees

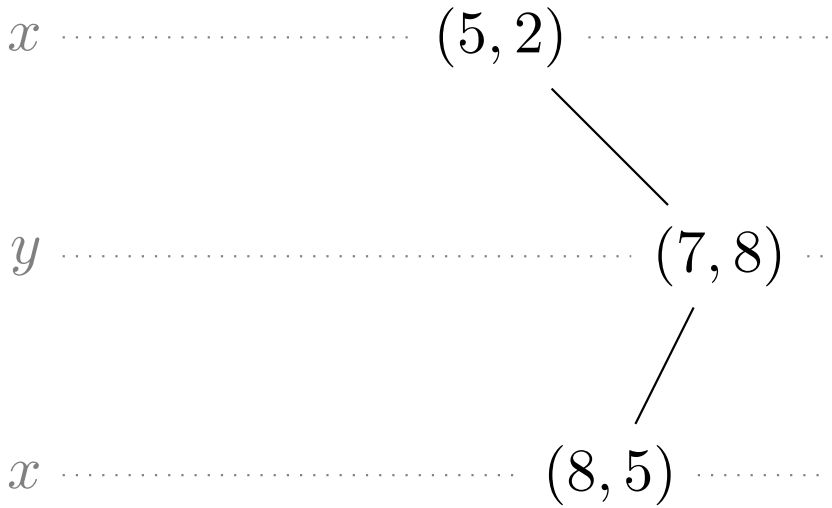
Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)

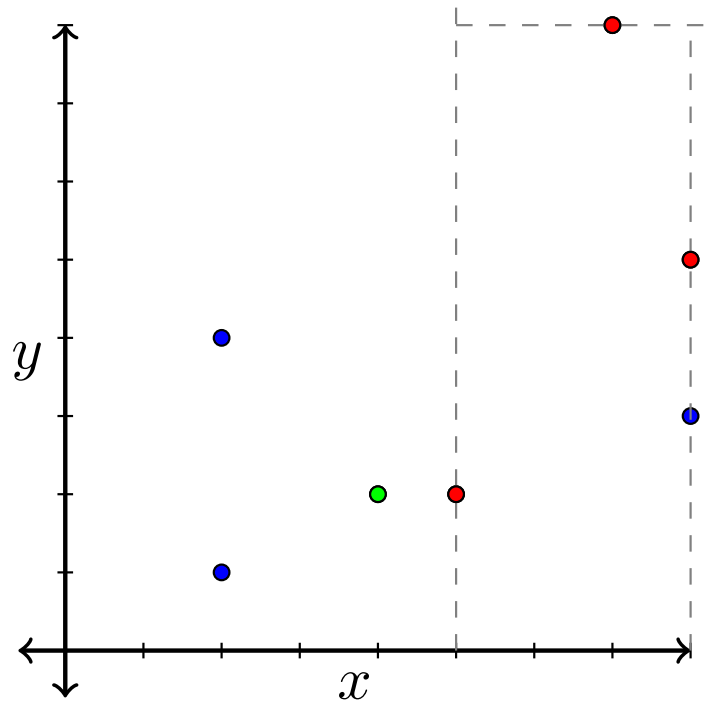




K-Dimensional Trees

Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)

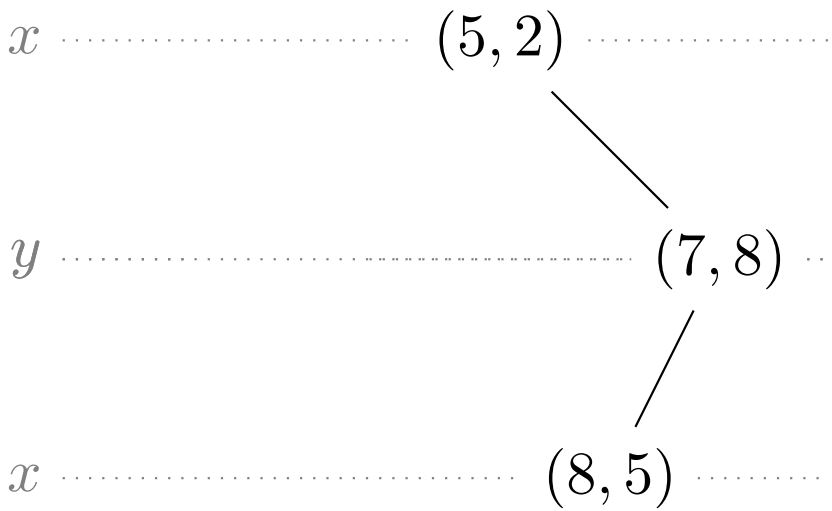


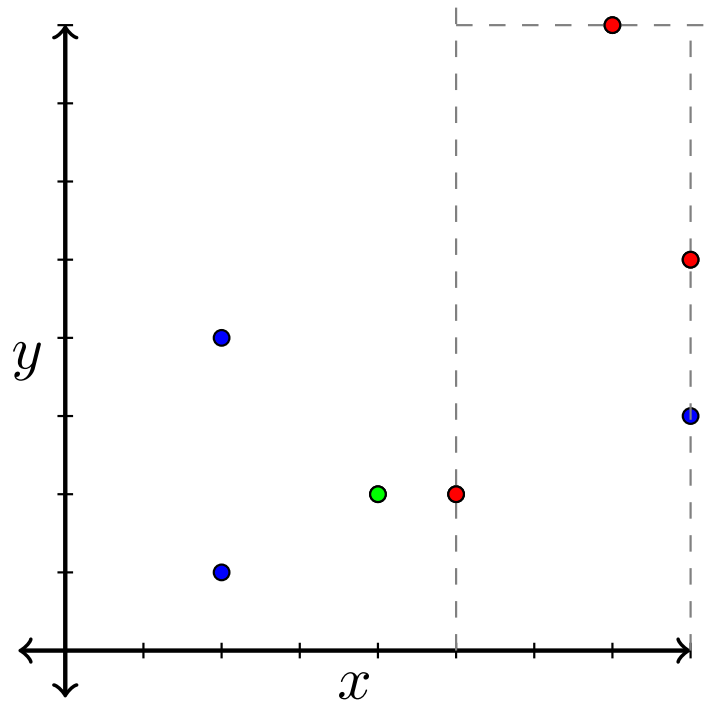


K-Dimensional Trees

Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)

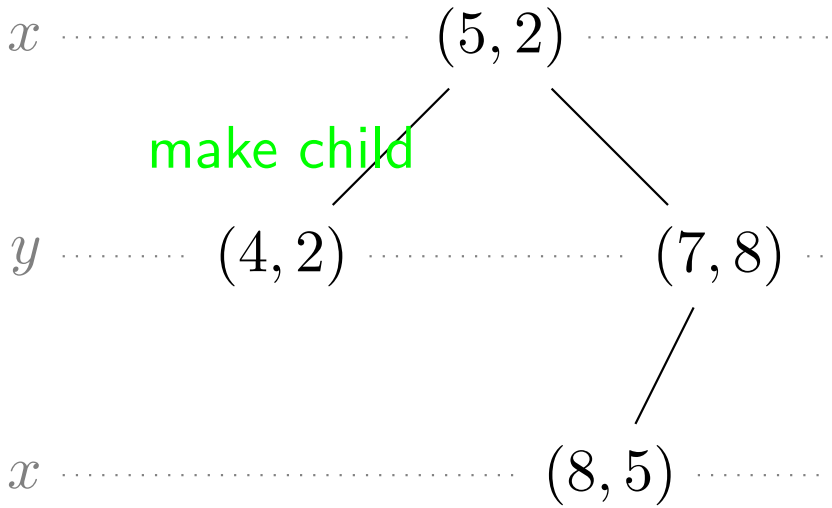
$(4, 2) < (5, 2)$

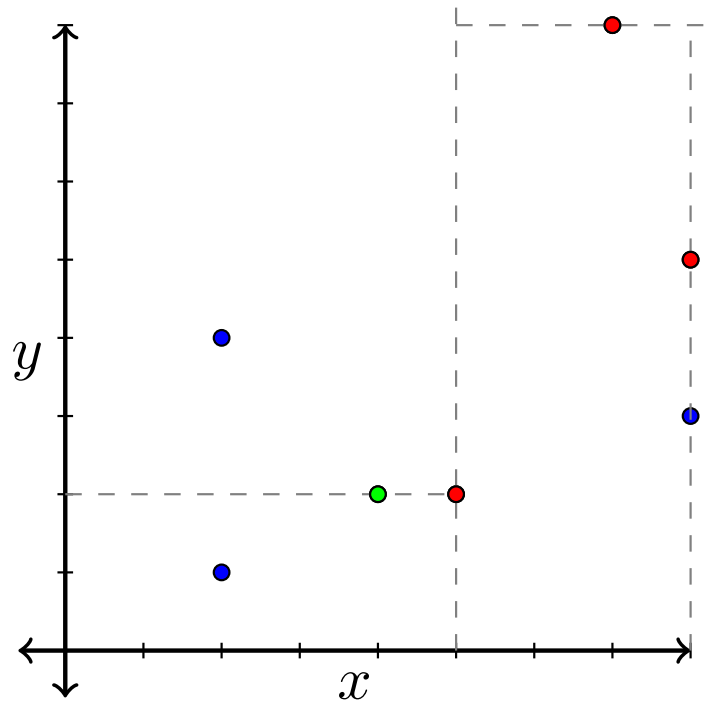




K-Dimensional Trees

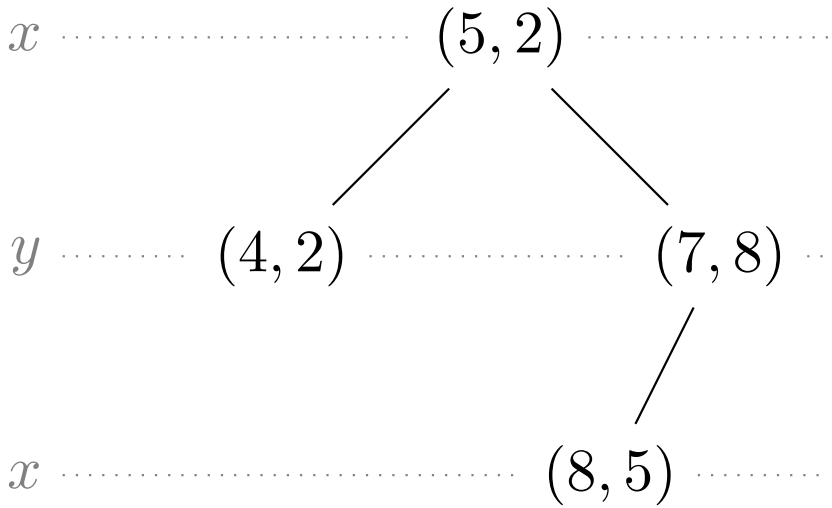
Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)

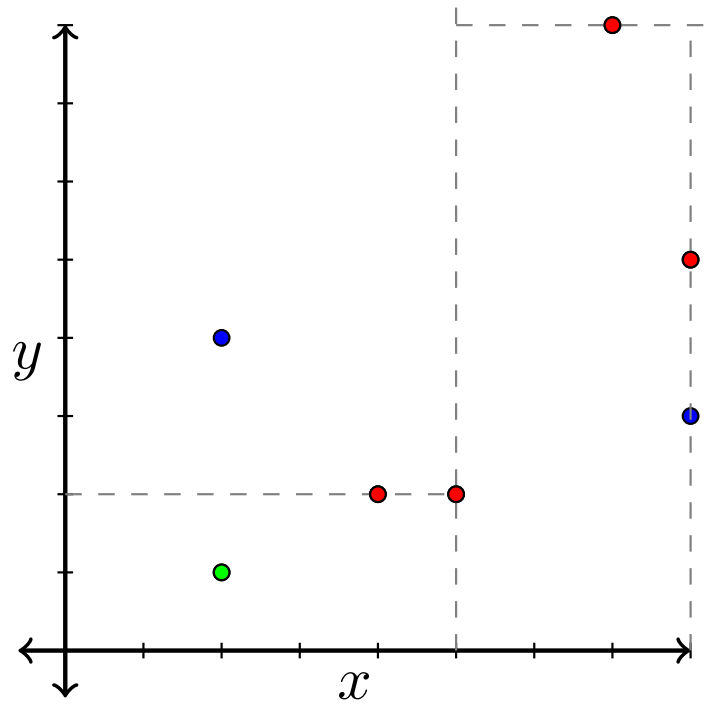




K-Dimensional Trees

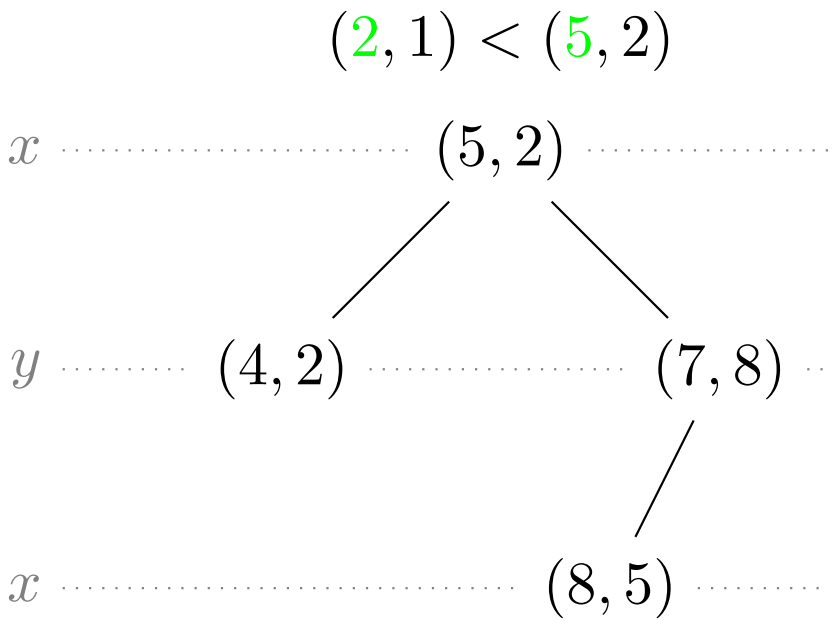
Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)

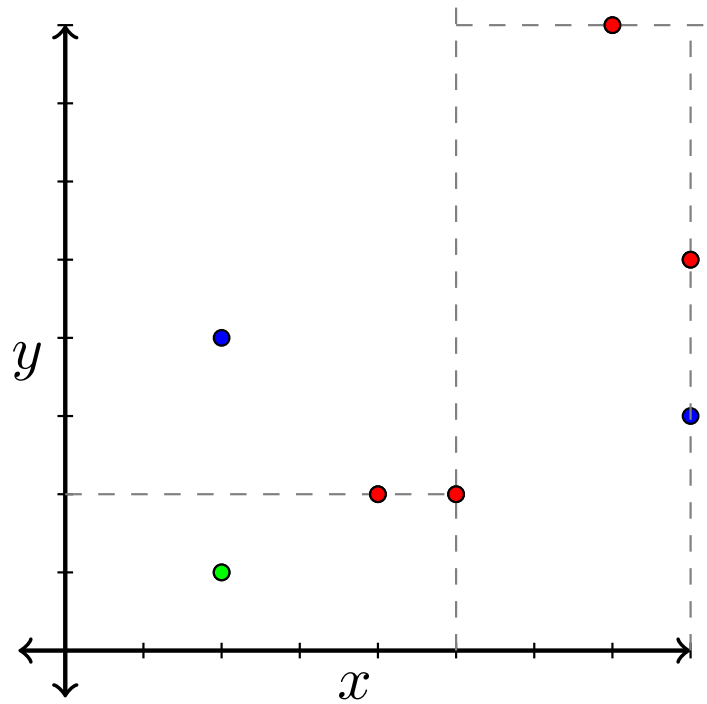




K-Dimensional Trees

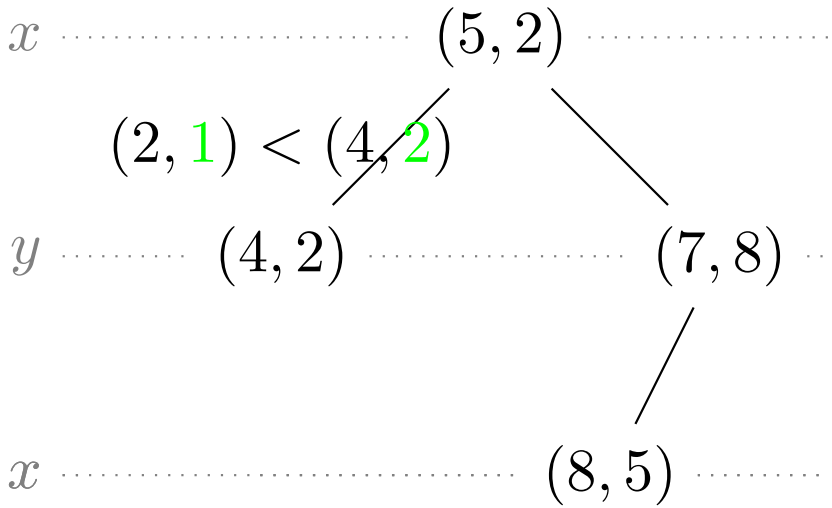
Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)

