FINAL REVIEW SOL.

Q 1. **Solution:** Let $x_{i,j}$ be the ith observation resulting from the jth group. Let the population means of Freshman, Sophomore, Junior classes' studying hours be μ_1, μ_2, and μ_3. $n_1 = 6$, $n_2 = 6$, and $n_3 = 7$.

- Hypothesis
  $H_0 : \mu_1 = \mu_2 = \mu_3$
  H_A : At least one population mean is different.

- Test Statistics
  Test statistics: $V.R = \frac{MSA}{MSW}$

- Rejection Region
  $V.R > F_{1-\alpha,2,16} = 3.63$

- Calculate the statistics

$$SSW = \sum_{j=1}^{3}\sum_{i=1}^{n_j}(x_{ij} - \bar{x}_{.j})^2 = 10.6369$$

$$SSA = \sum_{j=1}^{3} n_j(\bar{x}_{.j} - \bar{x}_{..})^2 = 30.0210$$

$$MSA = \frac{SSA}{k-1} = \frac{30.0210}{2} = 15.0105$$

$$MSW = \frac{SSW}{N-k} = \frac{10.6369}{16} = 0.6648$$

$$V.R = \frac{15.0105}{0.6648} = 22.5787$$

- Make Decision
  Our computed $V.R = 22.5787$ is greater than $F_{1-\alpha,2,16} = 3.63$, so we reject H_0.

Q 2. **Solution:** This is a two-tailed hypothesis test.

$$V.R. = \frac{S_s^2}{S_w^2} = \frac{64}{16} = 4.$$

The F distribution is not symmetric, so we will need to find $F_{1-\frac{\alpha}{2},n_1-1,n_2-1}$ and $F_{\frac{\alpha}{2},n_1-1,n_2-1}$

$$F_{1-\frac{\alpha}{2},n_1-1,n_2-1} = F_{0.95,12,15} = 2.48.$$
$$F_{\frac{\alpha}{2},n_1-1,n_2-1} = \frac{1}{F_{1-\frac{\alpha}{2},n_2-1,n1-1}} = \frac{1}{F_{0.95,15,12}} = \frac{1}{2.62} = 0.3817.$$

The rejection region is $[0, 0.3817] \cup [2.48, \infty)$. Since $V.R. = 4$ is outside the rejection region, we *fail to reject* H_0.

Q 3. **Solution:** This is a one-tailed hypothesis test.

We assume the H_0 to be true, thus we put $p_b - p_p = (p_b - p_p)_0 = 0$ in the test statistic. Moreover, we must find a common value $\bar{p}$ for the denominator

$$\bar{p} = \frac{28 + 43}{180 + 175} = 0.20.$$

$$T.S._{obs} = \frac{(\hat{p}_b - \hat{p}_p) - (p_b - p_p)_0}{\sqrt{\frac{\bar{p}(1-\bar{p})}{n_b} + \frac{\bar{p}(1-\bar{p})}{n_p}}} = \frac{(0.2389 - 0.16) - (0)}{\sqrt{\frac{0.20\times0.80}{180} + \frac{0.20\times0.80}{175}}} = 1.858 \approx 1.86.$$

Our p-value $= 1 - \Phi(z_{obs}) = 1 - \Phi(1.86) = 1 - 0.9686 = 0.0314$.
Since this value is less than $\alpha = 0.05$, we *reject* H_0.

Q 4. **Solution:** This is a one-tailed hypothesis test.

$$T.S. = \frac{(n-1)S^2}{\sigma_0^2}$$
$$\chi^2_{obs} = \frac{(n-1)s^2}{\sigma_0^2} = \frac{29 \times (0.09)}{0.14} = 18.64.$$

Find the value for $\chi^2_{n-1,\alpha} = \chi^2_{29,0.05} = 17.708$. Because $18.64 > 17.708$, it is not in the rejection region, and we *fail to reject* H_0.

Q 5. **Solution:** This is a two-tailed hypothesis test.

$$z_{obs} = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{69.2 - 68}{\frac{3.6}{\sqrt{36}}} = \frac{1.2}{\frac{3.6}{6}} = \frac{1.2}{0.6} = 2.0.$$

Our p-value $= 2 \times (1 - \Phi(z_{obs})) = 2 \times (1 - \Phi(2.0)) = 2 \times (1 - 0.9772) = 2 \times 0.0228 = 0.0456$. Since this value is greater than $\alpha = 0.01$, we *fail to reject* H_0.

