Southern Denmark University
IMADA
DM552, Part II

2017-2018

Chapter 1:
Propositional and first order logic

Exercise 01: Say whether the following expressions are well formed FOL formulas or terms?

1. a→ p(b).
2. r(x, b) → ∃y. q(y, y).
3. r(x, b) ∨ ∃y. g(y, b).
4. ¬ y ∨p(y).
5. ¬¬p(a).
6. ¬∀ x. ¬ p(x).
7. ∀x ∃y. (r(x, y) →r(y, x)).
8. ∀x ∃y. (r(x, y) → (r(y, x) ∨(f(a)=g(a, x)))).

Exercise 02: Find free variables in the following formulas?

1. P(x) ∧ ¬r(y, a).
2. ∃x. r(x, y).
3. ∀x. p(x) →∃y. ¬q(f(x), y, f(y)).
4. ∀x∃y. r(x, f(y)).

Exercise 03: What is the meaning of the following FOL formulas?

1. bought(Frank, dvd).
2. ∃x. bought(Frank, x).
3. ∀x. (bought(Frank, x) → bought(Susan, x)).
4. ∀x. bought(Frank, x) → ∀x. bought(Susan, x).
5. ∀x. ∃y. bought(x, y).
6. ∃x∀y. bought(x, y).

Exercise 04: Which of the following formulas is a formalization of the sentence?

"*There is a computer which is not used by any student"*

- ∃x. (Computer(x) ∧ ∀y. (¬Student(y) ∧ ¬Uses(y, x))).
- ∃x. (Computer(x) →∀y. (Student(y) → ¬Uses(y, x))).
- ∃x. (Computer(x) ∧ ∀y. (¬Student(y) → ¬Uses(y, x))).

Exercise 05: Evaluate the following formula ?

$$(\forall X)\ (P\ (g(X),\ a) \rightarrow Q\ (f(X),\ b))$$

Interpretation :

- ✓ D(X)={1, 2, 3}.
- ✓ g(x)= x+2 ; f(x)= x+3 ;
- ✓ a=2, b=3.
- ✓ P (a, b)= true if a <b, false otherwise.
- ✓ Q (a, b)=true if a>b, false otherwise.

Exercise 06: Define an appropriate language and formalize the following sentences using FOL formulas?

1. All Students are smart.
2. There exists a student.
3. There exists a smart student.
4. Every student loves some student.
5. Every student loves some other student.
6. There is a student who is loved by every other student.
7. Bill is a student.
8. Bill takes either Analysis or Geometry (but not both).
9. Bill takes Analysis and Geometry.
10. Bill doesn't take Analysis.
11. No students love Bill