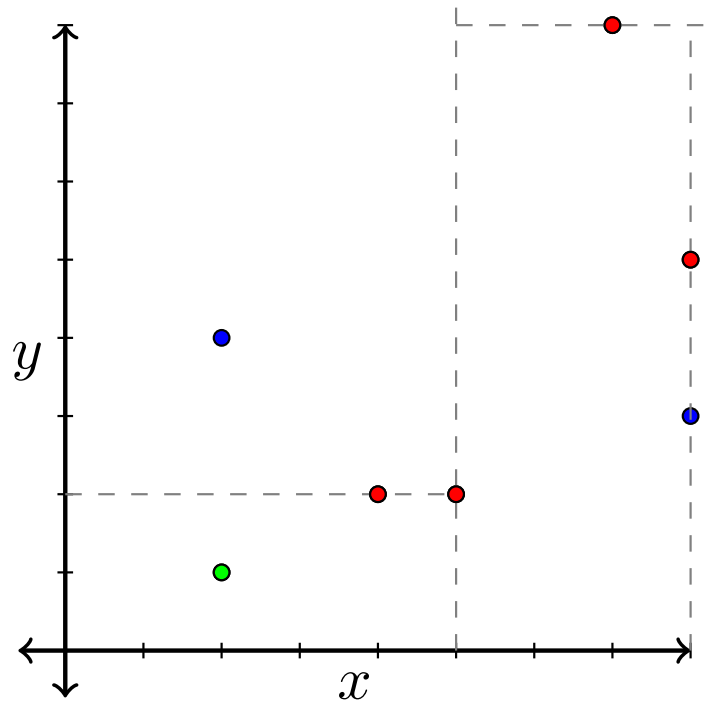




K-Dimensional Trees

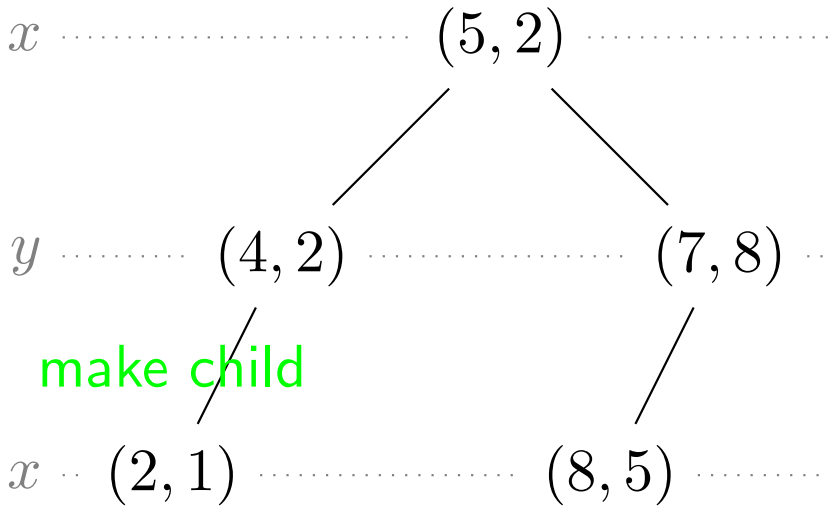
Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)

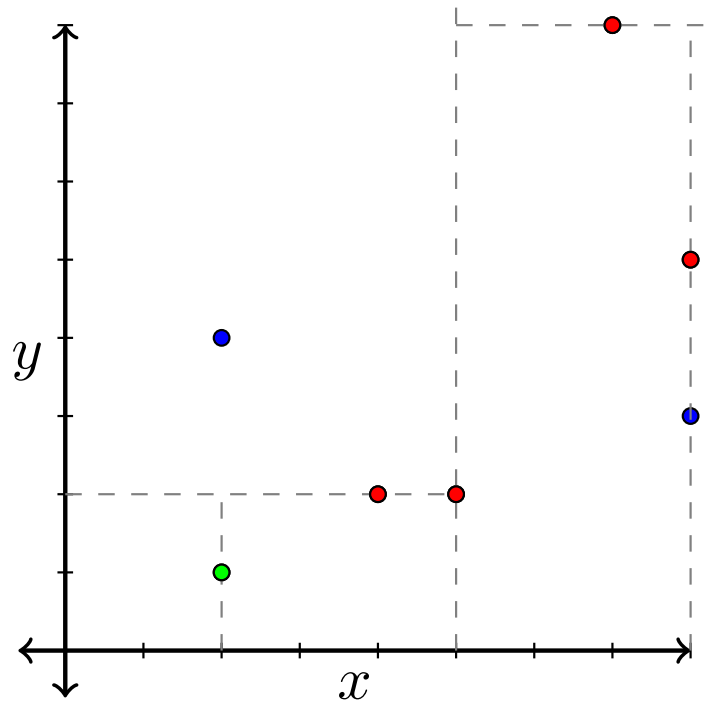




K-Dimensional Trees

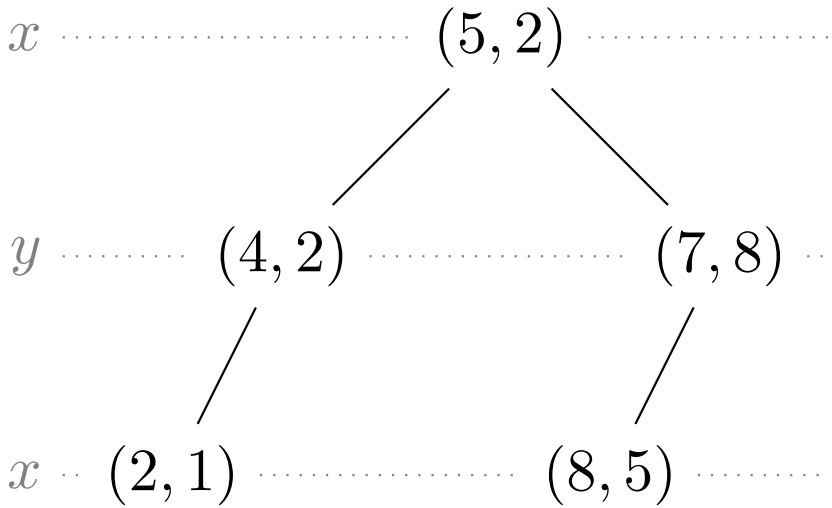
Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)

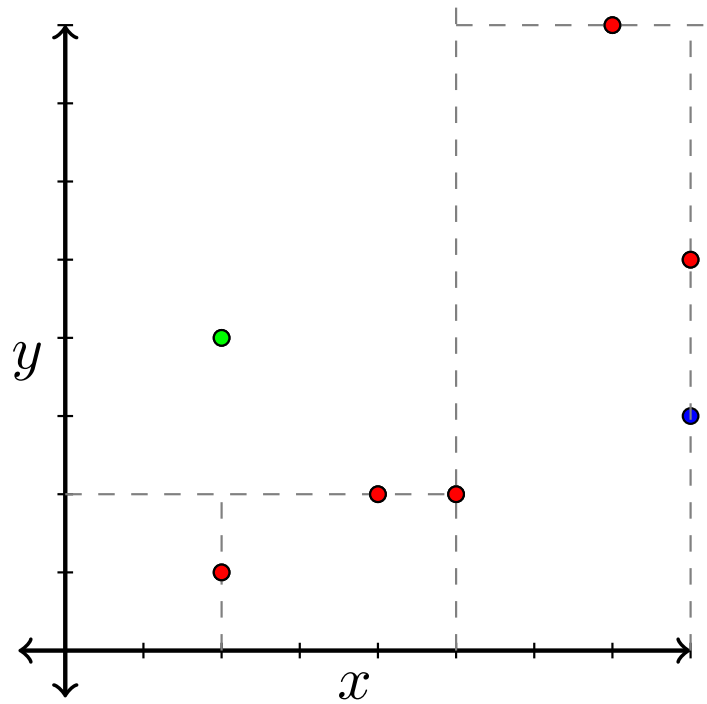




K-Dimensional Trees

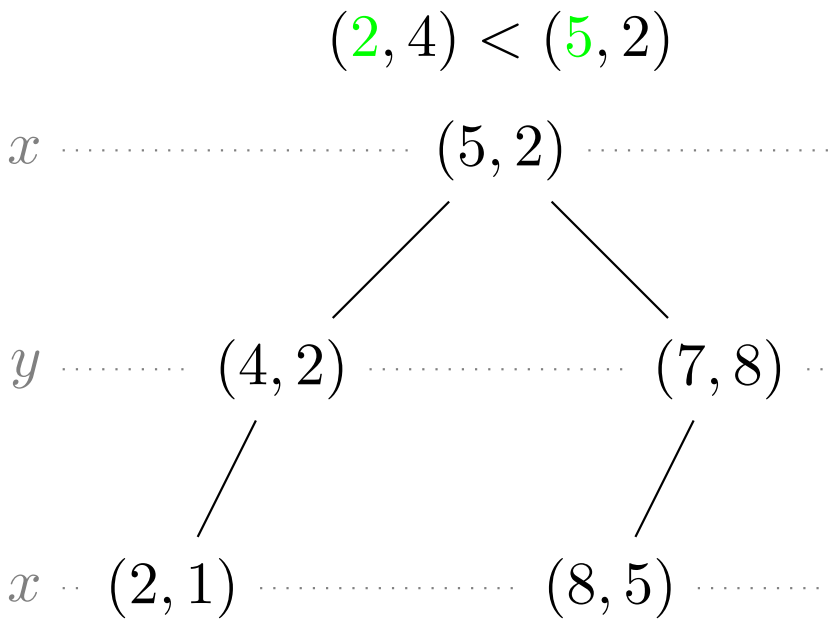
Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)

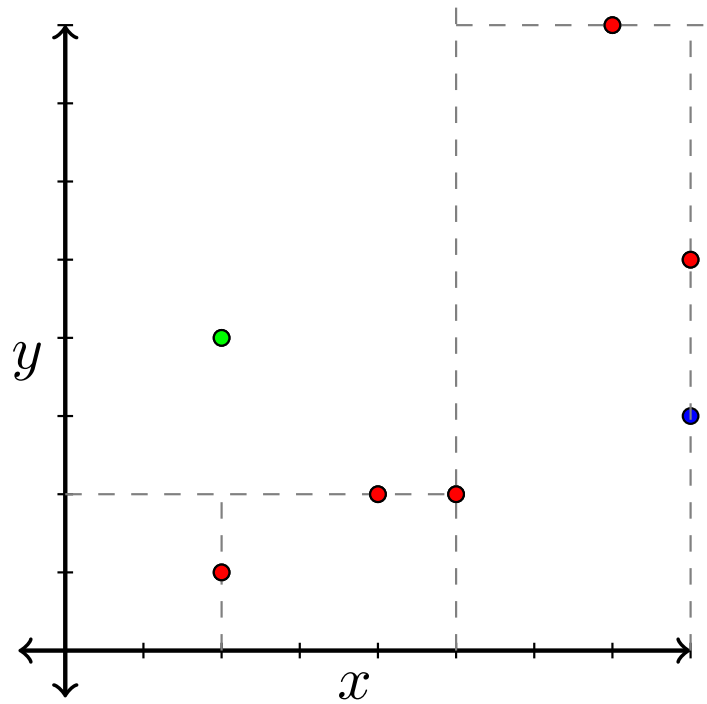




K-Dimensional Trees

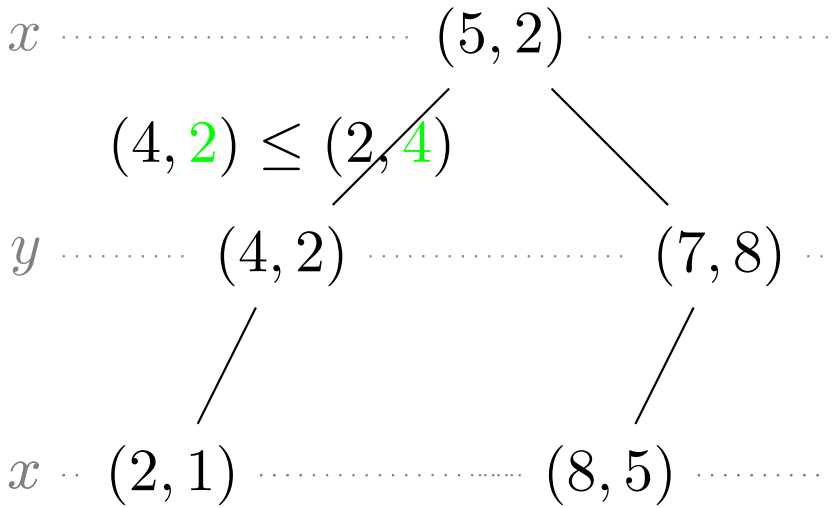
Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)

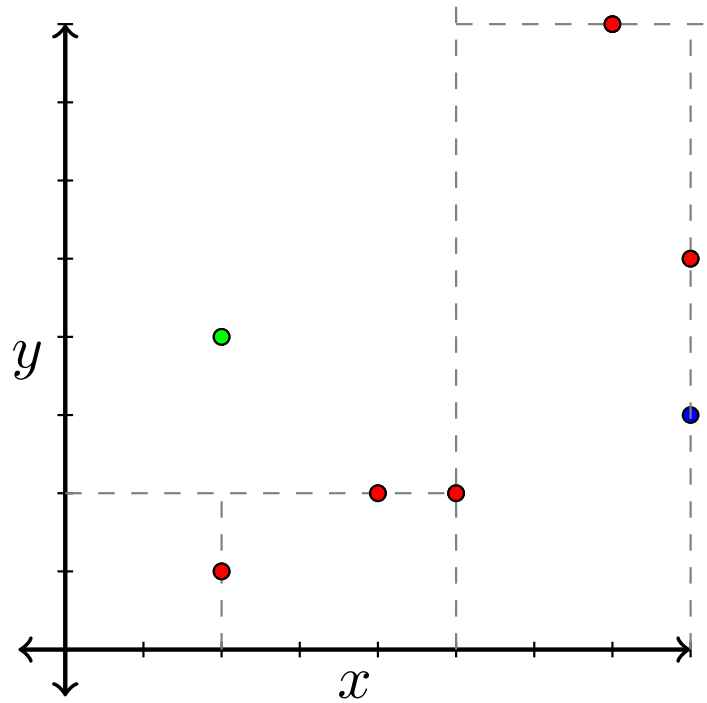




K-Dimensional Trees

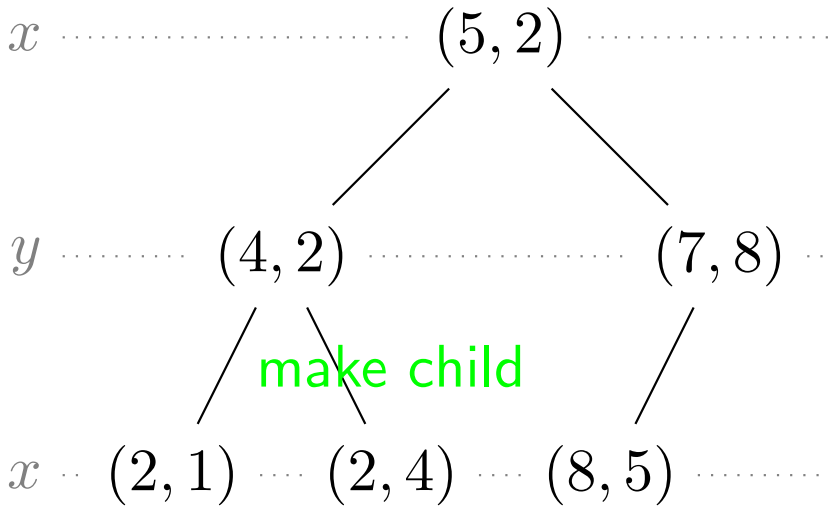
Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)

