Simulation Report 1

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Introduction

Suppose we have $z_i \sim N(0,1)$, $i = 1, 2$ under the null. Their correlation is ρ. We perform a one-sided test with rejection region $\Gamma = \{z \geq 1.645\}$.

We then estimate $FDR(\Gamma)$ by

$$F\hat{D}R(\Gamma) = \frac{\hat{\pi}_0 E[R^0(\Gamma)]}{R(\Gamma) \vee 1} \tag{1}$$

Since we have only two variables and $R(\Gamma) \vee 1$ is observable, we only focus on the numerator of equation(1).

Theoretical derivation

$$
\begin{aligned}
E[R^0(\Gamma)] &= P\{R^0(\Gamma)=1\}\times 1 + P\{R^0(\Gamma)=2\}\times 2 \\
&= p\{z_1 > 1.645 \textit{ and } z_2 \leq 1.645 \textit{ or } z_2 > 1.645 \textit{ and } z_1 \leq 1.645\} \\
&+ p\{z_1 > 1.645 \textit{ and } z_2 > 1.645\}\times 2 \\
&= \int_{1.645}^{\infty}\int_{-\infty}^{1.645} \frac{exp(-\frac{1}{2}(\boldsymbol{z}-\boldsymbol{\mu})'\Sigma^{-1}(\boldsymbol{z}-\boldsymbol{\mu}))}{\sqrt{(2\pi)^2|\Sigma|}} dz_1 dz_2 \times 2 \\
&+ \int_{1.645}^{\infty}\int_{1.645}^{\infty} \frac{exp(-\frac{1}{2}(\boldsymbol{z}-\boldsymbol{\mu})'\Sigma^{-1}(\boldsymbol{z}-\boldsymbol{\mu}))}{\sqrt{(2\pi)^2|\Sigma|}} dz_1 dz_2 \times 2
\end{aligned}
\tag{2}
$$

Theoretical derivation

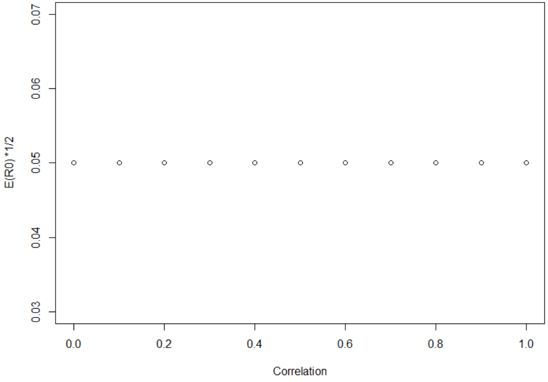


Figure 1: Numerator of equation(1) vs. correlation when m=2

Theoretical derivation

$$
\begin{aligned}
Var[R^0(\Gamma)] &= E[(R^0(\Gamma))^2] - (E[R^0(\Gamma)])^2 \\
&= p\{z_1 > 1.645 \text{ and } z_2 \leq 1.645 \text{ or } z_2 > 1.645 \text{ and} \\
&z_1 \leq 1.645\} + p\{z_1 > 1.645 \text{ and } z_2 > 1.645\} \times 2^2 \\
&- (E[R^0(\Gamma)])^2
\end{aligned} \tag{3}
$$

Theoretical derivation

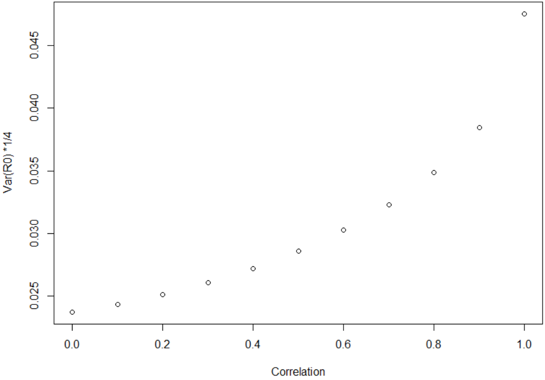
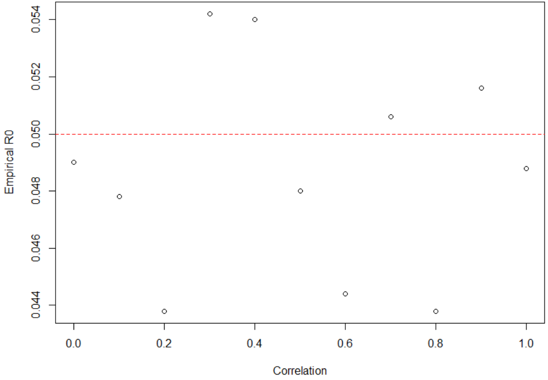


Figure 2: Exact variance of $R^0(\Gamma) \times 1/4$ vs. correlation when m=2

Simulation

Set up:

$z_1 \sim N(0,1), z_2 \sim N(0.5,1), \rho \in \{0, 0.1, ..., 1\}, n = 200, B = 5000$



Simulation

Empirical variance:

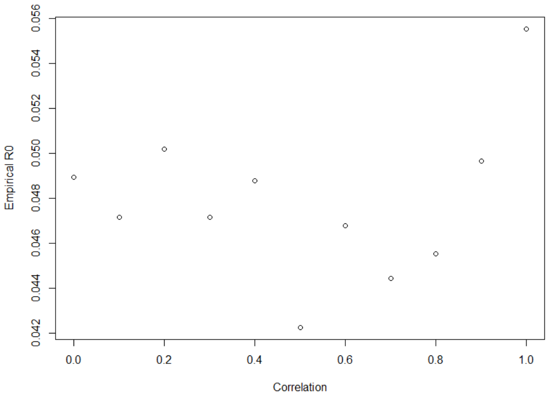


Figure 4: Variance of # false rejections vs. correlation when m=2

M=3

Exact mean:

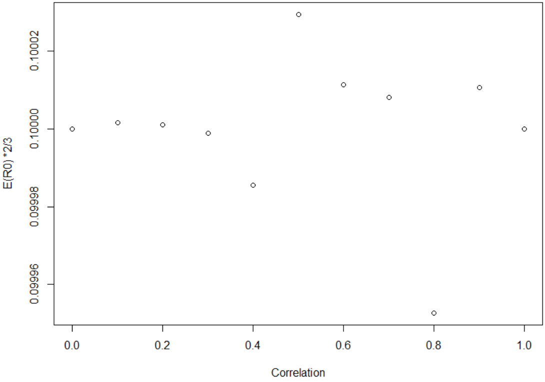


Figure 5: Numerator of equation(1) vs. correlation when m=3

M=3

Empirical mean:

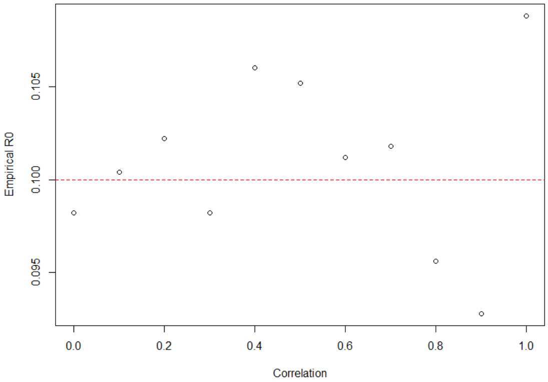


Figure 6: Mean of # false rejections vs. correlation when m=3

M=3

Exact variance:

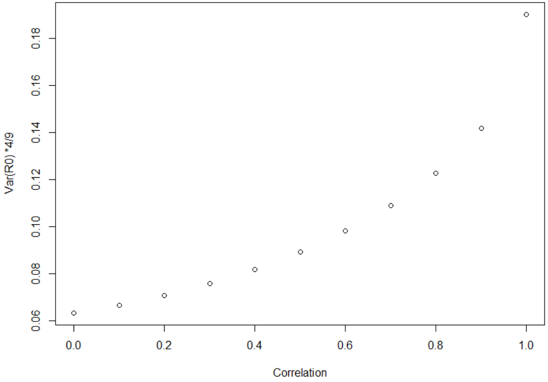


Figure 7: Exact variance of $R^0(\Gamma) \times 1/4$ vs. correlation when m=3

Empirical variance:

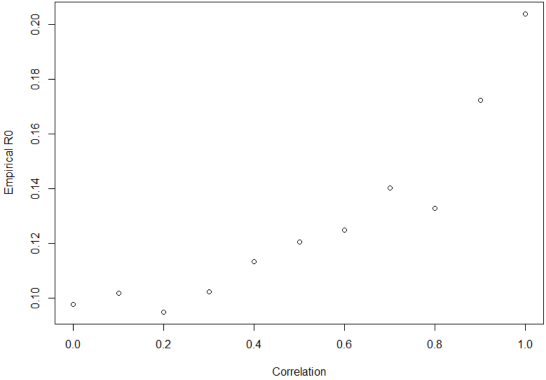


Figure 8: Variance of # false rejections vs. correlation when m=3

Simulation when M=100

Set up: $z_i \sim N(\mu_i, 1), \mu_i = 0$ *for* $i \in \{1, 2, ...90\}, \mu_i = 0.5$ *for* $i \in \{91, 92, ..., 100\}$. They have equal correlations $\rho \in \{0, 0.1, ..., 1\}$, $n = 200$, $B = 5000$.

Simulation when M=100

Empirical mean

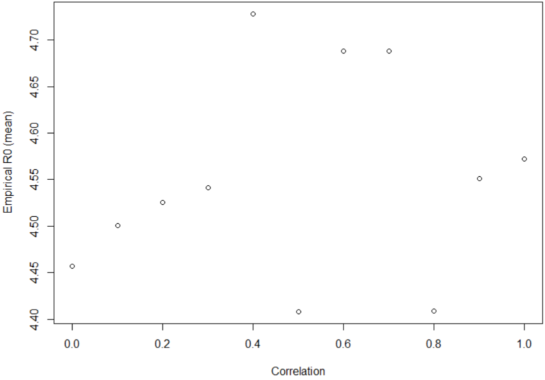


Figure 9: Mean of # false rejections vs. correlation when m=100

Simulation when M=100

Empirical variance

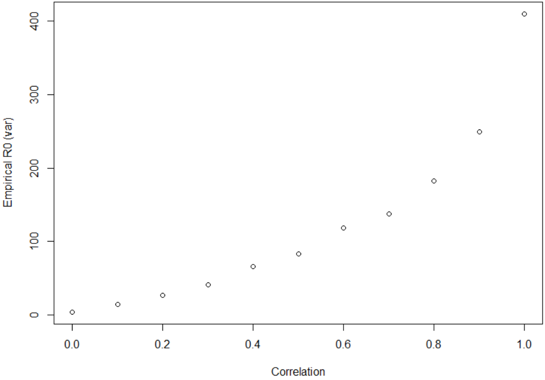


Figure 10: Variance of # false rejections vs. correlation when m=100