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DM559 – Linear and integer programming

Sheet 1, Spring 2018 [pdf format]

Exercise 1* Modelling

Flying with the wind, an airplane traveled 1365 miles in 3 hours. The plane then turned against the wind and traveled another 870 miles in 2 hours. Find the speed of the airplane and the speed of the wind assuming they remained constant in the 5 hour time lapse. Recall that the distance traveled is equal to the average speed times the time traveled at that rate, $D = r \cdot t$.

Exercise 2*

Use Gaussian elimination to solve the following system of equations

$$\begin{array}{rcrcrcr} x_1 & + & x_2 & + & x_3 & = & 6 \\ 2x_1 & + & 4x_2 & + & x_3 & = & 5 \\ 2x_1 & + & 3x_2 & + & x_3 & = & 6 \end{array}$$

Be sure to follow the algorithm to put the augmented matrix into row echelon form using row operations. Try then also the variant in which you put the augmented matrix in reduced row echelon form.

Exercise 3

Do the True False exercises on page 8, 20 and 30 of the book [AR].

Exercise 4*

Do exercises 1, 2, 3, 15, 16, 17, 57, 58, 59, 71, 223, 230 from the Exercise set 1 on pages 75-87 of [AR]. Below is a set of exercises taken from [Le], in case someone does not have the book [AR] yet. You do not need to look at this if you have the book.

1. In each part, determine whether the equation is linear in x_1, x_2, and x_3.

(a) $x_1 + 5x_2 - \sqrt{2}x_3 = 1$
(b) $x_1 + 3x_2 + x_1x_3 = 2$
(c) $x_1 = -7x_2 + 3x_3$
(d) $x_1^{-2} + x_2 + 8x_3 = 5$
(e) $x_1^{3/5} - 2x_2 + x_3 = 4$
(f) $\pi x_1 - \sqrt{2}x_2 = 7^{1/3}$

2. In each part, determine whether the equation is linear in x and y.

(a) $2^{1/3}x + \sqrt{3}y = 1$
(b) $2x^{1/3} + 3\sqrt{y} = 1$
(c) $\cos\left(\frac{\pi}{7}\right)x - 4y = \log 3$
(d) $\frac{\pi}{7}\cos x - 4y = 0$
(e) $xy = 1$
(f) $y + 7 = x$

3. Using the notation of Formula (7), write down a general linear system of

(a) two equations in two unknowns.

(b) three equations in three unknowns.

(c) two equations in four unknowns.

4. Write down the augmented matrix for each of the linear systems in Exercise 3.

► In each part of Exercises 5–6, find a linear system in the unknowns $x_1, x_2, x_3, \ldots,$ that corresponds to the given augmented matrix. ◄

5. (a) $\begin{bmatrix} 2 & 0 & 0 \\ 3 & -4 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 0 & -2 & 5 \\ 7 & 1 & 4 & -3 \\ 0 & -2 & 1 & 7 \end{bmatrix}$

6. (a) $\begin{bmatrix} 0 & 3 & -1 & -1 & -1 \\ 5 & 2 & 0 & -3 & -6 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & 0 & 1 & -4 & 3 \\ -4 & 0 & 4 & 1 & -3 \\ -1 & 3 & 0 & -2 & -9 \\ 0 & 0 & 0 & -1 & -2 \end{bmatrix}$

► In each part of Exercises 7–8, find the augmented matrix for the linear system. ◄

7. (a) $\begin{aligned} -2x_1 &= 6 \\ 3x_1 &= 8 \\ 9x_1 &= -3 \end{aligned}$ (b) $\begin{aligned} 6x_1 - x_2 + 3x_3 &= 4 \\ 5x_2 - x_3 &= 1 \end{aligned}$

(c) $\begin{aligned} 2x_2 - 3x_4 + x_5 &= 0 \\ -3x_1 - x_2 + x_3 &= -1 \\ 6x_1 + 2x_2 - x_3 + 2x_4 - 3x_5 &= 6 \end{aligned}$