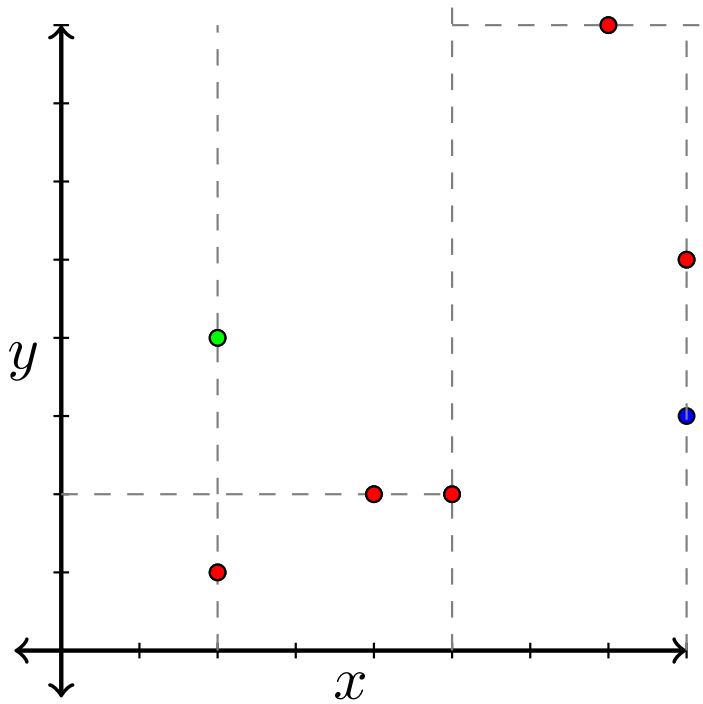




K-Dimensional Trees

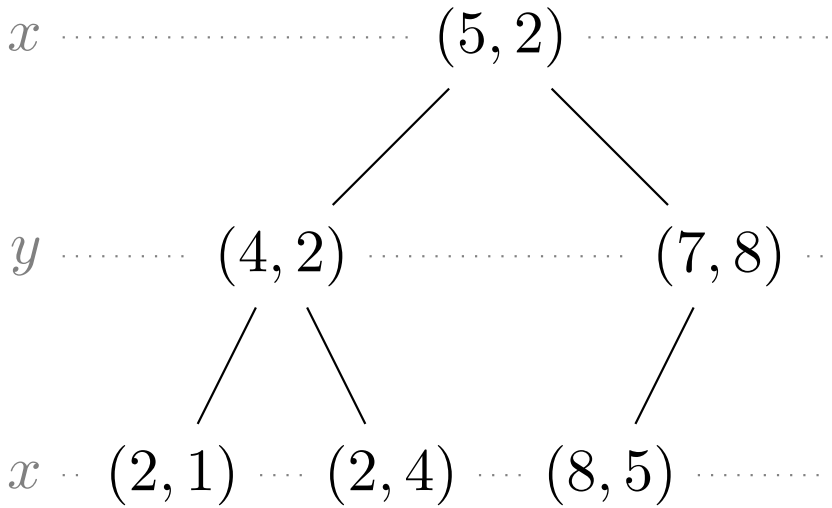
Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)

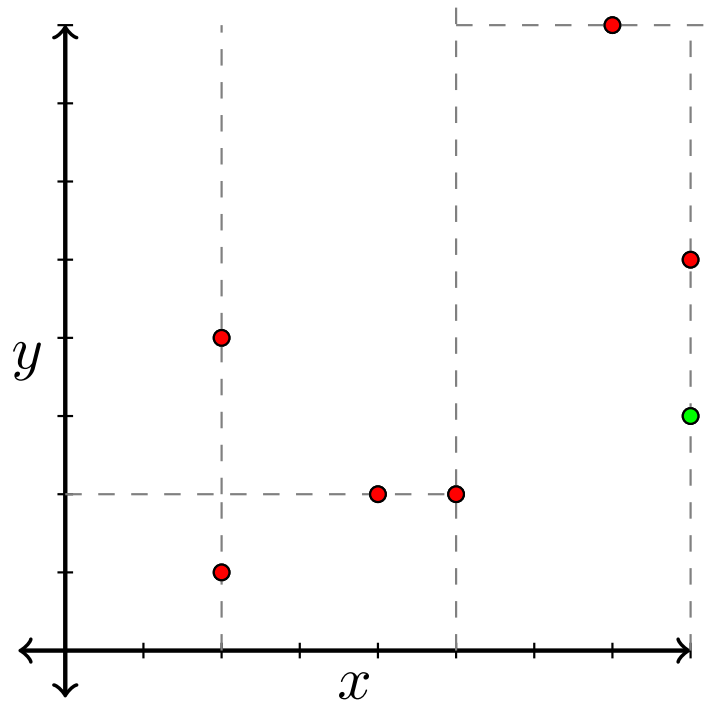




K-Dimensional Trees

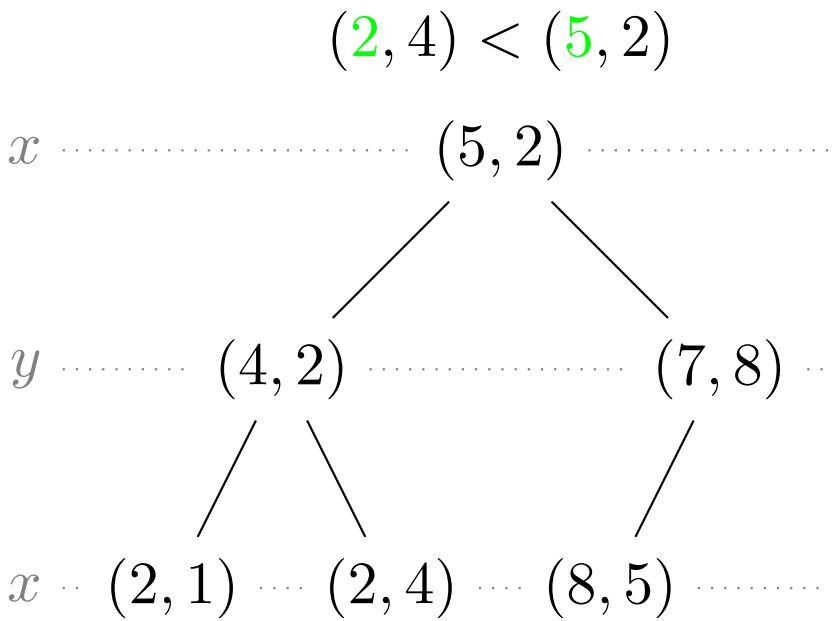
Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)

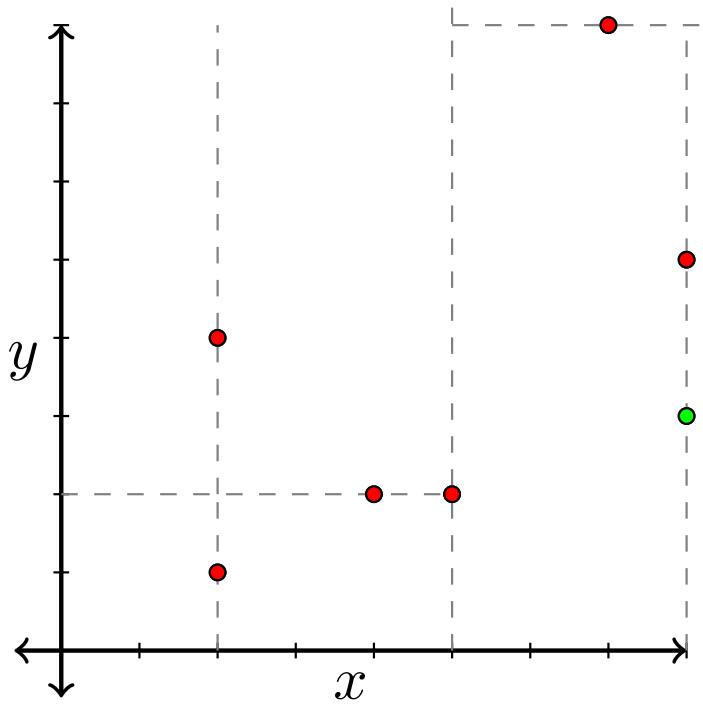




K-Dimensional Trees

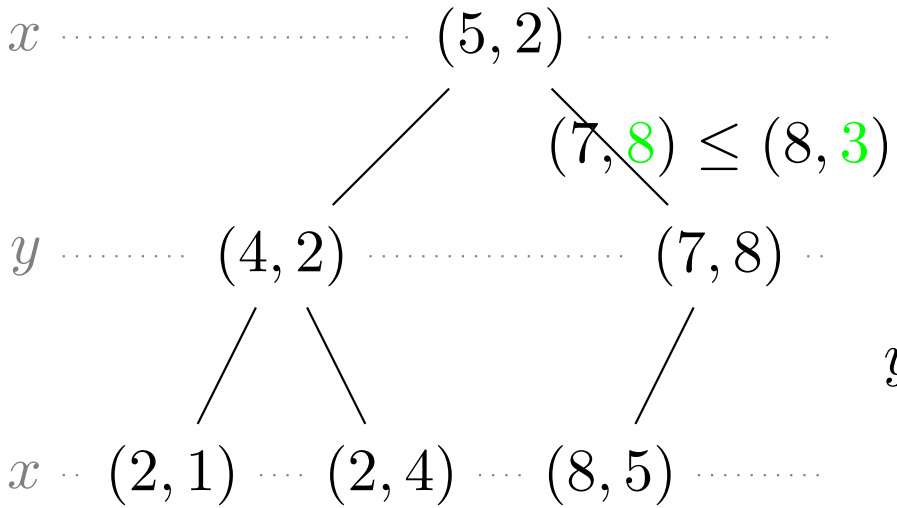
Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)

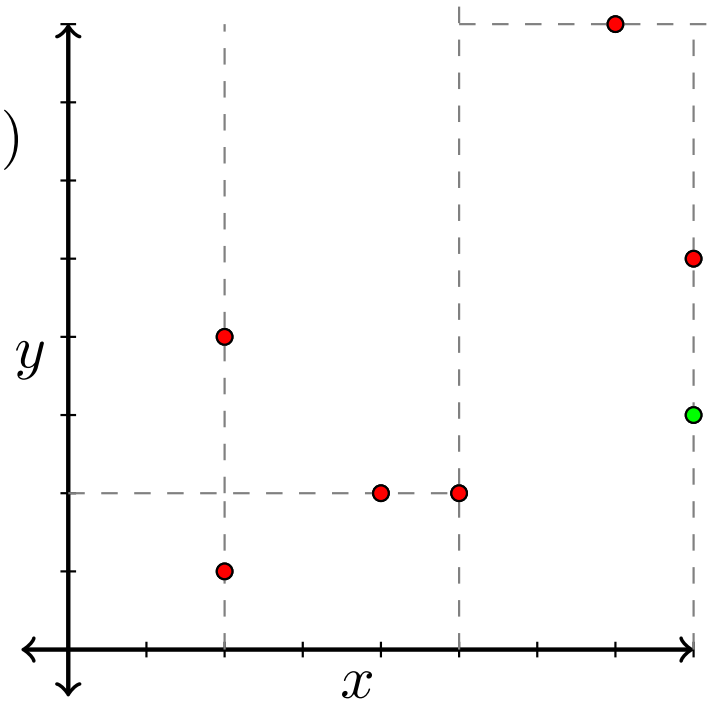




K-Dimensional Trees

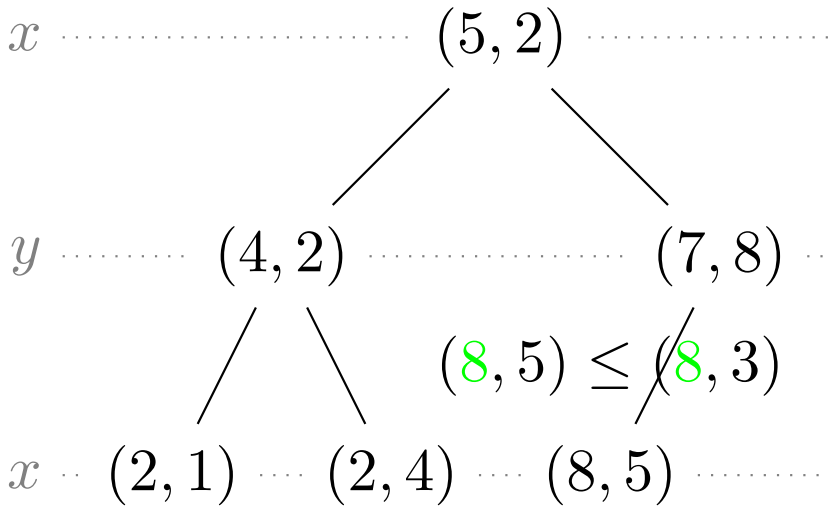
Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)

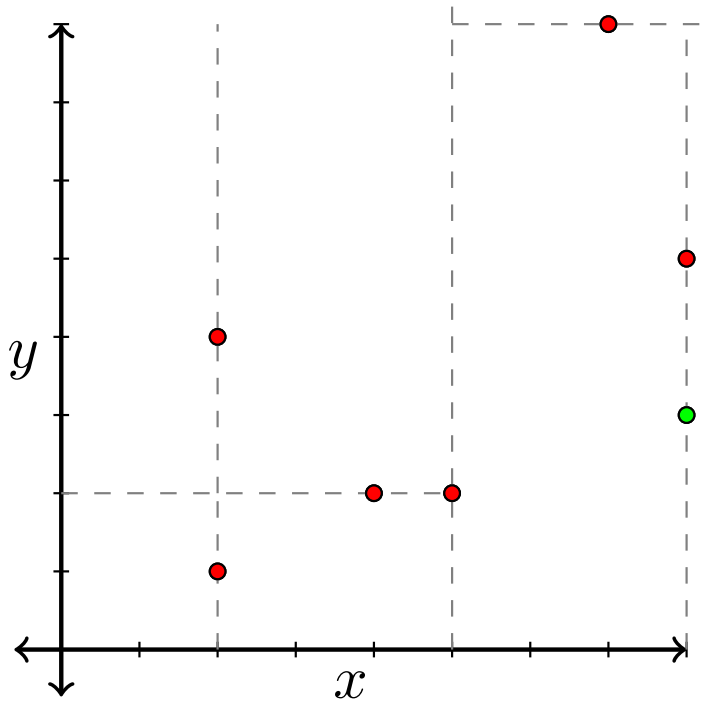




K-Dimensional Trees

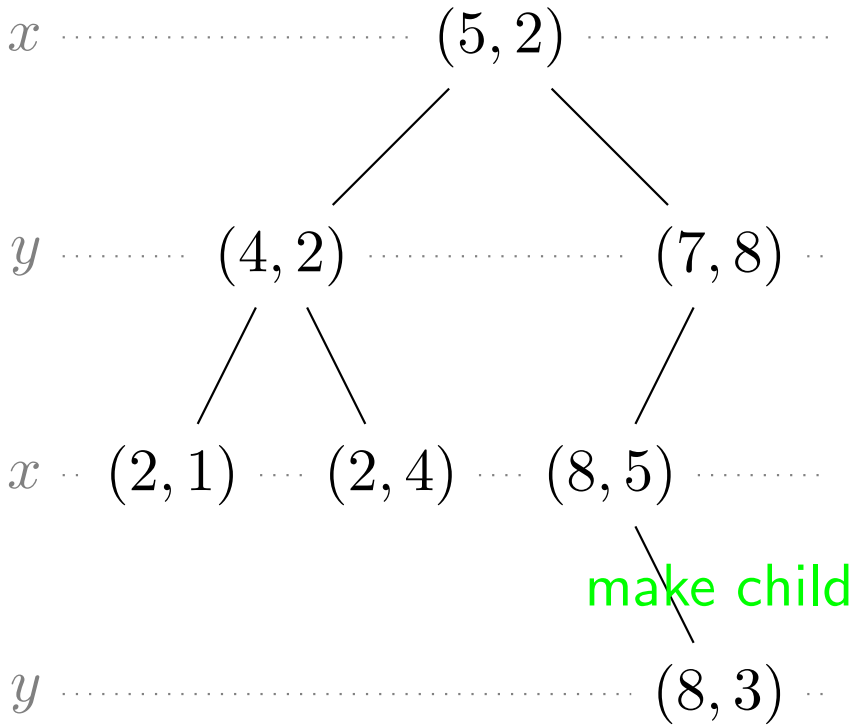
Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)