Q 6. **Solution:** This is a two-tailed hypothesis test. Since we do not know the variances but they are equal, we use a t-distribution, and need s_p^2:

$$s_p^2 = \frac{(n_A - 1)(s_A^2) + (n_B - 1)(s_B^2)}{n_A + n_B - 2} = \frac{194.75}{25} \approx 7.79.$$

Also note that $t_{1-\frac{\alpha}{2}, n_A+n_B-2} = t_{.99,25} = 2.485$, so $t_{.01,25} = -2.485$. Since, we assume the null hypothesis to be true, $\mu_A - \mu_B = 0$,

$$t_{obs} = \frac{(\bar{x}_A - \bar{x}_B) - (\mu_A - \mu_B)_0}{\sqrt{\frac{s_p^2}{n_A} + \frac{s_p^2}{n_B}}} = \frac{-4 - 0}{\sqrt{\frac{7.79}{15} + \frac{7.79}{12}}} = \frac{-4}{1.0810} \approx -3.67.$$

Since this observed t-value is in the rejection region, and we *reject* H_0.

Q 7. **Solution:** The confidence interval of the ratio of two population variances $\frac{\sigma_1^2}{\sigma_2^2}$ is

$\left(\frac{s_1^2/s_2^2}{F_{1-\frac{\alpha}{2}, n_1-1, n_2-2}}, \frac{s_1^2/s_2^2}{F_{\frac{\alpha}{2}, n_1-1, n_2-2}}\right)$ where $n_1 = 20, n_2 = 15$ and $s_1^2 = 100$, $s_2^2 = 120$.

For the 95% confidence interval, α is 0.05.

Since $F_{1-\frac{\alpha}{2}, n_1-1, n_2-1} = F_{0.975,19,14} = 2.86$,

and $F_{\frac{\alpha}{2}, n_1-1, n_2-1} = \frac{1}{F_{1-\frac{\alpha}{2}, n_2-1, n1-1}} = \frac{1}{F_{0.975,14,19}} = \frac{1}{2.65}$, the confidence interval of the ratio of two population variances is

$\left(\frac{100/120}{2.86}, \frac{100/120}{1/2.65}\right) = (0.2914, 2.2083);$

Q 8. **Solution:**

(a) $\alpha \geq 0.001$.

(b) $\alpha < 0.001$.

Q 9. **Solution:** Yes. The sample mean of any underlying population distribution for size $n > 30$ is normally distributed with mean μ and variance $\frac{\sigma^2}{n}$ by the central limit theorem.

Q 10. **Solution:** No. The population is not normally distributed, and the sample is too small to apply the central limit

Q 11. **Solution:** The $100(1-\alpha)$% confidence interval of the difference of population means is given by

$$\left(\bar{x}_1 - \bar{x}_2 - t_{1-\frac{\alpha}{2}, n_1+n_2-2} \times \sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}, \bar{x}_1 - \bar{x}_2 + t_{1-\frac{\alpha}{2}, n_1+n_2-2} \times \sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}\right)$$

where

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}.$$

Note that in this problem, students should realize $s_p = s_1 = s_2 = 0.5$. The degrees of freedom $df = n_1 + n_2 - 2 = 16 + 25 - 2 = 39$. We do not have the table for $df = 39$, thus we use values for $df = 40$.

(a) $CI_{95} = (6 - 5.5 - 2.0211 \times 0.16, 6 - 5.5 + 2.0211 \times 0.16) = (0.1766, 0.8234)$.

(b) $CI_{90} = (6 - 5.5 - 1.6839 \times 0.16, 6 - 5.5 + 1.6839 \times 0.16) = (0.2306, 0.7694)$.

Q 12. **Solution:**
Here, $n = 20, p = 0.25$, thus $\mu = np = 5$, and $\sigma^2 = np(1-p) = 20 \times 0.25 \times 0.75 = 3.75$. The continuity correction factor requires us to use 7.5 in order to include 8 since the inequality is weak and we want the region to the right.

$$\begin{aligned} P(X \geq 8) &\approx P\left(Z \geq \frac{7.5-\mu}{\sigma}\right) = P\left(Z \geq \frac{7.5-5}{\sqrt{3.75}}\right) \\ &= P(Z \geq 1.29) \\ &= 1 - P(Z \leq 1.29) \\ &= 1 - 0.9015 \\ &= 0.0985. \end{aligned}$$

Q 13. **Solution:**

(a) $\mu_{\hat{p}} = 0.30$

(b) $\sigma_{\hat{p}} = \sqrt{\frac{0.30 \times 0.70}{50}} = 0.0648$

(c)

$$\begin{aligned} P(\hat{p} \geq 0.4) &= P\left(Z \geq \frac{0.4-0.3}{0.0648}\right) \\ &= P(Z \geq 1.5432) \\ &= 1 - P(Z \leq 1.5432) \\ &= 1 - 0.9382 \\ &= 0.0168. \end{aligned}$$