FINAL REVIEW: CH5-8

Q 1. In a sociology graduate's senior thesis, she conducts a study on the number of hours per day JHU students spent on studying. Data is collected from the Freshman, Sophomore, and Junior classes and is listed below.

Freshman (F)	Sophomore (S)	Junior (J)
3.0	4.5	6.0
2.5	3.5	4.5
3.5	4.0	6.0
3.5	4.5	8.5
3.5	4.5	6.5
4.0	4.5	6.0
		6.5

Test the hypothesis that the means for all three groups are the same. Use $\alpha = 0.05$.

Q 2. We want to compare the number of seeds in strawberries and watermelons. Assume the populations are normally distributed. A sample of 13 strawberries had a sample variance of 64. A sample of 16 watermelons had a sample variance of 16. Test the null hypothesis at $\alpha = 0.10$ and find the full rejection region.

$$H_0 : \sigma_s^2 = \sigma_w^2$$
$$H_A : \sigma_s^2 \neq \sigma_w^2$$

Q 3. A study found that among a sample of 180 banana lovers, 43 of them ($\hat{p}_b = 0.2389$) reported to be avid collectors. Among 175 interviewed pineapple lovers, 28 of them ($\hat{p}_p = 0.16$) were avid collectors. Test the following at $\alpha = 0.05$. Find the p-value.

$$H_0 : p_b \leq p_p$$

$$H_A : p_b > p_p$$

Q 4. A study measured the diameter of strawberries. Assume population is normally distributed. A sample of 30 strawberries had a sample variance of 0.09. Test the null hypothesis at $\alpha = 0.05$.

$$H_0 : \sigma_0^2 \geq 0.14$$

$$H_A : \sigma_a^2 < 0.14$$

Q 5. A soda company states that the sugar content in their bottles is normally distributed. A sample of 36 soda bottles are tested for sugar content. It is found that this sample has an average of $\bar{x} = 69.2$ grams of sugar per bottle. Test the null hypothesis regarding the sugar contained in all soda bottles of this kind:

$$H_0 : \mu = 68$$

Versus the alternative hypothesis:

$$H_a : \mu \neq 68$$

It is known that the population standard deviation, $\sigma = 3.6$ grams. Use $\alpha = 0.01$, and find the p-value.

Q 6. Assume that the life expectancy are normally distributed in countries A and B. We do not know the variance of life length, but we know that they are equal in these nations. Suppose that 15 death certificates are obtained in Country A have $\bar{x}_A = 64$ years with $s_A = 3$, while 12 certificates in Country B have $\bar{x}_B = 68$ years and $s_B = 2.5$. Test the null hypothesis regarding the difference in means:

$$H_0 : \mu_A - \mu_B = 0$$

Versus the alternative hypothesis:

$$H_a : \mu_A - \mu_B \neq 0$$

Use $\alpha = 0.02$.

Q 7. The response of a new diabetic drug was measured for 20 patients at Johns Hopkins Hospital (group 1) and 15 control subjects (group 2). The sample variance for group 1 is $s_1^2 = 100$, and the sample variance for group 2 is $s_2^2 = 120$. Construct the 95 % confidence interval for the population variances: $\frac{\sigma_1^2}{\sigma_2^2}$.

Q 8. Given a p-value of 0.001:

(a) Find the range of α such that the null hypothesis can be rejected.

(b) Find the range of α such that the null hypothesis cannot be rejected.

Q 9. Consider a population quality of unknown distribution but with known $E(X) = \mu$ and $(X) = \sigma^2$. If we take a sample of size $n = 100$, is it true that:

$$\frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0, 1)?$$

Answer yes or no. If no, either give the correct distribution or state that it cannot be determined.

Q 10. Consider a population quality of unknown distribution but with known $E(X) = \mu$ and $(X) = \sigma^2$. If we take a sample of size $n = 10$, is it true that:

$$\frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0, 1)?$$

Answer yes or no. If no, either give the correct distribution or state that it cannot be determined.

Q 11. Now consider two different populations. We take one sample from each of sizes $n_1 = 16$ and $n_2 = 25$ to measure their heights in feet, and determine that the sample means $\bar{x}_1 = 6$ and $\bar{x}_2 = 5.5$, and the sample variances $s_1^2 = s_2^2 = 0.25$. Assume the population variances are equal.

(a) Compute the 95 % confidence interval for the difference of population mean heights.

(b) Compute the 90 % confidence interval for the difference of population mean heights.

Q 12. Let X be a binomial distribution with $n = 20$ and $p = 0.25$. What is the probability that $X \geq 8$? Find the answer using a normal approximation with a continuity correction.

Q 13. Suppose 30% of people in a population own dogs. Scientists drew a sample of size 50 from this population. Let $\hat{p}$ be the proportion of the sample who are dog owners. (You do not need to take into account continuity correction.)

(a) Find the mean of $\hat{p}$.

(b) Find the standard error of $\hat{p}$, i.e find $\sigma_{\hat{p}}$.

(c) What is the probability that $\hat{p} \geq 0.4$?