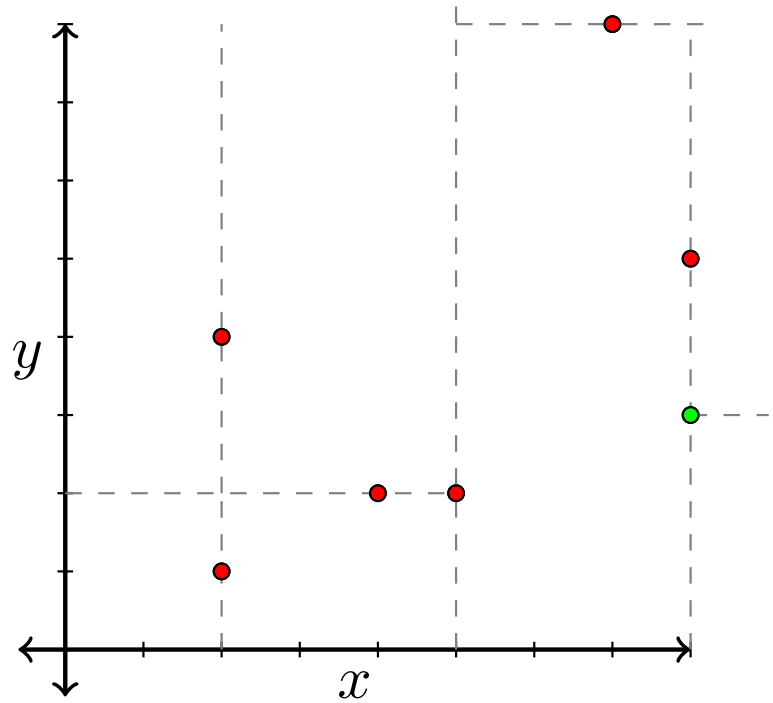




K-Dimensional Trees

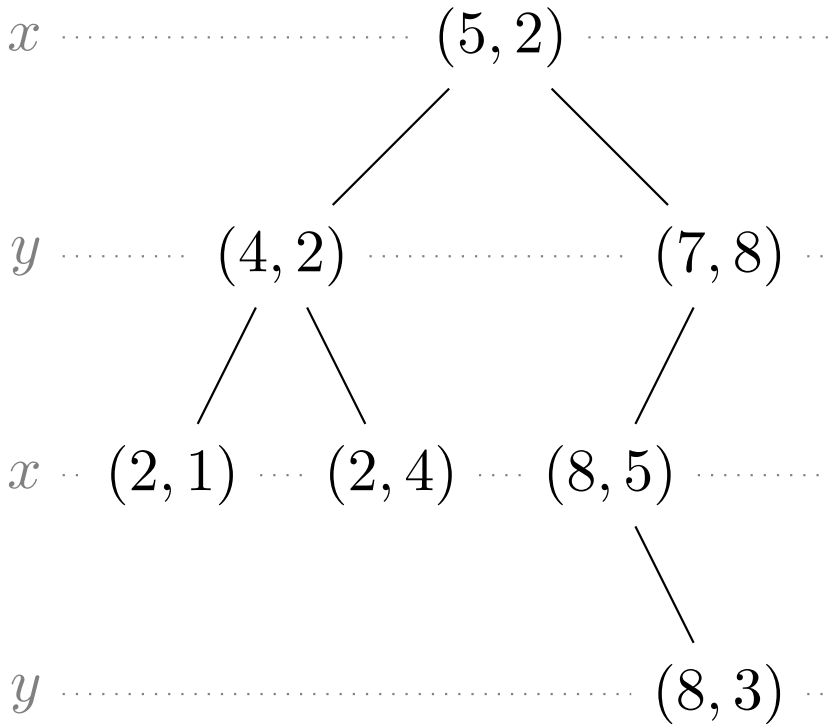
Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)

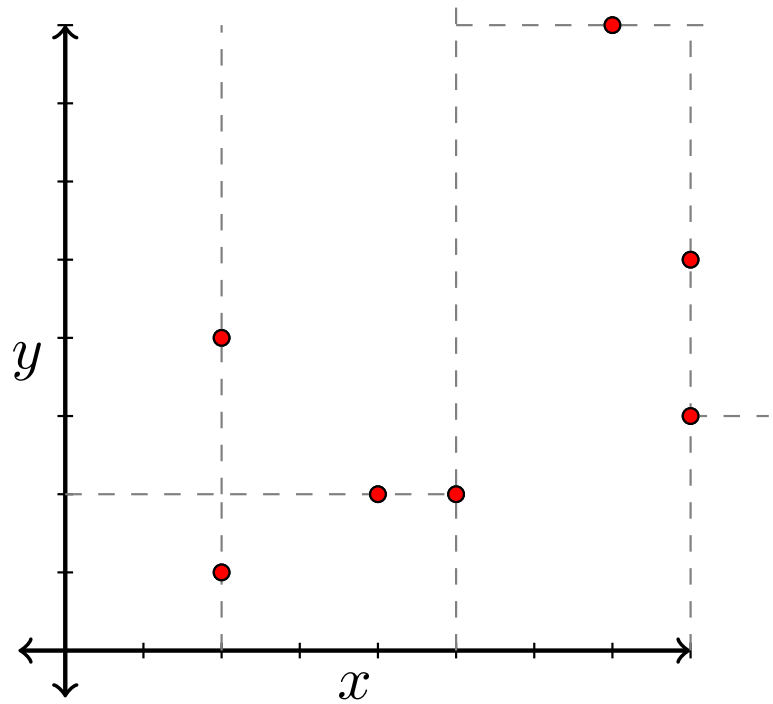




K-Dimensional Trees

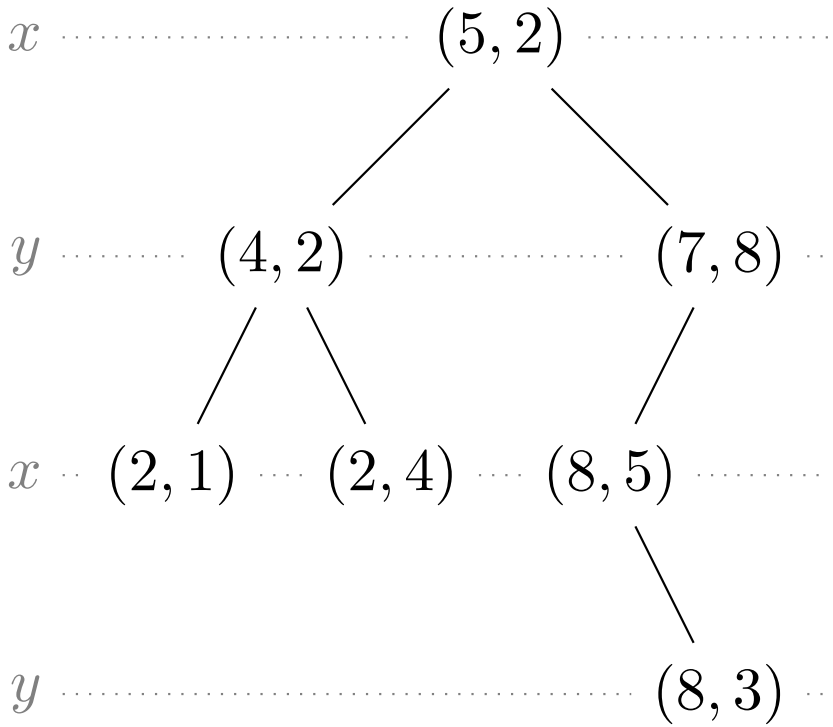
Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)

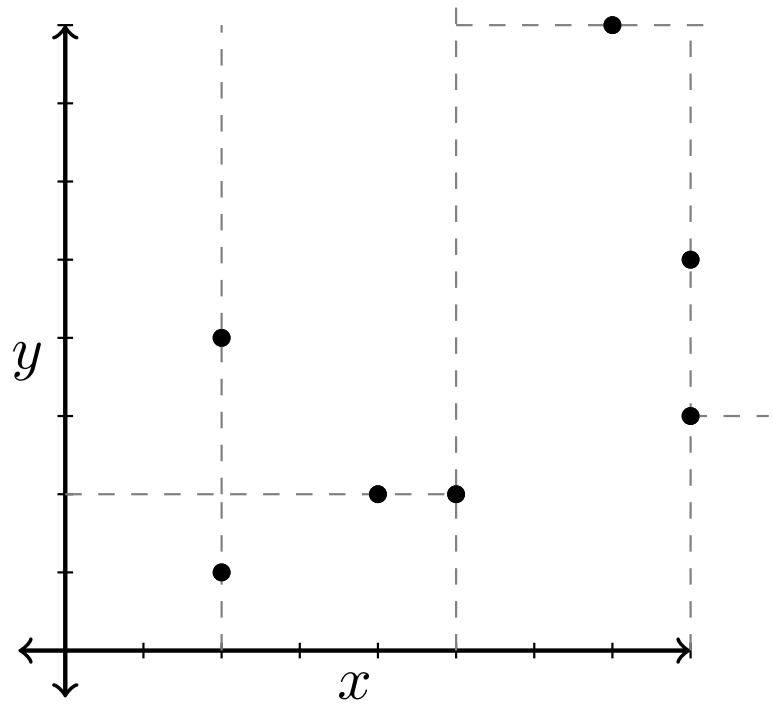




K-Dimensional Trees: Nearest Neighbor

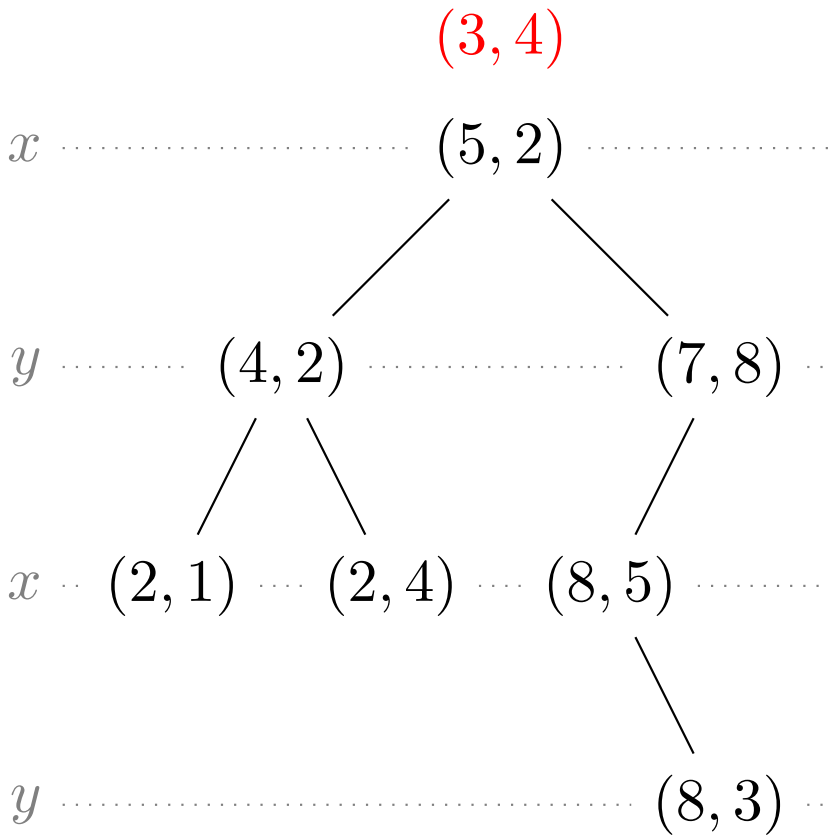
Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)

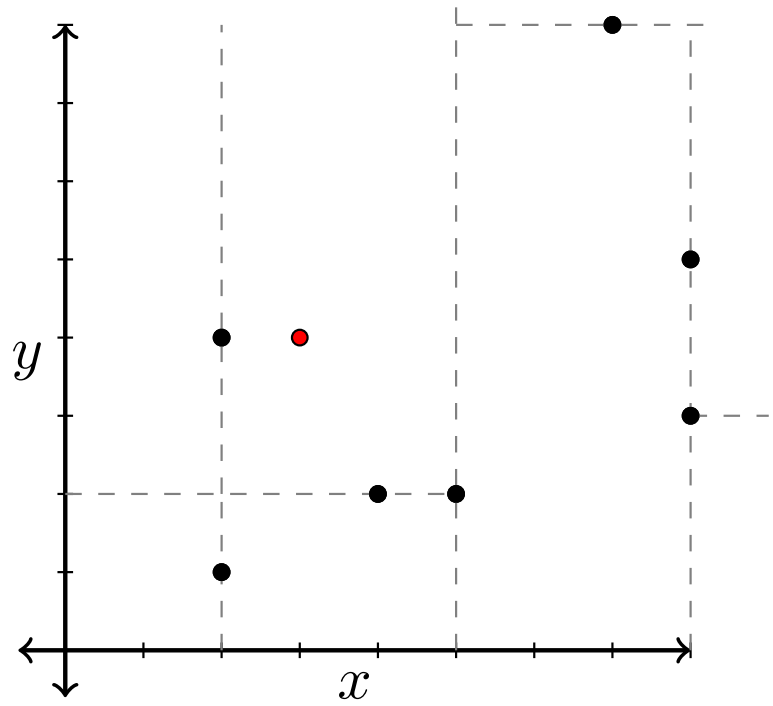




K-Dimensional Trees: Nearest Neighbor

Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)

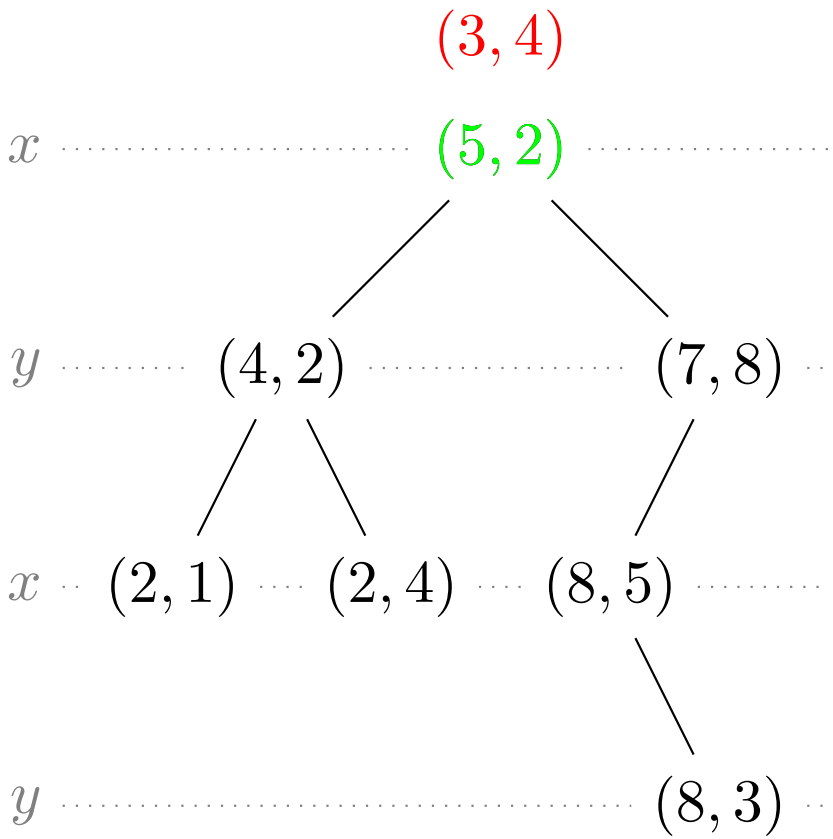


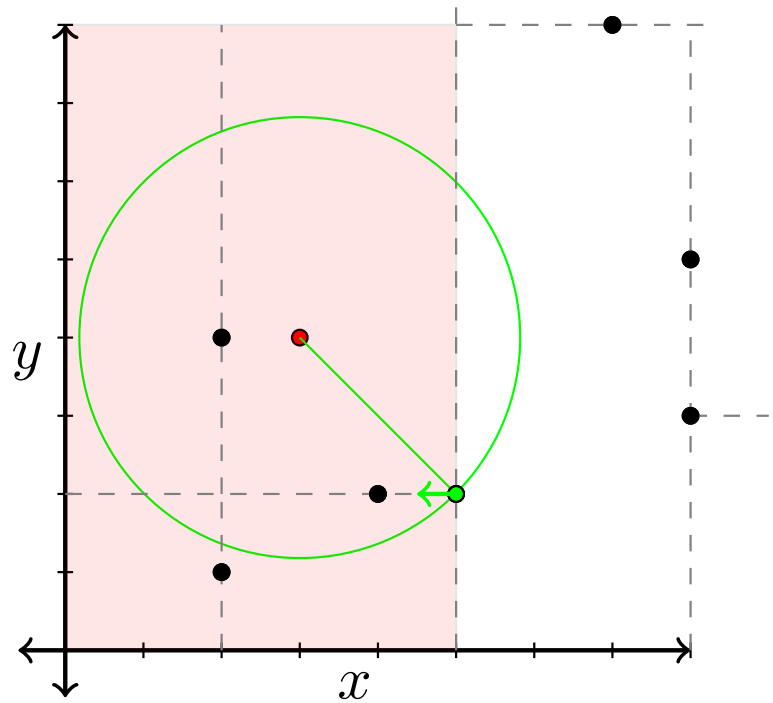


Find nearest neighbor to point $(3, 4)$

K-Dimensional Trees: Nearest Neighbor

Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)