8. (a) $\begin{aligned} 3x_1 - 2x_2 &= -1 \\ 4x_1 + 5x_2 &= 3 \\ 7x_1 + 3x_2 &= 2 \end{aligned}$ (b) $\begin{aligned} 2x_1 + 2x_3 &= 1 \\ 3x_1 - x_2 + 4x_3 &= 7 \\ 6x_1 + x_2 - x_3 &= 0 \end{aligned}$

(c) $\begin{aligned} x_1 &= 1 \\ x_2 &= 2 \\ x_3 &= 3 \end{aligned}$

9. In each part, determine whether the given 3-tuple is a solution of the linear system

$$\begin{aligned} 2x_1 - 4x_2 - x_3 &= 1 \\ x_1 - 3x_2 + x_3 &= 1 \\ 3x_1 - 5x_2 - 3x_3 &= 1 \end{aligned}$$

(a) $(3, 1, 1)$ (b) $(3, -1, 1)$ (c) $(13, 5, 2)$

(d) $\left(\frac{13}{2}, \frac{5}{2}, 2\right)$ (e) $(17, 7, 5)$

In Exercises 17–18, find a single elementary row operation that will create a 1 in the upper left corner of the given augmented matrix and will not create any fractions in its first row.

17. (a) $\begin{bmatrix} -3 & -1 & 2 & 4 \\ 2 & -3 & 3 & 2 \\ 0 & 2 & -3 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -1 & -5 & 0 \\ 2 & -9 & 3 & 2 \\ 1 & 4 & -3 & 3 \end{bmatrix}$

18. (a) $\begin{bmatrix} 2 & 4 & -6 & 8 \\ 7 & 1 & 4 & 3 \\ -5 & 4 & 2 & 7 \end{bmatrix}$ (b) $\begin{bmatrix} 7 & -4 & -2 & 2 \\ 3 & -1 & 8 & 1 \\ -6 & 3 & -1 & 4 \end{bmatrix}$

21. The curve $y = ax^2 + bx + c$ shown in the accompanying figure passes through the points (x_1, y_1), (x_2, y_2), and (x_3, y_3). Show that the coefficients a, b, and c form a solution of the system of linear equations whose augmented matrix is

$$\begin{bmatrix} x_1^2 & x_1 & 1 & y_1 \\ x_2^2 & x_2 & 1 & y_2 \\ x_3^2 & x_3 & 1 & y_3 \end{bmatrix}$$

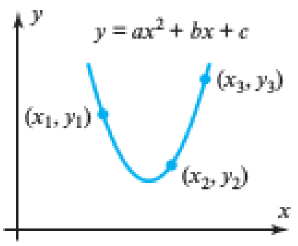

Figure Ex-21

Exercise 5*

34. The accompanying table shows a record of May and June unit sales for a clothing store. Let M denote the 4×3 matrix of May sales and J the 4×3 matrix of June sales.

(a) What does the matrix $M + J$ represent?

(b) What does the matrix $M - J$ represent?

(c) Find a column vector $\mathbf{x}$ for which $M\mathbf{x}$ provides a list of the number of shirts, jeans, suits, and raincoats sold in May.

(d) Find a row vector $\mathbf{y}$ for which $\mathbf{y}M$ provides a list of the number of small, medium, and large items sold in May.

(e) Using the matrices $\mathbf{x}$ and $\mathbf{y}$ that you found in parts (c) and (d), what does $\mathbf{y}M\mathbf{x}$ represent?

Table Ex-34

May Sales

	Small	Medium	Large
Shirts	45	60	75
Jeans	30	30	40
Suits	12	65	45
Raincoats	15	40	35

June Sales

	Small	Medium	Large
Shirts	30	33	40
Jeans	21	23	25
Suits	9	12	11
Raincoats	8	10	9