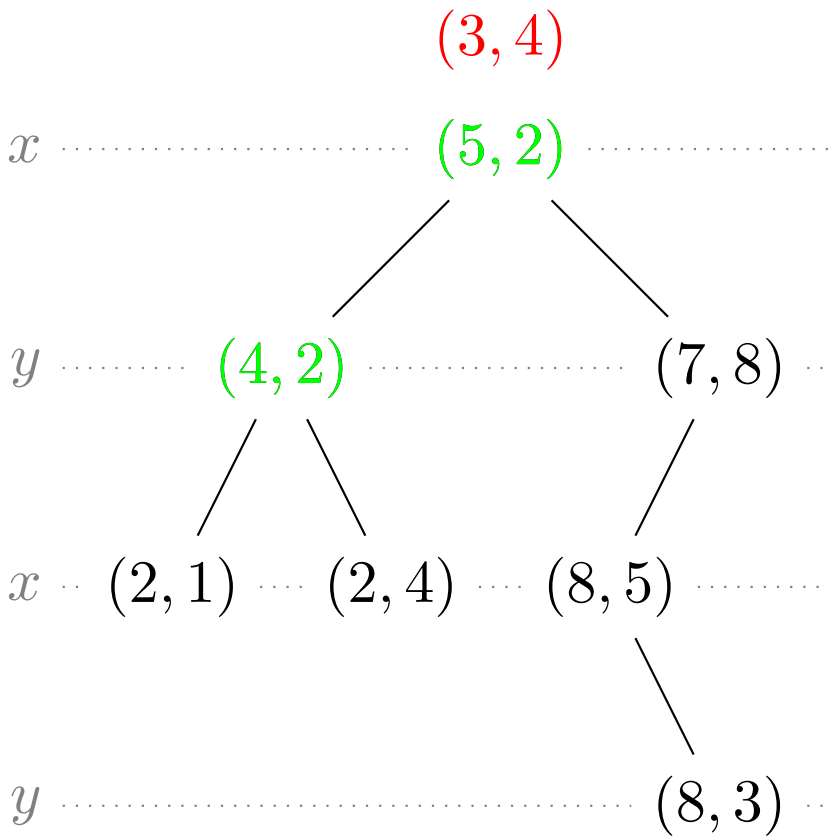


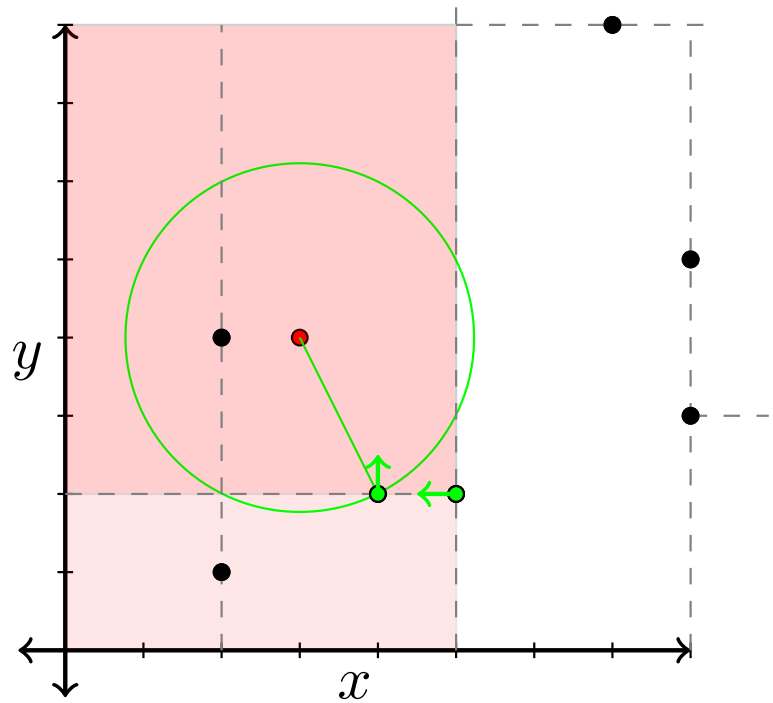


Save minimum distance to (5, 4)

K-Dimensional Trees: Nearest Neighbor

Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)

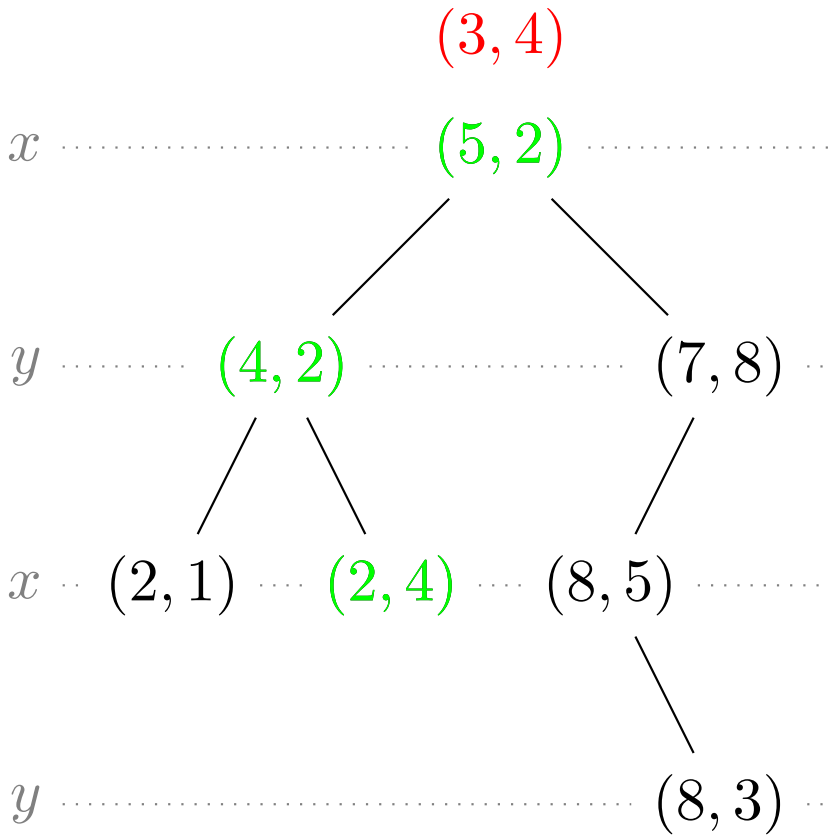


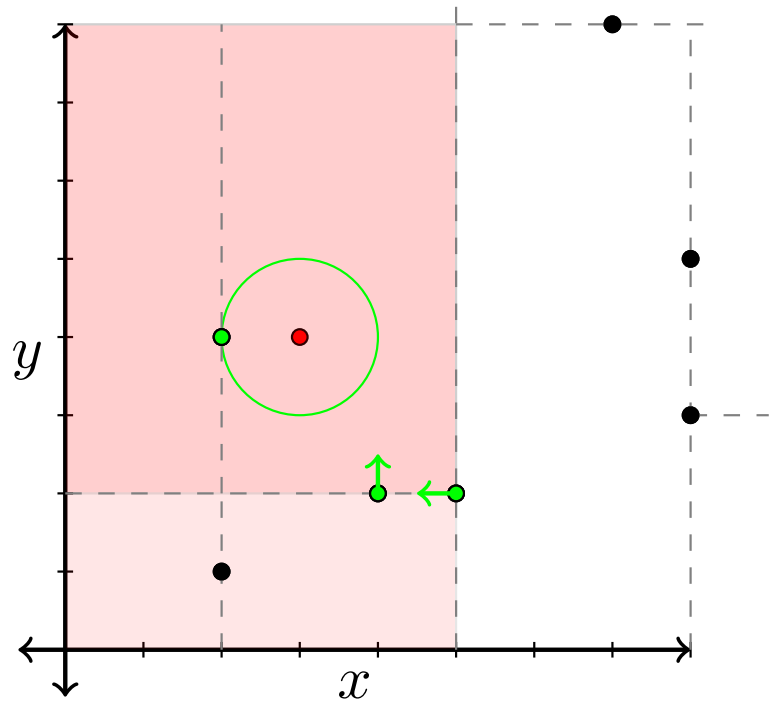


Save minimum distance to (4, 2)

K-Dimensional Trees: Nearest Neighbor

Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)

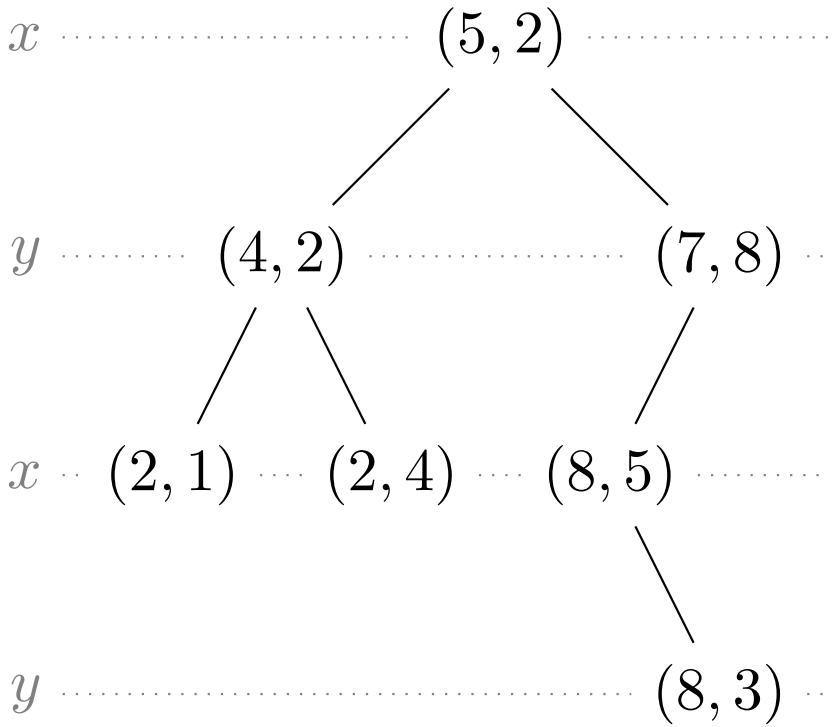


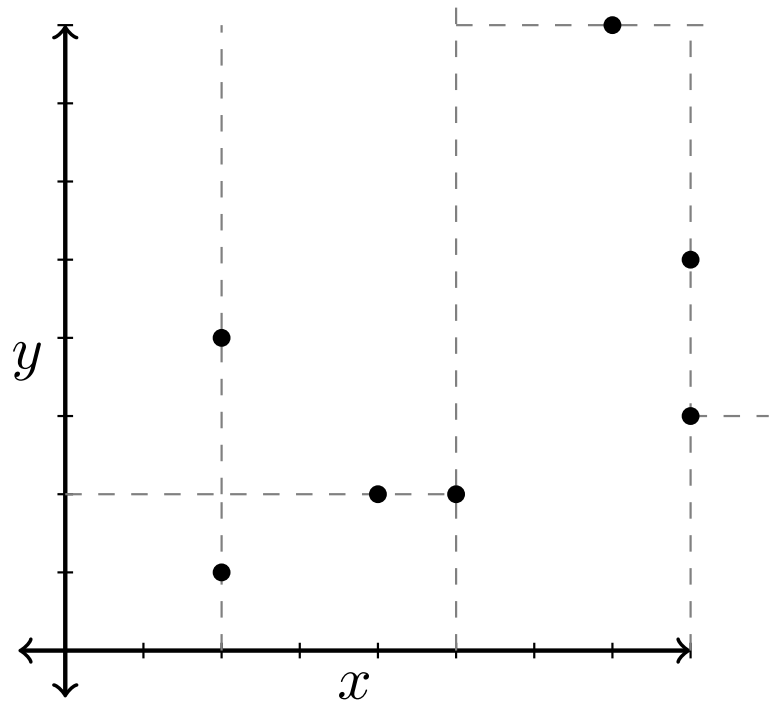


Save minimum distance to (2, 4) Is last leaf $(2, 4)$ always the nearest neighbor?

K-Dimensional Trees: Nearest Neighbor

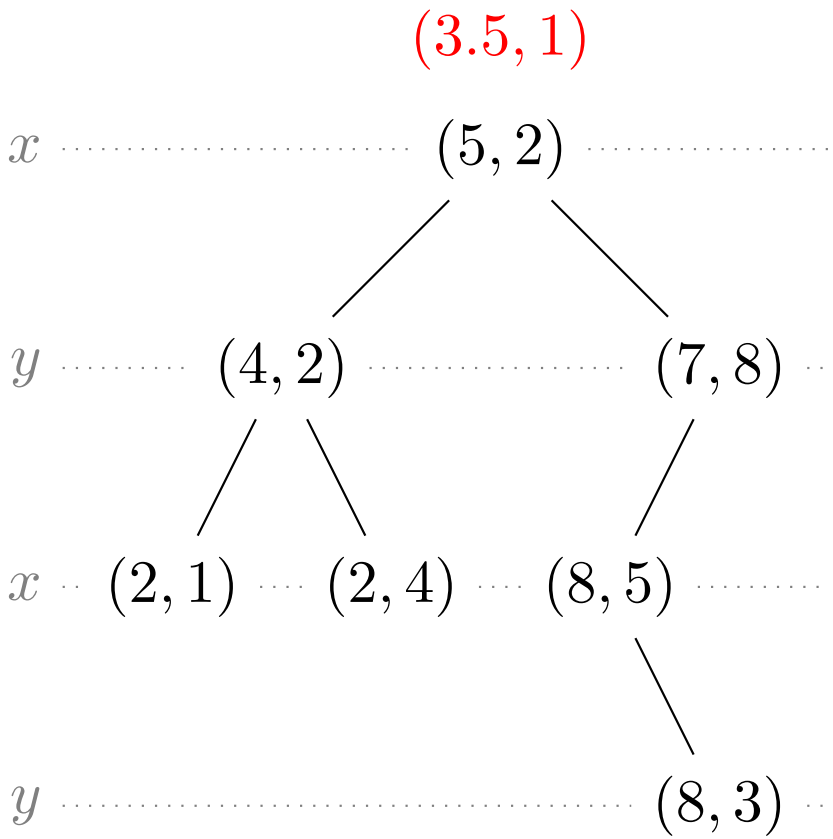
Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)

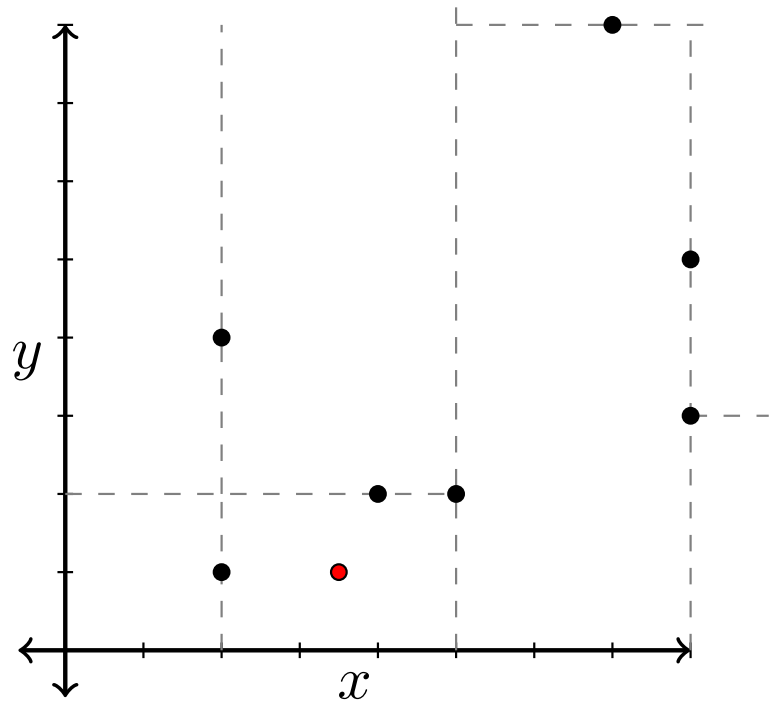




K-Dimensional Trees: Nearest Neighbor

Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)

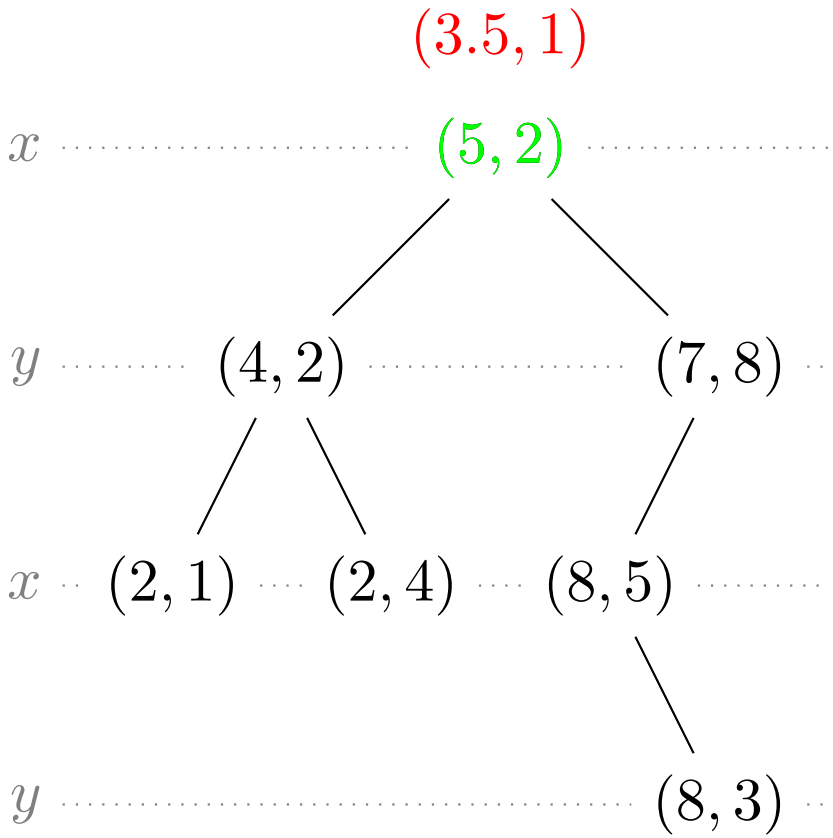


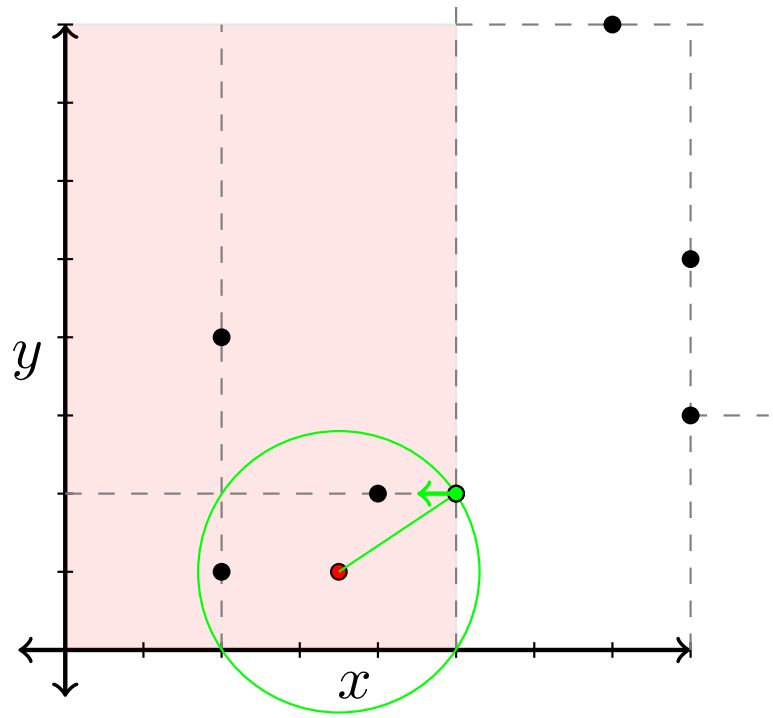


Find nearest neighbor to (3.5, 1)

K-Dimensional Trees: Nearest Neighbor

Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)

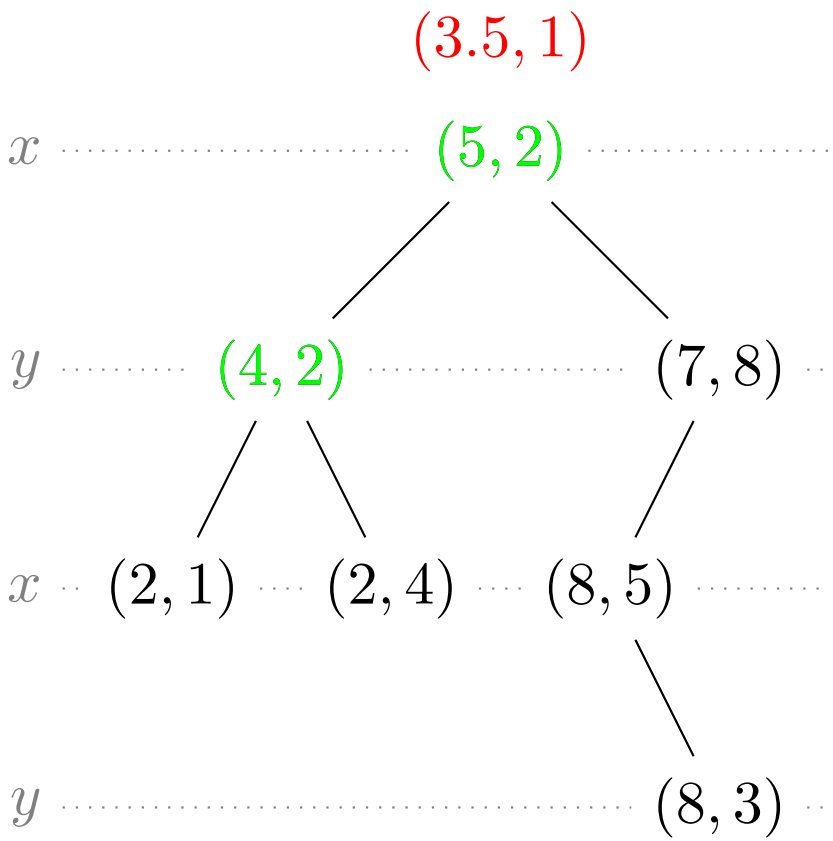


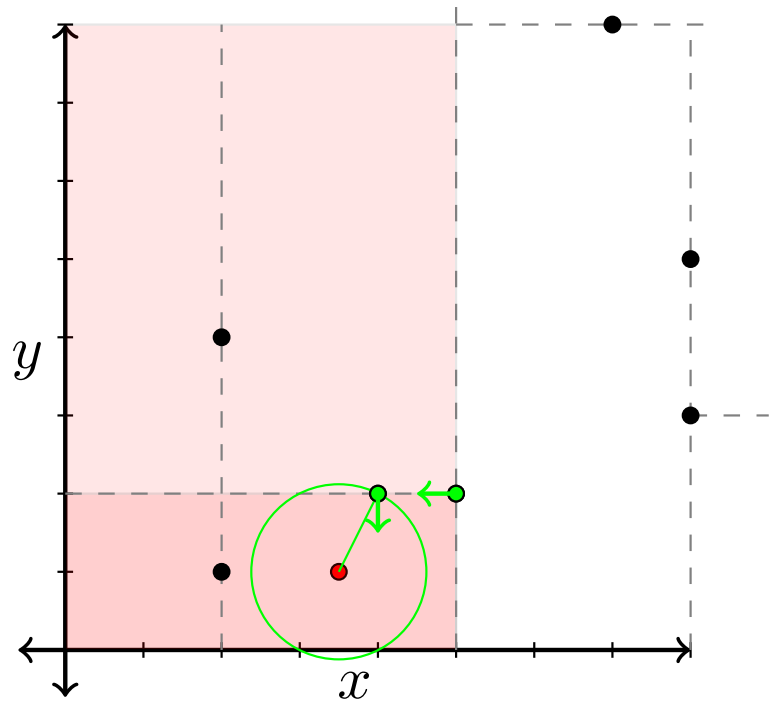


Set minimum distance; cannot prune

K-Dimensional Trees: Nearest Neighbor

Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)

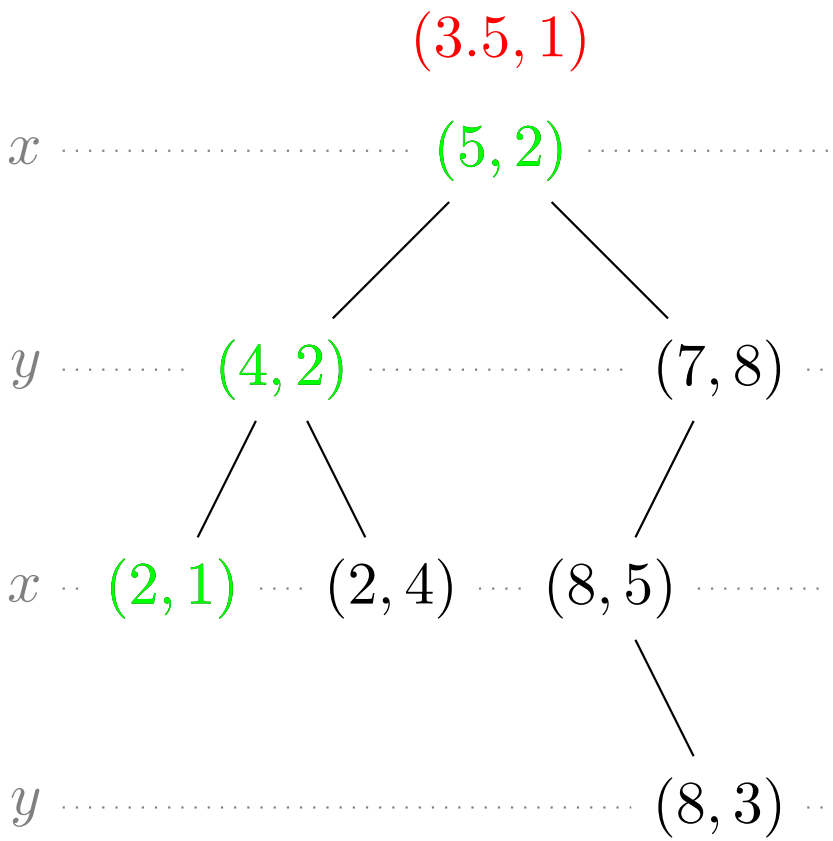


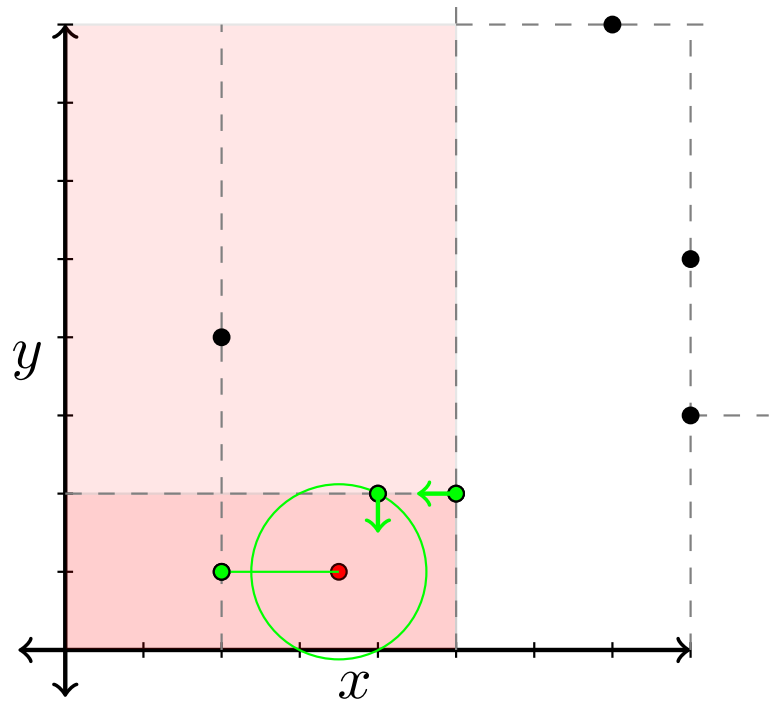


Set minimum distance: prune right subtree

K-Dimensional Trees: Nearest Neighbor

Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)

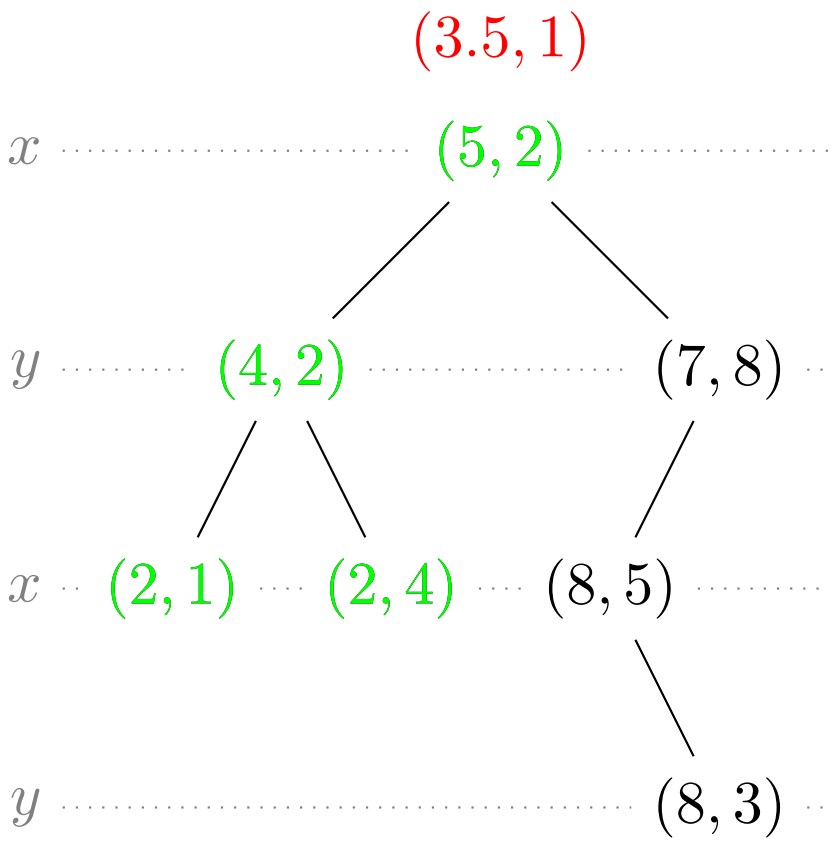


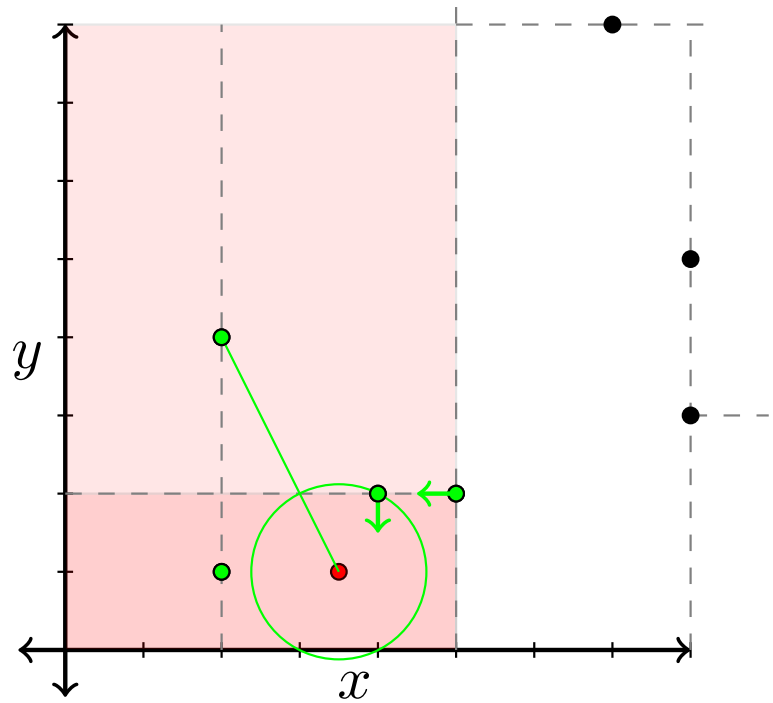


Do not change minimum distance

K-Dimensional Trees: Nearest Neighbor

Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3)

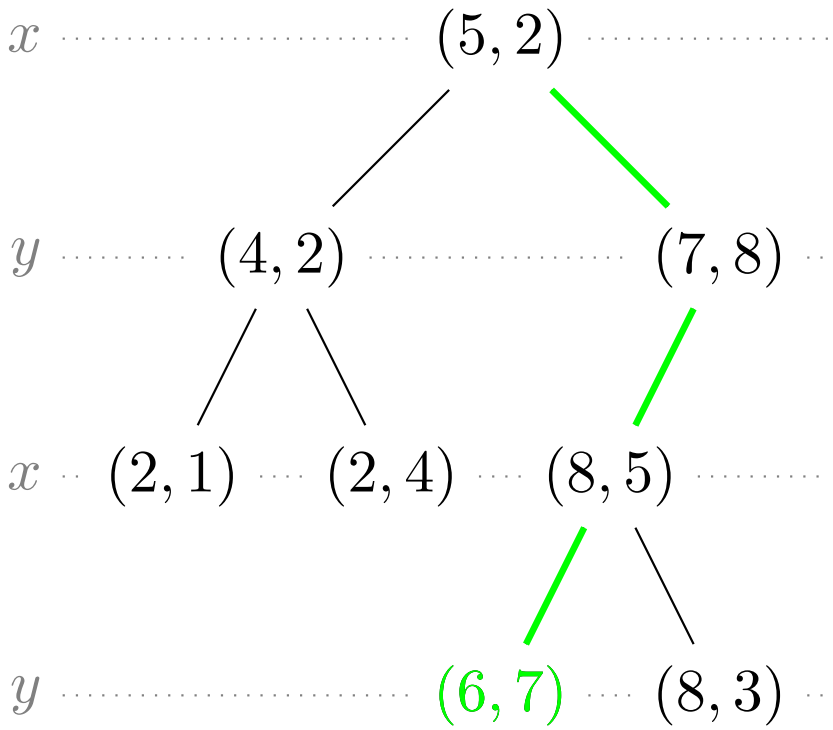


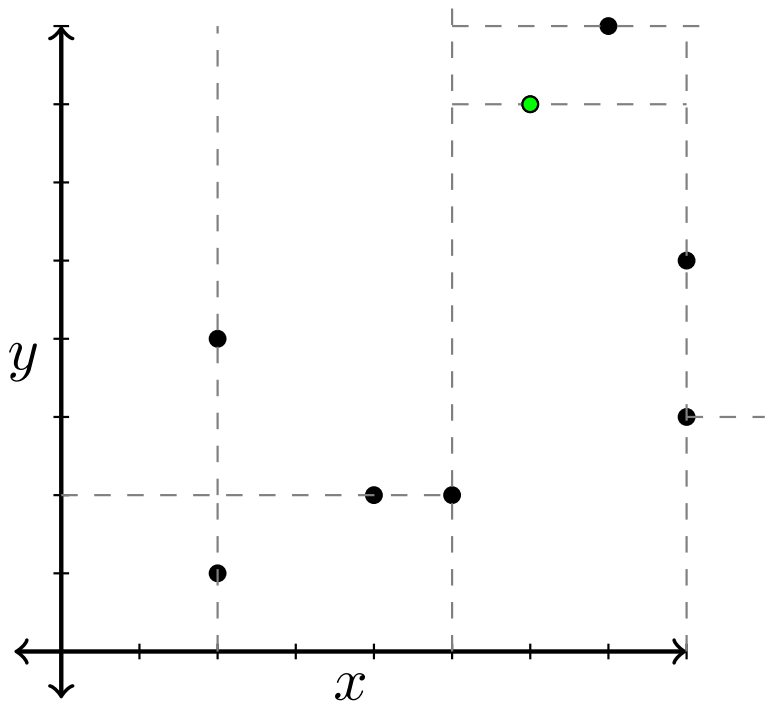


Do not change minimum distance

K-Dimensional Trees: Nearest Neighbor

Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3), (6, 7)

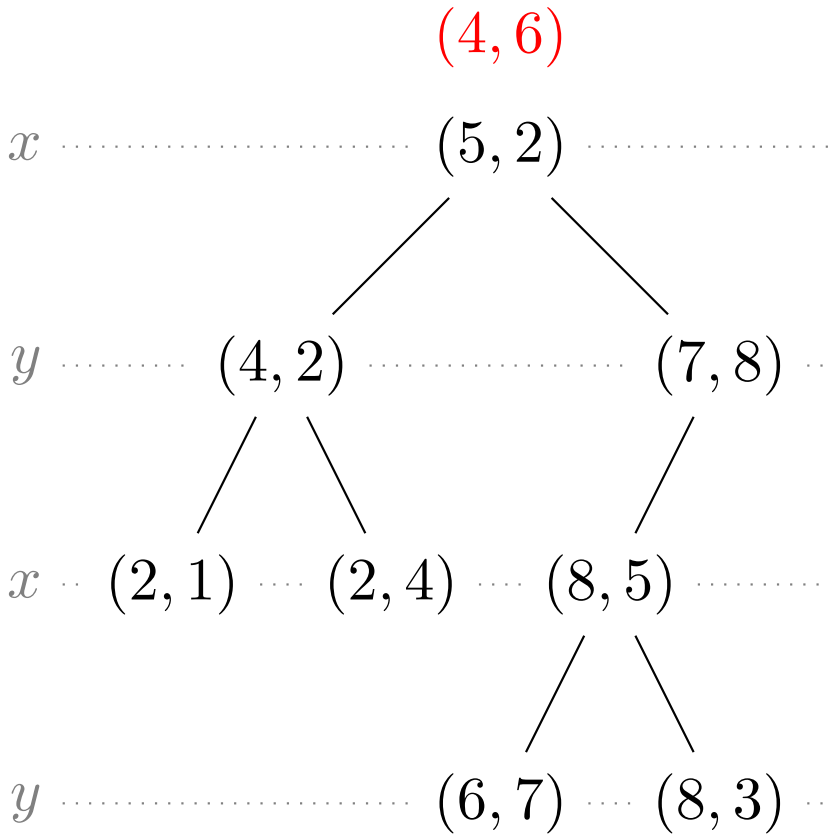


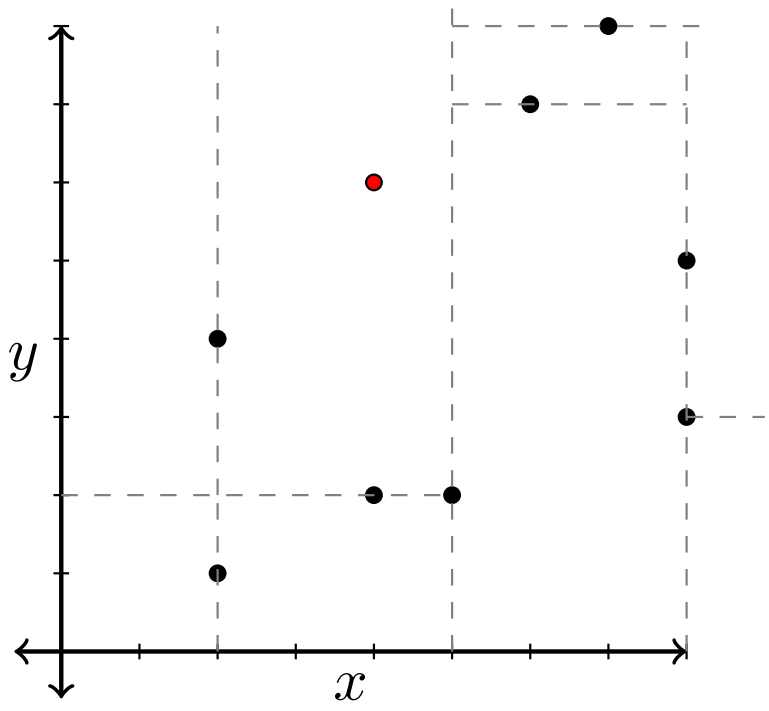


Consider new point (6, 7)

K-Dimensional Trees: Nearest Neighbor

Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3), (6, 7)

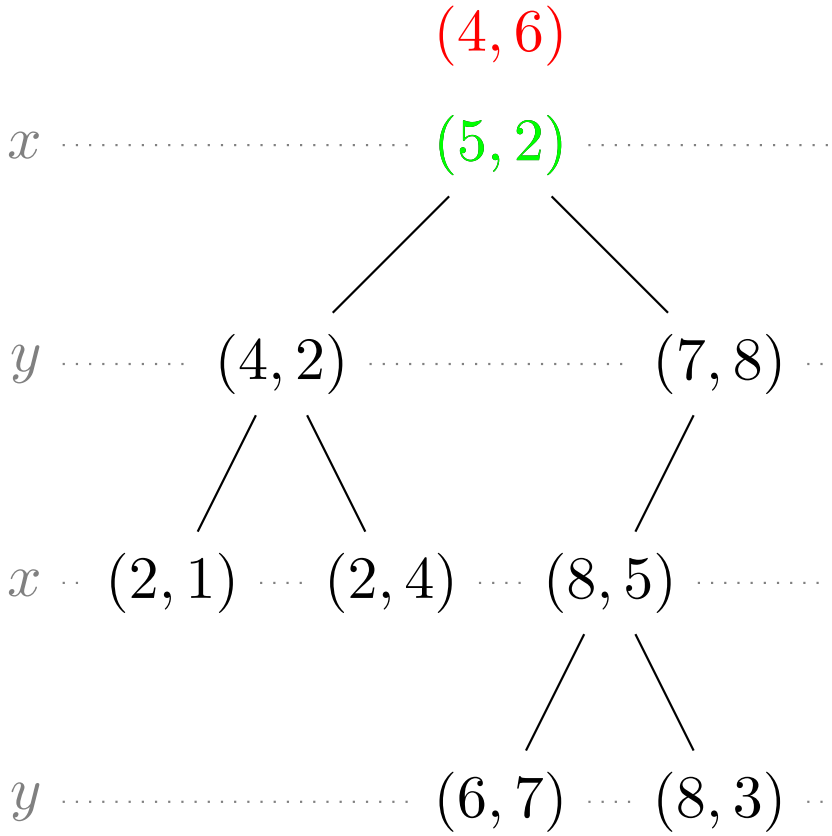


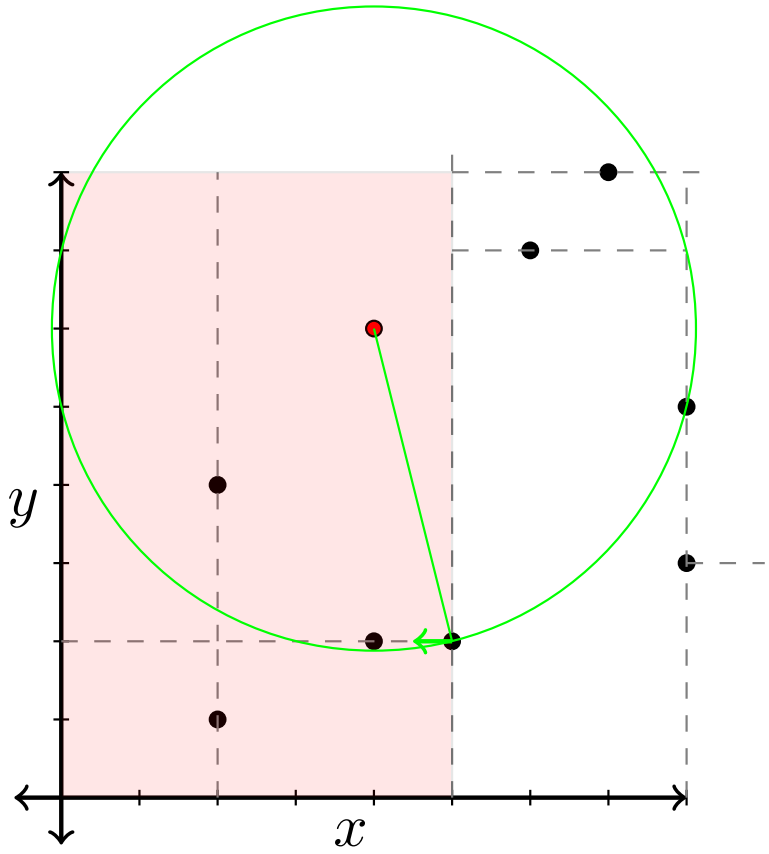


Find nearest neighbor to (4, 6)

K-Dimensional Trees: Nearest Neighbor

Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3), (6, 7)

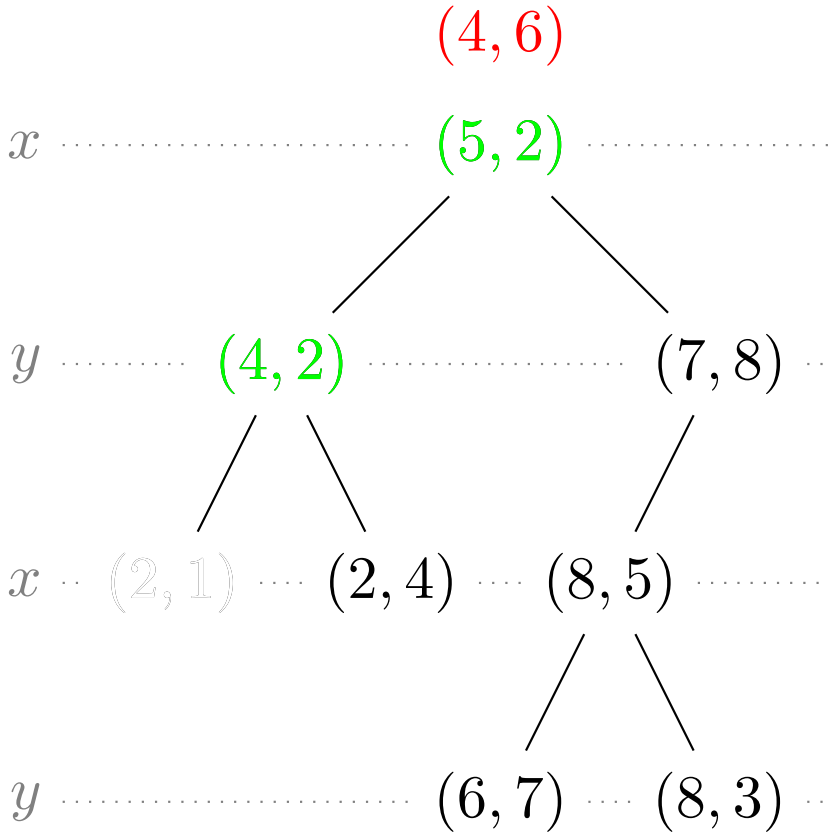


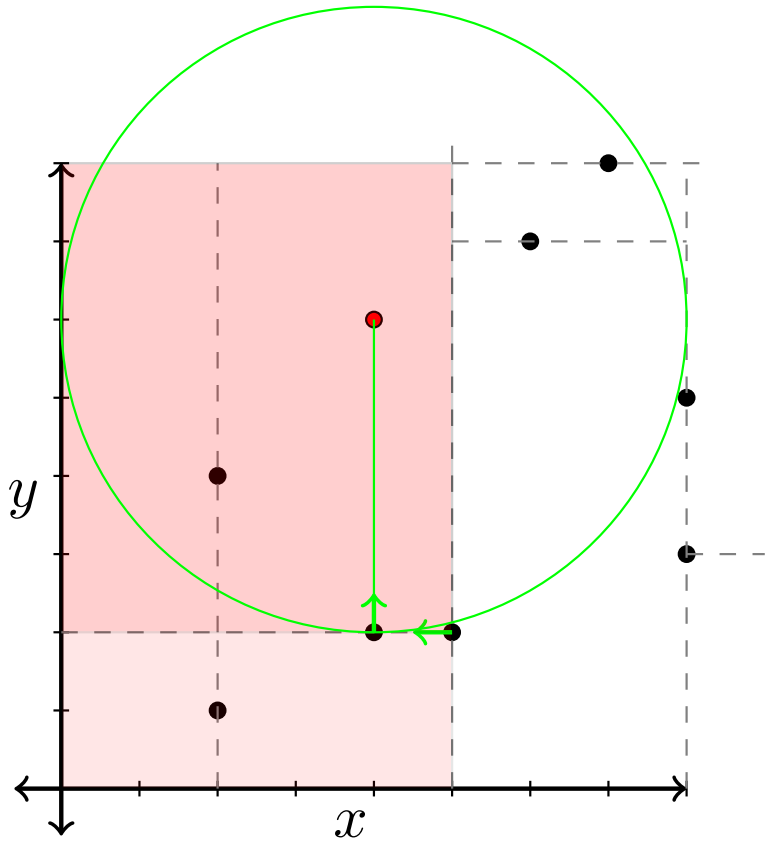


Set minimum distance: cannot prune

K-Dimensional Trees: Nearest Neighbor

Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3), (6, 7)

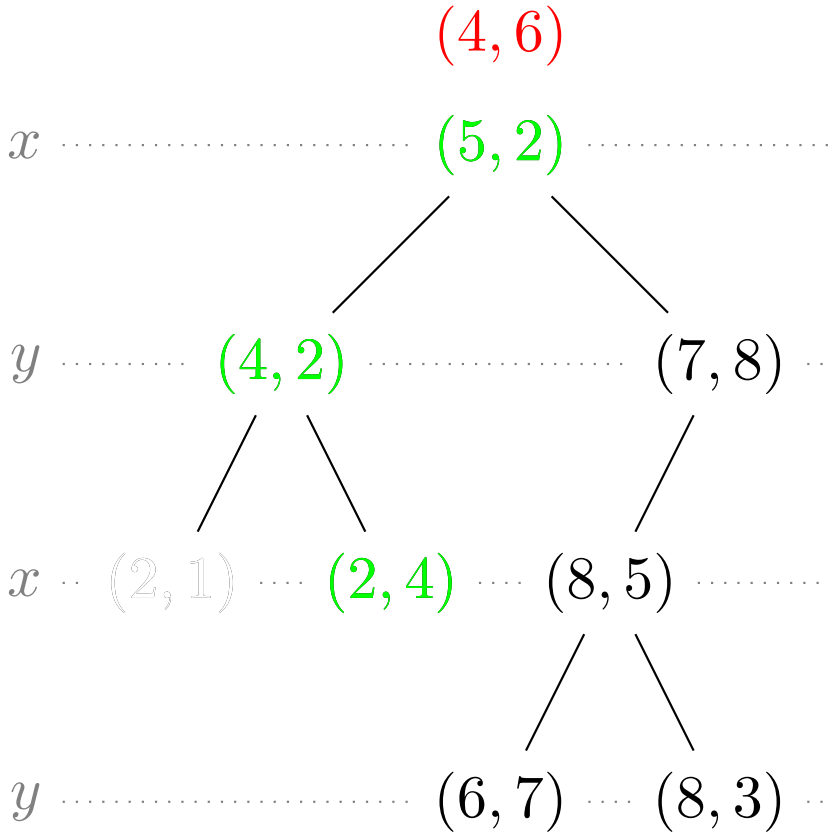


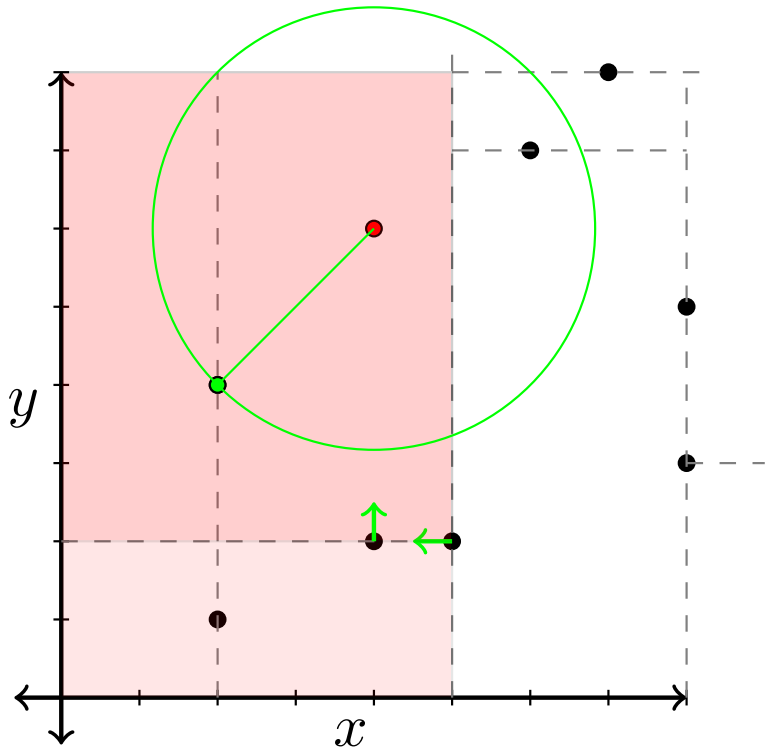


Set new minimum distance: prune

K-Dimensional Trees: Nearest Neighbor

Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3), (6, 7)

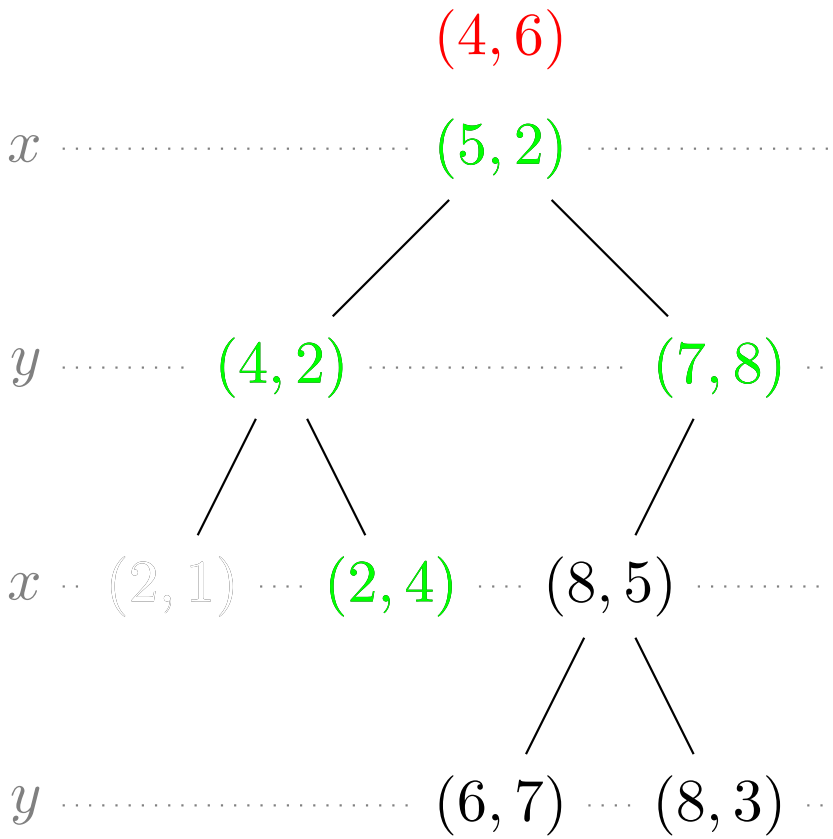


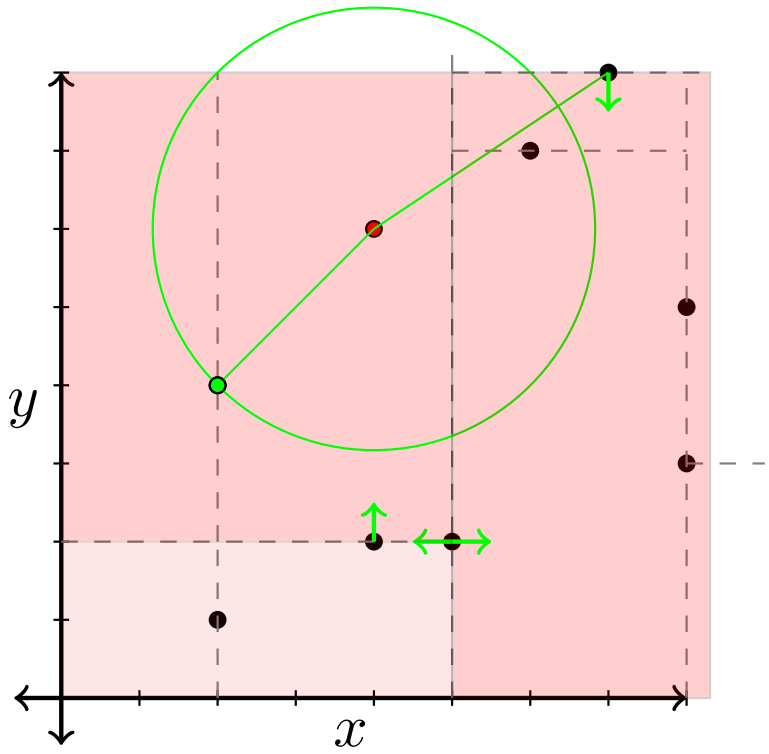


Set new minimum distance: cannot prune

K-Dimensional Trees: Nearest Neighbor

Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3), (6, 7)

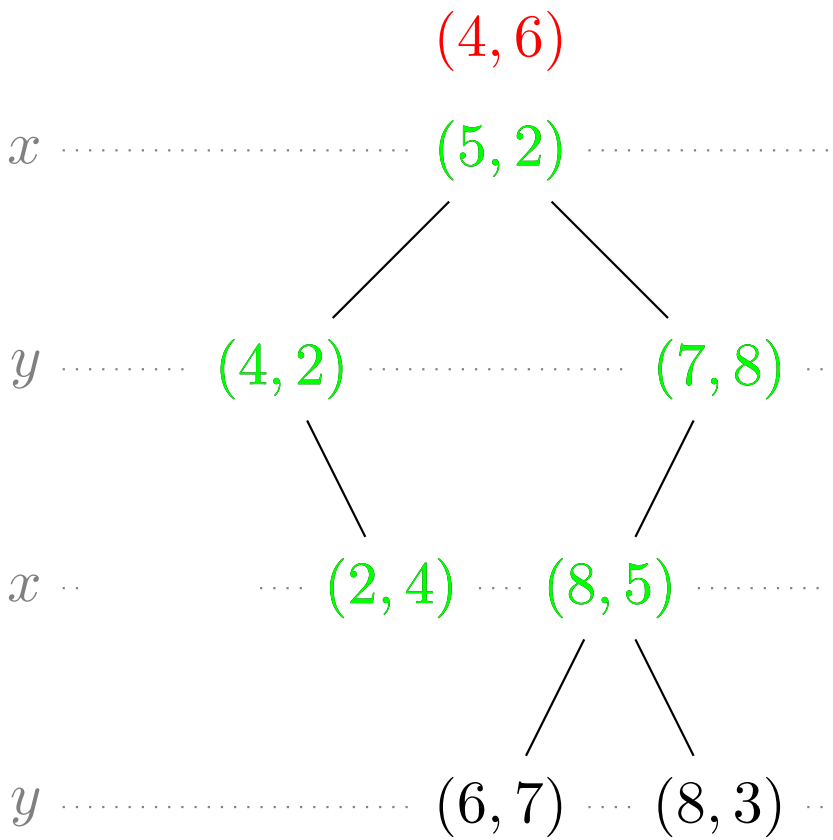


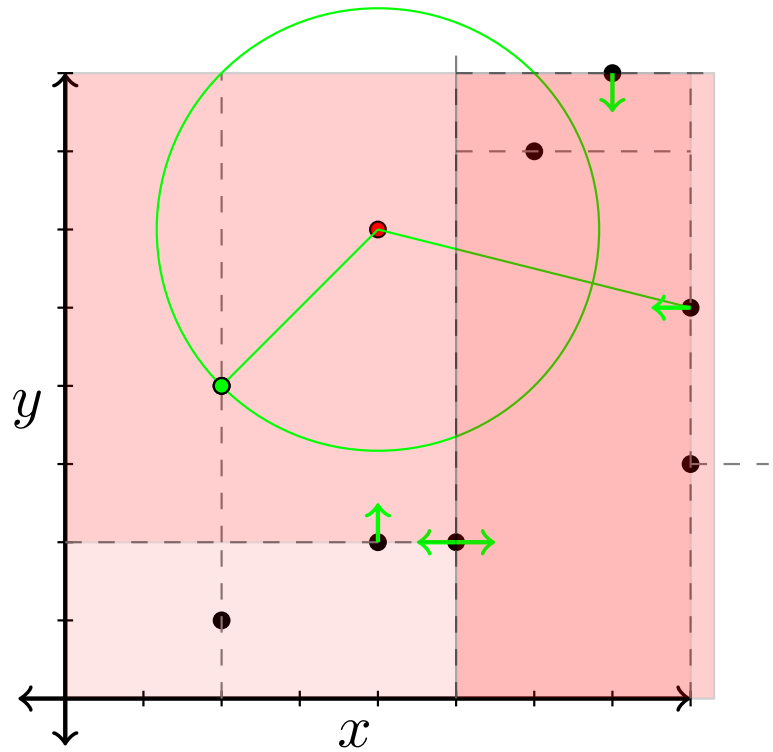


Traverse other side of tree

K-Dimensional Trees: Nearest Neighbor

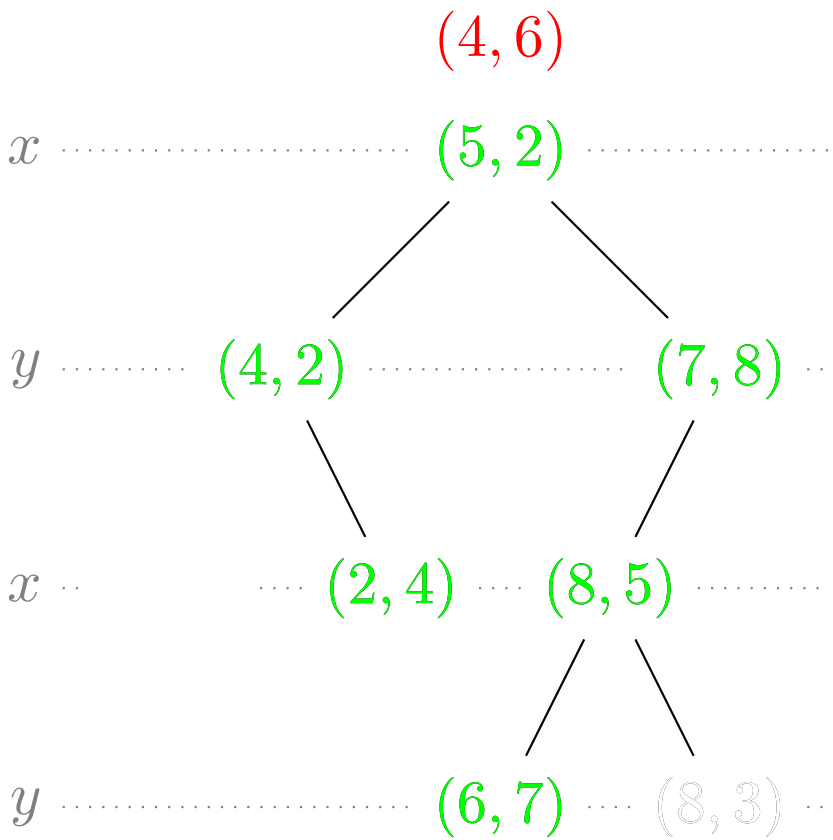
Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3), (6, 7)

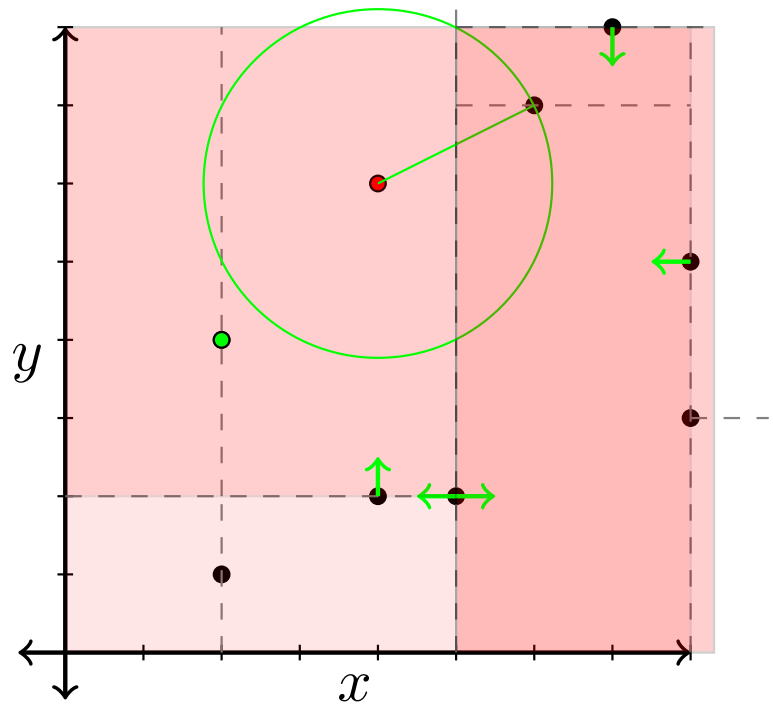




K-Dimensional Trees: Nearest Neighbor

Data: (5, 2), (7, 8), (8, 5), (4, 2), (2, 1), (2, 4), (8, 3), (6, 7)





Conclusion

- arrays are very powerful, but have limitations
- must think about what is important to our task to determine which data structure is optimal
- data structures are diverse and there are many ways to organize data to achieve our goals